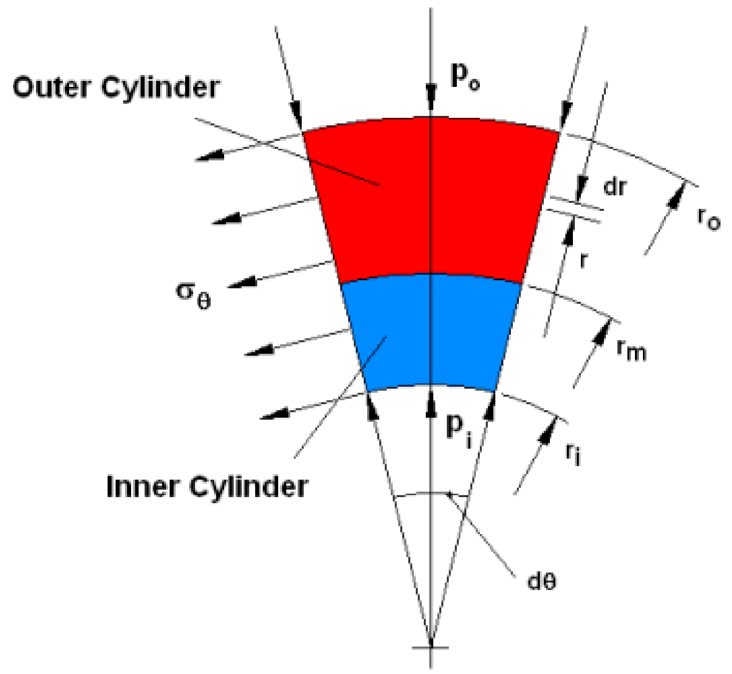
Thick-wall Cylinder Radial Interference Fit Analysis



External pressure ... $p_o := 0 \cdot MPa$

Internal pressure ... $p_i := 0 \cdot MPa$

Working temperature ... $T_w := 23 \cdot C$

Temperature difference ... $\Delta T := T_w - 23 \cdot C$ $\Delta T = 0\,C$

Cylinder properties ...	Inner Cylinder ...	Outer Cylinder ...
	Drg: 123456	Drg: ABCDEF
OD at room temperature ...	$OD_1 := 114 \cdot mm$	$OD_2 := 120 \cdot mm$
ID at room temperature ...	$ID_1 := 0 \cdot mm$	$ID_2 := 113.85 \cdot mm$
Elastic modulus ...	$E_1 := 210 \cdot GPa$	$E_2 := 210 \cdot GPa$
Poisson's ratio ...	$\upsilon_1 := 0.30$	$\upsilon_2 := 0.30$
Thermal expansion coefficient ...	$\alpha_1 := 12.3 \cdot 10^{-6} \cdot \frac{mm}{mm \cdot C}$	$\alpha_2 := 12.3 \cdot 10^{-6} \cdot \frac{mm}{mm \cdot C}$

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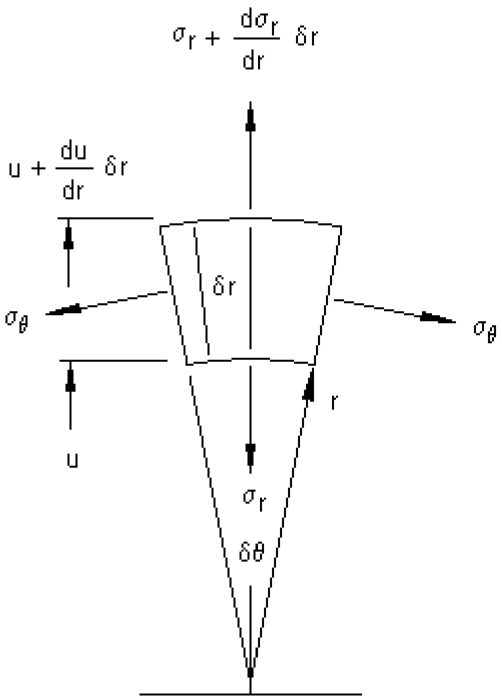
Cylinder Element Stresses and Radial Force Equilibrium



$$\left(\sigma_r + \frac{d}{dr}\sigma_r \cdot \delta r\right) \cdot (r + \delta r) \cdot \delta\theta - \sigma_r \cdot r \cdot \delta\theta - 2 \cdot \sigma_\theta \cdot \delta r \cdot \sin\left(\frac{\delta\theta}{2}\right) = 0$$

... force equilibrium in the radial direction

Expanding gives ... $\frac{d}{dr}\sigma_r + \frac{1}{r} \cdot \left(\sigma_r - \sigma_\theta\right) = 0$ ... force equilibrium equation

