F. ALSABBAGH

RIGID FRAME FORMULAS

Explicit Formulas

of all statical quantities for those single-panel frames which occur in practical steel, reinforced concrete. and timber construction

By

Prof. Dr.-Ing. A. KLEINLOGEL

114 rigid frame shapes with 1578 figures
General and special load conditions including temperature changes
Introduction and appendix with load terms
and illustrative examples

FREDERICK UNGAR PUBLISHING CO.
NEW YORK

Second American Edition

Translated from the German
RAHMENFORMELN
Twelfth Edition

By arrangement with Verlag Wilhelm Ernst & Sohn, Berlin

Seventh Printing, 1980

ISBN 0-8044-4551-6

Printed in the United States of America
Library of Congress Catalog Card Number 58-6789

Foreword to the First American Edition

By I. F. Morrison

Professor of Applied Mechanics, University of Alberta

The practical design of statically indeterminate structures is a trial and error process. Because the elastic equations are dependent on the substance, as well as on the form of the proposed structure, it is necessary to assume the size of each member in advance. This is based primarily on the experience of the designer, but these assumptions must then be justified by computation, and, as a rule, more than one trial is necessary to arrive at the final design.

The setting-up and solution of the elastic equations for the chosen redundant quantities involve much more work than the analysis of the comparatively simple statically determinate cases. Anything which will facilitate this work is therefore desirable, and such aids are often to be found in the algebraic formulas which are the solutions of the elastic equations in general terms.

Since the first appearance of Professor Kleinlogel's *Rahmenformeln* in 1913, this remarkable book has gone through eleven German editions. From time to time, it has been revised and enlarged from its initial form and now embraces nearly all of the practical single-span types of the rigid frame. This new English-language edition in one volume makes the book readily available to the structural engineer unfamiliar with German.

During the last fifty years, substantial progress has been made in structural analysis and design, but during the early part of this period there was some reluctance in practice to adopt indeterminate types. This was due chiefly to the difficult and often lengthy computations required and, so long as the numerical computations were time-consuming, the design office frowned on such procedures and preferred the more quickly computed statically determinate types. However, increasing costs and the more precise design of aircraft structures produced a demand for greater economy of material, and the advantages of continuity, stiffness and economy of the rigid frame, both in welded metal and reinforced concrete, came to be recognized. But, although systematic methods of stress analysis were developed, the demand for rapid computation, especially for preliminary design, still remained, and a handbook of reliable, compact formulas became more and more desirable.

The introduction of European methods of structural analysis, well developed there just before the turn of the century, came slowly on this continent and, until such methods came to be well established, there was little inclination among American engineers to indulge in the prodigious task of working out a large number of cases in algebraic form. And even today this has never been done to any great extent.

Owing to its pictorial character, this book is, in a sense, unique. The reproduction of the large numbers of diagrams of which it is comprised has made it necessary to retain the original notation, which differs somewhat from that familiar to the American engineer, but the use of J instead of I for the moment of inertia and of F instead of A for the area of the cross-section of a structural member should present no practical difficulty. Other features, such as the sign-convention, are also different, but these are fully explained in the text and will offer no handicap to those familiar with the subject.

The practical use of this handbook may be said to be three-fold. First, the formulas for the bending moments and reactions on rigid frames of a number of different types, and many loading conditions, may be used to secure results rapidly by the direct substitution of numerical values. Designers, even without advanced training in structural analysis, can avail themselves of the advantages of the rigid frame by its direct use and with but little added effort, influence lines, or tables, can be readily constructed as described in detail in the text.

Second, for those who are experienced in advanced analysis, the Mohr equation, aided by the diagrams in the book, will give a ready and rapid method for computing displacements of rigid frame structures. The moment-area theorems can be applied without difficulty.

Third, the rigid frames, themselves statically indeterminate, can be used as units in adopting a "primary structure" dealing with cases of more highly indeterminate frames, and so bring such structures within the range of easy computation by means of the Maxwell-Mohr work equation or, if one prefers, the slope-deflection equations. This extends considerably the field of practical application in the design of such structures and makes available an accurate and rapid method of analysis of structures which could heretofore be handled only by approximate methods or by lengthy numerical computations.

I. F. M.

Edmonton, Alberta
June, 1951

Preface to the 12th edition

The present 12th edition encompasses the same number of pages and frame shapes as the 11th edition. However, eleven frames have been omitted in order to create space for eleven entirely new frame shapes. The former can be easily obtained as special cases of the tabulated frames.

The eleven new frames are divided into three groups. Group I (frames 17 through 21) is a series of symmetrical triangular frames with tie rods and various end conditions of the diagonals; group II (frames 38 and 45) consists of a symmetrical and an unsymmetrical fixed rectangular frame with hinged knees; group III (frames 68-72) is a series of sheds with hinged or fixed bases and with or without ties at various levels. These new frames were added in response to the wishes of many users of the book.

With few exceptions, no changes have been made in the arrangement and form of the formulas; a small number of them have been transposed for easier use. All loading cases have been renumbered by a system X/Y. Here X denotes the frame shape (from 1 through 114) and Y the loading condition for that particular frame, each time starting out with 1.

For all 32 symmetrical frames, new antisymmetrical loading cases have been added to the symmetrical ones. This enables the user to obtain any unsymmetrical loading as the sum of a symmetrical and an antisymmetrical loading.

As before, no general normal loads on inclined members have been considered, because the corresponding formulas would not be simpler than the superposition of the formulas for the vertical and horizontal load components. Nonetheless, the triangular frames and some of the others contain loading conditions for normal loads on inclined members in line with some building code specifications.

The former $\mathfrak{M}_r$ and $\mathfrak{M}_l$ which denote the static moments of the load resultant S about the supports have been redesignated $\mathfrak{S}_r$ and $\mathfrak{S}_l$. This follows the notation used by other authors, and the former quantities will henceforth denote fixed-end moments (FEM) exclusively.

The Introduction has been considerably shortened. The derivation for the load terms $\mathfrak{L}$ and $\mathfrak{R}$ has been omitted because it proved too skimpy. The interested reader is referred to the volume *Belastungsglieder** for a complete explanation of these quantities and their application. The Appendix remains unchanged except for required modifications.

Planning and detailed execution of all the above changes was again in the hands of Mr. Arthur Haselbach, civil engineer, my co-worker of many years' standing, to whom I am greatly indebted.

Adolf Kleinlogel

Darmstadt, Germany
October, 1956

**Beam Formulas* by A. Kleinlogel. Translated, considerably expanded and adapted for American usage by Harold G. Lorsch (Ungar).

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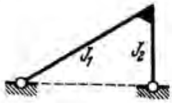

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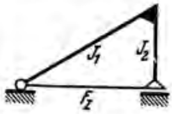

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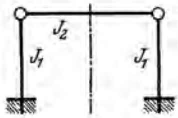

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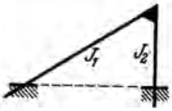

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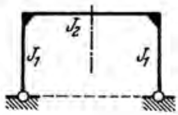

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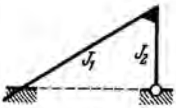

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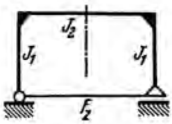

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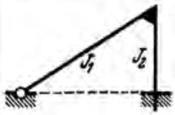

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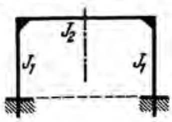

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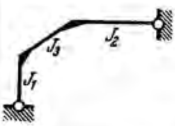

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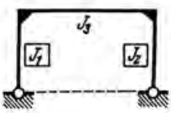

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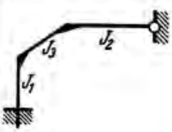

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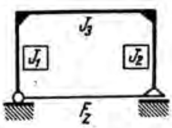

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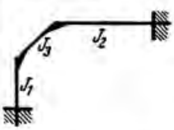

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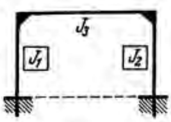

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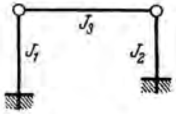

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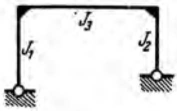

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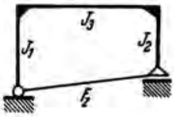

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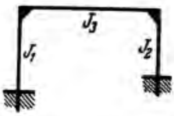

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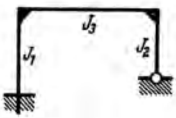

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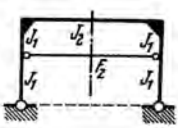

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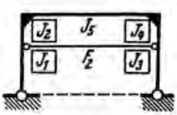

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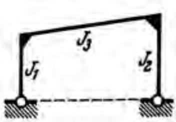

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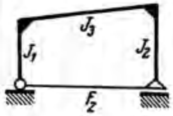

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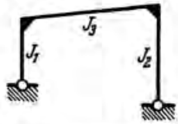

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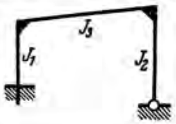

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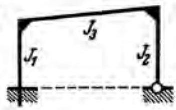

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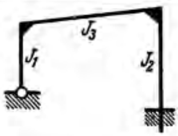

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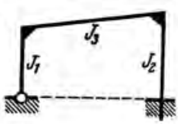

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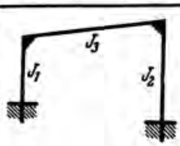

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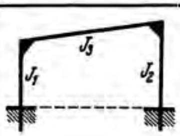

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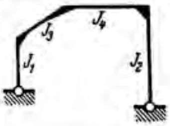

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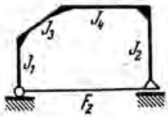

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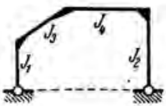

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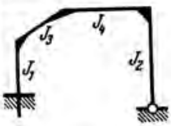

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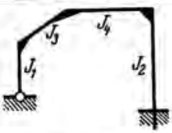

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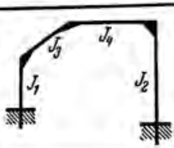

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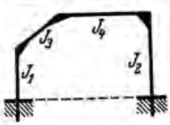

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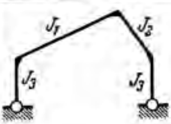

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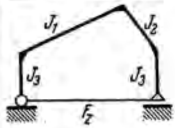

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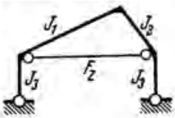

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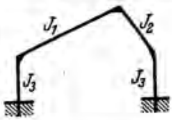

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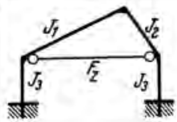

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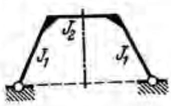

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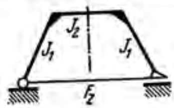

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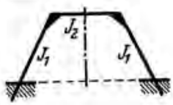

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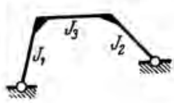

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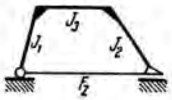

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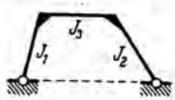

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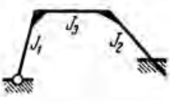

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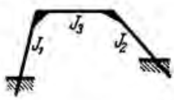

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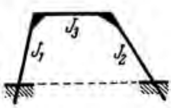

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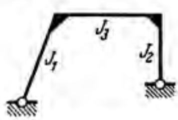

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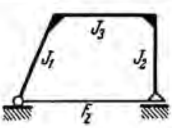

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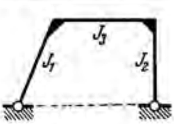

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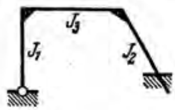

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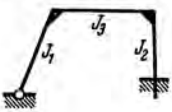
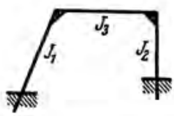
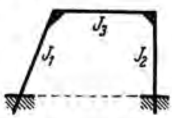


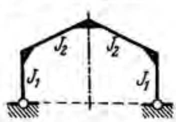
Frame 86
Pages 308, 309



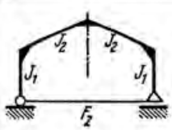
Frame 87
Pages 310 – 314



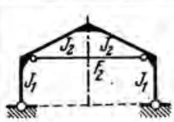
Frame 88
Page 315



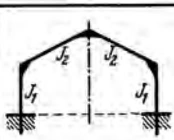
Frame 89
Pages 316 – 323



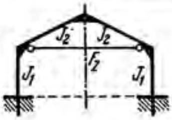
Frame 90
Pages 324 – 327



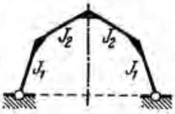
Frame 91
Pages 328, 329



Frame 92
Pages 330 – 337



Frame 93



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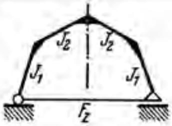

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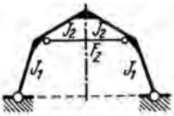
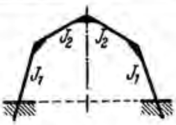


Frame 96



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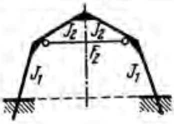
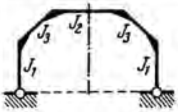
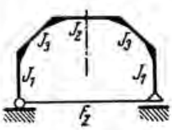


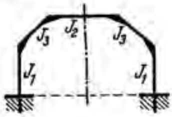
Frame 98



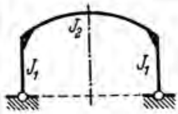
Frame 99



Frame 100



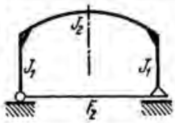
Frame 101



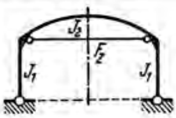
Frame 102

Auxiliary table to Frames 102 - 105

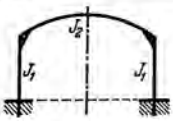
Belastungsglieder 𝔓



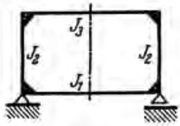
Frame 103



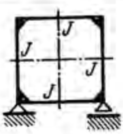
Frame 104



Frame 105



Frame 106



Frame 107

Appendix

Introduction

1. Organization of *Rigid Frame Formulas*

The 114 frames, shown pictorially in the index, are treated as 114 separate chapters.

Each type of frame is preceded by a full page which contains two sketches, the frame constants, and additional information if required. The left-hand sketch shows the type of frame, its support, the dimensions, the moments of inertia, and the joints notations. The right-hand sketch shows the positive direction of the reactions, the positive direction of the moment (tension on the face marked by a dashed line), and the coordinates of an arbitrary point.

Following this page are listed a certain number of loading conditions for each frame; *e.g.*, case 15/3. Here 15 denotes the number of the frame shape and 3 the loading condition for that particular frame. For all types of frames general loads on the members and a uniform temperature rise are covered first. The section on "Load Terms" in the Appendix is to be used in connection with the former. A varying number of special loading conditions are then given depending on the importance of a particular type of frame.

Each loading condition is again illustrated by two sketches. The left-hand sketch shows the frame and the load; the right-hand sketch shows the moment-diagram and the reactions. Formulas for temperature changes are shown with one sketch only.

For every loading condition formulas for vertical and horizontal reactions, moments at any point, and moments at the joints are given as a minimum. In cases where the computation of axial stresses and shearing stresses is complicated, the formulas for these stresses are given as well.

2. Arrangement of Formulas

As a rule, the formulas for the joint moments have been given directly at first. Usually it is then possible to compute reactions and moments at any point of the frame.

The kind of formula depends on the degree of statical indeterminacy and the shape of the rigid frame. Auxiliary coefficients X were introduced whenever the direct expressions for the statical quantities became too complicated or for other reasons of expediency. The X-values were represented in a convenient matrix form in the case of the more complicated rigid frames, statically indeterminate to the second or third degree (see pp. 235 and 236). The letter $\mathfrak{B}$ denotes "composite load terms" which occur in the equations for the X.

In the case of symmetrical frames two symmetrically located moments and forces have usually been combined into one double formula. The latter would have the typical form of

$$\left.\begin{matrix} G \\ G' \end{matrix}\right\rangle = Y_1 \pm Y_2$$

which represents the two forms

$$G = Y_1 + Y_2 \text{ and } G' = Y_1 - Y_2.$$

In these formulas Y_1 represents the influence of a symmetrical load, Y_2, the influence of an antisymmetrical load.

The letters A, B, C . . . designating a joint are used as indices for the M, V and H values (for example M_B, V_A, H_C).

The indices 1, 2, 3 are used in connection with the J or k values, to pertain to certain members (for example J_1, J_2).

The indices x and y are used in connection with the moment M and the shear Q at any point (for example M_x, Q_y).

3. The More Important Notations

A, B, C	Special points of the frame (support, joint, connection to the tie rod)
a, b, c l, h, s	Lengths of members and other dimensions
x, x'; y, y'	Variable dimensions (co-ordinates of any point on the frame)
J	Moment of inertia
k	Reciprocal of stiffness coefficient
α, β, γ m, n	Coefficients (explained on the first page of each chapter)
N, F; N_Z, L, G	Denominator in the formulas for determining statically indeterminate quantities
A, B, C, K, R, L	Constants (explained on the first page of each chapter)

P	External single concentrated load
q, p	Uniformly distributed load or triangular load per unit lengtl
M	Bending moment
V	Vertical reaction
H	Horizontal reaction
Q	Shear
S	Axial force
Z	Tension in tension rod
E	Modulus of elasticity
X	Constant (statically indeterminate moment)
n_{ik}	Coefficient
$\mathfrak{L}, \mathfrak{R}$	Load terms
$\boldsymbol{S}, \boldsymbol{W}$	Resultant of external loads for vertical and horizontal load respectively
$\mathfrak{S}_r, \mathfrak{S}_l$	Static moments of resultants of external loads[1]
M_x^0, M_y^0	Bending moments in a frame member considered as a sim supported beam under vertical and horizontal loads, respecti
$\mathfrak{B}$	Composite load term

4. Sign Conventions

General Rule: All computations must be carried out algebraicall hence every quantity must be used with its proper sign. The result wi then be automatically correct as to sign and magnitude.

Load: The direction of the external forces (single concentrated loa uniformly distributed load and moment) shown in the left-hand sketc for each type of frame is assumed to be positive. If the load acts in th opposite direction, its value is to be preceded by a negative sign whe substituting in formulas.

Reactions: The direction of the reaction shown in the right-han sketch for each type of frame is assumed to be positive. Therefore vertic reactions (V) are positive acting upward, and horizontal reactions (H are positive acting toward the structure.

Moment: A moment is positive if it causes tension in the face marke by a dashed line. There is no relationship between this sign conventio and the actual direction of rotation. Moment diagrams are drawn on th

[1]In previous editions of *Rigid Frame Formulas* and in *Beam Formulas* (translated an adapted for American usage by Harold G. Lorsch) these quantities are denoted by $\mathfrak{M}$

side of the member on which they cause tension. Hence, positive moments are shown on the dashed (inner) face of a member, negative moments on the face of the member opposite to the dashed one (outer face).

Unless otherwise noted on the first page of a chapter, the moment diagram in the right-hand sketch is approximately correct for the lengths shown and for equal moments of inertia of all members. Therefore the moment diagrams shown are to be used for general information only. For simple frames with normal variations of moments of inertia, however, the diagrams shown will usually be correct. For more complicated frames, for special dimensions, or for unusual variations in the moments of inertia, the actual moment diagrams can differ considerably from the moment diagrams shown, even to the extent of a change in sign.

Shear: The shear is positive if it is directed upward at the left end and downward at the right end (regular beam convention) of a member. The sign of the shear is independent of that of the moment and therefore independent of the dashed line.

Axial force: An axial force is positive if it causes compression; negative, if it causes tension.

Tie rod: A negative stress in a tie rod means that there is compression in the tie. A tension rod cannot take compression. If this compression is balanced by other tensile forces so that the final result is a tension force, the formulas used are correct. If the final stress remains compressive, the frame has to be figured by neglecting the tension rod completely. A rigid frame may then become a simple beam, if the force in the tie rod was the only redundant in the frame.

5. Assumptions Made in Deriving the Formulas

All formulas are based on the following assumptions: unyielding supports, no rotation or displacement of fixed supports, no displacement of hinged supports, and no vertical settlement of roller supports.

The influence of the bending moments alone was considered in the formulas for statically indeterminate quantities. The influence of axial and shear forces was neglected as being usually very small. Practical experience has shown that, except in special cases (short heavy legs of a rigid frame, etc.), the influence of axial forces may be neglected. This applies even more to shear forces. It must be remembered, however, that no "rule" is propounded to neglect these forces in the general analysis of statically indeterminate structures.

The influence of different moments of inertia is taken care of by using the stiffness coefficient k. It is assumed that the moment of inertia of any member remains constant.

The modulus of elasticity E is assumed to be the same for all members that are rigidly connected. It differs from the modulus of elasticity E_t

for tie rods. Both moduli of elasticity appear in the expressions for tie rod forces and temperature forces only.

The influence of a temperature change is computed by assuming a uniform temperature change for all members except the tie rods. Assuming that the tie rod is located between the supports, that the temperature change of the tie rod is the same as that of the other members, and that the coefficient of expansion of the tie rod is the same as that of the other members, a change of temperature does not cause any moment, axial stress, or shear stress in the structure. If a frame is statically determinate externally, a uniform change in temperature of all its members does not cause any stresses.

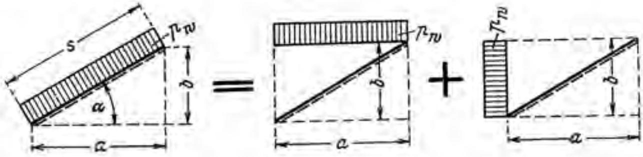
Special assumptions which pertain to individual frames alone are explained where they occur.

6. General Loads on Members

Introduction: A positive moment causes tension in the dashed face of a member. Positive moments are shown on the dashed side, negative moments on the opposite side. In order to distinguish between the left and the right end of a member, it must be looked at from the positive side. This book contains single-story frames only; the inside face of all members was dashed; hence the positive face of all members is the inside face. All members should be viewed from the inside in order to define "right" and "left" ends.

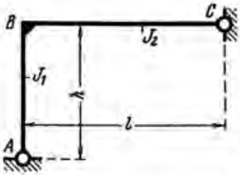
General unsymmetrical loads are always indicated by the dashed resultants of loads S or W. The load terms $\mathfrak{L}$ and $\mathfrak{R}$ are indicated by a double line || at the ends of the loaded members or their projections. The significance of the static moments $\mathfrak{S}_r$, $\mathfrak{S}_l$ and of the simple beam moments M_x^0 and M_y^0 is explained in the figure on page 440.

Loads do not generally act normally to the axes of inclined members (*e.g.*, snow or wind loads on inclined girders). The figure below shows that the uniformly distributed load of p_n lbs. per ft. acting normally to the axis of the inclined member of length s is equal to a vertical load p_n acting on the horizontal projection of the member and a horizontal load p_n acting on the vertical projection of the member. This will enable readers to use the present tables for horizontal, vertical, or inclined loads acting on horizontal, vertical, or inclined members.

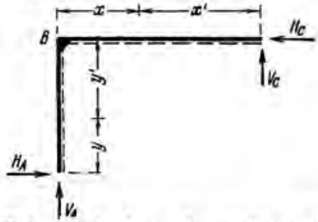


Frame 1

Single-leg, two-hinged rigid frame. Vertical leg. Horizontal girder.



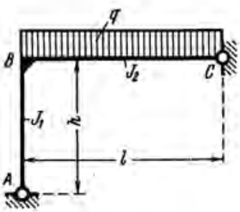
Shape of Frame
Dimensions and Notations

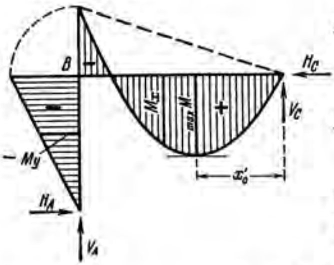


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \dfrac{J_2}{J_1} \cdot \dfrac{h}{l}$ $\qquad N = k + 1\,.$

Case 1/1: Rectangular load on the girder





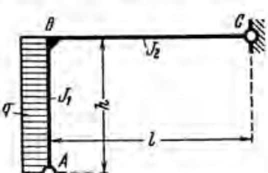
$$M_B = -\frac{q\,l^2}{8\,N}\,;$$

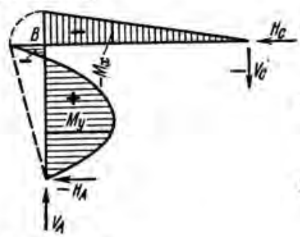
$$H_A = H_C = \frac{-M_B}{h}$$

$$\left.\begin{matrix} V_A \\ V_C \end{matrix}\right\rangle = \frac{q\,l}{2} \mp \frac{M_B}{l}\;;$$

$$M_x = \frac{q\,x\,x'}{2} + \frac{x'}{l}\,M_B \qquad x_0' = \frac{V_C}{q} \qquad \max M = \frac{V_C\,x_0'}{2} \qquad M_y = \frac{y}{h}\,M_B\,.$$

Case 1/2: Rectangular load on the leg





$$M_B = -\frac{q\,h^2\,k}{8\,N}\,;$$

$$V_A = -V_C = \frac{-M_B}{l}$$

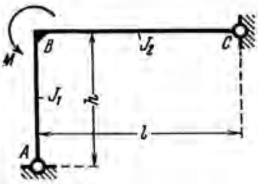
$$H_A = -\frac{q\,h}{2} - \frac{M_B}{h}$$

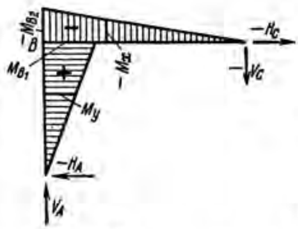
$$M_x = \frac{x'}{l}\,M_B \qquad M_y = \frac{q\,y\,y'}{2} + \frac{y}{h}\,M_B\,; \qquad H_C = \frac{q\,h}{2} - \frac{M_B}{h}\,.$$

FRAME 1

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $\qquad N = k + 1$

Case 1/3: The moment acts at joint *B*





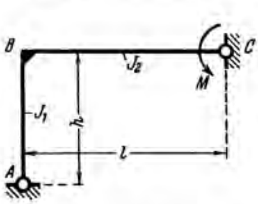
$$M_{B1} = + \frac{M}{N}$$

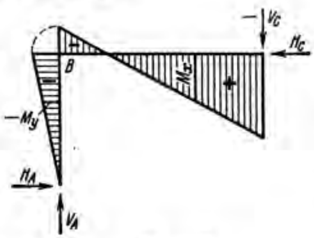
$$M_{B2} = - \frac{M\,k}{N}$$

$$(M_{B1} - M_{B2} = M);$$

$$H_A = H_C = - \frac{M_{B1}}{h} \qquad V_A = - V_C = \frac{- M_{B2}}{l}; \qquad M_x = \frac{x'}{l} M_{B2} \qquad M_y = \frac{y}{h} M_{B1}.$$

Case 1/4: The moment acts at hinge *C*





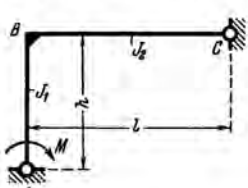
$$M_B = - \frac{M}{2\,N}$$

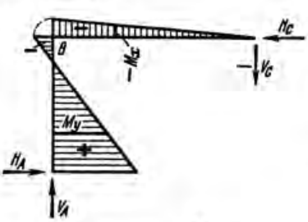
$$M_C = + M;$$

$$M_y = \frac{y}{h} M_B$$

$$H_A = H_C = \frac{- M_B}{h} \qquad V_A = - V_C = \frac{M - M_B}{l}; \qquad M_x = \frac{x}{l} M + \frac{x'}{l} M_B.$$

Case 1/5: The moment acts at hinge *A*





$$M_A = + M$$

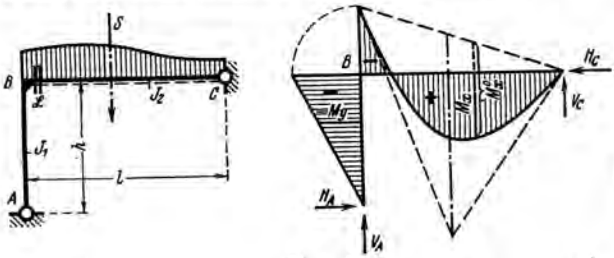
$$M_B = - \frac{M\,k}{2\,N};$$

$$M_x = \frac{x'}{l} M_B$$

$$H_A = H_C = \frac{M - M_B}{h} \qquad V_A = - V_C = \frac{- M_B}{l}; \qquad M_y = \frac{y'}{h} M + \frac{y}{h} M_B.$$

See Appendix A, Load Terms, pp. 440-445.

Case 1/6: Girder loaded by any type of vertical load

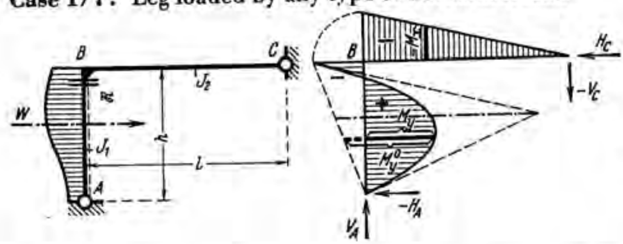


$$M_B = -\frac{\mathfrak{L}}{2N};$$

$$H_A = H_C = \frac{-M_B}{h}$$

$$V_A = \frac{\mathfrak{S}_r - M_B}{l} \qquad V_C = \frac{\mathfrak{S}_l + M_B}{l}; \qquad M_x = M_x^0 + \frac{x'}{l} M_B \qquad M_y = \frac{y}{h} M_B$$

Case 1/7: Leg loaded by any type of horizontal load

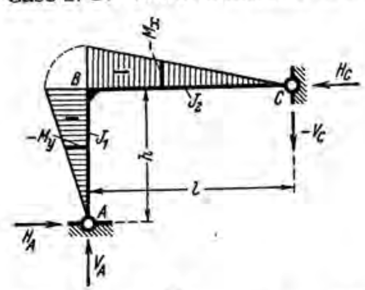


$$M_B = -\frac{\mathfrak{R}\,k}{2N};$$

$$V_A = -V_C = \frac{-M_B}{l}$$

$$H_C = \frac{\mathfrak{S}_l - M_B}{h} \qquad H_A = -(W - H_C); \qquad M_y = M_y^0 + \frac{y}{h} M_B \qquad M_x = \frac{x'}{l} M_B$$

Case 1/8: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

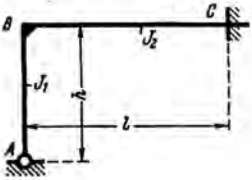
$$M_B = -\frac{3EJ_2\varepsilon t}{hN}\cdot\frac{l^2+h^2}{l^2};$$

$$H_A = H_C = \frac{-M_B}{h} \qquad V_A = -V_C = \frac{-M_B}{l}; \qquad M_x = \frac{x'}{l} M_B \qquad M_y = \frac{y}{h} M_B$$

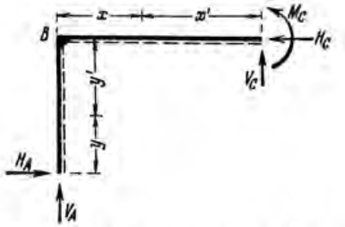
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 2

Single-leg, one-hinged rigid frame. Vertical leg, hinged at bottom. Horizontal girder.



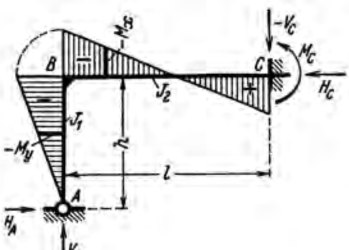
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $\qquad N = 4k + 3$.

ıse 2/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$T = \frac{6EJ_2\varepsilon t}{lN}, \qquad B = \frac{l^2 + h^2}{lh}.$$

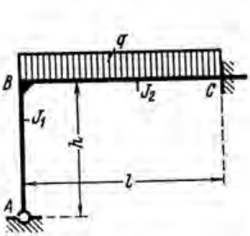
$$M_C = +T\left[B + \frac{2h(k+1)}{l}\right]$$

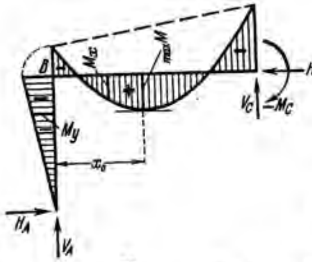
$$M_B = -T\left[2B + \frac{h}{l}\right]; \quad M_y = \frac{y}{h} M_B$$

$${}_A = H_C = \frac{-M_B}{h}; \qquad V_A = -V_C = \frac{M_C - M_B}{l}; \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C.$$

ıte: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 2/2: Rectangular load on the girder





$$M_B = -\frac{q l^2}{4N}$$

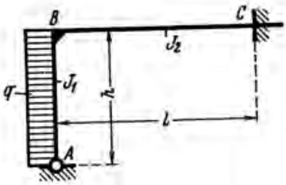
$$M_C = -\frac{q l^2 (2k+1)}{4N};$$

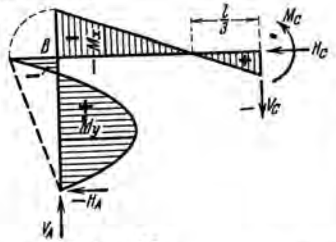
$$H_A = H_C = \frac{-M_B}{h};$$

$$V_A = \frac{q l}{2} - \frac{M_B - M_C}{l}; \qquad x_0 = \frac{V_A}{q} \qquad \max M = \frac{V_A x_0}{2} + M_B$$

$$V_C = \frac{q l}{2} + \frac{M_B - M_C}{l}; \qquad M_x = \frac{q x x'}{2} + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_y = \frac{y}{h} M_B.$$

Case 2/3: Rectangular load on the leg





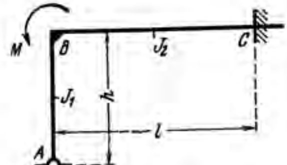
$$M_B = -\frac{q h^2 k}{2N}$$

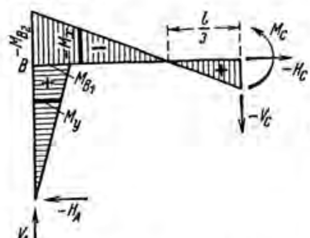
$$M_C = \frac{-M_B}{2}$$

$$H_A = -\frac{q h}{2} - \frac{M_B}{h} \qquad H_C = \frac{q h}{2} - \frac{M_B}{h} \qquad V_A = -V_C = \frac{3 M_C}{l}$$

$$M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_y = \frac{q y y'}{2} + \frac{y}{h} M_B.$$

Case 2/4: The moment acts at joint B





$$M_{B1} = \frac{3M}{N}$$

$$M_{B2} = -\frac{4Mk}{N}$$

$$(M_{B1} - M_{B2} = M)$$

$$M_C = \frac{-M_{B2}}{2};$$

$$H_A = H_C = -\frac{M_{B1}}{h}$$

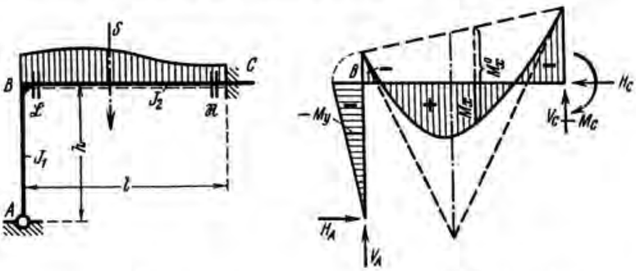
$$V_A = -V_C = \frac{3 M_C}{l}; \qquad M_x = \frac{x'}{l} M_{B2} + \frac{x}{l} M_C \qquad M_y = \frac{y}{h} M_{B1}.$$

AME 2

efficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $\qquad N = 4k + 3.$

See Appendix A, Load Terms, pp. 440-445.

se 2/5: Girder loaded by any type of vertical load

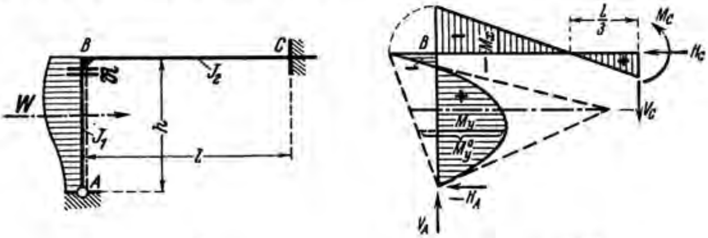


$$M_B = -\frac{2\mathfrak{L} - \mathfrak{R}}{N} \qquad M_C = -\frac{2\mathfrak{R}(k+1) - \mathfrak{L}}{N};$$

$$V_A = \frac{\mathfrak{S}_r - M_B + M_C}{l} \qquad V_C = S - V_A; \qquad H_A = H_C = \frac{-M_B}{h};$$

$$M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_y = \frac{y}{h} M_B.$$

se 2/6: Leg loaded by any type of horizontal load



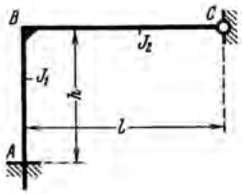
$$M_B = -\frac{2\mathfrak{R}k}{N} \qquad M_C = \frac{\mathfrak{R}k}{N};$$

$$V_A = -V_C = \frac{3M_C}{l}; \qquad H_C = \frac{\mathfrak{S}_l - M_B}{h} \qquad H_A = -(W - H_C);$$

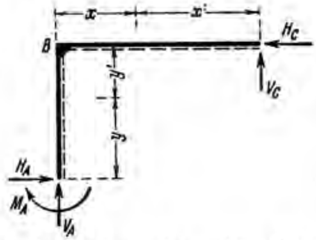
$$M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_y = M_y^0 + \frac{y}{h} M_B.$$

Frame 3

Single-leg, one-hinged rigid frame. Vertical leg. Horizontal girder, hinged at one end.



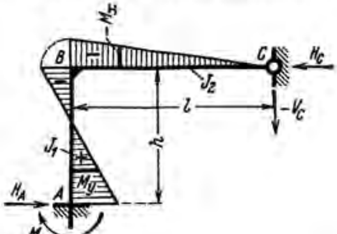
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \dfrac{J_2}{J_1} \cdot \dfrac{h}{l}$ $\qquad N = 3k + 4$.

Case 3/1: Uniform increase in temperature of the entire frame



$E =$ Modulus of elasticity
$\varepsilon =$ Coefficient of thermal expansion
$t =$ Change of temperature in degrees

Constants:

$$T = \frac{6 E J_2 \varepsilon t}{l N}, \qquad B = \frac{l^2 + h^2}{l h}.$$

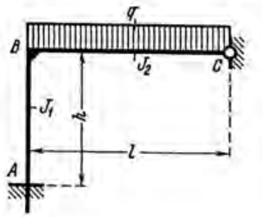
$$M_A = +T\left[\frac{2l(k+1)}{hk} + B\right] \qquad M_B = -T\left[\frac{l}{h} + 2B\right]; \quad M_x = \frac{x'}{l} M_B$$

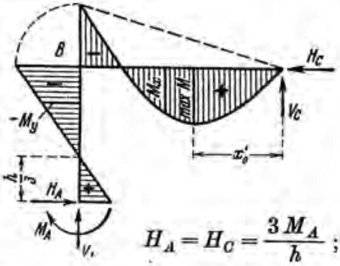
$$H_A = H_C = \frac{M_A - M_B}{h}; \qquad V_A = -V_C = \frac{-M_B}{l}; \qquad M_y = \frac{y'}{h} M_A + \frac{y}{h} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 3

Case 3/2: Rectangular load on the girder





$$M_B = -\frac{q\,l^2}{2\,N}$$

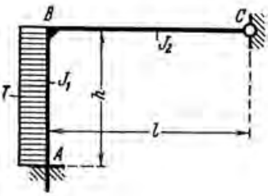
$$M_A = \frac{-M_B}{2};$$

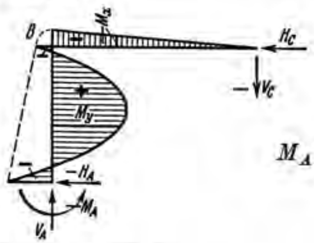
$$V_A = \frac{q\,l}{2} - \frac{M_B}{l}$$

$$H_A = H_C = \frac{3\,M_A}{h}; \qquad V_C = \frac{q\,l}{2} + \frac{M_B}{l};$$

$$x'_0 = \frac{V_C}{q} \qquad \max M = \frac{V_C\,x'_0}{2} \qquad M_x = \frac{q\,x\,x'}{2} + \frac{x'}{l} M_B \qquad M_y = \frac{y'}{h} M_A + \frac{y}{h} M_B.$$

Case 3/3: Rectangular load on the leg





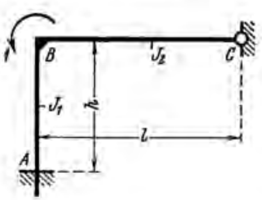
$$M_B = -\frac{q\,h^2\,k}{4\,N}$$

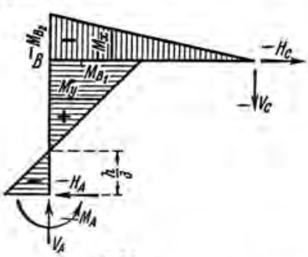
$$M_A = -\frac{q\,h^2\,(k+2)}{4\,N};$$

$$H_C = \frac{q\,h}{2} + \frac{M_A - M_B}{h} \qquad H_A = -(q\,h - H_C); \qquad V_A = -V_C = \frac{-M_B}{l};$$

$$M_x = \frac{x'}{l} M_B \qquad M_y = \frac{q\,y\,y'}{2} + \frac{y'}{h} M_A + \frac{y}{h} M_B.$$

Case 3/4: The moment acts at joint B





$$M_{B1} = \frac{4\,M}{N}$$

$$M_{B2} = -\frac{3\,M\,k}{N}$$

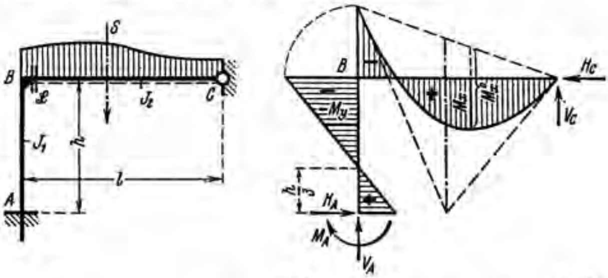
$$(M_{B1} - M_{B2} = M) \qquad M_A = -\frac{2\,M}{N}; \qquad H_A = H_C = \frac{3\,M_A}{h};$$

$$V_A = -V_C = \frac{-M_{B2}}{l}; \qquad M_x = \frac{x'}{l} M_{B2} \qquad M_y = \frac{y'}{h} M_A + \frac{y}{h} M_{B1}.$$

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $\qquad N = 3k + 4.$

See Appendix A, Load Terms, pp. 440-445.

