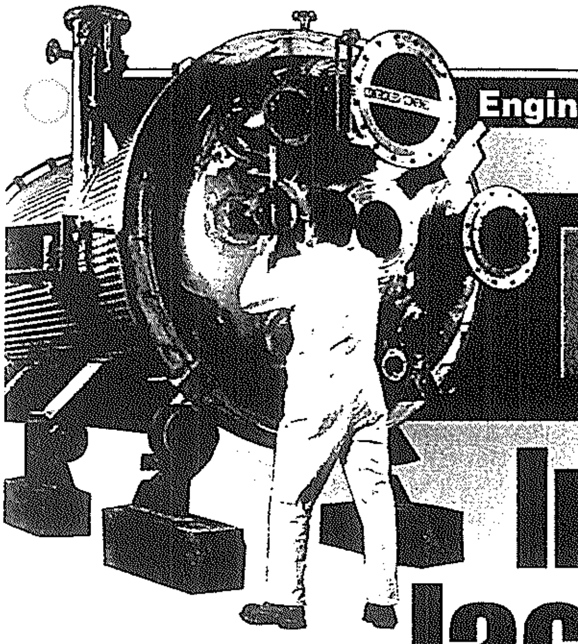

Engineering Practice

Heat Transfer In Agitated Jacketed Vessels

Robert F. Dream, P.E.
Lockwood Greene Engineers, Inc.

The principles are straightforward, but their application is tricky

Agitator-equipped vessels with jackets for heating or cooling are drawing increased attention in the process industries. Apart from a growing importance in biotechnology (box, p. 94), such vessels are widely used in a variety of other process applications. Accordingly, engineers can benefit from a working knowledge of how heat-transfer and temperature-control principles apply to such vessels.

The rate of heat transfer to or from an agitated liquid mass in a vessel is a function of the physical properties of that liquid and of the heating or cooling medium, the vessel geometry, and the degree of agitation. The type, and size of the agitator, as well as its location in the vessel, also affect the rate.

These values of the agitator parameters are set by the given mixing task (such as suspending or dispersing solids or gases, emulsifying an immiscible liquid, or fostering chemical reactions), usually before their effects upon heat transfer are considered. But if during operation, the course of the process proves to be governed mainly by the heat transfer, then such variables as log mean temperature difference and heat-transfer surface area will usually take on more significance than the agitation variables. In either case, the mixing can affect only the heat-transfer resistance on the inner vessel wall, which (as pointed out in Equation [6]) is but one of the resistances that determine the overall heat-transfer coefficient.

Many jacketed vessels are reactors, so exothermic or endothermic effects must be taken into account. Furthermore, in many applications employing jacketed vessels, successive batches of material are heated (or cooled) to a given temperature, so the heat transfer is unsteady-state.

Quick review sets the stage

In a vessel containing an agitated liquid, heat transfer takes place mainly through conduction and forced convection, as it does in heat exchangers. So, the starting point for heat-transfer calculations involving such vessels is the resistance or film theory that applies to exchangers. The heat flow and the calculation procedures may best be explained by building step by step upon the basic film-theory equation:

$$Rate = \frac{Driving\ Force}{Resistance} \quad (1)$$

where the heat-flow rate per unit area is in (for instance) Btu/(h)(ft^2), the driving force is the temperature difference in degrees Fahrenheit, the resistance is the reciprocal of heat conductance U, and U is in Btu/(h)(ft^2)(°F).

Equation (1) can be written as:

$$\frac{\Delta T}{\frac{1}{U}} = \frac{Q}{A} \quad (2)$$

where ΔT is the temperature difference in Fahrenheit degrees, Q is the heat-transfer rate in Btu/h, and A is the heat-transfer area in square feet.

Continuous operation

In the simplest, idealized situation, the vessel and its jacket each operate continuously under isothermal conditions. Under those circumstances, Equation (2) becomes transformed simply into

$$Q = UA\Delta T \quad (3)$$

and applied directly.

In the more realistic continuous situation, in which the vessel contents are at constant temperature but with different jacket inlet and outlet temperatures, the general equation becomes:

$$Q = UA\Delta T_{lm} \quad (4)$$

where ΔT_{lm} is the log mean temperature difference between the bulk temperature of the vessel contents, t, and the temperature in the jacket, T:

$$\Delta T_{lm} = \frac{(t_2 - T_2) - (t_1 - T_1)}{\ln[(t_2 - T_2)/(t_1 - T_1)]} \quad (5)$$

In this equation, Subscripts 1 and 2 refer to the temperatures of the entering and exiting fluids, respectively

