36th Conference of Slovak Society of Chemical Engineering
May 25 - 29, 2009 Tatranské Matliare

SHAPE AND RISING VELOCITY OF BUBBLES

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Everyting is known about bubbles

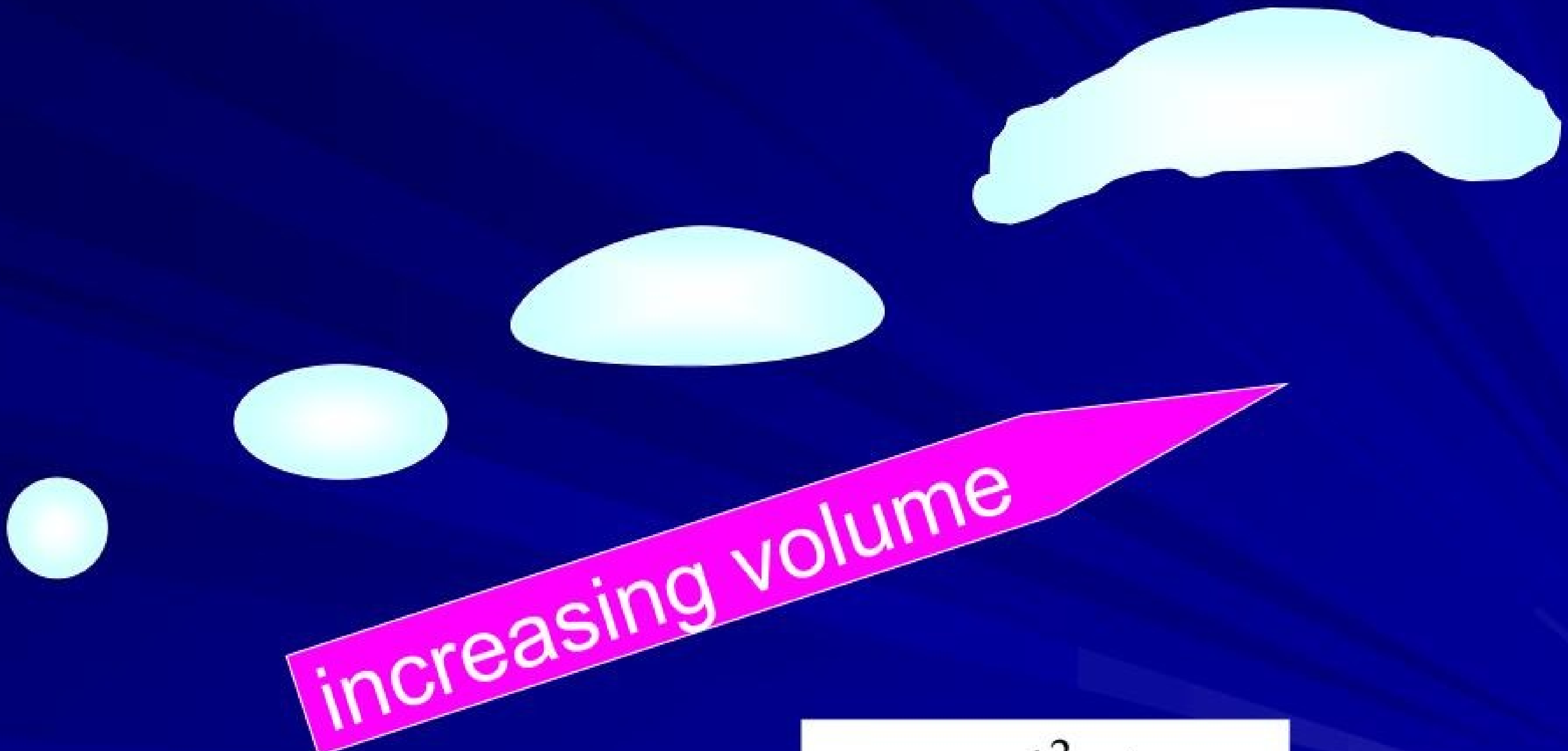
Clift, R., Grace, J.R. and Weber, M.E., 1978: *Bubbles, Drops, and Particles*. Academic Press, New York.

?

Classes of bubbles

- Small bubbles in high-viscosity liquids
- Small bubbles in low-viscosity liquids
- Medium bubbles in high-viscosity liquids
- Medium bubbles in pure low-viscosity liquids
- Medium bubbles in contamined low-viscosity liquids
- Large bubbles

SIZE OF RISING BUBBLES



$$Eo \equiv \frac{d_B^2 \; \Delta\rho \; g}{\sigma}$$

LIQUID VISCOSITY

Low– and medium-viscosity

…. $Re > 50$

$$Re \equiv \frac{d_B \, U_B \, \rho}{\mu}$$

PURE OR CONTAMINED LIQUID

- **Pure liquid** (surface-active components carefully removed)
  - Mobil surface
  - Lower drag resistance
  - Higher rising velocity
- **Contamined liquid**
  - Immobile surface
  - Drag like for solid bodies
  - Lower rising velocity

NO QUANTITATIVE PARAMETER !!!

Focus of our interest

- Small bubbles

  EASY EXPERIMENTS, SIMPLE THEORY
  Stokes (or Hadamard-Rybczinski) -law

- Medium bubbles in pure low-viscosity liquids

  COMPLICATED EXPERIMENTS

- Medium bubbles in contamined low-viscosity liquids

  ELLIPSOIDAL BUBBLES,
  THE MOST COMMON IN BUBBLE COLUMNS

- Large bubbles

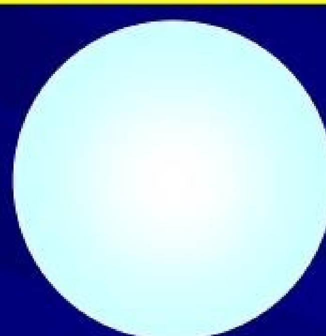
  PULSATING BUBBLE SHAPE AND
  VELOCITY, FREQUENT BREAKUP

LOWER DRAG (hypothetical)

Why not ??