Case 3/5: Girder loaded by any type of vertical load

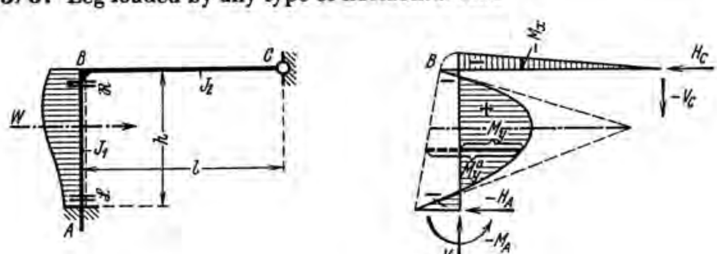


$$M_A = +\frac{\mathfrak{L}}{N} \qquad M_B = -\frac{2\mathfrak{L}}{N}; \qquad H_A = H_C = \frac{3M_A}{h};$$

$$V_A = \frac{\mathfrak{S}_r - M_B}{l} \qquad V_C = \frac{\mathfrak{S}_l + M_B}{l} \qquad (V_A + V_C = S);$$

$$M_x = M_x^0 + \frac{x'}{l} M_B \qquad M_y = \frac{y'}{h} M_A + \frac{y}{h} M_B.$$

Case 3/6: Leg loaded by any type of horizontal load



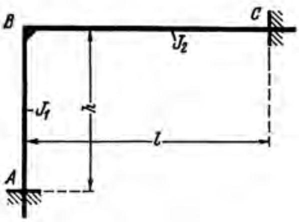
$$M_A = -\frac{2\mathfrak{L}(k+1) - \mathfrak{R}k}{N} \qquad M_B = -\frac{(2\mathfrak{R} - \mathfrak{L})k}{N};$$

$$V_A = -V_C = \frac{-M_B}{l}; \qquad H_C = \frac{\mathfrak{S}_l + M_A - M_B}{h} \qquad H_A = -(W - H_C);$$

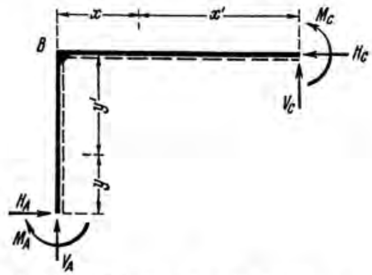
$$M_x = \frac{x'}{l} M_B \qquad M_y = M_y^0 + \frac{y'}{h} M_A + \frac{y}{h} M_B.$$

Frame 4

Single-leg, hingeless rigid frame. Vertical leg. Horizontal girder.



Shape of Frame
Dimensions and Notations

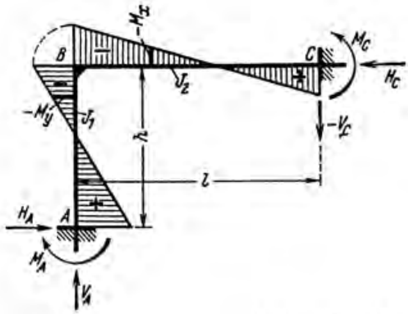


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{h}{l} \qquad N = k + 1 \, .$$

Case 4/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$T = \frac{3 E J_2 \varepsilon t}{l N}, \qquad B = \frac{l^2 + h^2}{l h}.$$

$$M_B = -2 T B$$

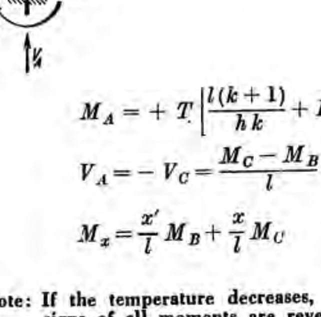
$$M_A = + T \left[\frac{l(k+1)}{h k} + B\right] \qquad M_C = + T \left[B + \frac{h(k+1)}{l}\right];$$

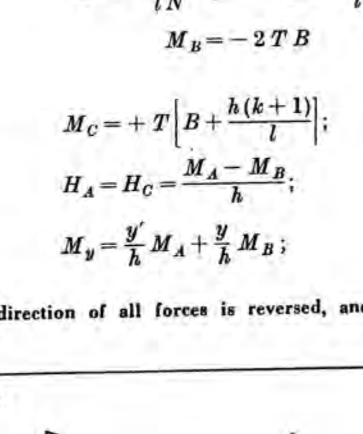
$$V_A = - V_C = \frac{M_C - M_B}{l} \qquad H_A = H_C = \frac{M_A - M_B}{h};$$

$$M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_y = \frac{y'}{h} M_A + \frac{y}{h} M_B;$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 4/2: The moment acts at joint B





$$M_{B1} = + \frac{M}{N} \qquad M_{B2} = - \frac{M k}{N} \qquad M_A = - \frac{M_{B1}}{2} \qquad M_C = \frac{- M_{B2}}{2}$$

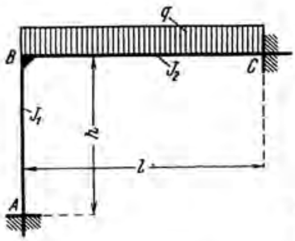
$$(M_{B1} - M_{B2} = M); \qquad V_A = - V_C = \frac{3 M_C}{l}; \qquad H_A = H_C = \frac{3 M_A}{h};$$

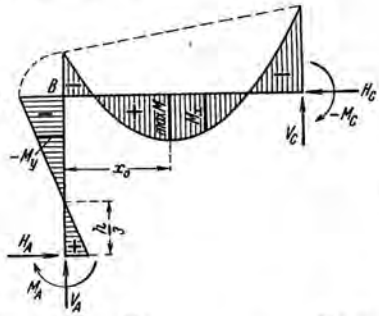
$$M_x = \frac{x'}{l} M_{B2} + \frac{x}{l} M_C \qquad M_y = \frac{y'}{h} M_A + \frac{y}{h} M_{B1}.$$

FRAME 4

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $\qquad N = k + 1$.

Case 4/3: Rectangular load on the girder





$$M_B = -\frac{q\,l^2}{12\,N}$$

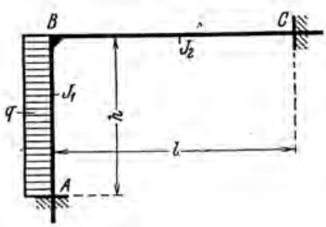
$$M_A = \frac{-M_B}{2}$$

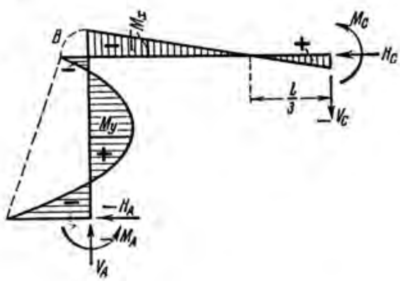
$$M_C = -\frac{q\,l^2(3\,k+2)}{24\,N} \qquad x_0 = \frac{V_A}{q} \qquad \max M = \frac{V_A\,x_0}{2} + M_B\,;$$

$$V_A = \frac{q\,l}{2} - \frac{M_B - M_C}{l} \qquad V_C = \frac{q\,l}{2} + \frac{M_B - M_C}{l} \qquad H_A = H_C = \frac{3\,M_A}{h};$$

$$M_x = \frac{q\,x\,x'}{2} + \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_y = \frac{y'}{h}M_A + \frac{y}{h}M_B\,.$$

Case 4/4: Rectangular load on the leg





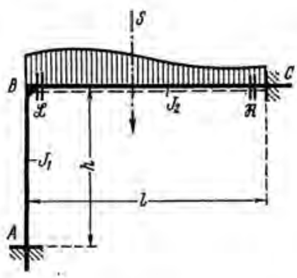
$$M_A = -\frac{q\,h^2(2\,k+3)}{24\,N} \qquad M_B = -\frac{q\,h^2\,k}{12\,N} \qquad M_C = \frac{-M_B}{2};$$

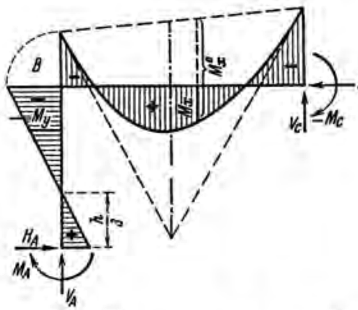
$$H_C = \frac{q\,h}{2} + \frac{M_A - M_B}{h} \qquad H_A = -(q\,h - H_C); \qquad V_A = -V_C = \frac{3\,M_C}{l};$$

$$M_y = \frac{q\,y\,y'}{2} + \frac{y'}{h}M_A + \frac{y}{h}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C\,.$$

See Appendix A, Load Terms, pp. 440-445.

Case 4/5: Girder loaded by any type of vertical load



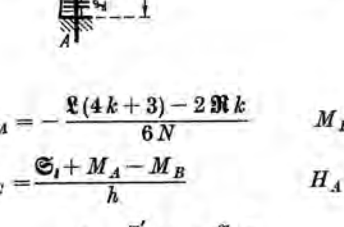


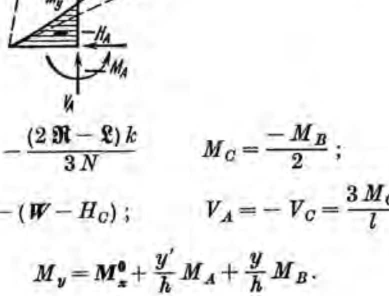
$$M_C = -\frac{\mathfrak{R}(3k+4) - 2\mathfrak{L}}{6N} \qquad M_B = -\frac{2\mathfrak{L} - \mathfrak{R}}{3N} \qquad M_A = \frac{-M_B}{2};$$

$$V_A = \frac{\mathfrak{S}_r - M_B + M_C}{l} \qquad V_C = \mathbf{S} - V_A; \qquad H_A = H_C = \frac{3M_A}{h};$$

$$M_x = \mathbf{M}_x^0 + \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_y = \frac{y'}{h}M_A + \frac{y}{h}M_B.$$

Case 4/6: Leg loaded by any type horizontal load





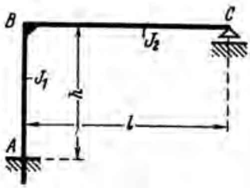
$$M_A = -\frac{\mathfrak{L}(4k+3) - 2\mathfrak{R}k}{6N} \qquad M_B = -\frac{(2\mathfrak{R} - \mathfrak{L})k}{3N} \qquad M_C = \frac{-M_B}{2};$$

$$H_C = \frac{\mathfrak{S}_l + M_A - M_B}{h} \qquad H_A = -(W - H_C); \qquad V_A = -V_C = \frac{3M_C}{l};$$

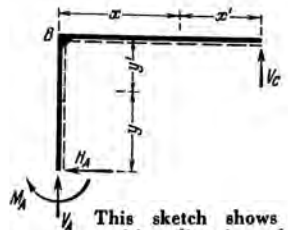
$$M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_y = \mathbf{M}_x^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B.$$

Frame 5

Single-leg, rigid frame. Vertical leg. Horizontal girder with roller at one end.



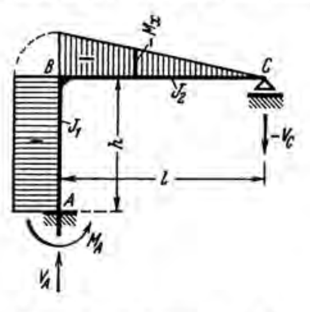
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions* and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $\qquad N = 3k + 1$.

Case 5/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_A = M_B = -\frac{3EJ_2\varepsilon t h}{l^2 N};$$

$$V_A = -V_C = \frac{-M_B}{l}; \qquad M_x = \frac{x'}{l} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

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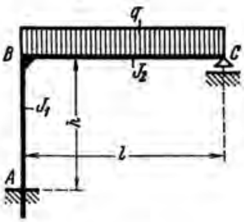
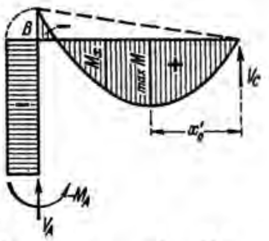
*Contrary to the sign convention used for all other frames, the positive direction of H_A has been chosen as shown.

Case 5/2: Rectangular load on the girder

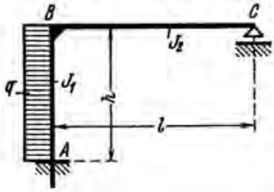


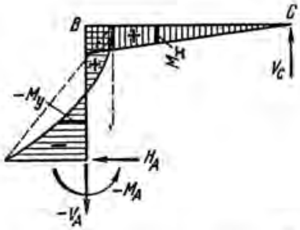


$$M_A = M_B = -\frac{q\,l^2}{8\,N} \qquad V_A = \frac{q\,l}{2} - \frac{M_B}{l} \qquad V_C = \frac{q\,l}{2} + \frac{M_B}{l}$$

$$x_0' = \frac{V_C}{q} \qquad \max M = \frac{V_C\,x_0'}{2} \qquad M_x = \frac{q\,x\,x'}{2} + \frac{x'}{l}\,M_B.$$

Case 5/3: Rectangular load on the leg

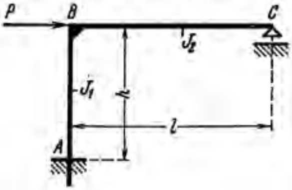


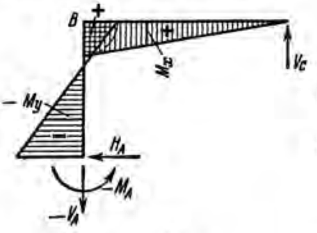


$$M_B = \frac{q\,h^2\,k}{2\,N} \qquad M_A = -\frac{q\,h^2}{2} + M_B \qquad V_A = -V_C = \frac{M_B}{l}$$

$$H_A = q\,h \qquad M_x = \frac{x'}{l}\,M_B \qquad M_y = -\frac{q\,y'^2}{2} + M_B.$$

Case 5/4: Horizontal concentrated load on the girder





$$M_B = \frac{3\,P\,h\,k}{2\,N}$$

$$M_A = -P\,h + M_B$$

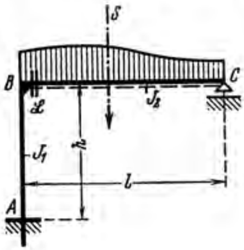
$$V_C = -V_A = \frac{M_B}{l} \qquad H_A = P \qquad M_x = \frac{x'}{l}\,M_B \qquad M_y = -P\,y' + M_B.$$

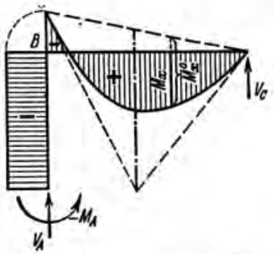
FRAME 5

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $N = 3k + 1$.

See Appendix A, Load Terms, pp. 440-445.

Case 5/5: Girder loaded by any type of vertical load

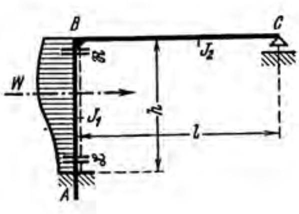


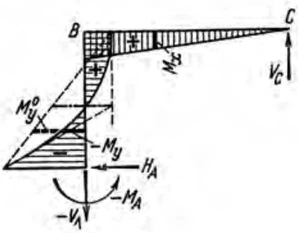


$$M_A = M_B = -\frac{\mathfrak{L}}{2N}; \qquad V_A = \frac{\mathfrak{S}_r - M_B}{l} \qquad V_C = \frac{\mathfrak{S}_l + M_B}{l};$$

$$M_x = M_x^0 + \frac{x'}{l} M_B.$$

Case 5/6: Leg loaded by any type of horizontal load





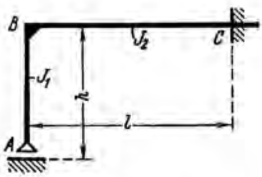
$$M_B = \frac{3\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})}{2} \cdot \frac{k}{N} \qquad M_A = -\mathfrak{S}_l + M_B;$$

$$V_C = -V_A = \frac{M_B}{l}; \qquad H_A = W;$$

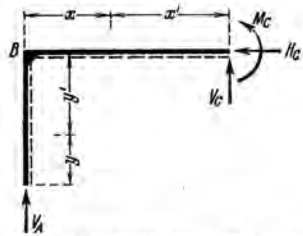
$$M_x = \frac{x'}{l} M_B \qquad M_y = M_y^0 + \frac{y'}{h} M_A + \frac{y}{h} M_B.$$

Frame 6

Single-leg, rigid frame. Vertical leg on roller. Horizontal girder.



Shape of Frame
Dimensions and Notation:



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Note: In this frame the bending moments are independent of the ratio of the moments of inertia of the members. Therefore k does not show in the formulas.

Case 6/1: Uniform increase in temperature of the entire frame[1]

E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_B = 0 \qquad M_C = \frac{3\,E J_2\,\varepsilon\,t\,h}{l^2}\,;$$

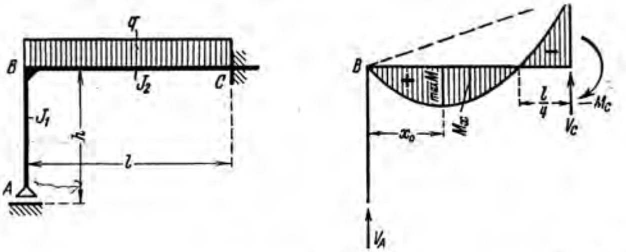
$$V_A = -V_C = \frac{M_C}{l} \qquad M_x = \frac{x}{l}\,M_C\,.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

[1] In this case only the change of temperature of the leg influences the moments and reactions.

FRAME 6

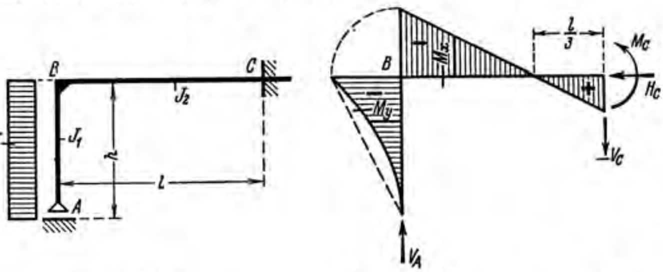
Case 6/2: Rectangular load on the girder



$M_B = 0$ $\qquad M_C = -\frac{q\,l^2}{8}$ $\qquad \max M = \frac{9\,q\,l^2}{128}$;

$V_A = \frac{3\,q\,l}{8}$ $\qquad V_C = \frac{5\,q\,l}{8}$; $\qquad M_x = \frac{q\,x}{2}\left(\frac{3\,l}{4} - x\right)$ $\qquad x_0 = \frac{3\,l}{8}$.

Case 6/3: Rectangular load on the leg



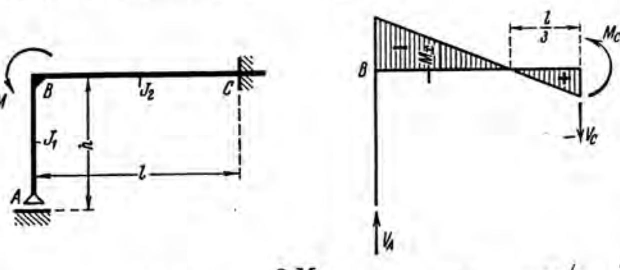
$M_B = -\frac{q\,h^2}{2}$

$M_C = +\frac{q\,h^2}{4}$;

$H_C = q\,h$

$V_A = -V_C = \frac{3\,M_C}{l}$; $\qquad M_y = -\frac{q\,y^2}{2}$ $\qquad M_x = -\frac{q\,h^2}{2}\left(1 - \frac{3\,x}{2\,l}\right)$.

Case 6/4: The moment acts at joint *B*



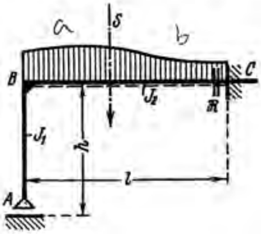
$M_B = -M$

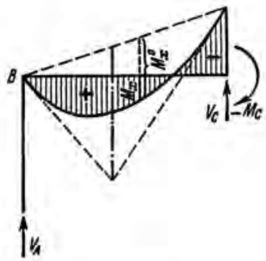
$M_C = \frac{M}{2}$;

$V_A = -V_C = \frac{3\,M}{2\,l}$; $\qquad M_x = -M\left(1 - \frac{3\,x}{2\,l}\right)$.

See Appendix A, Load Terms, pp. 440-445.

Case 6/5: Girder loaded by any type of vertical load

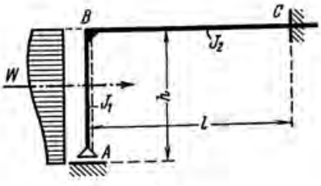


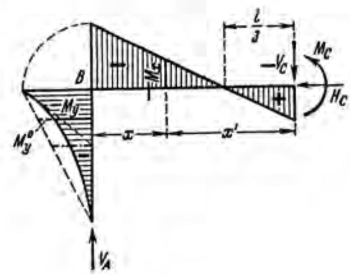


$$M_B = 0 \qquad M_C = -\frac{\mathfrak{R}}{2} \qquad M_x = M_x^0 + \frac{x}{l} M_C;$$

$$V_A = \frac{\mathfrak{S}_r + M_C}{l} \qquad V_C = \frac{\mathfrak{S}_l - M_C}{l}.$$

Case 6/6: Leg loaded by any type of horizontal load





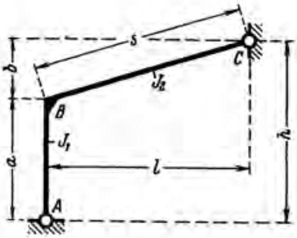
$$M_B = -\mathfrak{S}_r \qquad M_C = \frac{-M_B}{2};$$

$$V_A = -V_C = \frac{3\,M_C}{l} \qquad H_C = W;$$

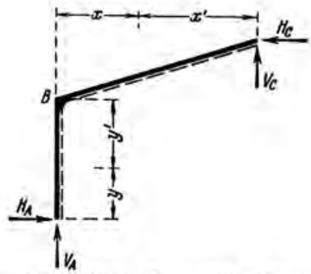
$$M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_y = M_y^0 + \frac{y}{h} M_B.$$

Frame 7

Single-leg, two-hinged rigid frame. Vertical leg. Inclined girder.



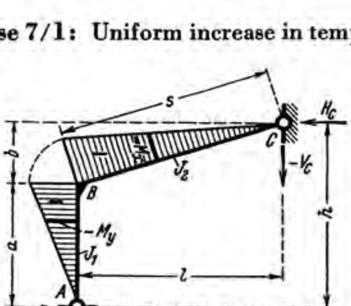
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k=\frac{J_2}{J_1}\cdot\frac{a}{s}$ $\qquad N=k+1$ $\qquad \alpha=\frac{h}{a}.$

Case 7/1: Uniform increase in temperature of the entire frame



$E =$ Modulus of elasticity
$\varepsilon =$ Coefficient of thermal expansion
$t =$ Change of temperature in degrees

$$M_B=-\frac{3\,E\,J_2\,\varepsilon\,t}{s\,N}\cdot\frac{l^2+h^2}{l\,a}$$

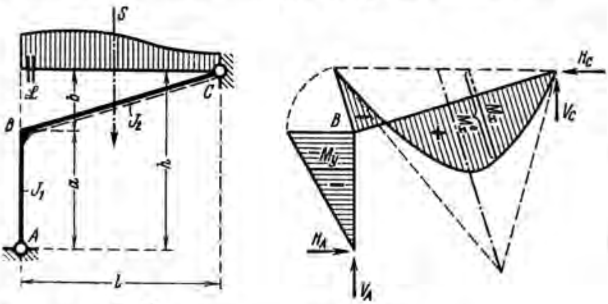
$$V_A=-V_C=\frac{-M_B\,\alpha}{l}$$

$$H_A=H_C=\frac{-M_B}{a}\qquad M_y=\frac{y}{a}M_B\qquad M_x=\frac{x'}{l}M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

See Appendix A, Load Terms, pp. 440-445.

Case 7/2: Girder loaded by any type of vertical load



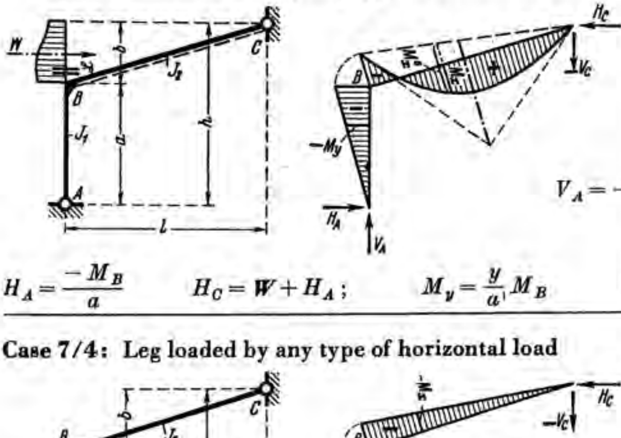
$$M_B = -\frac{\mathfrak{L}}{2N};$$

$$V_A = \frac{\mathfrak{S}_r - \alpha M_B}{l}$$

$$V_C = \frac{\mathfrak{S}_l + \alpha M_B}{l};$$

$$H_A = H_C = \frac{-M_B}{a}; \qquad M_y = \frac{y}{a} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B.$$

Case 7/3: Girder loaded by any type of horizontal load

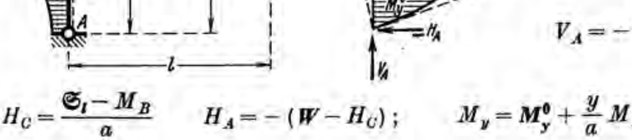


$$M_B = -\frac{\mathfrak{L}}{2N};$$

$$V_A = -V_C = \frac{\mathfrak{S}_r - \alpha M_B}{l};$$

$$H_A = \frac{-M_B}{a} \qquad H_C = W + H_A; \qquad M_y = \frac{y}{a} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B.$$

Case 7/4: Leg loaded by any type of horizontal load



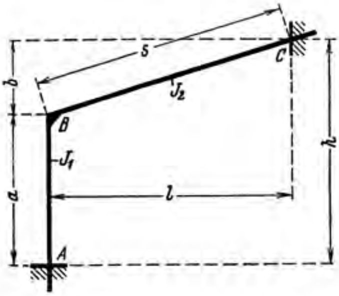
$$M_B = -\frac{\mathfrak{R}k}{2N};$$

$$V_A = -V_C = \frac{b\,\mathfrak{S}_l - h M_B}{l a};$$

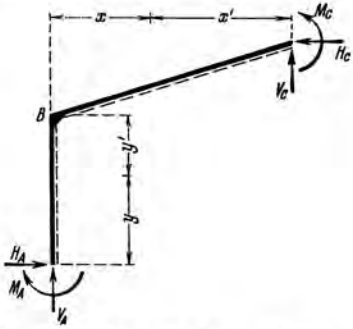
$$H_C = \frac{\mathfrak{S}_l - M_B}{a} \qquad H_A = -(W - H_C); \qquad M_y = M_y^0 + \frac{y}{a} M_B \qquad M_x = \frac{x'}{l} M_B.$$

Frame 8

Single-leg, hingeless rigid frame. Vertical leg. Inclined girder.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{a}{s} \qquad N = k + 1\,.$$

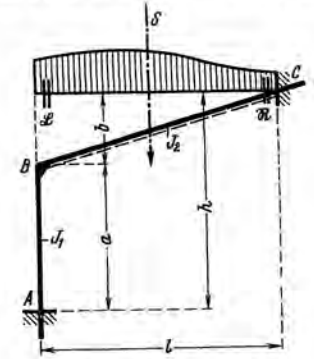
Variables:

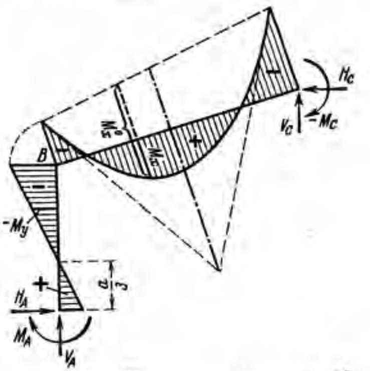
$$\xi = \frac{x}{l} \qquad \xi' = \frac{x'}{l}\,; \qquad \eta = \frac{y}{a} \qquad \eta' = \frac{y'}{a}\,;$$

$$(\xi + \xi' = 1)\,. \qquad (\eta + \eta' = 1)\,.$$

See Appendix A, Load Terms, pp. 440-445.

Case 8/1: Girder loaded by any type of vertical load



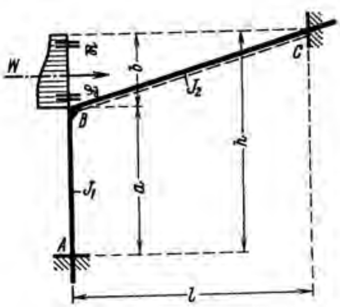


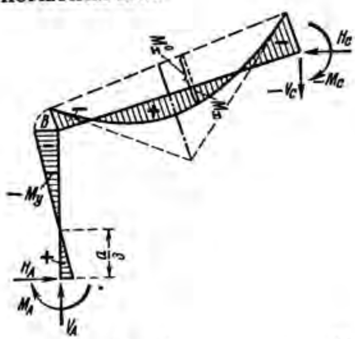
$$M_C = -\frac{\mathfrak{R}(3k+4) - 2\mathfrak{L}}{6N} \qquad M_B = -\frac{2\mathfrak{L} - \mathfrak{R}}{3N} \qquad M_A = \frac{-M_B}{2};$$

$$V_A = \frac{\mathfrak{S}_r + M_C}{l} + \frac{(2h+b)M_A}{la} \qquad V_C = S - V_A \qquad H_A = H_C = \frac{3M_A}{a};$$

$$M_x = M_x^0 + \xi' \cdot M_B + \xi \cdot M_C \qquad M_y = \eta' \cdot M_A + \eta \cdot M_B.$$

Case 8/2: Girder loaded by any type of horizontal load





$$M_C = -\frac{\mathfrak{R}(3k+4) - 2\mathfrak{L}}{6N} \qquad M_B = -\frac{2\mathfrak{L} - \mathfrak{R}}{3N} \qquad M_A = \frac{-M_B}{2};$$

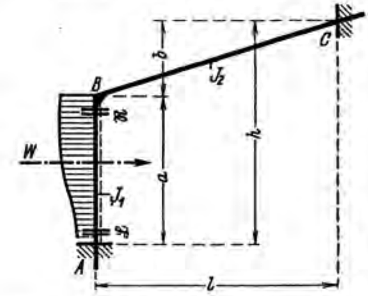
$$H_A = \frac{3M_A}{a} \qquad H_C = W + H_A \qquad V_A = -V_C = \frac{\mathfrak{S}_r + M_C}{l} + \frac{(2h+b)M_A}{la};$$

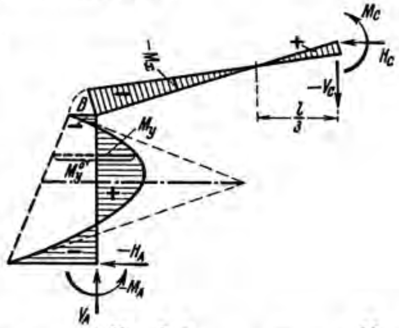
$$M_x = M_x^0 + \xi' \cdot M_B + \xi \cdot M_C \qquad M_y = \eta' \cdot M_A + \eta \cdot M_B.$$

FRAME 8

See Appendix A, Load Terms, pp. 440-445.

Case 8/3: Leg loaded by any type of horizontal load





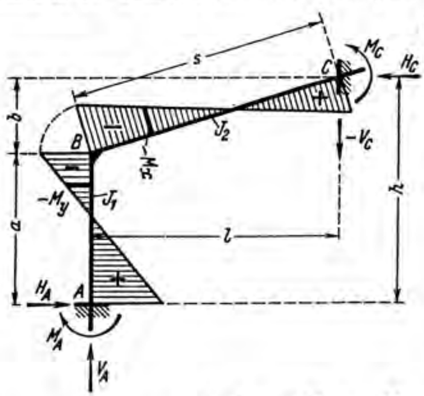
$$M_A = -\frac{\mathfrak{L}(4k+3) - 2\,\mathfrak{R}\,k}{6N} \qquad M_B = -\frac{(2\,\mathfrak{R} - \mathfrak{L})\,k}{3N} \qquad M_C = \frac{-M_B}{2};$$

$$H_C = \frac{\mathfrak{S}_l + M_A - M_B}{a} \qquad H_A = -(W - H_C)$$

$$V_A = -V_C = \frac{b(\mathfrak{S}_l + M_A) + (2h + a)\,M_C}{l\,a};$$

$$M_x = \xi' \cdot M_B + \xi \cdot M_C \qquad M_y = M_y^0 + \eta' \cdot M_A + \eta \cdot M_B.$$

Case 8/4: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants: $T = \frac{E\,J_2\,\varepsilon\,t}{s\,N}$

$$A = \frac{l^2 + h\,b}{l\,a}, \qquad B = \frac{l^2 + h^2}{l\,a},$$

$$M_A = +T\left[A\,\frac{4k+3}{k} + 2B + \frac{h}{l}\right]$$

$$M_B = -2T\left[A + 2B + \frac{h}{l}\right]$$

$$M_C = +T\left[A + 2B + \frac{h(3k+4)}{l}\right];$$

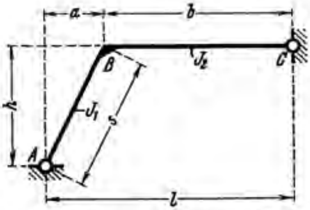
$$V_A = -V_C = \frac{b\,M_A - h\,M_B + a\,M_C}{l\,a} \qquad H_A = H_C = \frac{M_A - M_B}{a};$$

$$M_y = \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = \xi' \cdot M_B + \xi \cdot M_C.$$

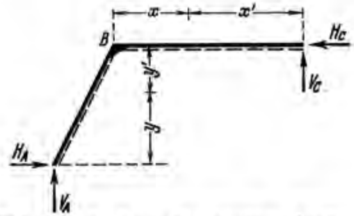
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 9

Single-leg, two-hinged rigid frame. Inclined leg. Horizontal girder.



Shape of Frame
Dimensions and Notations

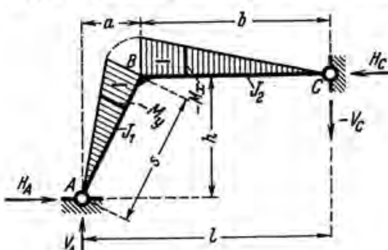


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{s}{b} \qquad N = k + 1 \qquad \beta = \frac{l}{b}.$$

Case 9/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_B = -\frac{3 E J_2 \varepsilon t}{h N} \cdot \frac{l^2 + h^2}{b^2};$$

$$V_A = -V_C = \frac{-M_B}{b} \qquad H_A = H_C = \frac{-M_B \beta}{h};$$

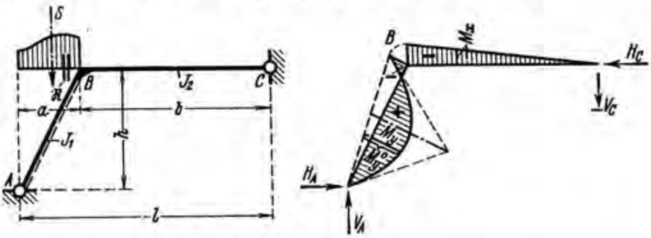
$$M_y = \frac{y}{h} M_B \qquad M_x = \frac{x'}{b} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 9

See Appendix A, Load Terms, pp. 440-445.

Case 9/2: Leg loaded by any type of vertical load



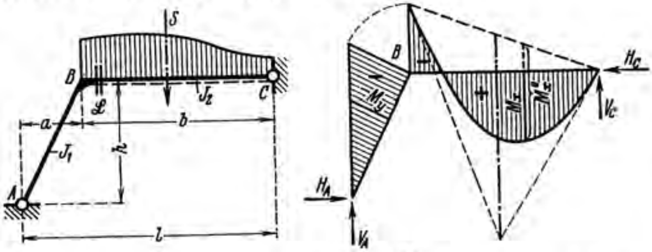
$$M_B = -\frac{\mathfrak{R}\,k}{2N};$$

$$V_A = S - \frac{M_B}{b}$$

$$V_C = \frac{M_B}{b};$$

$$H_A = H_C = \frac{\mathfrak{S}_l - \beta M_B}{h}; \qquad M_y = \mathbf{M}_y^0 + \frac{y}{h} M_B \qquad M_x = \frac{x'}{b} M_B.$$

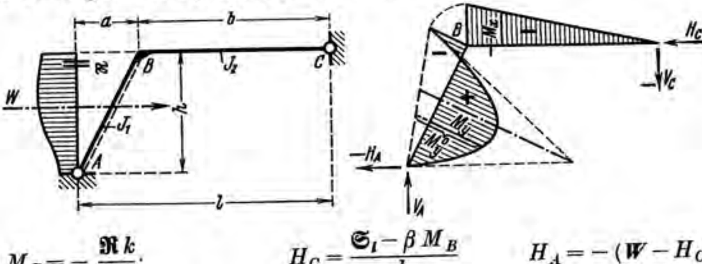
Case 9/3: Girder loaded by any type of vertical load



$$M_B = -\frac{\mathfrak{L}}{2N}; \qquad V_A = \frac{\mathfrak{S}_r - M_B}{b} \qquad V_C = S - V_A;$$

$$H_A = H_C = \frac{a\,\mathfrak{S}_r - l\,M_B}{h\,b}; \qquad M_y = \frac{y}{h} M_B \qquad M_x = \mathbf{M}_x^0 + \frac{x'}{b} M_B.$$

Case 9/4: Leg loaded by any type of horizontal load

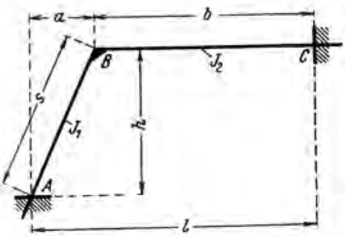


$$M_B = -\frac{\mathfrak{R}\,k}{2N}; \qquad H_C = \frac{\mathfrak{S}_l - \beta M_B}{h} \qquad H_A = -(W - H_C);$$

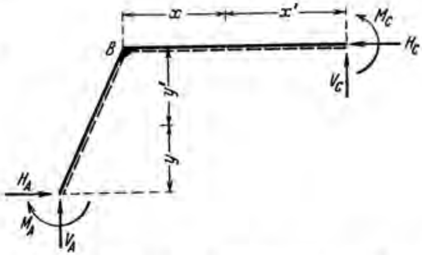
$$V_A = -V_C = \frac{-M_B}{b}; \qquad M_y = \mathbf{M}_y^0 + \frac{y}{h} M_B \qquad M_x = \frac{x'}{b} M_B.$$

Frame 10

Single-leg, hingeless rigid frame. Inclined leg. Horizontal girder.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{s}{b} \qquad N = k + 1 .$$

Variables:

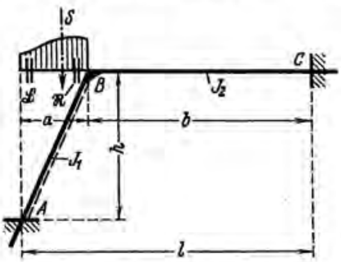
$$\xi = \frac{x}{b} \qquad \xi' = \frac{x'}{b} ; \qquad \eta = \frac{y}{h} \qquad \eta' = \frac{y'}{h} ;$$

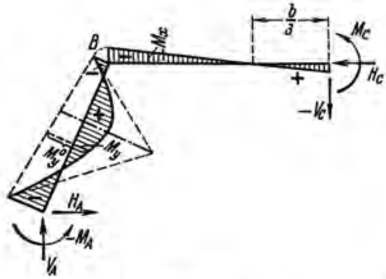
$$(\xi + \xi' = 1) . \qquad (\eta + \eta' = 1) .$$

FRAME 10

See Appendix A, Load Terms, pp. 440-445.

Case 10/1: Leg loaded by any type of vertical load



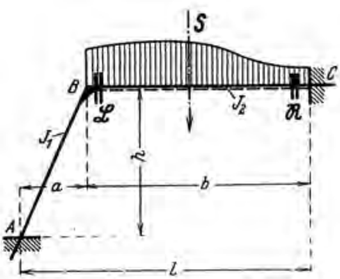


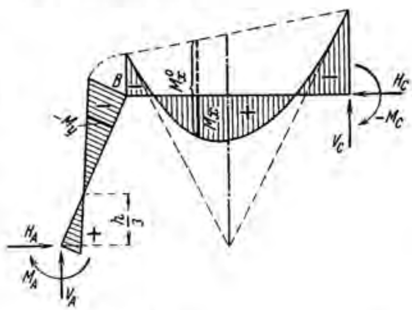
$$M_A = -\frac{\mathfrak{L}(4k+3) - 2\,\mathfrak{R}\,k}{6N} \qquad M_B = -\frac{(2\,\mathfrak{R} - \mathfrak{L})\,k}{3N} \qquad M_C = \frac{-M_B}{2}\,;$$

$$V_A = S + \frac{3M_C}{b} \qquad V_C = -\frac{3M_C}{b} \qquad H_A = H_C = \frac{\mathfrak{S}_l + M_A}{h} + \frac{(2l+a)\,M_C}{hb}\,;$$

$$M_y = M_y^0 + \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = \xi' \cdot M_B + \xi \cdot M_C\,.$$

Case 10/2: Girder loaded by any type of vertical load





$$M_C = -\frac{\mathfrak{R}(3k+4) - 2\,\mathfrak{L}}{6N} \qquad M_B = -\frac{2\,\mathfrak{L} - \mathfrak{R}}{3N} \qquad M_A = \frac{-M_B}{2}\,;$$

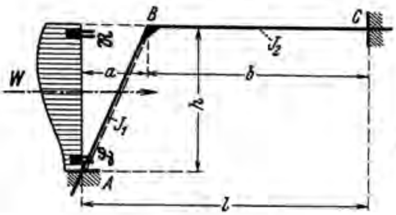
$$V_A = \frac{\mathfrak{S}_r - M_B + M_C}{b} \qquad V_C = S - V_A$$

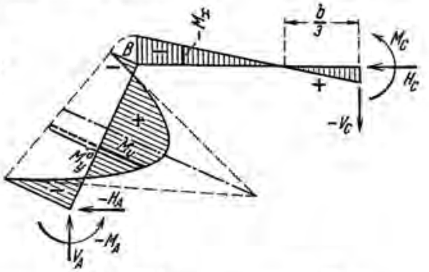
$$H_A = H_C = \frac{a(\mathfrak{S}_r + M_C) + (2l+b)\,M_A}{hb}\,;$$

$$M_y = \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = M_x^0 + \xi' \cdot M_B + \xi \cdot M_C\,.$$

See Appendix A, Load Terms, pp. 440-445.