The overall heat transfer coeffi-

Line No.	Agitator Type	Baffled?	N_{Re}	N_{Nu}	Remarks
TABLE 1. EQUATIONS FOR CALCULATING INSIDE FILM COEFFICIENTS (H_I) OF JACKETED AGITATED VESSELS					
1	Flat-blade turbine	Yes	>400	$0.74(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14}$	$D/D_T = 1/3$, $Z/D_T = 1.0$. Six-bladed turbine. Standard geometry. References (6, 7)
2	Flat-blade turbine	Yes/No	<400	$0.54(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14}$	$D/D_T = 1/3$, $Z/D_T = 1.0$. Six-bladed turbine. Standard geometry. References (6, 7)
3	Flat-blade turbine	Yes	>400	$0.85(N_{Re})^{0.66}(N_{Pr})^{0.33}(Z/D_T)^{-0.56} \times (D/D_T)^{0.13}(\mu/\mu_w)^{0.14}$	Non-standard geometry. General equation. Reference (8)
4	Retreating-blade turbine	No	No limitation*	$0.68(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14}$	For agitators with six retreating blades, refer to References (6, 11). For other related geometries, see References (9, 10)
5	Retreating-blade turbine	Yes	No limitation*	$0.33(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14}$	Glassed-steel impeller, three retreating blades. The lower constant (0.33) for the glassed-steel impeller is attributed to greater slippage around its curved surfaces than around the sharp corners of the alloy-steel impeller. Reference (9)
6	Retreating-blade turbine	Yes	No limitation*	$0.37(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.24}$	Alloy-steel impeller. Three retreating blades. Reference (9)
7	Propeller	Yes	No limitation*	$0.54(N_{Re})^{0.67}(N_{Pr})^{0.25}(\mu/\mu_w)^{0.14}$	45-deg-pitched, four-blade impeller. Equation is based on limited data with regard to propeller pitch and vessel baffling for design purposes. Divide the h_i obtained with this equation by a factor of about 1.3. Reference (12)
8	Paddle	Yes/No	>4,000	$0.36(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14}$	Vessel geometry is provided by Holland and Chapman, (11). Reference (13)
9	Paddle	Yes/No	$20<N_{Re}<4{,}000$	$0.415(N_{Re})^{0.67} \times (N_{Pr})^{0.33}(\mu/\mu_w)^{0.24}$	Vessel geometry is provided by Holland and Chapman (11). Reference (14)
10	Anchor	No	$30<N_{Re}<300$	$1.0(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.18}$	Vessel geometry is depicted in Reference (11). The overall coefficient, U, varies inversely with the anchor-to-wall clearance (15). Anchor-to-wall clearance is less than 1 in. Reference (14)
11	Anchor	No	$300<N_{Re}<4{,}000$	$0.33(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.18}$	Same as in Line 10
12	Anchor	No	$4{,}000<N_{Re}<37{,}000$	$0.55(N_{Re})^{0.67} \times (N_{Pr})^{0.25}(\mu/\mu_w)^{0.14}$	Vessel geometry is depicted in Reference (11). The overall coefficient, U, varies inversely to the anchor-to-wall clearance (15). Anchor to wall clearance of 1 to 5.125 in. Reference (12).
13	Helical ribbon	No	<130	$0.248(N_{Re})^{0.50}(N_{Pr})^{0.33} \times (\mu/\mu_w)^{0.14}(e/D)^{-0.22}(i/D)^{-0.28}$	e = Clearance, $(D_T - D)/2$, ft, D = impeller diameter, ft, i = agitator-ribbon pitch, ft. Reference (16)
14	Helical ribbon	No	>130	$0.238(N_{Re})^{0.67}(N_{Pr})^{0.33} \times (\mu/\mu_w)^{0.14}(i/D)^{-0.25}$	Same as Line 13

*The data provided do not mention any Reynolds number limitations for the use with the Nusselt expression.

cient, U, is found from the equation

$$\frac{1}{U} = \frac{1}{h_i} + ff_i + \frac{x}{k} + ff_j + \frac{1}{h_j} \qquad (6)$$

where ff_i and ff_j are respectively the fouling factors that apply inside the vessel and inside the jacket, both in (h)(ft^2)(°F)/Btu, h_i is the film coefficient on the process side (inside surface of the jacketed vessel) of the heat transfer area, in Btu/(h)(ft^2)(°F), h_j is the film coefficient on the inside surface of the jacket (in the same units), x is the thickness of the vessel wall, in feet, and k is the thermal conductivity of the wall, in Btu/(h)(ft^2)(°F/ft).

The values for the film coefficients h are found by rearranging the relationship $N_{Nu} = hD_T/k$, which is the defining equation for N_{Nu} the dimensionless Nusselt number (see box, p. 92), and determining the appropriate value for N_{Nu} via the relationships in Tables 1 and 2. The procedure is illustrated in the example at the end of this article.

In this discussion, we make two assumptions. First, we assume that the fluid specific heats do not vary with temperature. Second, we assume that the convection heat-transfer coefficients (h_i and h_j of Equation [6]) are constant throughout the heat transfer area.

In applying the equations in these tables, take care that the physical-property data are accurate. Of particular concern is the thermal conductivity, k, whose values may not always be readily available and can vary widely.

