Architecture 324

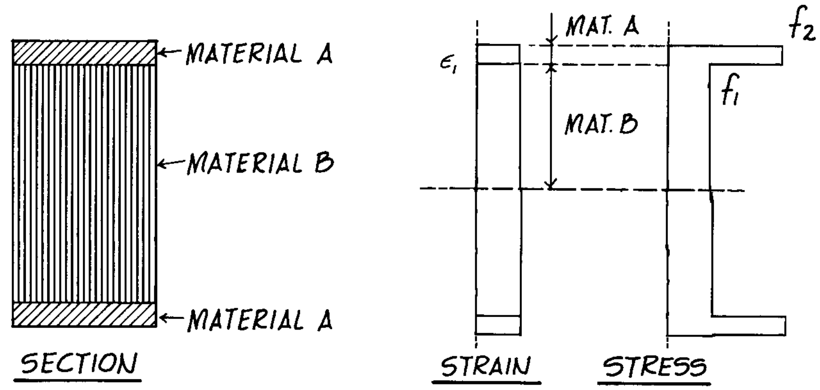
Structures II

Combined Materials

- Strain Compatibility
- Transformed Sections
- Flitched Beams

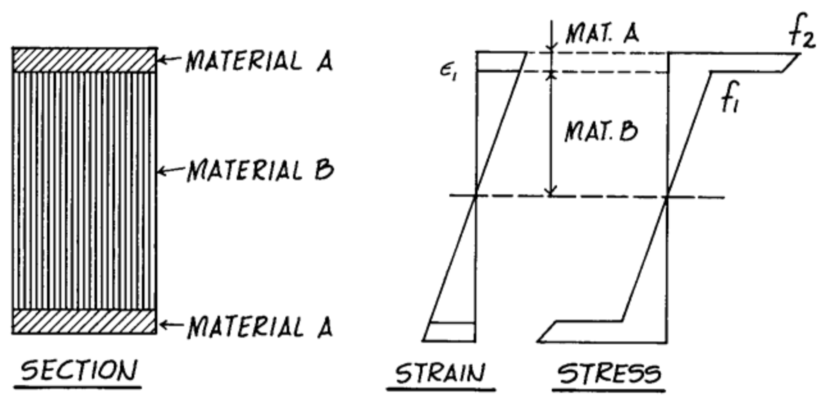
Strain Compatibility

With the two materials bonded together, both will act as one and the deformation in each is the same.



Therefore, the strains will be the same in each under axial load, and in flexure stains are the same as in a solid section, i.e. linear.

In flexure, if the two materials are at the same distance from the N.A., they will have the same strain at that point. Therefore, the strains are “compatible”.

