

Dimensional systems and systems of units in physics with special reference to chemical engineering

Part II. Practical rules for authors and readers and the need for their observance

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1. INTRODUCTION AND SUMMARY

In Part I the principles were discussed of constructing systems of units. The results of this study will now be applied to formulate a number of practical rules, the application of which should reduce the amount of misunderstanding in the field.

Most of these rules are intended for authors (see § 2). The main line of thought therein refers to the case where an equation in physics or engineering is derived from first principles and then numerically applied (rules 1-9).

Separate statements are given with reference to

empirical equations (rule 10),
dimensional analysis (rule 11),
dimensionless groups (rule 12),
equations that should be applicable in more than one dimensional system (rule 13).

In § 3 a few recommendations are given for the use of the readers.

In § 4 a number of literature quotations are analysed. This discussion shows that many authors put problems before the readers, which, it is suggested, they could have avoided.

Finally, a summary is presented of advantages and disadvantages of various systems, either derived by *a priori* considerations or from the analysis of the literature.

The weighing of these arguments is of course a subjective procedure. It is considered that following systems are preferable :

Metric : the practical system (m, kg (mass), sec., °C, practical electrical units),
British : the lb (mass), lb (force), ft., sec., BTU, °F system.

2. PRACTICAL RULES FOR THE AUTHOR

1. *The advice is given to consider letter symbols to stand for physical magnitudes and not for their numerical values only.*

This means that statements like : " the acceleration by gravity is g m/sec.² " are not allowed.

2. *Decide on a coherent set of fundamental quantities and dimensional constants and use it consistently.*

The sets of quantities discussed in Part I are :

In mechanics :

- (a) MLT* : dynamic systems.
- (b) FLT : static systems.
- (c) FMLT : English engineering system.

In mechanics + heat :

- (d) MLTQ θ , e.g.: cgs system : g_m , cm, sec., cal., °C. Brit. dyn. system : lb_m, ft., sec., BTU, °F.
- (e) MLT θ , e.g.: pract. system : kg_m, m, sec., °C.
- (f) FLTQ θ , e.g.: kg_f, m, sec., kcal., °C. lb_f, ft., sec., BTU, °F.
- (g) FMLTQ θ , e.g.: lb_f, lb_m, ft., sec., BTU, °F.

In mechanics + heat + electricity :

- (h) MLTQ θ , the cgs system : g_m , cm, sec., cal., °C.
- (i) MLT θ I, the practical system : kg_m, m, sec., °C, ampère.

The above list represents a choice of the most important combinations of wide applicability. Others, such as practical electrical units + mechanical cgs units, are also met.

* M = Mass, L = Length, T = Time, F = Force, Q = Heat, θ = Temperature, I = Electric current.

Excluding applications of Newton's First Law (mutual attraction of masses), the dimensional constants to be used with the above sets of quantities are :

In mechanics :

(a) none.

(b) none.

$$(c) \quad g_c = \left[\frac{ML}{FT^2} \right] \left\{ \begin{array}{l} \text{in lb}_f \text{ lb}_m \text{ ft. sec. system :} \\ \quad g_c = 32.2 \frac{\text{lb}_m \text{ ft.}}{\text{lb}_f \text{ sec.}^2} \\ \text{in lb}_f \text{ lb}_m \text{ ft. hr. system :} \\ \quad g_c = 4.17 \times 10^8 \frac{\text{lb}_m \text{ ft.}}{\text{lb}_f \text{ sec.}^2} \end{array} \right.$$

In mechanics + heat :

$$(d) \quad J = \left[\frac{ML^2 T^{-2}}{Q} \right] = \text{mech. equivalent of heat}$$

in cgs system :

$$J = 4.19 \times 10^7 \frac{\text{erg}}{\text{cal}}$$

in Brit. dyn. system :

$$J = 24.1 \frac{\text{ft. poundal}}{\text{BTU}}$$

(e) none.

$$(f) \quad J = \left[\frac{FL}{Q} \right], \text{ in techn. metric system :}$$

$$J = 427 \frac{\text{kg}_f \text{ m}}{\text{kcal}}$$

in lb_f ft. sec. BTU. °F system :

$$J = 778 \frac{\text{lb}_f \text{ ft.}}{\text{BTU}}$$

$$(g) \quad g_c = \left[\frac{ML}{FT^2} \right], \text{ as under (c).}$$

$$J = \left[\frac{FL}{Q} \right], \text{ in lb}_f \text{ lb}_m \text{ ft. sec. BTU. °F -}$$

$$\text{system :} \quad J = 778 \frac{\text{lb}_f \text{ ft.}}{\text{BTU}}$$

In mechanics + heat + electricity :

(h) one dimensional constant in some formulae in electricity and magnetism.

$$(i) \quad \mu_0 = \text{permeability of vacuum} = \frac{4\pi \text{ V sec.}}{10^7 \text{ A m}} \\ \epsilon_0 = \text{dielectric constant of vacuum} = \frac{10^7}{4\pi(8 \times 10^8)^2} \frac{\text{A sec.}}{\text{V m}} = 8.854 \times 10^{-12} \frac{\text{A sec.}}{\text{V m}}$$

Examples A 11 and A 12 in § 4 show difficulties that arise through unsystematic use of dimensional systems.

