

Tab. 8.9 — Reazioni dovute a cedimenti unitari e a carichi esterni assegnati		
	$W_A = \frac{4 E J}{l}; \quad W_B = \frac{2 E J}{l}$	
	$W_A = W_B = \frac{6 E J}{l^2}$	$V_A = V_B = \frac{6 E J}{l^2}$
	$W_A = \frac{3 E J}{l}$	$V_A = V_B = \frac{3 E J}{l^2}$
	$W_A = \frac{3 E J}{l^2}$	$V_A = V_B = \frac{3 E J}{l^3}$
	$W_A = W_B = \frac{P l}{8}$	$V_A = V_B = \frac{P}{2}$
	$W_A = W_B = \frac{P l^2}{12}$	$V_A = V_B = \frac{P l}{2}$

Le (98) esprimono le condizioni di equilibrio, rispettivamente alla traslazione orizzontale, a quella verticale e alla rotazione, del nodo 2. I risultati che precedono si possono estendere a una struttura generica con m gradi di vincolo in più. In tal caso il metodo degli spostamenti conduce a scrivere:

$$K_{11}x_1 + K_{12}x_2 + \dots + K_{1m}x_m = K_{10} \quad (99)$$

$$K_{m1}x_1 + K_{m2}x_2 + \dots + K_{mm}x_m = K_{m0} \quad (100)$$

Le (99) in forma matriciale diventano: $[K][x] = [K_0]$
dove il generico elemento K_{ij} della matrice $[K]$, detta «matrice delle rigidezze», si scrive:

$$K_{ij} = \int \left(\frac{M_i M_j}{E J} + \frac{N_i N_j}{E A} + \chi \frac{T_i T_j}{G A} \right) dx_3 \quad (101)$$

e il generico elemento K_{10} del vettore dei termini noti (K_0)

$$K_{10} = \int \left(\frac{M_0 M_i}{E J} + \frac{N_0 N_i}{E A} + \chi \frac{T_0 T_i}{G A} \right) dx_3 \quad (102)$$

Nel caso dei telai, quando si possano trascurare gli effetti delle reazioni assiale e tagliente rispetto a quelli della reazione flettente, le (101) e (102) diventano rispettivamente:

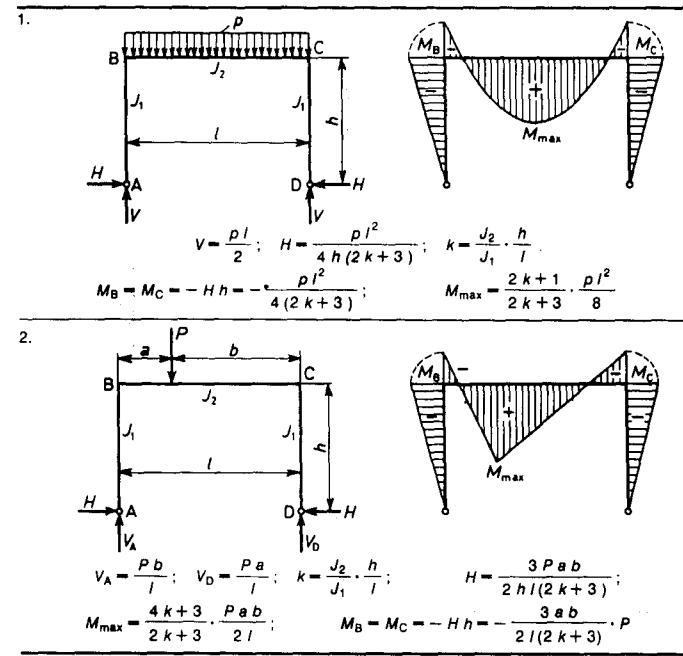
$$K_{ij} = \int \frac{M_i M_j}{E J} dx_3; \quad K_{10} = \int \frac{M_0 M_i}{E J} dx_3 \quad (103)$$

Nel caso di travature reticolari caricate ai nodi, le (101) e (102) diventano invece:

$$K_{ij} = \int \frac{N_i N_j}{E A} dx_3; \quad K_{10} = \int \frac{N_0 N_i}{E A} dx_3. \quad (104)$$

Nella tab. 8.9 sono riportate le reazioni dovute a cedimenti vincolari unitari e a forze esterne assegnate di alcuni casi strutturali semplici. Nella tab. 8.10 sono riportati diversi casi già risolti di telai.

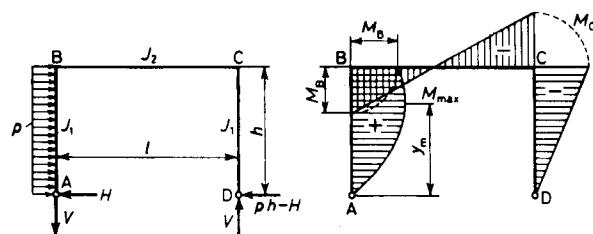
Tab. 8.10 — Telai



segue

seguito tab. 8.10

3.



$$V = \frac{P \cdot h^2}{24} ; \quad H = \frac{P \cdot h}{8} \cdot \frac{11k + 18}{2k + 3} ; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l}$$

$$M_C = -\frac{P \cdot h^2}{8} \cdot \frac{5k + 6}{2k + 3} ; \quad M_B = \frac{3}{8} P \cdot h^2 \cdot \frac{k + 2}{2k + 3}$$

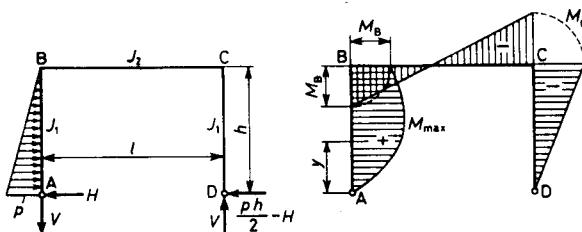
Per

$$y_m = \frac{h}{8} \cdot \frac{11k + 18}{2k + 3}$$

si ha:

$$M_{\max} = \frac{P \cdot h^2}{128} \left(\frac{11k + 18}{2k + 3} \right)^2$$

4.



$$V = \frac{P \cdot h^2}{6l} ; \quad H = \frac{P \cdot h}{40} \cdot \frac{31k + 50}{2k + 3} ; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l}$$

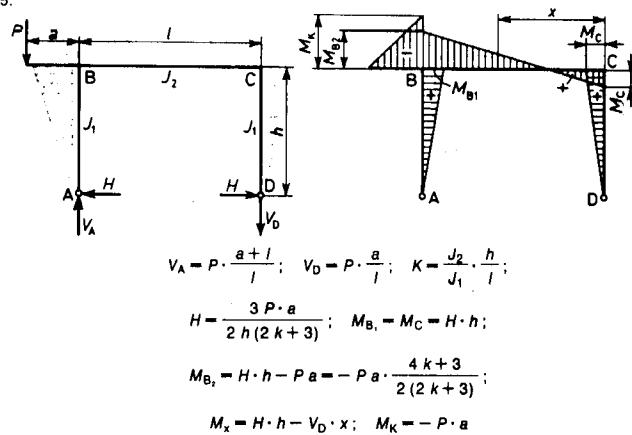
$$M_C = -\frac{P \cdot h^2}{40} \cdot \frac{9k + 10}{2k + 3} ; \quad M_B = \frac{P \cdot h^2}{120} \cdot \frac{13k + 30}{2k + 3}$$

$$M_y = H \cdot y - \frac{P \cdot y^2}{6h} (3h - y)$$

segue

seguito tab. 8.10

5.



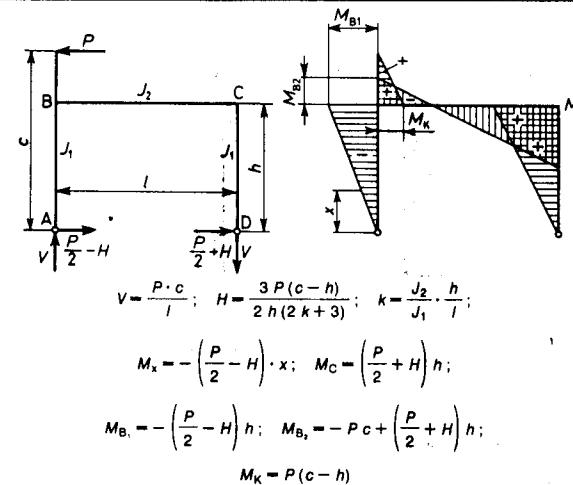
$$V_A = P \cdot \frac{a + l}{l} ; \quad V_D = P \cdot \frac{a}{l} ; \quad K = \frac{J_2}{J_1} \cdot \frac{h}{l}$$

$$H = \frac{3P \cdot a}{2h(2k + 3)} ; \quad M_{B1} = M_C = H \cdot h$$

$$M_{B2} = H \cdot h - Pa = -Pa \cdot \frac{4k + 3}{2(2k + 3)}$$

$$M_x = H \cdot h - V_D \cdot x ; \quad M_K = -P \cdot a$$

6.



$$V = \frac{P \cdot c}{l} ; \quad H = \frac{3P(c - h)}{2h(2k + 3)} ; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l}$$

$$M_x = -\left(\frac{P}{2} - H\right) \cdot x ; \quad M_C = \left(\frac{P}{2} + H\right) h$$

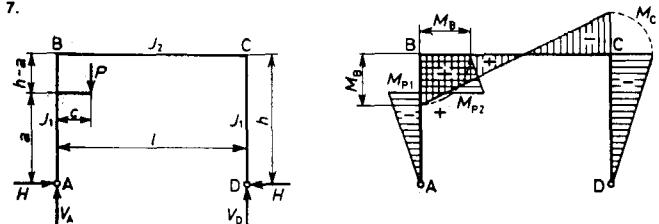
$$M_{B1} = -\left(\frac{P}{2} - H\right) h ; \quad M_{B2} = -Pc + \left(\frac{P}{2} + H\right) h$$

$$M_K = P(c - h)$$

segue

seguito tab. 8.10

7.

