WHEN SPLINES NEED STRESS CONTROL

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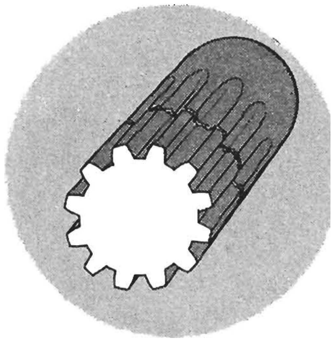
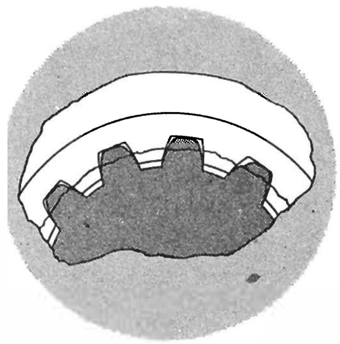
Correct diagnosis of spline-failure problems is a vital first step toward correct design. This article supplies charts and formulas for computing four important types of stresses that must not be exceeded.

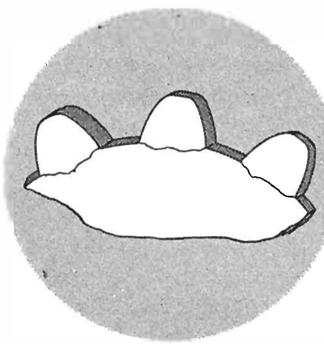
Splines may fail in one of these five ways:



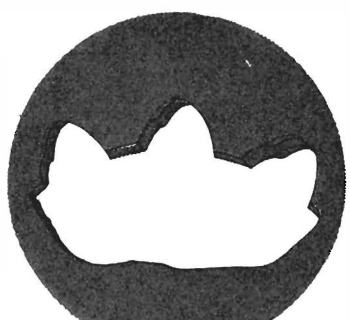
(A) shaft of externally toothed member breaks underneath spline teeth



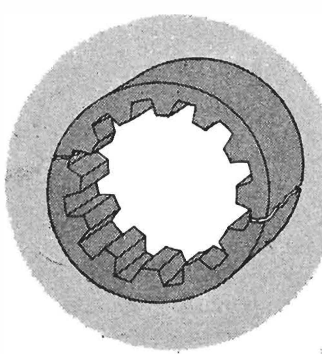
(B) teeth of spline shear off on pitch line



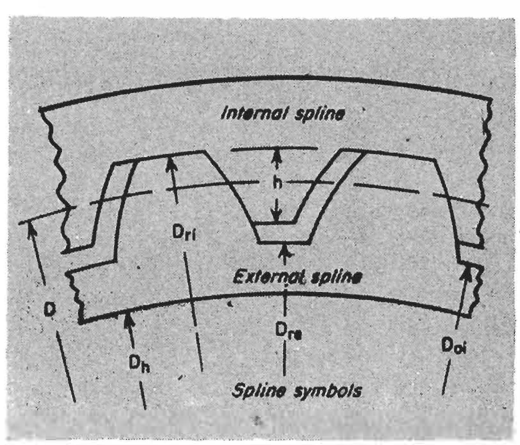
(C) teeth break at roots in a cantilever type failure similar to that of gear teeth



(D) contacting surface of spline teeth wear away by fretting corrosion



(E) shell of the internally toothed member ruptures



SPLINE SYMBOLS

A	height of crown
B	misalignment of spline, in./in.
D	pitch dia, in.
D_h	bore dia of shaft, in.
D_{oi}	outside dia of internally toothed part, in.
D_{ri}	major dia of internally toothed part, in.
D_{re}	root dia, also minor dia of external toothed member, in.
F	full face width, in.
F_e	effective face width, in. (may be less than F; see Editor's Note)
h	radial height of the tooth in contact, in.
K_a	application factor
K_m	load distribution factor
L_f	life factor limited by fatigue
L_w	life factor limited by wear
n	rpm
N	number of spline teeth
S_c	compressive stress, psi
S'_c	allowable compressive stress, psi
S_s	shear stress, psi
S'_s	allowable shear stress, psi
S_t	total stress, psi
t_c	chordal thickness at pitch line, in. (approximately equal to $D/2N$)
t_w	wall thickness, in.
T	torque, in.-lb = 63,000 hp/n
ϕ	pressure angle, deg