The system being designed should, if possible, be similar geometrically to the agitated vessel for which the applicable equations in Tables 1 and 2 were developed. For instance, all the equations presented in the tables are for vessels with dished bottoms. Other geometry-related comments appear in the Remarks column of the tables.

Batch heating and cooling

In batch operations, it is often necessary to calculate the time, θ, needed to heat or cool the contents of a jacketed vessel from temperature t_1 to t_2. In a simplified form, the relevant equations are as follows:

For heating:

$$\theta = \left(\frac{mc_p}{UA}\right)\ln\left(\frac{T-t_1}{T-t_2}\right) \quad (7)$$

And for cooling:

$$\theta = \left(\frac{mc_p}{UA}\right)\ln\left(\frac{t_1-T}{t_2-T}\right) \quad (8)$$

where T is the jacket temperature (see next paragraph), m the mass of material in the vessel and c_p the specific heat of this material.

Equations (7) and (8) assume that the jacket temperature, is constant. These equations can also be used in instances where the difference between the jacket inlet and outlet temperatures is not greater than 10% of the log mean temperature difference between the average temperature of the jacket and the temperature of the vessel's contents. In applying these equations to such instances, assign T the value of the average jacket temperature.

If, instead, the difference between jacket inlet and outlet temperature is greater than 10% of the just-mentioned log mean temperature difference, then apply Equation (9) or (10):

For heating:

$$\theta = \ln\left(\frac{T_1-t_1}{T_1-t_2}\right)\left(\frac{mc_p}{WC}\right)\left(\frac{k}{k-1}\right) \quad (9)$$

For cooling:

$$\theta = \ln\left(\frac{t_1-T_1}{t_2-T_1}\right)\left(\frac{mc_p}{WC}\right)\left(\frac{k}{k-1}\right) \quad (10)$$

where T_1 is the jacket inlet temperature, W the mass flow rate through the jacket, C the specific heat of the fluid in the jacket and k is defined by the following equation:

$$k = e^{\left(\frac{UA}{WC}\right)} \quad (11)$$

where A is the vessel surface area in contact with the process fluid.

In Equations (7) through (11), the coefficient U is assumed to be essentially constant. If the temperature range is large during heating or cooling of the vessel contents and U accordingly varies significantly, the range must be divided into small increments, and the time it takes to achieve each temperature increment must be calculated separately.

Correcting for viscosity

Many of the relationships in Tables 1 and 2 contain a viscosity-correction term, μ/μ_w, where μ is the viscosity at the bulk fluid temperature and μ_w the viscosity at the wall surface temperature. Although this term is often ignored in practice, we advise its use when the viscosity of the liquid being heated or cooled (whether the process fluid or the jacket fluid) varies significantly with temperature.

In such cases, a wall surface temperature must be estimated. This can be done by trial-and-error via Equation (12):

$$t_w = T - \left\{(T-t) / \left[1 + \left(\frac{h_j A_o}{h_i A_i}\right)\right]\right\} \quad (12)$$

where A_o is the jacketed area based on the outside vessel diameter and A_i the area based on the inside vessel diameter. This equation assumes steady heat flow through the jacket-side film and negligible temperature drop across the metal of the vessel wall. If, as is often the case, the difference be-

REYNOLDS, PRANDTL, ET AL

The mathematical expressions in the N_{Nu} column in Tables 1 and 2 are the starting points for calculating the film coefficients upon which heat transfer involving agitated and jacketed vessels is based. These expressions involve several dimensionless numbers. For convenience, we summarize here the usage of those numbers in the context of heat transfer in agitated, jacketed vessels.

Reynolds Number, N_{Re}: Two versions appear in the tables. The more familiar to most chemical engineers is:

$$N_{Re} = DV\rho/\mu$$

where D is diameter, V is velocity, ρ is density and μ is viscosity. This form of N_{Re} is used in the mathematical expressions in Table 2, as indicated in its Remarks column, with the diameter term consisting of D_e, equivalent diameter, as defined in that column.

The other N_{Re}, employed in Table 1, is known as the impeller Reynolds number:

$$N_{Re} = D^2N\rho/\mu$$

where D is impeller diameter and N is the rotational speed of the agitator.