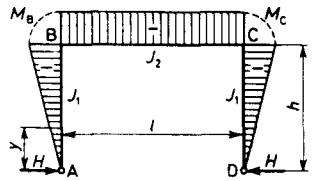


$$V_A = P \frac{l - c}{l}; \quad V_D = \frac{P c}{l}; \quad k = \frac{j_2 \cdot h}{j_1 \cdot l};$$

$$H = \frac{3}{2} P c \cdot \frac{k(h^2 - a^2) + h^2}{h^3(2k + 3)};$$

$$M_B = P c - H \cdot h; \quad M_C = -H \cdot h$$

8.

Diagramma dei momenti per un aumento Δt della temperatura.

$$k = \frac{j_2 \cdot h}{j_1 \cdot l}; \quad H = \frac{3 \cdot a \cdot E \cdot j_2 \cdot \Delta t}{h^2(3 + 2k)};$$

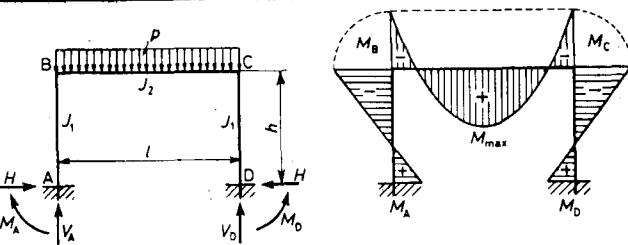
$$M_y = -H \cdot y; \quad M_B = M_C = -H \cdot h$$

 α = coefficiente di dilatazione lineare ($^{\circ}\text{C}^{-1}$). E = modulo di elasticità normale.

segue

seguito tab. 8.10

9.

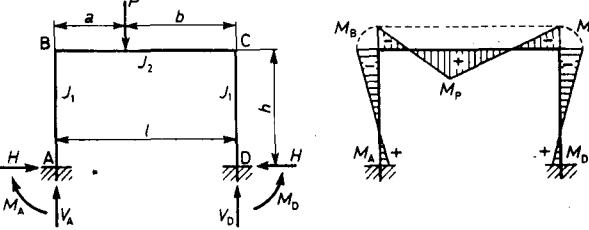


$$V_A = V_D = \frac{p \cdot l}{2}; \quad H = \frac{p \cdot l^2}{4h(k+2)}; \quad k = \frac{j_2 \cdot h}{j_1 \cdot l};$$

$$M_B = M_C = -\frac{p \cdot l^2}{6(k+2)}; \quad M_A = M_D = \frac{p \cdot l^2}{12(k+2)};$$

$$M_{\max} = \frac{p \cdot l^2}{8} - \frac{p \cdot l^2}{6(k+2)} = \frac{p \cdot l^2}{24} \cdot \frac{2 + 3k}{k+2}$$

10.



$$k = \frac{j_2 \cdot h}{j_1 \cdot l}; \quad b = \frac{a}{l}; \quad H = \frac{3P \cdot a \cdot b}{2h \cdot l(k+2)};$$

$$V_A = \frac{P \cdot b}{l} \cdot \frac{6k+1 + b - 2b^2}{6k+1}; \quad V_D = \frac{P a}{l} \cdot \frac{6k+3b-2b^2}{6k+1};$$

$$M_A = \frac{P a b}{2l} \cdot \frac{5k-1+2b(k+2)}{(k+2)(6k+1)};$$

$$M_D = \frac{P a b}{2l} \cdot \frac{3+7k-2b(k+2)}{(k+2)(6k+1)};$$

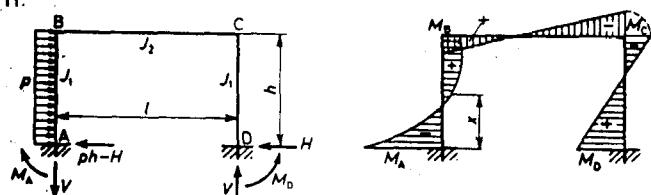
$$M_B = M_A - H \cdot h; \quad M_C = M_D - H \cdot h;$$

$$M_P = M_A - H \cdot h + V_A \cdot a$$

segue

seguito tab. 8.10

11.



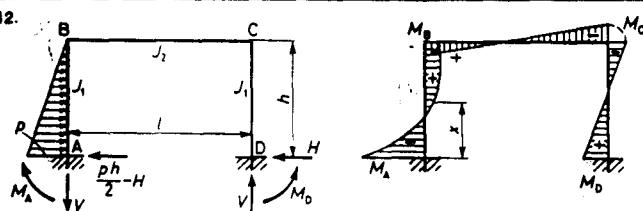
$$V = \frac{p h^2 k}{l(6k+1)}; \quad H = \frac{p h}{8} \cdot \frac{2k+3}{k+2}; \quad k = \frac{J_2 \cdot h}{J_1 \cdot l};$$

$$M_A = -\frac{p h^2}{24} \left(12 - \frac{5k+9}{k+2} - \frac{12k}{6k+1} \right); \quad M_D = \frac{p h^2}{24} \left(\frac{5k+9}{k+2} - \frac{12k}{6k+1} \right);$$

$$M_x = M_A + (p \cdot h - H) \cdot x - \frac{p x^2}{2};$$

$$M_B = M_A - H \cdot h + \frac{p h^2}{2}; \quad M_C = M_D - H \cdot h$$

12.



$$V = \frac{p \cdot k \cdot h^2}{4l(6k+1)}; \quad H = \frac{p h}{40} \cdot \frac{3k+4}{k+2}; \quad k = \frac{J_2 \cdot h}{J_1 \cdot l};$$

$$M_A = -\frac{p h^2}{120} \left(20 - \frac{7k+12}{k+2} - \frac{15k}{6k+1} \right);$$

$$M_D = \frac{p h^2}{120} \left(\frac{7k+12}{k+2} - \frac{15k}{6k+1} \right);$$

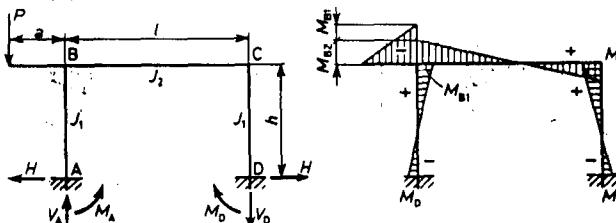
$$M_x = M_A + \left(\frac{p h}{2} - H \right) \cdot x - \frac{p x^2}{6h} (3h-x);$$

$$M_B = M_A - H \cdot h + \frac{p h^2}{6}; \quad M_C = M_D - H \cdot h$$

segue

seguito tab. 8.10

13.



$$k = \frac{J_2 \cdot h}{J_1 \cdot l}; \quad H = \frac{p a}{2h} \cdot \frac{3}{k+2};$$

$$V_A = P \cdot \frac{a+l}{l} + \frac{M_D - M_A}{l}; \quad V_D = P \cdot \frac{a}{l} + \frac{M_A - M_D}{l};$$

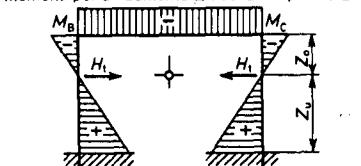
$$M_A = -\frac{P a}{2} \cdot \frac{5k+1}{(6k+1)(k+2)};$$

$$M_D = -\frac{P a}{2} \cdot \frac{7k+3}{(6k+1)(k+2)};$$

$$M_B = -\frac{P a}{2} \cdot \frac{13k+4}{(6k+1)(k+2)};$$

$$M_{B_1} = -\frac{P a}{2} \cdot \frac{k(12k+13)}{(6k+1)(k+2)};$$

$$M_C = -\frac{P a}{2} \cdot \frac{11k}{(6k+1)(k+2)}$$

14. Diagramma dei momenti per un aumento Δt della temperatura.

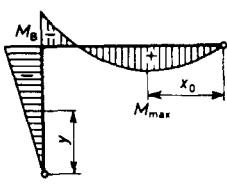
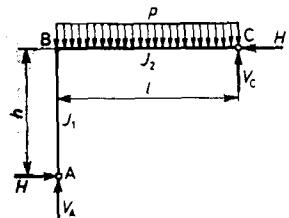
$$Z_0 = \frac{h k}{2k+1}; \quad Z_0 = \frac{h(k+1)}{2k+1}; \quad k = \frac{J_2 \cdot h}{J_1 \cdot l};$$

$$H_1 = \frac{3 E \cdot a \cdot \Delta t \cdot J_2 (2k+1)}{h^2 \cdot k (k+2)}$$

segue

seguito tab. 8.10

15.