3. Decide on factors fixing numerical constants.

Some of the alternatives have been mentioned in Part I, e.g. :

rationalization or non-rationalization in the field of electricity and magnetism ;

various ways of defining a Reynolds group (radius vs. diameter of pipe ; particle size vs. inverse of specific particle surface) ;

various ways of defining the friction factor for flow through a pipe (the American : $f =$ shear stress at wall divided by $\frac{1}{2}\rho v^2$, the European : $\lambda = 4f =$ pressure drop divided by $\frac{1}{2}\rho v^2 \cdot \frac{l}{d}$).

Deviations from accepted custom in this respect cannot be traced by dimensional checks.

It is impossible to give precise instructions. Rule 3 only serves as a reminder that in deriving physical relationships a number of conventions are used, often unconsciously.

4. Prepare a list of quantities and dimensional constants giving their symbols, names and dimensional formulae, and keep this list complete.

5. Write down equations in letter form, ensuring dimensional homogeneity by using appropriate dimensional constants. Carry out the necessary mathematical transformations and check dimensional homogeneity of the final result.

Remark—The final check is in principle superfluous but it is very useful to help in detecting mistakes.

6. With equations in letter form, do not prescribe the units to be used when this constitutes an unnecessary limitation.

7. Choose a unit for each fundamental quantity. Substitute the units in the dimensional formulae for the derived quantities in order to obtain the units of these derived quantities.

8. In order to prepare the equation for numerical use, replace constants (dimensional constants, physical quantities that happen to be constant in the given problem) by their numerical values. From this point onward, mention each time the units to be used.

It is convenient to mention the unit of the numerical factor in order that users of the equation may easily convert it into their own system.

THORNE and WALSHAW in their booklet "Engineering Units and the Stroud Convention" (1954) give prescriptions how to derive numerical equations from physical equations.

Their physical equations, which cover the fields of mechanics and heat are always written without the dimensional constants, g_c and J . This is how they come to the statement (p. 2) that physical equations are completely free from any restrictions as regards particular units or systems of units (see, however, Part I, page 132).

In a dimensionally homogeneous physical equation THORNE and WALSHAW then replace the letter symbols by products of numerics and units. No prescription to use a coherent system of units is given so that the resultant equation is "unitarily inhomogeneous." By inspection the so-called "unity brackets" are then chosen and added as extra factors (each being unity) to obtain unitary homogeneity. These factors cover :

the conversion of size of unit :

$$\text{e.g.: } \left[\frac{5^\circ\text{C}}{9^\circ\text{F}} \right] ; \left[\frac{3600 \text{ sec.}}{1 \text{ hr.}} \right]$$

the replacement of derived units by fundamental units :

$$\text{e.g.: } \left[\frac{\text{dyne sec.}^2}{\text{g cm}} \right] ; \left[\frac{\text{lb.}_f \text{ sec.}^2}{\text{slug ft.}} \right]$$

the change in fundamental unit :

$$\text{e.g.: } \left[\frac{32.2 \text{ lb.}_m \text{ ft.}}{\text{lb.}_f \text{ sec.}^2} \right] ; \left[\frac{778 \text{ ft. lb.}_f}{\text{BTU}} \right] ; \left[\frac{\text{knot hr.}}{6080 \text{ ft.}} \right]$$

This subdivision of "unity brackets" in three groups has been mentioned here for the purpose of seeing the relations with the present article. It is, however, artificial and we may just as well say that by not having systems of units, THORNE and WALSHAW leave the equations in physics full of "dimensional constants," which they do not write in equations in letter form and must introduce later.

9. Give the list sub 4 at the end of the article. If

desired the dimensional formulae can be replaced by the units.

Remarks—1. Obviously, such a list cannot allow alternatives.

2. In expressing the units it is not necessary to use fundamental units only. It may be more convenient to use some very important derived units, such as : dyne, erg, newton, joule, or, volt. They have precisely been given their names for that that purpose.

10. *Empirical equations that are not dimensionally homogeneous, should never be given without stating the units of the various letter factors.* Preferably the unit of any dimensional numerical factor should be mentioned in order to facilitate conversion.