Case 10/3: Leg loaded by any type of horizontal load



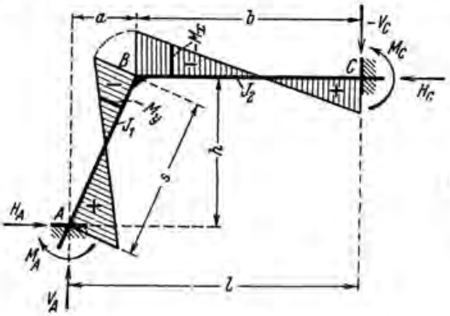


$$M_A = -\frac{\mathfrak{L}(4k+3) - 2\mathfrak{R}k}{6N} \qquad M_B = -\frac{(2\mathfrak{R} - \mathfrak{L})k}{3N} \qquad M_C = \frac{-M_B}{2};$$

$$V_A = -V_C = \frac{3M_C}{b} \qquad H_C = \frac{\mathfrak{S}_l + M_A}{h} + \frac{(2l+a)M_C}{hb} \qquad H_A = -(W - H_C);$$

$$M_y = M_y^0 + \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = \xi' \cdot M_B + \xi \cdot M_C.$$

Case 10/4: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degre

Constants: $T = \dfrac{EJ_2\varepsilon t}{bN}$,

$$B = \frac{l^2 + h^2}{hb}, \qquad C = \frac{la + h^2}{hb}.$$

$$M_A = +T\left[\frac{l(4k+3)}{hk} + 2B + C\right] \qquad M_y = \eta' \cdot M_A + \eta \cdot M_B$$

$$M_B = -2T\left[\frac{l}{h} + 2B + C\right] \qquad M_x = \xi' \cdot M_B + \xi \cdot M_C;$$

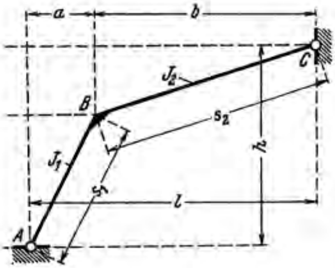
$$M_C = +T\left[\frac{l}{h} + 2B + C(3k+4)\right]; \qquad V_A = -V_C = \frac{M_C - M_B}{b}$$

$$H_A = H_C = \frac{bM_A - lM_B + aM_C}{hb}.$$

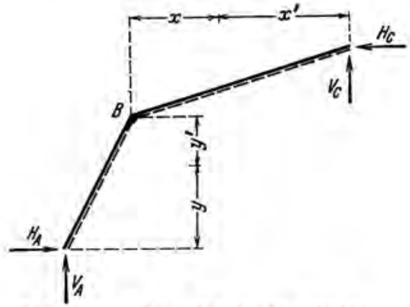
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 11

›ingle-leg, two-hinged rigid frame. Inclined leg. Inclined girder.



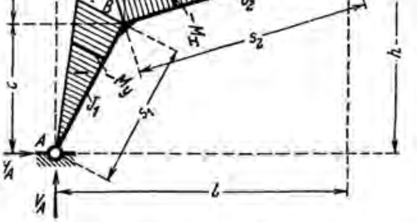
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

›efficients: $k = \frac{J_2}{J_1} \cdot \frac{s_1}{s_2}$ $N = k + 1$ $F = bc - ad.$

›ase 11/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_B = -\frac{3 E J_2 \varepsilon t}{s_2 N} \cdot \frac{l^2 + h^2}{F};$$

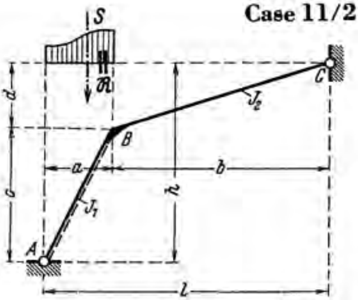
$$V_A = -V_C = \frac{-M_B h}{F}$$

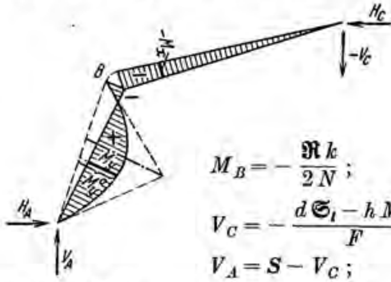
$$H_A = H_C = \frac{-M_B l}{F}; \qquad M_y = \frac{y}{c} M_B \qquad M_x = \frac{x'}{b} M_B.$$

›ote: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

See Appendix A, Load Terms, pp. 440-445.

Case 11/2: Leg loaded by any type of vertical load

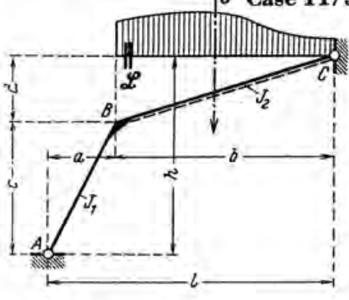


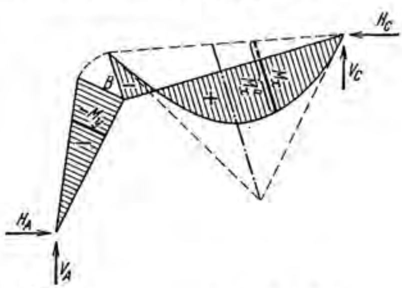


$$M_B = -\frac{\mathfrak{R}\,k}{2N}\,; \qquad V_C = -\frac{d\,\mathfrak{S}_l - h\,M_B}{F}\qquad V_A = S - V_C\,;$$

$$H_A = H_C = \frac{b\,\mathfrak{S}_l - l\,M_B}{F}\,; \qquad M_y = M_y^0 + \frac{y}{c}\,M_B \qquad M_x = \frac{x'}{b}\,M_B\,.$$

Case 11/3: Girder loaded by any type of vertical load

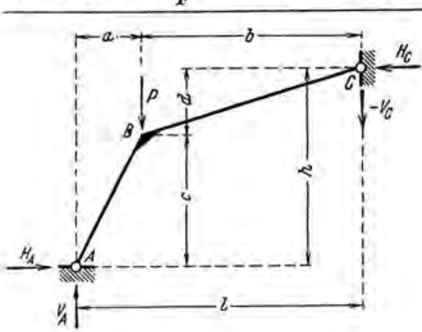




$$M_B = -\frac{\mathfrak{L}}{2N}\,; \qquad V_A = \frac{c\,\mathfrak{S}_r - h\,M_B}{F} \qquad V_C = S - V_A\,;$$

$$H_A = H_C = \frac{a\,\mathfrak{S}_r - l\,M_B}{F}\,; \qquad M_y = \frac{y}{c}\,M_B \qquad M_x = M_x^0 + \frac{x'}{b}\,M_B\,.$$

Case 11/4: Vertical concentrated load at joint B



There are no bending moments.

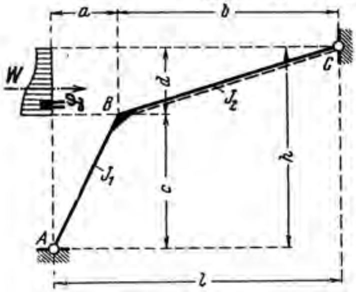
$$V_A = \frac{P\,b\,c}{F} \qquad V_C = -\frac{P\,a\,d}{F}$$

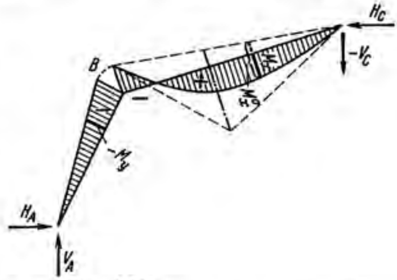
$$H_A = H_C = \frac{P\,a\,b}{F}\,.$$

FRAME 11

See Appendix A, Load Terms, pp. 440-445.

Case 11/5: Girder loaded by any type of horizontal load

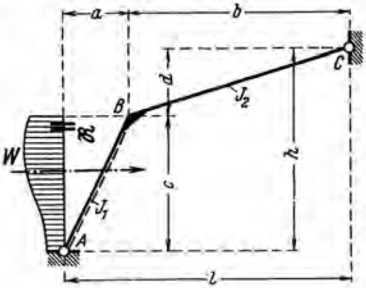


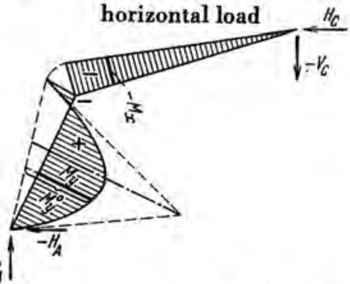


$$M_B = -\frac{\mathfrak{L}}{2N}\,; \qquad H_A = \frac{a\,\mathfrak{S}_r - l\,M_B}{F} \qquad H_C = W + H_A\,;$$

$$V_A = -V_C = \frac{c\,\mathfrak{S}_r - h\,M_B}{F}\,; \qquad M_y = \frac{y}{c}\,M_B \qquad M_x = M_x^0 + \frac{x'}{b}\,M_B\,.$$

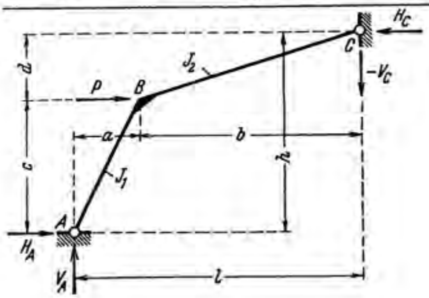
Case 11/6: Leg loaded by any type of horizontal load





$$M_B = -\frac{\mathfrak{R}\,k}{2N}\,; \qquad H_C = \frac{b\,\mathfrak{S}_l - l\,M_B}{F} \qquad H_A = -(W - H_C)\,;$$

$$V_A = -V_C = \frac{d\,\mathfrak{S}_l - h\,M_B}{F}\,; \qquad M_y = M_y^0 + \frac{y}{c}\,M_B \qquad M_x = \frac{x'}{b}\,M_B\,.$$



Case 11/7: Horizontal concentrated load at joint B

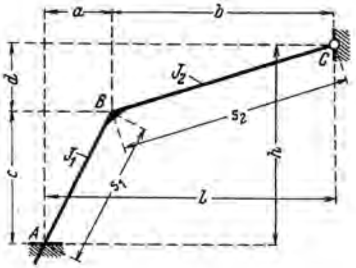
There are no bending moments.

$$H_A = \frac{P\,a\,d}{F} \qquad H_C = \frac{P\,b\,c}{F}$$

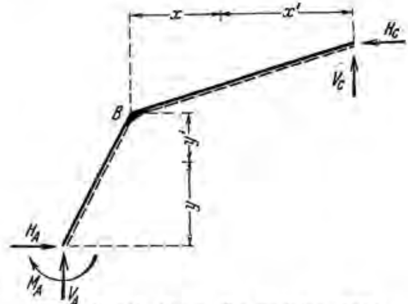
$$V_A = -V_C = \frac{P\,c\,d}{F}$$

FRAME 12

Single-leg, one-hinged rigid frame. Inclined leg. Inclined girder, hinged at one end.



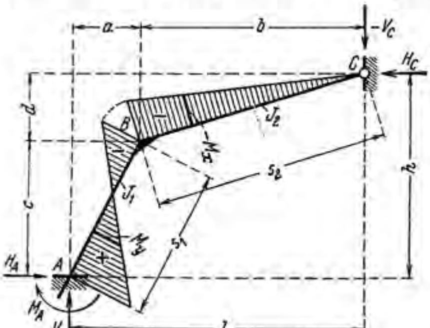
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{s_1}{s_2}$ $\qquad N = 3k + 4 \qquad F = bc - ad$

Case 12/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants: $T = \frac{6 E J_2 \varepsilon t}{s_2 N}$,

$$A = \frac{lb + hd}{F}, \qquad B = \frac{l^2 + h^2}{F}.$$

$$M_A = + T\left(2A\frac{k+1}{k} + B\right)$$

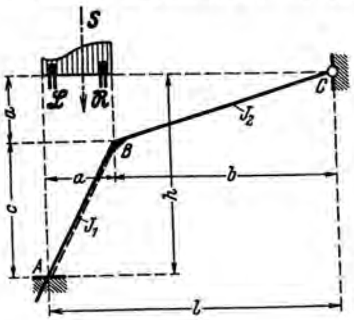
$$V_A = -V_C = \frac{d M_A - h M_B}{F}; \qquad M_B = -T(A + 2B)$$

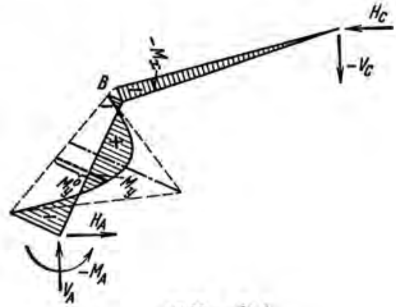
$$H_A = H_C = \frac{b M_A - l M_B}{F}; \qquad M_y = \frac{y'}{c} M_A + \frac{y}{c} M_B \qquad M_x = \frac{x'}{b} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

See Appendix A, Load Terms, pp. 440-445.

ase 12/2: Leg loaded by any type of vertical load





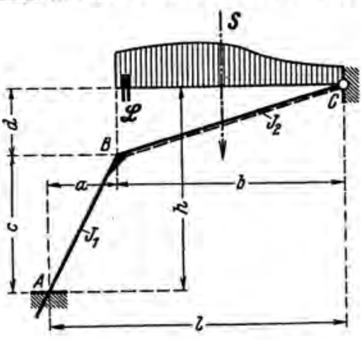
$$M_A = -\frac{2\,\mathfrak{L}(k+1) - \mathfrak{R}\,k}{N} \qquad M_B = -\frac{(2\,\mathfrak{R} - \mathfrak{L})\,k}{N};$$

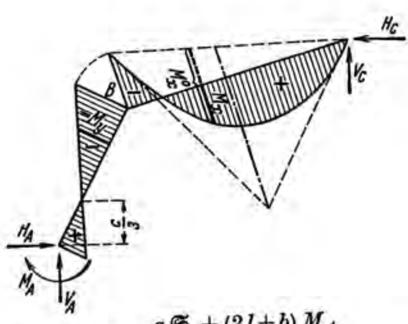
$$V_C = -\frac{d(\mathfrak{S}_l + M_A) - h\,M_B}{F} \qquad V_A = \mathbf{S} - V_C;$$

$$H_A = H_C = \frac{b(\mathfrak{S}_l + M_A) - l\,M_B}{F};$$

$$M_y = M_y^0 + \frac{y'}{c}\,M_A + \frac{y}{c}\,M_B \qquad M_x = \frac{x'}{b}\,M_B.$$

Case 12/3: Girder loaded by any type of vertical load





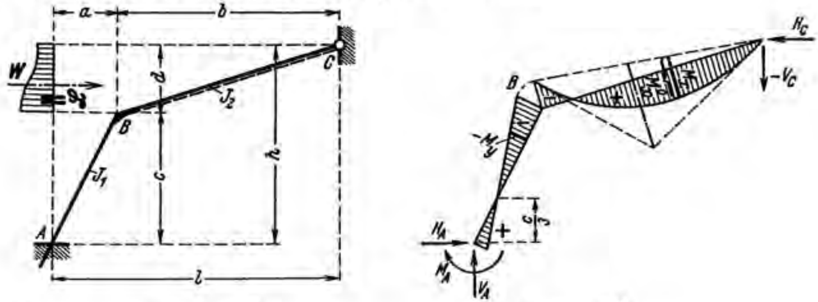
$$M_A = +\frac{\mathfrak{L}}{N} \qquad M_B = -\frac{2\,\mathfrak{L}}{N}; \qquad H_A = H_C = \frac{a\,\mathfrak{S}_r + (2\,l + b)\,M_A}{F};$$

$$V_A = \frac{c\,\mathfrak{S}_r + (2\,h + d)\,M_A}{F} \qquad V_C = \mathbf{S} - V_A;$$

$$M_y = \frac{y'}{c}\,M_A + \frac{y}{c}\,M_B \qquad M_x = M_x^0 + \frac{x'}{b}\,M_B.$$

See Appendix A, Load Terms, pp. 440-445.

Case 12/4: Girder loaded by any type of horizontal load



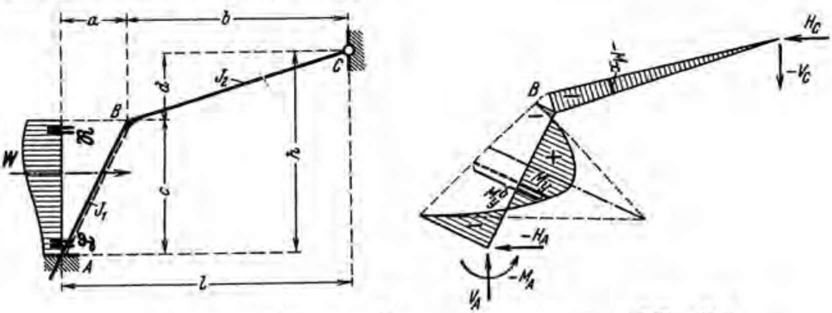
$$M_A = +\frac{\mathfrak{L}}{N} \qquad M_B = -\frac{2\,\mathfrak{L}}{N};$$

$$H_A = \frac{a\,\mathfrak{S}_r + (2l + b)\,M_A}{F} \qquad H_C = W + H_A;$$

$$V_A = -V_C = \frac{c\,\mathfrak{S}_r + (2h + d)\,M_A}{F};$$

$$M_y = \frac{y'}{c}\,M_A + \frac{y}{c}\,M_B \qquad M_x = \mathbf{M}_x^0 + \frac{x'}{b}\,M_B.$$

Case 12/5: Leg loaded by any type



$$M_A = -\frac{2\,\mathfrak{L}(k+1) - \mathfrak{R}\,k}{N} \qquad M_B = -\frac{(2\,\mathfrak{R} - \mathfrak{L})\,k}{N};$$

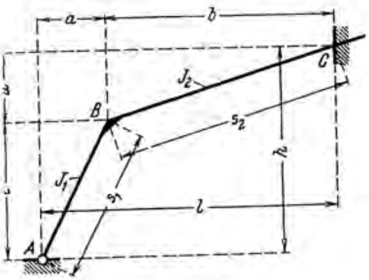
$$H_C = \frac{b\,(\mathfrak{S}_l + M_A) - l\,M_B}{F} \qquad H_A = -(W - H_C);$$

$$V_A = -V_C = \frac{d\,(\mathfrak{S}_l + M_A) - h\,M_B}{F};$$

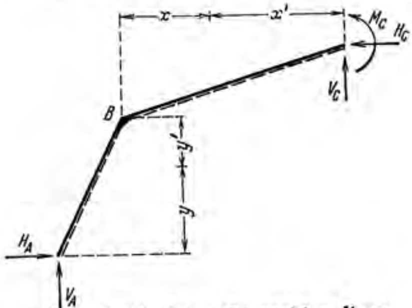
$$M_y = \mathbf{M}_y^0 + \frac{y'}{c}\,M_A + \frac{y}{c}\,M_B \qquad M_x = \frac{x'}{b}\,M_B.$$

Frame 13

Single leg, one-hinged, rigid frame. Inclined leg, hinged at bottom. Inclined girder.



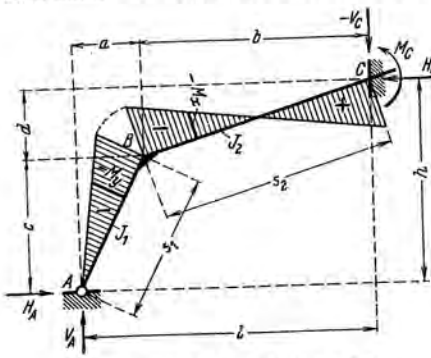
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{s_1}{s_2}$ $N = 4k + 3$ $F = bc - ad$.

Case 13/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants: $T = \frac{6 E J_2 \varepsilon t}{s_2 N}$,

$$B = \frac{l^2 + h^2}{F}, \qquad C = \frac{la + hc}{F}.$$

$$M_B = -T[2B + C]$$

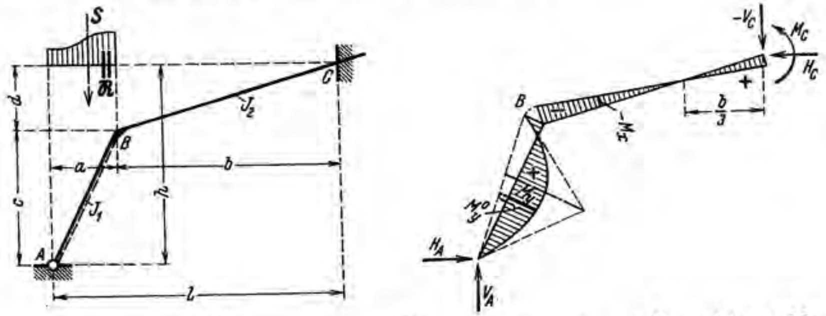
$$M_C = +T[B + 2C(k+1)]; \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C$$

$$V_A = -V_C = \frac{c M_C - h M_B}{F} \qquad H_A = H_C = \frac{a M_C - l M_B}{F}; \qquad M_y = \frac{y}{c} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

See Appendix A, Load Terms, pp. 440-445.

Case 13/2: Leg loaded by any type of vertical load

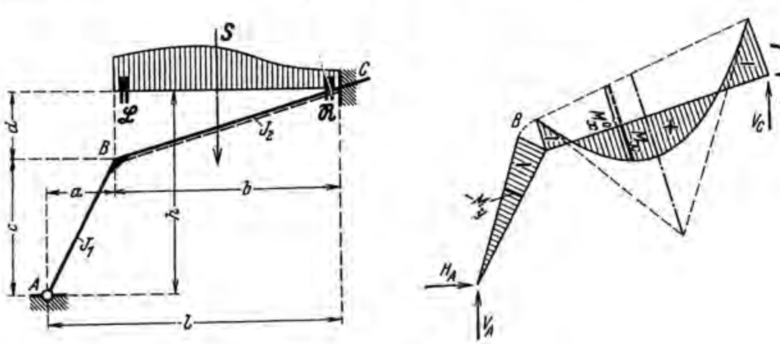


$$M_B = -\frac{2\,\mathfrak{R}\,k}{N} \qquad M_C = +\frac{\mathfrak{R}\,k}{N}; \qquad H_A = H_C = \frac{b\,\mathfrak{S}_l + (2\,l + a)\,M_C}{F};$$

$$V_C = -\frac{d\,\mathfrak{S}_l + (2\,h + c)\,M_C}{F} \qquad V_A = \mathbf{S} - V_C;$$

$$M_y = \mathbf{M}^0_y + \frac{y}{c}\,M_B \qquad M_x = \frac{x'}{b}\,M_B + \frac{x}{b}\,M_C.$$

Case 13/3: Girder loaded by any type of vertical load



$$M_B = -\frac{2\,\mathfrak{L} - \mathfrak{R}}{N} \qquad M_C = -\frac{2\,\mathfrak{R}\,(k+1) - \mathfrak{L}}{N}; \qquad V_A = \frac{c\,(\mathfrak{S}_r + M_C) - h\,M_B}{F}$$

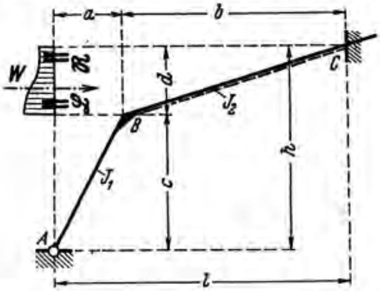
$$H_A = H_C = \frac{a\,(\mathfrak{S}_r + M_C) - l\,M_B}{F}; \qquad V_C = \mathbf{S} - V_A;$$

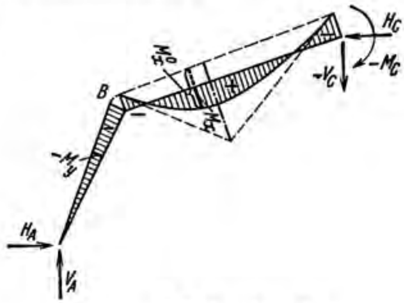
$$M_y = \frac{y}{c}\,M_B \qquad M_x = \mathbf{M}^0_x + \frac{x'}{b}\,M_B + \frac{x}{b}\,M_C.$$

RAME 13

See Appendix A, Load Terms, pp. 440-445.

ase 13/4: Girder loaded by any type of horizontal load





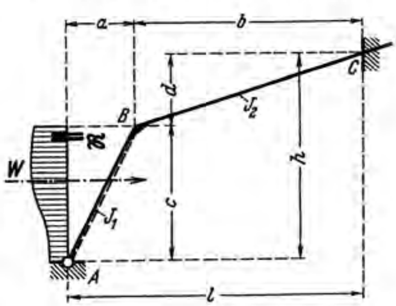
$$M_B = -\frac{2\,\mathfrak{L} - \mathfrak{R}}{N} \qquad M_C = -\frac{2\,\mathfrak{R}\,(k+1) - \mathfrak{L}}{N};$$

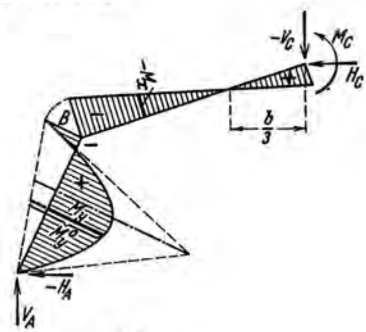
$$H_A = \frac{a\,(\mathfrak{S}_r + M_C) - l\,M_B}{F} \qquad H_C = W + H_A;$$

$$V_A = -V_C = \frac{c\,(\mathfrak{S}_r + M_C) - h\,M_B}{F};$$

$$M_y = \frac{y}{c}\,M_B \qquad M_x = M_x^0 + \frac{x'}{b}\,M_B + \frac{x}{b}\,M_C.$$

:ase 13/5: Leg loaded by any type of horizontal load





$$M_B = -\frac{2\,\mathfrak{R}\,k}{N} \qquad M_C = +\frac{\mathfrak{R}\,k}{N};$$

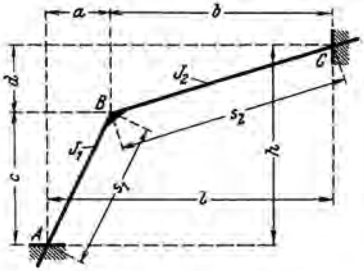
$$H_C = \frac{b\,\mathfrak{S}_l + (2\,l + a)\,M_C}{F} \qquad H_A = -(W - H_C)$$

$$V_A = -V_C = \frac{d\,\mathfrak{S}_l + (2\,h + c)\,M_C}{F};$$

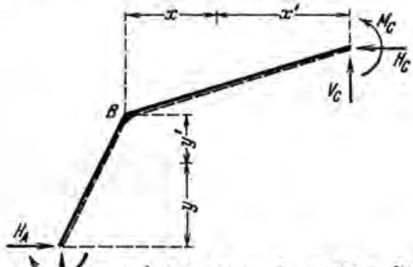
$$M_y = M_y^0 + \frac{y}{c}\,M_B \qquad M_x = \frac{x'}{b}\,M_B + \frac{x}{b}\,M_C.$$

Frame 14

Single-leg, hingeless rigid frame. Inclined leg. Inclined girder.



Shape of Frame
Dimensions and Notations

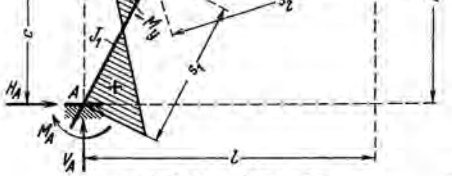


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{s_1}{s_2}$ $\qquad N = k + 1$ $\qquad F = b\,c - a\,d$

Variables: $\xi = \frac{x}{b}$ $\qquad \xi' = \frac{x'}{b};$ $\qquad \eta = \frac{y}{c}$ $\qquad \eta' = \frac{y'}{c}.$

Case 14/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$T = \frac{E J_2 \varepsilon t}{s_2 N}, \qquad A = \frac{l b + h d}{F},$$

$$B = \frac{l^2 + h^2}{F}, \qquad C = \frac{l a + h c}{F}.$$

$$M_A = +T\left[A\,\frac{4k+3}{k} + 2B + C\right]$$

$$M_B = -2T[A + 2B + C] \qquad M_C = +T[A + 2B + C(3k+4)];$$

$$V_A = -V_C = \frac{d M_A - h M_B + c M_C}{F} \qquad H_A = H_C = \frac{b M_A - l M_B + a M_C}{F};$$

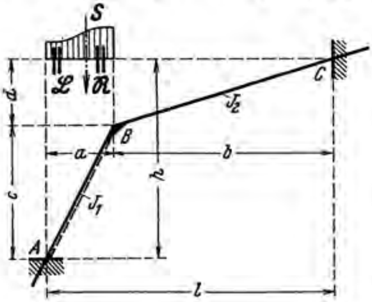
$$M_y = \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = \xi' \cdot M_B + \xi \cdot M_C.$$

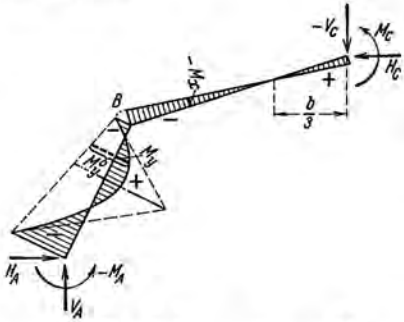
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 14

See Appendix A, Load Terms, pp. 440-445.

Case 14/2: Leg loaded by any type of vertical load





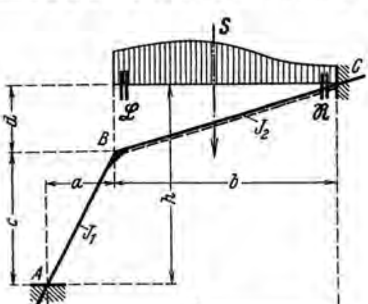
$$M_A = -\frac{\mathfrak{L}(4k+3) - 2\mathfrak{R}k}{6N} \qquad M_B = -\frac{(2\mathfrak{R} - \mathfrak{L})k}{3N} \qquad M_C = \frac{-M_B}{2};$$

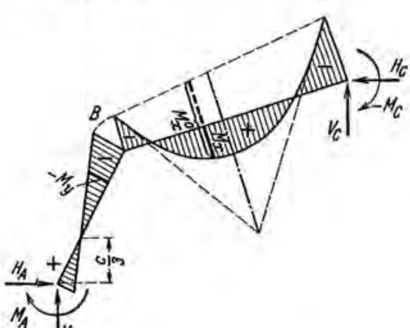
$$V_C = -\frac{d(\mathfrak{S}_l + M_A) + (2h+c)M_C}{F} \qquad V_A = \mathbf{S} - V_C;$$

$$H_A = H_C = \frac{b(\mathfrak{S}_l + M_A) + (2l+a)M_C}{F};$$

$$M_y = \mathbf{M}_y^0 + \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = \xi' \cdot M_B + \xi \cdot M_C.$$

Case 14/3: Girder loaded by any type of vertical load





$$M_C = -\frac{\mathfrak{R}(3k+4) - 2\mathfrak{L}}{6N} \qquad M_B = -\frac{2\mathfrak{L} - \mathfrak{R}}{3N} \qquad M_A = \frac{-M_B}{2};$$

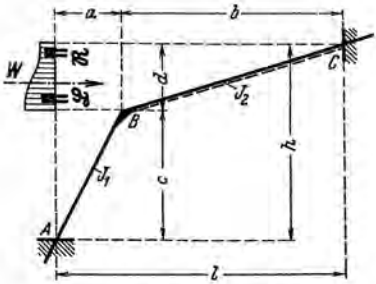
$$V_A = \frac{c(\mathfrak{S}_r + M_C) + (2h+d)M_A}{F} \qquad V_C = \mathbf{S} - V_A;$$

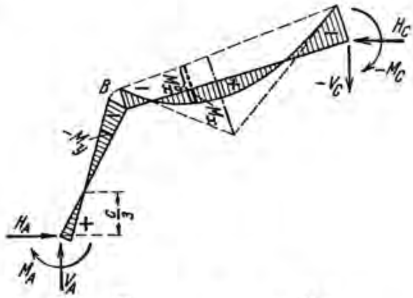
$$H_A = H_C = \frac{a(\mathfrak{S}_r + M_C) + (2l+b)M_A}{F};$$

$$\mathbf{M}_y = \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = \mathbf{M}_x^0 + \xi' \cdot M_B + \xi \cdot M_C.$$

See Appendix A, Load Terms, pp. 440-445.

Case 14/4: Girder loaded by any type of horizontal load





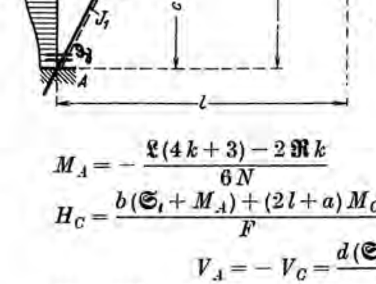
$$M_C = -\frac{\mathfrak{R}(3k+4) - 2\mathfrak{L}}{6N} \qquad M_B = -\frac{2\mathfrak{L} - \mathfrak{R}}{3N} \qquad M_A = \frac{-M_B}{2};$$

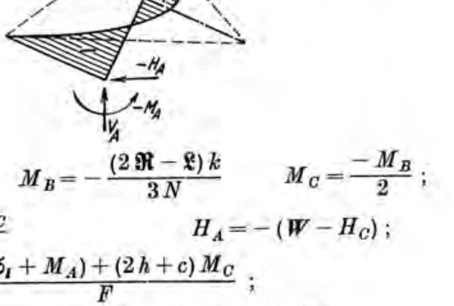
$$H_A = \frac{a(\mathfrak{S}_r + M_C) + (2l+b)M_A}{F} \qquad H_C = W + H_A;$$

$$V_A = -V_C = \frac{c(\mathfrak{S}_r + M_C) + (2h+d)M_A}{F};$$

$$M_y = \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = \mathbf{M}_x^0 + \xi' \cdot M_B + \xi \cdot M_C.$$

Case 14/5: Leg loaded by any type of horizontal load





$$M_A = -\frac{\mathfrak{L}(4k+3) - 2\mathfrak{R}k}{6N} \qquad M_B = -\frac{(2\mathfrak{R} - \mathfrak{L})k}{3N} \qquad M_C = \frac{-M_B}{2};$$

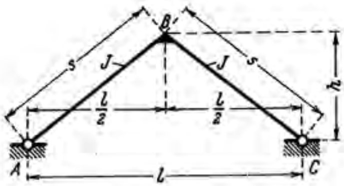
$$H_C = \frac{b(\mathfrak{S}_l + M_A) + (2l+a)M_C}{F} \qquad H_A = -(W - H_C);$$

$$V_A = -V_C = \frac{d(\mathfrak{S}_l + M_A) + (2h+c)M_C}{F};$$

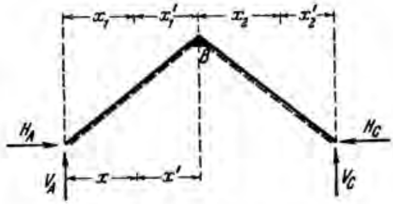
$$M_y = \mathbf{M}_y^0 + \eta' \cdot M_A + \eta \cdot M_B \qquad M_x = \xi' \cdot M_B + \xi \cdot M_C.$$

Frame 15

Symmetrical two-hinged, triangular rigid frame.

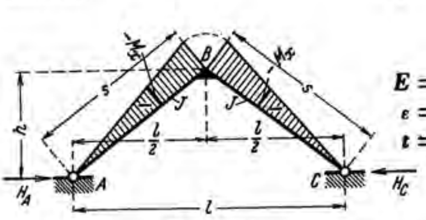


Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use x and x'. Positive bending moments cause tension at the face marked by a dashed line.

Case 15/1: Uniform increase in temperature of the entire frame

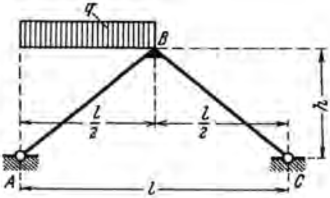


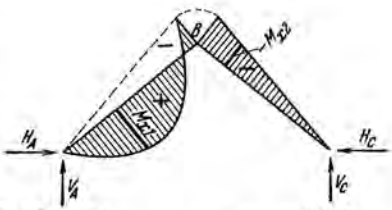
E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_B = -\frac{3EJ\varepsilon t l}{2sh} \qquad H_A = H_C = \frac{-M_B}{h} \qquad M_x = 2M_B\frac{x}{l}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 15/2: Rectangular load on the left leg

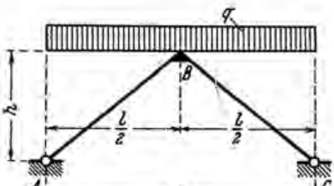


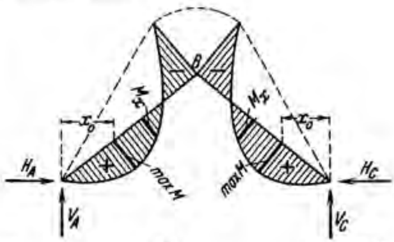


$$M_B = -\frac{q l^2}{64}; \qquad M_{x1} = \frac{q l x_1}{2}\left(\frac{7}{16} - \frac{x_1}{l}\right) \qquad M_{x2} = -\frac{q l x_2'}{32};$$

$$V_A = \frac{3 q l}{8} \qquad V_C = \frac{q l}{8}; \qquad H_A = H_C = \frac{5 q l^2}{64 h}; \qquad Q_{x1} = \frac{q l^2}{2 s}\left(\frac{7}{32} - \frac{x_1}{l}\right).$$

Case 15/3: Rectangular load over the entire frame

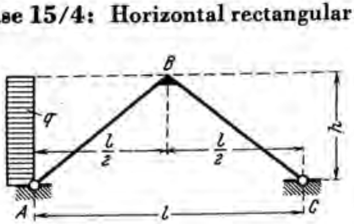


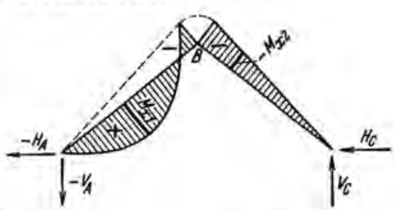


$$M_B = -\frac{q l^2}{32}; \qquad M_x = \frac{q l x}{2}\left(\frac{3}{8} - \frac{x}{l}\right); \qquad x_0 = \frac{3 l}{16} \qquad \max M = \frac{9 q l^2}{512};$$

$$V_A = V_C = \frac{q l}{2}; \qquad H_A = H_C = \frac{5 q l^2}{32 h}; \qquad Q_x = \frac{q l^2}{2 s}\left(\frac{3}{16} - \frac{x}{l}\right).$$

Case 15/4: Horizontal rectangular load from the left





$$M_B = -\frac{q h^2}{16}; \qquad M_{x1} = \frac{2 q h^2 x_1}{l}\left(\frac{7}{16} - \frac{x_1}{l}\right) \qquad M_{x2} = -\frac{q h^2 x_2'}{8 l};$$

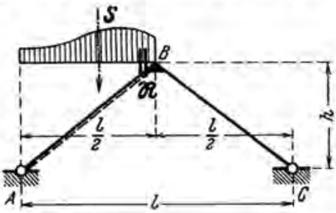
$$H_A = -\frac{11 q h}{16} \qquad H_C = \frac{5 q h}{16}; \qquad V_A = -V_C = -\frac{q h^2}{2 l};$$

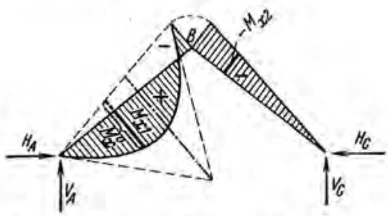
$$Q_{x1} = \frac{2 q h^2}{s}\left(\frac{7}{32} - \frac{x_1}{l}\right).$$

FRAME 15

See Appendix A, Load Terms, pp. 440-445.

Case 15/5: Left-hand member loaded by any type of vertical load

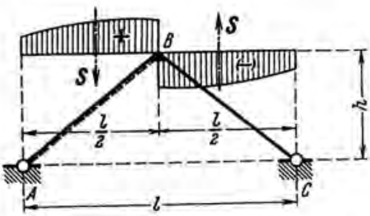


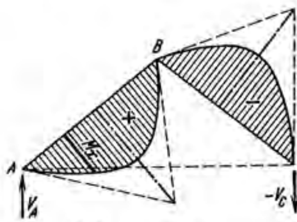


$$M_B = -\frac{\mathfrak{R}}{4}; \qquad M_{x1} = \mathbf{M}_x^0 + 2\,M_B\,\frac{x_1}{l} \qquad M_{x2} = 2\,M_B\,\frac{x_2'}{l};$$

$$V_C = \frac{\mathfrak{S}_l}{l} \qquad V_A = \mathbf{S} - V_C\,; \qquad H_A = H_C = \frac{\mathfrak{S}_l}{2h} - \frac{M_B}{h}.$$

Case 15/7: Both legs loaded by any type of antisymmetrical vertical load

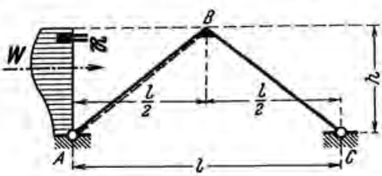


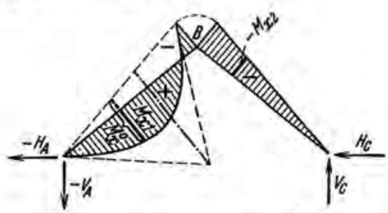


$$M_B = 0 \qquad M_x = \mathbf{M}_x^0; \qquad V_A = -V_C = \frac{2\,\mathfrak{S}_r}{l}; \qquad H_A = H_C = 0.$$

Note: All the load terms refer to the left member.