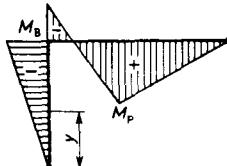
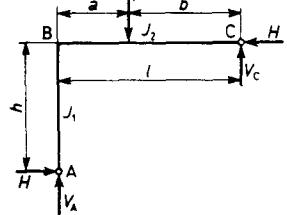
$$V_A = \frac{P l}{8} \cdot \frac{4 k + 5}{k + 1}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l};$$

$$V_C = \frac{P l}{8} \cdot \frac{4 k + 3}{k + 1}; \quad H = \frac{P l^2}{8 h (k + 1)};$$

$$M_B = -H h = -\frac{P l^2}{8 (k + 1)}; \quad M_y = -H y;$$

$$x_0 = \frac{l}{8} \cdot \frac{4 k + 3}{k + 1}; \quad M_{\max} = \frac{P l^2}{128} \left(\frac{4 k + 3}{k + 1} \right)^2$$

16.



$$H = \frac{P \cdot a \cdot b (l + b)}{2 h l^2 (k + 1)}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l};$$

$$V_A = \frac{P b + H h}{l}; \quad V_C = \frac{P a - H h}{l};$$

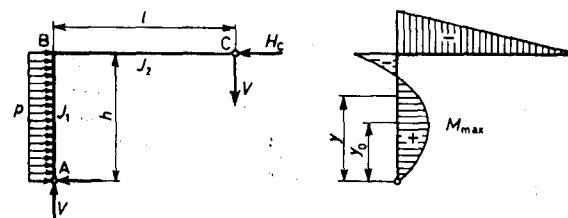
$$M_y = -H y; \quad M_B = -H h;$$

$$M_p = \frac{P a b}{2 l^3} \cdot \frac{2 k l^2 + 3 a l - a^2}{k + 1}$$

segue

seguito tab. 8.10

17.



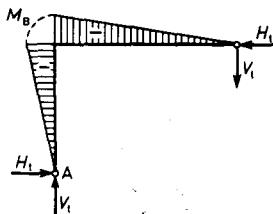
$$V = \frac{P h^2 k}{8 l (k + 1)}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l};$$

$$H_A = \frac{P h}{8} \cdot \frac{3 k + 4}{k + 1}; \quad H_C = \frac{P h}{8} \cdot \frac{5 k + 4}{k + 1};$$

$$M_B = -V l; \quad M_y = \frac{P y}{8} \cdot \frac{4(h - y) + k(3h - 4y)}{k + 1};$$

$$y_0 = \frac{3 k + 4}{k + 1} \cdot \frac{h}{8}; \quad M_{\max} = \frac{P h^2}{128} \left(\frac{3 k + 4}{k + 1} \right)^2$$

18.

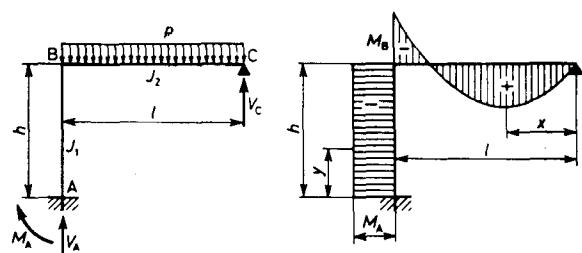
Diagramma dei momenti per un aumento Δt della temperatura.

$$H_t = \frac{2 E a \Delta t J_2}{h^2 (k + 1)}; \quad V_t = \frac{H_t h}{l}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l}$$

segue

seguito tab. 8.10

19.



$$V_C = \frac{3}{8} \cdot p \cdot l \cdot \frac{4k+1}{3k+1}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l};$$

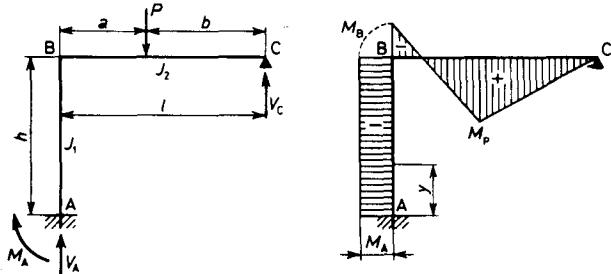
$$V_A = p \cdot l - V_C;$$

$$M_A = M_B = -\frac{p \cdot l^2}{8(3k+1)};$$

$$M_y = M_A = M_B = V_C \cdot l - \frac{p \cdot l^2}{2};$$

$$M_x = V_C \cdot x - \frac{p \cdot x^2}{2}$$

20.



$$V_C = P \cdot a \cdot \frac{a(3l-a)+6k \cdot l^2}{2l^3(3k+1)}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l};$$

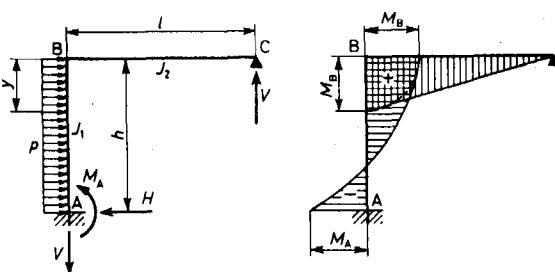
$$V_A = P - V_C; \quad M_y = M_A = M_B = V_C \cdot l - P \cdot a;$$

$$M_P = V_C \cdot b$$

segue

seguito tab. 8.10

21.

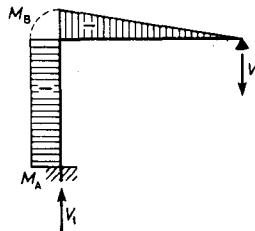


$$V = \frac{p \cdot h^2 \cdot k}{2l(3k+1)}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l};$$

$$H = p \cdot h; \quad M_A = Vl - \frac{p \cdot h^2}{2};$$

$$M_B = Vl$$

22.

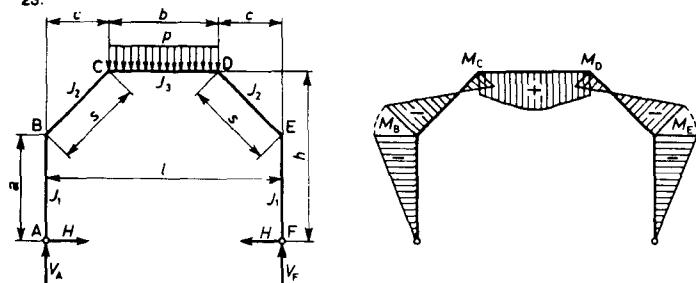
Diagramma dei momenti per un aumento Δt della temperatura.

$$V_t = \frac{3E\alpha\Delta t J_1 k}{l^2(3k+1)}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l}$$

segue

seguito tab. 8.10

23.



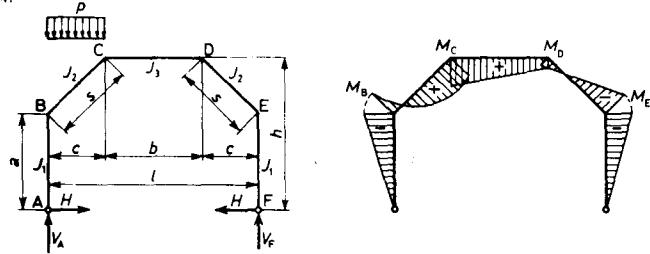
$$V_A = V_F = \frac{Pb}{2}; \quad k_1 = \frac{J_2}{J_1} \cdot \frac{a}{s}; \quad k_2 = \frac{J_2}{J_3} \cdot \frac{b}{s};$$

$$H = \frac{Pb}{4} \cdot \frac{2c(a+2h)+hk_2(l+4c)}{2a^2(k_1+1)+2ha+h^2(2+3k_2)};$$

$$M_B = M_E = -H \cdot a;$$

$$M_C = M_D = -H \cdot h + \frac{P \cdot b}{2} \cdot c; \quad M_{\max} = -H \cdot h + \frac{Pb}{4} \cdot \left(l - \frac{b}{2}\right)$$

24.



$$k_1 = \frac{J_2}{J_1} \cdot \frac{a}{s}; \quad k_2 = \frac{J_2}{J_3} \cdot \frac{b}{s}; \quad V_A = \frac{Pc(l-c/2)}{l}; \quad V_F = \frac{Pc^2}{2l};$$

$$H = \frac{Pc^2}{8} \cdot \frac{6hk_2+5h+3a}{2a^2(k_1+1)+2ha+h^2(2+3k_2)};$$

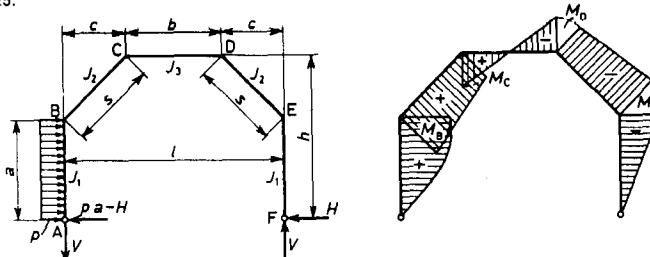
$$M_B = M_E = -Ha;$$

$$M_C = -H \cdot h - \frac{Pc^2}{2} + V_A \cdot c; \quad M_D = -H \cdot h + \frac{Pc^2}{2l}$$

segue

seguito tab. 8.10

25.