11. *Dimensional analysis need not be carried out with the fundamental units the author uses generally.* There is an advantage in using the maximum number of fundamental units compatible with the problem. No unnecessary dimensional constants must be included in the set of quantities to which dimensional analysis is applied.

Remarks—1. In heat transfer without conversion of heat in mechanical energy or vice versa, heat will be given a separate dimension – even if practical units are used elsewhere – without, however, introducing the mechanical equivalent of heat.

2. A statement by GUGGENHEIM [1] is highly appropriate :

"... the number of fundamental quantities having independent dimensions is, to some extent, a matter of choice. But, if in the same problem or set of problems two authors make a different choice, the one choosing the greater number is likely to be the more competent physicist."

12. *Use dimensionless groups to the utmost.*

All complete equations can be written as relations between dimensionless groups.

There is an advantage, in giving data and in presenting relationships in analytical or graphical form to use dimensionless groups to the greatest possible extent, since :

1. The numerical values of dimensionless groups are independent of the choice of the system of dimensions and the further choice of the sizes of the units ;
2. The number of variables is reduced thereby, so that equations assume their simplest form and the amount of experimentation to derive an empirical relationship is a minimum.

It is advantageous to introduce dimensionless groups at an early stage of the mathematical transformation, for instance to write already the governing differential equation and boundary conditions in dimensionless form.

In calculating numerical values of dimensionless groups coherent units must be used. Thus, if each symbol for a physical magnitude is replaced by the product of a numerical value and a unit, expressed in fundamental units, all units cancel.

Examples B 5 and 7-13 signal cases where the units as prescribed are inconsistent and yet the correct result is given. Thus, the author did not follow his own prescription.

In Examples B 1 and 6 the units are not coherent and the result deviates from the generally accepted one.

13. Multi-purpose formulae.

If formulae are to be ready for use with more than one system of units, following rules apply :

- (a) If all systems of units, in the field covered by the calculations, require the same numerical factors and dimensional constants, the equations in letter form are valid for all of them.
- (b) If numerical factors and/or dimensional constants differ, the equations should be provided with factors and constants for all cases, with precise instructions where to drop those.

The survey by KRONIG [2] of equations for rationalized practical units and for various types of cgs electrical units is given in this manner.

The combination of a dynamical system with the English Engineering system would require the introduction of g_c with the instruction to drop it for the dynamical systems. Failure to do so cannot be corrected by conversion of units (vide example A 8).

3. PRACTICAL RULES FOR THE READER

1. Check on observance of rules by author.
2. If desired, convert equations in letter form to the own preferred system by following methods :

(a) If the set of dimensional constants in the reader's preferred system is comprised in the set used by author :

Omit any superfluous dimensional constants.

Example :

Author : $g_c F = ma$ (Engl. Engineering).

Reader : $F = ma$ (static and dynamic systems).

(b) If the reader's set contains new dimensional constants :

Find by a dimensional check which dimensional constants are to be included.

Example :

Author : $pV = RT$ (practical system).

$$p = [\text{ML}^{-1} \text{T}^{-2}].$$

$$V = [\text{L}^3 \text{Mol}^{-1}].$$

$$R = [\text{ML}^2 \text{T}^{-2} \text{Mol}^{-1} \Theta^{-1}]$$

$$T = [\Theta].$$

Reader : $pV = JRT$ (cgs system + cal + °K).

$$p = [\text{ML}^{-1} \text{T}^{-2}].$$

$$V = [\text{L}^3 \text{Mol}^{-1}].$$

$$R = [\text{QMol}^{-1} \Theta^{-1}].$$

$$T = [\Theta].$$

so that : $J = [\text{ML}^2 \text{T}^{-2} \text{Q}^{-1}].$

Remark—Caution is necessary in the fields of electricity and magnetism in converting cgs into practical units and vice versa. Equations may differ by a numerical factor 4π , due to the fact that the practical system is rationalized and the cgs system is as a rule not. In cases of doubt, it is recommended to consult tables.

3. Conversion of units of physical quantities and constants in numerical equations.

- (a) Conversion within the same dimensional system.

If each quantity is considered to be the product of a numerical factor and a unit the conversion is easily carried out with the aid of the conversion factors for the fundamental units.

One method is, simply to replace each fundamental unit from one system by its equivalent in the other. Thus, for example,

$$100 \text{ lbs./cu.ft.} = 100 \times 0.454 \text{ kg}/(0.3048 \text{ m})^3 = 1602 \text{ kg/m}^3$$

If a ready conversion factor is used, it may be applied in the wrong direction unless one uses the principle adopted by VAN DIJCK [3] of heading tables in the following manner :

Quantity	Ex-pressed in	→ to multiply with → ← to divide by ←	Ex-pressed in
force	dyne	10^{-5}	newton

(b) Conversion to another dimensional system.