Prandtl Number, N_{Pr}: For all equations in Tables 1 and 2, this is defined straightforwardly as:

$$N_{Pr} = c_p\mu/k$$

where c_p is specific heat, μ is viscosity and k is thermal conductivity

Viscosity Number: This is defined as:

$$\mu/\mu_w$$

in which the numerator is the viscosity at the bulk-fluid temperature whereas the denominator is the viscosity at the wall surface temperature. For a discussion of its relevance, see the main text of the article

Grashof Number, N_{Gr}: This number, used in Line 18 of Table 2, is defined as:

$$N_{Gr} = D_e^3\rho^2g\beta\Delta t_G/\mu^2$$

where D_e is equivalent diameter (see above), g is acceleration due to gravity, β is coefficient of volumetric expansion, μ is viscosity, and Δt_G is the difference between the temperature at the wall and that in the bulk fluid

Nusselt Number, N_{Nu}: Each of the mathematical expressions in the N_{Nu} column in Tables 1 and 2 is numerically equal to N_{Nu}, which is basically defined as follows:

$$N_{Nu} = hD/k$$

where h is film coefficient, D is diameter and k is thermal conductivity. For Table 1, h is the inside film coefficient and D the vessel diameter. For Table 2 h is the outside (i.e., jacket) film coefficient, whereas D is as defined in the Remarks column. □

TABLE 2. EQUATIONS FOR CALCULATING OUTSIDE FILM COEFFICIENTS (H_O) OF JACKETED AGITATED VESSELS.

Line No.	Jacket Type	N_{Re}	N_{Nu}	Remarks
15	Annular jacket with spiral baffling	>10,000	$0.027(N_{Re})^{0.8}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14} \times (1 + 3.5D_e/D_c)$	For heat-transfer purposes, this jacket can be considered a special case of a helical coil if certain factors are incorporated into equations for calculating outside-film coefficients. In the equations at left and below, the equivalent heat transfer diameter, D_e, for a rectangular cross-section is equal to four times the width of the annular space, w, and D_c is the mean or centerline diameter of the coil helix. Velocities are calculated from the actual cross-section of the flow area, pw, where p is the pitch of the spiral baffle, and from the effective mass flow rate, W', through the passage. The leakage around spiral baffles is considerable, amounting to 35–50% of the total mass-flow rate, W (*20*). To get a conservative outside film coefficient and avoid testing, the effective mass-flow rate should be taken to be about 60% of the total mass-flow rate to the jacket: $W' \approx 0.6W$ The Nusselt number corresponding to the equation at left should be expressed in terms of D_e ($N_{Nu} = h_j D_e/k$), as should the Reynolds number ($N_{Re} = D_e V \rho/\mu$), k being thermal conductivity, V being velocity and ρ being density. Reference (*21*).
16	Annular jacket with spiral baffling	<2,100	$1.86((N_{Re})(N_{Pr})(D_e/L))^{0.33}(\mu/\mu_w)^{0.14}$	Same as for Line 15. In the equation at left, L is the length of coil or jacket passage, ft.
17	Annular jacket with spiral baffling	$2{,}100 < N_{Re} < 10{,}000$		Obtain N_{Nu} from Figure 4 of Reference (*29*) or, for greater accuracy, use the equation of Line 15 or 16, depending on the value of N_{Re}.
18	Annular jacket, no baffles	Laminar flow	$1.02(N_{Re})^{0.45}(N_{Pr})^{0.33}(D_e/L)^{0.4} \times (D_{jo}/D_{ji})^{0.8}(\mu/\mu_w)^{0.14}(N_{Gr})^{0.05}$	D_{jo} and D_{ji} are the outside and inside diamters of the jacket, respectively. For this equation, $D_e = D_{jo} - D_{ji}$. The Grashof number, N_{Gr}, must be evaluated from fluid properties at the bulk temperature. Reference (*22*).
19	Annular jacket with spiral baffling	<2,100	$1.86((N_{Re})(N_{Pr})(D_e/L))^{0.33}(\mu/\mu_w)^{0.14}$	Same as for Line 18. The Nusselt and Reynolds numbers must be calculated with D_e as the diameter term (as in Lines 15 and 16). Reference (*21*).
20	Annular jacket, no baffles	Turbulent flow	$0.027(N_{Re})^{0.8}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14}) \times (1 + 3.5D_e/D_c)$	For the equivalent heat-transfer diameter for turbulent flow, use: $D_e = ((D_{jo})^2 - (D_{ji})^2)/D_{ji}$., where D_{jo} and D_{ji} are as defined for Line 18. The cross-sectional flow area, A_x equals $\pi((D_{jo})^2 - (D_{ji})^2)/4$;. Reference (*21*).
21	Annular jacket with spiral baffling	$210 < N_{Re} < 10{,}000$		Obtain N_{Nu} from Figure 4 of Reference (*29*) or, for greater accuracy, use the equation of Line 19 or 20, depending on the value of N_{Re}.
22	Half-pipe coil jacket	Laminar flow	$1.86((N_{Re})(N_{Pr})(D_e/L))^{0.33}(\mu/\mu_w)^{0.14}$	When pipe coils are made with a semicircular cross-section, $D_e = \pi d_{ci}/2$, where d_{ci} is the inner diameter of the pipe, in feet. For calculating the velocity, the cross-sectional flow area equals $\pi d_{ci}^2/8$. When pipe coils are made with a 120-deg central angle, $D_e = 0.708\ d_{ci}$ and the cross-sectional area equals $0.154(d_{ci})^2$. Reference (*21*).
23	Half-pipe coil jacket	Turbulent flow	$0.027(N_{Re})^{0.2}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14} \times (1 + 3.5D_e/D_c)$	Same as for Line 22. D_c is the mean diameter of the coil.
24	Half-pipe coil jacket	Transition flow		Obtain N_{Nu} from Figure 4 of Reference (*29*) or, for greater accuracy, use the equation of Line 22 or 23, depending on the value of N_{Re}
25	Dimple jacket	Laminar flow	$1.86((N_{Re})(N_{Pr})(D_e/L))^{0.33}(\mu/\mu_w)^{0.14}$	The equivalent diameter, D_e, in a dimpled jacket equals 0.66 in. The cross-sectional flow area equals 1.98 $in.^2$ per foot of vessel circumference. Reference (*24*).
26	Dimple jacket	Turbulent flow	$0.27(N_{Re})^{0.8}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14}$	See Line 25. Because of turbulence created by the dimples in the flow stream, the coefficients so obtained are not very accurate, probably erring on the low side. If available, coefficients based on experimental data should be used. Reference (*24*).
27	Dimple jacket	Transition flow		Obtain N_{Nu} from Figure 4 of Reference (*29*) or, for greater accuracy, use the equation of Line 25 or 26, depending on the value of N_{Re}.