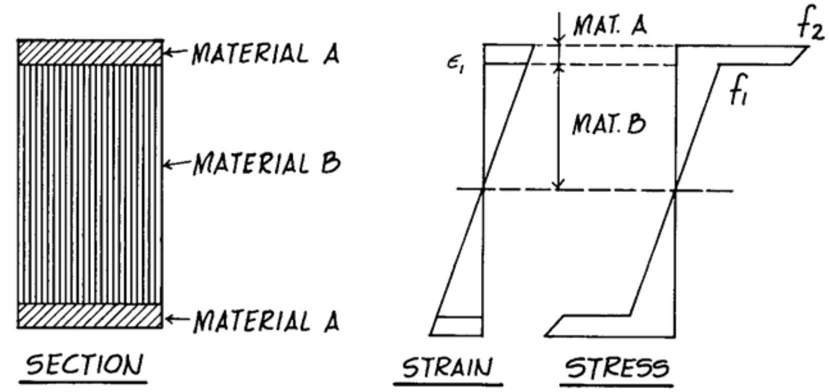


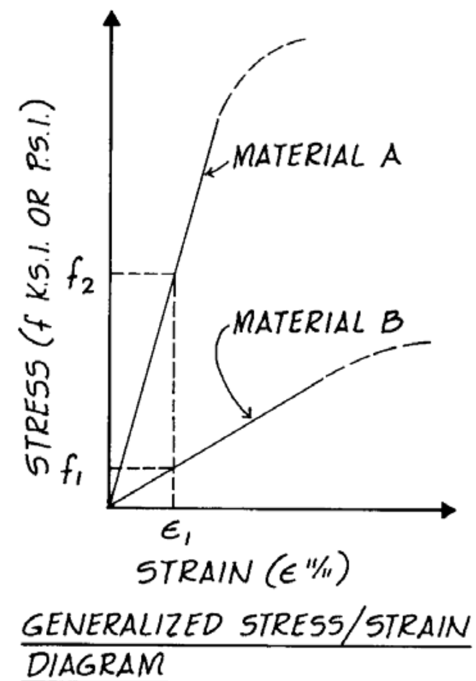
Strain Compatibility (cont.)

The stress in each material is determined by using Young's Modulus

$$\sigma = \mathrm{E}\varepsilon$$

Care must be taken that the elastic limit of each material is not exceeded, in either stress or strain.





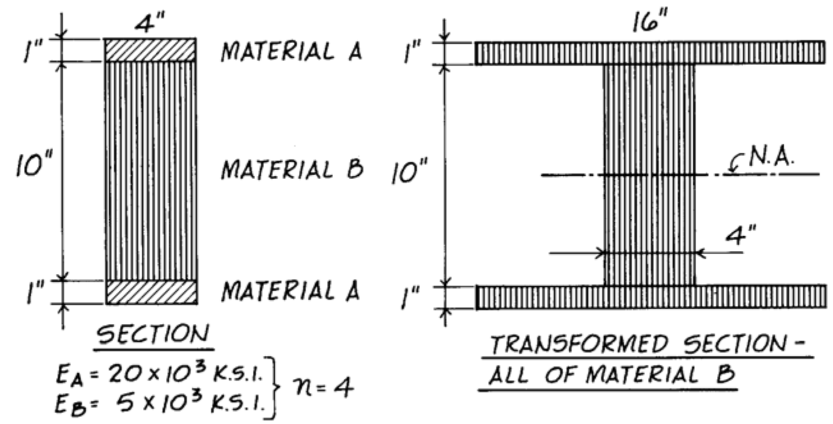
Transformed Sections

Because the material stiffness E can vary for the combined materials, the Moment of Inertia, I , needs to be calculated using a "transformed section".

In a transformed section, one material is transformed into an equivalent amount of the other material. The equivalence is based on the modular ratio, n.

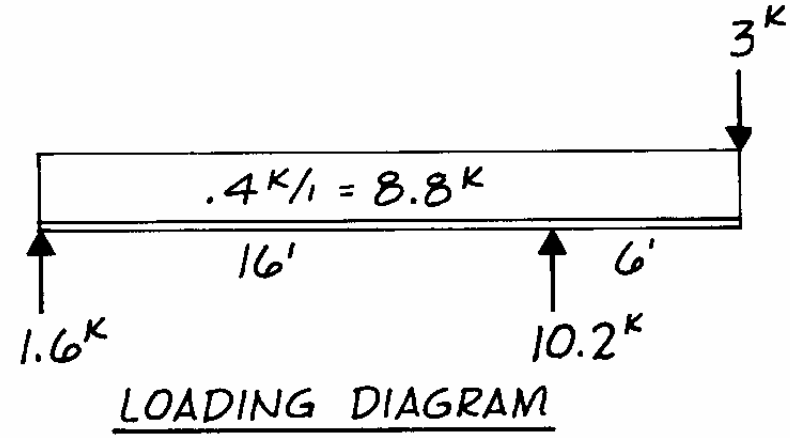
$$n = \frac{E_A}{E_B}$$

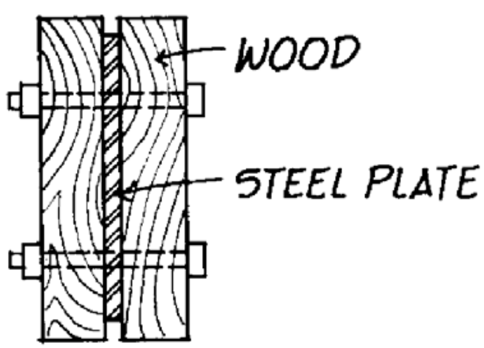
Based on the transformed section, I_{tr} can be calculated, and used to find flexural stress and deflection.

