1.2. Basic Heat Exchanger Equations

1.2.1. The Overall Heat Transfer Coefficient

Consider the situation in Fig. (1.18). Heat is being transferred from the fluid inside (at a local bulk or average temperature of T_i), through a dirt or fouling film, through the tube wall, through another fouling film to the outside fluid at a local bulk temperature of T_o. A_i and A_o are respectively inside and outside surface areas for heat transfer for a given length of tube. For a plain or bare cylindrical tube,

$$\frac{A_o}{A_i} = \frac{2\pi r_o L}{2\pi r_i L} = \frac{r_o}{r_i} \qquad (1.13)$$

The heat transfer rate between the fluid inside the tube and the surface of the inside fouling film is given by an equation of the form $Q/A = h(T_f - T_s)$ where the area is A_i and similarly for the outside convective process where the area is A_o . The values of h_i and h_o have to be calculated from appropriate correlations.

On most real heat exchanger surfaces in actual service, a film or deposit of sediment, scale, organic growth, etc., will sooner or later develop. A few fluids such as air or liquefied natural gas are usually clean enough that the fouling is absent or small enough to be neglected. Heat transfer across these films is predominantly by conduction, but the designer seldom knows enough about either the thickness or the thermal conductivity of the film to treat the heat transfer resistance as a conduction problem. Rather, the designer estimates from a table of standard values or from experience a fouling factor R_f. R_f is defined in terms of the heat flux Q/A and the temperature difference across the fouling ΔT_f by the equation:

$$R_f = \frac{\Delta T_f}{Q/A} \qquad (1.14)$$

The Overall Heat Transfer Coefficient

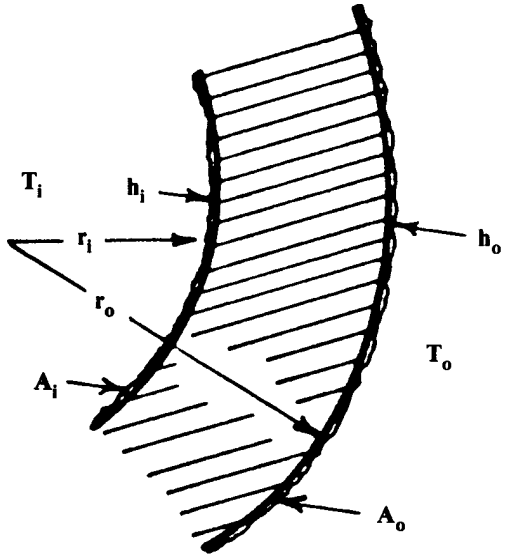

Fig. 1.18 Cross-section of Fluid-to-Fluid Heat Transfer Apparatus

From Eq.1.14, it is clear that R_f is equivalent to a reciprocal heat transfer coefficient for the fouling, h_f:

$$h_f = \frac{1}{R_f} = \frac{Q/A}{\Delta T_f} \qquad (1.15)$$

and in many books, the fouling is accounted for by a "fouling heat transfer coefficient," which is still an estimated quantity. The effect of including this additional resistance is to provide an exchanger somewhat larger than required when it is clean, so that the exchanger will still provide the desired service after it has been on stream for some time and some fouling has accumulated.

The rate of heat flow per unit length of tube must be the same across the inside fluid film, the inside dirt film, the wall, the outside dirt film, and the outside fluid film. If we require that the temperature differences across each of these resistances to heat transfer add up to the overall temperature difference, (T_i - T_o), we obtain for the case shown in Fig.1.18 the equation

$$Q = \frac{T_i - T_o}{\frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln(r_o / r_i)}{2\pi L k_w} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o}} \tag{1.16}$$

In writing Eq. (1.16), the fouling is assumed to have negligible thickness, so that the values of r_i, r_o, A_i and A_o are those of the clean tube and are independent of the buildup of fouling. Not only is this convenient – we don't know enough about the fouling to do anything else.

Now we define an overall heat transfer coefficient U^* based on *any* convenient reference area A^*:

$$Q \equiv U^* A^* (T_i - T_o) \tag{1.17}$$

Comparing the last two equations gives:

$$U^* = \frac{1}{\frac{A^*}{h_i A_i} + \frac{R_{fi} A^*}{A_i} + \frac{A^* \ln(r_o / r_i)}{2\pi L k_w} + \frac{R_{fo} A^*}{A_o} + \frac{A^*}{h_o A_o}} \tag{1.18}$$

Frequently, *but not always*, A^* is chosen to be equal to A_o, in which case $U^* = U_o$, and Eq. (1.18) becomes:

$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{R_{fi} A_o}{A_i} + \frac{A_o \ln(r_o / r_i)}{2\pi L k_w} + R_{fo} + \frac{1}{h_o}} \tag{1.19}$$

If the reference area A^* is chosen to be A_i, the corresponding overall heat transfer coefficient U_i is given by:

$$U_i = \frac{1}{\frac{1}{h_i} + R_{fi} + \frac{A_i \ln(r_o / r_i)}{2\pi L k_w} + \frac{R_{fo} A_i}{A_o} + \frac{A_i}{h_o A_o}} \tag{1.20}$$

The equation as written applies only at the particular point where (T_i - T_o) is the driving force. The question of applying the equation to an exchanger in which T_i and T_o vary from point to point is considered in the next section.

The wall resistance is ordinarily relatively small, and to a sufficient degree of precision for bare tubes, we may usually write

$$\frac{A_o \ell n(r_o / r_i)}{2\pi L k_w} \cong \frac{r_o \Delta X}{\frac{1}{2}(r_o + r_i)k_w}; \frac{A_i \ell n(r_o / r_i)}{2\pi L k_w} \cong \frac{r_i \Delta X}{\frac{1}{2}(r_o + r_i)k_w} \tag{1.21}$$

Inspection of the magnitudes of the terms in the denominator of Eqs. 1. 19 or 1.20 for any particular design case quickly reveals which term or terms (and therefore which heat transfer resistance) predominates. This term (or terms) controls the size of the heat exchanger and is the one upon which the designer should concentrate his attention. Perhaps the overall heat transfer coefficient can be significantly improved by a change in the design or operating conditions of the heat exchanger. In any case, the designer must give particular attention to calculating or estimating the value of the largest resistance, because any error or uncertainty in the data, the correlation, or the calculation of this term has a disproportionately large effect upon the size of the exchanger and/or the confidence that can be placed in its ability to do the job.

1.2.2. The Design Integral

In the previous section, we obtained an equation that related the rate of heat transfer to the local temperature difference (T-t) and the heat transfer area A, through the use of an overall heat transfer coefficient U. In most exchanger applications, however, one or both of the stream temperatures change from point to point through the flow paths of the respective streams. The change in temperature of each stream is calculated from the heat (enthalpy) balance on that stream and is a problem in thermodynamics.

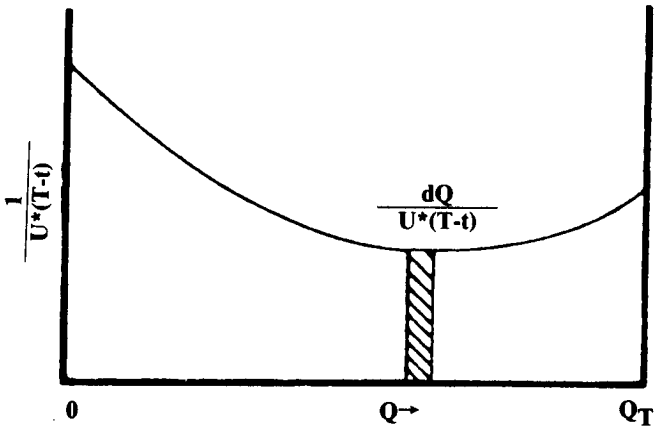

Fig. 1.19 Integration to Find Area.

Our next concern is to develop a method applying the equations already obtained to the case in which the temperature difference between the two streams is not constant. We first write Eq. (1. 17) in differential form

$$dA^* = \frac{dQ}{U^*(T-t)} \tag{1.22}$$

and then formally integrate this equation over the entire heat duty of the exchanger, Q_t :

$$A^* = \int_o^{Q_t} \frac{dQ}{U^*(T-t)} \tag{1.23}$$

This is the basic heat exchanger design equation, or the design integral.

U^* and A^* may be on any convenient consistent basis, but generally we will use U_o and A_o. U^* may be, and in practice sometimes is, a function of the amount of heat exchanged. If $1/U^*(T\text{-}t)$ may be calculated as a function of Q, then the area required may be calculated either numerically or graphically, as shown in Fig. (1.19).

The above procedure involving the evaluation of Eq. (1.23) is, within the stated assumptions, exact, and may always be used. It is also very tedious and time consuming. We may ask whether there is not a shorter and still acceptably accurate procedure that we could use. As it happens, if we make certain assumptions, Eq. (1.23) can be analytically integrated to the form of Eq. (1.24)

$$A^* = \frac{Q_t}{U^*(MTD)} \tag{1.24}$$

where U^* is the value (assumed constant) of the overall heat transfer coefficient and MTD is the "Mean Temperature Difference," which is discussed in detail in the following section.