Case 15/9: Left leg loaded by any type of horizontal load



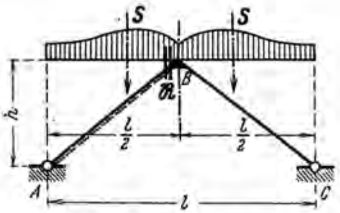


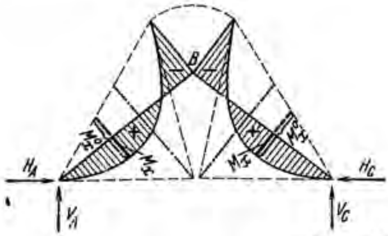
$$M_B = -\frac{\mathfrak{R}}{4}; \qquad M_{x1} = \mathbf{M}_x^0 + 2\,M_B\,\frac{x_1}{l} \qquad M_{x2} = 2\,M_B\,\frac{x_2'}{l};$$

$$V_A = -V_C = -\frac{\mathfrak{S}_l}{l}; \qquad H_C = \frac{\mathfrak{S}_l}{2h} - \frac{M_B}{h} \qquad H_A = -(\mathbf{W} - H_C).$$

(See Appendix A, Load Terms, pp. 440-445.)

Case 15/6: Both legs loaded by any type of symmetrical vertical load

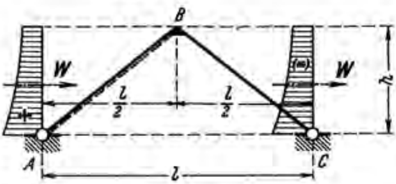


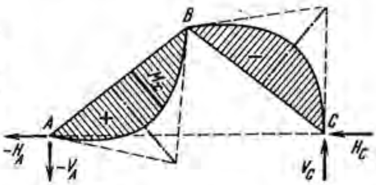


$M_B = -\dfrac{\mathfrak{R}}{2}$; $M_x = \mathbf{M}_x^0 + 2\,M_B\dfrac{x}{l}$; $V_A = V_C = \mathbf{S}$; $H_A = H_C = \dfrac{\mathfrak{S}_l - M_B}{h}$.

Note: All the load terms refer to the left leg.

Case 15/8: Both legs loaded by any type of antisymmetrical horizontal load

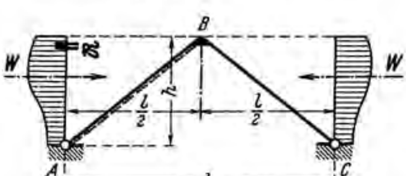


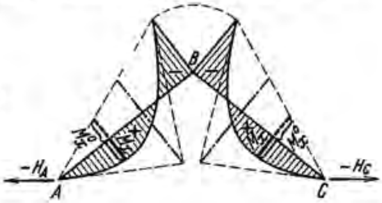


$M_B = 0$; $M_x = \mathbf{M}_x^0$; $V_C = -V_A = \dfrac{2\,\mathfrak{S}_l}{l}$; $H_C = -H_A = \mathbf{W}$.

Note: All the load terms refer to the left leg.

Case 15/10: Both legs loaded by any type of load, both carrying the same load



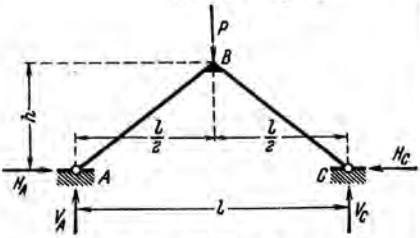


$M_B = -\dfrac{\mathfrak{R}}{2}$; $M_x = \mathbf{M}_x^0 + 2\,M_B\dfrac{x}{l}$; $V_A = V_C = 0$; $H_A = H_C = -\dfrac{\mathfrak{S}_r + M_B}{h}$.

Note: All the load terms refer to the left leg.

FRAME 15

Case 15/11: Vertical concentrated load at ridge B

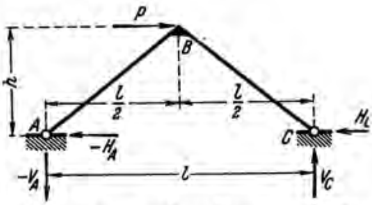


There are no bending moments

$$V_A = V_C = \frac{P}{2}$$

$$H_A = H_C = \frac{Pl}{4h}.$$

Case 15/12: Horizontal concentrated load at ridge B

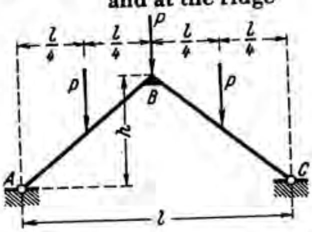


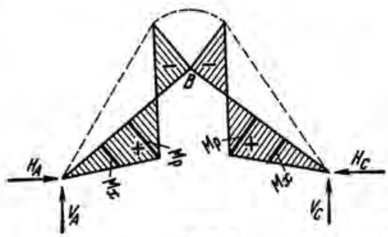
There are no bending moments

$$H_A = -H_C = -\frac{P}{2}$$

$$V_A = -V_C = -\frac{Ph}{l}.$$

Case 15/13: Three equal concentrated loads at the midpoints of the legs and at the ridge



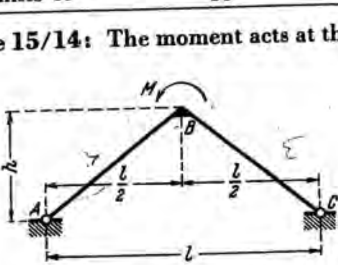


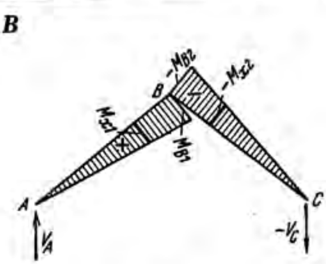
$$M_B = -\frac{3Pl}{32} \qquad V_A = V_C = \frac{3P}{2} \qquad H_A = H_C = \frac{19Pl}{32h} \qquad M_P = \frac{5Pl}{64}$$

Within the limits of AP: $M_x = \frac{5P}{16}x$

Within the limits of PB: $M_x = \frac{Pl}{4} - \frac{11P}{16}x.$

Case 15/14: The moment acts at the ridge B

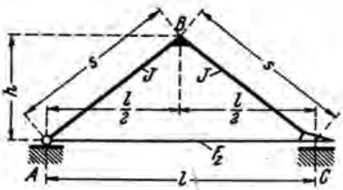




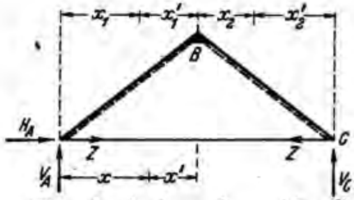
$$M_{B1} = +\frac{M}{2} \quad M_{B2} = -\frac{M}{2} \quad V_A = -V_C = \frac{M}{l} \quad M_{x1} = +\frac{x_1}{l}M \quad M_{x2} = -\frac{x_2'}{l}M.$$

FRAME 16

Symmetrical triangular rigid frame with tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use x and x'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$L = \frac{3J}{h^2 F_Z} \cdot \frac{l}{s} \cdot \frac{E}{E_Z} \qquad\qquad N_Z = 2 + L\,.$$

E = Modulus of elasticity of the material of the frame
E_z = Modulus of elasticity of the tie rod
F_z = Cross-sectional area of the tie rod

Note concerning cases of antisymmetrical load

The antisymmetric case 15/7, p. 44, is valid also for frame 16, since $Z = 0$ because of $H = 0$.

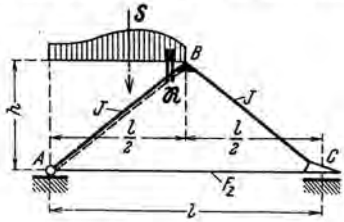
For the antisymmetric case 15/8, p. 45, with elastic tie-rod and hinged joint at A we have:

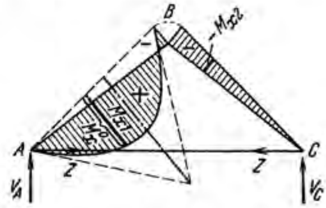
$$Z = \frac{2\,W}{N_Z} \qquad H_A = 2\,W; \qquad V_C = -V_A = \frac{2\,\mathfrak{S}_r}{l}; \qquad M_B = W h \cdot \frac{L}{N_Z}.$$

FRAME 16

See Appendix A, Load Terms, pp. 440-445.

Case 16/1: Left-hand member loaded by any type of vertical load

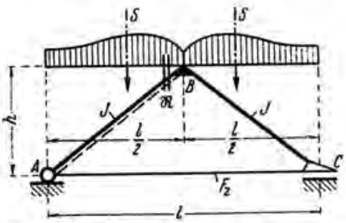


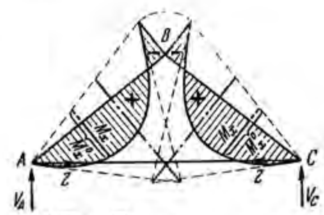


$$Z = \frac{\mathfrak{R} + 2\,\mathfrak{S}_l}{2\,h\,N_Z}; \qquad V_C = \frac{\mathfrak{S}_l}{l} \qquad V_A = \mathbf{S} - V_C\,; \qquad M_{x2} = 2\,M_B \frac{x_2'}{l}$$

$$M_B = \frac{\mathfrak{S}_l}{2} - Z\,h = \frac{L\,\mathfrak{S}_l - \mathfrak{R}}{2\,N_Z}; \qquad M_{x1} = \mathbf{M}_x^0 + 2\,M_B \frac{x_1}{l}.$$

Case 16/2: Both legs loaded by any type of symmetrical vertical load

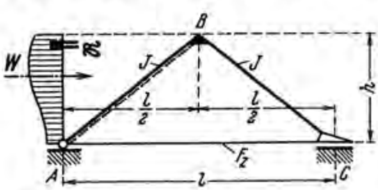


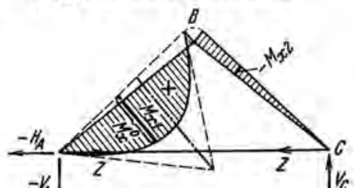


$$Z = \frac{\mathfrak{R} + 2\,\mathfrak{S}_l}{h\,N_Z}; \quad V_A = V_C = \mathbf{S}; \quad M_B = \mathfrak{S}_l - Z\,h = \frac{L\,\mathfrak{S}_l - \mathfrak{R}}{N_Z}; \quad M_x = \mathbf{M}_x^0 + 2\,M_B \frac{x}{l}.$$

Note: All the load terms refer to the left member.

Case 16/3: Left-hand member loaded by any type of horizontal load



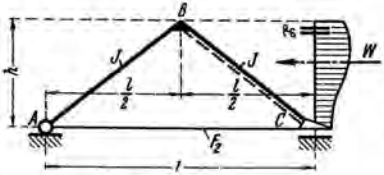


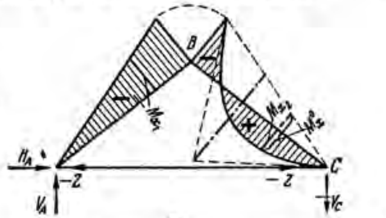
$$Z = \frac{\mathfrak{R} + 2\,\mathfrak{S}_l}{2\,h\,N_Z} \qquad H_A = -\,\mathbf{W}; \qquad V_A = -\,V_C = -\,\frac{\mathfrak{S}_l}{l}; \qquad M_{x2} = 2\,M_B \frac{x_2'}{l}$$

$$M_B = \frac{\mathfrak{S}_l}{2} - Z\,h = \frac{L\,\mathfrak{S}_l - \mathfrak{R}}{2\,N_Z}; \qquad M_{x1} = \mathbf{M}_x^0 + 2\,M_B \frac{x_1}{l}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 16/4: Right-hand member loaded by any type of horizontal load

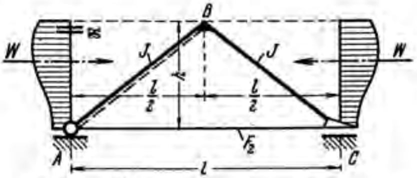


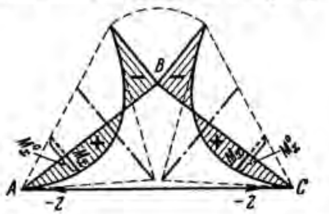


$$Z = -\left(\frac{2\,W}{N_Z} - \frac{\mathfrak{L} + 2\,\mathfrak{S}_r}{2\,h\,N_Z}\right)^* \qquad H_A = W \qquad V_A = -V_C = \frac{\mathfrak{S}_r}{l} \qquad M_{x1} = 2\,M_B\,\frac{x_1}{l}$$

$$M_B = -(W + Z)\,h + \frac{\mathfrak{S}_r}{2} = -\left(W\,h\,\frac{L}{N_Z} - \frac{L\,\mathfrak{S}_r - \mathfrak{L}}{2\,N_Z}\right) \qquad M_{x2} = M_x^0 + 2\,M_B\,\frac{x_2'}{l}.$$

Case 16/5: Both legs loaded by any type of horizontal load, both carrying the same load

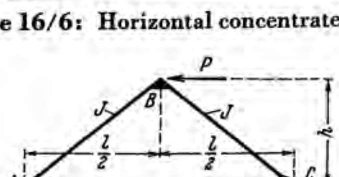


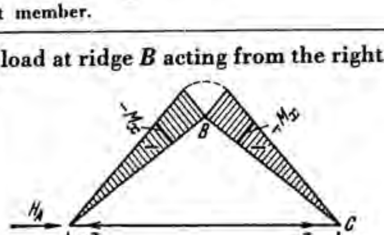


$$Z = -\frac{2\,\mathfrak{S}_r - \mathfrak{R}^*}{h\,N_Z} \qquad M_B = -(\mathfrak{S}_r + Z\,h) = -\frac{L\,\mathfrak{S}_r + \mathfrak{R}}{N_Z} \qquad M_x = M_x^0 + 2\,M_B\,\frac{x}{l}.$$

Note: All the load terms refer to the left member.

Case 16/6: Horizontal concentrated load at ridge *B* acting from the right



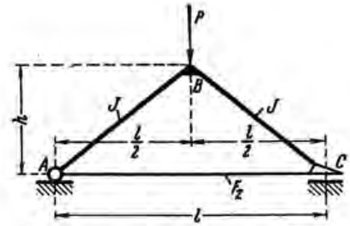


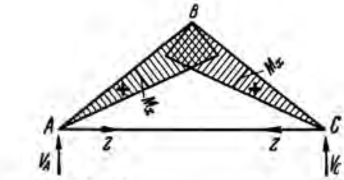
$$Z = -\frac{P^*}{N_Z} \qquad H_A = P \qquad V_A = -V_C = \frac{P\,h}{l} \qquad M_B = -\frac{P\,h}{2}\cdot\frac{L}{N_Z} \qquad M_x = 2\,M_B\,\frac{x}{l}.$$

*For the case of the above loading conditions and for a decrease in temperature (p. 50) *Z* becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

FRAME 16

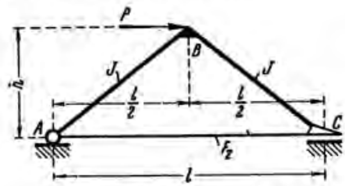
Case 16/7: Vertical concentrated load at ridge *B*

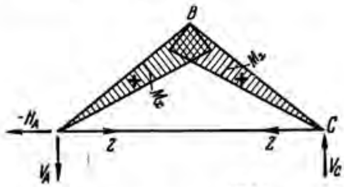




$$Z=\frac{Pl}{2hN_Z}\,; \qquad V_A=V_C=\frac{P}{2}\,; \qquad M_B=\frac{Pl}{4}\cdot\frac{L}{N_Z}\,; \qquad M_x=2\,M_B\frac{x}{l}\,.$$

Case 16/8: Horizontal concentrated load acting at ridge *B* from the left

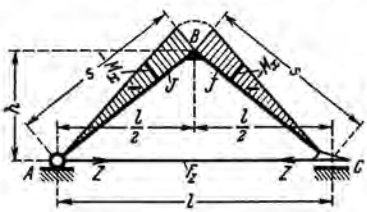




$$Z=\frac{P}{N_Z} \qquad H_A=-P\,; \qquad V_A=-V_C=-\frac{Ph}{l}\,;$$

$$M_B=\frac{Ph}{2}\cdot\frac{L}{N_Z}\,; \qquad M_x=2\,M_B\frac{x}{l}\,.$$

Case 16/9: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

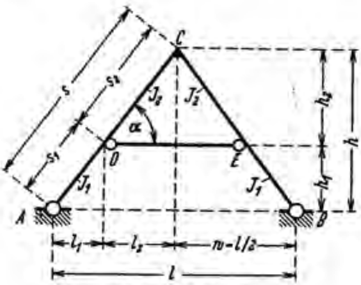
$$Z=\frac{3\,E\,J\,\varepsilon t l}{s\,h^2 N_Z}\,; \qquad M_B=-Zh\,; \qquad M_x=2\,M_B\frac{x}{l}\,.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

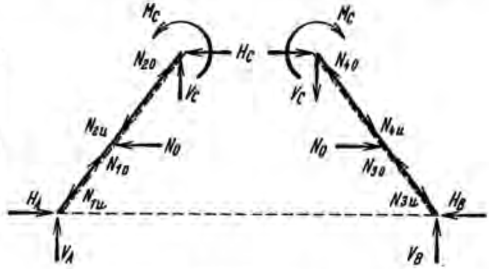
*See footnote on page 49.

FRAME 17

Symmetrical triangular two-hinged frame with hinged compression tie and with step-wise varying moments of inertia.[1]



Shape of Frame
Dimensions and Notations



Positive direction of all reactions at the ridge and all axial forces.[2]

Coefficients:

$$k=\frac{J_2}{J_1}\cdot\frac{s_1}{s_2}\ [1]; \quad \left(\frac{s_1}{s_2}=\frac{l_1}{l_2}=\frac{h_1}{h_2}\right); \quad \beta_1=\frac{l_1}{w}=\frac{h_1}{h}; \quad \beta_2=\frac{l_2}{w}=\frac{h_2}{h};$$

$$F=4k+3. \qquad (\beta_1+\beta_2=1).$$

Note: The moment diagrams shown for cases 17/1 through 17/6 were drawn for $J_1 = J_2$ and special case b: $q_1 = q_2$.

---

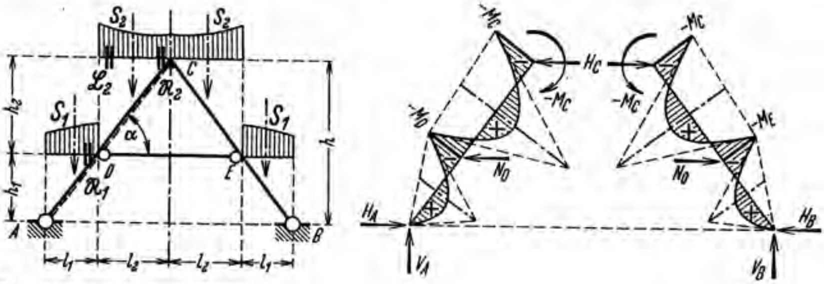
[1] If the moment of inertia is constant over s, i.e., if $J_1 = J_2$, then $k = s_1/s_2$.

[2] Positive bending moments M cause tension at the face marked by a dashed line. Positive axial forces are compression.

FRAME 17

See Appendix A, Load Terms, pp. 440-445.

Case 17/1: Entire frame loaded by any type of symmetrical vertical load



Constants and moments:

$$X = \frac{2\,\mathfrak{R}_1 k + (2\,\mathfrak{L}_2 - \mathfrak{R}_2)}{F}; \qquad M_C = \frac{-\mathfrak{R}_2 + X}{2} \qquad M_D = M_E = -X.$$

Reactions and Shears:

$$H_A = H_B = \frac{\mathfrak{S}_{l1} + S_2 l_1 - M_D}{h_1} \qquad H_C = \frac{\mathfrak{S}_{l2} - M_C + M_D}{h_2};$$

$$V_A = V_B = S_1 + S_2 \qquad V_C = 0; \qquad N_0 = H_A - H_C.$$

Axial forces:

$$N_{1u} = N_{3u} = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2o} = N_{4o} = H_C \cdot \cos\alpha$$

$$N_{1o} = N_{3o} = S_2 \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2u} = N_{4u} = H_C \cdot \cos\alpha + S_2 \cdot \sin\alpha.$$

Note: All the load terms refer to the left half of the frame.

Special case 17/1a: Symmetrical loads $(\mathfrak{R} = \mathfrak{L})$

$$H_A = H_B = \left(\frac{S_1}{2} + S_2\right) \cdot \cot\alpha - \frac{M_D}{h_1} \qquad H_C = \frac{S_2}{2} \cdot \cot\alpha + \frac{M_D - M_C}{h_2};$$

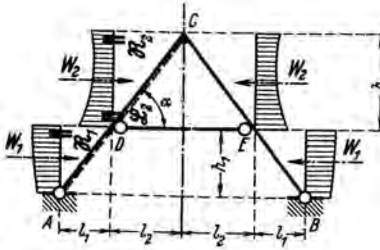
$$X = \frac{2\,\mathfrak{L}_1 k + \mathfrak{L}_2}{F}.$$ All other formulas same as above

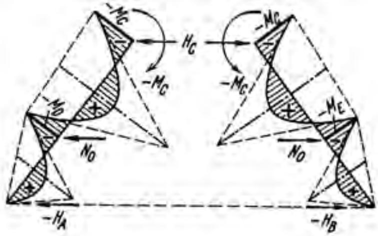
Special case 17/1b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$S_1 = q_1 l_1 \qquad S_2 = q_2 l_2; \qquad \mathfrak{L}_1 = \frac{S_1 l_1}{4} \qquad \mathfrak{L}_2 = \frac{S_2 l_2}{4}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 17/2: Entire frame loaded by any type of symmetrical horizontal load





Constants and moments:

$$X = \frac{2\,\mathfrak{R}_1 k + (2\,\mathfrak{L}_2 - \mathfrak{R}_2)}{F}; \qquad M_C = \frac{-\mathfrak{R}_2 + X}{2} \qquad M_D = M_E = -X.$$

Reactions and Shears:

$$H_A = H_B = -\frac{\mathfrak{S}_{r1} + M_D}{h_1} \qquad H_C = \frac{\mathfrak{S}_{l2} - M_C + M_D}{h_2};$$

$$V_A = V_B = 0 \qquad V_C = 0; \qquad N_0 = W_1 + W_2 + H_A - H_C.$$

Axial forces:

$$N_{1u} = N_{3u} = H_A \cdot \cos\alpha \qquad N_{2o} = N_{4o} = H_C \cdot \cos\alpha$$

$$N_{1o} = N_{3o} = (H_A + W_1) \cdot \cos\alpha \qquad N_{2u} = N_{4u} = (H_C - W_2) \cdot \cos\alpha.$$

Note: All the load terms refer to the left half of the frame.

Special case 17/2a: Symmetrical loads $(\mathfrak{R} = \mathfrak{L})$

$$H_A = H_B = -\frac{W_1}{2} - \frac{M_D}{h_1} \qquad H_C = \frac{W_2}{2} + \frac{M_D - M_C}{h_2};$$

$$X = \frac{2\,\mathfrak{L}_1 k + \mathfrak{L}_2}{F}.$$ **All other formulas same as above**

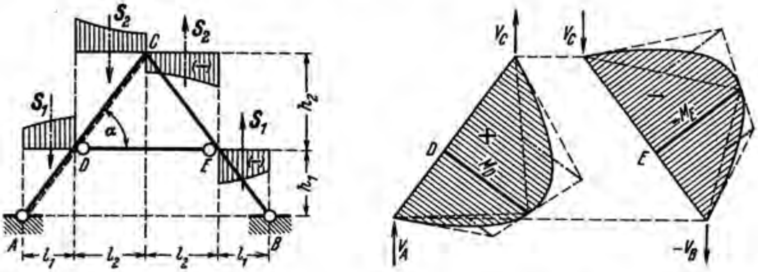
Special case 17/2b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$W_1 = q_1 h_1 \qquad W_2 = q_2 h_2; \qquad \mathfrak{L}_1 = \frac{W_1 h_1}{4} \qquad \mathfrak{L}_2 = \frac{W_2 h_2}{4}.$$

FRAME 17

See Appendix A, Load Terms, pp. 440-445.

Case 17/3: Entire frame loaded by any type of antisymmetrical vertical load



Moments:

$$M_C = 0 \qquad M_D = -M_E = \mathfrak{S}_{l1} \cdot \beta_2 + \mathfrak{S}_{r2} \cdot \beta_1 .$$

Reactions and Shears:

$$V_A = -V_B = \frac{\mathfrak{S}_{r1} + S_1 l_2 + \mathfrak{S}_{r2}}{w} \qquad V_C = \frac{\mathfrak{S}_{l1} + S_2 l_1 + \mathfrak{S}_{l2}}{w};$$

$$H_A = H_B = 0 \qquad H_C = 0; \qquad N_0 = 0 .$$

Axial forces:

$$N_{1u} = -N_{3u} = V_A \cdot \sin\alpha \qquad N_{2o} = -N_{4o} = -V_C \cdot \sin\alpha$$
$$N_{1o} = -N_{3o} = (V_A - S_1)\sin\alpha = N_{2u} = -N_{4u} = (S_2 - V_C)\sin\alpha .$$

Note: All the load terms refer to the left half of the frame.

Special case 17/3a: Symmetrical loads $(\mathfrak{S}_l = \mathfrak{S}_r)$

$$V_A = -V_B = \frac{S_1(1+\beta_2) + S_2 \cdot \beta_2}{2} \qquad V_C = \frac{S_1 \cdot \beta_1 + S_2(1+\beta_1)}{2};$$

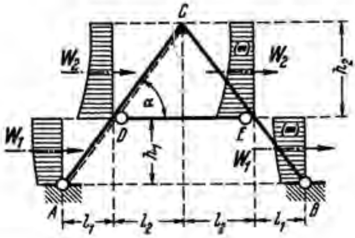
$$M_D = -M_E = \frac{(S_1 + S_2) l_1 l_2}{l} .$$ **All other formulas same as above**

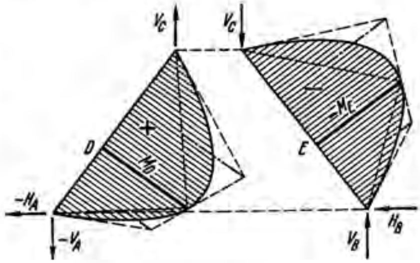
Special case 17/3b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$S_1 = q_1 l_1 \qquad S_2 = q_2 l_2 .$$

See Appendix A, Load Terms, pp. 440-445.

Case 17/4: **Entire frame loaded by any type of antisymmetrical vertical load**





Moments:

$$M_C = 0 \qquad M_D = -M_E = \mathfrak{S}_{l1} \cdot \beta_2 + \mathfrak{S}_{r2} \cdot \beta_1 .$$

Reactions and Shears:

$$V_B = V_C = -V_A = \frac{\mathfrak{S}_{l1} + W_2 h_1 + \mathfrak{S}_{l2}}{w}$$

$$H_B = -H_A = W_1 + W_2 \qquad H_C = 0 \qquad N_0 = 0 .$$

Axial forces:

$$N_{3u} = -N_{1u} = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha \qquad N_{4o} = -N_{2o} = V_C \cdot \sin\alpha$$

$$N_{3o} = -N_{1o} = N_{3u} - W_1 \cdot \cos\alpha = N_{4u} = -N_{2u} = V_C \cdot \sin\alpha + W_2 \cdot \cos\alpha .$$

Note: All the load terms refer to the left half of the frame.

Special case 17/4a: Symmetrical loads, ($\mathfrak{S}_l = \mathfrak{S}_r$)

$$M_D = -M_E = \frac{(W_1 + W_2) h_1 h_2}{2h} ; \qquad V_B = V_C = -V_A = \frac{W_1 h_1 + W_2 (h + h_1)}{l} .$$

All other formulas same as above

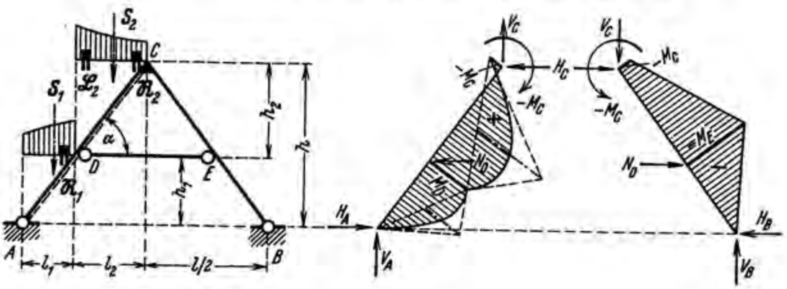
Special case 17/4b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$W_1 = q_1 h_1 \qquad W_2 = q_2 h_2 .$$

FRAME 17

See Appendix A, Load Terms, pp. 440-445.

Case 17/5: Left half of frame loaded by any type of vertical load



Moments (constant X same as case 17/1, p. 52):

$$M_C = \frac{-\mathfrak{R}_2 + X}{4} \qquad \begin{matrix} M_D \\ M_E \end{matrix} \Big\rangle = -\frac{X}{2} \pm \frac{\mathfrak{S}_{l1}\cdot\beta_2 + \mathfrak{S}_{r2}\cdot\beta_1}{2}.$$

Reactions and Shears:

$$V_B = V_C = \frac{\mathfrak{S}_{l1} + S_2 l_1 + \mathfrak{S}_{l2}}{l} \qquad V_A = S_1 + S_2 - V_B;$$

$$H_A = H_B = \frac{V_B\cdot l_1 - M_E}{h_1} \qquad H_C = \frac{V_C\cdot l_2 - M_C + M_E}{h_2}; \qquad N_0 = H_B - H_C.$$

Axial forces:

$$N_{1u} = V_A\cdot\sin\alpha + H_A\cdot\cos\alpha \qquad N_{2o} = -V_C\cdot\sin\alpha + H_C\cdot\cos\alpha$$
$$N_{1o} = N_{1u} - S_1\cdot\sin\alpha; \qquad N_{2u} = N_{2o} + S_2\cdot\sin\alpha;$$
$$N_3 = V_B\cdot\sin\alpha + H_B\cdot\cos\alpha \qquad N_4 = V_C\cdot\sin\alpha + H_C\cdot\cos\alpha.$$

Special case 17/5a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$\begin{matrix} M_D \\ M_E \end{matrix} \Big\rangle = -\frac{X}{2} \pm \frac{(S_1 + S_2) l_1 l_2}{2l}; \qquad V_B = V_C = \frac{S_1\cdot\beta_1 + S_2(1+\beta_1)}{4}.$$

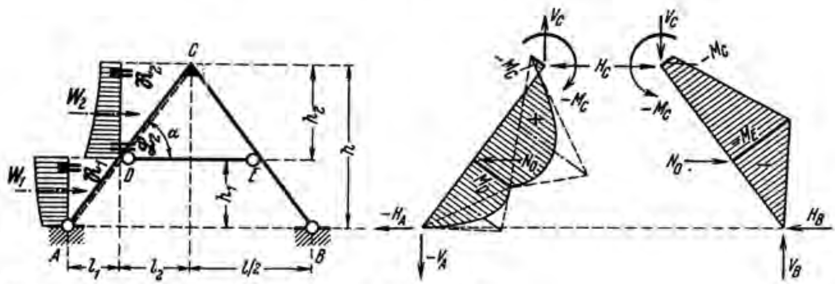
All other formulas same as above (Auxiliary value X exactly as in case 17/1, p. 52):

Special case 17/5b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$S_1 = q_1 l_1 \qquad S_2 = q_2 l_2; \qquad (\mathfrak{L}_1 = S_1 l_1/4 \quad \mathfrak{L}_2 = S_2 l_2/4).$$

See Appendix A, Load Terms, pp. 440-445.

Case 17/6: Left half of frame loaded by any type of horizontal load



Moments (constant X same as case 17/2, p. 53):

$$M_C = \frac{-\mathfrak{R}_2 + X}{4} \qquad \left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{X}{2} \pm \frac{\mathfrak{S}_{l1} \cdot \beta_2 + \mathfrak{S}_{r2} \cdot \beta_1}{2}.$$

Reactions and Shears:

$$V_B = V_C = -V_A = \frac{\mathfrak{S}_{l1} + W_2 h_1 + \mathfrak{S}_{l2}}{l}; \qquad H_C = \frac{V_C \cdot l_2 - M_C + M_E}{h_2}$$

$$H_B = \frac{V_B \cdot l_1 - M_E}{h_1} \qquad H_A = -W_1 - W_2 + H_B; \qquad N_0 = H_B - H_C.$$

Axial forces:

$$N_{1u} = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2o} = -V_C \cdot \sin\alpha + H_C \cdot \cos\alpha$$

$$N_{1o} = N_{1u} + W_1 \cdot \cos\alpha; \qquad N_{2u} = N_{2o} - W_2 \cdot \cos\alpha;$$

$$N_3 = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha \qquad N_4 = V_C \cdot \sin\alpha + H_C \cdot \cos\alpha.$$

Special case 17/6a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

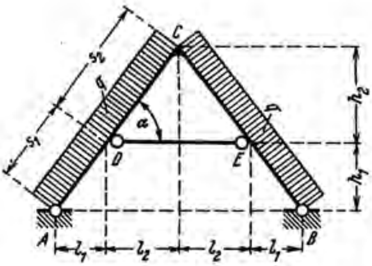
$$\left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{X}{2} \pm \frac{(W_1 + W_2) h_1 h_2}{4h}; \qquad V_B = V_C = -V_A = \frac{W_1 h_1 + W_2 (h + h_1)}{2l}.$$

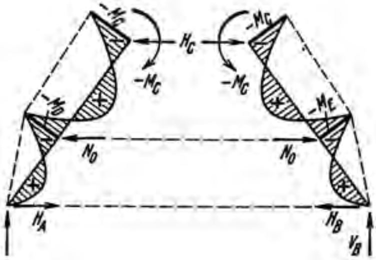
All other formulas same as above (Auxiliary value X-exactly as in case 17/2, p. 53):

Special case 17/6b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$W_1 = q_1 h_1 \qquad W_2 = q_2 h_2; \qquad (\mathfrak{L}_1 = W_1 h_1 / 4 \qquad \mathfrak{L}_2 = W_2 h_2 / 4).$$

Case 17/7: Full uniform symmetrical load, acting normally to the inclined members





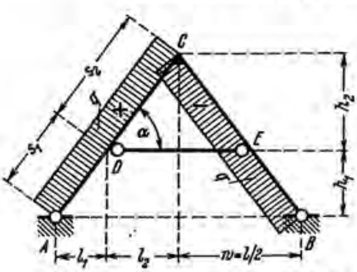
$$M_D = M_E = -\frac{q\,(2\,k \cdot s_1^2 + s_2^2)}{4\,F} \qquad M_C = -\frac{q\,s_2^2}{8} - \frac{M_D}{2}.$$

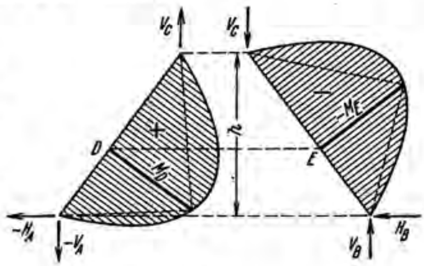
$$H_A = H_B = \frac{q\,(l\,l_1 - s_1^2)}{2\,h_1} - \frac{M_D}{h_1} \qquad H_C = \frac{q\,s_2^2}{2\,h_2} + \frac{M_D - M_C}{h_2};$$

$$V_A = V_B = q\,w \qquad V_C = 0; \qquad N_0 = q\,h + H_A - H_C.$$

$$N_1 = N_3 = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_2 = N_4 = H_C \cdot \cos\alpha.$$

Case 17/8: Full uniform antisymmetrical load, acting normally to the inclined members (Pressure and suction)





$$M_D = -M_E = \frac{q\,s_1\,s_2}{2} \qquad M_C = 0.$$

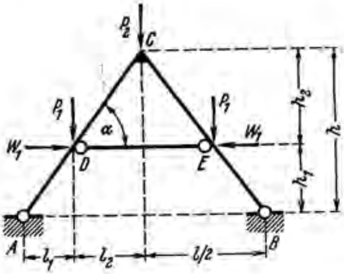
$$V_B = -V_A = \frac{q\,(h^2 - w^2)}{l} \qquad V_C = \frac{q\,s^2}{l};$$

$$H_B = -H_A = q\,h \qquad H_C = 0; \qquad N_0 = 0.$$

Axial forces:

$$N_3 = N_4 = -N_1 = -N_2 = \frac{q\,s\,h}{l}.$$

Case 17/9: Symmetrical arrangement of concentrated load



There are no bending moments.

$$(M_C = M_D = M_E = 0).$$

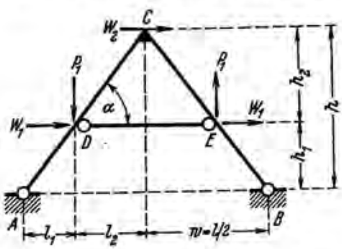
$$V_A = V_B = P_1 + \frac{P_2}{2} \qquad V_C = 0.$$

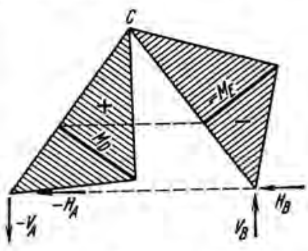
$$H_A = H_B = V_A \cdot \cot\alpha \qquad H_C = \frac{P_2}{2} \cdot \cot\alpha\,; \qquad N_0 = P_1 \cdot \cot\alpha + W_1.$$

Axial forces: $$N_1 = N_3 = \frac{V_A}{\sin\alpha} \qquad N_2 = N_4 = \frac{P_2}{2\sin\alpha}.$$

Note: The horizontal loads W_1 merely cause an additional axial load W_1 in the tie rod.

Case 17/10: Antisymmetrical arrangement of concentrated load





$$M_D = -M_E = (P_1 l_1 + W_1 h_1)\beta_2 \qquad M_C = 0.$$

$$V_C = \frac{P_1 l_1 + W_1 h_1}{w} + \frac{W_2 h}{l} \qquad V_B = -V_A = V_C - P_1\,;$$

$$H_B = -H_A = W_1 + \frac{W_2}{2} \qquad H_C = 0\,; \qquad N_0 = 0.$$

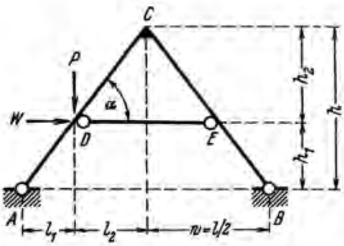
Axial forces:

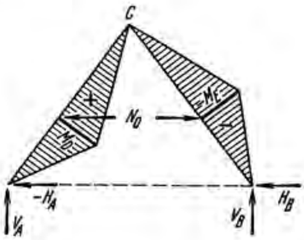
$$N_3 = -N_1 = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha \qquad N_4 = -N_2 = V_C \cdot \sin\alpha + \frac{W_2}{2} \cdot \cos\alpha.$$

There are no bending moments.

FRAME 17

Case 17/11: Unsymmetrical arrangement of concentrated load





$$V_B = V_C = \frac{P l_1 + W h_1}{l} \qquad V_A = P - V_C\,; \qquad M_D = -M_E = V_C \cdot l_2$$

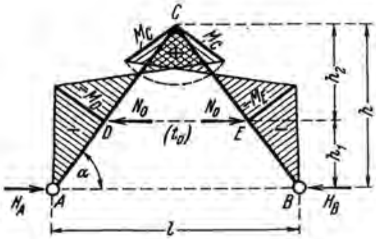
$$H_A = \frac{P w}{2h} - \frac{W}{2} \qquad H_B = N_0 = \frac{P w}{2h} + \frac{W}{2} \qquad H_C = 0\,; \qquad M_C = 0.$$

Axial forces:

$$N_1 = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_4 = V_C \cdot \sin\alpha$$

$$N_2 = -V_C \cdot \sin\alpha\,; \qquad N_3 = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha\,.$$

Case 17/12: Uniform increase in temperature of the tie *DE* by t_0 degrees



Constant: $T = \frac{3 E J_2 \cdot \varepsilon}{s_2 F} \cdot \frac{l}{h}$;

E = Modulus of elasticity
ε = Coefficient of thermal expansion

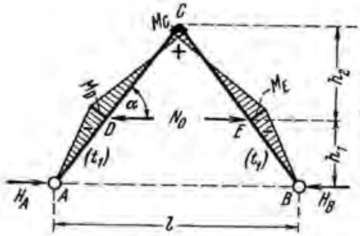
$$M_D = M_E = -T t_0 \cdot \left(2 \frac{h_2}{h_1} + 3\right) \qquad M_C = +T t_0 \cdot \left(\frac{h_2}{h_1} + 2k + 3\right).$$

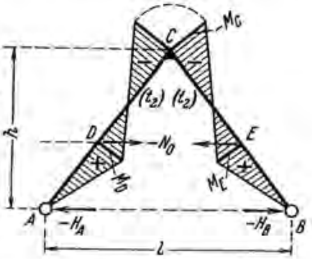
$$V_A = V_B = V_C = 0\,; \qquad H_A = H_B = \frac{-M_D}{h_1} \qquad H_C = \frac{M_D - M_C}{h_2};$$

$$N_0 = H_A - H_C\,. \qquad N_1 = N_3 = H_A \cdot \cos\alpha \qquad N_2 = N_4 = H_C \cdot \cos\alpha\,.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 17/13: Uniform increase in temperature of the lower diagonal bars by t_1, and the upper diagonal bars by t_2 degrees (Symmetrical case)





Constants T, E, and ε same as case 17/12, p. 60.

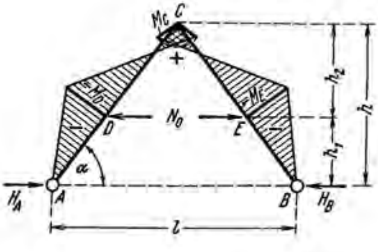
$$M_D = M_E = T \cdot [-2\,t_1 + 3\,t_2] \qquad M_C = T \cdot [+\,t_1 - (2\,k + 3)\,t_2].$$

Formulas for all V-, H-, and N-forces same as case 17/12

Case 17/14: Unsymmetrical increase in temperature

If the temperature increase t_1 or t_2 occurs in the left half or the right half of the frame only, all moments and forces are one-half of those for case 17/13. The moment diagram remains symmetrical.