Spline design follows a simple procedure. Calculate the various stresses, then adjust size of the spline so that all stresses come within design limits. These four types of stresses are the major ones:

1. Shear stresses in spline shaft
2. Shear stresses in spline teeth
3. Compressive stresses in spline teeth
4. Bursting stresses in internal spline parts

SHAFT STRESSES

The externally toothed spline is cut on either solid or hollow shaft. Shear stresses induced in a solid shaft when transmitting a torque are:

$$S_s = \frac{16T}{\pi D_{re}^3} \tag{1}$$

For a hollow shaft:

$$S_s = \frac{16TD_{re}}{\pi(D_{re}^4 - D_h^4)} \tag{2}$$

Whether or not the shaft can withstand these induced stresses depends on the shaft material. Table I gives the maximum allowable shear stress S_s' for various types of steel. Computed stresses S_s from Eq (1) and (2) must not exceed the allowable stress S_s' in Table I, modified by application factor K_a and fatigue life factor L_f:

$$S_s' \geq S_s \frac{K_a}{L_f} \tag{3}$$

Application factors are listed in Table II. Note that relatively high service factors are used where both the driving and driven apparatus are relatively rough-running. Under normal conditions a generator represents a smooth drive; but a short circuit through a generator may develop a severe overload torque—and this possibility requires a high value of K_a in the shear-stress formula.

Fatigue-life factor L_f is based on the number of torque cycles. Generally, spline teeth get a cycle of load on and load off only when a machine is started up, and then later turned off. But some machinery—for example, power shovels, bulldozers, planers, shapers and airplane rudder controls—subject a spline drive to more frequent cycles of loading. Life factors for different torque-cycle values are shown in Table III. Unity life factor is taken as 10,000 cycles. If the torque cycles are known to be frequent, but difficult to determine accurately, a life factor for 10 million cycles should be used.

The allowable stresses given in tables represent the author's "middle of the road" recommendation for design standards. At present there are no generally recognized standards for intensity of spline loading. Perhaps these published values may become the basis for effective standards.

SHEAR STRESSES IN TEETH

Assuming now that the spline shaft can transmit the desired torque, the next potential failure spot is in the teeth. There are two ways they can fail: They may shear; or they may wear excessively because of high compressive stresses. Both possibilities must be examined.

Teeth are assumed to shear at the pitch line. The

TABLE I—ALLOWABLE SHEAR STRESSES FOR SPLINES

Material	Hardness Brinell	Hardness Rockwell C	Max Allowable Shear Stress, S_s', psi
Steel	160–200	.....	20,000
Steel	230–260	.....	30,000
Steel	302–351	33–38	40,000
Surface-hardened steel	....	48–53	40,000
Case-hardened steel	....	58–63	50,000
Thru-hardened steel (aircraft quality)	....	42–46	45,000

TABLE II—SPLINE APPLICATION FACTORS, K_a

Power Source	Type of Load: Uniform (generator, fan)	Light Shock (oscillating pumps, etc.)	Intermittent Shock (actuating pumps, etc.)	Heavy Shock (punches, shears, etc.)
Uniform (turbine, motor)	1.0	1.2	1.5	1.8
Light shock (hydraulic motor)	1.2	1.3	1.8	2.1
Medium shock (internal combustion engine)	2.0	2.2	2.4	2.8

TABLE III—FATIGUE-LIFE FACTOR FOR SPLINES

No. of Torque, Cycles	Life Factor, L_f Unidirectional	Fully-reversed
1,000	1.8	1.8
10,000	1.0	1.0
100,000	0.5	0.4
1,000,000	0.4	0.3
10,000,000	0.3	0.2

TABLE IV—LOAD DISTRIBUTION FACTOR FOR SPLINES

Misalignment	Factor K_m: ½-in. Face Width	1-in. Face Width	2-in. Face Width	4-in. Face Width
0.001 in./in.	1	1	1	1½
0.002 in./in.	1	1	1½	2
0.004 in./in.	1	1½	2	2½
0.008 in./in.	1½	2	2½	3