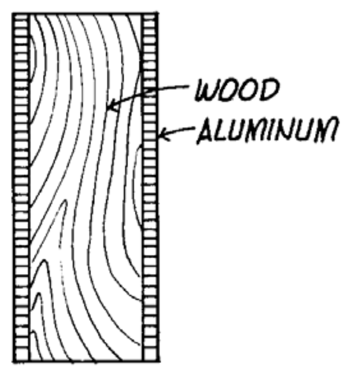


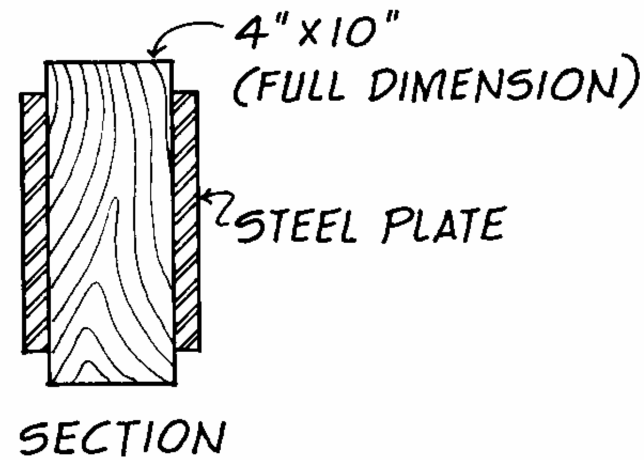
Flitched Beams

- Compatible with wood structure, i.e. can be nailed
- Lighter weight than steel section
- Less deep than wood alone
- Stronger than wood alone
- Allow longer spans
- Can be designed over partial span as optimized section (scab plates)

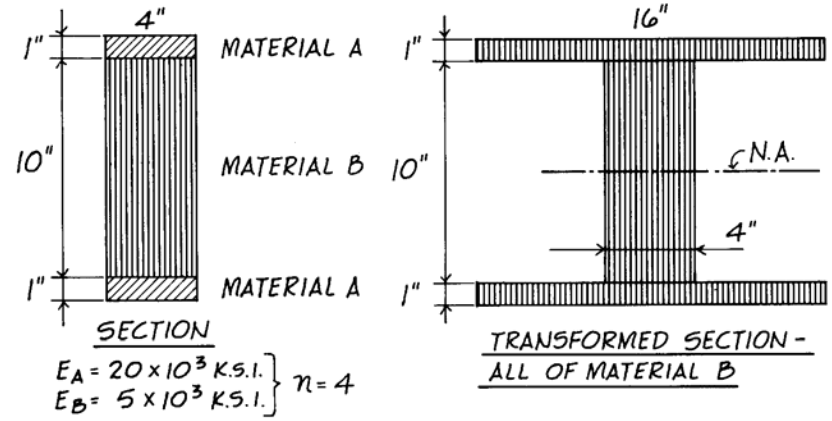








Analysis Procedure:



1. Determine the modular ratio(s). Usually the weaker (lower E) material is used as a base.

2. Construct a transformed section by scaling the width of the material by its modular, n.

3. Determine the Moment of Inertia of the transformed section.

4. Calculate the flexural stress in each material using

$$f_b = \frac{Mc\ n}{I_{tr}}$$

$$n = \frac{E_A}{E_B}$$

Transformed material

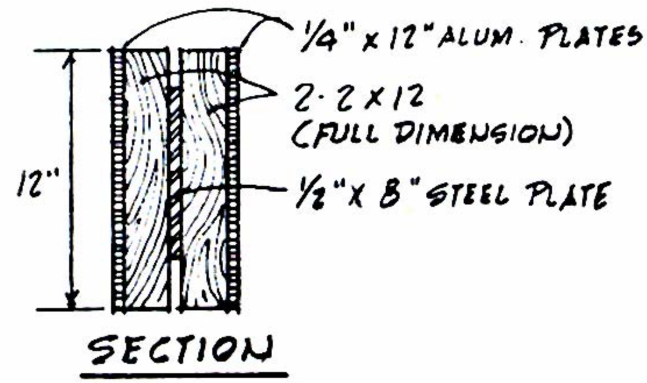
Base material

Analysis Example:

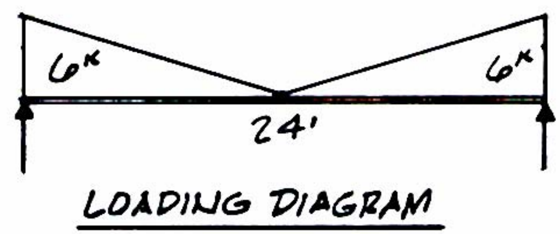
For the composite section, find the maximum flexural stress level in each laminate material.

Determine the modular ratios for each material.

Use wood (the lowest E) as base material.



WOOD: $E = 1.5 \times 10^3$ K.S.I.
STEEL: $E = 30 \times 10^3$ K.S.I.
ALUM: $E = 12 \times 10^3$ K.S.I



$$n_{WOOD} = \frac{1.5}{1.5} = 1.0$$

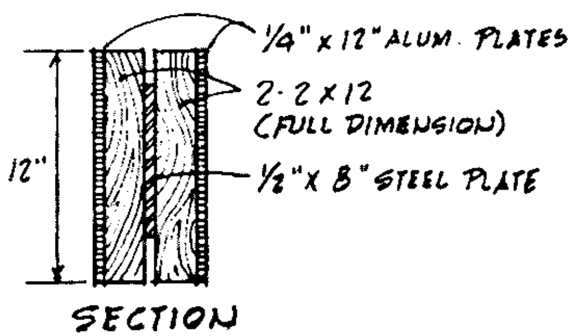
$$n_{AL} = \frac{12}{1.5} = 8.0$$

$$n_{ST} = \frac{30}{1.5} = 20.$$

Analysis Example cont.:

Determine the transformed width of each material.

Construct a transformed section.



ALUM.

$t = 1/4''$

$t_{tr} = 1/4 \times n_{AL}$

$= 1/4\,(8.0) = 2.0''$

STEEL

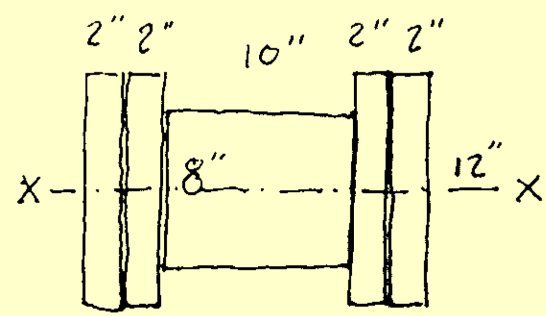
$t = 1/2''$

$t_{tr} = 1/2 \times n_{ST}$

$= 1/2\,(20) = 10.''$

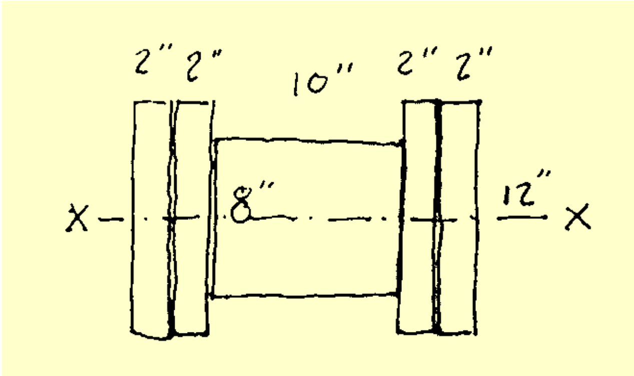
WOOD

$t = t_{tr} = 2''$



Analysis Example cont.:

Construct a transformed section.



Calculate the Moment of Inertia for the transformed section.