Case 17/15: Uniform increase in temperature of the entire frame (including the tie DE) by t degrees



E = Modulus of elasticity
ε = Coefficient of thermal expansion

$$M_D = M_E = -\frac{6\,E\,J_2 \cdot \varepsilon}{s_2\,F} \cdot \frac{l}{h_1} \cdot t$$

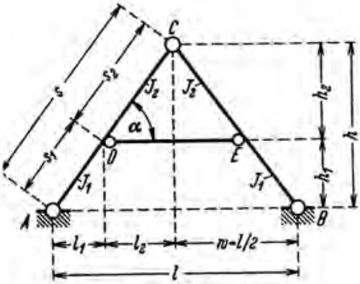
$$M_C = \frac{-\,M_D}{2}.$$

Formulas for all V-, H-, and N-forces same as case 17/12

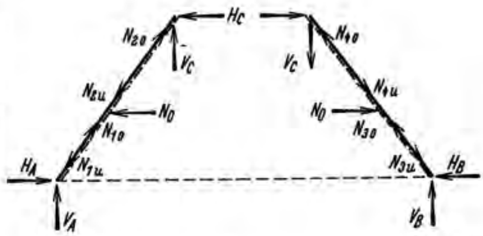
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 18

Symmetrical triangular three-hinged frame with hinged tie-rod and variable moment of inertia[1]



Shape of Frame
Dimensions and Notations



Positive direction of all reactions at the ridge and all axial forces.[2]

Coefficients:

$$k=\frac{J_2}{J_1}\cdot\frac{s_1}{s_2}\text{[1]};\qquad \left(\frac{s_1}{s_2}=\frac{l_1}{l_2}=\frac{h_1}{h_2}\right);\qquad \beta_1=\frac{l_1}{w}=\frac{h_1}{h}\qquad \beta_2=1-\beta_1.$$

Note: The numbering of the cases for frames 17 and 18 is identical. Hence 18/3, 4, 9, 10, and 11 are not repeated because they are identical with 17/3, 4, 9, 10, and 11 on account of $M_C = 0$.

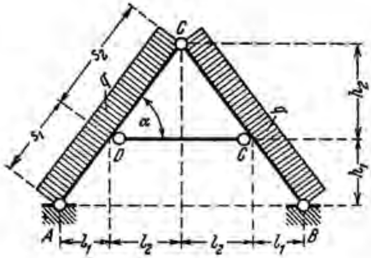
Note: The moment diagrams shown for cases 18/1, 2, 5, and 6 were drawn for $J_1 = J_2$ and special case b: $q_1 = q_2$.

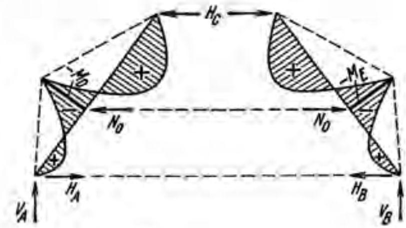
---

[1]If the moment of inertia is constant over s, i.e., if $J_1 = J_2$, then $k = s_1/s_2$.

[2]Positive bending moments cause tension at the face marked by a dashed line. Positive axial forces are compression.

Case 18/7: Full uniform symmetrical load, acting normally to the inclined members





$$M_D = M_E = -\frac{q\left(k \cdot s_1^2 + s_2^2\right)}{8(k+1)}.$$

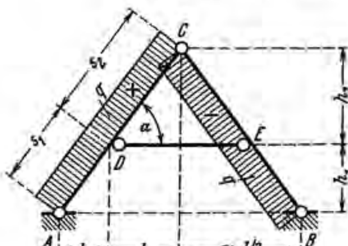
$$H_A = H_B = \frac{q\,(l\,l_1 - s_1^2)}{2\,h_1} - \frac{M_D}{h_1} \qquad H_C = \frac{q\,s_2^2}{2\,h_2} + \frac{M_D}{h_2};$$

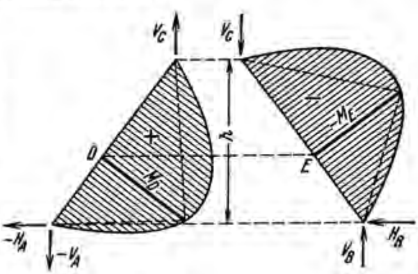
$$V_A = V_B = q\,w \qquad V_C = 0; \qquad N_0 = q\,h + H_A - H_C.$$

Axial forces:

$$N_1 = N_3 = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_2 = N_4 = H_C \cdot \cos\alpha.$$

Case 18/8: Full uniform antisymmetrical load, acting normally to the inclined members (Pressure and suction)





$$M_D = -M_E = \frac{q\,s_1\,s_2}{2}. \qquad V_B = -V_A = \frac{q\,(h^2 - w^2)}{l} \qquad V_C = \frac{q\,s^2}{l};$$

$$H_B = -H_A = q\,h \qquad H_C = 0; \qquad N_0 = 0;$$

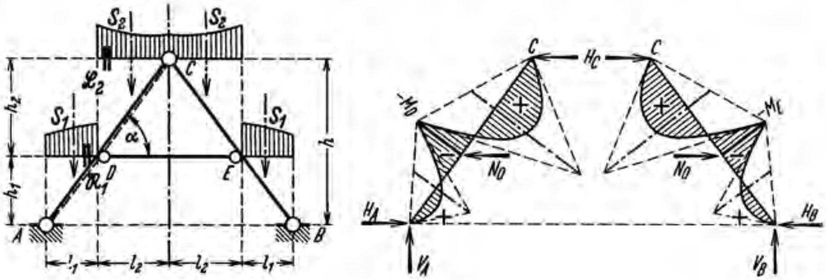
Axial forces:

$$N_3 = N_4 = -N_1 = -N_2 = \frac{q\,s\,h}{l}.$$

FRAME 18

See Appendix A; Load Terms, pp. 440-445.

Case 18/1: Entire frame loaded by any type of symmetrical vertical load



Constants, Reactions and Shears:

$$M_D = M_E = -\frac{\mathfrak{R}_1 k + \mathfrak{L}_2}{2(k+1)}. \qquad H_A = H_B = \frac{\mathfrak{S}_{l1} + S_2 l_1 - M_D}{h_1}$$

$$H_C = \frac{\mathfrak{S}_{l2} + M_D}{h_2}; \qquad N_0 = H_A - H_C. \qquad V_A = V_B = S_1 + S_2 \qquad V_C = 0.$$

Axial forces:

$$N_{1u} = N_{3u} = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2o} = N_{4o} = H_C \cdot \cos\alpha$$

$$N_{1o} = N_{3o} = S_2 \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2u} = N_{4u} = H_C \cdot \cos\alpha + S_2 \cdot \sin\alpha.$$

Note: All the load terms refer to the left half of the frame.

Special case 18/1a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$H_A = H_B = \left(\frac{S_1}{2} + S_2\right) \cdot \cot\alpha - \frac{M_D}{h_1} \qquad H_C = \frac{S_2}{2} \cdot \cot\alpha + \frac{M_D}{h_2}.$$

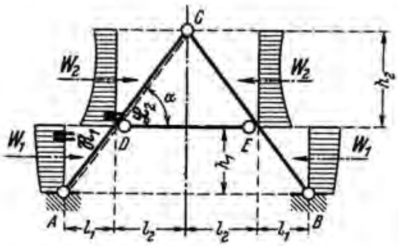
All other formulas same as above

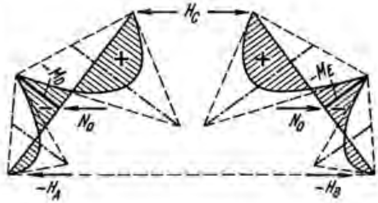
Special case 18/1b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$S_1 = q_1 l_1 \qquad S_2 = q_2 l_2; \qquad \mathfrak{L}_1 = \frac{S_1 l_1}{4} \qquad \mathfrak{L}_2 = \frac{S_2 l_2}{4}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 18/2: Entire frame loaded by any type of symmetrical horizontal load





Constants, Reactions and Shears:

$$M_D = M_E = -\frac{\mathfrak{R}_1 k + \mathfrak{L}_2}{2(k+1)}. \qquad H_A = H_B = -\frac{\mathfrak{S}_{r1} + M_D}{h_1}$$

$$H_C = \frac{\mathfrak{S}_{l2} + M_D}{h_2}; \qquad N_0 = W_1 + W_2 + H_A - H_C; \qquad V_A = V_B = V_C = 0.$$

Axial forces:

$$N_{1u} = N_{3u} = H_A \cdot \cos\alpha \qquad N_{2o} = N_{4o} = H_C \cdot \cos\alpha$$
$$N_{1o} = N_{3o} = (H_A + W_1) \cdot \cos\alpha; \qquad N_{2u} = N_{4u} = (H_C - W_2) \cdot \cos\alpha.$$

Note: All the load terms refer to the left half of the frame.

Special case 18/2a: Symmetrical loads $(\mathfrak{R} = \mathfrak{L})$

$$H_A = H_B = -\frac{W_1}{2} - \frac{M_D}{h_1} \qquad H_C = \frac{W_2}{2} + \frac{M_D}{h_2}.$$

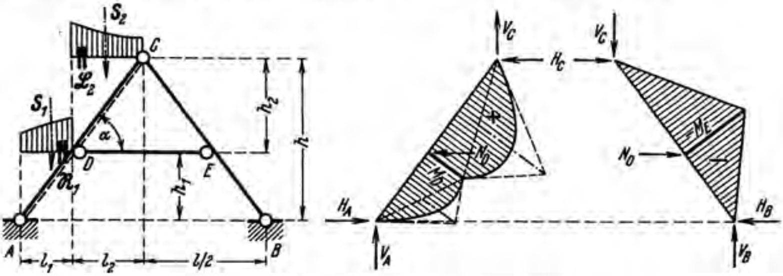
All other formulas same as above

Special case 18/2b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$W_1 = q_1 h_1 \qquad W_2 = q_2 h_2; \qquad \mathfrak{L}_1 = \frac{W_1 h_1}{4} \qquad \mathfrak{L}_2 = \frac{W_2 h_2}{4}$$

See Appendix A, Load Terms, pp. 440-445.

Case 18/5: Left half of frame loaded by any type of vertical load



Moments:

$$\left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{\mathfrak{R}_1 k + \mathfrak{L}_2}{4(k+1)} \pm \frac{\mathfrak{S}_{l1} \cdot \beta_2 + \mathfrak{S}_{r2} \cdot \beta_1}{2}.$$

Reactions and Shears:

$$V_B = V_C = \frac{\mathfrak{S}_{l1} + S_2 l_1 + \mathfrak{S}_{l2}}{l} \qquad V_A = S_1 + S_2 - V_B;$$

$$H_A = H_B = \frac{V_B \cdot l_1 - M_E}{h_1} \qquad H_C = \frac{V_C \cdot l_2 + M_E}{h_2}; \qquad N_0 = H_B - H_C.$$

Axial forces:

$$N_{1u} = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2o} = -V_C \cdot \sin\alpha + H_C \cdot \cos\alpha$$

$$N_{1o} = N_{1u} - S_1 \cdot \sin\alpha; \qquad N_{2u} = N_{2o} + S_2 \cdot \sin\alpha;$$

$$N_3 = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha \qquad N_4 = V_C \cdot \sin\alpha + H_C \cdot \cos\alpha.$$

Special case 18/5a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$\left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{\mathfrak{L}_1 k + \mathfrak{L}_2}{4(k+1)} \pm \frac{(S_1 + S_2) l_1 l_2}{2l};$$

$$V_B = V_C = \frac{S_1 \cdot \beta_1 + S_2 (1 + \beta_1)}{4}.$$

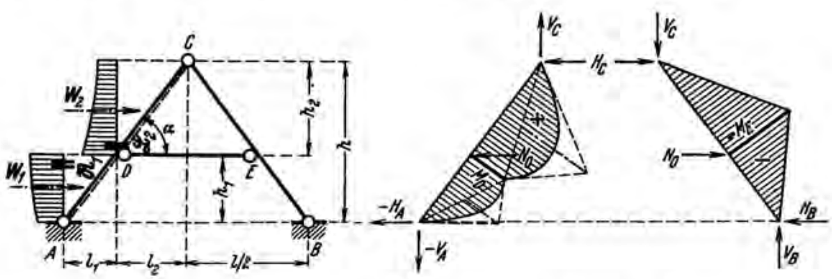
All other formulas same as above

Special case 18/5b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$S_1 = q_1 l_1 \qquad S_2 = q_2 l_2; \qquad (\mathfrak{L}_1 = S_1 l_1 / 4 \qquad \mathfrak{L}_2 = S_2 l_2 / 4).$$

See Appendix A, Load Terms, pp. 440-445.

Case 18/6: Left half of frame loaded by any type of horizontal load



Moments:

$$\left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{\mathfrak{R}_1 k + \mathfrak{L}_2}{4(k+1)} \pm \frac{\mathfrak{S}_{l1}\cdot\beta_2 + \mathfrak{S}_{r2}\cdot\beta_1}{2}$$

Reactions and Shears:

$$V_B = V_C = -V_A = \frac{\mathfrak{S}_{l1} + W_2 h_1 + \mathfrak{S}_{l2}}{l}; \qquad H_C = \frac{V_C\cdot l_2 + M_E}{h_2}$$

$$H_B = \frac{V_B\cdot l_1 - M_E}{h_1} \qquad H_A = -W_1 - W_2 + H_B; \qquad N_0 = H_B - H_C.$$

Axial forces:

$$N_{1u} = V_A\cdot\sin\alpha + H_A\cdot\cos\alpha \qquad N_{2o} = -V_C\cdot\sin\alpha + H_C\cdot\cos\alpha$$

$$N_{1o} = N_{1u} + W_1\cdot\cos\alpha; \qquad N_{2u} = N_{2o} - W_2\cdot\cos\alpha;$$

$$N_3 = V_B\cdot\sin\alpha + H_B\cdot\cos\alpha \qquad N_4 = V_C\cdot\sin\alpha + H_C\cdot\cos\alpha.$$

Special case 18/6a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$\left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{\mathfrak{L}_1 k + \mathfrak{L}_2}{4(k+1)} \pm \frac{(W_1 + W_2) h_1 h_2}{4h}; \quad V_B = V_C = -V_A = \frac{W_1 h_1 + W_2 (h + h_1)}{2l}.$$

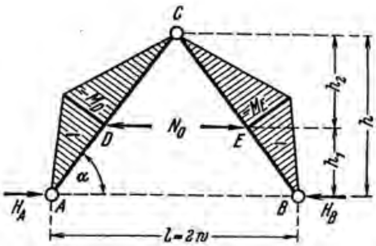
All other formulas same as above

Special case 18/6b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$W_1 = q_1 h_1 \qquad W_2 = q_2 h_2; \qquad (\mathfrak{L}_1 = W_1 h_1/4 \quad \mathfrak{L}_2 = W_2 h_2/4).$$

FRAME 18

Case 18/12: Uniform increase in temperature of the tie DE by t_0 degrees*



E = Modulus of elasticity
ε = Coefficient of thermal expansion

$$M_D = M_E = -\frac{3\,E\,J_2\cdot\varepsilon}{s_2\,(k+1)}\cdot\frac{w}{h_1}\cdot t_0\,;$$

$$H_A = H_B = \frac{-M_D}{h_1} \qquad H_C = \frac{M_D}{h_2}\,; \qquad N_0 = H_A - H_C = \frac{-M_D\cdot h}{h_1 h_2}\,;$$

$$V_A = V_B = V_C = 0\,. \qquad N_1 = N_3 = H_A\cdot\cos\alpha \qquad N_2 = N_4 = H_C\cdot\cos\alpha.$$

Case 18/13: Uniform increase in temperature of the lower diagonal bars by t_1 and the upper diagonal bars by t_2 degrees (Symmetrical case)*

$$M_D = M_E = \frac{3\,E\,J_2\cdot\varepsilon}{s_2\,(k+1)}\cdot\frac{l_1}{h_1}\cdot(t_2-t_1)^{**}\;; \qquad E \text{ and } \varepsilon \text{ as above.}$$

All other formulas same as for case 18/12.

Moment diagrams similar to case 18/12.

Case 18/14: Unsymmetrical increase in temperature*
If the diagonal bars of the left or the right half of the frame suffer temperature increases of t_1, resp. t_2, degrees, all moments and forces are one-half of those for case 18/13 and remain symmetrical.

Case 18/15: Uniform increase in temperature of the entire frame (including the tie DE) by t degrees*

$$M_D = M_E = -\frac{3\,E\,J_2\cdot\varepsilon}{s_2\,(k+1)}\cdot\frac{w}{h_1}\cdot t\,.$$ All other formulas same as for case 18/12.

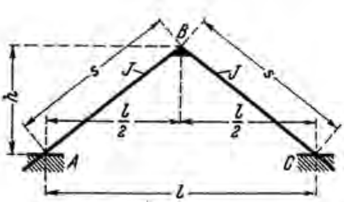
---

* With a decrease in temperature all moments and forces reverse their direction.

** With simultaneous operation of $(t_1 = t_2) = t$, $M_D = M_E = 0$.

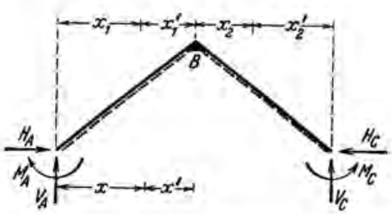
FRAME 19

Fully fixed symmetrical triangular frame



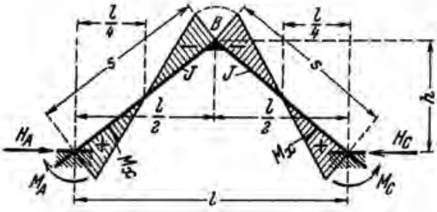
Shape of Frame
Dimensions and Notation:

$$\frac{l}{2} = w.$$



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use x and x'. Positive bending moments cause tension at the face marked by a dashed line.

Case 19/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degree

$$M_A = M_C = -M_B = \frac{3EJ\varepsilon t}{s} \cdot \frac{l}{h}$$

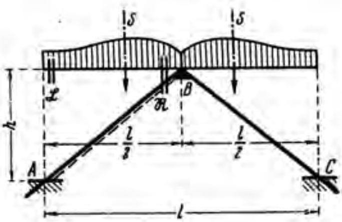
$$H_A = H_C = \frac{2M_A}{h} \qquad M_x = \frac{M_A}{w}(x' - x).$$

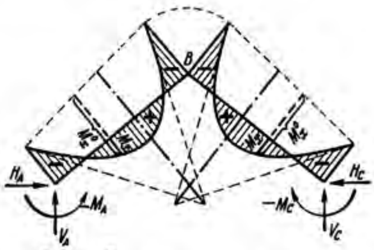
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 19

See Appendix A, Load Terms, pp. 440-445.

Case 19/2: Both members loaded by any type of symmetrical vertical load



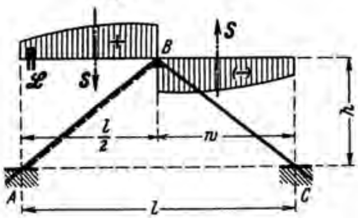


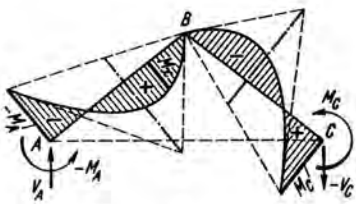
$$M_A = M_C = -\frac{2\mathfrak{L} - \mathfrak{R}}{3} = \mathfrak{M}_l \qquad M_B = -\frac{2\mathfrak{R} - \mathfrak{L}}{3} = \mathfrak{M}_r; \qquad V_A = V_C = S;$$

$$H_A = H_C = \frac{\mathfrak{S}_l - \mathfrak{L} + \mathfrak{R}}{h}; \qquad M_x = M_x^0 + \frac{x'}{w} M_A + \frac{x}{w} M_B.$$

Note: All the load terms refer to the left member.

Case 19/4: Both members loaded by any type of antisymmetrical vertical load



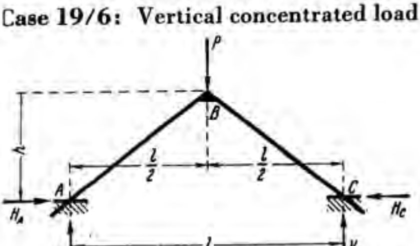


$$M_A = -M_C = -\frac{\mathfrak{L}}{2} \qquad M_B = 0; \qquad H_A = H_C = 0;$$

$$V_A = -V_C = \frac{\mathfrak{S}_r - M_A}{w}; \qquad M_x = M_x^0 + \frac{x'}{w} M_A.$$

Note: All the load terms refer to the left member.

Case 19/6: Vertical concentrated load at the ridge



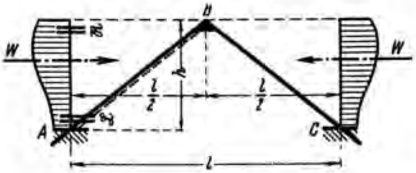
There are no bending moments.

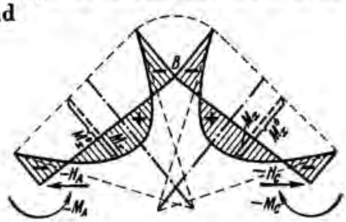
$$V_A = V_C = \frac{P}{2}$$

$$H_A = H_C = \frac{P l}{4 h}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 19/3: Both members loaded by any type of horizontal load, both members carrying the same load



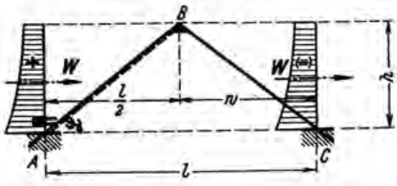


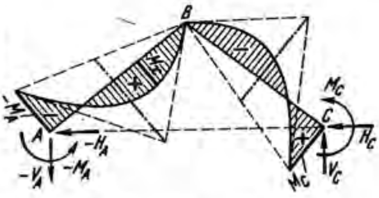
$$M_A = M_C = -\frac{2\mathfrak{L} - \mathfrak{R}}{3} = \mathfrak{M}_l \qquad M_B = -\frac{2\mathfrak{R} - \mathfrak{L}}{3} = \mathfrak{M}_r;$$

$$H_A = H_C = -\frac{\mathfrak{S}_r + \mathfrak{L} - \mathfrak{R}}{h}; \qquad M_x = M_x^0 + \frac{x'}{w} M_A + \frac{x}{w} M_B.$$

Note: All terms refer to the left leg.

Case 19/5: Both members loaded by any type of antisymmetrical horizontal load



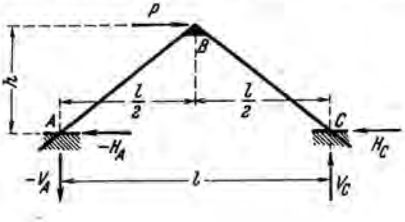


$$M_C = -M_A = \frac{\mathfrak{L}}{2} \qquad M_B = 0; \qquad H_C = -H_A = W;$$

$$V_C = -V_A = \frac{\mathfrak{S}_l - M_C}{w}; \qquad M_x = M_x^0 + \frac{x'}{w} M_A.$$

Note: All terms refer to the left leg.

Case 19/7: Horizontal concentrated load at the ridge



There are no bending moments.

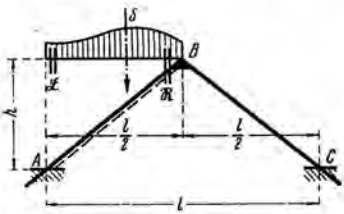
$$H_C = -H_A = \frac{P}{2}$$

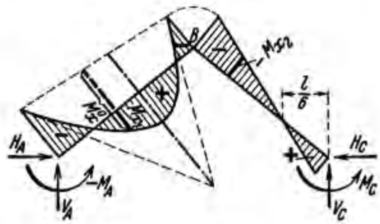
$$V_C = -V_A = \frac{Ph}{l}.$$

FRAME 19

See Appendix A, Load Terms, pp. 440-445.

Case 19/8: Left-hand member loaded by any type of vertical load





$$M_A = -\frac{7\,\mathfrak{L} - 2\,\mathfrak{R}}{12} \qquad M_B = -\frac{2\,\mathfrak{R} - \mathfrak{L}}{6} \qquad M_C = \frac{-M_B}{2};$$

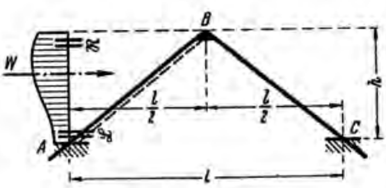
$$H_A = H_C = \frac{\mathfrak{S}_l - \mathfrak{L} + \mathfrak{R}}{2h} \qquad V_C = \frac{\mathfrak{S}_l}{l} - \frac{\mathfrak{L}}{2l} \qquad V_A = S - V_C.$$

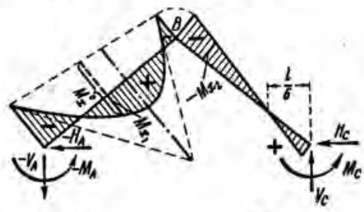
Special case 19/8a: Symmetrical load $(\mathfrak{R} = \mathfrak{L})$

$$M_A = -\frac{5\,\mathfrak{L}}{12} \qquad M_B = -\frac{\mathfrak{L}}{6} \qquad M_C = +\frac{\mathfrak{L}}{12};$$

$$H_A = H_C = \frac{Sl}{8h} \qquad V_C = \frac{S}{4} - \frac{\mathfrak{L}}{2l} \qquad V_A = S - V_C.$$

Case 19/9: Left-hand member loaded by any type of horizontal load





$$M_A = -\frac{7\,\mathfrak{L} - 2\,\mathfrak{R}}{12} \qquad M_B = -\frac{2\,\mathfrak{R} - \mathfrak{L}}{6} \qquad M_C = \frac{-M_B}{2};$$

$$V_A = -V_C = -\frac{\mathfrak{S}_l}{l} + \frac{\mathfrak{L}}{2l} \qquad H_C = \frac{\mathfrak{S}_l - \mathfrak{L} + \mathfrak{R}}{2h} \qquad H_A = -(W - H_C).$$

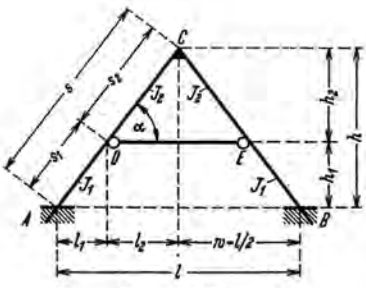
Special case 19/9a: Symmetrical load $(\mathfrak{R} = \mathfrak{L})$

$$M_A = -\frac{5\,\mathfrak{L}}{12} \qquad M_B = -\frac{\mathfrak{L}}{6} \qquad M_C = +\frac{\mathfrak{L}}{12};$$

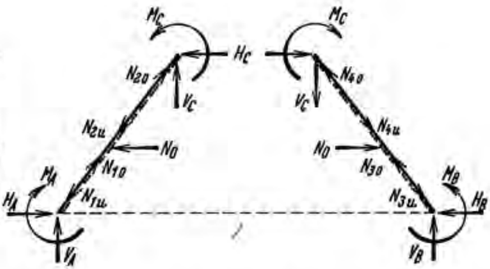
$$V_C = -V_A = \frac{Wh - \mathfrak{L}}{2l} \qquad H_C = \frac{W}{4} \qquad H_A = -\frac{3\,W}{4}.$$

FRAME 20

Symmetrical triangular fixed frame with hinged tie-rod and variable moment of inertia[1]



Shape of Frame
Dimensions and Notations



Positive direction of all reactions at the ridge and all axial forces.[2]

Coefficients:

$$k=\frac{J_2}{J_1}\cdot\frac{s_1}{s_2}\text{ }^{1)};\qquad \left(\frac{s_1}{s_2}=\frac{l_1}{l_2}=\frac{h_1}{h_2}\right);\qquad \beta_1=\frac{l_1}{w}=\frac{h_1}{h}\qquad \beta_2=1-\beta_1.$$

$$K_1=k+2\beta_2(k+1)\qquad K_2=k(2+\beta_2);\qquad G=K_1\beta_2+K_2$$

Note: The moment diagrams shown for cases 20/1 through 20/6 were drawn for $J_1 = J_2$ and special case b: $q_1 = q_2$.

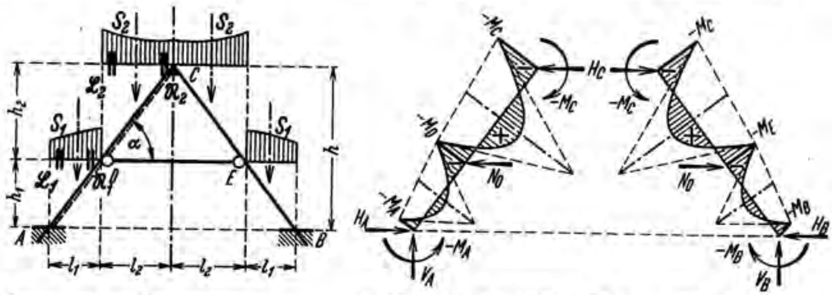
[1] If the moment of inertia is constant over s, i.e., if $J_1 = J_2$, then $k = s_1/s_2$.

[2] Positive bending moments M cause tension at the face marked by a dashed line. Positive axial forces are compression.

FRAME 20

See Appendix A, Load Terms, pp. 440-445.

Case 20/1: Entire frame loaded by any type of symmetrical vertical load



Constants and moments:

$$X = \frac{(2\,\mathfrak{R}_1 - \mathfrak{L}_1)\,k + (2\,\mathfrak{L}_2 - \mathfrak{R}_2)}{3\,(k+1)} = -\frac{\mathfrak{M}_{r1}\,k - \mathfrak{M}_{l2}}{k+1};$$

$$M_A = M_B = \frac{-\,\mathfrak{L}_1 + X}{2} \qquad M_D = M_E = -\,X \qquad M_C = \frac{-\,\mathfrak{R}_2 + X}{2}.$$

Reactions and Shears:

$$H_A = H_B = \frac{\mathfrak{S}_{l1} + S_2\,l_1 + M_A - M_D}{h_1} \qquad H_C = \frac{\mathfrak{S}_{l2} - M_C + M_D}{h_2};$$

$$V_A = V_B = S_1 + S_2 \qquad V_C = 0; \qquad N_0 = H_A - H_C.$$

Axial forces:

$$N_{1u} = N_{3u} = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2o} = N_{4o} = H_C \cdot \cos\alpha$$

$$N_{1o} = N_{3o} = S_2 \cdot \sin\alpha + H_A \cdot \cos\alpha; \qquad N_{2u} = N_{4u} = H_C \cdot \cos\alpha + S_2 \cdot \sin\alpha.$$

Note: All the load terms refer to the left half of the frame.

Special case 20/1a: Symmetrical loads $(\mathfrak{R} = \mathfrak{L})$

$$H_A = H_B = \left(\frac{S_1}{2} + S_2\right) \cdot \cot\alpha + \frac{M_A - M_D}{h_1} \qquad H_C = \frac{S_2}{2} \cdot \cot\alpha + \frac{M_D - M_C}{h_2};$$

$$X = \frac{\mathfrak{L}_1\,k + \mathfrak{L}_2}{3\,(k+1)}.$$

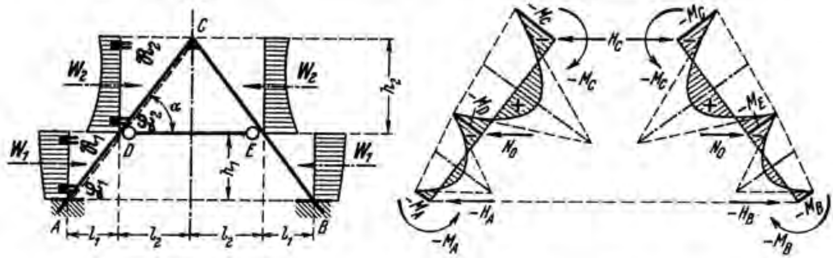
All other formulas same as above

Special case 20/1b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$S_1 = q_1\,l_1 \qquad S_2 = q_2\,l_2; \qquad \mathfrak{L}_1 = \frac{S_1\,l_1}{4} \qquad \mathfrak{L}_2 = \frac{S_2\,l_2}{4}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 20/2: Entire frame loaded by any type of symmetrical horizontal load



Constants and moments:

$$X = \frac{(2\,\mathfrak{R}_1 - \mathfrak{L}_1)\,k + (2\,\mathfrak{L}_2 - \mathfrak{R}_2)}{3\,(k+1)} = -\frac{\mathfrak{M}_{r1}\,k + \mathfrak{M}_{l2}}{k+1};$$

$$M_A = M_B = \frac{-\mathfrak{L}_1 + X}{2} \qquad M_D = M_E = -X \qquad M_C = \frac{-\mathfrak{R}_2 + X}{2}.$$

Reactions and Shears:

$$H_A = H_B = \frac{-\mathfrak{S}_{r1} + M_A - M_D}{h_1} \qquad H_C = \frac{\mathfrak{S}_{l2} - M_C + M_D}{h_2};$$

$$V_A = V_B = 0 \qquad V_C = 0; \qquad N_0 = W_1 + W_2 + H_A - H_C.$$

Axial forces:

$$N_{1u} = N_{3u} = H_A \cdot \cos\alpha \qquad N_{2o} = N_{4o} = H_C \cdot \cos\alpha$$

$$N_{1o} = N_{3o} = (H_A + W_1) \cdot \cos\alpha; \qquad N_{2u} = N_{4u} = (H_C - W_2) \cdot \cos\alpha.$$

Note: All the load terms refer to the left half of the frame.

Special case 20/2a: Symmetrical loads $(\mathfrak{R} = \mathfrak{L})$

$$H_A = H_B = -\frac{W_1}{2} + \frac{M_A - M_D}{h_1} \qquad H_C = \frac{W_2}{2} + \frac{M_D - M_C}{h_2};$$

$$X = \frac{\mathfrak{L}_1\,k + \mathfrak{L}_2}{3\,(k+1)}.$$

All other formulas same as above

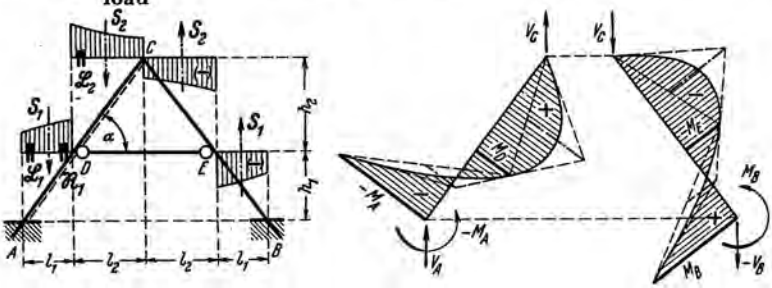
Special case 20/2b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$W_1 = q_1\,h_1 \qquad W_2 = q_2\,h_2; \qquad \mathfrak{L}_1 = \frac{W_1\,h_1}{4} \qquad \mathfrak{L}_2 = \frac{W_2\,h_2}{4}.$$

FRAME 20

See Appendix A, Load Terms, pp. 440-445.

Case 20/3: **Entire frame loaded by any type of antisymmetrical vertical load**



Constants and moments:

$$\mathfrak{S} = \mathfrak{S}_{l1} \cdot \beta_2 + \mathfrak{S}_{r2} \cdot \beta_1 \qquad \mathfrak{B} = (\mathfrak{L}_1 + \mathfrak{R}_1 \beta_2)\, k + \mathfrak{L}_2 \beta_2\,;$$

$$M_A = -M_B = -\frac{\mathfrak{S} \cdot K_1 + \mathfrak{B}}{G} \qquad M_D = -M_E = \frac{\mathfrak{S} \cdot K_2 - \mathfrak{B} \cdot \beta_2}{G} \qquad M_C = 0\,.$$

Reactions and Shears:

$$V_A = -V_B = \frac{\mathfrak{S}_{r1} + S_1 l_2 + \mathfrak{S}_{r2} - M_A}{w} \qquad V_C = S_1 + S_2 - V_A\,;$$

$$H_A = H_B = 0 \qquad H_C = 0\,; \qquad N_0 = 0\,.$$

Axial forces:

$$N_{1u} = -N_{3u} = V_A \cdot \sin\alpha \qquad N_{2o} = -N_{4o} = -V_C \cdot \sin\alpha$$

$$N_{1o} = -N_{3o} = (V_A - S_1)\sin\alpha \quad = \quad N_{2u} = -N_{4u} = (S_2 - V_C)\sin\alpha\,.$$

Note: All the load terms refer to the left half of the frame.

Special case 20/3a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$\mathfrak{S} = \frac{(S_1 + S_2)\, l_1 l_2}{l} \qquad \mathfrak{B} = \mathfrak{L}_1 k\, (1 + \beta_2) + \mathfrak{L}_2 \beta_2\,;$$

$$V_A = -V_B = \frac{S_1 (1 + \beta_2) + S_2 \beta_2}{2} - \frac{M_A}{w}\,.$$

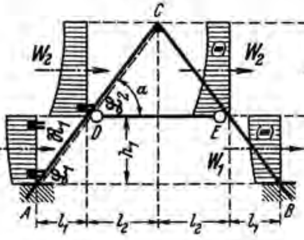
All other formulas same as above

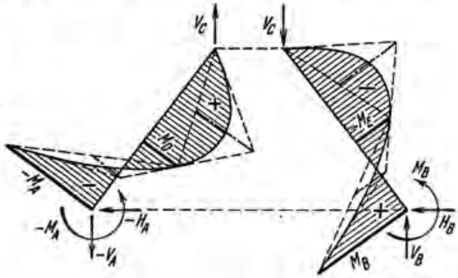
Special case 20/3b: **Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:**

$$S_1 = q\, l_1 \qquad S_2 = q_2 l_2\,; \qquad \mathfrak{L}_1 = \frac{S_1 l_1}{4} \qquad \mathfrak{L}_2 = \frac{S_2 l_2}{4}\,.$$

See Appendix A, Load Terms, pp. 440-445.

Case 20/4: Entire frame loaded by any type of antisymmetrical horizontal load





Constants and moments:

$$\mathfrak{S} = \mathfrak{S}_{l1} \cdot \beta_2 + \mathfrak{S}_{r2} \cdot \beta_1 \qquad \mathfrak{B} = (\mathfrak{L}_1 + \mathfrak{R}_1 \beta_2)\, k + \mathfrak{L}_2 \beta_2;$$

$$M_A = -M_B = -\frac{\mathfrak{S} \cdot K_1 + \mathfrak{B}}{G} \qquad M_D = -M_E = \frac{\mathfrak{S} \cdot K_2 - \mathfrak{B} \cdot \beta_2}{G} \qquad M_C = 0.$$

Reactions and Shears:

$$V_B = V_C = -V_A = \frac{\mathfrak{S}_{l1} + W_2 h_1 + \mathfrak{S}_{l2} + M_A}{w}$$

$$H_B = -H_A = W_1 + W_2 \qquad H_C = 0; \qquad N_0 = 0.$$

Axial forces:

$$N_{3u} = -N_{1u} = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha \qquad N_{4o} = -N_{2o} = V_C \cdot \sin\alpha$$

$$N_{3o} = -N_{1o} = N_{3u} - W_1 \cdot \cos\alpha = N_{4u} = -N_{2u} = V_C \cdot \sin\alpha + W_2 \cdot \cos\alpha.$$

Note: All the load terms refer to the left half of the frame.

Special case 20/4a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$\mathfrak{S} = \frac{(W_1 + W_2)\, h_1 h_2}{2h} \qquad \mathfrak{B} = \mathfrak{L}_1 k\,(1 + \beta_2) + \mathfrak{L}_2 \beta_2;$$

$$V_B = V_C = -V_A = \frac{W_1 h_1 + W_2 (h + h_1)}{l} + \frac{M_A}{w}.$$

All other formulas same as above

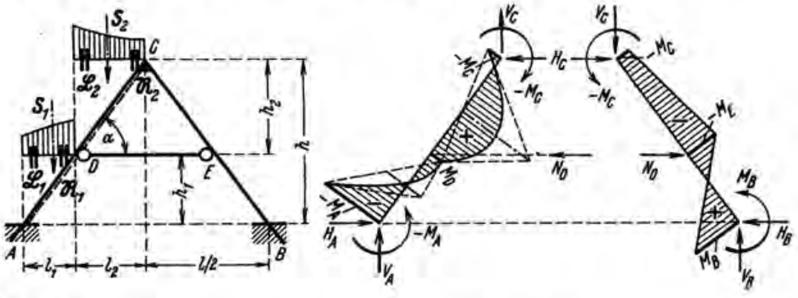
Special case 20/4b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$W_1 = q_1 h_1 \qquad W_2 = q_2 h_2 \qquad \mathfrak{L}_1 = \frac{W_1 h_1}{4} \qquad \mathfrak{L}_2 = \frac{W_2 h_2}{4}.$$

FRAME 20

See Appendix A, Load Terms, pp. 440-445.

Case 20/5: Left half of frame loaded by any type of vertical load



Moments:

Constants X, $\mathfrak{S}$ and $\mathfrak{B}$ same as in case 20/1 and 20/3.

$$M_C = \frac{-\mathfrak{R}_2 + X}{4}$$

$$\left.\begin{matrix} M_A \\ M_B \end{matrix}\right\rangle = \frac{-\mathfrak{L}_1 + X}{4} \mp \frac{\mathfrak{S}\cdot K_1 + \mathfrak{B}}{2\,G} \qquad \left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{X}{2} \pm \frac{\mathfrak{S}\cdot K_2 - \mathfrak{B}\cdot\beta_2}{2\,G}.$$

Reactions and Shears:

$$V_B = V_C = \frac{\mathfrak{S}_{l1} + S_2 l_1 + \mathfrak{S}_{l2}}{l} + \frac{M_A - M_B}{l} \qquad V_A = S_1 + S_2 - V_B;$$

$$H_A = H_B = \frac{V_B\cdot l_1 + M_B - M_E}{h_1} \qquad H_C = \frac{V_B\cdot l_2 - M_C + M_E}{h_2}; \quad N_0 = H_B - H_C.$$

Axial forces:

$$N_{1u} = V_A\cdot\sin\alpha + H_A\cdot\cos\alpha$$
$$N_{1o} = N_{1u} - S_1\cdot\sin\alpha;$$
$$N_3 = V_B\cdot\sin\alpha + H_B\cdot\cos\alpha$$

$$N_{2o} = -V_C\cdot\sin\alpha + H_C\cdot\cos\alpha$$
$$N_{2u} = N_{2o} + S_2\cdot\sin\alpha;$$
$$N_4 = V_C\cdot\sin\alpha + H_C\cdot\cos\alpha.$$

Special case 20/5a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

Constants X, $\mathfrak{S}$ and $\mathfrak{B}$ same as special case 20/1a and 20/3a.