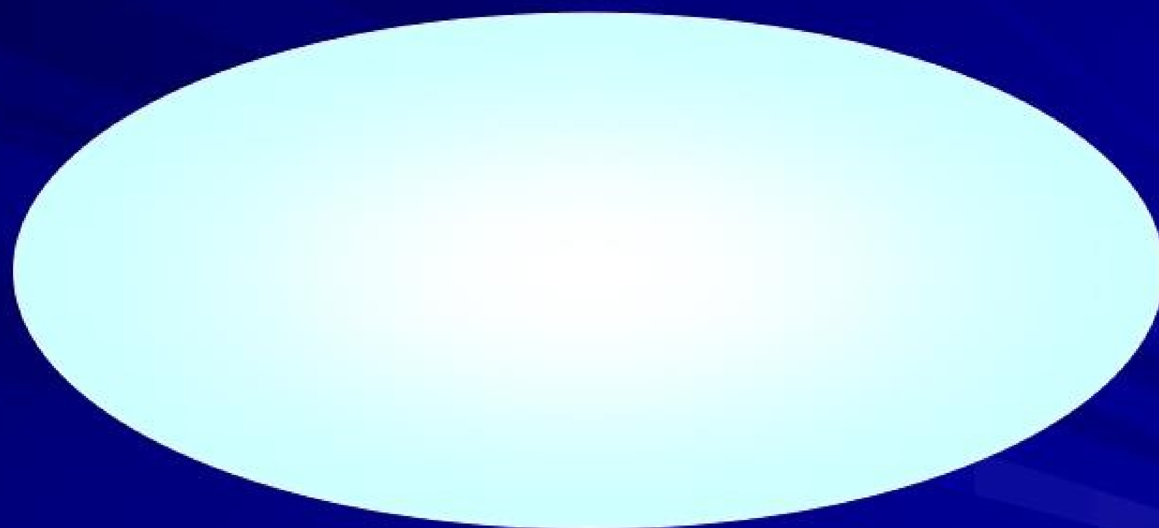
BUBBLE SHAPE

Surface tension



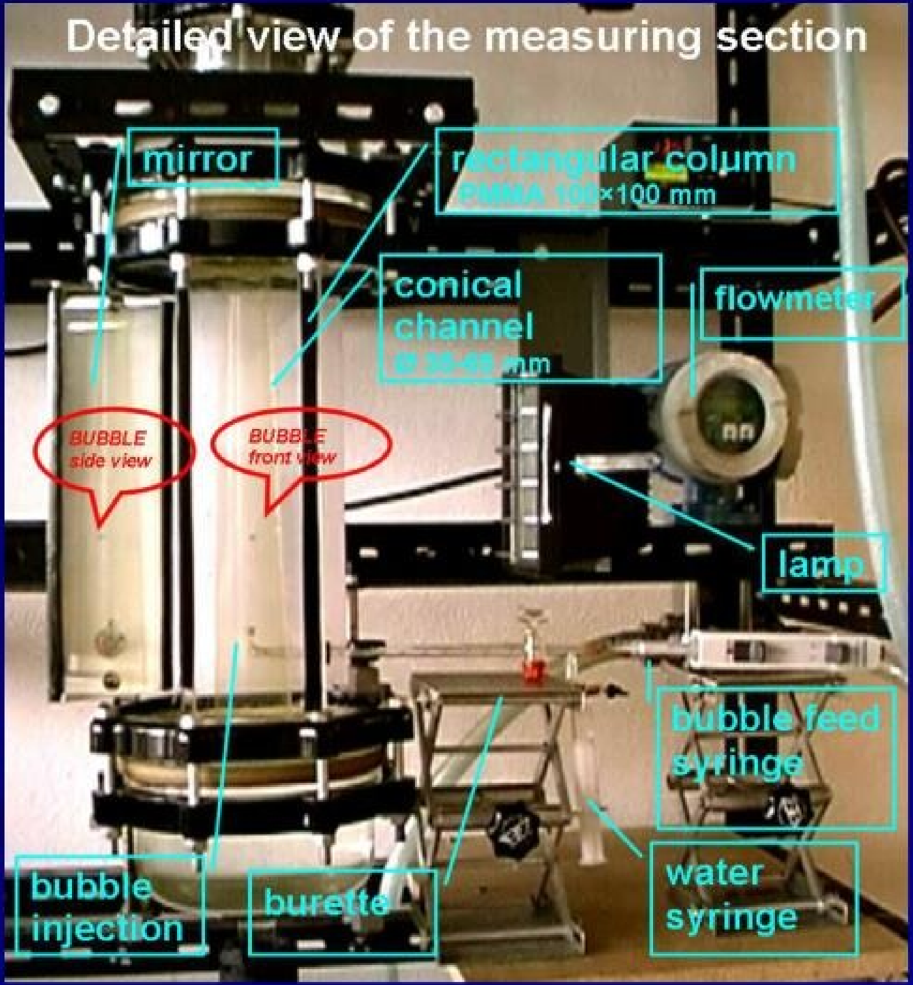
Hydrostatics

SYMMETRIC OBLATE BUBBLE



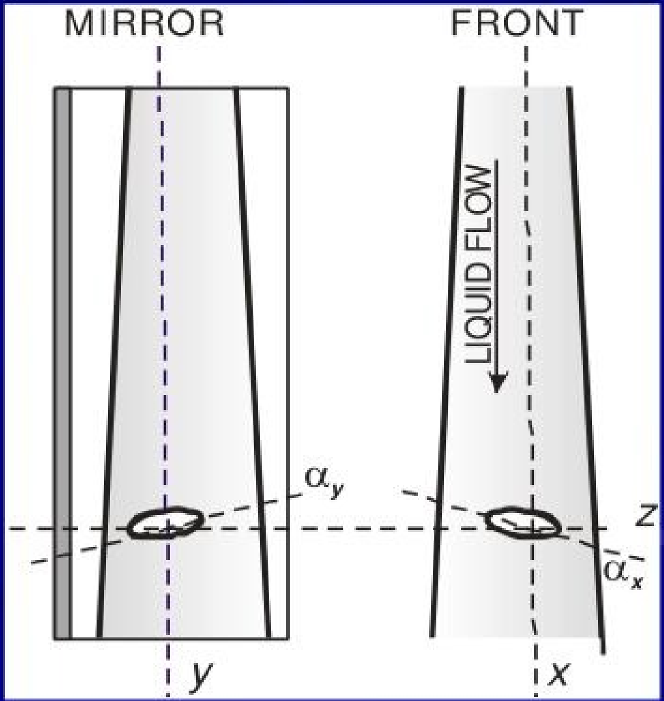

Our laboratories

DETERMINATION OF THE SHAPE OF BUBBLES

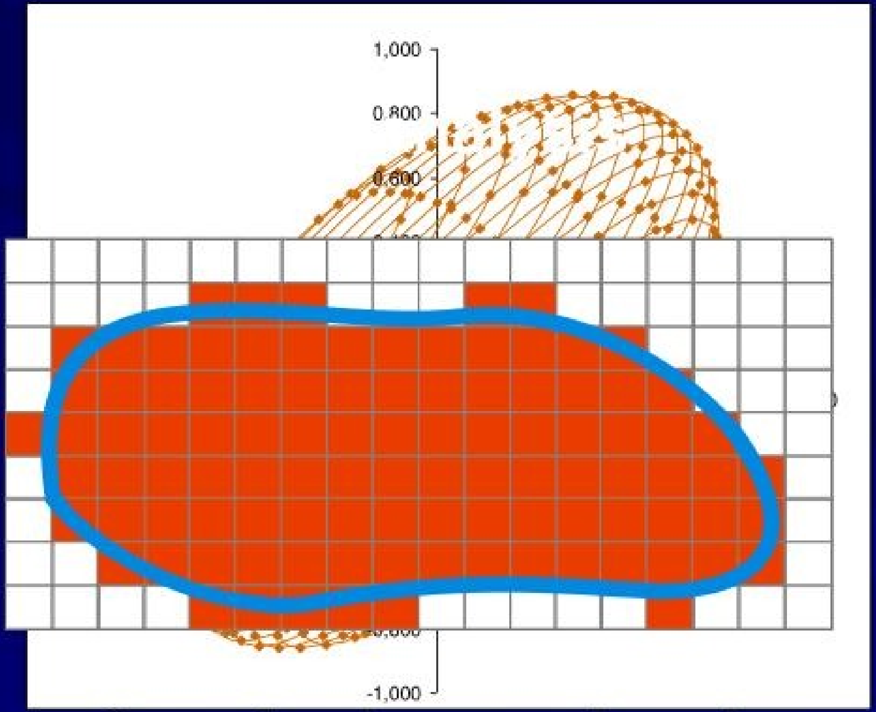


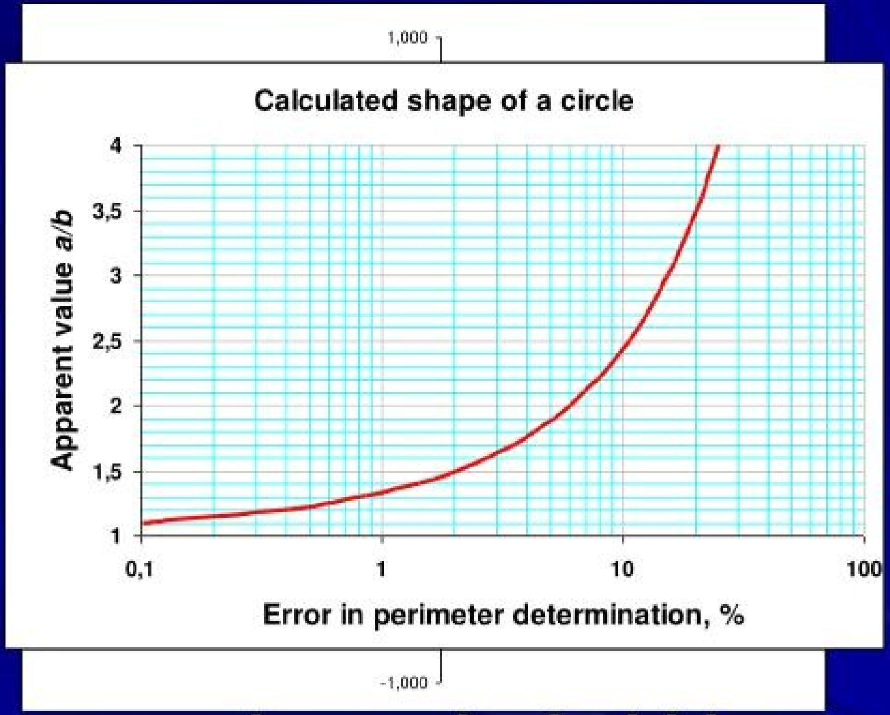
Bubble levitating

in dowstream flow



IDEAL OBLATE ELLIPSOID
semiaxes *a, b*

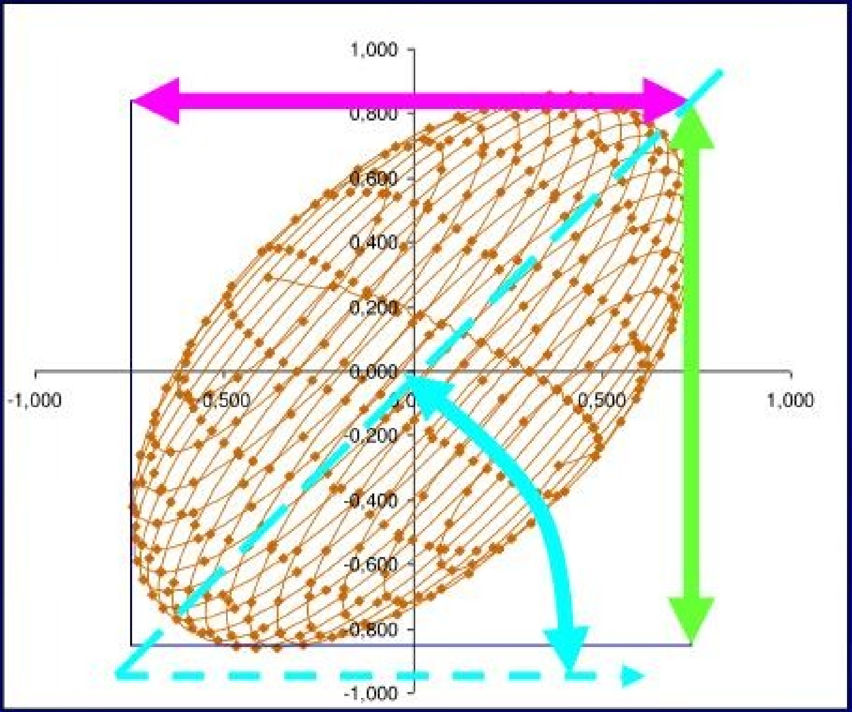




Standard procedure how to approximate the bubble:

- **Determination of the projected bubble area**
- **Determination of perimeter (usually overestimated)**