tween A_i and A_o is negligible, Equation (12) simplifies to:

$$t_w = T - \left\{ (T-t) / \left[1 + \left(\frac{h_j}{h_i} \right) \right] \right\} \quad (13)$$

In the first trial to estimate h as required for this viscosity-correction calculation, assume (μ/μ_w) does equal 1.0 when using the N_{Nu} equations in Tables 1 or 2. Unless the term varies greatly from 1.0, one iteration should be sufficient to establish t_w. The viscosity at the wall, μ_w, is then taken from viscosity-vs-temperature data. We assume that the ratio (μ/μ_w) stays constant as the temperature within the vessel rises or falls during the heating or cooling.

When a liquid is being heated, the correction term will be greater than 1.0, because liquid viscosity decreases with increasing temperature; accordingly, the corrected value of h will be larger than the uncorrected one. When a liquid is being cooled, the converse will occur.

SOME COMPLICATIONS IN BIOTECHNOLOGY

In biotechnological manufacture, heat may be added to or removed from an agitated, jacketed vessel for any of several process reasons. Among the the main reasons are the following ones:

- Sterilization, in which case the temperature must be high enough to kill essentially all organisms during the total holding time
- Adding heat to reduce the water content of a cell cake by drying
- Adding or removing heat in order to maintain isothermal conditions. For example, an anaerobic sewage-sludge digester that operates between 55 and 60°C may require heat addition, whereas fermentation may require heat removal to keep the active cells viable. And although less heat transfer is involved, heat of metabolism must also be removed from cell cultures

Cells are extremely sensitive to temperature; in most cases, a cell culture must be held within about 0.1°C of its optimal temperature. Especially temperature-sensitive are animal cells, usually kept at around 37°C.

Unfortunately, maintaining and monitoring the temperature of many cultures in agitated vessels is complicated, because the stirring is slow and therefore the vessel-side heat-transfer film coefficient is low. In such cases, the engineer must guard against an unacceptable temperature gradient being set up between the vessel wall and the interior portion of the contents.

Water, heated or cooled as necessary, is the usual temperature-control medium in biotechnological manufacture. The temperature of the water itself may be adjusted either by direct injection of steam within the jacket, or by inclusion of a small heat exchanger in the heat-transfer-fluid circuit. The latter option is usually the better, because direct steam injection causes knocking and noises, and is difficult to control.

Although the accompanying article focuses on jacketed vessels, keep in mind that there are other options for heat transfer in biotechnological (or other) operations. One is the use of a piping coil immersed in the culture. And, small reactors under 30 L may be heated or cooled by water passing through hollow baffles within the vessel, or heated by a low-wattage electric-heater mat wrapped around the vessel.

The use of internal coils that present a large surface area may transfer the greatest amount of heat. On the other hand, they introduce a site for shearing forces, and make it hard to clean the inside of the vessel. When hollow baffles are used, it is difficult to maintain the proper amount of heat transfer during operations in which the viscosity or other properties of the culture are changing. Baffles also entail a great deal of shear. When baffles are nevertheless the choice for a given design, they should be removable and easy to clean, and should not be installed against the vessel wall. □

Control, tracking, calorimetry

Temperature control — and, thus, the heat-flow control — for a jacketed-vessel system usually requires two sensing elements, one in the vessel itself and the other in the heat-transfer medium. Resistance temperature detectors (RTDs) are preferable to thermocouples (*CE*, May 1998, pp. 90ff). By use of a cascade loop, plant operators can control the temperature differential between the vessel and the jacket, and thereby prevent harm to temperature-sensitive materials (such as biochemical fluids; see box, this page), particularly the portion near the vessel wall.