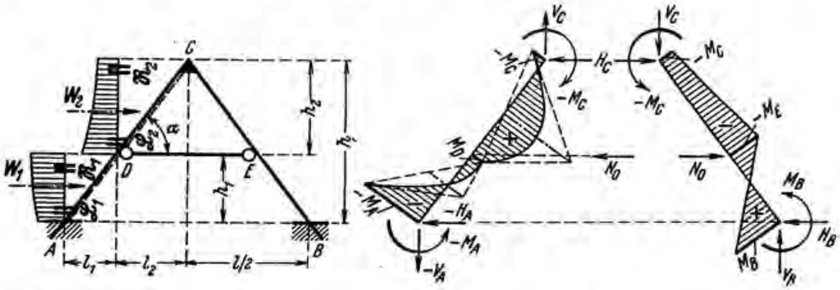
$$V_B = V_C = \frac{S_1\cdot\beta_1 + S_2(1+\beta_1)}{4} + \frac{M_A - M_B}{l}.$$

All other formulas same as above

Special case 20/5b: See the special cases 20/1b and 20/3b

See Appendix A, Load Terms, pp. 440-445.

Case 20/6: Left half of frame loaded by any type of horizontal load



Moments:

Constants X, $\mathfrak{S}$ and $\mathfrak{B}$ same as in case 20/2 and 20/4.

$$M_C = \frac{-\mathfrak{R}_2 + X}{4}$$

$$\left.\begin{matrix} M_A \\ M_B \end{matrix}\right\rangle = \frac{-\mathfrak{L}_1 + X}{4} \mp \frac{\mathfrak{S} \cdot K_1 + \mathfrak{B}}{2G} \qquad \left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{X}{2} \pm \frac{\mathfrak{S} \cdot K_2 - \mathfrak{B} \cdot \beta_2}{2G}$$

Reactions and Shears:

$$V_B = V_C = -V_A = \frac{\mathfrak{S}_{l1} + W_2 h_1 + \mathfrak{S}_{l2}}{l} + \frac{M_A - M_B}{l}; \qquad N_0 = H_B - H_C;$$

$$H_B = \frac{V_B \cdot l_1 + M_B - M_E}{h_1} \quad H_C = \frac{V_B \cdot l_2 - M_C + M_E}{h_2} \quad H_A = -W_1 - W_2 + H_B.$$

Axial forces:

$$N_{1u} = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha$$
$$N_{1o} = N_{1u} + W_1 \cdot \cos\alpha;$$
$$N_3 = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha$$

$$N_{2o} = -V_C \cdot \sin\alpha + H_C \cdot \cos\alpha$$
$$N_{2u} = N_{2o} - W_2 \cdot \cos\alpha;$$
$$N_4 = V_C \cdot \sin\alpha + H_C \cdot \cos\alpha.$$

Special case 20/6a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

Constants X, $\mathfrak{S}$ and $\mathfrak{B}$ same as special case 20/2a and 20/4a.

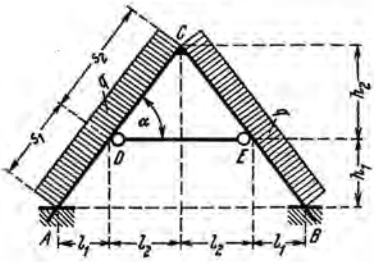
$$V_B = V_C = -V_A = \frac{W_1 l_1 + W_2 (h + h_1)}{2l} + \frac{M_A - M_B}{l}.$$

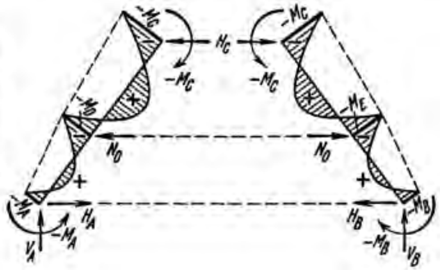
All other formulas same as above

Special case 20/6b: See the special cases 20/2b and 20/4b

FRAME 20

Case 20/7: Full uniform symmetrical load, acting normally to the inclined members





$$M_D = M_E = -\frac{q\,(k \cdot s_1^2 + s_2^2)}{12\,(k+1)} \qquad M_A = M_B = -\frac{q\,s_1^2}{8} - \frac{M_D}{2} \qquad M_C = -\frac{q\,s_2^2}{8} - \frac{M_D}{2};$$

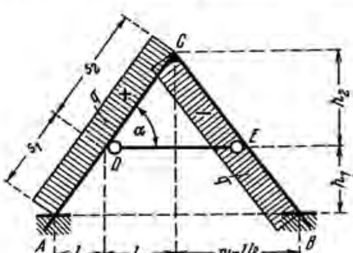
$$H_A = H_B = \frac{q\,(l\,l_1 - s_1^2)}{2\,h_1} + \frac{M_A - M_D}{h_1} \qquad H_C = \frac{q\,s_2^2}{2\,h_2} + \frac{M_D - M_C}{h_2};$$

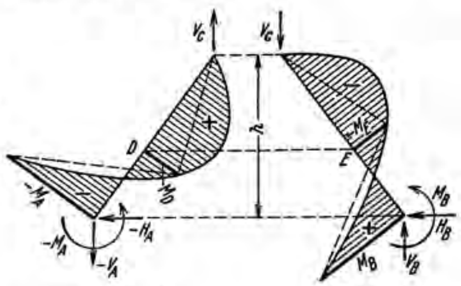
$$V_A = V_B = q\,w \qquad V_C = 0; \qquad N_0 = q\,h + H_A - H_C.$$

Axial forces:

$$N_1 = N_3 = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_2 = N_4 = H_C \cdot \cos\alpha.$$

Case 20/8: Full uniform antisymmetrical load, acting normally to the inclined members (Pressure and suction)





$$\mathfrak{S} = \frac{q\,s_1\,s_2}{2} \qquad \mathfrak{B} = \frac{q\,s_1^2}{4}\left[k\,(1+\beta_2) + \frac{s_2^2}{s_1^2}\cdot\beta_2\right];$$

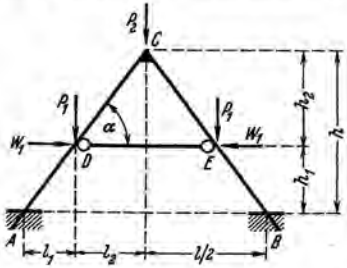
$$M_B = -M_A = \frac{\mathfrak{S}\cdot K_1 + \mathfrak{B}}{G} \qquad M_D = -M_E = \frac{\mathfrak{S}\cdot K_2 - \mathfrak{B}\cdot\beta_2}{G} \qquad M_C = 0.$$

$$V_B = -V_A = \frac{q\,(h^2 - w^2)}{l} - \frac{M_B}{w} \qquad V_C = \frac{q\,s^2}{l} - \frac{M_B}{w};$$

$$H_B = -H_A = q\,h \qquad H_C = 0; \qquad N_0 = 0.$$

Axial forces: $N_3 = N_4 = -N_1 = -N_2 = \dfrac{q\,s\,h}{l}.$

Case 20/9: Symmetrical arrangement of concentrated load



There are no bending moments

$$(M_A = M_B = M_C = M_D = M_E = 0.)$$

$$V_A = V_B = P_1 + \frac{P_2}{2} \qquad V_C = 0.$$

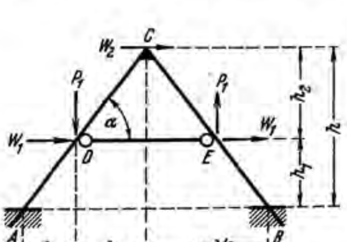
$$H_A = H_B = V_A \cdot \cot\alpha \qquad H_C = \frac{P_2}{2} \cdot \cot\alpha; \qquad N_0 = P_1 \cdot \cot\alpha + W_1.$$

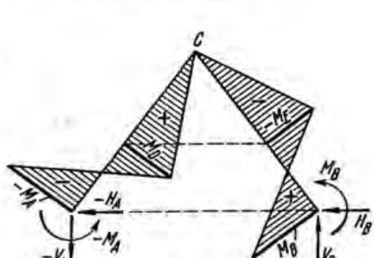
Axial forces:

$$N_1 = N_3 = \frac{V_A}{\sin\alpha} \qquad N_2 = N_4 = \frac{P_2}{2\sin\alpha}.$$

Note: The horizontal loads W_1 merely cause an additional axial load W_1 in the tie rod.

Case 20/10: Antisymmetrical arrangement of concentrated load





$$\mathfrak{S} = (P_1 l_1 + W_1 h_1)\beta_2;$$

$$M_B = -M_A = \mathfrak{S} \cdot \frac{K_1}{G} \qquad M_D = -M_E = \mathfrak{S} \cdot \frac{K_2}{G} \qquad M_C = 0.$$

$$V_C = \frac{P_1 l_1 + W_1 h_1 - M_B}{w} + \frac{W_2 h}{l} \qquad V_B = -V_A = V_C - P_1;$$

$$H_B = -H_A = W_1 + \frac{W_2}{2} \qquad H_C = 0; \qquad N_0 = 0.$$

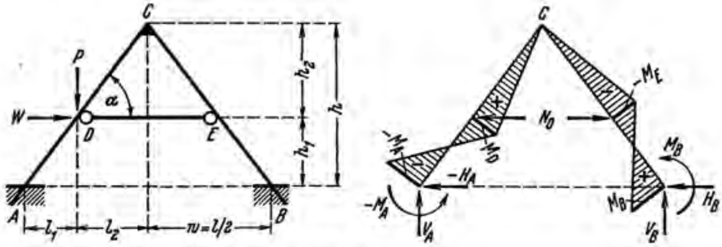
Axial forces:

$$N_3 = -N_1 = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha \qquad N_4 = -N_2 = V_C \cdot \sin\alpha + \frac{W_2}{2} \cdot \cos\alpha.$$

There are no bending moments.

FRAME 20

Case 20/11: Unsymmetrical arrangement of concentrated load



$$\mathfrak{S} = (P l_1 + W h_1)\beta_2;$$

$$M_B = -M_A = \mathfrak{S}\cdot\frac{K_1}{2G} \qquad M_D = -M_E = \mathfrak{S}\cdot\frac{K_2}{2G} \qquad M_C = 0.$$

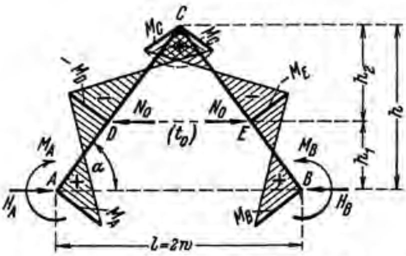
$$V_B = V_C = \frac{P l_1 + W h_1 - 2M_B}{l} \qquad V_A = P - V_B; \qquad N_0 = H_B - H_C.$$

$$H_B = \frac{V_B\cdot l_1 + M_B - M_E}{h_1} \qquad H_C = \frac{V_B\cdot l_2 + M_E}{h_2} \qquad H_A = H_B - W.$$

Axial forces:

$$N_1 = V_A\cdot\sin\alpha + H_A\cdot\cos\alpha \qquad N_4 = V_C\cdot\sin\alpha + H_C\cdot\cos\alpha$$
$$N_2 = -V_C\cdot\sin\alpha + H_C\cdot\cos\alpha \qquad N_3 = V_B\cdot\sin\alpha + H_B\cdot\cos\alpha.$$

Case 20/12: Uniform increase in temperature of the tie *DE* by t_0 degrees



E = Modulus of elasticity
ε = Coefficient of thermal expansion

Constant:

$$T = \frac{3EJ_2\cdot\varepsilon}{s_2(k+1)}\cdot\frac{w}{h}.$$

$$M_A = M_B = +Tt_0\left[\left(\frac{1}{k}+2\right)\frac{h_2}{h_1}+1\right] \qquad M_D = M_E = -2Tt_0\left[\frac{h_2}{h_1}+1\right]$$

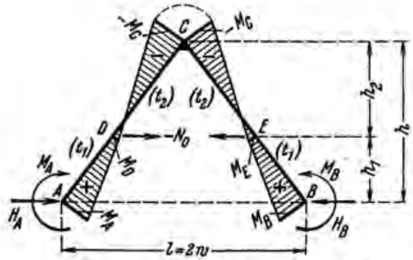
$$M_C = +Tt_0\left[\frac{h_2}{h_1}+k+2\right]. \qquad V_A = V_B = V_C = 0;$$

$$H_A = H_B = \frac{M_A - M_D}{h_1} \qquad H_C = \frac{M_D - M_C}{h_2}; \qquad N_0 = H_A - H_C.$$

Axial forces: $N_1 = N_3 = H_A\cdot\cos\alpha \qquad N_2 = N_4 = H_C\cdot\cos\alpha.$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 20/13: Symmetrical increase in temperature of the inclined members



t_1 in degrees for rods s_1,

t_2 in degrees for rods s_2.

Constant T as well as E and ε same as for case 20/12, p. 82.

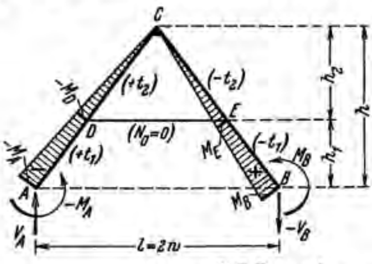
$$M_C = T[+t_1 - (k+2)t_2]$$

$$M_A = M_B = T\left[\left(\frac{1}{k}+2\right)t_1 - t_2\right] \qquad M_D = M_E = 2T[-t_1 + t_2].$$

Formulas for all V-, H-, and N-forces same as case 20/12.

Note: For the special case $t_1 = t_2$ we have $M_D = M_E = 0$, and $M_A : (-M_C) = 1 : k$.

Case 20/14: Antisymmetrical change in temperature of the inclined members



t_1 and t_2 as before, but negative for the right half of the frame. E and ε same as case 20/12.

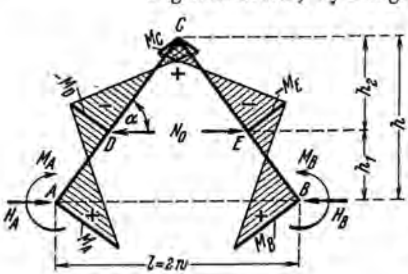
$$H_A = H_B = H_C = 0\,; \qquad N_0 = 0\,.$$

$$M_E = -M_D = \beta_2 \cdot M_B \qquad M_C = 0;$$

$$M_B = -M_A = \frac{6EJ_2\cdot\varepsilon}{s_2 G}\cdot\frac{h}{w}(\beta_1 t_1 + \beta_2 t_2); \qquad V_A = -V_B = -V_C = \frac{M_B}{w}\,.$$

Axial forces: $N_1 = N_2 = -N_3 = -N_4 = V_A \cdot \sin\alpha\,.$

Case 20/15: Uniform increase in temperature of the entire frame (including the tie DE) by t degrees



E and ε as in case 20/12.

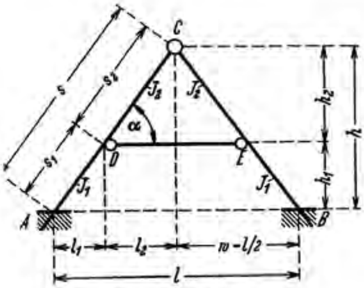
$$M_D = M_E = -\frac{3EJ_2\cdot\varepsilon}{s_2(k+1)}\cdot\frac{l}{h_1}\cdot t$$

$$M_A = M_B = \frac{-M_D}{2}\left(\frac{1}{k}+2\right) \quad M_C = \frac{-M_D}{2}$$

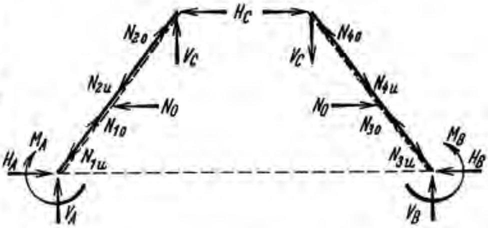
Formulas for all V-, H-, and N-forces same as in case 20/12.

Frame 21

Symmetrical triangular one-hinged frame with hinged tie-rod and variable moment of inertia[1]



Shape of Frame
Dimensions and Notations



Positive direction of all reactions at the ridge and all axial forces.[2]

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{s_1}{s_2}\,^{1)}; \qquad \left(\frac{s_1}{s_2} = \frac{l_1}{l_2} = \frac{h_1}{h_2}\right); \qquad \beta_1 = \frac{l_1}{w} = \frac{h_1}{h} \qquad \beta_2 = \frac{l_2}{w} = \frac{h_2}{h};$$

$$F = 3k + 4. \qquad (\beta_1 + \beta_2 = 1).$$

$$K_1 = k + 2\beta_2(k+1) \qquad K_2 = k(2 + \beta_2); \qquad G = K_1\beta_2 + K_2.$$

Note: The numbering of the cases for frames 20 and 21 is identical. Hence 21/3, 4, 9, 10, 11 and 14 are not repeated because they are identical with 20/3, 4, 9, 10, 11, and 14 on account of $M_C = 0$.

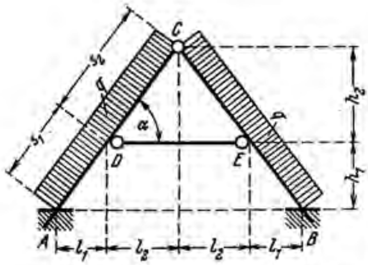
Note: The moment diagrams shown for cases 21/1, 2, 5, and 6 were drawn for $J_1 = J_2$ and special case b: $q_1 = q_2$.

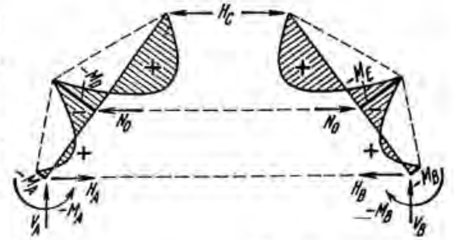
---

[1] If the moment of inertia is constant over s, i.e., if $J_1 = J_2$, then $k = s_1/s_2$.

[2] Positive bending moments M cause tension at the face marked by a dashed line. Positive axial forces are compression.

Case 21/7: **Full uniform symmetrical load, acting normally to the inclined members**





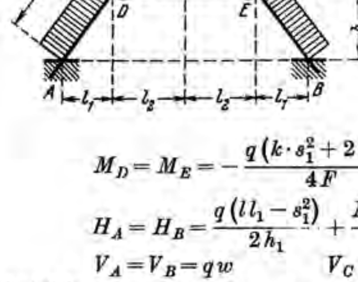
$$M_D = M_E = -\frac{q\left(k \cdot s_1^2 + 2 \cdot s_2^2\right)}{4F} \qquad M_A = M_B = -\frac{q s_1^2}{8} - \frac{M_D}{2}.$$

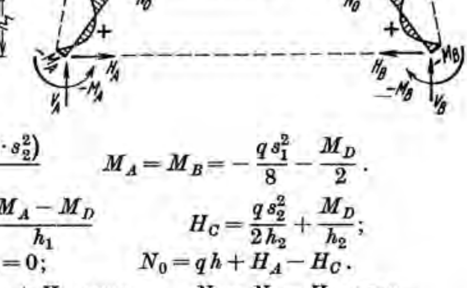
$$H_A = H_B = \frac{q\left(l l_1 - s_1^2\right)}{2h_1} + \frac{M_A - M_D}{h_1} \qquad H_C = \frac{q s_2^2}{2h_2} + \frac{M_D}{h_2};$$

$$V_A = V_B = q w \qquad V_C = 0; \qquad N_0 = q h + H_A - H_C.$$

Axial forces: $N_1 = N_3 = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_2 = N_4 = H_C \cdot \cos\alpha.$

Case 21/8: **Full uniform antisymmetrical load, acting normally to the inclined members (Pressure and suction)**





$$\mathfrak{S} = \frac{q s_1 s_2}{2} \qquad \mathfrak{B} = \frac{q s_1^2}{4}\left[k\left(1+\beta_2\right) + \frac{s_2^2}{s_1^2} \cdot \beta_2\right];$$

$$M_B = -M_A = \frac{\mathfrak{S} \cdot K_1 + \mathfrak{B}}{G} \qquad M_D = -M_E = \frac{\mathfrak{S} \cdot K_2 - \mathfrak{B} \cdot \beta_2}{G}$$

$$V_B = -V_A = \frac{q\left(h^2 - w^2\right)}{l} - \frac{M_B}{w} \qquad V_C = \frac{q s^2}{l} - \frac{M_B}{w};$$

$$H_B = -H_A = q h \qquad H_C = 0; \qquad N_0 = 0.$$

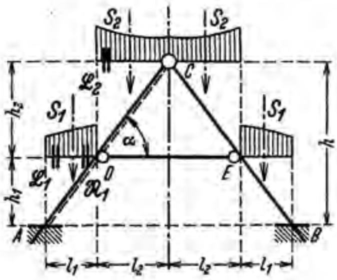
Axial forces:

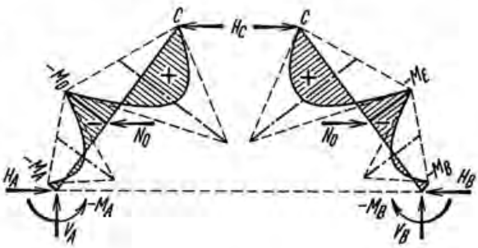
$$N_3 = N_4 = -N_1 = -N_2 = \frac{q s h}{l}.$$

FRAME 21

See Appendix A, Load Terms, pp. 440-445.

Case 21/1: Entire frame loaded by any type of symmetrical vertical load





Constants and moments:

$$X = \frac{(2\,\mathfrak{R}_1 - \mathfrak{L}_1)\,k + 2\,\mathfrak{L}_2}{F}; \qquad M_A = M_B = \frac{-\mathfrak{L}_1 + X}{2} \qquad M_D = M_E = -X.$$

Reactions and Shears:

$$H_A = H_B = \frac{\mathfrak{S}_{l1} + S_2\,l_1 + M_A - M_D}{h_1} \qquad H_C = \frac{\mathfrak{S}_{l2} + M_D}{h_2};$$

$$V_A = V_B = S_1 + S_2 \qquad V_C = 0; \qquad N_0 = H_A - H_C.$$

Axial forces:

$$N_{1u} = N_{3u} = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2o} = N_{4o} = H_C \cdot \cos\alpha$$

$$N_{1o} = N_{3o} = S_2 \cdot \sin\alpha + H_A \cdot \cos\alpha; \qquad N_{2u} = N_{4u} = H_C \cdot \cos\alpha + S_2 \cdot \sin\alpha.$$

Note: All the load terms refer to the left half of the frame.

Special case 21/1a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$H_A = H_B = \left(\frac{S_1}{2} + S_2\right) \cdot \cot\alpha + \frac{M_A - M_D}{h_1} \qquad H_C = \frac{S_2}{2} \cdot \cot\alpha + \frac{M_D}{h_2};$$

$$X = \frac{\mathfrak{L}_1\,k + 2\,\mathfrak{L}_2}{F}.$$

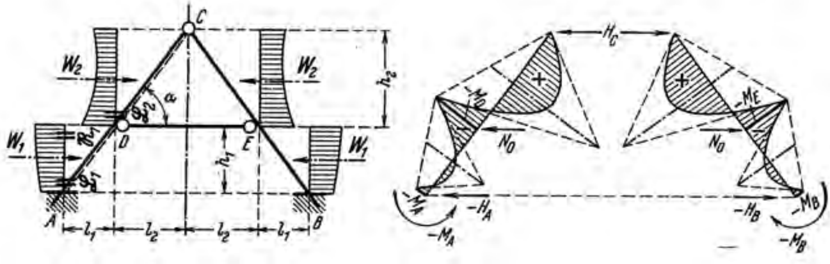
All other formulas same as above

Special case 21/1b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

$$S_1 = q_1\,l_1 \qquad S_2 = q_2\,l_2; \qquad \mathfrak{L}_1 = \frac{S_1\,l_1}{4} \qquad \mathfrak{L}_2 = \frac{S_2\,l_2}{4}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 21/2: Entire frame loaded by any type of symmetrical horizontal load



Constants and moments:

$$X = \frac{(2\,\mathfrak{R}_1 - \mathfrak{L}_2)\,k + 2\,\mathfrak{L}_2}{F}\,; \qquad M_A = M_B = \frac{-\,\mathfrak{L}_1 + X}{2} \qquad M_D = M_E = -X\,.$$

Reactions and Shears:

$$H_A = H_B = \frac{-\,\mathfrak{S}_{r1} + M_A - M_D}{h_1} \qquad H_C = \frac{\mathfrak{S}_{l2} + M_D}{h_2}\,;$$

$$V_A = V_B = 0 \qquad V_C = 0\,; \qquad N_0 = W_1 + W_2 + H_A - H_C\,.$$

Axial forces:

$$N_{1u} = N_{3u} = H_A \cdot \cos\alpha \qquad N_{2o} = N_{4o} = H_C \cdot \cos\alpha$$

$$N_{1o} = N_{3o} = (H_A + W_1) \cdot \cos\alpha\,; \qquad N_{2u} = N_{4u} = (H_C - W_2)\cos\alpha\,.$$

Note: All the load terms refer to the left half of the frame.

Special case 21/2a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$X = \frac{\mathfrak{L}_1 k + 2\,\mathfrak{L}_2}{F}\,; \qquad H_A = H_B = -\frac{W_1}{2} + \frac{M_A - M_D}{h_1} \qquad H_C = \frac{W_2}{2} + \frac{M_D}{h_2}\,.$$

All other formulas same as above

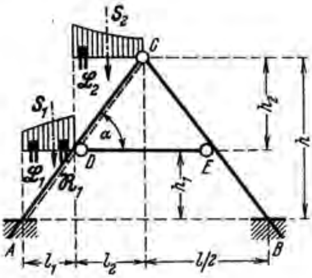
Special case 21/2b: Uniformly distributed loads q_1 and q_2. By substitution in the previous formulas:

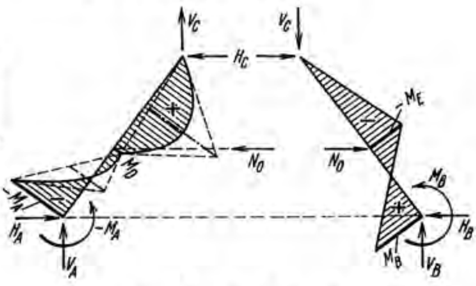
$$W_1 = q_1 h_1 \qquad W_2 = q_2 h_2\,; \qquad \mathfrak{L}_1 = \frac{W_1 h_1}{4} \qquad \mathfrak{L}_2 = \frac{W_2 h_2}{4}\,.$$

FRAME 21

See Appendix A, Load Terms, pp. 440-445.

Case 21/5: Left half of frame loaded by any type of vertical load





Constants and moments:

$$X = \frac{(2\,\mathfrak{R}_1 - \mathfrak{L}_1)\,k + 2\,\mathfrak{L}_2}{F};$$

$$\mathfrak{S} = \mathfrak{S}_{l1}\cdot\beta_2 + \mathfrak{S}_{r2}\cdot\beta_1$$

$$\mathfrak{B} = (\mathfrak{L}_1 + \mathfrak{R}_1\beta_2)\,k + \mathfrak{L}_2\beta_2\,.$$

$$\left.\begin{matrix} M_A \\ M_B \end{matrix}\right\rangle = \frac{-\mathfrak{L}_1 + X}{4} \mp \frac{\mathfrak{S}\cdot K_1 + \mathfrak{B}}{2G}$$

$$\left.\begin{matrix} M_D \\ M_E \end{matrix}\right\rangle = -\frac{X}{2} \pm \frac{\mathfrak{S}\cdot K_2 - \mathfrak{B}\cdot\beta_2}{2G}\,.$$

Reactions and Shears:

$$V_B = V_C = \frac{\mathfrak{S}_{l1} + S_2 l_1 + \mathfrak{S}_{l2}}{l} + \frac{M_A - M_B}{l} \qquad V_A = S_1 + S_2 - V_B\,;$$

$$H_A = H_B = \frac{V_B\cdot l_1 + M_B - M_E}{h_1} \qquad H_C = \frac{V_B\cdot l_2 + M_E}{h_2}\,; \qquad N_0 = H_B - H_C\,.$$

Axial forces:

$$N_{1u} = V_A\cdot\sin\alpha + H_A\cdot\cos\alpha$$
$$N_{1o} = N_{1u} - S_1\cdot\sin\alpha;$$
$$N_3 = V_B\cdot\sin\alpha + H_B\cdot\cos\alpha$$

$$N_{2o} = -V_C\cdot\sin\alpha + H_C\cdot\cos\alpha$$
$$N_{2u} = N_{2o} + S_2\cdot\sin\alpha;$$
$$N_4 = V_C\cdot\sin\alpha + H_C\cdot\cos\alpha\,.$$

Special case 21/5a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$X = \frac{\mathfrak{L}_1 k + 2\,\mathfrak{L}_2}{F}\,; \qquad \mathfrak{S} = \frac{(S_1 + S_2)\,l_1 l_2}{l} \qquad \mathfrak{B} = \mathfrak{L}_1\,k\,(1 + \beta_2) + \mathfrak{L}_2\beta_2\,.$$

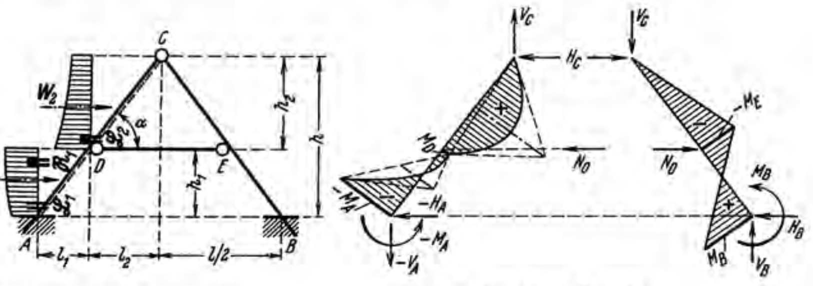
$$V_B = V_C = \frac{S_1\cdot\beta_1 + S_2(1 + \beta_1)}{4} + \frac{M_A - M_B}{l}\,.$$

All other formulas same as above

Special case 21/5b: Use expressions for special case 21/1b

See Appendix A, Load Terms, pp. 440-445.

Case 21/6: Left half of frame loaded by any type of horizontal load



Constants and moments:

$$\mathfrak{S} = \mathfrak{S}_{l1} \cdot \beta_2 + \mathfrak{S}_{r2} \cdot \beta_1$$

$$X = \frac{(2\,\mathfrak{R}_1 - \mathfrak{L}_1)\,k + 2\,\mathfrak{L}_2}{F}; \qquad \mathfrak{B} = (\mathfrak{L}_1 + \mathfrak{R}_1\,\beta_2)\,k + \mathfrak{L}_2\,\beta_2 .$$

$$\begin{matrix} M_A \\ M_B \end{matrix} = \frac{-\mathfrak{L}_1 + X}{4} \mp \frac{\mathfrak{S} \cdot K_1 + \mathfrak{B}}{2\,G} \qquad \begin{matrix} M_D \\ M_E \end{matrix} = -\frac{X}{2} \pm \frac{\mathfrak{S} \cdot K_2 - \mathfrak{B} \cdot \beta_2}{2\,G} .$$

Reactions and Shears:

$$V_B = V_C = -V_A = \frac{\mathfrak{S}_{l1} + W_2\,h_1 + \mathfrak{S}_{l2}}{l} + \frac{M_A - M_B}{l}; \qquad N_0 = H_B - H_C ;$$

$$H_B = \frac{V_B \cdot l_1 + M_B - M_E}{h_1} \quad H_C = \frac{V_B \cdot l_2 + M_E}{h_2} \quad H_A = -W_1 - W_2 + H_B .$$

Axial forces:

$$N_{1u} = V_A \cdot \sin\alpha + H_A \cdot \cos\alpha \qquad N_{2o} = -V_C \cdot \sin\alpha + H_C \cdot \cos\alpha$$

$$N_{1o} = N_{1u} + W_1 \cdot \cos\alpha; \qquad N_{2u} = N_{2o} - W_2 \cdot \cos\alpha;$$

$$N_3 = V_B \cdot \sin\alpha + H_B \cdot \cos\alpha \qquad N_4 = V_C \cdot \sin\alpha + H_C \cdot \cos\alpha .$$

Special case 21/6a: Symmetrical loads ($\mathfrak{R} = \mathfrak{L}$)

$$X = \frac{\mathfrak{L}_1\,k + 2\,\mathfrak{L}_2}{F}; \qquad \mathfrak{S} = \frac{(W_1 + W_2)\,h_1\,h_2}{2\,h} \qquad \mathfrak{B} = \mathfrak{L}_1\,k\,(1 + \beta_2) + \mathfrak{L}_2\,\beta_2;$$

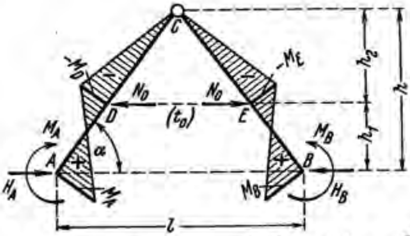
$$V_B = V_C = -V_A = \frac{W_1\,l_1 + W_2\,(h + h_2)}{2\,l} + \frac{M_A - M_B}{l} .$$

All other formulas same as above

Special case 21/6b: Use expressions for special case 21/2b

FRAME 21

Case 21/12: Uniform increase in temperature of the tie DE by t_0 degrees*



E = Modulus of elasticity
ε = Coefficient of thermal expansion

Constants:

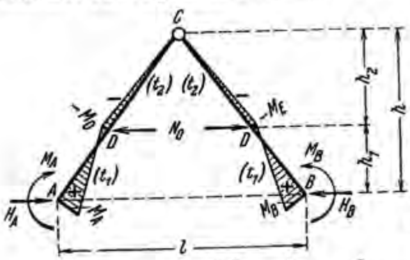
$$T=\frac{3EJ_2\cdot\varepsilon}{s_2F}\cdot\frac{l}{h}.$$

$$M_A=M_B=+T\,t_0\left[\left(\frac{2}{k}+3\right)\frac{h_2}{h_1}+1\right]\qquad M_D=M_E=-T\,t_0\left[3\frac{h_2}{h_1}+2\right];$$

$$H_A=H_B=\frac{M_A-M_D}{h_1}\quad H_C=\frac{M_D}{h_2};\quad N_0=H_A-H_C;\quad V_A=V_B=V_C=0.$$

Axial forces: $N_1=N_3=H_A\cdot\cos\alpha$ $\qquad N_2=N_4=H_C\cdot\cos\alpha$.

Case 21/13: Symmetrical increase in temperature of the inclined members*



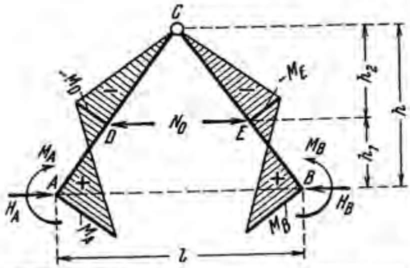
t_1 in degrees for rods s_1,
t_2 in degrees for rods s_2.

Constant T as well as E and ε same as for case 21/12.

Formulas for all V-, H-, and N-forces same as in case 21/12.

$$M_A=M_B=T\left[\left(\frac{2}{k}+3\right)t_1-t_2\right]\qquad M_D=M_E=T[-3t_1+2t_2].$$

Case 21/15: Uniform increase in temperature of the entire frame (including the tie DE) by t degrees*



E and ε as in case 21/12.

$$M_D=M_E=-\frac{9EJ_2\cdot\varepsilon}{s_2F}\cdot\frac{l}{h_1}\cdot t$$

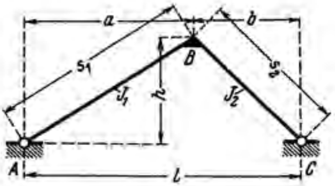
$$M_A=M_B=-M_D\cdot\left(\frac{2}{3k}+1\right)$$

Formulas for all V-, H-, and N-forces same as in case 21/12.

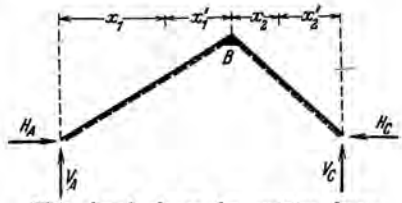
* With a decrease in temperature all moments and forces reverse their direction.

Frame 22

Unsymmetrical two-hinged, triangular rigid frame. Hinges at same elevation.



Shape of Frame
Dimensions and Notations

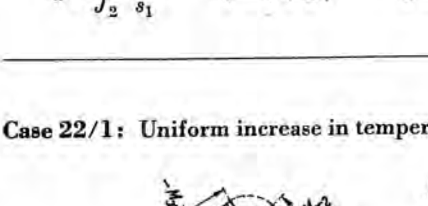


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_1}{J_2} \cdot \frac{s_2}{s_1} \qquad N = 1 + k\,; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l} \qquad (\alpha + \beta = 1)\,.$$

Case 22/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_B = -\frac{3EJ_1\varepsilon t l}{s_1 h N}\,;$$

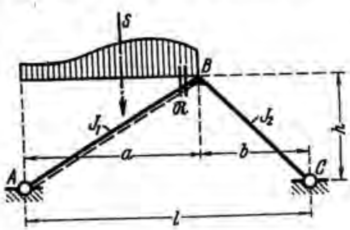
$$H_A = H_C = \frac{-M_B}{h}\,; \qquad M_{x1} = \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B\,.$$

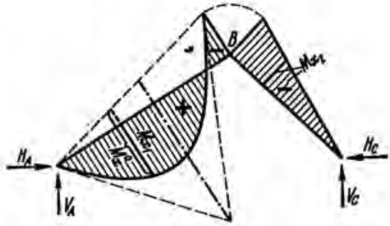
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 22

See Appendix A, Load Terms, pp. 440-445.

Case 22/2: Left-hand member loaded by any type of vertical load

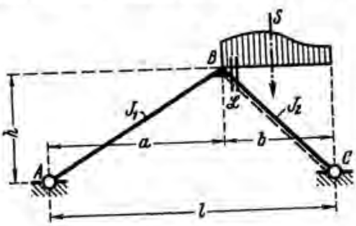


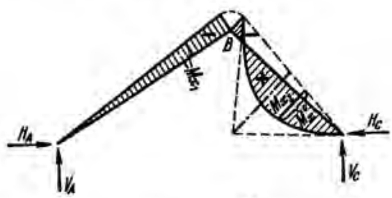


$$M_B = -\frac{\mathfrak{R}}{2N}; \qquad M_{x1} = \mathbf{M}_x^0 + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B;$$

$$V_C = \frac{\mathfrak{S}_l}{l} \qquad V_A = \mathbf{S} - V_C; \qquad H_A = H_C = \frac{\beta\,\mathfrak{S}_l - M_B}{h}.$$

Case 22/4: Right-hand member loaded by any type of vertical load

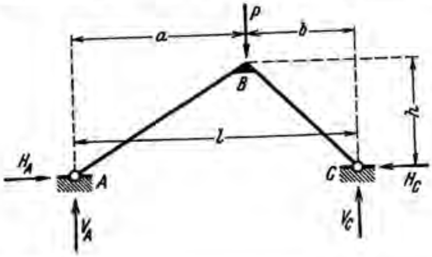




$$M_B = -\frac{\mathfrak{L}\,k}{2N}; \qquad M_{x1} = \frac{x_1}{a} M_B \qquad M_{x2} = \mathbf{M}_x^0 + \frac{x_2'}{b} M_B;$$

$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_C = \mathbf{S} - V_A; \qquad H_A = H_C = \frac{\alpha\,\mathfrak{S}_r - M_B}{h}.$$

Case 22/6: Vertical concentrated load at the ridge B



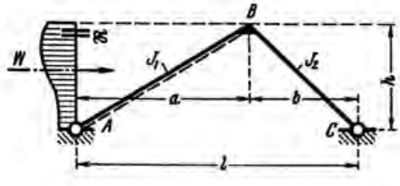
There are no bending moments.

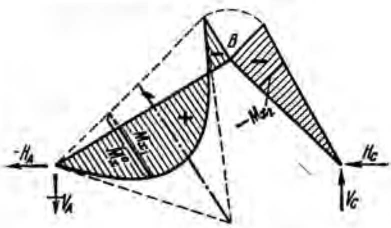
$$V_A = \beta P \qquad V_C = \alpha P;$$

$$H_A = H_C = \frac{P\,a\,b}{l\,h}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 22/3: Left-hand member loaded by any type of horizontal load

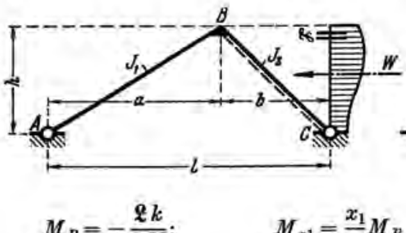


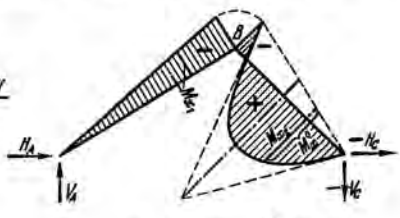


$$M_B = -\frac{\mathfrak{R}}{2N}; \qquad M_{x1} = M_x^0 + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B;$$

$$H_C = \frac{\beta \mathfrak{S}_l - M_B}{h} \qquad H_A = -(W - H_C); \qquad V_A = -V_C = -\frac{\mathfrak{S}_l}{l}.$$

Case 22/5: Right-hand member loaded by any type of horizontal load

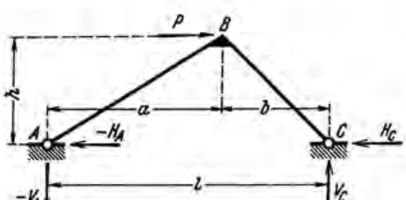




$$M_B = -\frac{\mathfrak{L} k}{2N}; \qquad M_{x1} = \frac{x_1}{a} M_B \qquad M_{x2} = M_x^0 + \frac{x_2'}{b} M_B;$$

$$H_A = \frac{\alpha \mathfrak{S}_r - M_B}{h} \qquad H_C = -(W - H_A); \qquad V_A = -V_C = \frac{\mathfrak{S}_r}{l}.$$

Case 22/7: Horizontal concentrated load at the ridge *B*



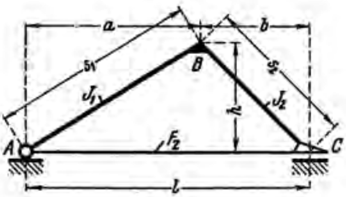
There are no bending moments.

$$H_A = -\alpha P \qquad H_C = \beta P;$$

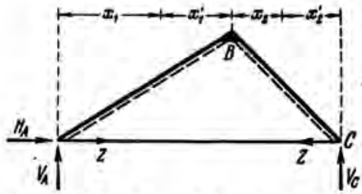
$$V_A = -V_C = -\frac{P h}{l}.$$

Frame 23

Unsymmetrical triangular rigid frame with horizontal tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_1}{J_2} \cdot \frac{s_2}{s_1}; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l} \qquad (\alpha + \beta = 1);$$

$$N = 1 + k \qquad L = \frac{3 J_1}{h^2 F_Z} \cdot \frac{l}{s_1} \cdot \frac{E}{E_Z} \qquad N_Z = N + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

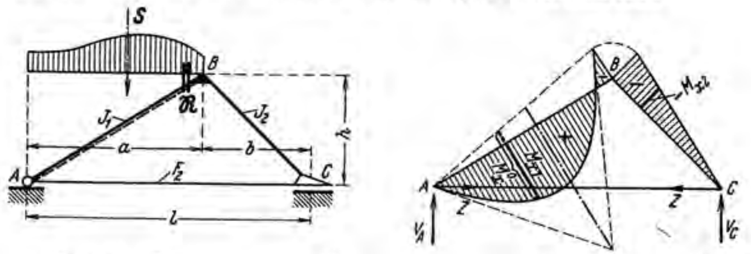
Formulas to case 23/3, p. 95:

$$Z = \frac{\mathfrak{R} + 2 N \beta \mathfrak{S}_l}{2 h N_Z}; \quad H_A = -\mathfrak{W} \quad V_A = -V_C = -\frac{\mathfrak{S}_l}{l}; \quad M_{x2} = \frac{x_2'}{b} M_B$$

$$M_B = \beta \mathfrak{S}_l - Z h = \frac{2 L \beta \mathfrak{S}_l - \mathfrak{R}}{2 N_Z}; \qquad M_{x1} = M_x^0 + \frac{x_1}{a} M_B.$$

See Appendix A, Load Terms, pp. 440-445.