It is probably safer not to attempt to convert to units of other dimensional systems but to change the dimensional system already in the equation in letter symbols, as mentioned in para. 2.

Example A 8 indicates the sort of difficulties one might otherwise encounter.

4. EXAMPLES

In most cases the authors make no numerical mistakes. Yet their prescriptions, if duly carried out, would often lead one to do so.

It is the aim of the following discussion to prove that insufficient attention in this respect places an extra burden on the reader.

The examples are grouped under the following headings :

- A. Unsystematic use of dimensional formulae and dimensional constants and incorrect statements thereon.
- B. Inconsistencies in the substitution of numerical values in equations.

Regarding subject B, the substitution of numerical values in equations, it is noted that the author may :

1. give an incomplete description of the units to be used in his equations ;
2. give an overcomplete description, i.e. leave alternatives to choose from ;
3. describe an inconsistent set of units.

In this respect it is most dangerous to omit a dimensional constant from a coherent set, when the incoherence easily escapes attention ;

4. describe no units at all.

This is in many cases acceptable, viz. when no numerical application is given, when it is clear what dimensional system has been used and it is left to the reader to choose the sizes of the fundamental units.

A. Unsystematic use of dimensional formulae and dimensional constants, incorrect statements on dimensional systems.

A 1. Author A 1* under a heading " cgs system " mentions erroneously :

$$\text{conversion factor } g_c = 980.6 \frac{g_m \text{ cm}}{g_f \text{ sec.}^2}$$

$$\text{unit for the gas constant } R : \frac{g_f \text{ cm}}{\text{mol } ^\circ\text{C}}$$

In a column " English " analogous definitions are given together with a statement that the unit of force is the poundal. However, lb_f and poundal are incompatible.

Elsewhere g_c is incorrectly given as $32.17 (\text{lb}_m/\text{lb}_f)/(\text{ft./sec.})$.

A 2. Authors A 2 in their notation use F, M, L, T, Q and Θ as fundamental quantities and give corresponding dimensional formulae, such as pressure drop $[\text{FL}^{-2}]$ and density $[\text{ML}^{-3}]$. This is the English Engineering system, new version.

The text, however, equates a pressure drop to a density times a length, which means that F and M have been equated, as described on page 186, Part I (English Engineering system, old version.)

A 3 and 4. At one place the author of A 3 presents an admirable treatise on the use of the English Engineering system (new version) with the dimensional constant g_c . At other places there are some misstatements regarding the unit in which this constant is expressed, viz.:

$$\text{A 3, elsewhere, } g_c = 4.18 \times 10^8 \text{ lb force} \times \text{ft./}(\text{pounds fluid})(\text{hr.})(\text{hr.})$$

$$\text{A 4, same author, } g_c = 4.17 \times 10^8 \frac{\text{lb matter ft.}}{\text{hr.}^2}$$

A 5. The authors of A 5 make no distinct

* The list of authors has been submitted to the Editor of this Journal but will not be published.

choice between the three systems mentioned in § 2, rule 2, a, b, and c, having

<i>fundamental units</i>	<i>derived units</i>
(a) lb. _m , ft., sec.	poundal
(b) lb. _f , ft., sec.	slug
(c) lb. _m , lb. _f , ft., sec.	—

They define four fundamental dimensions (system (c)) but use the units sub. (b) for them, one of which is a derived one. Newton's Law is then used to derive a relation between the fundamental dimensions (logically impossible!) in order to decrease their number to three (system (a) or (b)).

The notation is unsystematic, e.g.

power	ft.lb./sec.	occurs in system (b) and (c)
density	slug/cu.ft. or lb. _m /cu.ft.	(b) or (a) and (c)
viscosity	lb. _m /ft.sec.	(a) and (c)
force	lb. _f	(b) and (c)
mass	slug	(b)

A 6. Author A 6 defines a permeability K_1 of a porous bed measuring pressure in g/cm²; viscosity in poises, length in cm and linear velocity in cm/sec. Thus the quantity defined is the product of the permeability in cm² (usual definition) and the dimensional constant g in cm/sec.²

As a result of the above, an equation is obtained relating permeability to porosity (dimensionless) and specific surface S_0 (cm⁻¹) of following form :

$$K_1 = \frac{g \epsilon^3}{5 S_0^2 (1 - \epsilon)^2}$$

where g makes a mysterious appearance.

The List of Symbols says that all units are consistent cgs units except where otherwise stated. It omits to give a unit for K_1 , and this is definitely not a cgs unit.

