1.3. The Mean Temperature Difference

1.3.1. The Logarithmic Mean Temperature Difference

1. **Basic Assumptions.** In the previous section, we observed that the design equation could be solved much easier if we could define a "Mean Temperature Difference" (MTD) such that:

$$A^{*} = \frac{Q_t}{U^{*}(MTD)} \tag{1.25}$$

In order to do so, we need to make some assumptions concerning the heat transfer process.

One set of assumptions that is reasonably valid for a wide range of cases and leads to a very useful result is the following:

1. All elements of a given stream have the same thermal history.
2. The heat exchanger is at a steady state.
3. Each stream has a constant specific heat.
4. The overall heat transfer coefficient is constant.
5. There are no heat losses from the exchanger.
6. There is no longitudinal heat transfer within a given stream.
7. The flow is either cocurrent or counter-current.

The first assumption is worthy of some note because it is often omitted or stated in a less definitive way. It simply means that all elements of a given stream that enter an exchanger follow paths through the exchanger that have the same heat transfer characteristics and have the same exposure to heat transfer surface. In fact, in most heat exchangers, there are some flow paths that have less flow resistance than others and also present less heat transfer surface to the fluid. Then the fluid preferentially follows these paths and undergoes less heat transfer. Usually the differences are small and do not cause serious error, but occasionally the imbalance is so great that the exchanger is very seriously deficient. Detailed analysis of the problem is too complex to treat there, but the designer learns to recognize potentially troublesome configurations and avoid them.

The second, third, fourth, fifth, and sixth assumptions are all straight- forward and are commonly satisfied in practice. It should be noted that an isothermal phase transition (boiling or condensing a pure component at constant pressure) corresponds to an infinite specific heat, which in turn satisfies the third assumption very well.

The seventh assumption requires some illustration in terms of a common and simple heat exchanger configuration, the double pipe exchanger.

2. **The Double Pipe Heat Exchanger**. A double pipe heat exchanger essentially consists of one pipe concentrically located inside a second, larger one, as shown in Fig. 1.20. One fluid flows in the annulus between the inner and outer pipes and the other in the inner pipe. In Fig. 1.20, the two fluids are shown as entering at the same end, flowing in the same direction, and leaving at the other end; this configuration is called *cocurrent*. In Fig. 1.21, possible temperature profiles are drawn for the temperatures of the

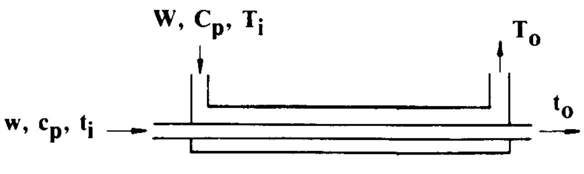

Fig. 1.20 Schematic Diagram of a Double Pipe Heat Exchanger, in Cocurrent Flow.

fluids in this exchanger. (We have shown the hot fluid in the annulus and the cold fluid in the inner pipe, but the reverse situation is equally possible.)

Notice that the outlet temperatures can only approach equilibrium with one another, sharply limiting the possible temperature change. If we had plotted the local temperatures vs. quantity of heat transferred, we would get straight lines, a consequence of the assumption that the specific heats are constant.

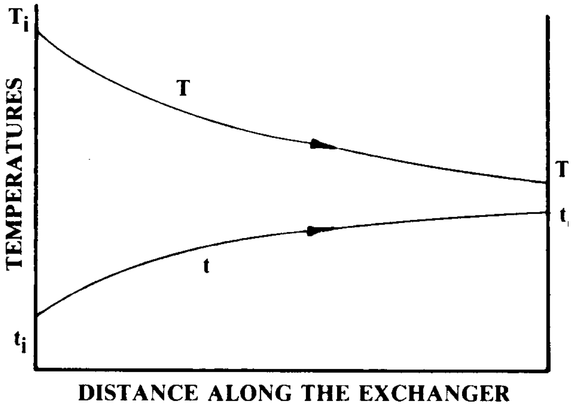

Fig. 1.21 Thermal Profiles in a Cocurrent Heat Exchanger.