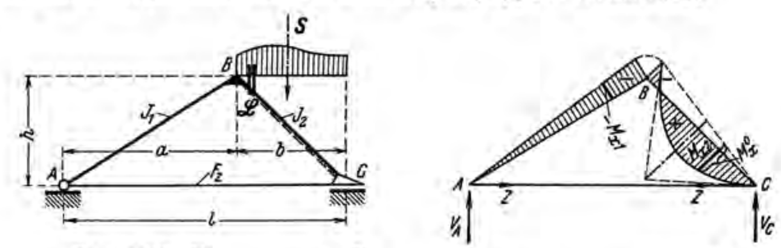
Case 23/1: Left-hand member loaded by any type of vertical load



$$Z = \frac{\mathfrak{R} + 2N\beta\mathfrak{S}_l}{2hN_Z}; \qquad V_C = \frac{\mathfrak{S}_l}{l} \qquad V_A = S - V_C; \qquad M_{x2} = \frac{x_2'}{b} M_B$$

$$M_B = \beta\mathfrak{S}_l - Zh = \frac{2L\beta\mathfrak{S}_l - \mathfrak{R}}{2N_Z}; \qquad M_{x1} = M_x^0 + \frac{x_1}{a} M_B.$$

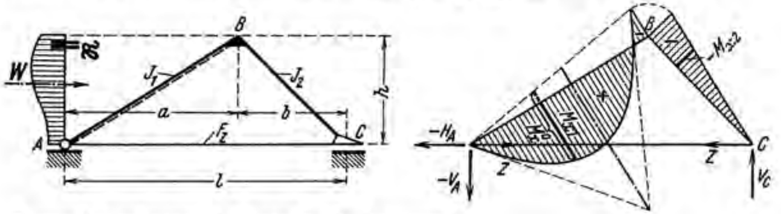
Case 23/2: Right-hand member loaded by any type of vertical load



$$Z = \frac{\mathfrak{L}k + 2N\alpha\mathfrak{S}_r}{2hN_Z}; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_C = S - V_A; \qquad M_{x1} = \frac{x_1}{a} M_B$$

$$M_B = \alpha\mathfrak{S}_r - Zh = \frac{2L\alpha\mathfrak{S}_r - \mathfrak{L}k}{2N_Z}; \qquad M_{x2} = M_x^0 = \frac{x_2'}{b} M_B.$$

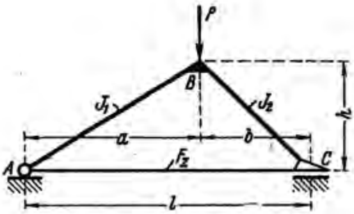
Case 23/3: Left-hand member loaded by any type of horizontal load

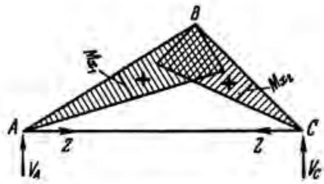


Formulas to case 23/3 see p. 94 bottom.

FRAME 23

Case 23/4: Vertical concentrated load at ridge B

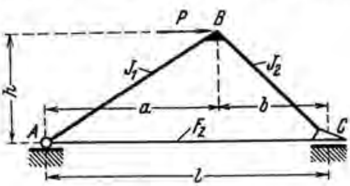


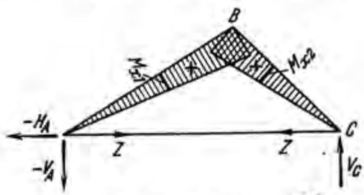


$$Z=\frac{Pab}{lh}\cdot\frac{N}{N_Z};\qquad V_A=\beta P\qquad V_C=\alpha P;$$

$$M_B=\frac{Pab}{l}\cdot\frac{N}{N_Z};\qquad M_{x1}=\frac{x_1}{a}M_B\qquad M_{x2}=\frac{x_2'}{b}M_B.$$

Case 23/5: Horizontal concentrated load at ridge B acting from the left

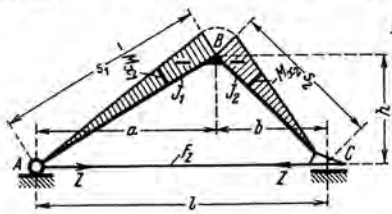




$$Z=\beta P\frac{N}{N_Z};\qquad H_A=-P\qquad V_A=-V_C=-\frac{Ph}{l};$$

$$M_B=\beta Ph\frac{L}{N_Z};\qquad M_{x1}=\frac{x_1}{a}M_B\qquad M_{x2}=\frac{x_2'}{b}M_B.$$

Case 23/6: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z=\frac{3EJ_1\cdot\varepsilon t l}{s_1h^2N_Z};$$

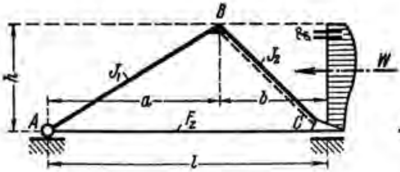
$$M_B=-Zh;\qquad M_{x1}=\frac{x_1}{a}M_B\qquad M_{x2}=\frac{x_2'}{b}M_B.$$

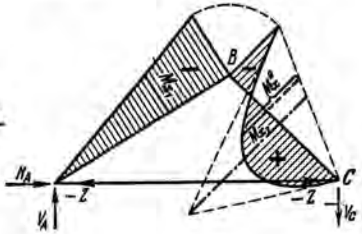
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

*See footnote on page 97.

Case 23/7: Right-hand member loaded by any type of horizontal load

See Appendix A, Load Terms, pp. 440-445.



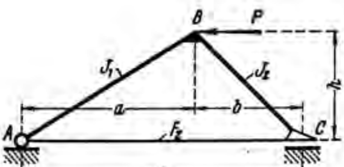


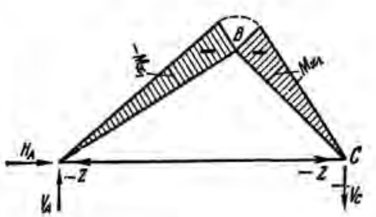
$$Z = -\left(W\frac{N}{N_Z} - \frac{\mathfrak{L}k + 2N\alpha\mathfrak{S}_r}{2hN_Z}\right)^* ; \qquad H_A = W; \qquad V_A = -V_C = \frac{\mathfrak{S}_r}{l};$$

$$M_B = -(W+Z)h + \alpha\mathfrak{S}_r = -\left(Wh\frac{L}{N_Z} - \frac{2L\alpha\mathfrak{S}_r - \mathfrak{L}k}{2N_Z}\right);$$

$$M_{x1} = \frac{x_1}{a}M_B \qquad M_{x2} = M_x^0 + \frac{x_2'}{b}M_B.$$

Case 23/8: Horizontal concentrated load at ridge *B* acting from the right





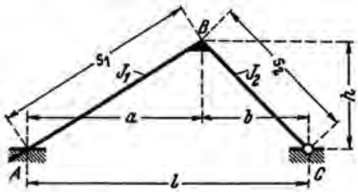
$$Z = -\beta P\frac{N^*}{N_Z} ; \qquad H_A = P; \qquad V_A = -V_C = \frac{Ph}{l}$$

$$M_B = -\beta Ph\frac{L}{N_Z}; \qquad M_{x1} = \frac{x_1}{a}M_B \qquad M_{x2} = \frac{x_2'}{b}M_B.$$

* For the above loading conditions and for decrease in temperature (p. 96) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

Frame 24

Unsymmetrical triangular rigid frame. One support fixed, one support hinged; both supports at the same elevation.



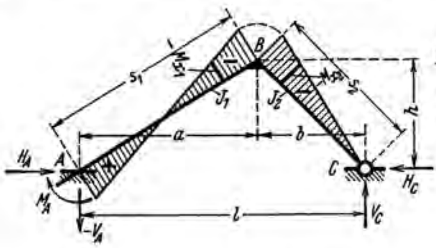
Shape of Frame
Dimensions and Notations

This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* same as for frame 27 (pp. 103-105) with the following simplifications:

$$(h_1 = h_2) = h \qquad v = 0 \qquad F = l\,h\,.$$

Case 24/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $\quad T = \dfrac{6\,E\,J_1\,\varepsilon\,t}{s_1\,N}.$

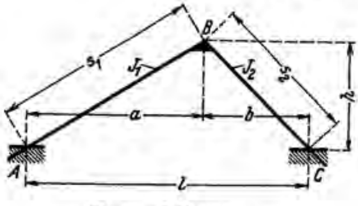
$$M_A = +\,T \cdot \frac{2(1+k)\,b + l}{h} \qquad M_B = -\,T \cdot \frac{2l + b}{h}; \qquad M_{x2} = \frac{x_2'}{b}\,M_B$$

$$V_C = -V_A = \frac{M_A}{l} \qquad H_A = H_C = \frac{b\,M_A - l\,M_B}{l\,h}; \qquad M_{x1} = \frac{x_1'}{a}\,M_A + \frac{x_1}{a}\,M_B.$$

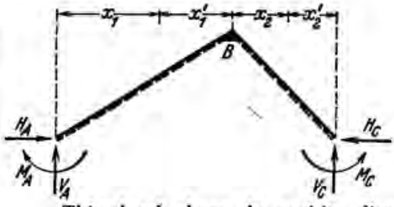
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 25

Unsymmetrical hingeless, triangular rigid frame. Both supports at the same elevation.



Shape of Frame
Dimensions and Notations

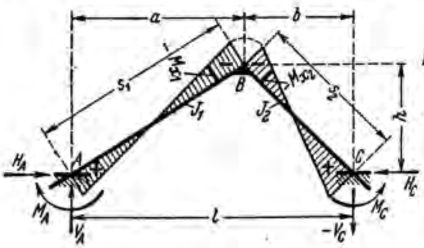


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* same as for frame 28 (pp. 106-108) with the following simplifications:

$$(h_1 = h_2) = h \qquad v = 0 \qquad F = l h .$$

Case 25/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degree

Constant: $\quad T = \dfrac{3 E J_1 \varepsilon t}{s_1 N}.$

$$M_A = + T \cdot \frac{N b + l}{h} \qquad M_B = -2 T \cdot \frac{l}{h} \qquad M_C = + T \cdot \frac{l + a \cdot N / k}{h};$$

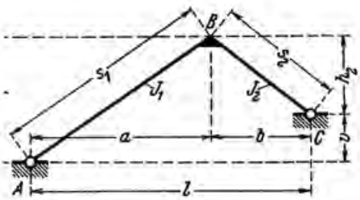
$$V_A = -V_C = \frac{M_C - M_A}{l} \qquad H_A = H_C = \frac{b M_A - l M_B + a M_C}{l h};$$

$$M_{x1} = \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B + \frac{x_2}{b} M_C .$$

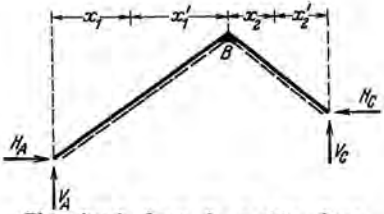
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 26

Unsymmetrical two-hinged triangular rigid frame. Supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$v = h_1 - h_2$* $k = \frac{J_1}{J_2} \cdot \frac{s_2}{s_1}$; $N = 1 + k$ $F = b h_1 + a h_2$.

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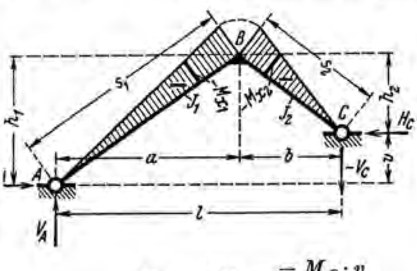
ase 26/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_B = -\frac{3 E J_1 \varepsilon t}{s_1 N} \cdot \frac{l^2 + v^2}{F};$$

$$V_A = -V_C = \frac{-M_B \cdot v}{F} \qquad H_A = H_C = \frac{-M_B \cdot l}{F};$$

$$M_{x1} = \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

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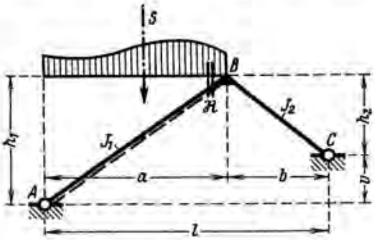
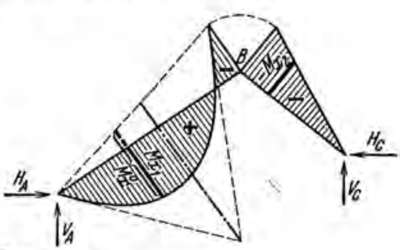
*When $h_2 > h_1$, v becomes negative.

See Appendix A, Load Terms, pp. 440-445.

Case 26/2: Left-hand member loaded by any type of vertical load

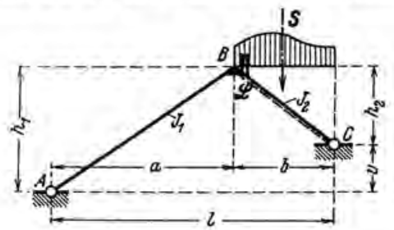


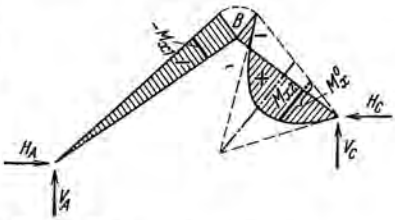


$$M_B = -\frac{\mathfrak{R}}{2N}; \qquad V_C = \frac{h_2 \mathfrak{S}_l + v M_B}{F} \qquad V_A = S - V_C;$$

$$H_A = H_C = \frac{b \mathfrak{S}_l - l M_B}{F}; \quad M_{x1} = M_x^0 + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B.$$

Case 26/3: Right-hand member loaded by any type of vertical load

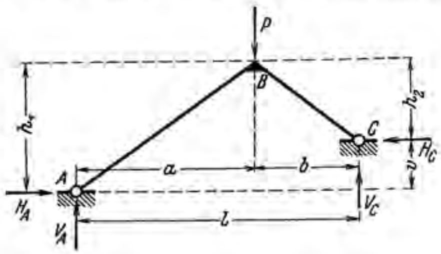




$$M_B = -\frac{\mathfrak{L} k}{2N}; \qquad V_A = \frac{h_1 \mathfrak{S}_r - v M_B}{F} \qquad V_C = S - V_A;$$

$$H_A = H_C = \frac{a \mathfrak{S}_r - l M_B}{F}; \quad M_{x1} = \frac{x_1}{a} M_B \qquad M_{x2} = M_x^0 + \frac{x_2'}{b} M_B.$$

Case 26/4: Vertical concentrated load at the ridge *B*



There are no bending moments

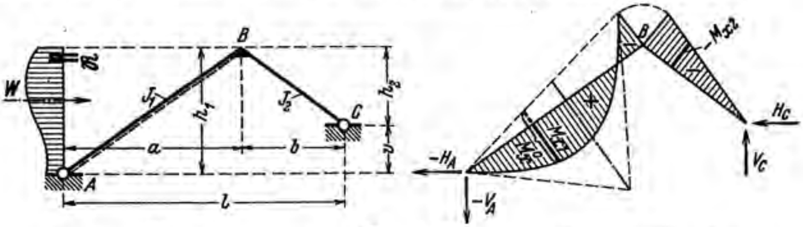
$$H_A = H_C = \frac{P a b}{F};$$

$$V_A = \frac{P b h_1}{F} \qquad V_C = \frac{P a h_2}{F}.$$

FRAME 26

See Appendix A, Load Terms, pp. 440-445.

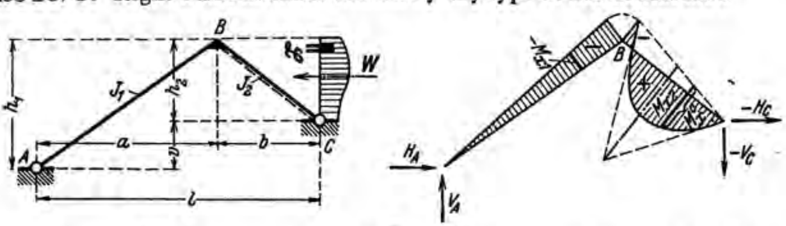
Case 26/5: Left-hand member loaded by any type of horizontal load



$$M_B = -\frac{\mathfrak{R}}{2N}; \qquad H_C = \frac{b\,\mathfrak{S}_l - l\,M_B}{F} \qquad H_A = -(W - H_C);$$

$$V_C = -V_A = \frac{h_2\,\mathfrak{S}_l + v\,M_B}{F}; \qquad M_{x1} = M_x^0 + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B.$$

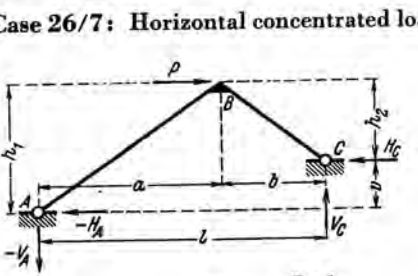
Case 26/6: Right-hand member loaded by any type of horizontal load



$$M_B = -\frac{\mathfrak{L}\,k}{2N}; \qquad H_A = \frac{a\,\mathfrak{S}_r - l\,M_B}{F} \qquad H_C = -(W - H_A);$$

$$V_A = -V_C = \frac{h_1\,\mathfrak{S}_r - v\,M_B}{F}; \qquad M_{x1} = \frac{x_1}{a} M_B \qquad M_{x2} = M_x^0 + \frac{x_2'}{b} M_B.$$

Case 26/7: Horizontal concentrated load at the ridge B



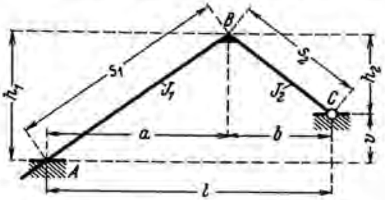
There are no bending moments

$$V_C = -V_A = \frac{P\,h_1\,h_2}{F};$$

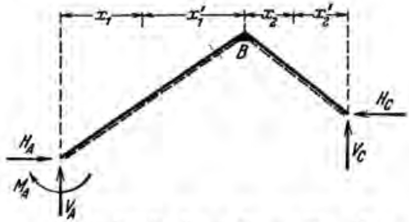
$$H_A = -\frac{P\,a\,h_2}{F} \qquad H_C = \frac{P\,b\,h_1}{F}.$$

Frame 27

Unsymmetrical triangular rigid frame. One support fixed, one support hinged; supports at different elevations.



Shape of Frame
Dimensions and Notations

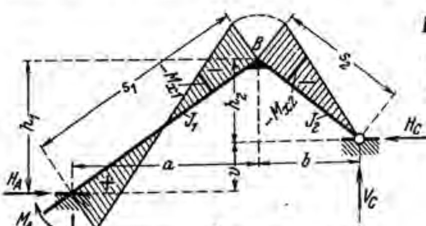


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$v = h_1 - h_2^{*} \qquad k = \frac{J_1}{J_2} \cdot \frac{s_2}{s_1}; \qquad N = 3 + 4k \qquad F = b h_1 + a h_2 .$$

Case 27/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants: $\quad T = \dfrac{6 E J_1 \varepsilon t}{s_1 N},$

$$A = \frac{l b - v h_2}{F}, \qquad B = \frac{l^2 + v^2}{F}.$$

$$M_A = + T[2A(1+k) + B] \qquad M_B = - T[A + 2B];$$

$$V_A = - V_C = \frac{- h_2 M_A - v M_B}{F}; \qquad H_A = H_C = \frac{b M_A - M_B}{F};$$

$$M_{x1} = \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B .$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

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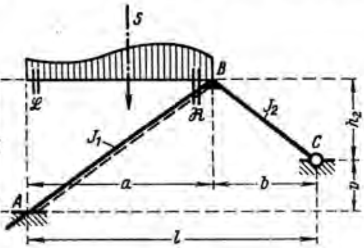
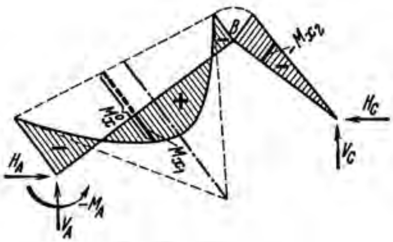
* When $h_2 > h_1$, v becomes negative.

FRAME 27

See Appendix A, Load Terms, pp. 440-445.

Case 27/2: Left-hand member loaded by any type of vertical load





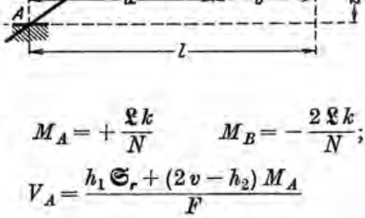
$$M_A = -\frac{2\,\mathfrak{L}(1+k) - \mathfrak{R}}{N} \qquad M_B = -\frac{2\,\mathfrak{R} - \mathfrak{L}}{N};$$

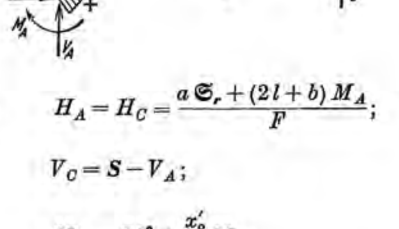
$$V_C = \frac{h_2(\mathfrak{S}_l + M_A) + v\,M_B}{F} \qquad V_A = S - V_C;$$

$$H_A = H_C = \frac{b(\mathfrak{S}_l + M_A) - l\,M_B}{F};$$

$$M_{x1} = M_x^0 + \frac{x_1'}{a}M_A + \frac{x_1}{a}M_B \qquad M_{x2} = \frac{x_2'}{b}M_B.$$

Case 27/3: Right-handed member loaded by any type of vertical load





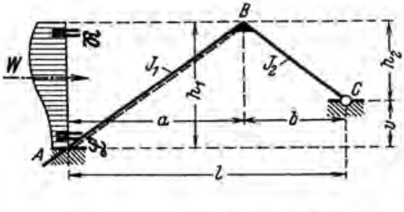
$$M_A = +\frac{\mathfrak{L}\,k}{N} \qquad M_B = -\frac{2\,\mathfrak{L}\,k}{N}; \qquad H_A = H_C = \frac{a\,\mathfrak{S}_r + (2l+b)\,M_A}{F};$$

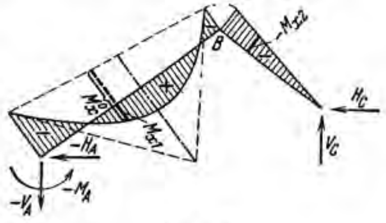
$$V_A = \frac{h_1\,\mathfrak{S}_r + (2v - h_2)\,M_A}{F} \qquad V_C = S - V_A;$$

$$M_{x1} = \frac{x_1'}{a}M_A + \frac{x_1}{a}M_B \qquad M_{x2} = M_x^0 + \frac{x_2'}{b}M_B.$$

See Appendix A, Load Terms, pp. 440-445.

Case 27/4: Left-hand member loaded by any type of horizontal load





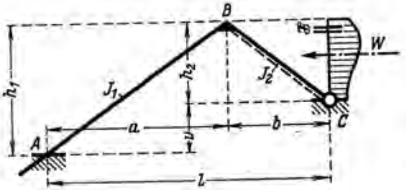
$$M_A = -\frac{2\,\mathfrak{L}(1+k) - \mathfrak{R}}{N} \qquad M_B = -\frac{2\,\mathfrak{R} - \mathfrak{L}}{N};$$

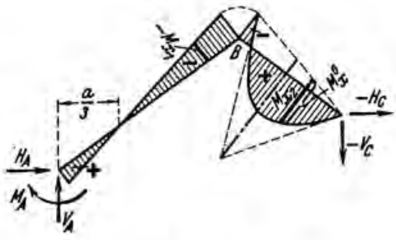
$$H_C = \frac{b(\mathfrak{S}_l + M_A) - l\,M_B}{F} \qquad H_A = -(W - H_C);$$

$$V_C = -V_A = \frac{h_2(\mathfrak{S}_l + M_A) + v\,M_B}{F};$$

$$M_{x1} = M_x^0 + \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B .$$

Case 27/5: Right-hand member loaded by any type of horizontal load





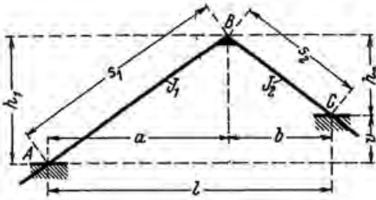
$$M_A = +\frac{\mathfrak{L}\,k}{N} \qquad M_B = -\frac{2\,\mathfrak{L}\,k}{N} \qquad H_A = \frac{a\,\mathfrak{S}_r + (2l+b)\,M_A}{F}$$

$$V_A = -V_C = \frac{h_1\,\mathfrak{S}_r + (2v - h_2)\,M_A}{F} \qquad H_C = -(W - H_A)$$

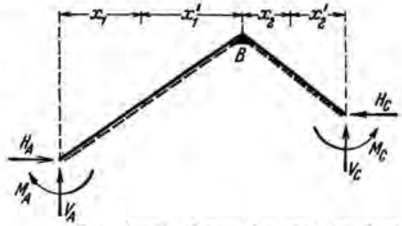
$$M_{x1} = \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = M_x^0 + \frac{x_2'}{b} M_B .$$

Frame 28

Unsymmetrical hingleless, triangular rigid frame. Supports at different elevations.



Shape of Frame Dimensions and Notations

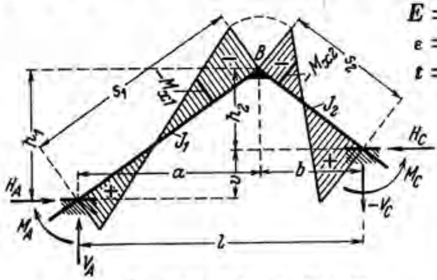


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$v = h_1 - h_2$ * $k = \dfrac{J_1}{J_2} \cdot \dfrac{s_2}{s_1}$; $N = 1 + k$ $F = b\,h_1 + a\,h_2$.

Case 28/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$T = \frac{E J_1 \varepsilon t}{s_1 N}, \qquad A = \frac{l b - v h_2}{F},$$

$$B = \frac{l^2 + v^2}{F}, \qquad C = \frac{l a + v h_1}{F}.$$

$$M_A = +T[A(4+3k) + 2B + C]$$
$$M_B = -2T[A + 2B + C]$$
$$M_C = +T\left[A + 2B + C\frac{3+4k}{k}\right];$$

$$V_A = -V_C = \frac{-h_2 M_A - v M_B + h_1 M_C}{F}$$
$$H_A = H_C = \frac{b M_A - l M_B + a M_C}{F};$$

$$M_{x1} = \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B + \frac{x_2}{b} M_C.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

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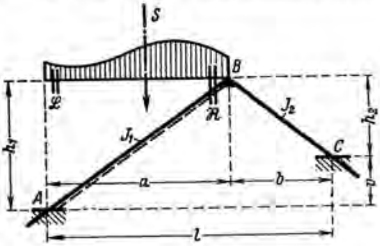
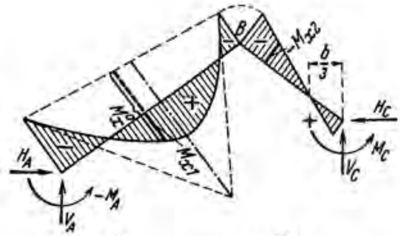
*When $h_2 > h_1$, v becomes negative.

See Appendix A, Load Terms, pp. 440-445.

Case 28/2: Left-hand member loaded by any type of vertical load





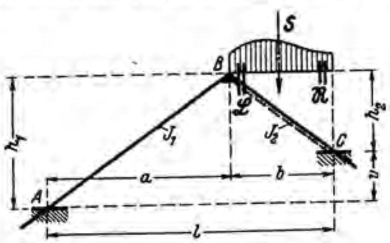
$$M_A = -\frac{\mathfrak{L}(4+3k) - 2\mathfrak{R}}{6N} \qquad M_B = -\frac{2\mathfrak{R} - \mathfrak{L}}{3N} \qquad M_C = \frac{-M_B}{2};$$

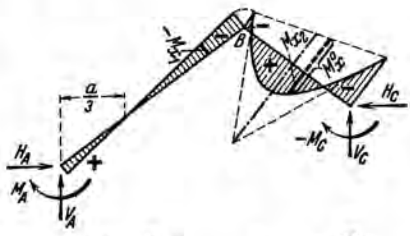
$$V_C = \frac{h_2(\mathfrak{S}_l + M_A) - (2v + h_1) M_C}{F} \qquad V_A = S - V_C;$$

$$H_A = H_C = \frac{b(\mathfrak{S}_l + M_A) + (2l + a) M_C}{F};$$

$$M_{x1} = M_x^0 + \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B + \frac{x_2}{b} M_C.$$

Case 28/3: Right-hand member loaded by any type of vertical load





$$M_C = -\frac{\mathfrak{R}(3+4k) - 2\mathfrak{L}k}{6N} \qquad M_B = -\frac{(2\mathfrak{L} - \mathfrak{R})k}{3N} \qquad M_A = \frac{-M_B}{2};$$

$$V_A = \frac{h_1(\mathfrak{S}_r + M_C) + (2v - h_2) M_A}{F} \qquad V_C = S - V_A;$$

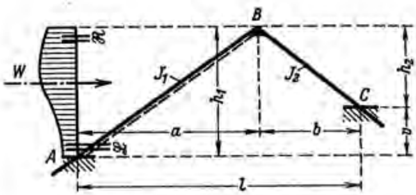
$$H_A = H_C = \frac{a(\mathfrak{S}_r + M_C) + (2l + b) M_A}{F};$$

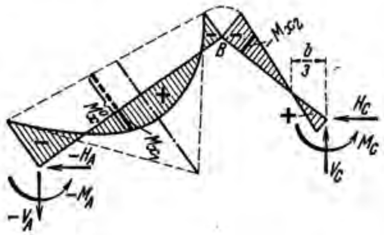
$$M_{x1} = \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = M_x^0 + \frac{x_2'}{b} M_B + \frac{x_2}{b} M_C.$$

FRAME 28

See Appendix A, Load Terms, pp. 440-445.

Case 28/4: Left-hand member loaded by any type of horizontal load





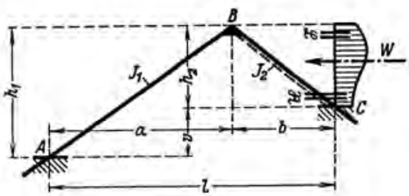
$$M_A = -\frac{\mathfrak{L}(4+3k) - 2\,\mathfrak{R}}{6N} \qquad M_B = -\frac{2\,\mathfrak{R} - \mathfrak{L}}{3N} \qquad M_C = \frac{-M_B}{2};$$

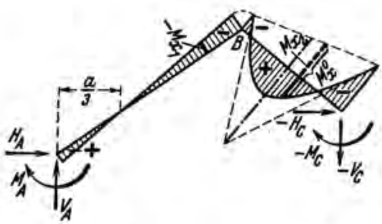
$$H_C = \frac{b(\mathfrak{S}_l + M_A) + (2l + a)\,M_C}{F} \qquad H_A = -(W - H_C);$$

$$V_C = -V_A = \frac{h_2(\mathfrak{S}_l + M_A) - (2v + h_1)\,M_C}{F};$$

$$M_{x1} = \mathbf{M}_x^0 + \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = \frac{x_2'}{b} M_B + \frac{x_2}{b} M_C .$$

Case 28/5: Right-hand member loaded by any type of horizontal load





$$M_C = -\frac{\mathfrak{R}(3+4k) - 2\,\mathfrak{L}k}{6N} \qquad M_B = -\frac{(2\,\mathfrak{L} - \mathfrak{R})\,k}{3N} \qquad M_A = \frac{-M_B}{2};$$

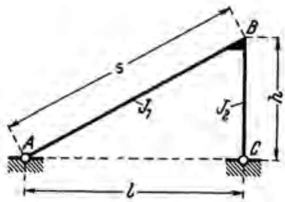
$$H_A = \frac{a(\mathfrak{S}_r + M_C) + (2l + b)\,M_A}{F} \qquad H_C = -(W - H_A);$$

$$V_A = -V_C = \frac{h_1(\mathfrak{S}_r + M_C) + (2v - h_2)\,M_A}{F};$$

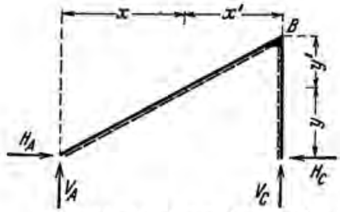
$$M_{x1} = \frac{x_1'}{a} M_A + \frac{x_1}{a} M_B \qquad M_{x2} = \mathbf{M}_x^0 + \frac{x_2'}{b} M_B + \frac{x_2}{b} M_C .$$

Frame 29

Unsymmetrical two-hinged, triangular rigid frame. One leg vertical. Both supports at the same elevation.



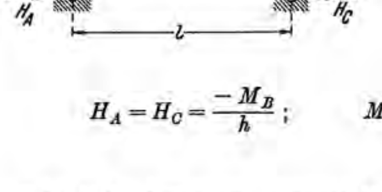
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_1}{J_2} \cdot \frac{h}{s}$ $\qquad N = 1 + k.$

Case 29/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_B = -\frac{3EJ_1\varepsilon t l}{shN};$$

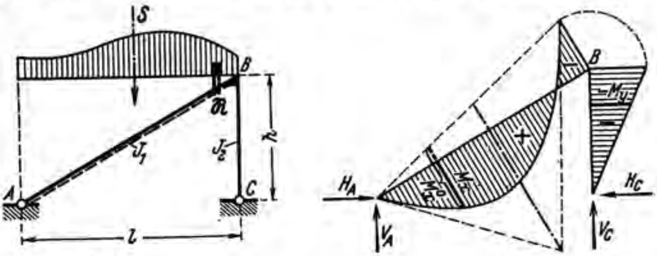
$$H_A = H_C = \frac{-M_B}{h}; \qquad M_x = \frac{x}{l} M_B \qquad M_y = \frac{y}{h} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 29

See Appendix A, Load Terms, pp. 440-445.

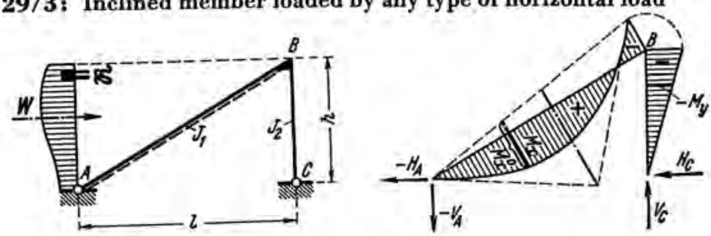
Case 29/2: Inclined member loaded by any type of vertical load



$$M_B = -\frac{\mathfrak{R}}{2N}; \qquad M_x = M_x^0 + \frac{x}{l} M_B \qquad M_y = \frac{y}{h} M_B;$$

$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_C = \frac{\mathfrak{S}_l}{l}; \qquad H_A = H_C = \frac{-M_B}{h}.$$

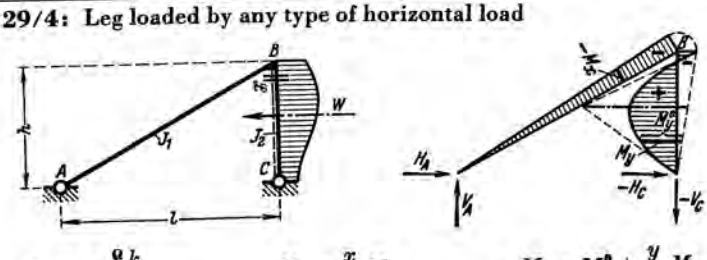
Case 29/3: Inclined member loaded by any type of horizontal load



$$M_B = -\frac{\mathfrak{R}}{2N}; \qquad M_x = M_x^0 + \frac{x}{l} M_B \qquad M_y = \frac{y}{h} M_B;$$

$$H_C = \frac{-M_B}{h} \qquad H_A = -(W - H_C); \qquad V_C = -V_A = \frac{\mathfrak{S}_l}{l}.$$

Case 29/4: Leg loaded by any type of horizontal load

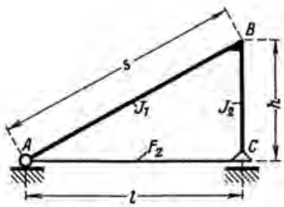


$$M_B = -\frac{\mathfrak{L}k}{2N}; \qquad M_x = \frac{x}{l} M_B \qquad M_y = M_y^0 + \frac{y}{h} M_B;$$

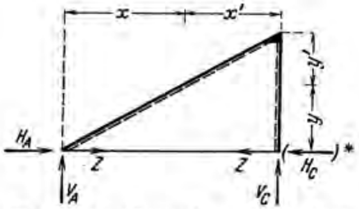
$$H_A = \frac{\mathfrak{S}_r - M_B}{h} \qquad H_C = -(W - H_A); \qquad V_A = -V_C = \frac{\mathfrak{S}_r}{l}.$$

Frame 30

Triangular rigid frame with horizontal tie-rod. Externally simply supported. One leg vertical.



Shape of Frame Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_1}{J_2} \cdot \frac{h}{s}; \qquad L = \frac{3J_1}{h^2 F_Z} \cdot \frac{l}{s} \cdot \frac{E}{E_Z} \qquad N = 1 + k; \qquad N_Z = N + L.$$

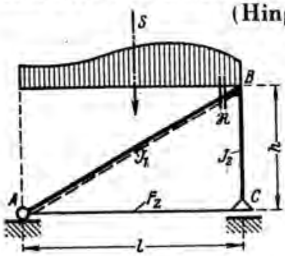
E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

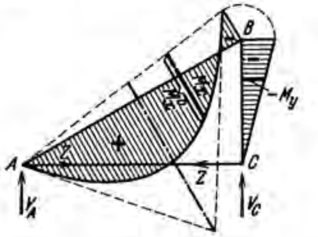
*H_C occurs when the hinged support is at C.

FRAME 30

See Appendix A, Load Terms, pp. 440-445.

Case 30/1: Inclined member loaded by any type of vertical load (Hinged support at A or C)

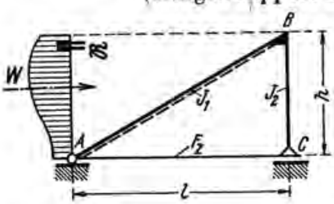


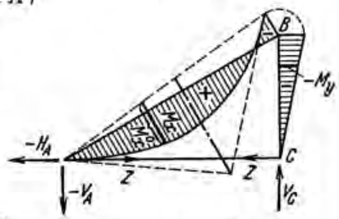


$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_C = \frac{\mathfrak{S}_l}{l};$$

$$Z = \frac{\mathfrak{R}}{2hN_Z}; \qquad M_B = -Zh; \qquad M_x = M_x^0 + \frac{x}{l}M_B \qquad M_y = -Zy.$$

Case 30/2: Inclined member loaded by any type of horizontal load (Hinged support at A)



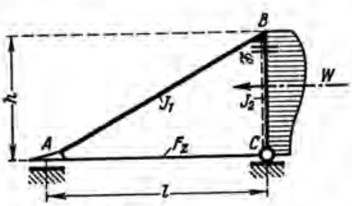


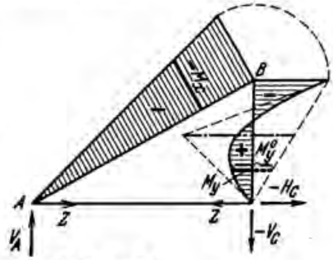
$$Z = \frac{\mathfrak{R}}{2hN_Z}$$

$$M_B = -Zh;$$

$$H_A = -W; \qquad V_C = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad M_x = M_x^0 + \frac{x}{l}M_B \qquad M_y = -Zy.$$

Case 30/3: Leg loaded by any type of horizontal load (Hinged support at C)



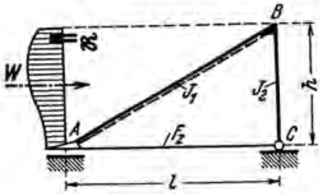


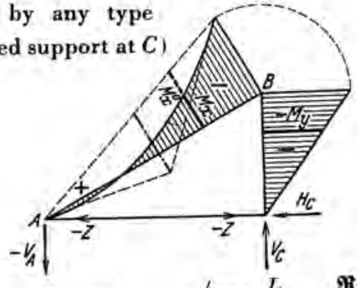
$$Z = \frac{\mathfrak{L}k + 2N\mathfrak{S}_r}{2hN_Z}; \qquad M_B = \mathfrak{S}_r - Zh = \frac{2L\mathfrak{S}_r - \mathfrak{L}k}{2N_Z}; \qquad H_C = -W;$$

$$V_A = -V_C = \frac{\mathfrak{S}_r}{l}; \qquad M_x = \frac{x}{l}M_B \qquad M_y = M_y^0 + \frac{y}{h}M_B.$$

See Appendix A, Load Terms, pp. 440-445.