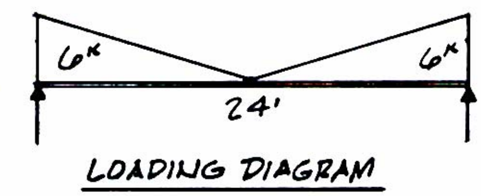
$$I_{tr} = \frac{8\ 12^3}{12} + \frac{10\ 8^3}{12}$$

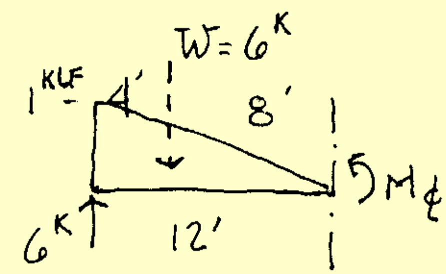
$$= 1152 + 426$$

$$= 1578\ in^4$$

Analysis Example cont.:

Find the maximum moment.





$$\Sigma M_{@℄} = 0 =$$

$$6(12') - 6(8') - M_{℄} = 0$$

$$M_{℄} = 24^{K-'}$$

Analysis Example cont.:

Calculate the stress for each material using the modular ratio to convert I.

$$I_{tr} / n = I$$

Compare the stress in each material to limits of yield stress or the safe allowable stress.

$$f_{AL} = \frac{Mc(n)}{I_{tr}} = \frac{24(12)(6'')\,8}{1578}$$

$$= 8.76 \text{ KSI} \quad (f_y \approx 35 \text{ KSI})$$

$$f_{ST} = \frac{Mc(n)}{I_{tr}} = \frac{24(12)(4'')\,20}{1578}$$

$$= 14.6 \text{ KSI} \quad (f_y \approx 36 \text{ KSI})$$

$$f_{WD} = \frac{Mcn}{I_{tr}} = \frac{24(12)(6'')\,1.0}{1578}$$

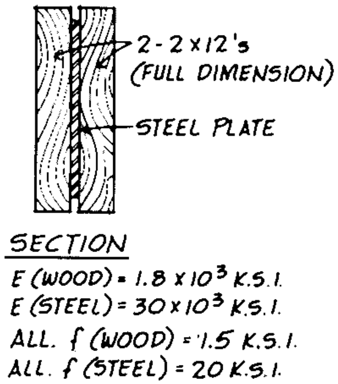
$$= 1.09 \text{ KSI} \quad (f_y \approx 1.5)$$

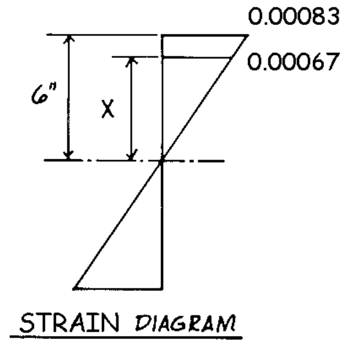
Design Procedure:

Given: Span and load conditions
Material properties
Wood dimensions

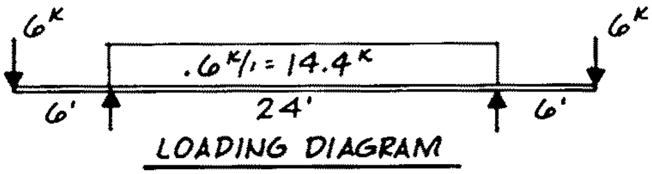
Req'd: Steel plate dimensions

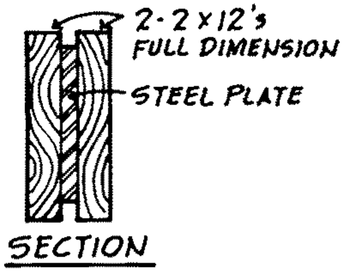
1. Determine the required moment.
2. Find the moment capacity of the wood.
3. Determine the required capacity for steel.
4. Based on strain compatibility with wood, find the largest d for steel where $\epsilon_s < \epsilon_y$.
5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.