IDEAL OBLATE ELLIPSOID
semiaxes *a, b*

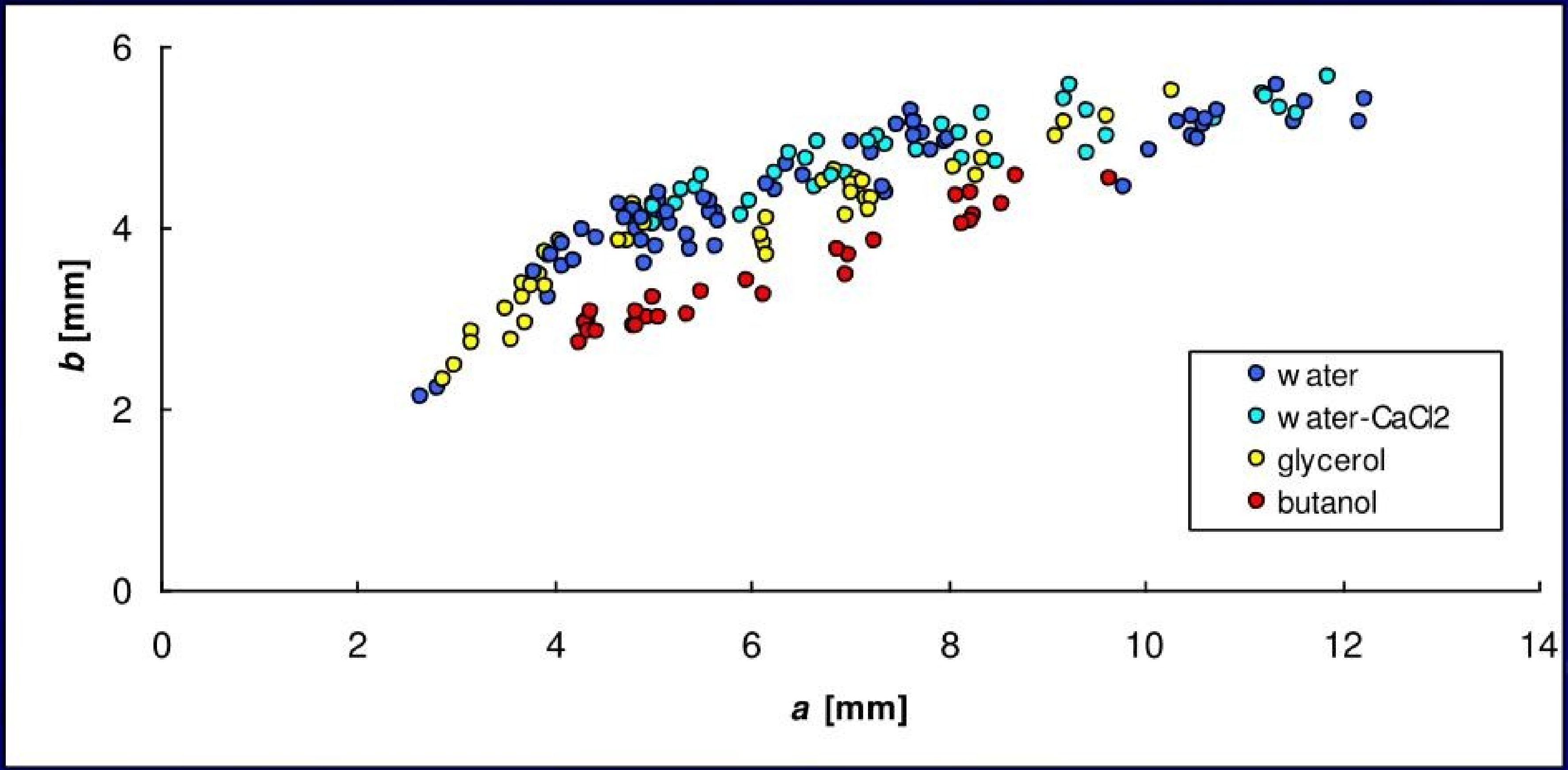


Improved procedure:

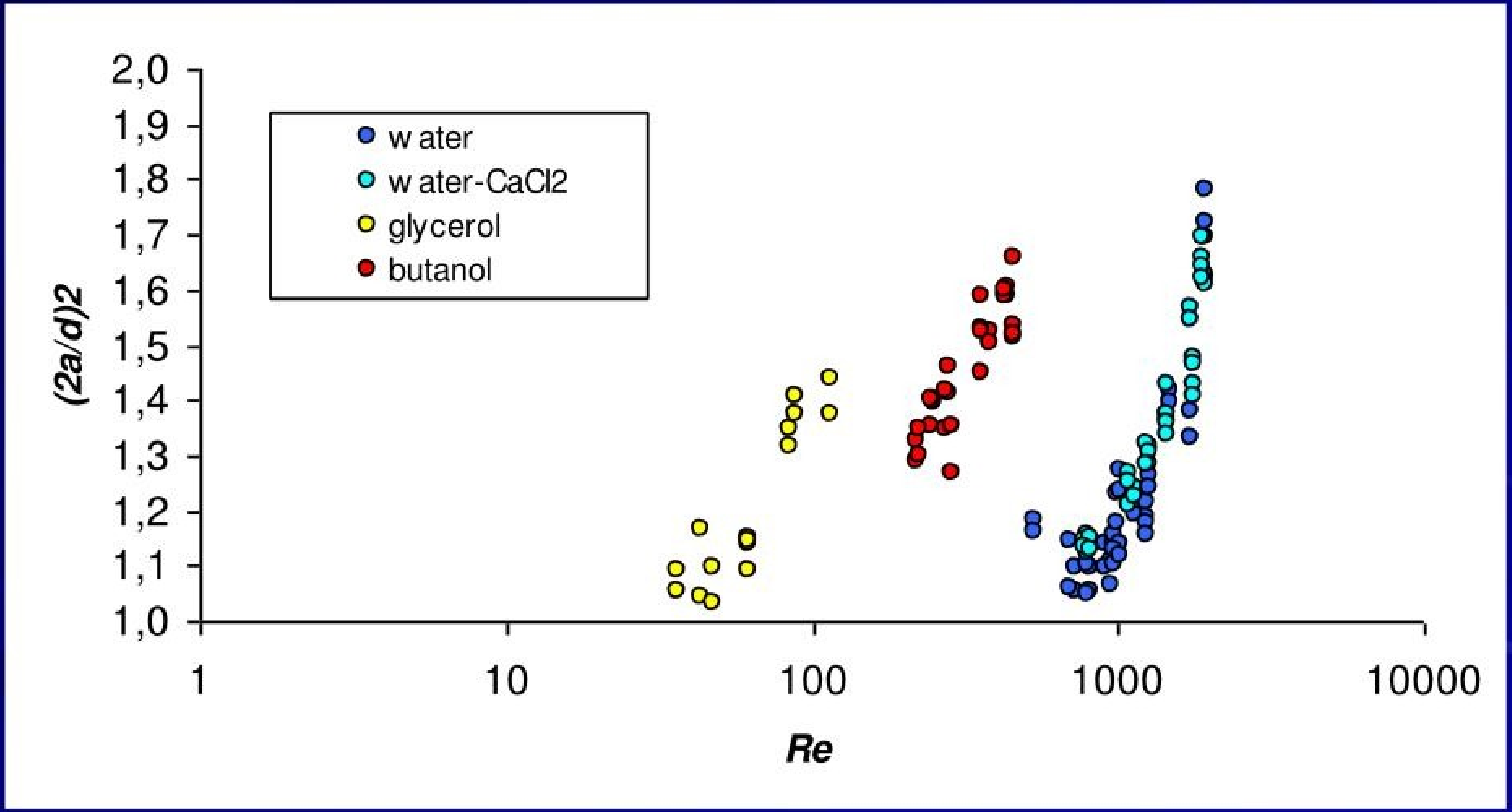
Determination of the object width, height and inclination