A countercurrent heat exchanger is diagrammed in Fig.1.22 and a possible set of temperature profiles as a function of length is shown in Fig. 1.23. Also observe that the maximum temperature change is limited by one of the outlet temperatures equilibrating with the inlet temperature of the other stream, giving a basically more efficient heat exchanger for otherwise identical inlet conditions compared to the cocurrent arrangement. For this reason, the designer will almost always choose a countercurrent flow arrangement where possible.

If one stream is isothermal, the two cases are equivalent and the choice of cocurrent or countercurrent flow is immaterial, at least on grounds of temperature profiles.

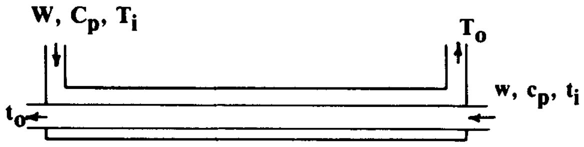

Fig. 1.22 Schematic Diagram of a Double Pipe Heat Exchanger in Countercurrent Flow.

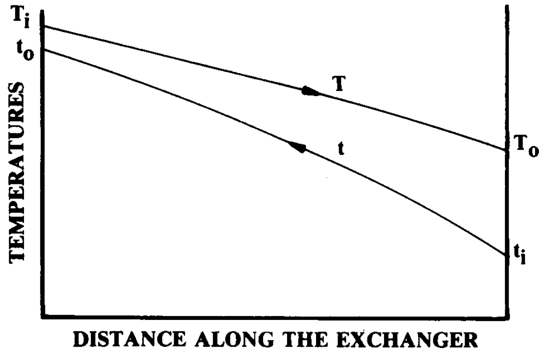

Fig. 1.23 Thermal Profiles in a Countercurrent Heat Exchanger.

3. **The Logarithmic Mean Temperature Difference.** The analytical evaluation of the design integral Eq. 1.23 can be carried out for both cocurrent and countercurrent flow if the basic assumptions are valid. The details of the derivation are not relevant here and can be found in a number of standard textbooks (e.g. Ref. 6). For the cocurrent exchanger, the result is:

$$MTD = \frac{(T_i - t_i) - (T_o - t_o)}{\ln \dfrac{(T_i - t_i)}{(T_o - t_o)}} \qquad (1.26)$$

and for the countercurrent case,

$$MTD = \frac{(T_i - t_o) - (T_o - t_i)}{\ln \dfrac{(T_i - t_o)}{(T_o - t_i)}} \qquad (1.27)$$

For the special case that $(T_i - t_o) = (T_o - t_i)$, eqn. (1.27) reduces to:

$$MTD = (T_i - t_o) = (T_o - t_i) \qquad (1.28)$$

The definitions of MTD's given in Eqns. (1.26) and (1.27) are the logarithmic means of the terminal temperature differences in each case. Because of its widespread importance in heat exchanger design, Eq. (1.27) is commonly referred to as "the logarithmic mean temperature difference," abbreviated as LMTD.

1.3.2. Configuration Correction Factors on the LMTD

1. **Multiple Tube Side Passes.** One of the assumptions of the LMTD derivation was that the flow was either completely cocurrent or completely countercurrent. For a variety of reasons, mixed, reversed or crossflow exchanger configurations may be preferred. A common case is shown in Fig. 1.24 - a one-shell-pass, two-tube-pass design (a 1-2 exchanger, for short):

Note that on the first tube side pass, the tube fluid is in countercurrent flow to the shell-side fluid, whereas on the second tube pass, the tube fluid is in cocurrent flow with the shell-side fluid. A possible set of temperature profiles for this exchanger is given in Fig. 1.25.

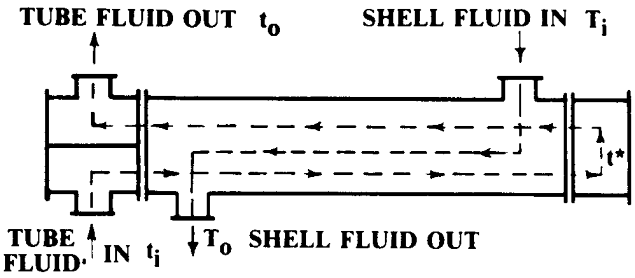

Fig. 1.24 Diagram of a 1-2 Exchanger.