A 7. Authors A 7 give author's A 6 equation in the following form :

$$S_0 = 14 \sqrt{\frac{\epsilon^3}{K_1(1 - \epsilon)^2}}$$

Commenting thereon, in this equation the factor 14 is a dimensional constant $\left(\sqrt{\frac{g}{5}}\right)$. Again

the dimension of K_1 is omitted in the Notation. The equation in which K_1 is defined is not of much avail, since the prescription is given to insert the viscosity in centipoises instead of poises.

A 8. Authors A 8 use formulae valid in dynamical systems, i.e. not including g_c . They wanted to make the result applicable with the English Engineering system (using hours) as well, by indicating conversion factors in the list of symbols.

The pressure drop in N/m² then has to be replaced by g_c times the pressure drop in lb_f/sq.ft. To say Δp = pressure drop N/m² ($= 8.7 \times 10^6$ lb. force/ft.²) is incorrect.

A 9. Author A 9 gives in one article formulae valid for the metric equivalent of the English Engineering system (old version), involving a dimensional constant g (in cm.sec.⁻²), and formulae valid for the cgs system. The Table of Symbols says that pressure is expressed in cm water gauge or g cm⁻², which is incompatible with the cgs formulae.

A 10. Authors A 10 have expressed the pressure difference over a pipe section, Δp , in static units (lb/sq.ft.) and the shearing stress F at the wall of the pipe in dynamic units (lb/ft.hr.²) in accordance with their list of symbols. When using the English Engineering system a choice must be made, as mentioned in Part I (page 136) whether quantities are expressed in the one way or the other. It is very unfortunate that in defining the units for two so closely related quantities as a normal and a tangential force per unit area, different routes have been used.

Consequently the equation relating Δp , F and the pipe dimensions (d and l) contains g as a dimensional constant, viz.:

$$\Delta p \cdot \frac{\pi d^2}{4} = \frac{F}{g} \cdot \pi l d$$

with $g = 4.18 \times 10^8$ ft./hr.²

However, later on, again for pressure drop along a pipe, eq. (41) reads

$$p_1 - p_2 = \frac{4l}{d} F \text{ (note absence of } g)$$

Here the context shows that pressure is expressed in the dynamic unit lb./ft.hr.²

A 11. Author A 11 in discussing the fine-structure constant $\frac{ch}{2\pi e^2}$ (c = velocity of light, h = Planck's constant, e = charge of electron) says: "it is frequently referred to as "dimensionless," for example when EDDINGTON states that when we say that its value is 137 "we are eliminating all reference to our artificial standards." It is clear, however, that this is not so."

We should like to say that the fine-structure constant, being the ratio of two like quantities (the velocity of light and the velocity of the electron in the first orbit in a hydrogen atom) is truly dimensionless so that its value is independent of our standards.

It is only the expression for it that does depend on our standards (electrostatic cgs system: $\frac{ch}{2\pi e^2}$, practical system: $\frac{2ch\epsilon_0}{e^2}$).

The fact that the fine-structure constant is dimensionless should not be checked by substituting in the e.s. cgs definitional equation the dimensional formulae for the various quantities from another system, involving an ϵ_0 , as it has been done.

A 12. An interesting paradox is mentioned by POHL [4].

Any charge can be expressed in e.s. cgs units or in e.m. cgs units and it is therefore possible to equate the two, to give e.g.

$$1 \text{ e.m. cgs unit of charge} = 3 \times 10^{10} \text{ e.s. cgs units of charge} \quad (1)$$

Now the e.m. unit and the e.s. unit are related to fundamental units as follows:

$$1 \text{ e.m. cgs unit of charge} = 1 \text{ cm}^{1/2} \text{ g}^{1/2} \quad (2)$$

$$1 \text{ e.s. cgs unit of charge} = 1 \text{ cm}^{3/2} \text{ g}^{1/2} \text{ sec}^{-1} \quad (3)$$

Inserting (2) and (3) in (1) gives:

$$1 \text{ sec.} = 3 \times 10^{10} \text{ cm} \quad (4)$$

VERMEULEN [5] says the cause is to be sought in the essentially different symbolic nature of these products.

Following statement elaborates hereon: Eq. (2) holds within the dimensional system used in

forming e.m. units and *within* that system the product formations and so on can be regarded as normal mathematical routine.

Eq. (3) similarly holds within the dimensional system used in forming e.s. units.

In view of these restrictions the eq. (2) and (3) are in general incompatible and cannot be combined to give eq. (4).

The e.s. and e.m. systems would become identical if such units of length and time were used that the velocity of light, $c = 1$ (see Part I, page 134). This is precisely what is found in eq. (4).