- **In front view**
- **In side view**

Untreated data

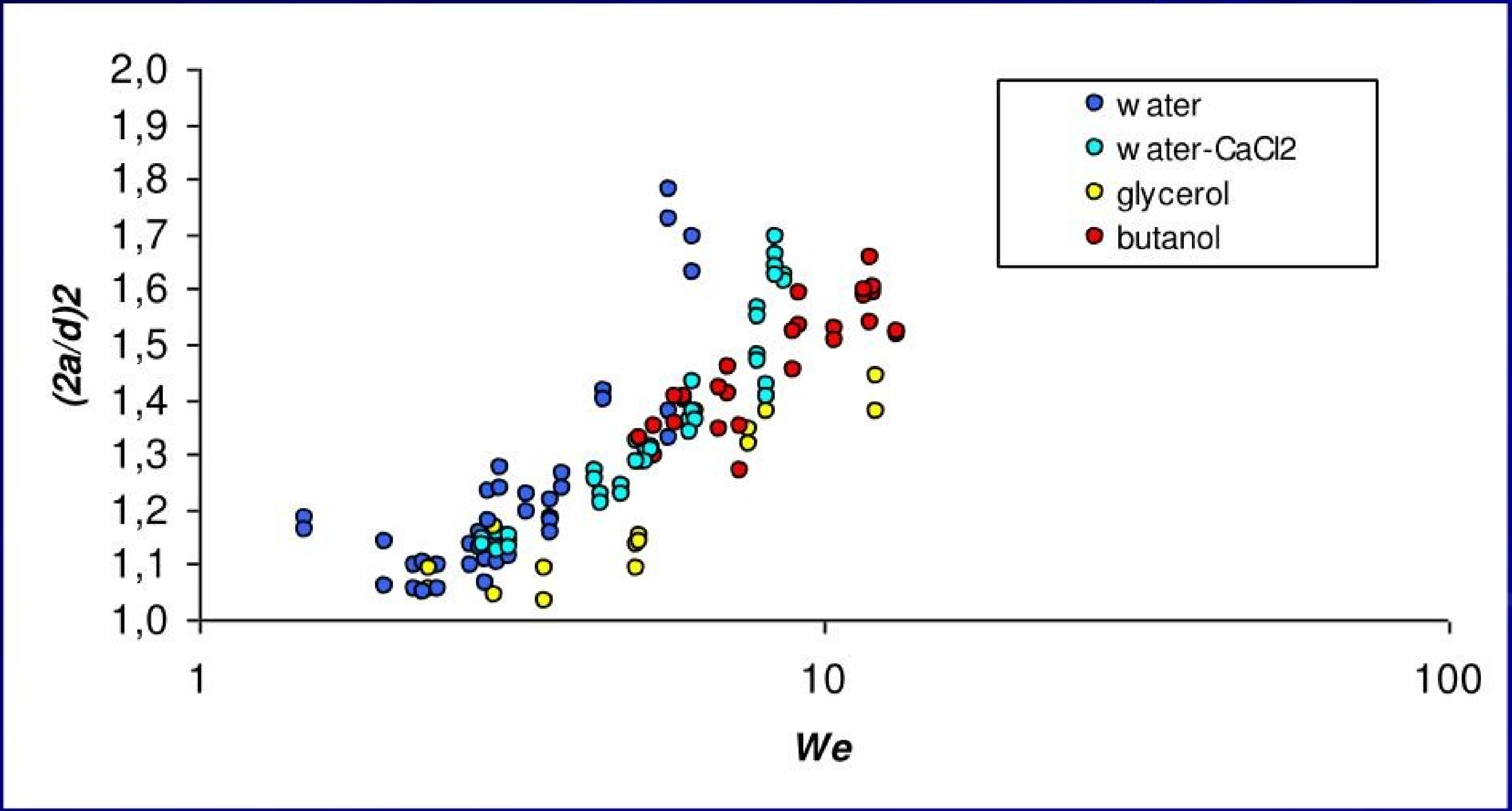


Reynolds number ?



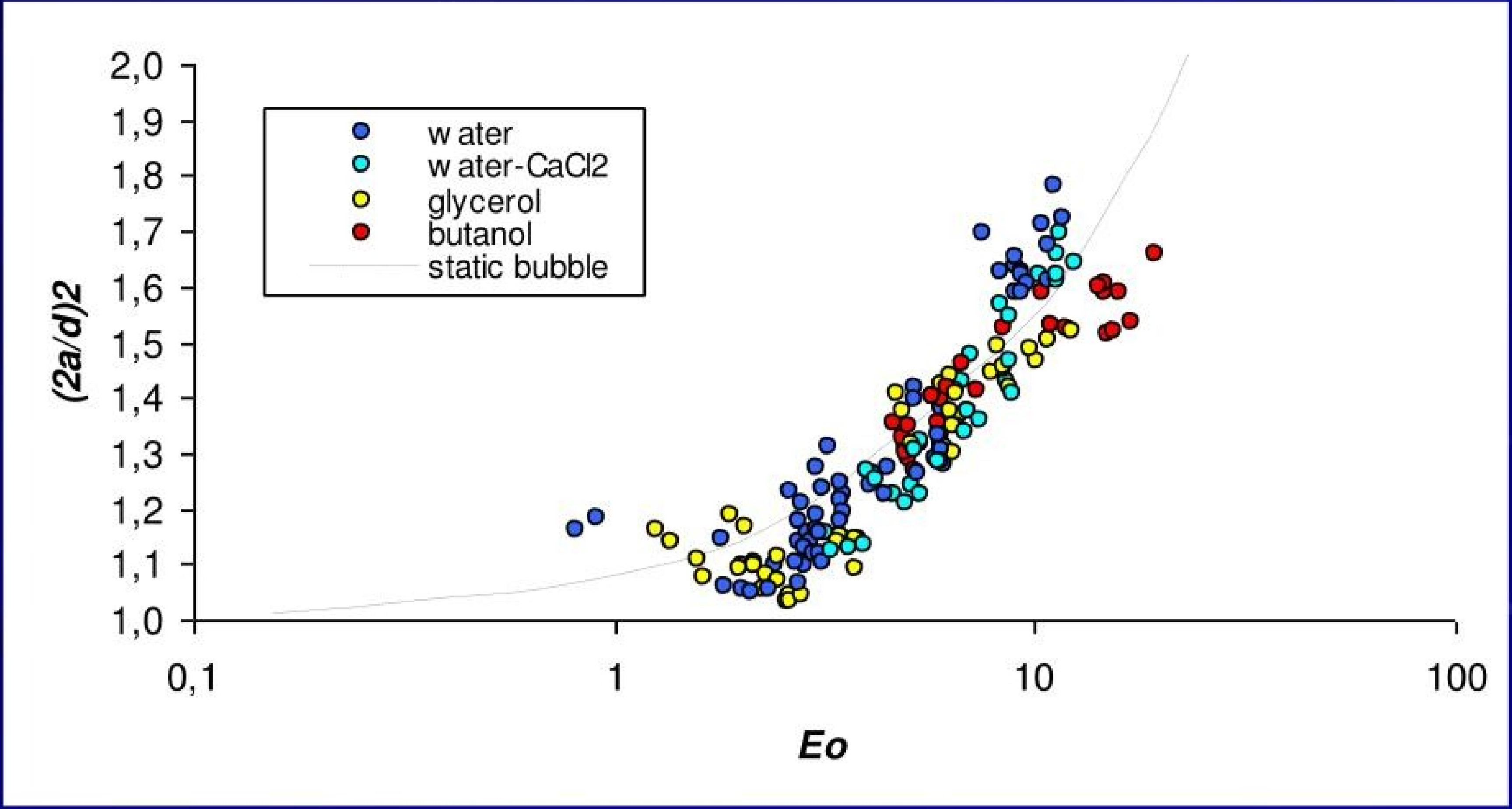
Weber number ?