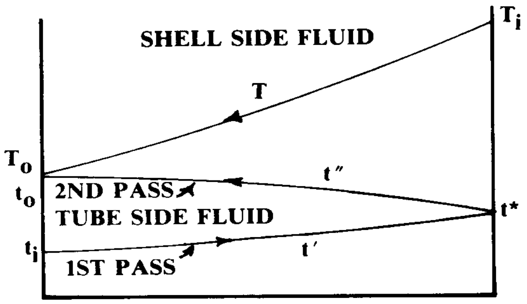

Fig. 1.25 Possible Set of Temperature Profiles for 1-2 Exchanger.

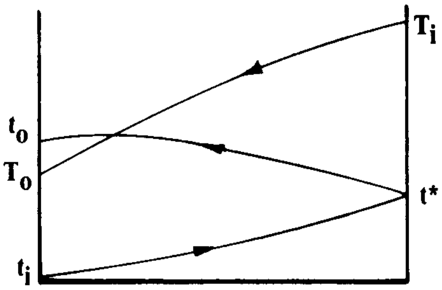

Fig. 1.26 Temperature Cross in a 1-2 Exchanger.

Note that it is possible for the outlet tube side temperature to be somewhat greater than the outlet shell-side temperature. The resulting temperature profiles then might look like Fig. 1.26.

The maximum possible tube outlet temperature that can be achieved in this case, assuming constant overall heat transfer coefficient, is

$$t_{o,\max} = 2T_o - t_i \tag{1.29}$$

Since this requires infinite area and all of the other assumptions being rigorously true, one would ordinarily stay well below this limit or look for another configuration.

An alternative arrangement of a 1-2 exchanger is shown in Fig. 1.27, and a possible set of temperature profiles is given in Fig. 1.28. In this case t^* cannot exceed T_o.

In spite of the very different appearance of these two cases, it turns out that they give identical values of the effective temperature difference for identical temperatures.

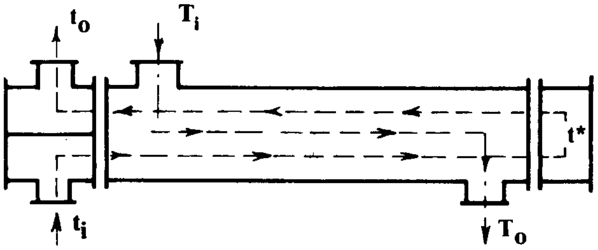

Fig. 1.27 Diagram of Another 1-2 Exchanger.

The problem of computing an effective mean temperature difference for this configuration can be carried out along lines very similar to those used to obtain the LMTD. The basic assumptions are the same (except for the pure cocurrent or countercurrent limitation), though in addition it is assumed that each pass has the same amount of heat transfer area. Rather than calculate the MTD directly however, it is preferable to compute a correction factor F on the LMTD calculated assuming pure countercurrent flow, i.e.

$$F = \frac{MTD}{LMTD} \qquad (1.30)$$

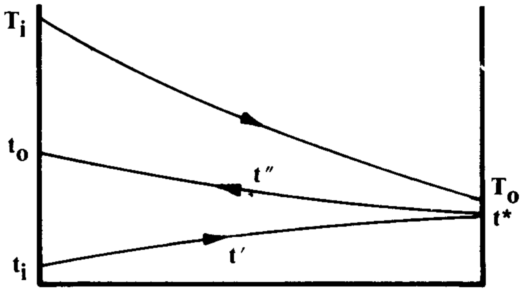

Fig. 1.28 Possible Temperature Profiles for Exchanger in Fig. 1.27.

where F = 1 indicates the flow situation is equivalent to countercurrent flow, and lower values very clearly and directly show what penalty (ultimately expressed in area required) is being paid for the 1-2 configuration. It is important to remember that the LMTD used in Eq. (1.30) is to be calculated for the countercurrent flow case, Eq. (1.27).

The correction factor F is shown in Fig. 1.29 for a 1-2 exchanger as a function of two parameters R and P defined as (in terms of the nomenclature given on the chart):

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{Range\ of\ shell\ fluid}{Range\ of\ tube\ fluid} \qquad (1.31a)$$

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{Range\ of\ tube\ fluid}{Maximum\ temperature\ difference} \qquad (1.31b)$$

The chart given here is adapted from the Standards of the Tubular Exchanger Manufacturers Association (9) and is almost identical to the one in Kern (7). The corresponding chart in McAdams (8) uses entirely different symbols, but is in fact identical to the one given here. However, there are other different (but finally equivalent) formulations and each one should be used carefully with its own definitions.