Conversely, monitoring the temperatures of the vessel and jacket contents can enable the engineer or plant operator to track the course of the reaction or physical transformation taking place. Such tracking may be especially useful for assessing the results of changes in operating parameters, such as the reactant feed rates or the choice of catalyst.

Determining the effects of such parameter variations is also, of course, important in plant design. Test runs for that purpose, including such additional findings as reaction pathways and the heats of reaction are commonly made in laboratory-scale calorimeters.

Carrying out a jacketed-calorimeter study usually requires a continuous measurement of the difference between the temperature of the fluid in the laboratory calorimetry reactor and that of the heat transfer medium in the calorimetry-reactor jacket. The jacket temperature is adjusted by a thermostat.

The amount of heat flow through the calorimetry-reactor wall depends not only on the temperature difference between vessel and jacket but also on the heat-exchange area and the overall heat-transfer coefficient. In a calorimeter, neither the area nor the coefficient can be assumed to be constant. The heat exchange area, A, is the area of the calorimetric-reactor wall wetted by the liquid phase, which depends not merely on the volume present but also on the stirring speed. The overall transfer coefficient, U, depends on numerous parameters specific to the material and instrument, such as stirrer speed, temperature, and physical properties of the reactor and jacket contents.

To lessen these complications, U and A are in practice treated as a product. The value of the product, UA, is determined through calorimetric calibration with the aid of electrical calibration heating, by introducing a known amount of thermal power, q_{cal}, into the reactor. The difference in temperature between reactor and jacket is noted. Then UA is calculated on the basis of the relation:

$$UA = \frac{q_{cal}}{(T_r - T_j)} \quad (14)$$

where T_r is the temperature of the reactor contents and T_j is that of the jacket fluid.

A new value for UA is calculated for every calibration, and stored together with the other measured values. During an experiment, any number of calibrations can be carried out, to take into account known or likely changes in UA. They should take place not only before and after every reaction, but also during the reaction if it is a relatively slow one — for instance, a cell-

AN EXAMPLE MAKES IT CLEARER

Several of the points developed in the main text are illustrated in this sample problem:

An agitated, jacketed vessel having an 8-ft diameter (D_T) contains 3,000 gal (401 ft³) of process fluid having the properties listed below. Its agitator is a retreating-blade turbine, 3 ft in diameter (D), turning at a speed N of 50 rpm. Neither the vessel nor the jacket contains baffles. The inside and outside diameters of the jacket, D_{ji} and D_{jo}, are 8 ft and 8.5 ft, respectively. The process fluid is being heated by a jacket fluid consisting of a mixture of ethylene glycol and water, having the properties shown below.

Calculate the overall heat-transfer coefficient, assuming that the fouling factors and the vessel-wall resistance (x/k in Equation [6]) can be ignored and that (μ/μ_w) equals 1.0 for the process fluid and heat-transfer fluid alike. Also, calculate the time required to heat the process fluid from 20 to 120°C, assuming that the heat-transfer fluid enters at 130°C, leaves at 124°C, and flows at a rate of 100 gal/min (0.22 ft³/s).

The process fluid has these properties:

- Density, $\rho = 45$ lb/ft³
- Viscosity, $\mu = 10$ lb/(ft)(h)
- Specific heat, $c_p = 0.7$ Btu/(lb)(°F)
- Thermal conductivity, $k = 0.42$ Btu/(h)(ft²)(°F/ft)

The properties of the heat-transfer fluid are as follows:

- Density, $\rho = 62.427$ lb/ft³
- Viscosity, $\mu = 0.03$ lb/(ft)(h)
- Specific heat, $C = 0.905$ Btu/(lb)(°F)
- Thermal conductivity, $k = 0.13$ Btu/(h)(ft²)(°F/ft)

Solution:

1a. We first calculate h_i, thus working with the process-fluid properties and assuming turbulent flow. From Table 1, Line 4, we have the following:

$$h_i D_T/k = 0.68(N_{Re})^{0.67}(N_{Pr})^{0.33}(\mu/\mu_w)^{0.14}$$

where

$$N_{Re} = D^2 N\rho/\mu$$

$$= (3)^2(50 \text{ revs/min} \times 60 \text{ min/h})(45)/10$$
$$= 1.21 \times 10^5 \text{ (so the flow is indeed turbulent) and}$$

$$N_{Pr} = c_p\mu/k = (0.7)(10)/0.42 = 16.67$$

Thus, substituting in the Table 1, Line 4 equation, we have:

$$h_i(8)/0.42 = 0.68(1.21 \times 10^5)^{0.67} \times (16.67)^{0.33}(1)^{0.14},$$

so $h_i = 230$ Btu/(h)(ft²)(°F)