$$We \equiv \frac{d_B\ u_B^2\ \rho_L}{\sigma}$$

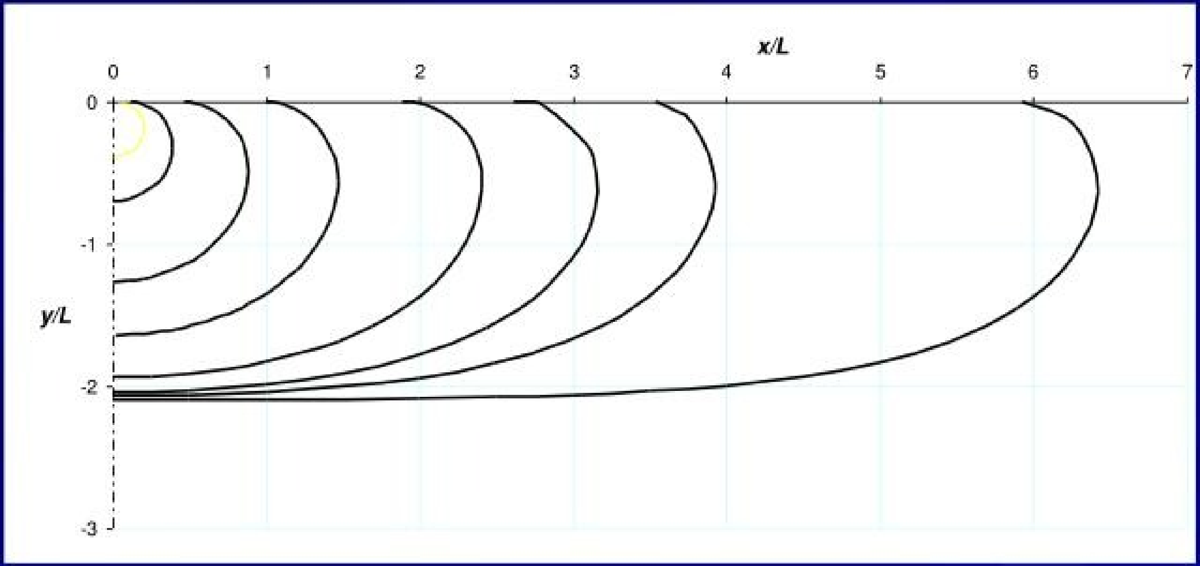


Eötvös number

$$Eo \equiv \frac{d_B^2 \, \Delta\rho \, g}{\sigma}$$



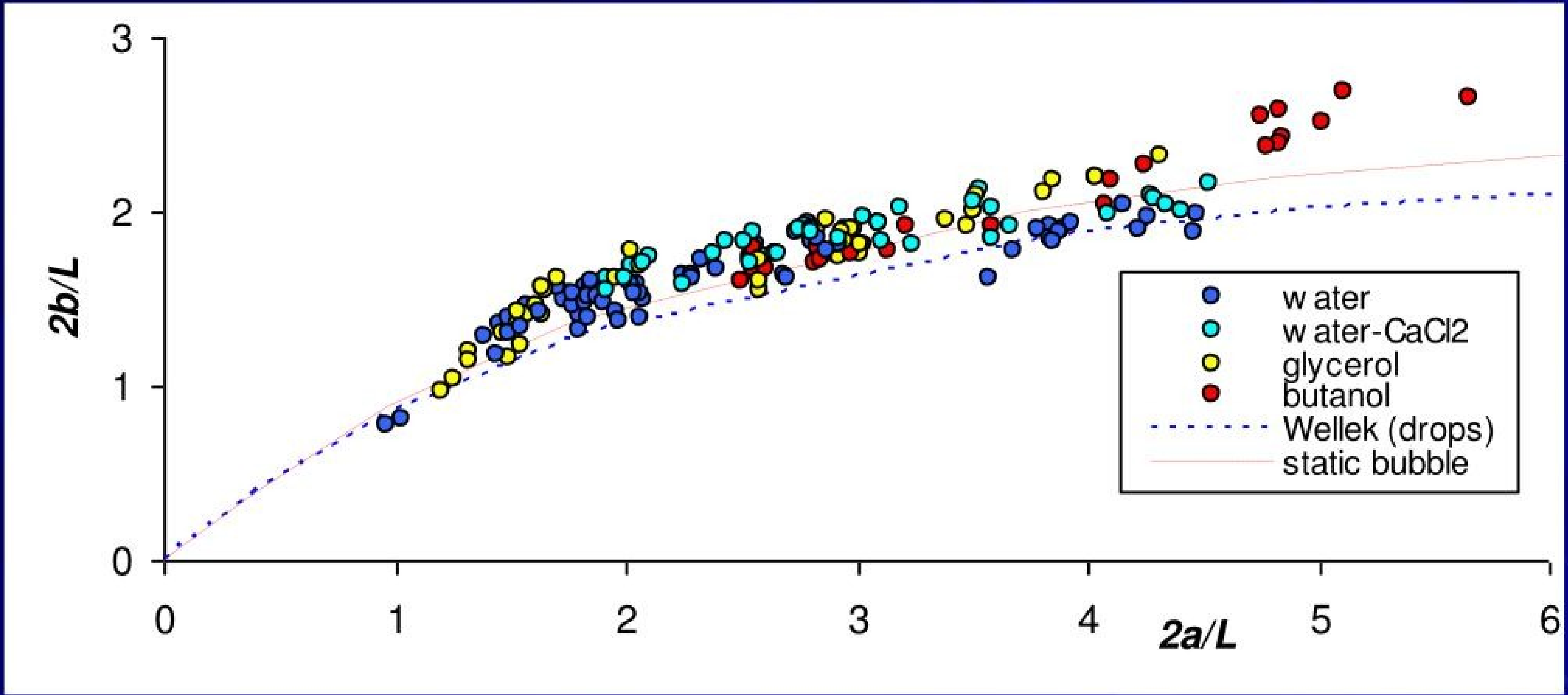
Static bubble profile (under a wetted plate)



$$L \equiv \sqrt{\frac{\sigma}{\Delta\rho\, g}}$$

„Laplace length!“

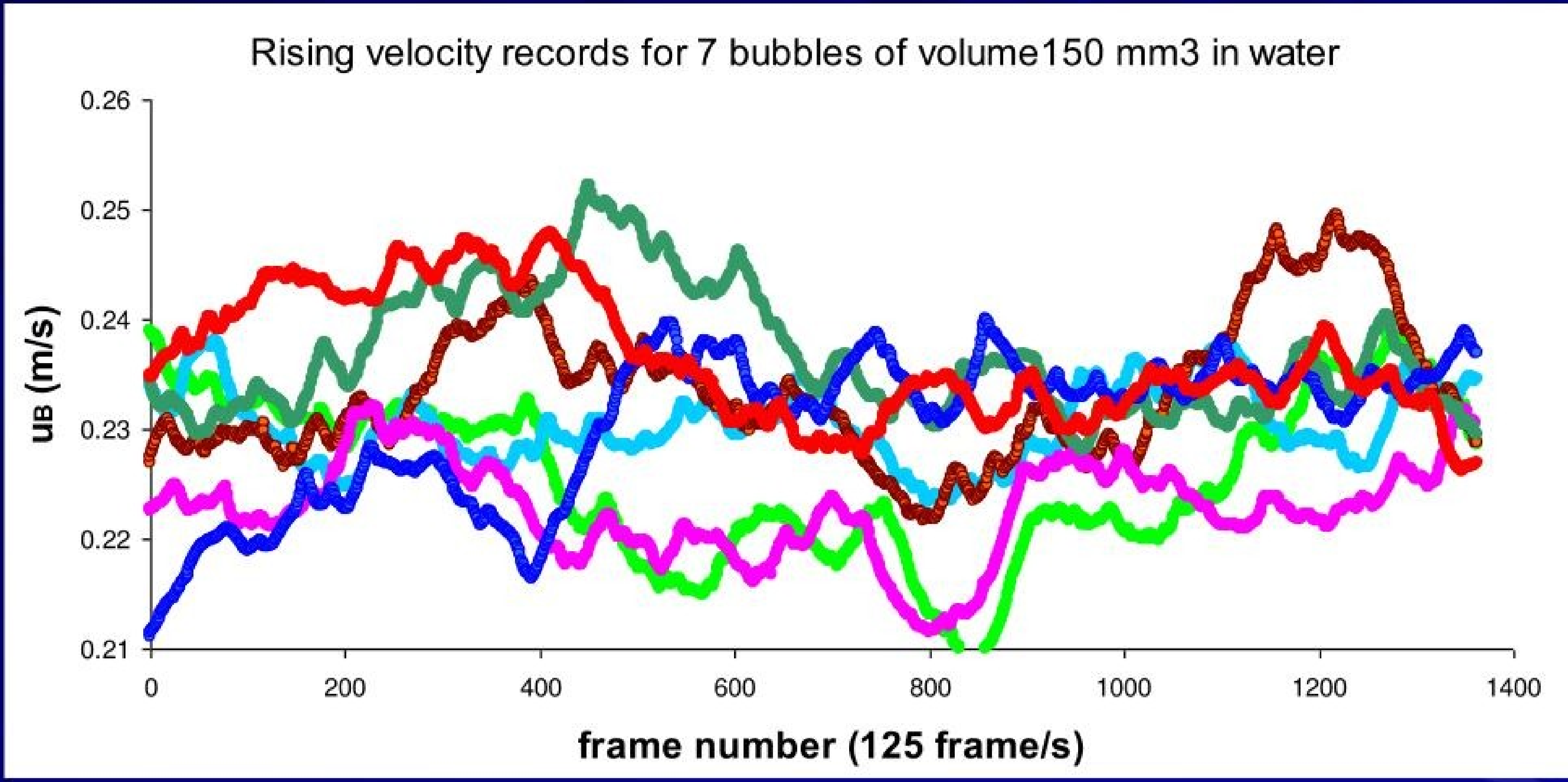
Normalized semiaxes of rising bubbles



Theoretical prediction for static bubbles

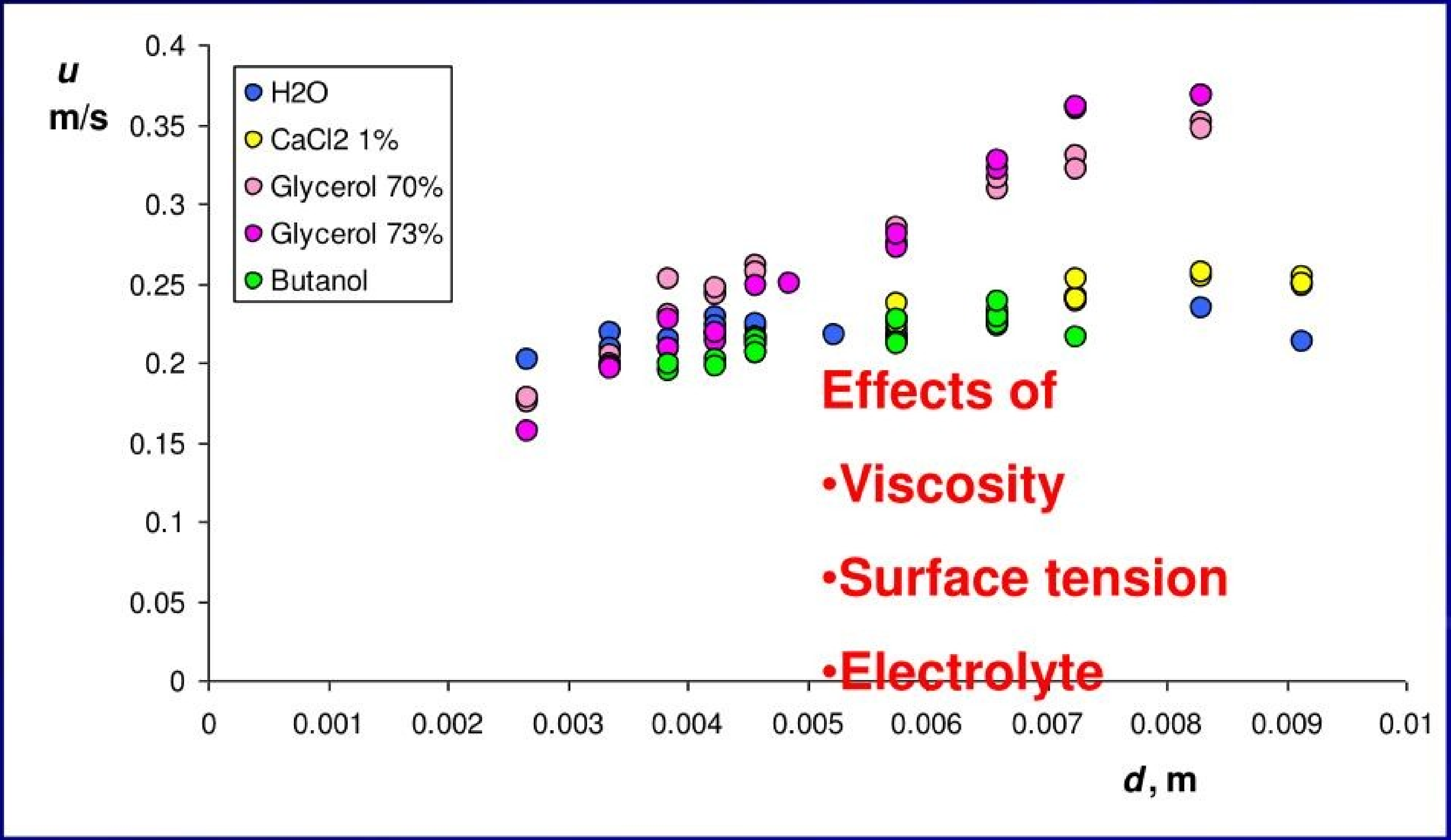
$$\left(\frac{2a}{d}\right)^2 = 1 + 0.095 Eo^{0.75} \quad \text{for} \quad Eo < 20$$