A 18. Authors A 18 say that the diameter D_b of bubbles formed in boiling is proportional to

$$\sqrt{\frac{2 g_0 \sigma}{g (\rho_l - \rho_v)}}$$

and their list of symbols says:

D_b = diameter of bubble, ft.

g = acceleration of gravity.

g_0 = conversion factor

σ = surface tension, dynes/cm or lb./ft.

ρ_l, ρ_v = densities of liquid and vapour, lb./cu.ft.

If lb_m and lb_f are not distinguished, g and g_0 are identical.

B. *Inconsistencies in the substitution of numerical values in equations.*

B 1. Authors B 1 in studying the resistance of pipes calculate a group $\frac{\Delta P}{\rho u^2} \frac{D}{L}$ and plot it against a Reynolds number.

The units for ΔP and ρ being derived from lb_f and lb_m respectively the above group should have been made dimensionless by including a factor $g_c = 32.2 \frac{\text{lb}_m \text{ ft.}}{\text{lb}_f \text{ sec.}^2}$.

The "friction factor" - Re graph as given, accordingly differs from those obtained by a consequent application of coherent units.

See also example B 6.

B 2. Authors B 2 define the coefficient of friction ψ for the turbulent flow of an incompressible fluid in a smooth pipe by

$$p_1 - p_2 = \psi \frac{l}{d} \cdot \frac{1}{2} \rho w^2$$

with $p_1 - p_2$ = pressure drop (in cm water) over the length l (in cm)

d = diameter in cm

w = mean velocity in cm/sec.

ρ = density in g/cm³

ψ = dimensionless.

With the other units as prescribed, $p_1 - p_2$ should have been expressed in dynes/cm² = g cm⁻¹ sec.⁻². The ratio between a pressure difference and a liquid head is ρg , i.e. for water, 981.

B 3. Authors B 3 describe the additivity of resistances in mass transfer between phases. With the units given, k_L is in cm/hr. and the product of the Henry coefficient with K_G is in litre/hr. cm².

B 4. Authors B 4 give the following equation for the heat transport through a very dilute gas between parallel walls at temperatures T_1 and T_2 :

$$Q = \frac{8}{8} p \sqrt{\frac{8R}{MT}} (T_1 - T_2)$$

where : p = pressure

T = average temperature

M = molecular weight

R = gas constant.

Q is not defined but the general list of symbols says : Q = quantity of heat, with dimension : kcal.

The Q in the above equation represents heat per area time but Q and R can only be expressed in mechanical units.

B 5. Author B 5 gives for water at 100°C : specific heat

$$c_p = 1.011 \text{ kcal kg}^{-1} \text{ }^\circ\text{C}^{-1}$$

viscosity

$$\mu = 8.0 \times 10^{-9} \text{ kg hr. m}^{-2} \text{ (i.e. kg}_t \text{ hr. m}^{-2})$$

thermal conductivity

$$k = 0.588 \text{ kcal hr.}^{-1} \text{ m}^{-1} \text{ }^\circ\text{C}^{-1}$$

and N_{Pr} (i.e. Prandtl group)

$$= 1.75 \text{ (dimensionless).}$$

The Prandtl group had been previously defined as $\frac{\mu c_p}{k}$. The above data would give $\frac{\mu c_p}{k}$

$\frac{1.75}{1.80 \times 10^8} \frac{\text{kg m}}{\text{kg}_t \text{ hr.}^2}$. It is immaterial whether the c_p is thought to be referred to kg_t or kg_m , the difference being that the forgotten multiplier is to be interpreted as g or g_c .

In order to arrive at the correct result : either the dimensional system should be adjusted to the units by the inclusion of the dimensional constant g or g_c , or the units adjusted to a dynamical dimensional system by expressing μ in $\text{kg}_m \text{ m}^{-1} \text{ hr.}^{-1}$.

Remark—This case is closely related to B 11 and B 12.

B 6. Author B 6 defines the Reynolds group by

$$Re = \frac{Dus}{z}$$

where : D = diameter of pipe (inches)

u = velocity (ft./sec.)

s = specific gravity referred to water at 60°F (dimensionless)

z = absolute viscosity in centipoises.

The value differs from the usual one by a factor $\frac{1}{7740}$. The friction factor versus Re graph changes accordingly.

It is felt that by this procedure the great advantage of dimensionless groups, their universality, is lost.

B 7 and B 8. Authors B 7 and author B 8 prescribe to express diameter in ft., velocity in ft./sec., density in lb./cu.ft. and viscosity in lb./sec.hr. The Reynolds group thus calculated would be a factor 3600 low.