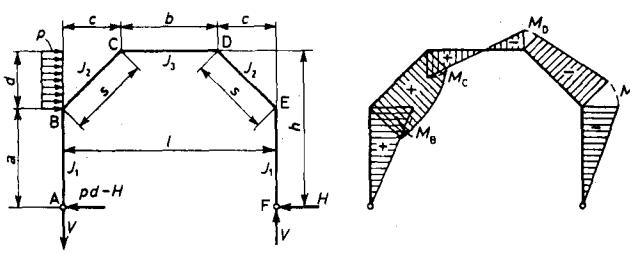
$$V = \frac{Pa^2}{2l}; \quad k_1 = \frac{J_2}{J_1} \cdot \frac{a}{s}; \quad k_2 = \frac{J_2}{J_3} \cdot \frac{b}{s};$$

$$H = \frac{Pa^2}{8} \cdot \frac{6h(k_2+1)+a(5k_1+6)}{2a^2(k_1+1)+2ha+h^2(2+3k_2)};$$

$$M_B = (pa-H)a - \frac{pa^2}{2}; \quad M_C = V(l-c) - H \cdot h;$$

$$M_D = -H \cdot h + V \cdot c; \quad M_E = -H \cdot a$$

26.



$$V = \frac{pd(h-d/2)}{l}; \quad k_1 = \frac{J_2}{J_1}; \quad k_2 = \frac{J_2}{J_3} \cdot \frac{b}{s};$$

$$H = \frac{pd}{8} \cdot \frac{a^2(8k_1+9)+h[2a(5+3k_2)+h(5+6k_2)]}{2a^2(k_1+1)+2ha+h^2(2+3k_2)};$$

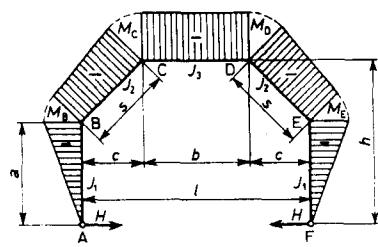
$$M_B = (pd-H)a; \quad M_C = V(l-c) - H \cdot h;$$

$$M_D = -H \cdot h + V \cdot c; \quad M_E = -H \cdot a$$

segue

seguito tab. 8.10

27.

Diagramma dei momenti per aumento Δt della temperatura.

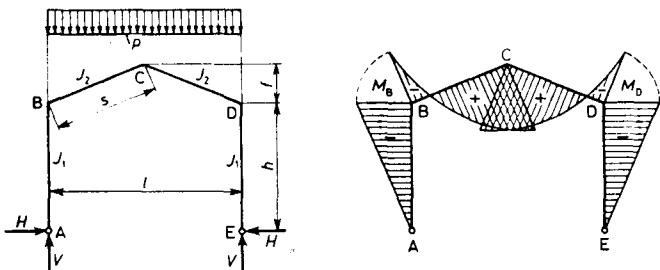
$$k_1 = \frac{J_2}{J_1} \cdot \frac{b}{s}; \quad k_2 = \frac{J_2}{J_3} \cdot \frac{b}{s};$$

$$H = \frac{3l\alpha\Delta t E J_2}{s[2a^2(k_1+1) + 2ha + h^2(2+3k_2)]};$$

$$M_C = -H \cdot h$$

$$M_B = -H \cdot a$$

28.



$$V = \frac{pI}{2}; \quad H = \frac{pI^2}{32} \cdot \frac{8h+5l}{h^2(k+3)+l(3h+l)};$$

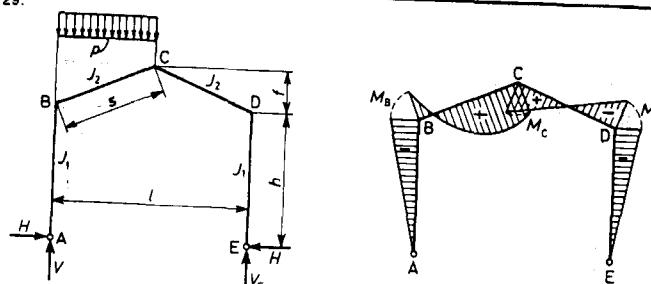
$$k = \frac{J_2}{J_1} \cdot \frac{h}{s};$$

$$M_B = -H \cdot h; \quad M_C = \frac{pI^2}{8} - H(h+l)$$

segue

seguito tab. 8.10

29.

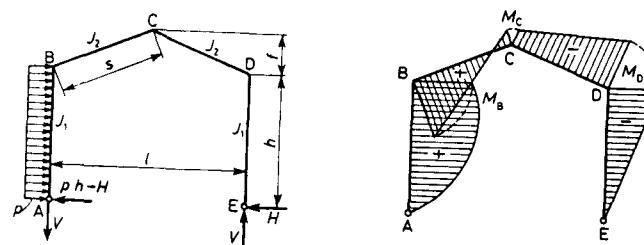


$$V_A = \frac{3pI}{8}; \quad V_E = \frac{pI}{8}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{s};$$

$$H = \frac{pI^2}{64} \cdot \frac{8h+5l}{h^2(k+3)+l(3h+l)};$$

$$M_B = M_D = -H \cdot h; \quad M_C = V_E \cdot l/2 - H(h+l)$$

30.



$$k = \frac{J_2}{J_1} \cdot \frac{h}{s};$$

$$V = \frac{qh^2}{2I}; \quad H = \frac{ph^2}{16} \cdot \frac{5hk+6(2h+l)}{h^2(k+3)+l(3h+l)};$$

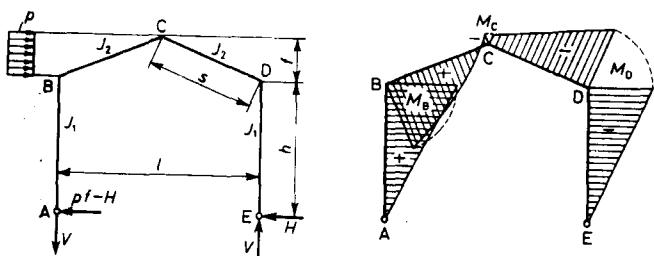
$$M_C = \frac{ph^2}{4} - H(h+l);$$

$$M_D = -H \cdot h; \quad M_B = \frac{ph^2}{2} - H \cdot h$$

segue

seguito tab. 8.10

31.



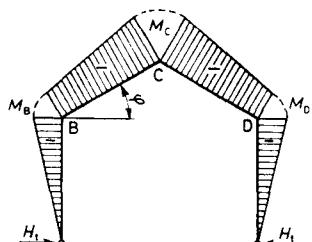
$$k = \frac{J_2}{J_1} \cdot \frac{h}{s}; \quad V = \frac{p f (2h + f)}{2l};$$

$$H = \frac{p f^2}{16} \cdot \frac{8h^2(k+3) + 5f(4h+f)}{h^2(k+3) + f(3h+f)};$$

$$M_B = (p f - H) h; \quad M_D = -H \cdot h;$$

$$M_C = V \cdot \frac{l}{2} - H(h+f)$$

32.

Diagramma dei momenti per un aumento Δt della temperatura.

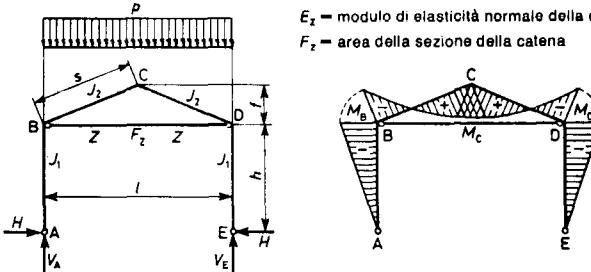
$$k = \frac{J_2}{J_1} \cdot \frac{h}{s};$$

$$H_t = \frac{3E \cdot a \cdot \Delta t \cdot J_2 \cos \varphi}{h^2(k+3) + f(3h+f)}$$

segue

seguito tab. 8.10

33.



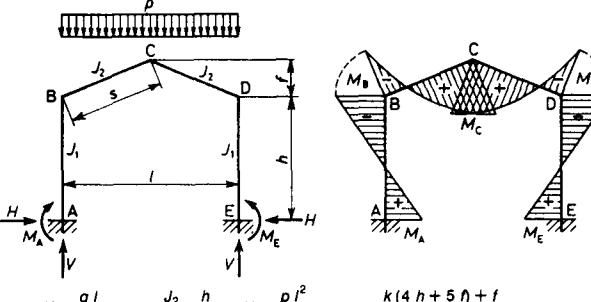
$$k = \frac{J_2}{J_1} \cdot \frac{h}{s}; \quad m = \frac{E J_2}{E_z F_z}; \quad \gamma = \frac{1}{1 + \frac{3m}{2f^2 s}}; \quad V = \frac{p l}{2};$$

$$H = \frac{p l^2}{16} \cdot \frac{h(16 - 15\gamma) + 10f(1-\gamma)}{h^2(4k + 2 - 9\gamma) + 12hf(1-\gamma) + 4f^2(1-\gamma)};$$

$$\text{Tensione nella catena } Z = (Z_0 - Z_a \cdot H), \text{ con } Z_0 = \frac{5p l^2}{32f} \cdot \gamma; \quad Z_a = \left(\frac{3h}{2f} + 1 \right) \cdot \gamma$$

$$\text{Per catene rigide } \gamma = 1. \quad M_B = M_B = -H \cdot h; \quad M_C = \frac{p l^2}{8} - H(h+f) - Z_l$$

34.



$$V = \frac{q l}{2}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{s}; \quad H = \frac{p l^2}{8} \cdot \frac{k(4h + 5f) + f}{(hk + f)^2 + 4k(h^2 + hf + f^2)};$$

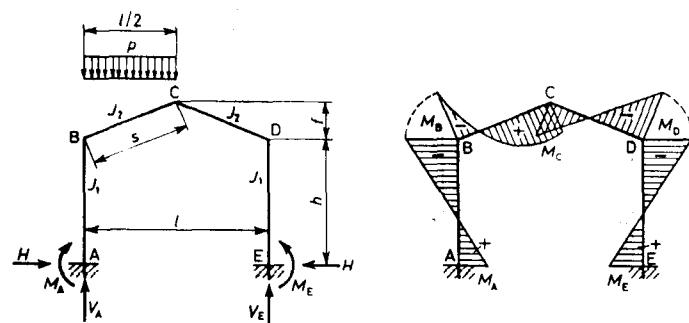
$$M_A = M_E = \frac{p l^2}{48} \cdot \frac{kh(8h + 15f) + f(6h - f)}{(kh + f)^2 + 4k(h^2 + hf + f^2)};$$

$$M_B = M_D = -H \cdot h + M_A; \quad M_C = -H(h+f) + M_A + \frac{p l^2}{8}$$

segue

seguito tab. 8.10

35.