Examination of the chart reveals that for each value of R, the curve becomes suddenly and extremely steep at some value of P. This is due to the tube-side temperature approaching one of the thermodynamic limits discussed above. It is extremely dangerous to design an exchanger on or near this steep region, because even a small failure of one of the basic assumptions can easily render the exchanger thermodynamically incapable of rendering the specified performance no matter how much excess surface is provided; the first assumption is especially critical in this case. Therefore, there is a generally accepted rule-of-thumb that no exchanger will be designed to F < 0.75. Besides, lower values of F result in large additional surface requirements and there is almost always some way to do it better.

The discussion to this point has centered on the 1-2 exchanger. Larger numbers of tube-side passes are possible and frequently used. Kern discusses the problem briefly and points out that correction factors for any even number of tube-side passes are within about 2 percent of those for two passes, so it is common practice to use Fig. 1.29 for all 1-n exchangers where n is any even number. Other configurations will be discussed later. Kern, McAdams, and Perry's Handbook (10) give fairly extensive collections of F charts.

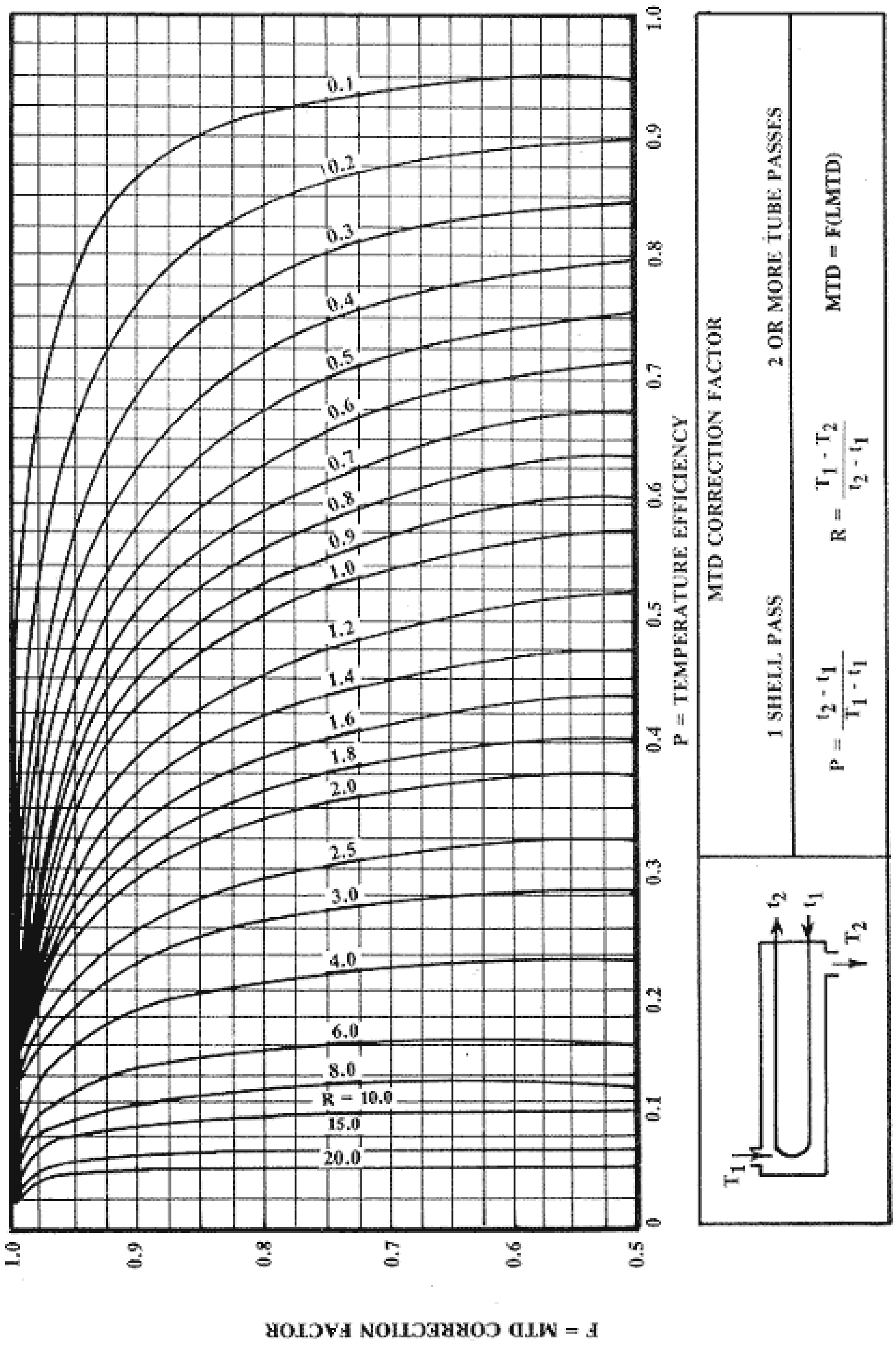

Fig. 1.29 F Correction Factor for a 1-n Exchanger.
(Adapted from Ref. (9)).

2. **Multiple Shell-Side Passes.