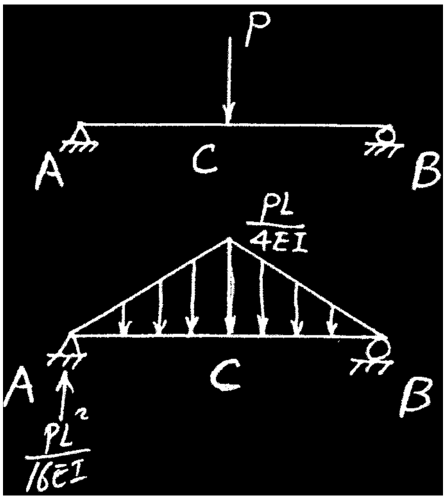
Example 6

$$\theta_A = -\underline{V}_A = -\frac{1}{2}\,\frac{L}{2}\,\frac{PL}{4EI} = -\frac{PL^2}{16EI} \qquad (\circlearrowright)$$

$$\delta_C = -\underline{M}_C = -\left(\frac{PL^2}{16EI}\,\frac{L}{2} - \frac{1}{2}\,\frac{L}{2}\,\frac{PL}{4EI}\,\frac{L}{6}\right)$$

$$= -\frac{PL^3}{EI}\left(\frac{1}{32} - \frac{1}{96}\right) = -\frac{PL^3}{48EI} \qquad (\downarrow)$$

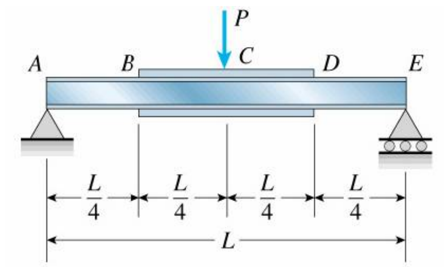


9.7 Nonprismatic Beam

$EI \neq$ constant

Example 9-13

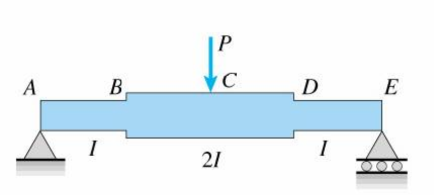
a beam *ABCDE* is supported a concentrated load *P* at midspan as shown



(a)

$$I_{BD} = 2\,I_{AB} = 2\,I_{DE} = 2\,I$$

determine the deflection curve, θ_A and δ_C

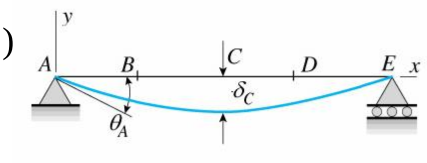


(b)

$$M = \frac{Px}{2} \qquad \left(0 \leqq x \leqq \frac{L}{2}\right)$$

then $\quad EIv'' = Px/2 \qquad (0 \leqq x \leqq L/4)$

$$E(2I)v'' = Px/2 \qquad (L/4 \leqq x \leqq L/2)$$



(c)

thus $\quad v' = \dfrac{Px^2}{4EI} + C_1 \quad (0 \leqq x \leqq L/4)$

$$v' = \frac{Px^2}{8EI} + C_2 \quad (L/4 \leqq x \leqq L/2)$$

$\because \quad v' = 0 \quad$ at $\quad x = L/2 \quad$ (symmetric)

$$\therefore \quad C_2 = -\frac{PL^2}{32EI}$$

continuity condition $\quad v'(L/4)^- = v'(L/4)^+$

$$C_1 = -\frac{5PL^2}{128EI}$$

therefore $\quad v' = -\dfrac{P}{128EI}(5L^2 - 32x^2) \qquad (0 \leqq x \leqq L/4)$

$$v' = -\frac{P}{32EI}(L^2 - 4x^2) \qquad (L/4 \leqq x \leqq L/2)$$

the angle of rotation θ_A is

$$\theta_A = v'(0) = -\frac{5PL^2}{128EI} \qquad (\circlearrowright)$$

integrating the slope equation and obtained

$$v = -\frac{P}{128EI}\left(5L^2x - \frac{32x^3}{3}\right) + C_3 \qquad (0 \leqq x \leqq L/4)$$

$$v = -\frac{P}{32EI}\left(L^2x - \frac{4x^3}{3}\right) + C_4 \qquad (L/4 \leqq x \leqq L/2)$$