Design Example:

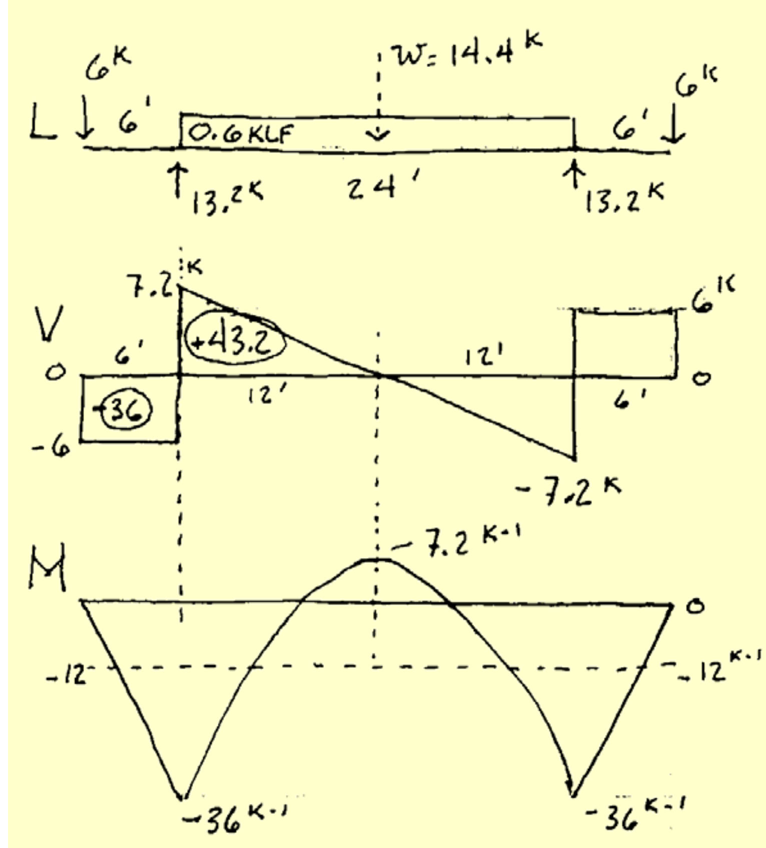




WOOD: E = 2000 K.S.I. f ALL. = 1.5 K.S.I.
STEEL: E = 30000 K.S.I. f ALL. = 18 K.S.I.

(A) DETERMINE THE DIMENSIONS OF THE STEEL PLATE REQUIRED FOR NEGATIVE MOMENT.

(B) DETERMINE THE LENGTH OF THE PLATES REQUIRED FOR NEGATIVE AND POSITIVE MOMENT.



1. Determine the required moment.
2. Find the moment capacity of the wood.
3. Determine the required capacity for steel.

WOOD

$b = 2''$ $d = 12''$

$S_x = \frac{bd^2}{6} = \frac{2(144)}{6} = 48 \text{ in}^3$

$\times 2$ pcs. $S_{WOOD} = 96 \text{ in}^3$

$M_{WOOD} = F_b S_x$

$= 1.5^{KSI} \, 96 \text{ in}^3 = 144^{K-''}$

$= 12^{K-'}$

$M_{TOTAL} = M_{WOOD} + M_{STEEL} = 36^{K-'}$

$M_{STEEL} = 36^{K-'} - 12^{K-'} = 24^{K-'}$

Design Example cont:

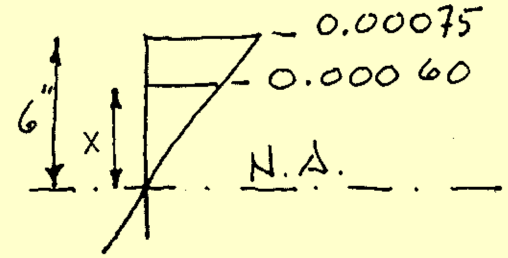
4. Based on strain compatibility with wood, find the largest d for steel where $\in_s < \in_y$.

ALLOWABLE STRAINS

$$\epsilon_w = \frac{f}{E} = \frac{1.5}{2000} = 0.00075$$

$$\epsilon_s = \frac{f}{E} = \frac{18}{30000} = 0.00060$$

STRAIN DIAGRAM



$$\frac{75}{6} = \frac{60}{x} \qquad x = 4.8''$$

$\therefore$ d STEEL PLATE = 9.6"

Design Example cont:

5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.