RISING VELOCITY



Time = 10 s , (corresponding path 2,3 m)

RISING VELOCITY
for medium contaminated bubbles



Our data compared with that of Clift

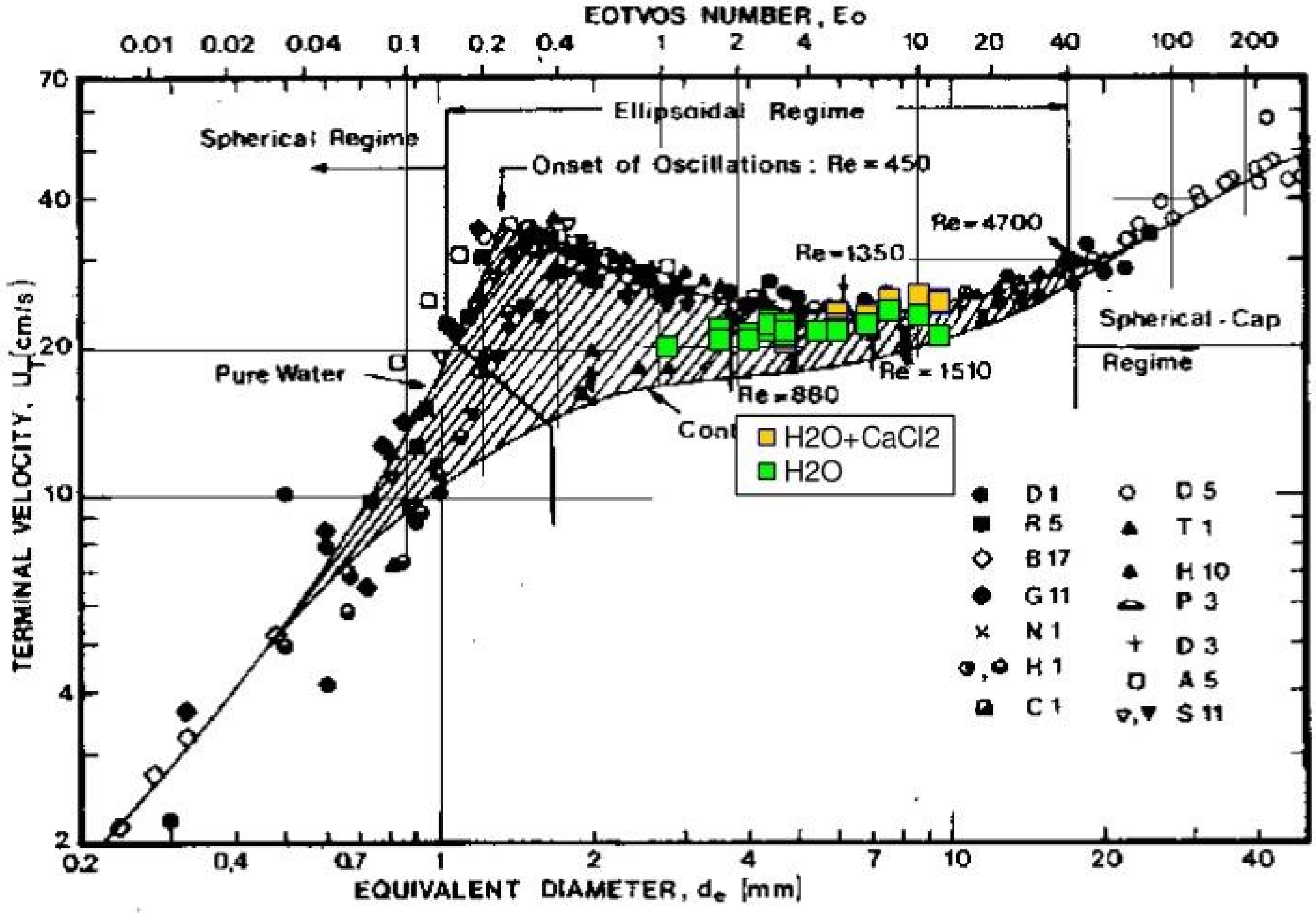


FIG. 7.3 Terminal velocity of air bubbles in water at 20° C.

Our data compared with that of Tomiyama

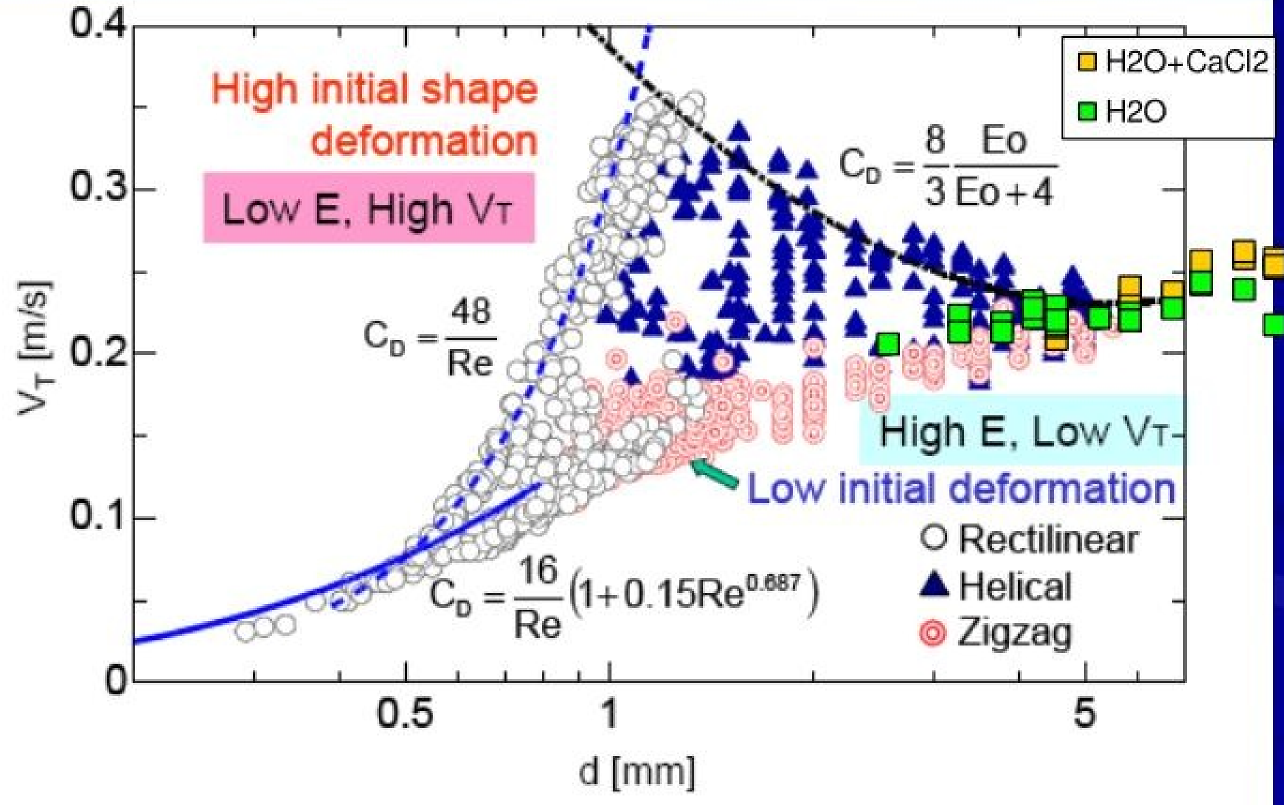


Fig. 4 Measured V_T of single air bubbles in distilled stagnant water

RISING VELOCITY
dimensional analysis

$$u = f\left(d, g, \rho, \Delta\rho, \mu, \sigma\right)$$

Rising velocity

- **Bubble equivalent diameter**
- **Gravity acceleration**
- **Liquid density**
- **Density difference**
- **Liquid viscosity**
- **Surface tension**

