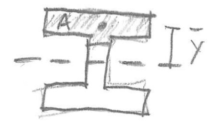
e) Lattice Walls



A truss, a framework or some other lattice structure may occasionally constitute one or more wall elements of a thin-walled, closed box member. Such a framework may be replaced in the analysis by an equivalent wall element of constant thickness t^* which, as in the previous example, may be obtained from strain energy consideration.

Consider the shaft shown in Fig. 2.5a. The wall elements consist of three plates and one lattice truss system. The strain energy U of the shaft of length a is:

$$U = \tfrac{1}{2}\sum_{i=1}^{4} a \int_{F_i} \tau_i \gamma_i \, dF_i$$

$$= \frac{a}{2G}\sum_{i=1}^{4} \int_{F_i} \tau_i^2 \, dF_i .$$

$q = \frac{VQ}{I} \leftarrow Q = A\bar{y}$

If each of the wall elements is of constant thickness t_i and of width b_i, then $\tau_i = q/t_i$ and $F_i = b_i t_i$ and thus:

$$U = \frac{aq^2}{2G}\sum_{i=1}^{4} \frac{b_i}{t_i} .$$

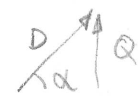
$q = [N/mm]$

The contribution of the fictitious wall element to the strain energy is therefore:

$$\Delta U = \frac{aq^2}{2G}\,\frac{b}{t^*} .$$

The fictitious wall thickness t^* will now be determined from the condition that the contribution ΔU has to be equal to the strain energy in the truss element of length a.

$\sin\alpha = \frac{Q}{D}$



The shear flow q results in a total shear force Q in the plane of the truss of $Q = qb$.

The shear Q causes the force $D = Q/\sin\alpha$ in the diagonal of length d. Since $\sin\alpha = b/d$, it is simply $D = qd$.

Fig. 2.5b shows the upper chord separated from the adjacent wall element. The shear flow q acts along the line of separation. It is introduced by the diagonals into the gusset plates in the form of concentrated forces. These concentrated forces cause axial forces in the chords which may be assumed to vary linearly within the distance a, from zero up to the maximum value of $\Delta F = qa$, $|\Delta F_0| = |\Delta F_u| = |q|\,a$.

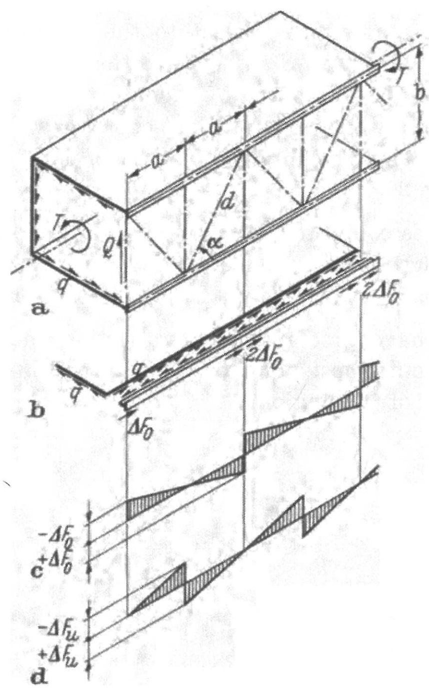


Fig. 2.5. Closed Box Member with one Wall Element in the Form of a Truss.

The strain energy in a chord of length l and area of cross section F subjected to the axial load P is $P^2 l/2EF$ and for the case where the axial force varies

Normal notation: $U = \frac{F^2 L}{2EA}$ [J] [N·m]

$U = \frac{q^2 a^2 a}{2E F_0}$

linearly within the distance l from zero to P, $P^2 l/6EF$. Using these results, the strain energy of a truss element of length a may be written down as follows:

Contribution of the upper chord: $\frac{1}{2E}\left(\frac{qa}{F_o}\right)^2 \frac{a}{3} F_o = \frac{q^2}{2E}\frac{a^3}{3F_o}$. $\frac{(qa)^2 a}{6E F_o} \Rightarrow \frac{q^2 a^3}{6EF_o}$

Contribution of the diagonal: $\frac{1}{2E}\left(\frac{qd}{F_d}\right)^2 dF_d = \frac{q^2}{2E}\frac{d^3}{F_d}$. $\frac{(qd)^2 d}{2EF_d} \Rightarrow \frac{q^2 d^3}{2EF_d}$

Contribution of the post: free of stress

Contribution of the lower chord: $\frac{1}{2E}\left(\frac{qa}{F_u}\right)^2 \frac{a}{3} F_o = \frac{q}{2E}\frac{a^3}{3F_u}$. $\frac{q^2 a^3}{6EF_u}$

Total for truss of length a: $= \frac{q^2}{2E}\left(\frac{d^3}{F_d} + \frac{a^3}{3}\left(\frac{1}{F_o} + \frac{1}{F_u}\right)\right)$. ✓ Sum

$$\frac{aq^2 b}{2Gt^*} = \frac{q^2}{2E}\left[\frac{d^3}{F_d} + \frac{a^3}{3}\left(\frac{1}{F_o} + \frac{1}{F_u}\right)\right]$$

The thickness t^* of the fictitious wall element is obtained when this sum is set equal to the expression for ΔU obtained above:

$$t^* = \frac{E}{G}\frac{ab}{\frac{d^3}{F_d} + \frac{a^3}{3}\left(\frac{1}{F_o} + \frac{1}{F_u}\right)}. \checkmark \tag{2.13}$$

$$t^* = \frac{abE}{G}\left[\frac{1}{\frac{d^3}{F_d} + \frac{a^3}{3}\left(\frac{1}{F_o} + \frac{1}{F_u}\right)}\right]$$

The following remarks are made with respect to the determination of the chord cross sections F_o and F_u: The wall S adjacent to the chord G will assist in carrying the force ΔF. This may be considered by including a portion of the wall area F_S with the chord area F_G to yield the equivalent chord area F^*. An estimate of the contribution of the wall S may be obtained from a consideration of the two limiting cases presented in Fig. 2.6: $t^* =$

a) The wall element which is connected to the wall S on the opposite side of the chord G has an area F which is much smaller than F_S, i.e. $F \ll F_S$.
b) The area F of the wall element exceeds by far the area F_S, i.e. $F \gg F_S$.