$$V_A = \frac{3 p l}{8} + \frac{M_E - M_A}{l};$$

$$V_E = \frac{p l}{8} - \frac{M_E - M_A}{l}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{s};$$

$$H = \frac{p l^2}{16} \cdot \frac{k(4h+5f)+f}{(hk+f)^2+4k(h^2+hf+f^2)};$$

$$M_A = \frac{p l^2}{96} \left[\frac{kh(8h+15f)+f(6h-f)}{(kh+f)^2+4k(h^2+hf+f^2)} - \frac{3}{2(3k+1)} \right];$$

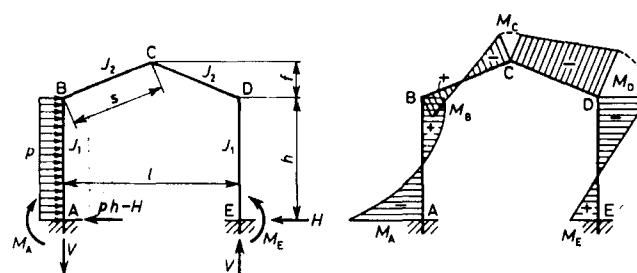
$$M_E = \frac{p l^2}{96} \left[\frac{kh(8h+15f)+f(6h-f)}{(kh+f)^2+4k(h^2+hf+f^2)} + \frac{3}{2(3k+1)} \right];$$

$$M_B = -Hh + M_A; \quad M_D = -Hh + M_E;$$

$$M_C = -H(h+f) + M_E + V_E \frac{l}{2}$$

seguito tab. 8.10

36.



$$V = \frac{p h^2}{2l} + \frac{M_A - M_E}{l}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{s};$$

$$H = \frac{p h^2}{4} \cdot \frac{k^2 h + k(3h+2f)}{(hk+f)^2 + 4k(h^2+hf+f^2)};$$

$$M_A = -\frac{p h^2}{24} \left[\frac{h^2 k(k+6) + kf(15h+16f)+6f^2}{(hk+f)^2 + 4k(h^2+hf+f^2)} + \frac{12k+6}{3k+1} \right];$$

$$M_E = \frac{h^2}{24} \left[-\frac{k^2 k(k+6) + kf(15h+16f)+6f^2}{(hk+f)^2 + 4k(h^2+hf+f^2)} + \frac{12k+6}{3k+1} \right];$$

$$M_D = -H \cdot h + M_E;$$

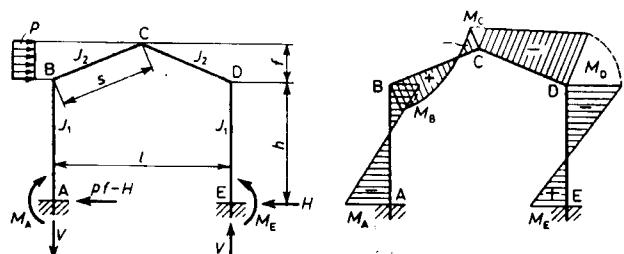
$$M_B = (ph-H)h + M_A - \frac{p h^2}{2};$$

$$M_C = -H(h+f) + M_E + \frac{p h^2}{4} \cdot \frac{k}{3k+1}$$

segue

seguito tab. 8.10

37.



$$V = \frac{P f (h + l/2)}{l} + \frac{M_A - M_E}{l}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{s};$$

$$H = \frac{P f}{4} \cdot \frac{5 f k (2 h + l) + 2 h^2 k (k + 4) + l^2}{(h k + l)^2 + 4 k (h^2 + h f + f^2)};$$

$$M_A = -\frac{P f}{24} \left[\frac{f (h k (4 h + 9 l) + f (6 h + l))}{(h k + l)^2 + 4 k (h^2 + h f + f^2)} + \frac{12 h (3 k + 2) + 3 l}{6 k + 2} \right];$$

$$M_E = \frac{P f}{24} \left[-\frac{f (h k (4 h + 9 l) + f (6 h + l))}{(h k + l)^2 + 4 k (h^2 + h f + f^2)} + \frac{12 h (3 k + 2) + 3 l}{6 k + 2} \right];$$

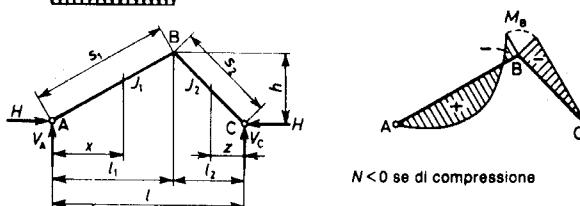
$$M_D = -H \cdot h + M_E; \quad M_B = (P f - H) h + M_A;$$

$$M_C = -H (h + l) + M_E + V_E \cdot \frac{l}{2}$$

segue

seguito tab. 8.10

38.

 $N < 0$ se di compressione

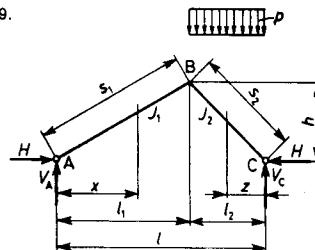
$$V_A = \frac{P l_1}{2 l} (l + l_2); \quad V_C = \frac{P l_1^2}{2 l}; \quad k = \frac{J_2}{J_1} \cdot \frac{s_1}{s_2};$$

$$H = \frac{P l_1^2}{8 h l} \cdot \frac{4 l_2 + (l + 4 l_2) k}{k + 1}; \quad M_B = V_C \cdot l_2 - H \cdot h;$$

$$M_x = V_A \cdot x - H \cdot \frac{h x}{l_1} - \frac{P x^2}{2}; \quad N_x = -(V_A - P x) \frac{h}{s_1} - H \cdot \frac{l_1}{s_1};$$

$$M_z = V_C \cdot z - H \cdot \frac{h \cdot z}{l_2}; \quad N_{BC} = -V_C \cdot \frac{h}{s_2} - H \cdot \frac{l_2}{s_2}$$

39.



$$V_A = \frac{P l_2^2}{2 l}; \quad V_C = \frac{P l_2}{2 l} \cdot (l + l_1); \quad k = \frac{J_2}{J_1} \cdot \frac{s_1}{s_2};$$

$$H = \frac{P l_2^2}{8 h l} \cdot \frac{l + 4 l_1 (k + 1)}{(k + 1)};$$

$$M_x = V_A \cdot x - H \cdot \frac{h x}{l_1}; \quad M_B = V_A \cdot l_1 - H \cdot h;$$

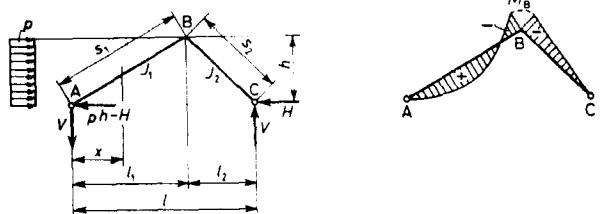
$$M_z = V_C \cdot z - H \cdot \frac{h z}{l_2} - \frac{P z^2}{2};$$

$$N_{AB} = -V_A \cdot \frac{h}{s_1} - H \cdot \frac{l_1}{s_1}; \quad N_z = -(V_C - P z) \frac{h}{s_2} - H \cdot \frac{l_2}{s_2}$$

segue

seguito tab. 8.10

40.



$$V = \frac{P h^2}{2 l}; \quad k = \frac{j_2}{j_1} \cdot \frac{s_1}{s_2};$$

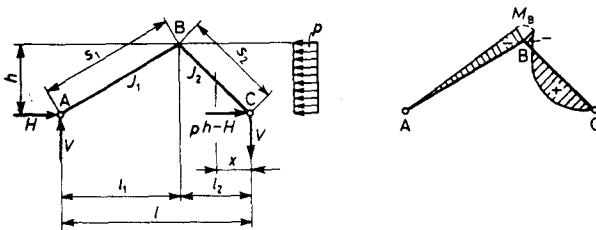
$$H = \frac{p \cdot h}{8 l} \cdot \frac{4 l_2 + (l + 4 l_2) k}{k + 1};$$

$$M_x = (p \cdot h - H) \frac{h x}{l_1} - V \cdot x - \frac{p \cdot h^2 x^2}{2 l_1^2}; \quad M_B = V l_2 - H \cdot h;$$

$$N_x = V \cdot \frac{h}{s_1} + \left(p \cdot h - H - \frac{p \cdot h \cdot x}{l_1} \right) \frac{l_1}{s_1};$$

$$N_{BC} = -V \cdot \frac{h}{s_2} - H \cdot \frac{l_2}{s_2}$$

41.



$$V = \frac{P h^2}{2 l}; \quad H = \frac{p \cdot h}{8 l} \cdot \frac{l + 4 l_1 (k + 1)}{k + 1}; \quad M_B = +V \cdot l_1 - H \cdot h;$$

$$k = \frac{j_2}{j_1} \cdot \frac{s_1}{s_2}; \quad M_x = (p \cdot h - H) \frac{h x}{l_2} - V \cdot x - \frac{p \cdot h^2 x^2}{2 l_2^2};$$

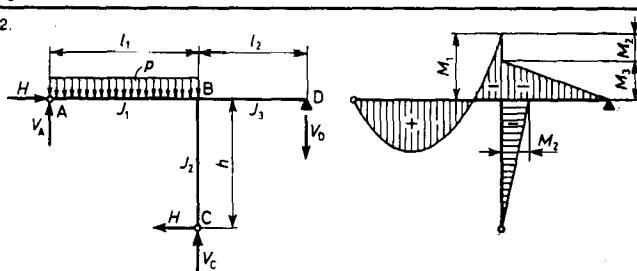
$$N_{AB} = -V \cdot \frac{h}{s_1} - H \cdot \frac{l_1}{s_1};$$

$$N_x = +V \cdot \frac{h}{s_2} + \left(p \cdot h - H - \frac{p \cdot h \cdot x}{l_2} \right) \frac{l_2}{s_2}$$

segue

seguito tab. 8.10

42.