** In an attempt to offset the disadvantage of values of F less than 1.0 resulting from the multiple tube side passes, some manufacturers regularly design shell and tube exchangers with longitudinal shell-side baffles as shown in Fig. 1.30. If one traces through the flow paths, one sees that the two streams are always countercurrent to one another, therefore superficially giving F = 1.0. The principle could be extended to multiple shell side passes to match multiple tube side passes but this is seldom or never done in practice.

Even the provision of a single shell-side longitudinal baffle poses a number of fabrication, operation and maintenance problems. Without discussing all of the possibilities, we may observe that there may be, unless very special precautions are taken, will be, thermal leakage from the hot shell-side pass through the baffle to the other (cold) pass, which violates the 6th assumption. Further, there may even be physical leakage of fluid from the first shell-side pass to the second because of the pressure difference, and this violates the 1st assumption.

A recent analysis has been made of the problem (Rozenmann and Taborek, Ref. 11), which warns one when the penalty may become severe.

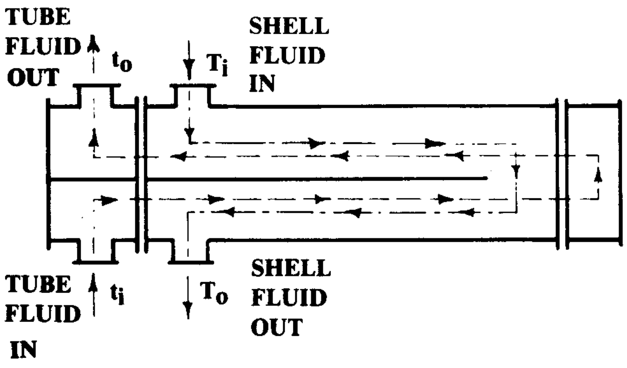

Fig. 1.30 Diagram of a 2-2 Exchanger.

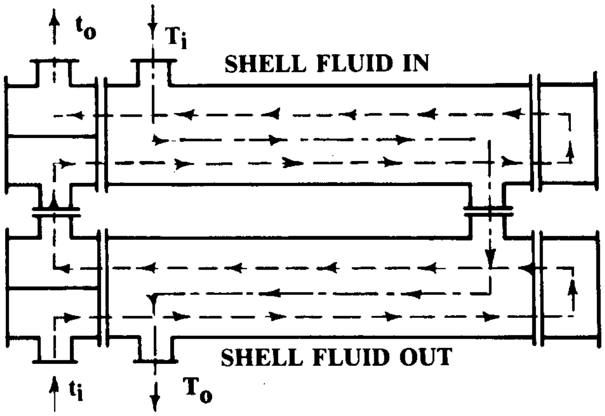

Fig. 1.31 Two 1-2 Exchangers in Series.

3. **Multiple Shells in Series.** If we need to use multiple tube side passes (as we often do), and if the single shell pass configuration results in too low a value of F (or in fact is thermodynamically inoperable), what can we do?

The usual solution is to use multiple shells in series, as diagrammed in Fig. 1.31 for a very simple case.

More than two tube passes per shell may be used. The use of up to six shells in series is quite common, especially in heat recovery trains, but sooner or later pressure drop limits on one stream or the other limit the number of shells.