Strain displacement equations ... $\varepsilon_r = \frac{d}{dr}u$ ... radial strain, where u is the radial displacement function to be derived

$$\varepsilon_\theta = \frac{2 \cdot \pi \cdot (r + u) - 2 \cdot \pi \cdot r}{2 \cdot \pi \cdot r} = \frac{u}{r}$$

... circumferential strain

Note, u is the radial displacement as a function of r, i.e. u = f(r)

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Stress-strain relationships ... $\varepsilon_r = \frac{\sigma_r}{E} - \frac{\nu \cdot \sigma_\theta}{E}$ ... radial strain

$\varepsilon_\theta = \frac{\sigma_\theta}{E} - \frac{\nu \cdot \sigma_r}{E}$ ... circumferential strain

In matrix format ...

$$\begin{pmatrix} \varepsilon_r \\ \varepsilon_\theta \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & \frac{-\nu}{E} \\ \frac{-\nu}{E} & \frac{1}{E} \end{pmatrix} \cdot \begin{pmatrix} \sigma_r \\ \sigma_\theta \end{pmatrix}$$

Solving for stresses ...

$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & \frac{-\nu}{E} \\ \frac{-\nu}{E} & \frac{1}{E} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \varepsilon_r \\ \varepsilon_\theta \end{pmatrix}$$

gives ...

$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \end{pmatrix} = \frac{E}{\left(1 - \nu^2\right)} \cdot \begin{pmatrix} \varepsilon_r + \nu \cdot \varepsilon_\theta \\ \nu \cdot \varepsilon_r + \varepsilon_\theta \end{pmatrix}$$

and substituting ...

$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \end{pmatrix} = \frac{E}{\left(1 - \nu^2\right)} \cdot \begin{pmatrix} \frac{d}{dr}u + \nu \cdot \frac{u}{r} \\ \nu \cdot \frac{d}{dr}u + \frac{u}{r} \end{pmatrix}$$

$$\frac{d}{dr}\left(\frac{d}{dr}u + \nu \cdot \frac{u}{r}\right) + \frac{1}{r} \cdot \left[\frac{d}{dr}u + \nu \cdot \frac{u}{r} - \left(\nu \cdot \frac{d}{dr}u + \frac{u}{r}\right)\right] = 0$$

... substituting for stresses in force equilibrium equation

$$\frac{d^2}{dr^2}u + \nu \cdot \frac{d}{dr}\frac{u(r)}{r} + \frac{(1 - \nu)}{r} \cdot \left(\frac{d}{dr}u - \frac{u}{r}\right) = 0$$

... expanding and simplifying

$$\frac{d^2}{dr^2}u + \frac{1}{r} \cdot \frac{d}{dr}u - \frac{u}{r^2} = 0$$

... differential equation for radial displacement in cylinder wall

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Let ... $u = A \cdot r + \frac{B}{r}$ ... general solution (GS)

$$\frac{d^2}{dr^2}\left(A \cdot r + \frac{B}{r}\right) + \frac{1}{r} \cdot \frac{d}{dr}\left(A \cdot r + \frac{B}{r}\right) - \frac{1}{r^2} \cdot \left(A \cdot r + \frac{B}{r}\right) = 0$$ ... substituting GS into differential equ'n

$$\frac{d}{dr}u = A - \frac{B}{r^2}$$

$$\sigma_\theta = \frac{E}{\left(1 - \nu^2\right)} \cdot \left(\nu \cdot \frac{d}{dr}u + \frac{u}{r}\right)$$ ... circumferential stress

$$\sigma_\theta = \frac{E}{\left(1 - \nu^2\right)} \cdot \left[A \cdot (1 + \nu) + \frac{B}{r^2} \cdot (1 - \nu)\right]$$

$$\sigma_r = \frac{E}{\left(1 - \nu^2\right)} \cdot \left(\frac{d}{dr}u + \nu \cdot \frac{u}{r}\right)$$ ... radial stress

$$\sigma_r = \frac{E}{\left(1 - \nu^2\right)} \cdot \left[A \cdot (1 + \nu) - \frac{B}{r^2} \cdot (1 - \nu)\right]$$

Equations for Inner Cylinder

Radial stress ...

$$f_{r1}(r, A_1, B_1) := \frac{E_1}{\left(1 - \upsilon_1^{2}\right)} \cdot \begin{cases} \left[A_1 \cdot \left(1 + \upsilon_1\right) - \frac{B_1}{r^2} \cdot \left(1 - \upsilon_1\right)\right] & \text{if } ID_1 > 0 \cdot mm \\ A_1 \cdot \left(1 + \upsilon_1\right) & \text{otherwise} \end{cases}$$