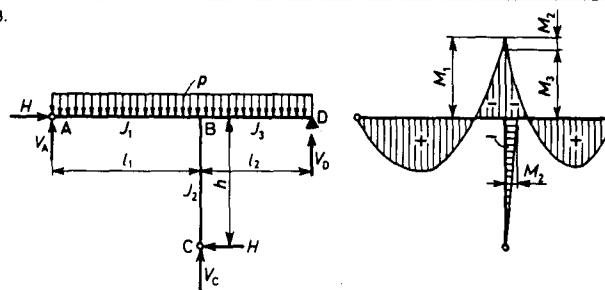
$$V_A = \frac{p l_1}{2} + \frac{M_1}{l}; \quad V_C = \frac{p l_1}{2} - \frac{M_1}{l_1} - \frac{M_2}{l_2};$$

$$V_D = -\frac{M_3}{l_2}; \quad k_1 = \frac{j_2}{j_1} \cdot \frac{l_1}{h}; \quad k_2 = \frac{j_2}{j_3} \cdot \frac{l_2}{h};$$

$$M_1 = -\frac{p l_1^2}{8} \cdot \frac{k_1 (k_2 + 1)}{k_1 + k_1 k_2 + k_2}; \quad M_3 = -\frac{p l_1^2}{8} \cdot \frac{k_1}{k_1 + k_1 k_2 + k_2};$$

$$M_2 = M_1 - M_3; \quad H = -\frac{M_2}{h}$$

43.



$$k_1 = \frac{j_2}{j_1} \cdot \frac{l_1}{h}; \quad k_2 = \frac{j_2}{j_3} \cdot \frac{l_2}{h}; \quad n = \frac{l_2}{l_1}$$

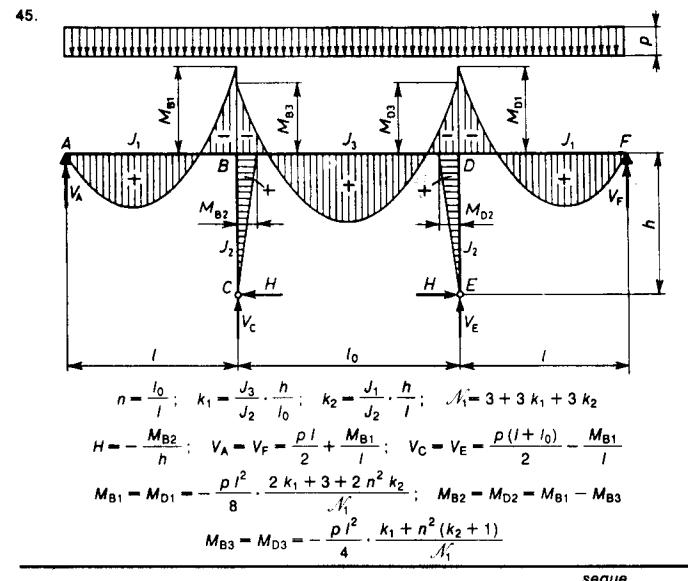
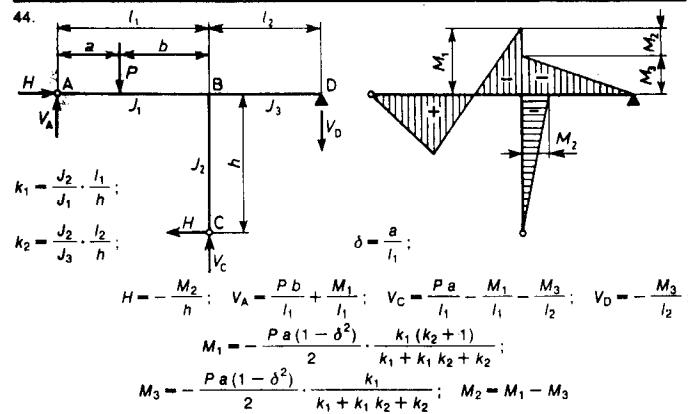
$$V_A = \frac{p l_1}{2} + \frac{M_1}{l_1}; \quad V_C = \frac{p (l_1 + l_2)}{2} - \frac{M_1}{l_1} - \frac{M_3}{l_2}; \quad V_D = \frac{p l_2}{2} + \frac{M_3}{l_2};$$

$$M_1 = -\frac{p l_1^2}{8} \cdot \frac{k_1 (k_2 + 1) + n^2 k_2}{k_1 + k_1 k_2 + k_2}; \quad M_3 = -\frac{p l_1^2}{8} \cdot \frac{k_1 + n^2 k_2 (k_1 + 1)}{k_1 + k_1 k_2 + k_2};$$

$$M_2 = M_1 - M_3; \quad H = -\frac{M_2}{h}$$

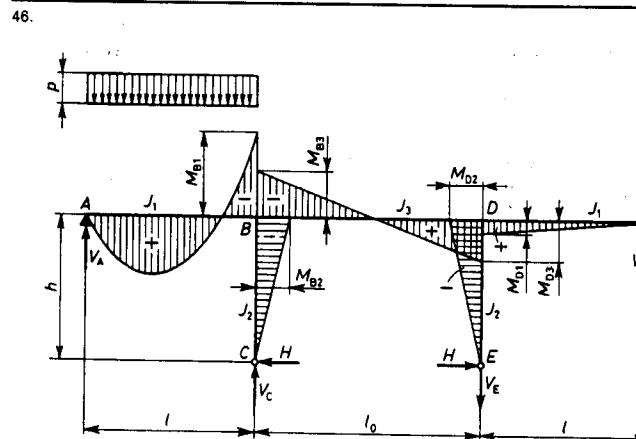
segue

seguito tab. 8.10



segue

seguito tab. 8.10



$$k_1 = \frac{J_3}{J_2} \cdot \frac{h}{l_0}; \quad k_2 = \frac{J_1}{J_2} \cdot \frac{h}{l}; \quad \mathcal{N}_1 = 3 + 2k_1 + 3k_2; \\ \mathcal{N}_2 = 1 + 2k_1 + k_2;$$

$$H_C = \frac{M_{B2}}{h}; \quad H_E = -\frac{M_{D2}}{h};$$

$$V_A = \frac{Pi}{2} + \frac{M_{B1}}{l}; \quad V_F = \frac{M_{D1}}{l};$$

$$V_C = \frac{Pi}{2} - \frac{M_{B1}}{l} + \frac{M_{D3} - M_{B3}}{l_0};$$

$$V_E = -\frac{M_{D3} - M_{B3}}{l_0} - \frac{M_{D1}}{l};$$

$$\left. \begin{array}{l} M_{B1} \\ M_{D1} \end{array} \right\} = \mp \frac{pi^2}{16} \left(\frac{2k_1 + 1}{\mathcal{N}_2} \pm \frac{2k_1 + 3}{\mathcal{N}_1} \right);$$

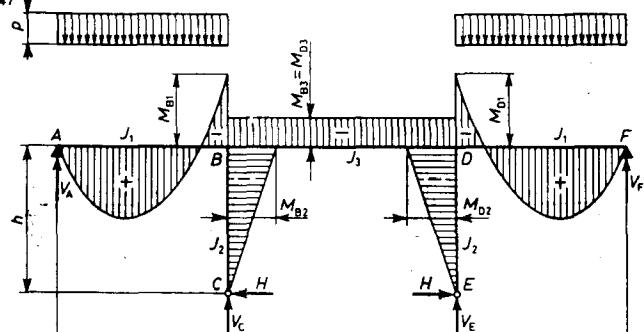
$$\left. \begin{array}{l} M_{B3} \\ M_{D3} \end{array} \right\} = \mp \frac{pi^2 k_1}{8} \left(\frac{1}{\mathcal{N}_2} \pm \frac{1}{\mathcal{N}_1} \right);$$

$$M_{B2} = M_{B1} - M_{B3}; \quad M_{D2} = M_{D1} - M_{D3}$$

segue

seguito tab. 8.10

47.