Qualitatively, we may observe that the overall flow arrangement of the two streams is countercurrent, even though the flow within each shell is still mixed. Since, however, the temperature change of each stream in one shell is only a fraction of the total change, the departure from true countercurrent flow is less. A little reflection will show that as the number of shells in series becomes infinite, the heat transfer process approaches true counter- current flow and F → 1.0.

It is possible to analyze the thermal performance of a series of shells each having one shell pass and an even number of tube passes, by using heat balances and Fig. 1.29 applied to each shell. Such calculations quickly become very tedious and it is much more convenient to use charts derived specifically for various numbers of shells in series. Such charts are included in Chapter 2 of this Manual.

4. **The Mean Temperature Difference in Crossflow Exchangers**. Many heat exchangers - especially air-cooled heat exchangers (Fig. 1.32) - are arranged so that one fluid flows crosswise to the other fluid.

The mean temperature difference in crossflow exchangers is calculated in much the same way and using the same assumptions as for shell and tube exchangers. That is,

$$\text{MTD} = F\,(\text{LMTD}) \qquad (1.32)$$

where F is taken from Figure 1.33 for the configuration shown in Fig. 1.32. Recall that the LMTD is calculated on the basis that the two streams are in countercurrent flow.

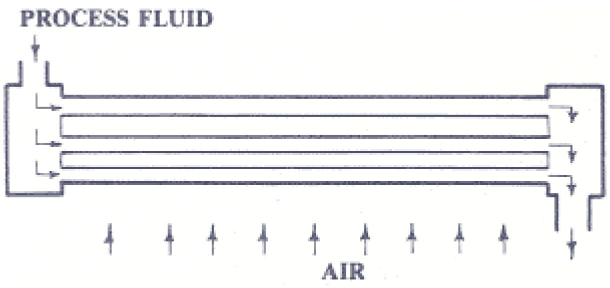

Fig. 1.32 Typical Arrangement of a Heat Exchanger Using Air in Crossflow.

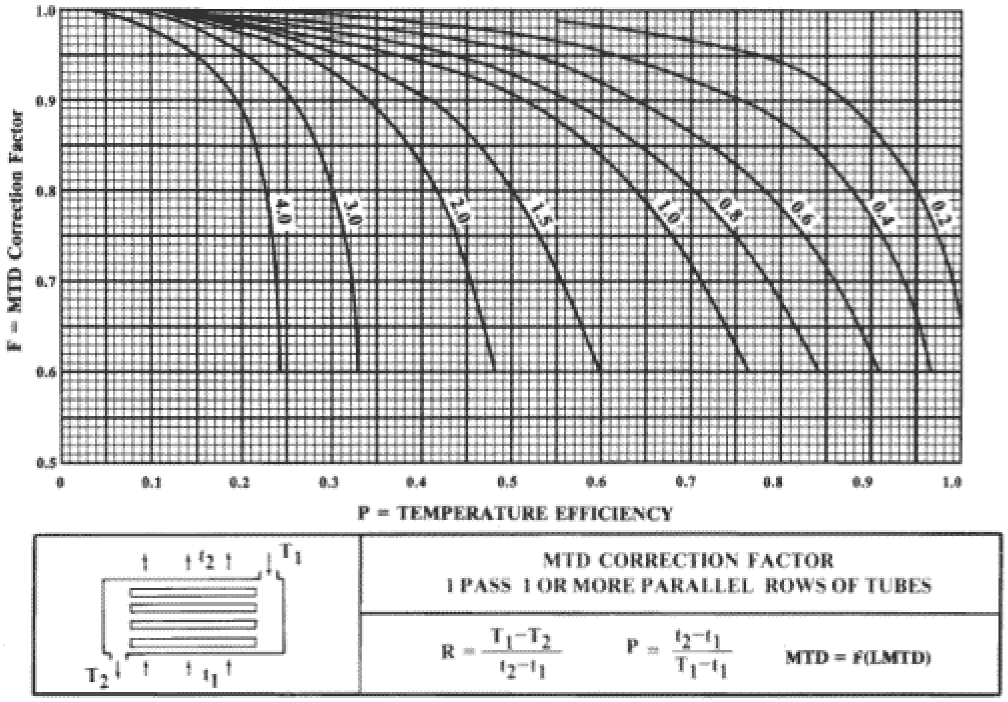

Fig. 1.33 MTD Configuration Correction Factor for Crossflow to 1 or More Parallel Rows of Tubes. (Adapted from Ref. (22)).