B 9. Authors B 9 = B 7 in their Notation prescribe noncoherent units, e.g. :

particle diameter D_p , mm

gas velocity U , ft.sec.⁻¹

density ρ , g.cm⁻³

viscosity, μ , centipoise

diffusivity D_o , sq.ft. hr.⁻¹

These cannot have served in calculating the dimensionless groups $Re = \frac{D_p U \rho}{\mu}$ and $Sc = \frac{\mu}{\rho D_o}$, the numerical values of which are given correctly.

B 10. Authors B 10 in their Notation leave alternative choices, viz. diffusivity D in sq.cm/sec. or sq.cm/day; viscosity μ in centipoises or lb./sec.ft. The density ρ is expressed in lb./cu.ft.

It is also impossible to select a coherent set of data to substitute in $\frac{\mu}{\rho D}$ = the Schmidt group.

B 11. Author B 11 defines $Pr = \frac{\nu}{a} \left(= \frac{\eta c_p}{\lambda} \right)$ and gives following data in his tables :

$$\begin{aligned} c_p &= \text{in kcal/kg } ^\circ\text{C} \\ \eta &= \text{in kg sec./m}^2 \\ \lambda &= \text{in kcal/m hr. } ^\circ\text{C} \\ Pr &= \text{dimensionless} \end{aligned}$$

However, with these units $\frac{\eta c_p}{\lambda}$ is expressed in $\frac{\text{hr. sec.}}{\text{m.}}$ and the multiplication with $3600 \frac{\text{sec.}}{\text{hr.}} \times 9.81 \frac{\text{m}}{\text{sec.}^2}$ is left to the reader.

B 12. Author B 12 does the same as author B 11.

B 13. Author B 13 defines $Pr = \frac{c_p \mu}{k}$ with the following units :

$$\begin{aligned} c_p &\text{ in BTU/lb. mole } ^\circ\text{F} \\ \mu &\text{ in lb./sec.ft.} \\ k &\text{ in BTU/hr. ft. } ^\circ\text{F} \end{aligned}$$

Note inconsistencies : lb. mole and lb., hr. and sec.

The Sc group cannot be calculated either.

5. THE SUITABILITY OF VARIOUS SYSTEMS OF UNITS TO THE CHEMICAL ENGINEER

In this section we shall :

- Enumerate the properties, desirable in a system of units ;
- Discuss these desiderata and examine in how far they can be reached ;
- Compare various systems.

A. Desiderata

For convenience and clarity the following is considered desirable :

General requirements

- To use a coherent system of units in the particular field in which the chemical engineer is working.
- To let such a coherent system of units cover at least the fields of mechanics and heat.
- To strive to a one-to-one correspondence between the quantities and their units and/or dimensional formulae, i.e.
 - Each quantity should have a single dimensional formula and unit.
 - Each dimensional formula and unit should correspond to a single quantity only (see FLEISCHMANN [7]).
 - As a logical consequence of 3b physical quantities should not be dimensionless.
- To use a small number of dimensional constants.
- To employ such dimensional formulae and to express the derived units in such a manner that clarity is best served. This means that it should be obvious how the chosen expression is related to a definitional equation for the quantity under consideration. In this way it is easiest to remember the unit for the given quantity and vice versa.
- To choose such a system that the acceleration by gravity appears only when gravity actually plays a role.

Specific requirements re units

- Units should be well defined.
- Use *must* be made of following units :

	<i>metric</i>	<i>British</i>
length	m or cm	ft.
mass and/or force	kg or g	lb.
time	sec.	sec. or hr.
temperature	$^{\circ}\text{C}$	$^{\circ}\text{F}$ or $^{\circ}\text{C}$

- Due consideration must be given to the systems used by physicists, mechanical engineers and electrical engineers, since their equations and data will be used.

B. Discussion of desiderata

1. Coherent units

A great majority of physicists and engineers use a coherent system of units. If their basic data

are given in other units, they carry out the conversions before substituting them in the equations.

It should however, be placed on record that it is not necessary to do so (see discussion on THORNE and WALSHAW's method, Part II, page 169).

2. Width of field

Special systems are sometimes used in a restricted field. Permeability of porous media, for instance, is expressed in darcy's. We might say that there is an "atm./cm; cm/sec.; centipoise; darcy system" for the use in Darcy's Law only.

As long as this is only applied in this particular field it works satisfactorily. If, however, permeability is related to particle size, shape and porosity it becomes more practical to express it in m^2 or cm^2 .

Likewise, in heat transfer there is a preference for hours, in other fields for seconds. Inconsistencies in the use of seconds and hours become apparent in examples B 7-8-9-11-12 and 13. It is considered that a consistent system for mechanics and heat is to be preferred and that it should be based on the second.

The various static, dynamic and engineering systems, amplified with a unit of temperature and in some cases with a heat unit, as discussed, all fulfil this requirement.