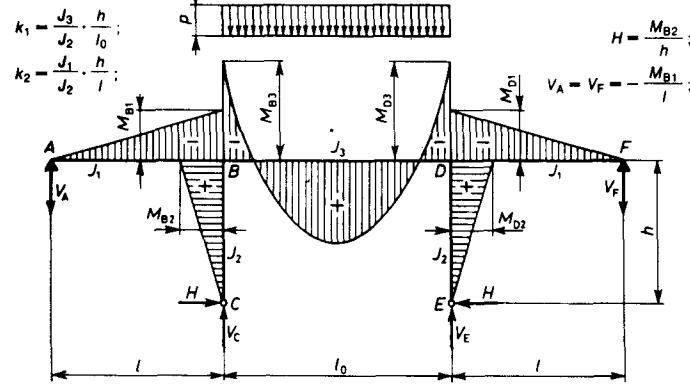


$$k_1 = \frac{J_3}{J_1} \cdot \frac{h}{l_0}; \quad k_2 = \frac{J_1}{J_2} \cdot \frac{h}{l}; \quad \mathcal{N}_1 = 3 + 2k_1 + 3k_2; \quad H = -\frac{M_{B2}}{h};$$

$$V_A = V_F = \frac{pI}{2} + \frac{M_{B1}}{l}; \quad V_C = V_E = \frac{pI}{2} - \frac{M_{B1}}{l}; \quad M_{B2} = M_{D2} = M_{B1} - M_{B3};$$

$$M_{B1} = M_{D1} = -\frac{pI^2}{8} \cdot \frac{2k_1 + 3}{k_1}; \quad M_{B3} = M_{D3} = -\frac{pI^2}{4} \cdot \frac{k_1}{J_1}.$$

48.



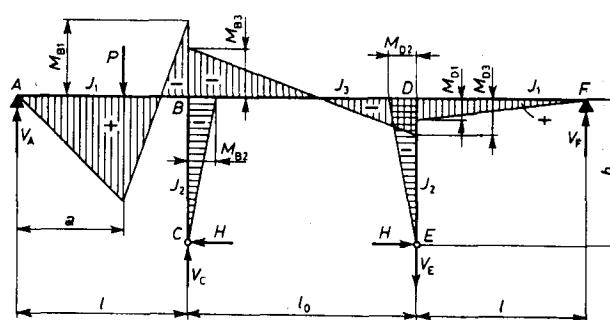
$$\mathcal{N}_1 = 3 + 2k_1 + 3k_2; \quad V_C = V_E = \frac{pI_0}{2} - \frac{M_{B1}}{l};$$

$$M_{B1} = M_{D1} = -\frac{pI_0^2k_2}{4J_1}; \quad M_{B2} = M_{D2} = -\frac{pI_0^2}{4J_1}; \quad M_{B3} = M_{D3} = -\frac{pI_0^2(k_2 + 1)}{4\mathcal{N}_1}$$

segue

seguito tab. 8.10

49.



$$k_1 = \frac{J_3}{J_2} \cdot \frac{h}{l_0}; \quad k_2 = \frac{J_1}{J_2} \cdot \frac{h}{l}; \quad \mathcal{N}_1 = 3 + 2k_1 + 3k_2;$$

$$\mathcal{N}_2 = 1 + 2k_1 + k_2; \quad \delta = \frac{a}{l};$$

$$H_C = -\frac{M_{B2}}{h}; \quad H_E = -\frac{M_{D2}}{h};$$

$$V_A = P(1 - \delta) + \frac{M_{B1}}{l}; \quad V_F = \frac{M_{D1}}{l};$$

$$V_C = P\delta - \frac{M_{B1}}{l} + \frac{M_{D3} - M_{B3}}{l_0};$$

$$V_E = \frac{M_{D3} - M_{B3}}{l_0} + \frac{M_{D1}}{l};$$

$$\left. \begin{array}{l} M_{B1} \\ M_{D1} \end{array} \right\} = \mp \frac{Pa(1 - \delta^2)}{4} \left(\frac{2k_1 + 1}{\mathcal{N}_2} \pm \frac{2k_1 + 3}{\mathcal{N}_1} \right);$$

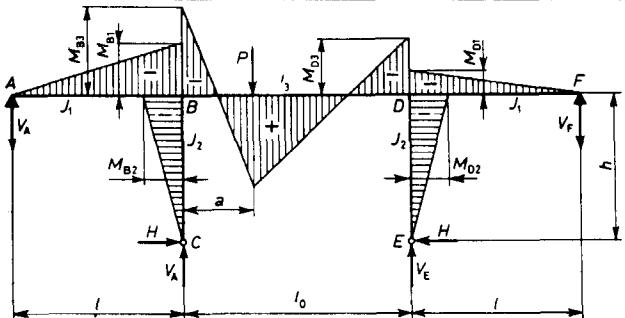
$$\left. \begin{array}{l} M_{B3} \\ M_{D3} \end{array} \right\} = \mp \frac{Pa(1 - \delta^2)k_1}{2} \left(\frac{1}{\mathcal{N}_2} \pm \frac{1}{\mathcal{N}_1} \right);$$

$$M_{B2} = M_{B1} - M_{B3}; \quad M_{D2} = M_{D1} - M_{D3}$$

segue

seguito tab. 8.10

50.



$$k_1 = \frac{J_3}{J_2} \cdot \frac{h}{l_0}; \quad k_2 = \frac{J_1}{J_2} \cdot \frac{h}{l}; \quad J'_1 = 3 + 2k_1 + 3k_2; \quad J'_2 = 1 + 2k_1 + k_2;$$

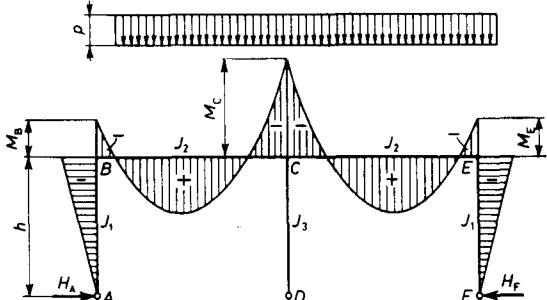
$$\delta = \frac{a}{l_0}; \quad H_C = -\frac{M_{B2}}{h}; \quad H_E = -\frac{M_{D2}}{h}; \quad V_A = -\frac{M_{B1}}{l};$$

$$V_C = P(1-\delta) - \frac{M_{B1}}{l} + \frac{M_{D3}-M_{B3}}{l_0}; \quad V_E = P\delta + \frac{M_{B3}-M_{D3}}{l_0} - \frac{M_{D1}}{l}; \quad V_F = -\frac{M_{D1}}{l};$$

$$\begin{aligned} M_{B1} &= -\frac{P a (1-\delta) k_2}{2} \left(\frac{3}{J_1} \pm \frac{1-2\delta}{J_2} \right); \quad M_{B3} = -\frac{P a (1-\delta) \cdot (k_2+1)}{2} \left(\frac{3}{J_1} \pm \frac{1-2\delta}{J_2} \right); \\ M_{D1} &= \end{aligned}$$

$$M_{B2} = M_{B3} - M_{B1}; \quad M_{D2} = M_{D3} - M_{D1}.$$

51.



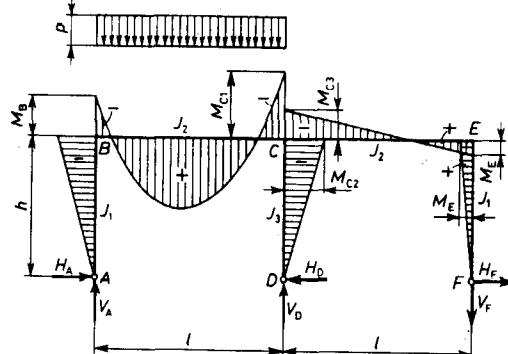
$$V_A = V_E = \frac{p l}{2} + \frac{M_C - M_B}{l}; \quad V_F = p l - \frac{2(M_C - M_B)}{l}; \quad k = \frac{J_2}{J_1} \cdot \frac{h}{l};$$

$$H = \frac{p l^2}{4(3+4k)h}; \quad M_B = M_D = -H \cdot h; \quad M_C = -\frac{p l^2}{4} \cdot \frac{1+2k}{3+4k}.$$

segue

seguito tab. 8.10

52.



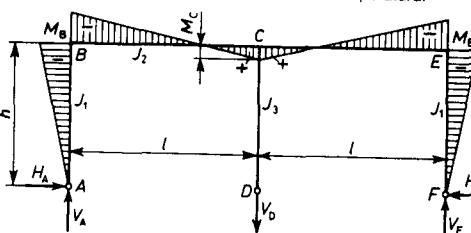
$$V_A = \frac{p l}{2} + \frac{M_C^s - M_B}{l}; \quad V_D = \frac{p l}{2} - \frac{M_C^s - M_B + M_C^d - M_E}{l}; \quad V_F = -\frac{M_E + M_C^d}{l};$$

$$k_1 = \frac{J_2}{J_1} \cdot \frac{h}{l}; \quad k_2 = \frac{J_2}{J_3} \cdot \frac{h}{l}; \quad H_1 = \frac{p l^2}{8h} \left(\frac{1}{3+k_1+2k_2} + \frac{1}{3+4k_1} \right);$$

$$H_2 = \frac{p l^2}{8h} \cdot \frac{2}{3+k_1+2k_2}; \quad H_3 = \frac{p l^2}{8h} \left(\frac{1}{3+k_1+2k_2} - \frac{1}{3+4k_1} \right);$$

$$M_B = -H_1 \cdot h; \quad M_C = -H_2 \cdot h; \quad M_E = +H_2 \cdot h;$$

$$M_C^s = -\frac{p l^2}{8} \left(\pm \frac{1}{3+k_1+2k_2} + \frac{1+2k_1}{3+4k_1} \right)$$