1b. Next, we calculate h_j, working with the jacket geometry and the heat-transfer-fluid properties and assuming turbulent flow. From Table 2, Line 20 (and recognizing that h_j corresponds to h_o),

$$h_o D_e/k = 0.027(N_{Re})^{0.8}(N_{Pr})^{0.33} \times (\mu/\mu_w)^{0.14}(1 + 3.5D_e/D_c)$$

where in this case

$$N_{Re} = D_e V\rho/\mu$$

and

$$D_e = (D_{jo}^2 - D_{ji}^2)/D_{ji}$$
$$= [(8.5)^2 - (8)^2]/8 = 1.03 \text{ ft}$$

The jacket cross-section area, A_x, is found as follows:

$$A_x = \pi(D_{jo}^2 - D_{ji}^2)/4 = 6.48 \text{ ft}^2,$$

so

$$V = Q/A_x = 0.22/6.48 = 0.034 \text{ ft/s}$$

Therefore,

$$N_{Re} = [(1.03)(0.034 \text{ ft/s} \times 3{,}600 \text{ s/h}) \times 62.43]/0.03$$
$$= 2.6 \times 10^5 \text{ (so the flow is indeed turbulent), and}$$

$$N_{Pr} = c_p\mu/k = (0.905)(0.03)/0.13 = 0.21$$

Thus, substituting in the Table 2, Line 20, equation, we have:

$$h_o(1.03)/0.13 = [0.027(2.6 \times 10^5)^{0.8} \times (0.21)^{0.33}(1)^{0.14}][1 + (3.5)(1.03)/8.25]$$

so $h_o = h_j = 63$ Btu/(h)(ft²)(°F)

1c. Based on h_i and h_j, and the assumptions that x/k and the ff terms can be disregarded, we calculate U by substituting into Equation (6):

$$1/U = 1/h_i + 1/h_j = 1/230 + 1/63,$$

so $U = 49$ Btu/(h)(ft²)(°F)

2. To calculate the time required for heating, we first compare the log-mean temperature difference ΔT_{lm} between vessel and jacket with the temperature drop ΔT within the jacket itself.

$$\Delta T_{lm} = [(t_2 - T_2) - (t_1 - T_1)] \div \ln[(t_2 - T_2)/(t_1 - T_1)]$$
$$= [(121 - 124) - (20 - 130)] \cdot 4 \ln[(121 - 124)/(20 - 130)]$$
$$= -29.71 \text{ deg.}$$

whereas $\Delta T = 124 - 130 = -6$ degrees

Since ΔT is more than 10% of ΔT_{lm}, we must use Equation (9) rather than Equation (7):

$$\theta = \{\ln[(T_1 - t_1)/(T_1 - t_2)]\}(mc_p/WC) \times \{k/(k-1)\}$$

Now W, the mass flow through the jacket, equals the volumetric flow, 0.22 ft³/s, multiplied by the fluid density:

$$W = (0.22 \text{ ft}^3/\text{s})(3{,}600 \text{ s/h})(62.43)$$
$$= 49{,}445 \text{ lb/h}$$

and m, the mass within the vessel, is found similarly:

$$m = [3{,}000 \text{ gal}][1/(7.48 \text{ gal/ft}^3)] \times [45 \text{ lb/ft3}] = 18{,}048 \text{ lb}$$

The height of process fluid in the vessel equals 401 ft³/[π(8 ft)²/4], or 7.98 ft. Therefore, the wall area in contact with this fluid equals (7.98)(8)π, or 200.56 ft². The floor area equals π(8 ft)²/4, or 50.27 ft². Accordingly, from Equation (11), k is calculated as follows:

$$k = e^{UA/WC}$$
$$= \exp(49)(200.56 + 50.27)/49{,}445)(0.905)$$
$$= \exp(0.275) = 1.316$$

Therefore,

$$\theta = \{\ln[(130 - 20)/(130 - 121)]\} \times \{(18{,}048)(0.7)/(49{,}445)(0.905)\} \times \{(1.316)/(1.316 - 1)\}$$
$$= (2.503)(0.282)(4.16) = 2.94 \text{ h}$$

❑

culture growth that takes place over a period of days.

The bases for mathematically manipulating calorimetric data are the mass and heat balances around the calorimeter system. The mass balance is as follows:

Reaction mass present at time t = algebraic sum of all dosing and sampling operations up to time t.

The heat balance for the system is (on a rate basis) as follows:

$$q_r + q_{cal} = q_{flow} + q_{accu} + q_{dos} + q_{loss} + q_{add} \quad (15)$$

where:

- q_r = sum of the heat-production rates of all physical processes and chemical reactions during the run
- q_{cal} = calibration heating; namely, the effective electrical power supplied to the reactor
- q_{flow} = heat flow through the reactor wall, equal to $UA(T_r - T_j)$
- q_{accu} = heat accumulation, equal to $mc_p(dT_r/dt)$
- q_{dos} = heat flow due to dosing of reactants at a temperature not equal to T_r; equal to $(dm_{dos}/dt)(c_{p,dos})(T_r - T_{dos})$, where the subscript *dos* pertains to the reactants dosed
- q_{loss} = heat losses due to dissipation via the internal fittings, a function of $(T_r - T_{room})$;
- q_{add} = additional heat flows, such as those involving reflux condensers

In the calculation of the reaction heat flow, q_r, that term can be considered either collectively or individually in terms of its components. Depending on the problem, this allows the required cooling power, the heat output of the actual heat reaction or other heat outputs to be determined.* ■

Edited by Nicholas P. Chopey

*For a more detailed discussion of reaction calorimetry, see *CE*, May 1997, pp. 92ff.