Hoop stress ...

$$f_{\theta 1}(r, A_1, B_1) := \frac{E_1}{\left(1 - \upsilon_1^{2}\right)} \cdot \begin{cases} \left[A_1 \cdot \left(1 + \upsilon_1\right) + \frac{B_1}{r^2} \cdot \left(1 - \upsilon_1\right)\right] & \text{if } ID_1 > 0 \cdot mm \\ A_1 \cdot \left(1 + \upsilon_1\right) & \text{otherwise} \end{cases}$$

Inner cylinder radial displacement ...

$$u_1(r, A_1, B_1) := A_1 \cdot r + \frac{B_1}{r}$$

Rate of change in radial displacement ...

$$du_1(r, A_1, B_1) := A_1 - \frac{B_1}{r^2}$$

Equations for Outer Cylinder

Radial stress ...

$$f_{r2}(r, A_2, B_2) := \frac{E_2}{\left(1 - \upsilon_2^{2}\right)} \cdot \left[A_2 \cdot \left(1 + \upsilon_2\right) - \frac{B_2}{r^2} \cdot \left(1 - \upsilon_2\right)\right]$$

Hoop stress ...

$$f_{\theta 2}(r, A_2, B_2) := \frac{E_2}{\left(1 - \upsilon_2^{2}\right)} \cdot \left[A_2 \cdot \left(1 + \upsilon_2\right) + \frac{B_2}{r^2} \cdot \left(1 - \upsilon_2\right)\right]$$

Outer cylinder radial displacement ...

$$u_2(r, A_2, B_2) := A_2 \cdot r + \frac{B_2}{r}$$

Rate of change in radial displacement ...

$$du_2(r, A_2, B_2) := A_2 - \frac{B_2}{r^2}$$

Thermal expansion ... $ID_1 := ID_1 \cdot (1 + \Delta T \cdot \alpha_1)$ $ID_1 = 0.000\,mm$

$OD_1 := OD_1 \cdot (1 + \Delta T \cdot \alpha_1)$ $OD_1 = 114.000\,mm$

$ID_2 := ID_2 \cdot (1 + \Delta T \cdot \alpha_2)$ $ID_2 = 113.850\,mm$

$OD_2 := OD_2 \cdot (1 + \Delta T \cdot \alpha_2)$ $OD_2 = 120.000\,mm$

Radial interference ... $\delta_r := 0.5 \cdot (OD_1 - ID_2)$ $\delta_r = 75.000\,\mu m$

Radial distance to mating faces ...

$$\Delta u_{12}(A_1, B_1, A_2, B_2) := \frac{|du_2(0.5 \cdot ID_2, A_2, B_2)|}{|du_2(0.5 \cdot ID_2, A_2, B_2)| + |du_1(0.5 \cdot OD_1, A_1, B_1)|}$$

...ratio of rate of change in outer cylinder displacement w.r.t total rate of cheange

$$r_m(A_1, B_1, A_2, B_2) := 0.5 \cdot [ID_2 + (OD_1 - ID_2) \cdot \Delta u_{12}(A_1, B_1, A_2, B_2)]$$

... radial distance to mating surfaces

Given the following boundary conditions ...

$$B_1 = \begin{cases} B_1 & \text{if } ID_1 > 0 \cdot mm \\ 0 \cdot mm^2 & \text{otherwise} \end{cases}$$

... setting variable B_1 based on geometry conditions

$$f_{r1}(0.5 \cdot ID_1, A_1, B_1) = \begin{cases} -p_i & \text{if } ID_1 > 0 \cdot mm \\ f_{r1}(0.5 \cdot ID_1, A_1, B_1) & \text{otherwise} \end{cases}$$

... equating stress to internal pressure at inner cylinder ID (including geometry conditions)

$$f_{r2}(0.5 \cdot OD_2, A_2, B_2) = -p_o$$

... equating stress to external pressure at outer cylinder OD

$$f_{r1}(r_m(A_1, B_1, A_2, B_2), A_1, B_1) = f_{r2}(r_m(A_1, B_1, A_2, B_2), A_2, B_2)$$

... equating radial stress at mating surfaces

$$u_2(r_m(A_1, B_1, A_2, B_2), A_2, B_2) - u_1(r_m(A_1, B_1, A_2, B_2), A_1, B_1) = \delta_r$$