RISING VELOCITY
effect of surface tension

Morton number

$$Mo \equiv \frac{\mu^4 g}{\sigma^3 \rho}$$

Dimensionless bubble diameter

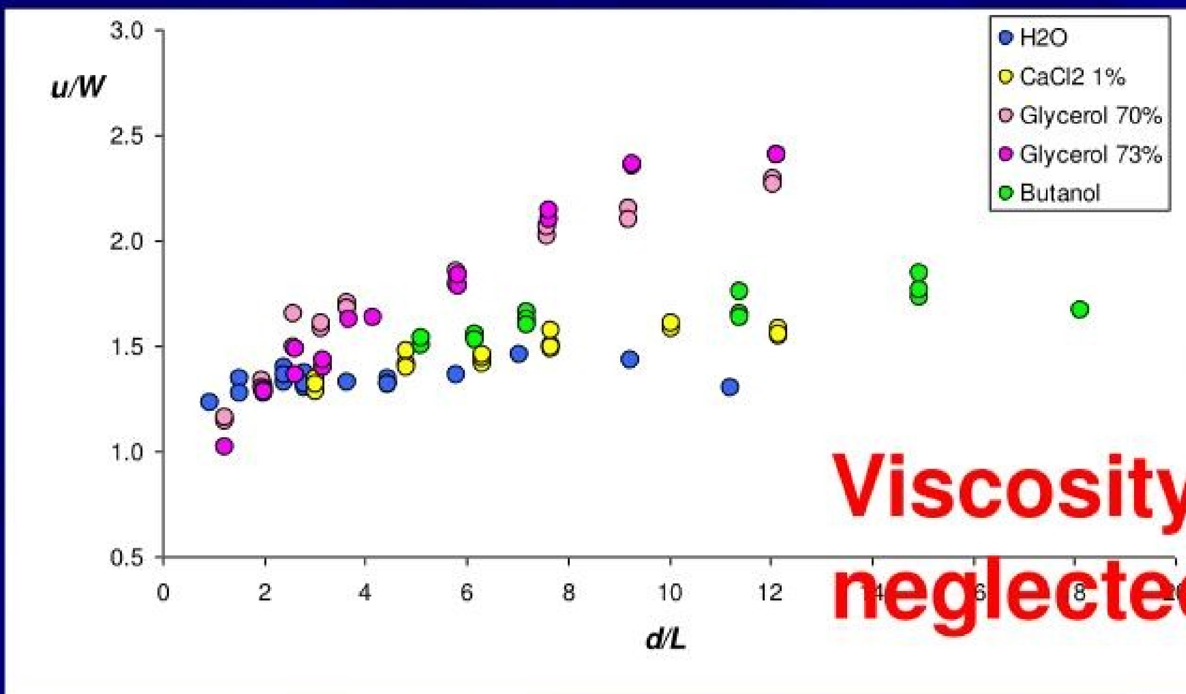
$$D_\sigma \equiv \frac{d}{\left(\frac{\sigma}{\rho g}\right)^{1/2}} = \frac{d}{L} = Eo^{1/2}$$

„Laplace velocity“

$$W \equiv \left(\frac{\sigma g}{\rho}\right)^{1/4}$$

Dimensionless bubble velocity

$$U_\sigma \equiv \frac{u}{W} = \frac{Re\, Mo^{1/4}}{Eo^{1/2}}$$

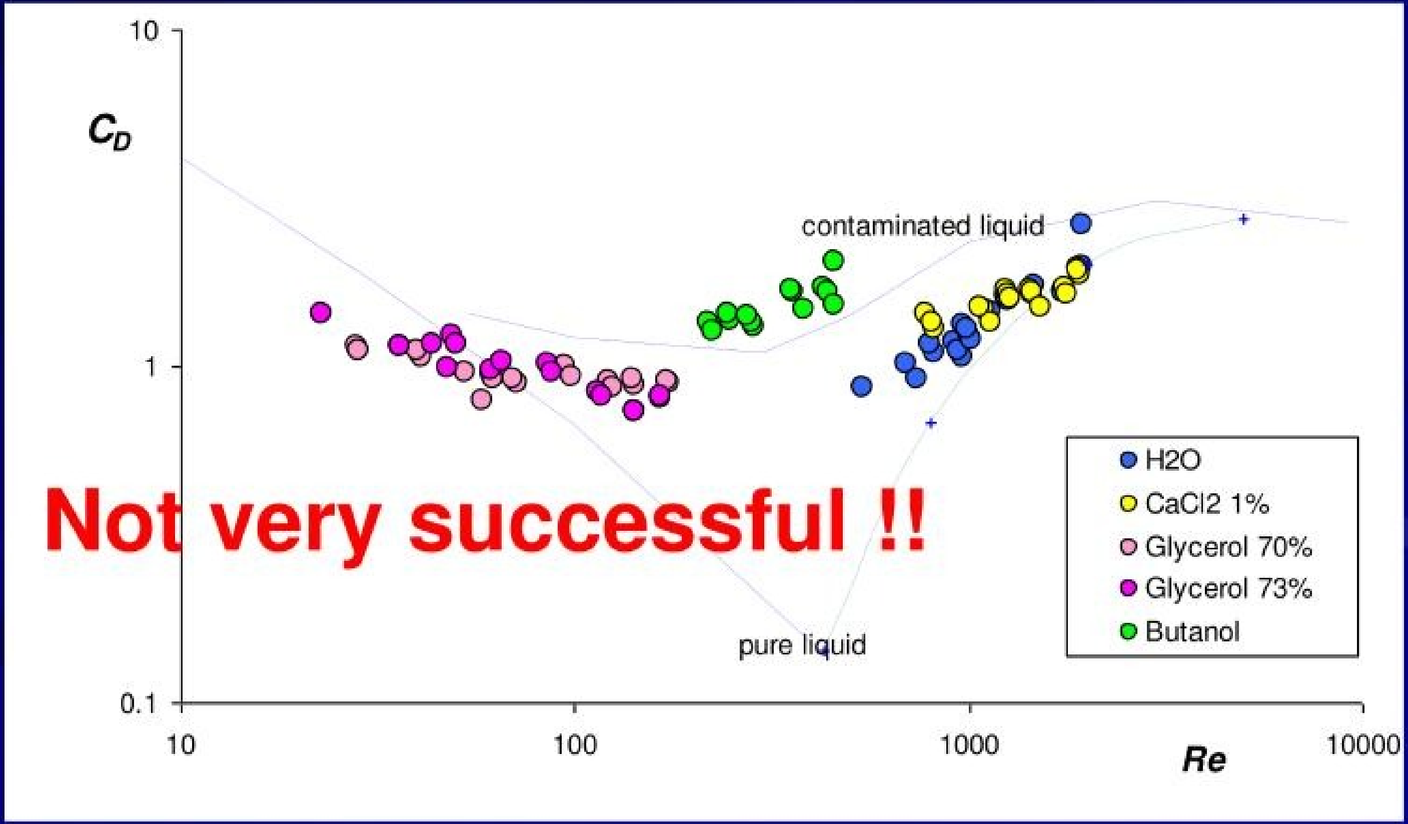


Viscosity cannot be neglected !!

RISING VELOCITY
classical variables

Drag coefficient

$$C_D = \frac{4dg}{3u^2}\frac{\Delta\rho}{\rho} \approx \frac{4dg}{3u^2}$$



Not very successful !!

RISING VELOCITY
effect of viscosity

Drag coefficient

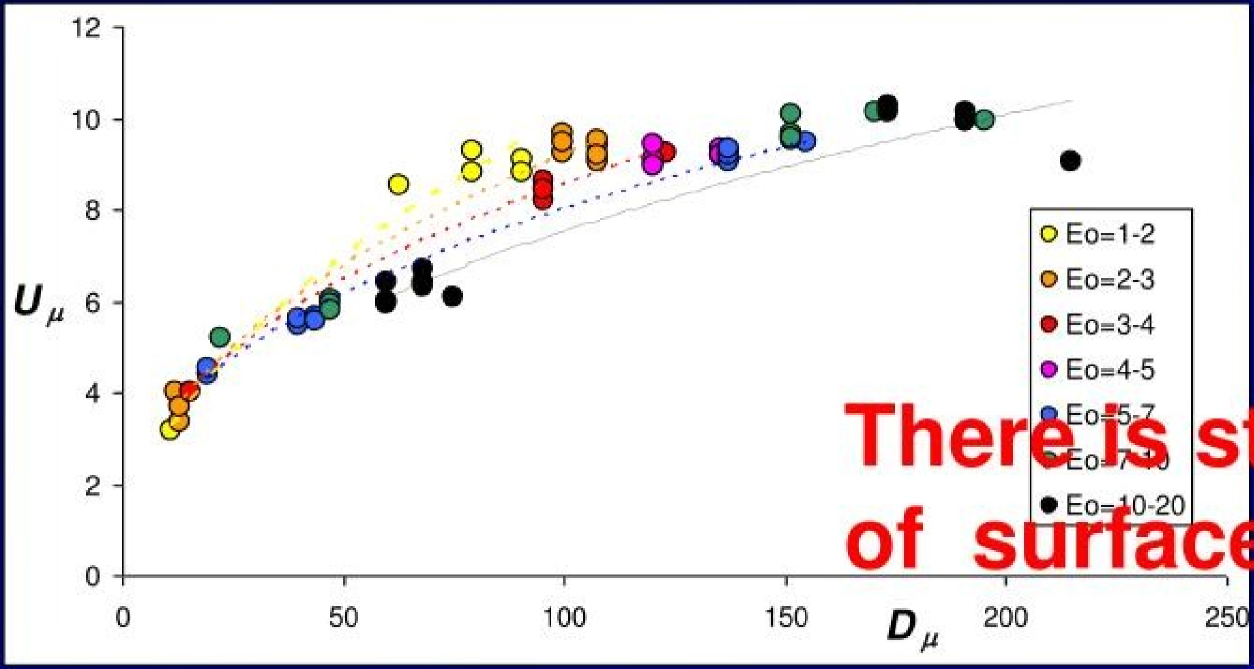
$$C_D = \frac{4dg}{3u^2}\frac{\Delta\rho}{\rho} \approx \frac{4dg}{3u^2}$$

Dimensionless bubble diameter

$$D_\mu \equiv \frac{d}{\left(\frac{\mu}{\rho}\right)^{2/3}\left(\frac{4g}{3}\right)^{1/3}} = Re^{2/3} C_D^{1/3}$$

Dimensionless bubble velocity

$$U_\mu \equiv \frac{u}{\left(\frac{4\mu g}{3\rho}\right)^{1/3}} = \left(\frac{Re}{C_D}\right)^{1/3}$$



There is still an effect of surface tension!!