boundary condition $v(0) = 0$

we get $C_3 = 0$

continuity condition $v(L/4)^- = v(L/4)^+$

we get $C_4 = - \frac{PL^3}{768EI}$

therefore the deflection equations are

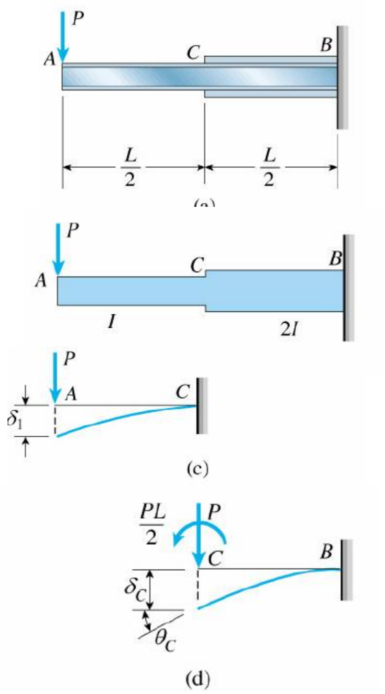
$$v = - \frac{Px}{384EI} (15L^2 - 32x^2) \qquad (0 \leqq x \leqq L/4)$$

$$v = - \frac{P}{768EI} (L^3 + 24L^2x - 32x^3) \qquad (L/4 \leqq x \leqq L/2)$$

the midpoint deflection is obtained

$$\delta_C = - v(L/2) = \frac{3PL^3}{256EI} \qquad (\downarrow)$$

moment-area method and conjugate beam methods can also be used



Example 9-14

a cantilever beam ABC supports a concentrated load P at the free end

$I_{BC} = 2 I_{AB} = 2 I$

determine δ_A

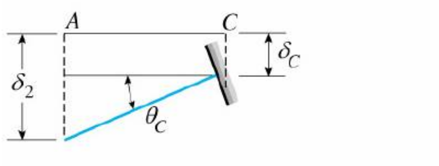
denote δ_1 the deflection of A due to C fixed

$$\delta_1 = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\text{and} \quad \delta_C = \frac{P(L/2)^3}{3E(2I)} + \frac{(PL/2)(L/2)^2}{2E(2I)} = \frac{5PL^3}{96EI}$$

$$\theta_C = \frac{P(L/2)^2}{2E(2I)} + \frac{(PL/2)(L/2)}{E(2I)} = \frac{PL^2}{16EI}$$

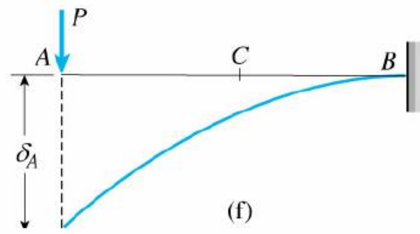
addition deflection at A due to δ_C and θ_C



(e)

$$\delta_2 = \delta_C + \theta_C \frac{L}{2} = \frac{5PL^3}{48EI}$$

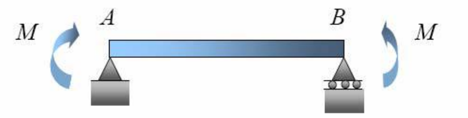
$$\delta_A = \delta_1 + \delta_2 = \frac{5PL^3}{16EI}$$



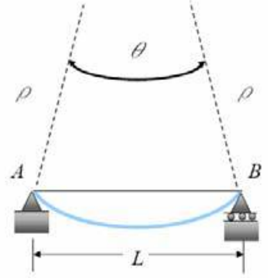
(f)

moment-area method and conjugate beam methods can also be used

9.8 Strain Energy of Bending

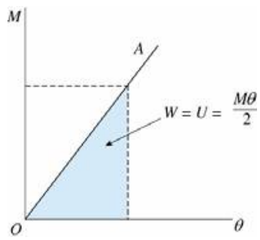


consider a simple beam AB subjected to pure bending under the action of two couples M



the angle θ is

$$\theta = \frac{L}{\rho} = \kappa L = \frac{ML}{EI}$$



if the material is linear elastic, M and θ has linear relation, then

$$W = U = \frac{M\theta}{2} = \frac{M^2L}{2EI} = \frac{EI\theta^2}{2L}$$