... equating sum of radial displacement to interference

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Stress results ...	Inner Cylinder ...	Outer Cylinder ....
ID radial stress ...	$f_{r\_ID\_1} = -13.50\,MPa$	$f_{r\_ID\_2} = -13.50\,MPa$
OD radial stress ...	$f_{rOD\_1} = -13.50\,MPa$	$f_{r\_OD\_2} = 0.00\,MPa$
ID hoop stress ...	$f_{\theta\_ID\_1} = -13.50\,MPa$	$f_{\theta\_ID\_2} = 262.85\,MPa$
OD hoop stress ...	$f_{\theta\_OD\_1} = -13.50\,MPa$	$f_{\theta\_OD\_2} = 249.34\,MPa$

Radial interference ... $u_2(r'_m, A_2, B_2) - u_1(r'_m, A_1, B_1) = 75.00\,\mu m$ where $\delta_r = 75.00\,\mu m$

where ... $u_1(r'_m, A_1, B_1) = -2.57\,\mu m$ and $u_2(r'_m, A_2, B_2) = 72.43\,\mu m$

$r'_m = 56.993\,mm$ ... radial distance to mating faces

$\Delta u_{12}(A_1, B_1, A_2, B_2) = 90.76\,\%$ ... outer cylinder proportion of interference deflection

$$\frac{2 \cdot r'_m - ID_2}{OD_1 - ID_2} = 90.76\,\%$$ ... check

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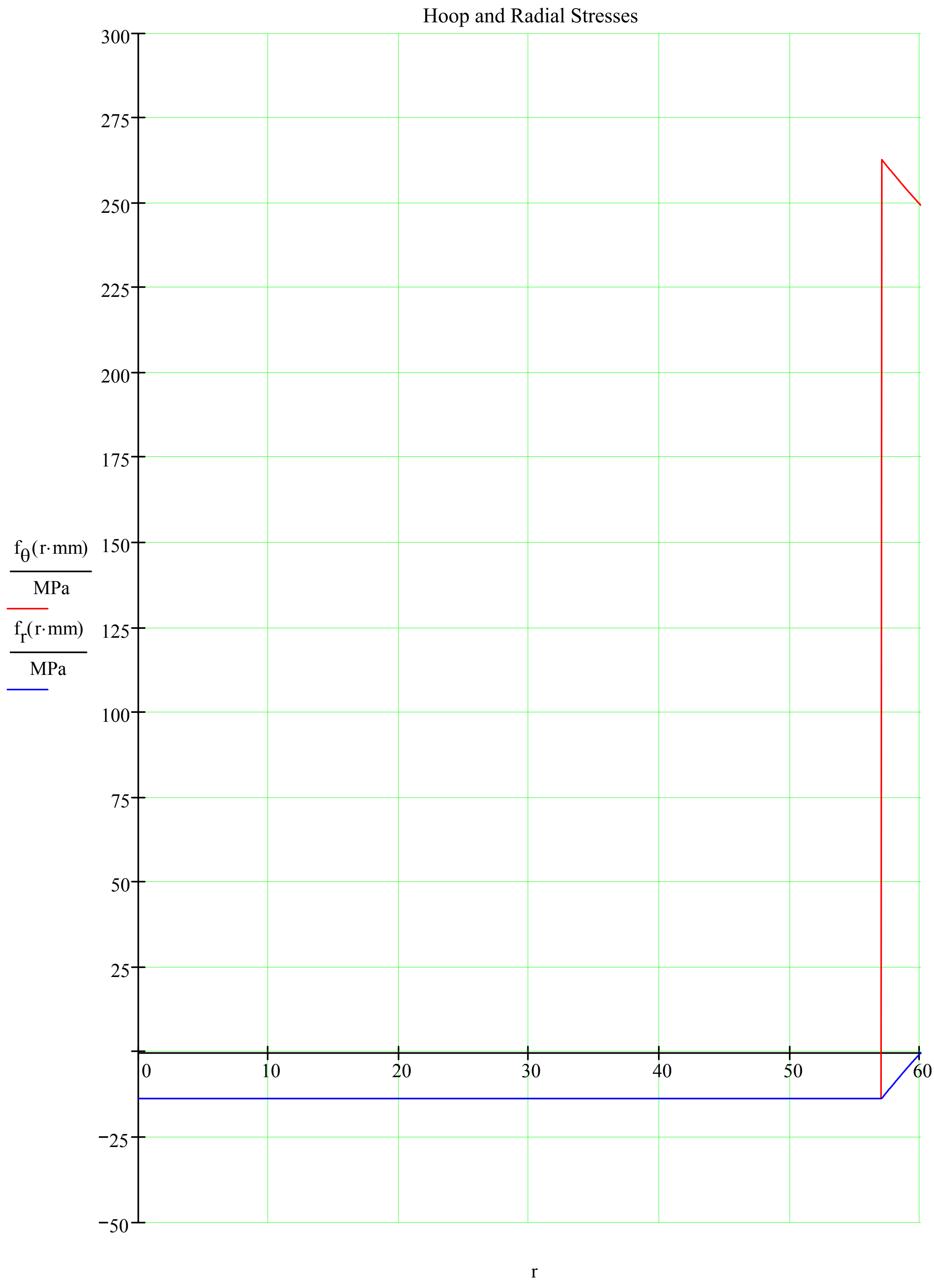