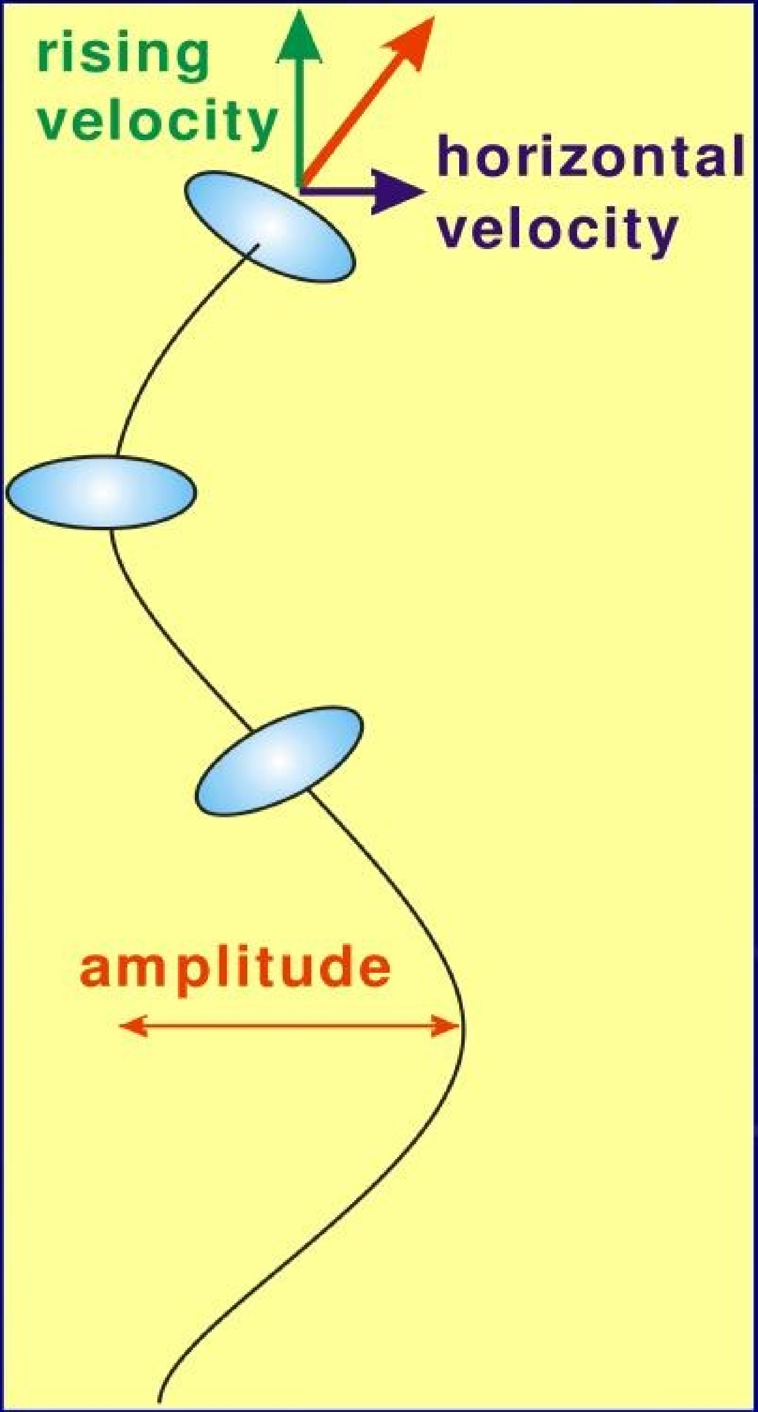
RISING BUBBLE

Oscillatory movement of a bubble

Bubble front area is
$S = \pi a^2$

New definition of the drag coefficient

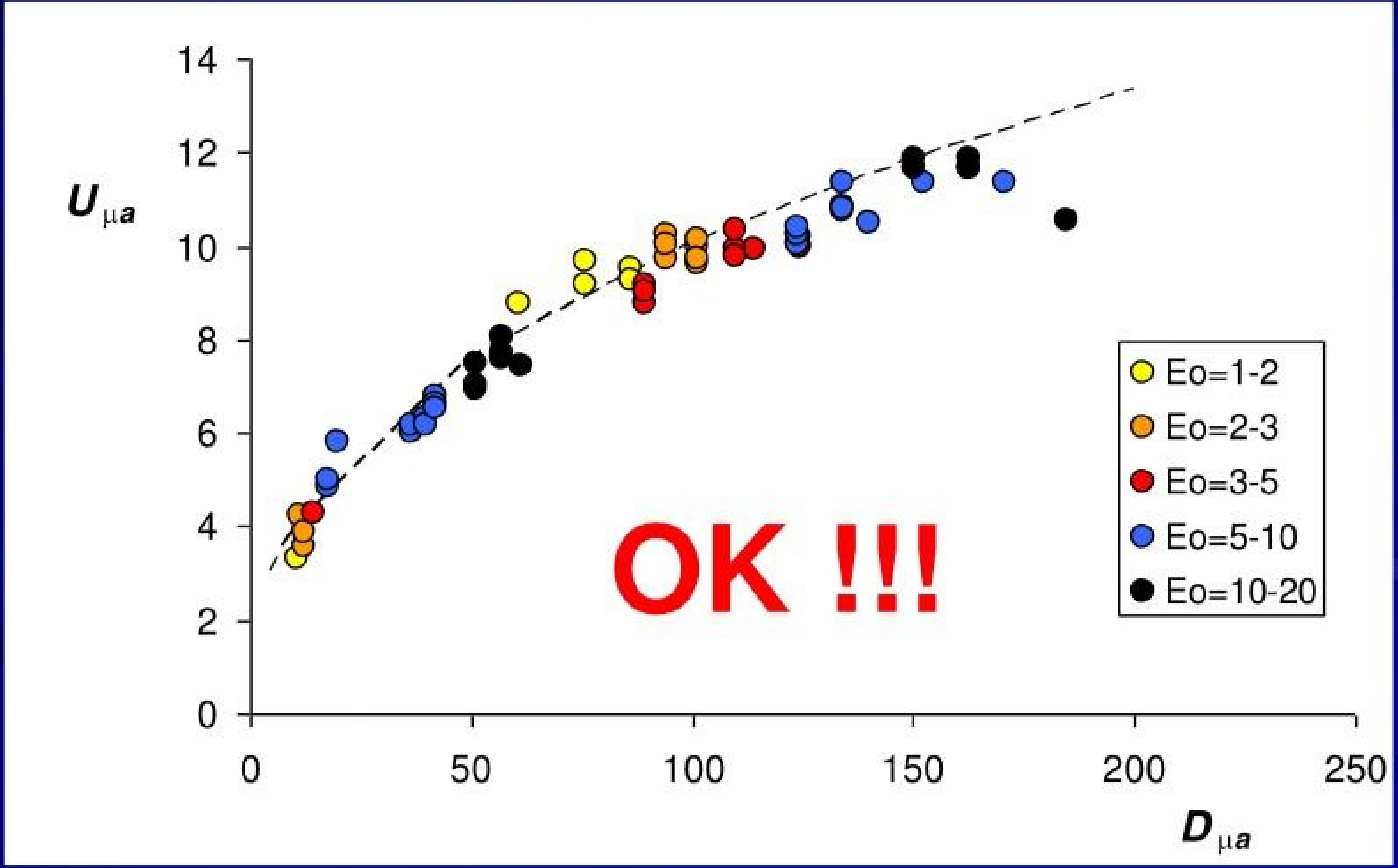
$$C_A \equiv C_D \left(\frac{d}{2a}\right)^2 \approx \frac{4d\ g}{3u^2}\left(\frac{d}{2a}\right)^2$$



New dimensionless variables

$$D_{\mu a} = D_{\mu}\left(\frac{d}{2a}\right)^{2/3}$$

$$U_{\mu a} \equiv U_{\mu}\left(\frac{d}{2a}\right)^{-2/3}$$



RESULTING FORMULAS

- Correlation of the bubble shape

$$\left(\frac{2a}{d}\right)^2 = 1 + 0.095Eo^{0.75} \quad \text{for} \quad Eo < 20$$

- Correlation of the bubble velocity

$$C_D = 0.365Re^{0.143}\left(1 + 0.095Eo^{0.75}\right)$$
$$\text{for } 1 < Eo < 20, \; Re \geq 30$$

This correlation fits well the data for medium size bubbles in contaminated low- and medium- viscosity liquids. In carefully prepared pure liquids, the rising velocity of can be somewhat higher.

Thank you for your attention

Acknowledgments

We gratefully acknowledge financial support by the grant No.104/07/1110 from the Grant Agency of the Czech Republic.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