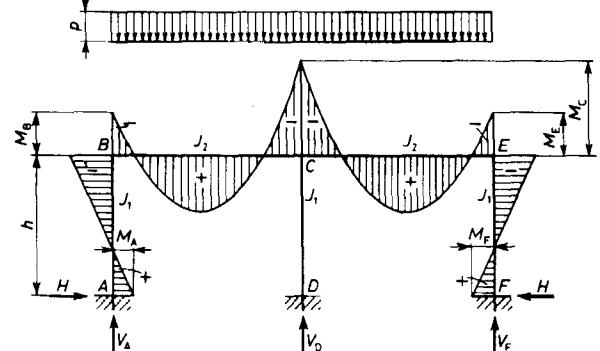
53. Diagramma dei momenti per un aumento Δt della temperatura.

$$k = \frac{J_2}{J_1} \cdot \frac{h}{l}; \quad H = \frac{12 \cdot E \cdot a \cdot \Delta t \cdot J_2}{h^2 (3+4k)}; \quad M_B = M_D = -H \cdot h; \quad M_C = +\frac{1}{2} M_B.$$

segue

seguito tab. 8.10

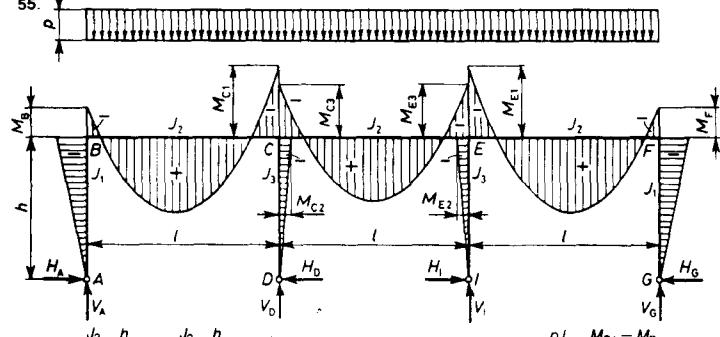
54.



$$V_A = V_E = \frac{p l}{2} + \frac{M_C - M_B}{l}; \quad V_F = p l - 2 \frac{M_C - M_B}{l}; \quad H = \frac{M_A + M_B}{h}; \quad k = \frac{j_2 \cdot h}{j_1 \cdot l};$$

$$M_A = M_E = \frac{p l^2}{24(1+k)}; \quad M_B = M_D = -\frac{p l^2}{12(1+k)}; \quad M_C = -\frac{2+3k}{3(1+k)} \cdot \frac{p l^2}{8}$$

55.



$$k_1 = \frac{j_2 \cdot h}{j_1 \cdot l}; \quad k_2 = \frac{j_2 \cdot h}{j_3 \cdot l}; \quad l = 12k_1 + 20k_2 + 18k_3; \quad V_A = V_G = \frac{p l}{2} + \frac{M_{C1} - M_B}{l};$$

$$V_D = V_E = p l + \frac{M_B - M_{C1}}{l}; \quad H_A = H_G = -\frac{M_B}{h}; \quad H_D = H_E = -\frac{M_{C2}}{h};$$

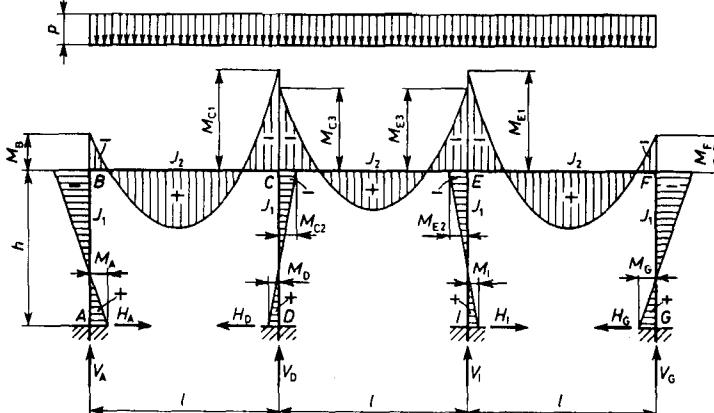
$$M_B = M_F = -\frac{p l^2}{4} \cdot \frac{6k_2 + 3}{l^3}; \quad M_{C1} = M_{E1} = -\frac{p l^2}{4} \cdot \frac{6k_1 + 8k_1k_2 + 2k_2 + 7}{l^3};$$

$$M_{C3} = M_{E3} = -\frac{p l^2}{4} \cdot \frac{4k_1 + 8k_1k_2 + 6k_2 + 3}{l^3}; \quad M_{C2} = M_{E2} = M_{C1} - M_{C3}$$

segue

seguito tab. 8.10

56.



$$k = \frac{j_2 \cdot h}{j_1 \cdot l}; \quad l = 3(4 + 10k^2 + k^2);$$

$$V_A = V_G = \frac{p l}{2} + \frac{M_{C1} - M_B}{l};$$

$$V_D = V_E = p l + \frac{M_B + M_{E2}}{l} - \frac{M_{C1} + M_{C3}}{l};$$

$$H_A = H_G = \frac{M_A + M_B}{h}; \quad H_D = H_E = \frac{M_D + M_{C2}}{h};$$

$$M_A = M_G = +\frac{p l^2}{2} \cdot \frac{2+3k}{l^3};$$

$$M_B = M_F = -\frac{p l^2}{2} \cdot \frac{2+3k}{l^3};$$

$$M_{C1} = M_{E1} = -\frac{p l^2}{2} \cdot \frac{2+6k+3k^2}{l^3};$$

$$M_{C3} = M_{E3} = -\frac{p l^2}{2} \cdot \frac{2+5k+3k^2}{l^3};$$

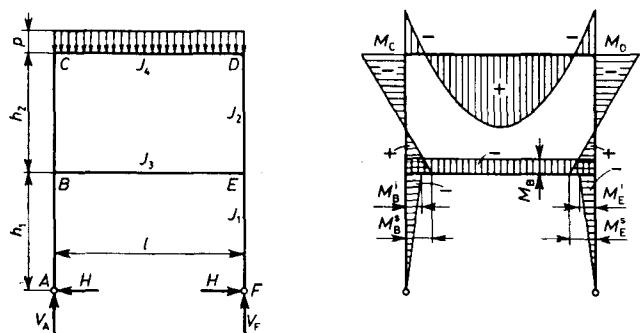
$$M_{C2} = M_{E2} = M_{C1} - M_{C3};$$

$$M_D = M_I = \frac{M_{C2}}{2}$$

segue

seguito tab. 8.10

57.



$$V_A = V_F = \frac{p l}{2}; \quad k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{l}; \quad k_2 = \frac{J_4}{J_2} \cdot \frac{h_2}{l};$$

$$k_3 = \frac{J_3}{J_2} \cdot \frac{h_3}{l}; \quad H = \frac{M_B^i}{h_1};$$

$$M_B^i = M_E^i = -\frac{p l^2}{4} \cdot \frac{k_3}{2 k_1 (3 + 2 k_2) + k_3 (2 + k_2) (3 + 2 k_1)};$$

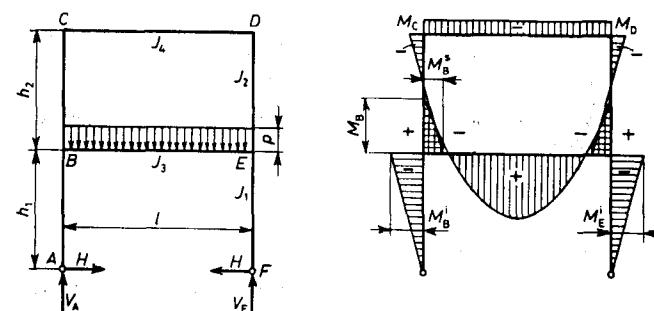
$$M_B = M_E = -\frac{p l^2}{12} \cdot \frac{k_1 k_3}{2 k_1 (3 + 2 k_2) + k_3 (2 + k_2) (3 + 2 k_1)};$$

$$M_B^s = M_E^s = +\frac{p l^2}{12} \cdot \frac{k_3 (3 + 2 k_1)}{2 k_1 (3 + 2 k_2) + k_3 (2 + k_2) (3 + 2 k_1)};$$

$$M_C = M_D = -\frac{p l^2}{6} \cdot \frac{k_1 (3 + 2 k_3) + 3 k_2}{2 k_1 (3 + 2 k_2) + k_3 (2 + k_2) (3 + 2 k_1)}$$

seguito tab. 8.10

58.



$$V_A = V_F = \frac{p l}{2}; \quad k_1 = \frac{J_2}{J_1} \cdot \frac{h_1}{l}; \quad k_2 = \frac{J_4}{J_2} \cdot \frac{h_2}{l};$$

$$k_3 = \frac{J_3}{J_2} \cdot \frac{h_3}{l}; \quad H = -\frac{M_B^i}{h_1};$$

$$M_B^i = M_E^i = -\frac{p l^2}{4} \cdot \frac{k_3 (2 + k_2)}{2 k_1 (3 + 2 k_2) + k_3 (2 + k_2) (3 + 2 k_1)};$$

$$M_B = M_E = -\frac{p l^2}{6} \cdot \frac{k_1 k_3}{2 k_1 (3 + 2 k_2) + k_2 (2 + k_2) (3 + 2 k_1)};$$

$$M_B^s = M_E^s = \frac{p l^2}{12} \cdot \frac{k_3 (3 + 2 k_1)}{2 k_1 (3 + 2 k_2) + k_3 (2 + k_2) (3 + 2 k_1)};$$

$$M_C = M_D = -\frac{p l^2}{6} \cdot \frac{k_1 (3 + 2 k_3) + 3 k_2}{2 k_1 (3 + 2 k_2) + k_3 (2 + k_2) (3 + 2 k_1)}$$

segue