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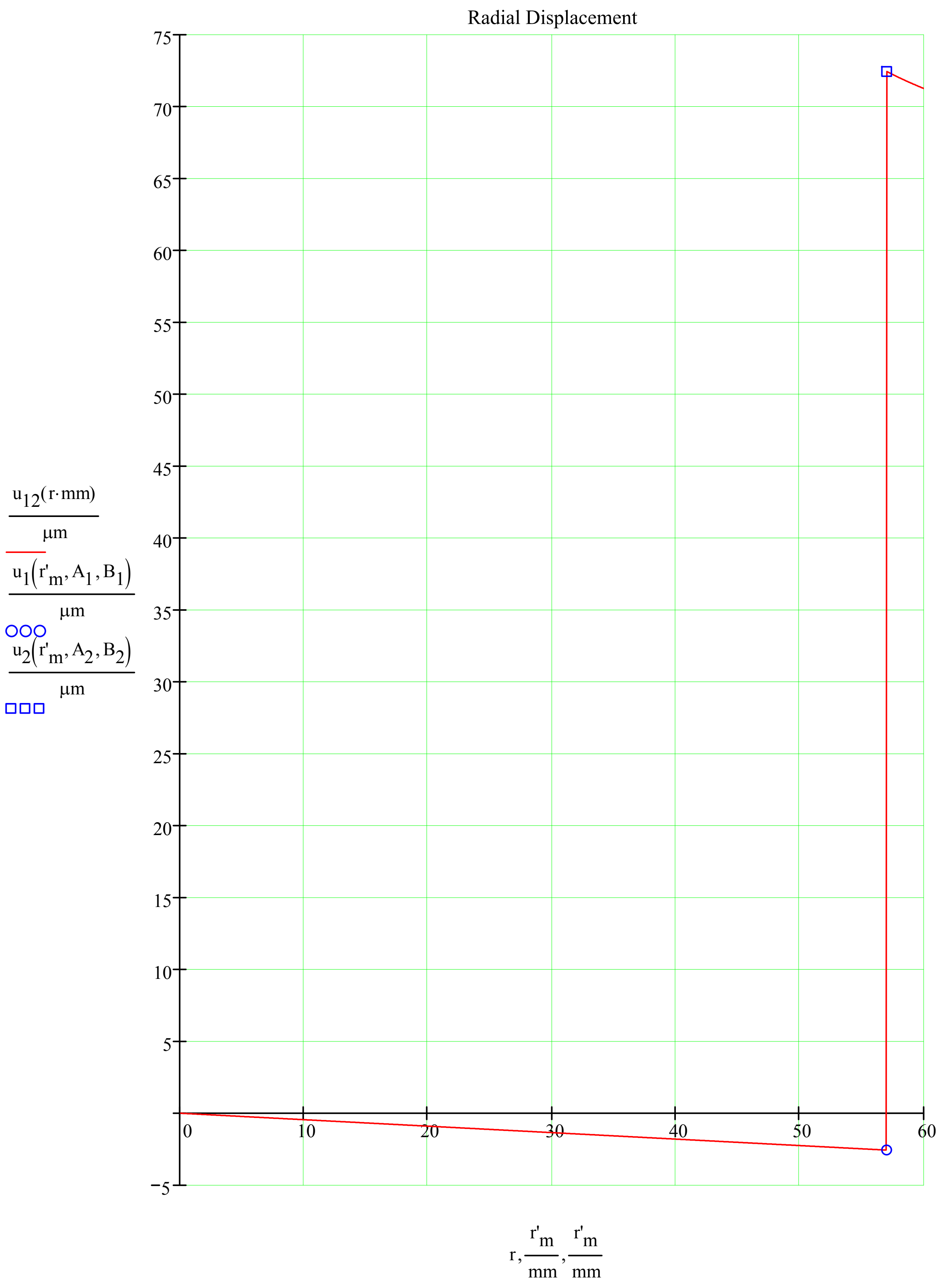

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