In the following it is taken for granted that such a coherent system will be used.

3. "One-to-one" correspondence, and

4. Small number of dimensional constants

Rule 8a asks for a large number of dimensional constants; a compromise with rule 4 must thus be found.

Notable violations of 8a are :

cgs system applied to electricity and magnetism, giving two sets of units ;

English Engineering system (old and new), giving alternative choices for various units.

Notable violations of 8b are that the same unit is employed

<i>in</i>	<i>for</i>
cgs e.s. system	electric capacity and length

<i>in</i>	<i>for</i>
cgs e.m. system	inductance and length
English Engin. system, old version	force and mass

Notable violations of 8c are :

cgs e.s. system	dielectric constant
cgs e.m. system	magnetic permeability
heat units described in Part I, p. 138-9, ad 3	specific heat, specific entropy

The necessity to use a small number of dimensional constants can be underlined by reference to frequent conversions requiring the mechanical equivalents of heat and electrical energy and to a considerable amount of misunderstanding about the conversion factor g_c (Par. 4, Ex. A 1, 3, 4, 13).

The practical system of course introduces μ_0 and ϵ_0 but avoids the conversion from practical electrical units to cgs units and back.

Further refinement in the sense of requirement 3 would be possible by distinguishing the three length co-ordinates in dimensional analysis and in units, or perhaps by introducing vector concepts. This is considered too complicated for general use, i.e. it is considered inevitable that torque and work have the same unit and that an angle (= arc/radius) will be dimensionless.

5. Clarity of dimensional formulae

Regarding clarity of dimensional formulae it is essential to introduce one new fundamental unit when entering a new field of physics, i.e.:

temperature in the field of heat (all systems discussed, see Part I, p. 138) ;

one electrical unit in the field of electricity and magnetism (practical system, see Part I, p. 139).

This is contrary to the principle of making so-called absolute units, based on mechanics alone, which for a long time has been the ultimate aim of physicists.

Requirements 3 and 5 are closely related, both acting contrarily to No. 4.

6. The place of "g" in equations

Requirement 6 disallows static systems (for the

reason explained in Part I, pp. 137-8) and the old version of the English Engineering system (Part I, p. 136).

7. Definitions of units

In order to properly define a unit of force, with the aid of a standard kilogram, or pound, it is necessary to specify a standard acceleration of gravity ($g = 980.665 \text{ cm/sec.}^2$ at 45° latitude at sea level). Consequently the force of gravity acting on 1 lb_m elsewhere in general is *not* 1 lb_f . This reduces the supposed simplicity of static systems.

8. Use of existing units

All systems discussed in this paper conform to this requirement.

9. Units used in physics and various branches of engineering

In general physicists prefer dynamic systems and mechanical engineers static systems.

Static systems have the advantage of having well understood units for force, pressure, etc. (derived from kg_f , g_f , lb_f) but the disadvantage of not having suitable mass units. This condemns them for use in general physics.

Dynamic systems have suitable mass units but the unit of force (newton, dyne, poundal) is considered less suited by mechanical engineers.

The English Engineering system allows conventional units for both force and mass, however, at the expense of conformity to requirement 3a.

C. Comparison of systems

It is considered that the English Engineering system (new version) has more advantages than disadvantages when compared to static systems and to the English Engineering system (old version).

Hence the choice is one between the English Engineering system (new version) – or its metric equivalent – and dynamic systems.

It should be noted that E. SCHMIDT declares himself in favour of a kg_f , kg_m , m, sec. system but does not use it since there is no further literature in it.

The comparison between engineering systems (new version) and dynamic systems is different for British and metric units.

In British units the argument to be in line with physics does not count so much, much of the physical data being in metric units. The entire field of electricity is in the metric system. Therefore the English Engineering system may be a very suitable compromise to satisfy the mechanical engineers.

Its disadvantages were noted to be the following:

1. Alternative possibilities of defining dimensions of physical quantities with consequent alternatives where to place the dimensional constant g_c .
2. Difficulty to grasp the significance of g_c .

A variety of false statements have been found (examples A 2, 3 and 4).

It is necessary that the pounds (mass and force) are well distinguished. Simplest, but not self-evident, are lb and Lb . [6]. In this paper, lb_m and lb_f have been used. The abbreviations lb (mass) and lb (force) are also employed.

In metric units contact with physics counts much more. Here a dynamical system is considered to be at its place. The choice between cgs and practical units in physics is largely governed by the fact that practical units are indispensable in electricity and signs are that cgs units will be completely abolished in this field. There is accordingly a strong current to do the same through all physics. Hence it is considered that the metric chemical engineer should use the practical system.

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