TABLE V—ALLOWABLE COMPRESSIVE STRESS FOR SPLINES

Material	Hardness: Brinell	Hardness: Rockwell C	Max Allowable Compressive stress, S'_c, psi: Straight	Max Allowable Compressive stress, S'_c, psi: Crowned
Steel	160–200	—	1,500	6,000
Steel	230–260	—	2,000	8,000
Steel	302–351	33–38	3,000	12,000
Surface-hardened steel	—	48–53	4,000	16,000
Case-hardened steel	—	58–63	5,000	20,000

TABLE VI—WEAR LIFE OF FLEXIBLE SPLINES

No. of Revolutions	Life Factor, L_w
10,000	4.0
100,000	2.8
1 million	2.0
10 million	1.4
100 million	1.0
1 billion	0.7
10 billion	0.5

TABLE VII—ALLOWABLE TENSILE STRESS FOR SPLINES

Material	Hardness: Brinell	Hardness: Rockwell C	Max Allowable Stress S'_t, psi
Steel	160–200	—	22,000
Steel	230–260	—	32,000
Steel	302–351	33–38	45,000
Surface-hardened steel	—	48–53	45,000
Case-hardened steel	—	58–63	55,000
Thru-hardened steel	—	42–46	50,000

TABLE VIII—COMPARISON OF SPLINE MANUFACTURING METHODS

Method	Used on: External	Used on: Internal	General Comments
Broaching	No	Yes	Low cost for quantity production; high accuracy
Shaping	Yes	Yes	Low cost for small lot production
Hobbing	Yes	No	Low cost for small lot production
Shaving	Yes	Yes	For high accuracy, high speed couplings
Grinding	Yes	Yes	For highest capacity couplings
Casting	Yes	Yes	For small-dia nonmetal splines
Cold-forming	Yes	No	High-production, low-cost process for automotive parts

induced shear stresses are:

$$S_s = \frac{4TK_m}{DNF_e L_e} \qquad (4)$$

In a spline, contrasted with a gear, tooth failure cannot stop the drive until all teeth are broken on both members. The constant 4 in the above equation assumes that, because of spacing errors, only half the teeth carry the load. With poor manufacturing accuracies, it is best to increase the factor to 6. Values for load distribution factor K_m, given in Table IV, are based on the amount of misalignment; K_m is 1.0 for a fixed spline (misalignment in a fixed spline is zero).

After calculating tooth shear stress, Eq (3) can again be used—to relate the calculated stresses with the allowable stresses.

COMPRESSIVE STRESSES

Compressive stresses act on the sides of the spline teeth and can be calculated from:

$$S_c = \frac{2TK_m}{DNF_e h} \qquad (5)$$

Factor h is radial height in inches of the contacting tooth. If the teeth do not have a substantial tip radius or chamfer, then h is the sum of the addendum of the external tooth and the addendum of the internal tooth. If with chamfer or radius, subtract its average radial height from the sum of the addendums.

The constant 2 in Eq (5) assumes all teeth to be working, which becomes true after some initial wear.

Computed compressive stress should be compared with the allowable compressive stresses in Table V, by using these equations:

$$\text{Flexible splines,} \quad S'_c \geqq \frac{S_c K_a}{L_w} \qquad (6)$$

$$\text{Fixed splines,} \quad S'_c \geqq \frac{S_c K_a}{9L_f} \qquad (7)$$

The allowable compressive stress values are very much lower than those used for gear teeth because splines do not distribute the load uniformly between teeth. With typical misalignment, straight splines (a term used to differentiate from crowned splines) have only one end of the tooth loaded at a time. The full surface area of the spline is rather inefficiently used. Exceeding the allowable compressive-stress values results in fretting corrosion. The life factor L_w for Eq (6) is a wear-life factor, Table VI; the life factor L_f for Eq (7) is a fatigue-life factor, Table III. Wear life factors are based on revolutions of the spline—not cycles of torque on and off. Each time a flexible spline makes a revolution, there is a back-and-forth rubbing of the teeth which causes wear. In Eq (7), factor 9 indicates that a fixed spline has about nine times more ability to carry compressive stress than a flexible spline.