If the wall S is assumed to have a rectangular cross section, then in case a the neutral axis goes through the outermost point of the core and in case b it goes through the opposite boundary of the section.

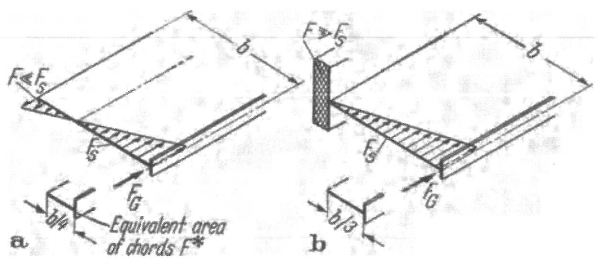


Fig. 2.6. Limiting Cases for the Equivalent Area of the Chords.

In the stress determination, the portion of the wall S which should be added to the area of the chords in the first case (a) is $F_S/4$ and in the second case (b) $F_S/3$ [see inequalities (12.18a)]. These are limiting cases and the true equivalent area of the chords is likely to be somewhere in between. Furthermore, the force F_G changes over a length which is of the order of magnitude of the width of the wall S. It is therefore possible that the classical beam theory overestimates the contribution of the wall S. Consequently, it is better to base the computation of F_o and F_u in Eq. (2.13) on the lower limiting value as sketched in Fig. 2.6a, e.g.:

$$F^* = F_G + \tfrac{1}{4}F_S. \tag{2.14}$$

Exercise 2.1. *Lattice Walls.* Compute the fictitous wall thicknesses t^* of the lattice walls shown below.

Results to Exercise 2.1

P = qa or qd

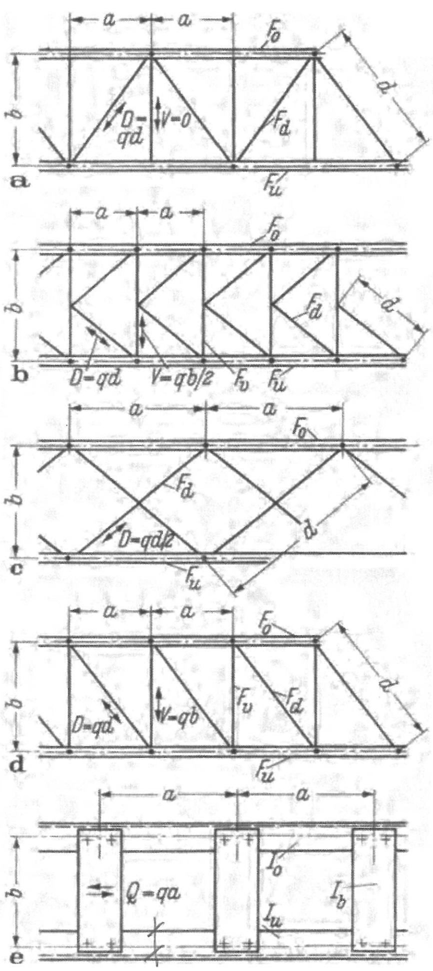
Case a): $$t^* = \frac{E}{G}\,\frac{ab}{\dfrac{d^3}{F_d} + \dfrac{a^3}{3}\left(\dfrac{1}{F_o} + \dfrac{1}{F_u}\right)}$$

Case b): $$t^* = \frac{E}{G}\,\frac{ab}{\dfrac{2d^3}{F_d} + \dfrac{b^3}{4F_v} + \dfrac{a^3}{12}\left(\dfrac{1}{F_o} + \dfrac{1}{F_u}\right)}$$

Case c): $$t^* = \frac{E}{G}\,\frac{ab}{\dfrac{d^3}{2F_d} + \dfrac{a^3}{12}\left(\dfrac{1}{F_o} + \dfrac{1}{F_u}\right)}$$

Case d): $$t^* = \frac{E}{G}\,\frac{ab}{\dfrac{d^3}{F_d} + \dfrac{b^3}{F_v} + \dfrac{a^3}{12}\left(\dfrac{1}{F_o} + \dfrac{1}{F_u}\right)}$$

Case e): $$t^* = \frac{E}{G}\,\frac{1}{\dfrac{ab^2}{12I_b} + \dfrac{a^2b}{48}\left(\dfrac{1}{I_o} + \dfrac{1}{I_u}\right)}.$$



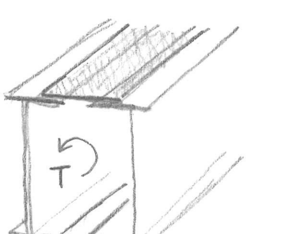
Q = qa

t

~~**2.3 Multicellular Box Section Members**~~

~~**a) General Remarks**~~

~~The problem is the analysis of a closed box section consisting of more than one longitudinal cell. The problem involves essentially the superposition of individual closed box member solutions.~~



$$M_u = \frac{\omega_2 \pi}{L}\left[\sqrt{E_s I_y G_s J + \left(\frac{\pi E}{L}\right)^2 I_y C_w}\right.$$

S16 Steel Code

based on t^*

Paul S. 1985 Waterloo