Author

Robert F. Dream, P.E. is a Senior Manager, Pharmaceutical Technology, with Lockwood Greene Engineers, Inc., Suite 500, 11311 Reed Cornell Park Drive, Cincinnati, OH 45242; Phone: 513-530-0030, Ext. 230; Fax 513-530-5541; E-mail: rdream@lg.com. He has more than 20 years of industry experience related to biotechnology and pharmaceutical manufacture. A registered professional engineer, he is an active member of the International Soc. of Pharmaceutical Engineers (ISPE) and of the Parenteral Drug Assn. He has published several papers and made presentations for ISPE and several other associations, as well as at universities. He holds B.S. and M.S. degrees from Illinois Institute of Technology, and has completed academic requirements toward a Ph.D. at Drexel University.

References

1. Parker, N. H., *Chem. Eng.*, June 8, 1964 p. 165.
2. Hicks, E. W., et al, *Chem. Eng.*, Apr. 26, 1976, p. 102.
3. Gates, L. E., et al, *Chem. Eng.*, May 24, 1976, p. 144.
4. Hicks, R. W., and Gates, L. E., *Chem. Eng.*, July 19, 1976, p. 141.
5. Kern, D. Q., "Process Heat Transfer," McGraw-Hill, New York, 1950.
6. Holland, F. A., and Chapman, F. S., *Chem. Eng.*, Feb. 15, 1965, p. 175.
7. Brooks, G., and Su, G. D., *Chem. Eng. Prog.*, October 1959, p. 54.
8. Dickey, D., and Hicks, R. W., *Chem. Eng.*, Feb. 2, 1976, p. 93.
9. Ackley, E. J., *Chem. Eng.*, Aug. 22, 1960, p. 133.
10. Cummings, G. H., and West, A. S., *Ind. Eng. Chem.*, Vol. 42, 1950, p. 2303.
11. Holland, F. A., and Chapman, F. S., *Chem. Eng.*, Jan. 18, 1965, p. 153.
12. Brown, R. W., Scott, R., and Toyne, C., *Trans. IChE*, vol. 25, 1947, P. 181.
13. Chilton, T. H., et al., *Ind. Eng. Chem.*, Vol. 36, 1944, p. 510.
14. Uhl, V. W., *Chem. Eng. Prog.* Symposium Series, Vol. 51, 1954, p. 93.
15. Uhl, V. W., and Voznick, H. P., *Chem. Eng. Prog.*, March 1960, p. 172.
16. Blazinski, H., and Kuncewicz, C., *Intl. Chem. Eng.*, Vol. 21, 1981, p. 679.
17. Oldshue, J. Y., and Gretton, A. T., *Chem. Eng. Prog.*, Vol. 50, 1954, p. 615.
18. Rushton, J. H., et al., *Ind. Eng. Chem.*, Vol. 40, 1948, p. 1082.
19. Dunlap, I. R., and Rushton, J. H., *Chem. Eng. Prog.* Symposium Series, Vol. 49, 1953, p. 5.
20. Bollinger, D. H., *Chem. Eng.*, Sept. 20, 1982, p.95.
21. Perry, R. H., and Chilton, C. H., (Ed.), "Chemical Engineers' Handbook," 5th ed., pp. 10–17, McGraw-Hill, New York, 1973.
22. Chen, C. Y., et al, *Trans. ASME*, Vol. 68, 1946, p. 99.
23. Martinelli, R. C., et al, *Trans. AIChE*, Vol.38, 1942, p. 493.
24. Markovitz, R. E., *Chem. Eng.*, Nov. 15, 1971, p. 156.
25. Crane Co., "Flow of Fluids," Technical Paper No. 410, 1969.
26. *ASME Trans., J. of Basic Engineering*, Vol. 81, 1959, p. 126.
27. Kaferle, J. A., *Chem. Eng.*, Nov. 24, 1975, p. 86.
28. Sieder, E. N., and Tate, G. E., *Ind. Eng. Chem.*, Vol. 28, 1936, p. 1429.
29. Ludwig, E. E., "Applied Process Design for Chemical and Petrochemical Plants," Vol.3, pp. 57-58, Gulf Pub. Co., Houston, 1965.
30. Fleischaker, R. J., et al, *Adv. Appl. Microbiol.* 27, 137-167 (1981a).