BURSTING STRESSES

Internally toothed spline parts tend to burst because of three different kinds of tensile stresses: (1) bursting stress caused by the radial force component at the pitch line, (2) bursting stress caused by centrifugal force, (3) tensile stress caused by the tangential force component at the pitch line. The bursting stress caused by radial forces is:

$$S_1 = \frac{T \tan \phi}{\pi D t_w F} \tag{8}$$

The wall thickness of the internally toothed member is obtained by subtracting the major dia from the OD of the coupling sleeve, and dividing the result by two.

Bursting stress caused by centrifugal force may be estimated by the formula for a simple cylinder which assumes that the face width is less than one-third the pitch dia.

$$S_2 = 0.828(10^{-6})(n^2)(2D^2_{oi} + 0.424D^2_{ri})$$

The tensile stress due to beam loading of the teeth is

$$S_3 = \frac{4T}{D^2 F_e Y} \tag{10}$$

The Y-factor is the same as in the Lewis equation for gear design. Internal teeth of splines have a rather high Y-factor compared with gear teeth; usually equal to 1.5 or more. In most spline designs, it is not worth the trouble to make layouts to get an exact value for the Y-factor. Factor 4 in Eq (10) compensates for the load being carried on half the teeth.

All of the above stresses are tensile stresses at the root dia of the internally toothed part. The total stress tending to burst the rim is

$$S_t = K_a K_m(S_1 + S_3) + S_2 \tag{11}$$

This total can be compared with allowable tensile stress values in Table VII using the relation,

$$S'_t \geqq S_t/L_f \tag{12}$$

CROWNED TEETH

Crowned teeth on splines can handle a high amount of misalignment. Radii in the illustration can be obtained from these equations:

$$r_1 = 0.90(D/2)(\tan \phi) \tag{13}$$
$$r_2 = r_1/\tan \phi \tag{14}$$

If the spline is fully crowned, the contact will stay fairly close to the tooth center even when the spline rocks through an appreciable angle. Fully crowned splines are successful with even as much as 3° misalignment.

If only slight misalignment is expected, the curvature of the teeth may be reduced, providing that dimension A is made greater than the quantity $BF/2$. This means that a spline of 1 in. face width misaligned 0.002 in. per in. should have a height of crown equal to at least 0.001 in.

When the crown is specified by the height of crown method, the approximate radius, r_2, may be calculated by

$$r_2 = F^2/8A \tag{15}$$

When a flexible coupling has sufficient crown, the width of the band of contact remains constant; Eq (5) is no longer valid for calculating the surface compressive stress. Instead a Hertz-stress formula must be used.

$$S_c = 2290\sqrt{\frac{2T}{DNhr_2}} \tag{16}$$

The computed value of S_c obtained above should be compared with the allowable values of S'_c for crowned splines, using Table V and Eq (6). Crowned splines never exceed the capacity of precisely aligned straight splines; the crown is only valuable in helping a spline carry a high amount of misalignment.

HOW DO SPLINES FAIL?

Splines transmit more torque for their size than any other type of coupler or joint. Frequently they are operating with a relatively small-dia shaft. Thus, the most common type of fixed spline failure is that of the shaft shearing.

Next most common is failure of tooth shear. Generally, the "fixed" spline (which mounts a gear, pulley wheel, turbine wheel, etc., in contrast with the "flexible" spline used as a coupling between shafts) has no relative motion between internal and external teeth to cause wear.

Fixed splines frequently carry high compressive (or "bearing") stresses and yet show no wear. When there is wear, it usually results from abnormal vibrations of the apparatus, or severe misalignment.

Fixed-spline teeth seldom fail in bending fatigue as in gear teeth because spline teeth are usually shorter and stubbier. Also a gear tooth is subjected to fatigue loading in which it receives a full on-and-off load each revolution—unlike fixed-spline teeth which do not have much change in loading per revolution.

Flexible splines are usually employed as couplings and do not carry as much torque per dia. There is little danger of the teeth shearing off. But there is some relative motion between teeth which makes them vulnerable to wear.

In larger spline couplings, the internally toothed member may burst because of applied torque. In addition, high speeds create severe centrifugal forces which add to bursting stresses. Even in small couplings, a thin wall on the internal member, a high tooth pressure angle, or a high torque loading for the size of the coupling can burst the internal member.

Crowned-tooth splines . . .

permit high misalignments—are often needed in couplings. Design procedure is same as straight splines except when determining radii define amount of crown.