STEEL

$$M_{STEEL} = 24^{K\text{-}'} = 288^{K\text{-}''}$$

$$F_{ST} = 18^{KSI} \quad \text{(GIVEN)}$$

$$S'_x = \frac{M}{F} = \frac{288}{18} = 16 \text{ in}^3$$

STEEL PLATE

$$S_x \text{ REQ'D} = 16 \text{ in}^3 = \frac{bd^2}{6}$$

$$b = \frac{S_x\, 6}{d^2} = \frac{16(6)}{9.6^2} = 1.042''$$

ROUND TO $\frac{1}{8}''$ = $1\frac{1}{8}''$ (SAFE)

OR $1''$ (~OK)

∴ USE

$9.6'' \times 1''$

OR

$9.5'' \times 1\frac{1}{8}''$

Design Example cont:

8. Determine required length and location of plate.

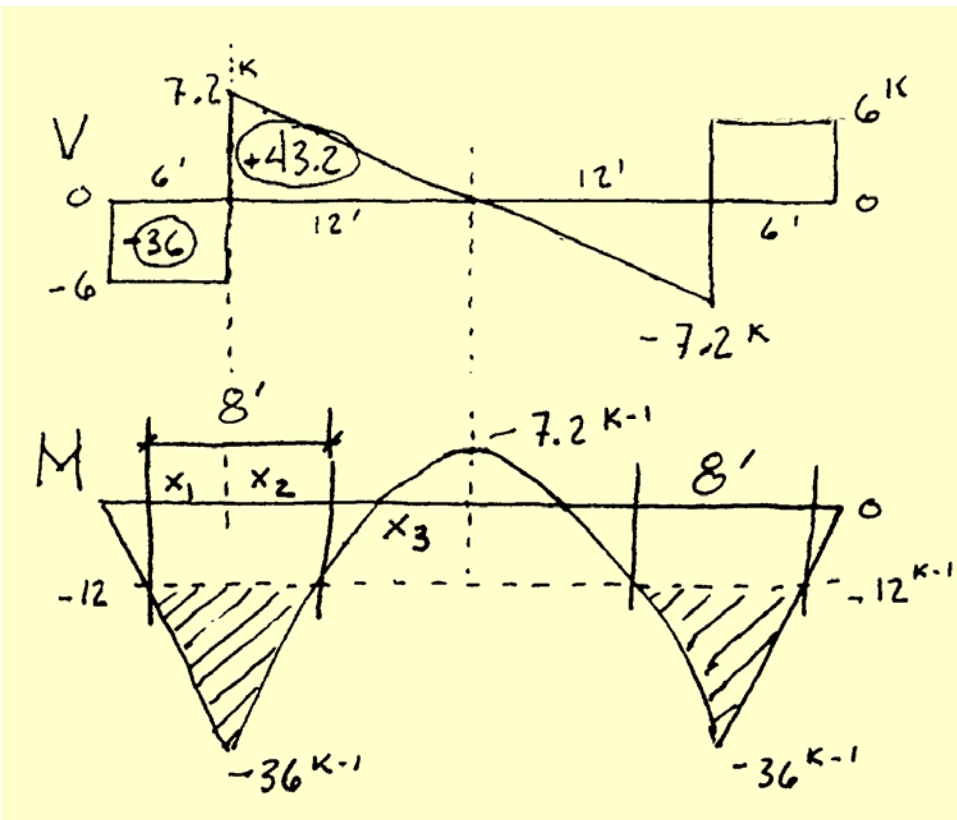
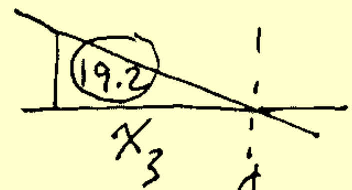


PLATE LENGTH

$\frac{36}{24} \quad \frac{6}{x_1} \qquad x_1 = 4'$

SHEAR AREA = 24

$43.2 - 24 = 19.2$



$\frac{bd}{2} = 19.2$

$\frac{x_3\left(\frac{7.2}{12} x_3\right)}{2} = 19.2$

$x_3^2 = 64 \qquad x_3 = 8'$

$x_2 = 12 - x_3 = 4'$

$\therefore$ PLATE LENGTH = 8'

Applications:

Applications:

Broadford Farm Pavilion, Hailey, Idaho
Lake/Flato Architects, San Antonio