Case 30/4: Inclined member loaded by any type of horizontal load (Hinged support at C)

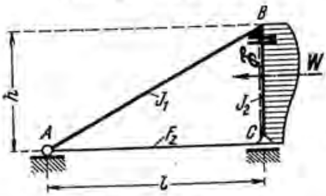


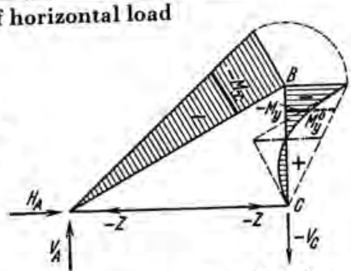


$$Z = -\left(W\frac{N}{N_Z} - \frac{\mathfrak{M}}{2hN_Z}\right)^* ; \qquad M_B = -(W + Z)h = -\left(Wh\frac{L}{N_Z} + \frac{\mathfrak{M}}{2N_Z}\right);$$

$$V_C = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_C = W; \qquad M_x = M_x^0 + \frac{x}{l}M_B \qquad M_y = \frac{y}{h}M_B.$$

Case 30/5: Leg loaded by any type of horizontal load (Hinged support at A)

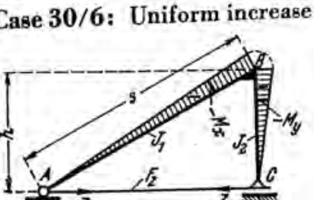




$$Z = -\frac{2N\mathfrak{S}_l - \mathfrak{L}k^*}{2hN_Z} ; \qquad M_B = -\mathfrak{S}_l - Zh = -\frac{\mathfrak{L}k + 2L\mathfrak{S}_l}{2N_Z};$$

$$V_A = -V_C = \frac{\mathfrak{S}_r}{l}; \qquad H_A = W; \qquad M_x = \frac{x}{l}M_B \qquad M_y = M_y^0 + \frac{y}{h}M_B.$$

Case 30/6: Uniform increase in temperature of the entire frame



E = Modulus of elasticity

ε = Coefficient of thermal expansion

t = Change of temperature in degrees

$$Z = \frac{3EJ_1\varepsilon tl}{sh^2N_Z}; \qquad M_B = -Zh;$$

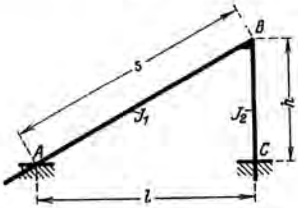
$$M_x = \frac{x}{l}M_B \qquad M_y = \frac{y}{h}M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

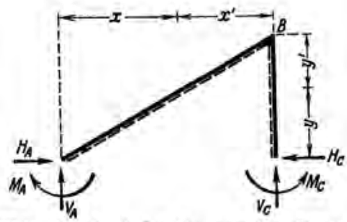
*For the case of the above loading conditions and for a decrease in temperature Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

Frame 31

Triangular hingeless rigid frame. One leg vertical. Both supports at the same elevation.



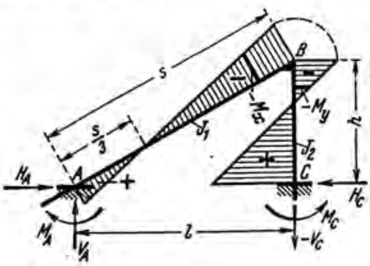
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \dfrac{J_1}{J_2} \cdot \dfrac{h}{s}$ $\qquad N = 1 + k\,.$

Case 31/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $\quad T = \dfrac{3 E J_1 \varepsilon t l}{s h N}\,.$

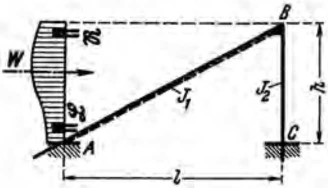
$$M_A = +T \qquad M_B = -2T \qquad M_C = +T\frac{1+2k}{k}; \qquad H_A = H_C = \frac{M_C - M_B}{h};$$

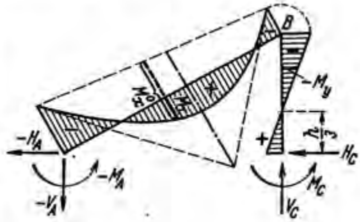
$$V_A = -V_C = \frac{M_C - M_A}{l}; \qquad M_x = \frac{x'}{l} M_A + \frac{x}{l} M_B \qquad M_y = \frac{y}{h} M_B + \frac{y'}{h} M_C\,.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

See Appendix A, Load Terms, pp. 440-445.

Case 31/2: Inclined member loaded by any type of horizontal load



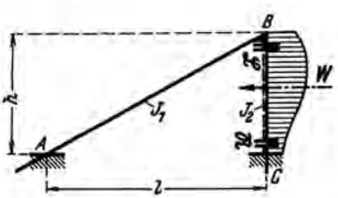


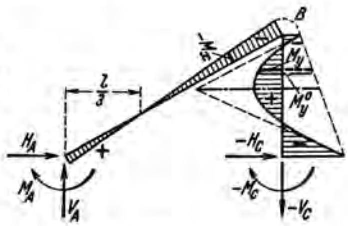
$$M_A = -\frac{\mathfrak{L}(4+3k) - 2\,\mathfrak{R}}{6N} \qquad M_B = -\frac{2\,\mathfrak{R} - \mathfrak{L}}{3N} \qquad M_C = \frac{-M_B}{2};$$

$$H_C = \frac{3\,M_C}{h} \qquad H_A = -(W - H_C); \qquad V_C = -V_A = \frac{\mathfrak{S}_l + M_A - M_C}{l};$$

$$M_x = M_x^0 + \frac{x'}{l} M_A + \frac{x}{l} M_B \qquad M_y = \frac{y}{h} M_B + \frac{y'}{h} M_C.$$

Case 31/3: Leg loaded by any type of horizontal load





$$M_C = -\frac{\mathfrak{R}(3+4k) - 2\,\mathfrak{L}k}{6N} \qquad M_B = -\frac{(2\,\mathfrak{L} - \mathfrak{R})k}{3N} \qquad M_A = \frac{-M_B}{2};$$

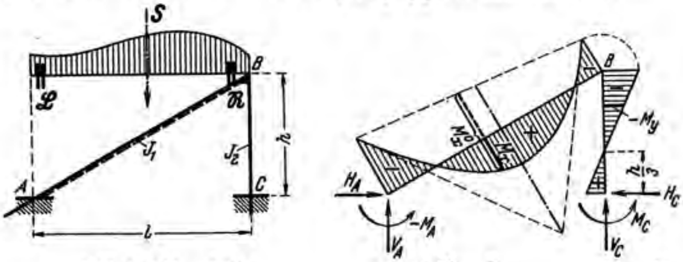
$$H_A = \frac{\mathfrak{S}_r - M_B + M_C}{h} \qquad H_C = -(W - H_A);$$

$$V_A = -V_C = \frac{\mathfrak{S}_r - M_A + M_C}{l};$$

$$M_x = \frac{x'}{l} M_A + \frac{x}{l} M_B \qquad M_y = M_y^0 + \frac{y}{h} M_B + \frac{y'}{h} M_C.$$

See Appendix A, Load Terms, pp. 440-445.

e 31/4: Inclined member loaded by any type of vertical load

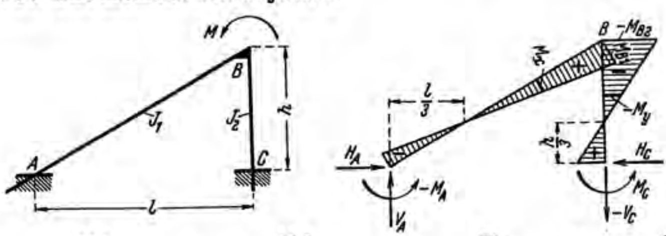


$$M_A = -\frac{\mathfrak{L}(4+3k) - 2\,\mathfrak{R}}{6N} \qquad M_B = -\frac{2\,\mathfrak{R} - \mathfrak{L}}{3N} \qquad M_C = \frac{-M_B}{2};$$

$$V_C = \frac{\mathfrak{S}_l + M_A - M_C}{l} \qquad V_A = S - V_C; \qquad H_A = H_C = \frac{3M_C}{h};$$

$$M_x = M_x^0 + \frac{x'}{l} M_A + \frac{x}{l} M_B \qquad M_y = \frac{y}{h} M_B + \frac{y'}{h} M_C.$$

se 31/5: The moment acts at joint B

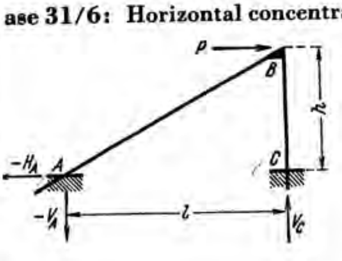


$$M_A = -\frac{Mk}{2N} \qquad M_{B1} = +\frac{Mk}{N} \qquad M_{B2} = -\frac{M}{N} \qquad M_C = +\frac{M}{2N};$$

$$(M_{B1} - M_{B2} = M); \qquad H_A = H_C = \frac{3M}{2hN}; \qquad V_A = -V_C = \frac{3M}{2l};$$

$$M_x = \frac{x'}{l} M_A + \frac{x}{l} M_{B1} \qquad M_y = \frac{y}{h} M_{B2} + \frac{y'}{h} M_C.$$

ase 31/6: Horizontal concentrated load at joint B

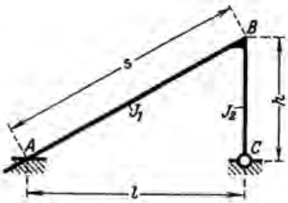


There are no bending moments

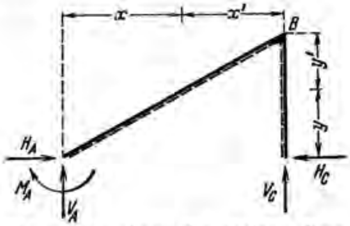
$$V_C = -V_A = \frac{Ph}{l} \qquad H_A = -P.$$

Frame 32

Triangular rigid frame. One leg vertical, hinged at bottom. Other support fixed. Both supports at the same elevation.



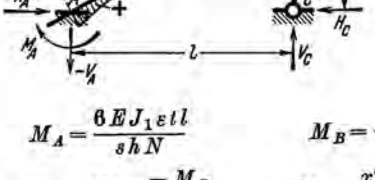
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_1}{J_2} \cdot \frac{h}{s}$ $N = 3 + 4k.$

Case 32/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_A = \frac{6EJ_1\varepsilon t l}{s h N} \qquad M_B = -2M_A; \qquad V_C = -V_A = \frac{M_A}{l};$$

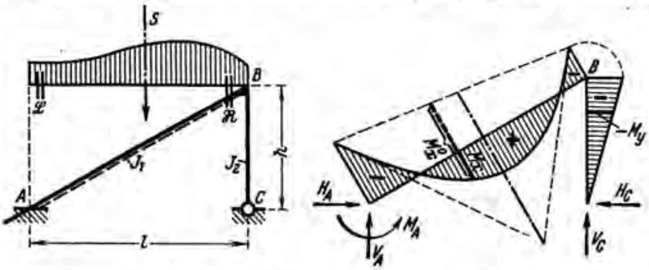
$$H_A = H_C = \frac{-M_B}{h}; \qquad M_x = \frac{x'}{l} M_A + \frac{x}{l} M_B \qquad M_y = \frac{y}{h} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 32

See Appendix A, Load Terms, pp. 440-445.

Case 32/2: Inclined member loaded by any type of vertical load

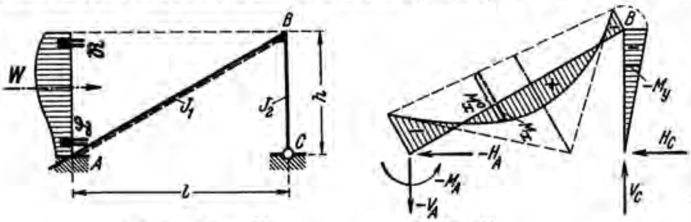


$$M_y = \frac{y}{h} M_B$$

$$M_A = -\frac{2\mathfrak{L}(1+k) - \mathfrak{R}}{N} \qquad M_B = -\frac{2\mathfrak{R} - \mathfrak{L}}{N}; \qquad H_A = H_C = \frac{-M_B}{h};$$

$$V_C = \frac{\mathfrak{S}_l + M_A}{l} \qquad V_A = S - V_C; \qquad M_x = M_x^0 + \frac{x'}{l} M_A + \frac{x}{l} M_B.$$

Case 32/3: Inclined member loaded by any type of horizontal load

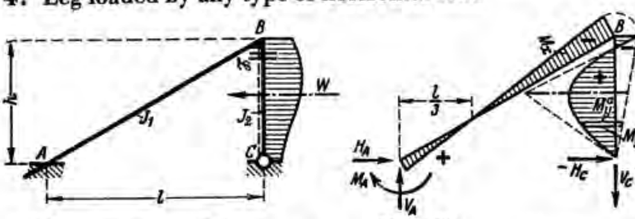


$$M_y = \frac{y}{h} M_B$$

$$M_A = -\frac{2\mathfrak{L}(1+k) - \mathfrak{R}}{N} \qquad M_B = -\frac{2\mathfrak{R} - \mathfrak{L}}{N}; \qquad V_C = -V_A = \frac{\mathfrak{S}_l + M_A}{l};$$

$$H_C = \frac{-M_B}{h} \qquad H_A = -(W - H_C); \qquad M_x = M_x^0 + \frac{x'}{l} M_A + \frac{x}{l} M_B.$$

Case 32/4: Leg loaded by any type of horizontal load

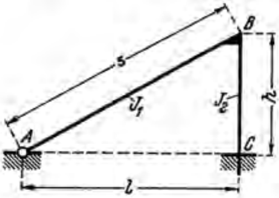


$$M_A = +\frac{\mathfrak{L}k}{N} \qquad M_B = -\frac{2\mathfrak{L}k}{N}; \qquad H_A = \frac{\mathfrak{S}_r - M_B}{h} \qquad H_C = -(W - H_A);$$

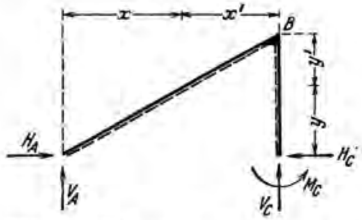
$$V_A = -V_C = \frac{\mathfrak{S}_r - M_A}{l}; \qquad M_x = \frac{x'}{l} M_A + \frac{x}{l} M_B \qquad M_y = M_x^0 + \frac{y}{h} M_B.$$

Frame 33

Triangular rigid frame. One leg vertical, fixed at bottom. Other support hinged. Both supports at the same elevation.



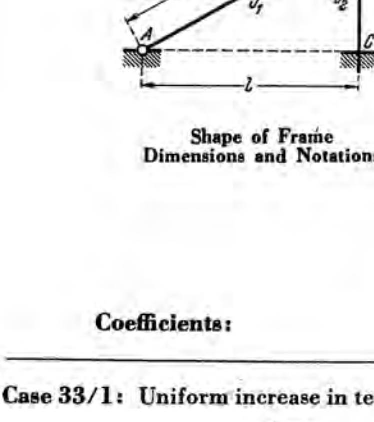
Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_1}{J_2} \cdot \frac{h}{s}$ $N = 4 + 3k.$

Case 33/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $T = \frac{6 E J_1 \varepsilon t l}{s h N}.$

$$M_B = -3T \qquad M_C = +T\frac{3k+2}{k}; \qquad V_A = -V_C = \frac{M_C}{l}$$

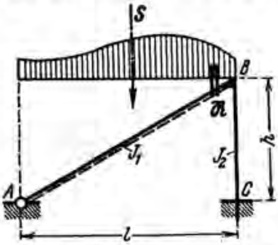
$$H_A = H_C = \frac{M_C - M_B}{h}; \qquad M_x = \frac{x}{l} M_B \qquad M_y = \frac{y}{h} M_B + \frac{y'}{h} M_C.$$

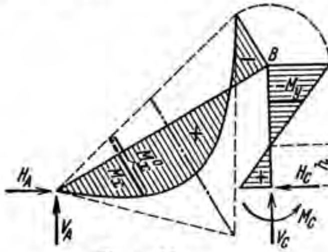
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 33

See Appendix A, Load Terms, pp. 440-445.

Case 33/2: Inclined member loaded by any type of vertical load

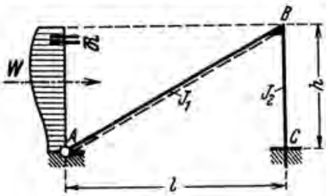


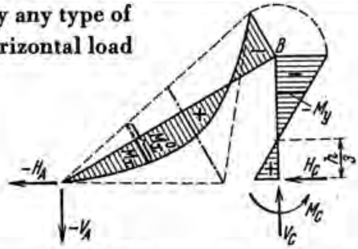


$$M_B = -\frac{2\,\mathfrak{R}}{N} \qquad M_C = +\frac{\mathfrak{R}}{N}\,; \qquad V_A = \frac{\mathfrak{S}_r + M_C}{l} \qquad V_C = S - V_A\,;$$

$$H_A = H_C = \frac{3\,M_C}{h}\,; \qquad M_x = M_x^0 + \frac{x}{l}\,M_B \qquad M_y = \frac{y}{h}\,M_B + \frac{y'}{h}\,M_C\,.$$

Case 33/3: Inclined member loaded by any type of horizontal load

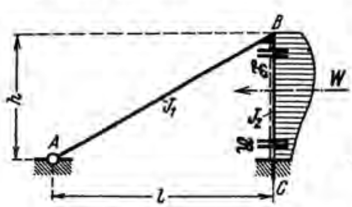


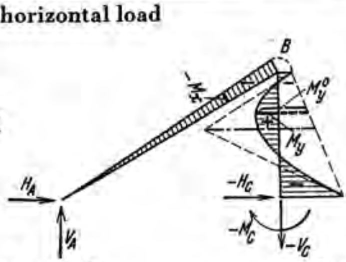


$$M_B = -\frac{2\,\mathfrak{R}}{N} \qquad M_C = +\frac{\mathfrak{R}}{N}\,; \qquad H_C = \frac{M_C - M_B}{h} \qquad H_A = -(W - H_C)\,;$$

$$V_C = -V_A = \frac{\mathfrak{S}_l - M_C}{l}\,; \qquad M_x = M_x^0 + \frac{x}{l}\,M_B \qquad M_y = \frac{y}{h}\,M_B + \frac{y'}{h}\,M_C\,.$$

Case 33/4: Leg loaded by any type of horizontal load



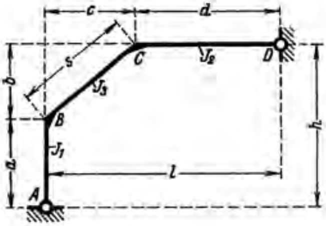


$$M_B = -\frac{(2\,\mathfrak{L} - \mathfrak{R})\,k}{N} \qquad M_C = -\frac{2\,\mathfrak{R}\,(1+k) - \mathfrak{L}\,k}{N}\,; \qquad V_A = -V_C = \frac{\mathfrak{S}_r + M_C}{l}\,.$$

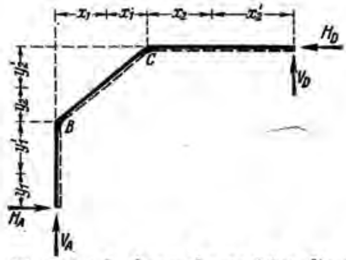
$$H_A = \frac{\mathfrak{S}_r - M_B + M_C}{h} \qquad H_C = -(W - H_A)\,; \qquad M_y = M_y^0 + \frac{y}{h}\,M_B + \frac{y'}{h}\,M_C\,.$$

Frame 34

Single-leg, two-hinged rigid frame. Vertical leg. Horizontal girder. Skew corner.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{a}{s} \qquad \alpha = \frac{a}{h} \qquad \beta = \frac{b}{h} \qquad (\alpha + \beta = 1)$$

$$k_2 = \frac{J_3}{J_2} \cdot \frac{d}{s} \qquad \gamma = \frac{c}{l} \qquad \delta = \frac{d}{l} \qquad (\gamma + \delta = 1);$$

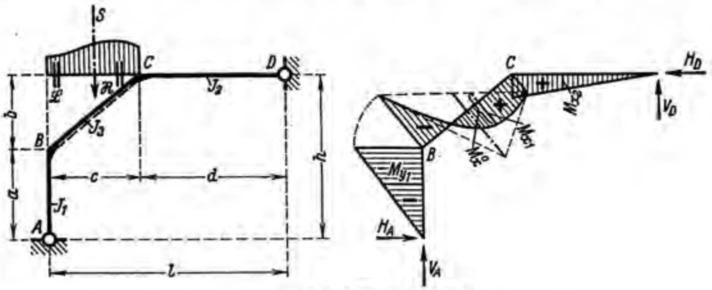
$$B = 2\alpha(k_1 + 1) + \delta \qquad C = \alpha + 2\delta(1 + k_2); \qquad N = \alpha B + C\delta.$$

For the inclined member, the coordinates x are used for the vertical load, y for the horizontal load. Their relationship can be stated as follows: $y_1 : x_1 = y'_1 : x'_1 = b : c$.

FRAME 34

See Appendix A, Load Terms, pp. 440-445.

Case 34/1: Inclined member loaded by any type of vertical load



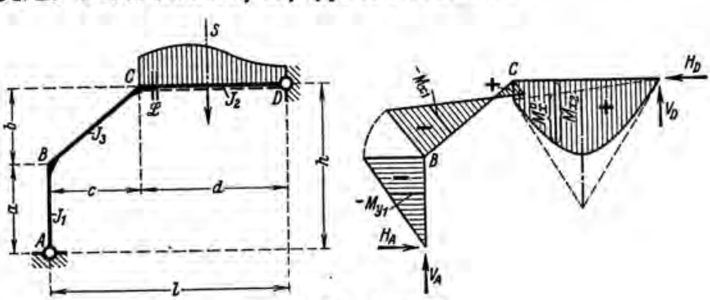
Constant: $$X = \frac{\alpha\,\mathfrak{L} + \delta\,\mathfrak{R} + \delta\,C\,\mathfrak{S}_l}{N}.$$

$$M_B = -\alpha X \qquad M_C = \delta(\mathfrak{S}_l - X);$$

$$V_D = \frac{\mathfrak{S}_l - X}{l} \qquad V_A = S - V_D; \qquad H_A = H_D = \frac{X}{h};$$

$$M_{y1} = \frac{y_1}{a} M_B \qquad M_{x1} = \mathbf{M}_x^0 + \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

Case 34/2: Girder loaded by any type of vertical load



Constant: $$X = \frac{\delta\,\mathfrak{L}\,k_2 + \gamma\,C\,\mathfrak{S}_r}{N}.$$

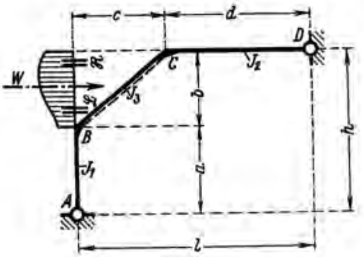
$$M_B = -\alpha X \qquad M_C = \gamma\,\mathfrak{S}_r - \delta X;$$

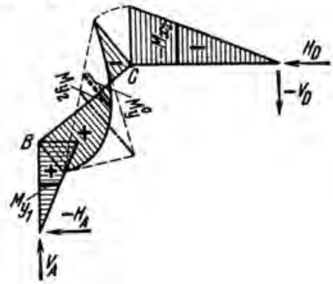
$$V_A = \frac{\mathfrak{S}_r + X}{l} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{X}{h};$$

$$M_{y1} = \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \mathbf{M}_x^0 + \frac{x_2'}{d} M_C.$$

See Appendix A, Load Terms, pp. 440-445.

Case 34/3: Inclined member loaded by any type of horizontal load





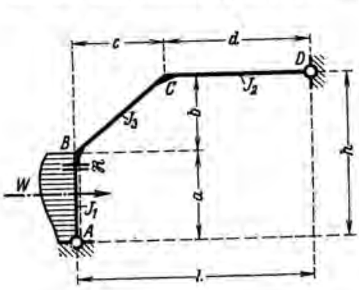
Constant: $$X = \frac{\alpha\,\mathfrak{L} + \delta\,\mathfrak{R} + \alpha\,B\,\mathfrak{S}_r}{N}.$$

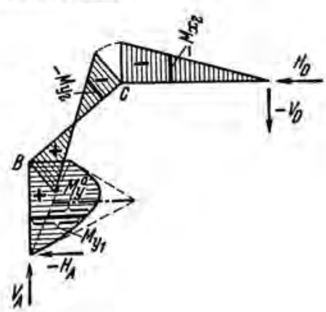
$$M_B = \alpha(\mathfrak{S}_r - X) \qquad M_C = -\delta X;$$

$$H_A = -\frac{\mathfrak{S}_r - X}{h} \qquad H_D = W + H_A; \qquad V_A = -V_D = \frac{X}{l};$$

$$M_{y1} = \frac{y_1}{a} M_B \qquad M_{y2} = M_y^0 + \frac{y_2'}{b} M_B + \frac{y_2}{b} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

Case 34/4: Leg loaded by any type of horizontal load





Constant: $$X = \frac{\alpha\,\mathfrak{R}\,k_1 + \beta\,B\,\mathfrak{S}_l}{N}.$$

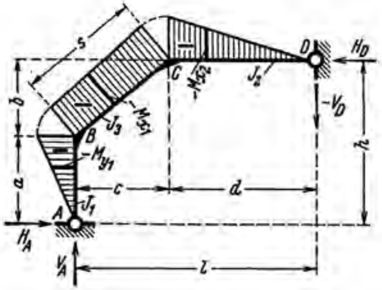
$$M_B = \beta\,\mathfrak{S}_l - \alpha X \qquad M_C = -\delta X;$$

$$H_D = \frac{\mathfrak{S}_l + X}{h} \qquad H_A = -(W - H_D); \qquad V_A = -V_D = \frac{X}{l};$$

$$M_{y1} = M_y^0 + \frac{y_1}{a} M_B \qquad M_{y2} = \frac{y_2'}{b} M_B + \frac{y_2}{b} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

FRAME 34

Case 34/5: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant:

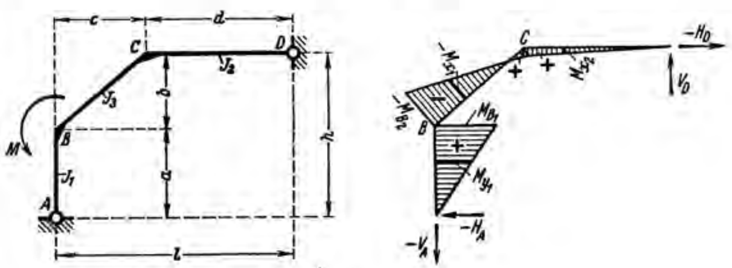
$$X = \frac{6EJ_3\varepsilon t}{sN}\left(\frac{h}{l} + \frac{l}{h}\right).$$

$$M_B = -\alpha X \qquad M_C = -\delta X; \qquad H_A = H_D = \frac{X}{h}; \qquad V_A = -V_D = \frac{X}{l};$$

$$M_{y1} = \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 34/6: The moment acts at joint B



Constant:

$$X = \frac{M}{N}[\alpha(B-2) - \delta].$$

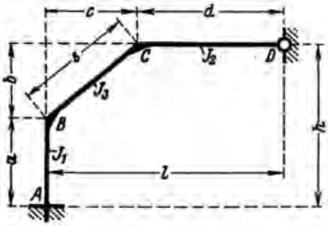
$$H_A = H_D = -\frac{M - X}{h} \qquad V_A = -V_D = \frac{X}{l};$$

$$M_{B1} = \alpha(M - X) \qquad M_{B2} = -M + M_{B1} \qquad M_C = -\delta X;$$

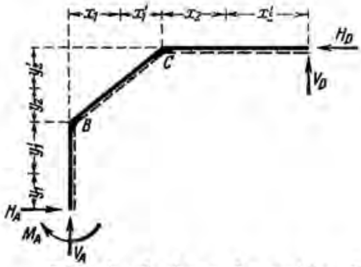
$$M_{y1} = \frac{y_1}{a} M_{B1} \qquad M_{x1} = \frac{x_1'}{c} M_{B2} + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

Frame 35

Single-leg, one-hinged rigid frame. Vertical leg. Horizontal girder, hinged at one end. Skew corner.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k_1 = \frac{J_3}{J_1} \cdot \frac{a}{s}$ $\qquad k_2 = \frac{J_3}{J_2} \cdot \frac{d}{s}$;

$\alpha = \frac{a}{h}$ $\qquad \beta = 1 - \alpha$ $\qquad \delta = \frac{d}{l}$ $\qquad \gamma = 1 - \delta$;

$$B_1 = 3k_1 + 2 + \delta \qquad C_1 = 1 + 2\delta(1 + k_2)$$

$$B_2 = 2\alpha(k_1 + 1) + \delta \qquad C_2 = \alpha + 2\delta(1 + k_2) ;$$

$$R_1 = 3k_1 + B_1 + \delta C_1 \qquad R_2 = \alpha B_2 + \delta C_2 \qquad K = \alpha B_1 + \delta C_1 ;$$

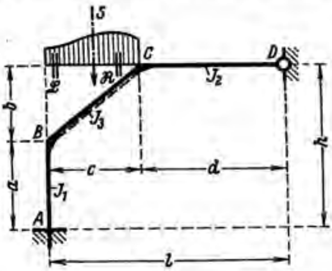
$$N = R_1 R_2 - K^2 ;$$

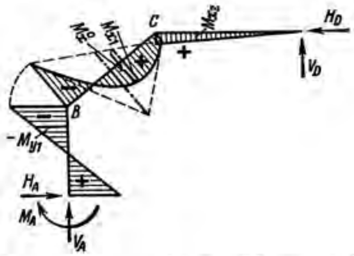
$$n_{11} = \frac{R_2}{N} \qquad n_{12} = n_{21} = \frac{K}{N} \qquad n_{22} = \frac{R_1}{N} .$$

FRAME 35

See Appendix A, Load Terms, pp. 440-445.

Case 35/1: Inclined member loaded by any type of vertical load

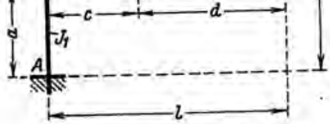


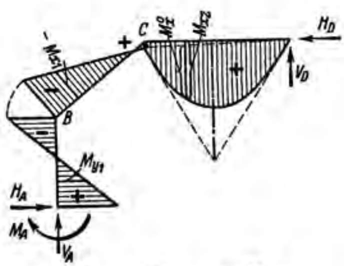


Constant:
$$\mathfrak{B}_1 = \delta C_1 \mathfrak{S}_l + \mathfrak{L} + \delta \mathfrak{R} \qquad X_1 = -\mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = \delta C_2 \mathfrak{S}_l + \alpha \mathfrak{L} + \delta \mathfrak{R}; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

$$M_A = X_1 \qquad M_B = X_1 - \alpha X_2 \qquad M_C = \delta(\mathfrak{S}_l + X_1 - X_2);$$
$$V_D = \frac{\mathfrak{S}_l + X_1 - X_2}{l} \qquad V_A = S - V_D; \qquad H_A = H_D = \frac{X_2}{h};$$
$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{x1} = M_x^0 + \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

Case 35/2: Girder loaded by any type of vertical load

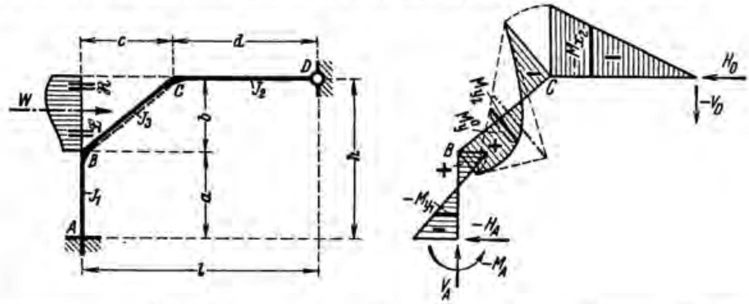




Constant:
$$\mathfrak{B}_1 = \gamma C_1 \mathfrak{S}_r + \delta \mathfrak{L} k_2 \qquad X_1 = -\mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = \gamma C_2 \mathfrak{S}_r + \delta \mathfrak{L} k_2; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$
$$M_A = X_1 \qquad M_B = X_1 - \alpha X_2 \qquad M_C = \gamma \mathfrak{S}_r + \delta(X_1 - X_2);$$
$$V_A = \frac{\mathfrak{S}_r - X_1 + X_2}{l} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{X_2}{h};$$
$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = M_x^0 + \frac{x_2'}{d} M_C.$$

See Appendix A, Load Terms, pp. 440-445.

Case 35/3: Inclined member loaded by any type of horizontal load



Constants:

$$\mathfrak{B}_1 = \delta C_1 \mathfrak{S}_r - (\mathfrak{L} + \delta \mathfrak{R}) \qquad X_1 = -\mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21}$$

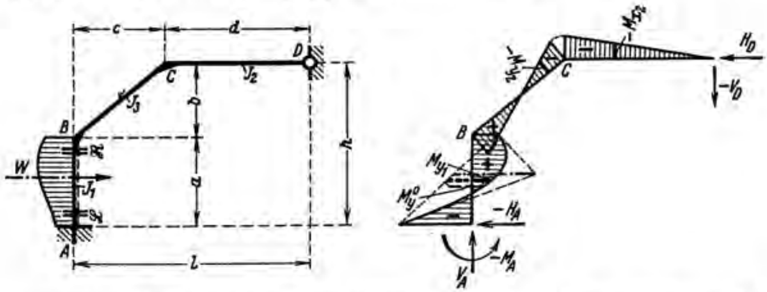
$$\mathfrak{B}_2 = \delta C_2 \mathfrak{S}_r - (\alpha \mathfrak{L} + \delta \mathfrak{R}); \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

$$M_A = -X_1 \qquad M_B = \alpha X_2 - X_1 \qquad M_C = -\delta(\mathfrak{S}_r + X_1 - X_2);$$

$$H_A = -\frac{X_2}{h} \qquad H_D = W + H_A; \qquad V_A = -V_D = \frac{\mathfrak{S}_r + X_1 - X_2}{l};$$

$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{y2} = \mathbf{M}_y^0 + \frac{y_2'}{b} M_B + \frac{y_2}{b} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

Case 35/4: Leg loaded by any type of horizontal load



Constant:

$$\mathfrak{B}_1 = (B_1 + \delta C_1)\mathfrak{S}_l + (\mathfrak{L} + \mathfrak{R}) k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

$$\mathfrak{B}_2 = (B_2 + \delta C_2)\mathfrak{S}_l + \alpha \mathfrak{R} k_1; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

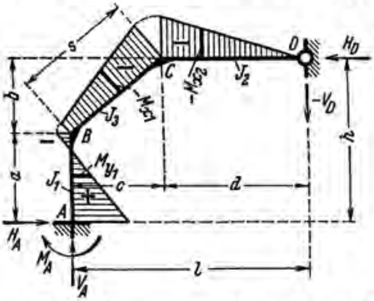
$$M_A = -X_1 \qquad M_B = \mathfrak{S}_l - X_1 - \alpha X_2 \qquad M_C = \delta(\mathfrak{S}_l - X_1 - X_2);$$

$$H_D = \frac{X_2}{h} \qquad H_A = -(W - H_D); \qquad V_A = -V_D = -\frac{\mathfrak{S}_l - X_1 - X_2}{l};$$

$$M_{y1} = \mathbf{M}_y^0 + \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{y2} = \frac{y_2'}{b} M_B + \frac{y_2}{b} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

FRAME 35

Case 35/5: Uniform increase in temperature of the entire frame



$E =$ Modulus of elasticity
$\varepsilon =$ Coefficient of thermal expansion
$t =$ Change of temperature in degrees

Constants:

$$T = \frac{6 E J_3 \varepsilon t h}{s l} \qquad \lambda = \frac{l^2 + h^2}{h^2};$$

$$X_1 = T(-n_{11} + \lambda n_{21})$$

$$X_1 = T(-n_{12} + \lambda n_{22}).$$

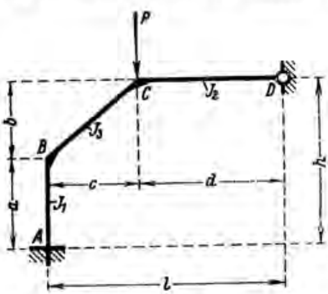
$$M_A = X_1 \qquad M_B = X_1 - \alpha X_2 \qquad M_C = -\delta(X_2 - X_1);$$

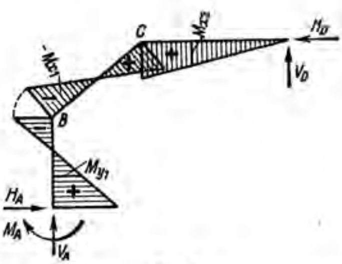
$$H_A = H_D = \frac{X_2}{h}; \qquad V_A = -V_D = \frac{X_2 - X_1}{l};$$

$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 35/6: Vertical concentrated load at joint C





Constants: $\quad X_1 = \frac{P c d}{l}(-C_1 n_{11} + C_2 n_{21}) \qquad X_2 = \frac{P c d}{l}(-C_1 n_{12} + C_2 n_{22});$

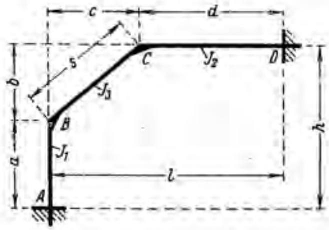
$$M_A = X_1 \qquad M_B = X_1 - \alpha X_2 \qquad M_C = \frac{P c d}{l} - \delta(X_2 - X_1);$$

$$V_A = \delta P + \frac{X_2 - X_1}{l} \qquad V_D = P - V_A; \qquad H_A = H_D = \frac{X_2}{h};$$

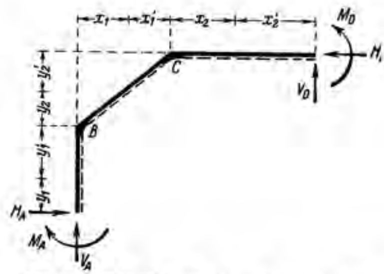
$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C.$$

Frame 36

Single-leg, one-hinged rigid frame. Vertical leg, hinged at bottom. Horizontal girder. Skew corner.



Shape of Frame Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k_1 = \frac{J_3}{J_1} \cdot \frac{a}{s}$ $\qquad k_2 = \frac{J_3}{J_2} \cdot \frac{d}{s};$

$$\alpha = \frac{a}{h} \qquad \beta = 1 - \alpha \qquad \delta = \frac{d}{l} \qquad \gamma = 1 - \delta;$$

$$B_1 = 3k_1 + 2 + \delta \qquad B_3 = 2\alpha(k_1 + 1) + \delta;$$

$$C_1 = 1 + 2\delta(1 + k_2) \qquad C_2 = 2\gamma(1 + k_2) + k_2 \qquad C_3 = \alpha + 2\delta(1 + k_2)$$

$$R_1 = 3k_1 + B_1 + \delta C_1 \qquad K_1 = \alpha\gamma + \delta C_2$$

$$R_2 = (\gamma + 2)k_2 + \gamma C_2 \qquad K_2 = \alpha B_1 + \delta C_1$$

$$R_3 = \alpha B_3 + \delta C_3 \qquad K_3 = \gamma + \delta C_2;$$

$$N = R_1 R_2 R_3 + 2K_1 K_2 K_3 - R_1 K_1^2 - R_2 K_2^2 - R_3 K_3^2;$$

$$n_{11} = \frac{R_2 R_3 - K_1^2}{N} \qquad n_{12} = n_{21} = \frac{K_1 K_2 - R_3 K_3}{N}$$

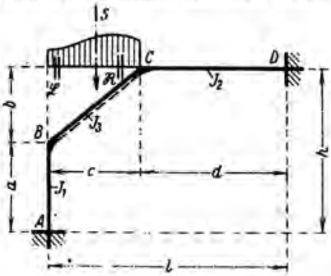
$$n_{22} = \frac{R_1 R_3 - K_2^2}{N} \qquad n_{13} = n_{31} = \frac{R_2 K_2 - K_1 K_3}{N}$$

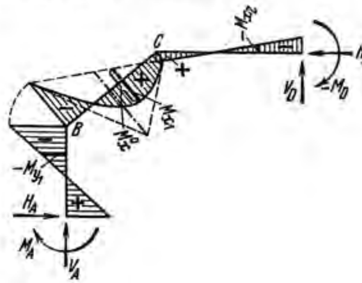
$$n_{33} = \frac{R_1 R_2 - K_3^2}{N} \qquad n_{23} = n_{32} = \frac{R_1 K_1 - K_2 K_3}{N}$$

FRAME 36

See Appendix A, Load Terms, pp. 440-445.

Case 36/1: Inclined member loaded by any type of vertical load



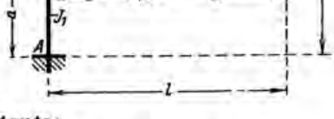


Constants:

$$\mathfrak{B}_1=\delta C_1\mathfrak{S}_l+\mathfrak{L}+\delta\mathfrak{R} \qquad X_1=-\mathfrak{B}_1 n_{11}-\mathfrak{B}_2 n_{21}+\mathfrak{B}_3 n_{31}$$
$$\mathfrak{B}_2=\delta C_2\mathfrak{S}_l+\gamma\mathfrak{R} \qquad X_2=+\mathfrak{B}_1 n_{12}+\mathfrak{B}_2 n_{22}-\mathfrak{B}_3 n_{32}$$
$$\mathfrak{B}_3=\delta C_3\mathfrak{S}_l+\alpha\mathfrak{L}+\delta\mathfrak{R}; \qquad X_3=-\mathfrak{B}_1 n_{13}-\mathfrak{B}_2 n_{23}+\mathfrak{B}_3 n_{33}.$$
$$M_A=X_1 \qquad M_C=\delta(\mathfrak{S}_l+X_1-X_3)-\gamma X_2$$
$$M_B=X_1-\alpha X_3 \qquad M_D=-X_2;$$
$$V_D=\frac{\mathfrak{S}_l+X_1+X_2-X_3}{l} \qquad V_A=S-V_D; \qquad H_A=H_D=\frac{X_3}{h}.$$

Formulas for M_y and M_x same as for case 36/5. For M_{x1} add M_x^0 to these.

Case 36/2: Girder loaded by any type of vertical load



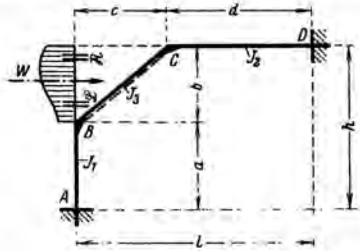
Constants:

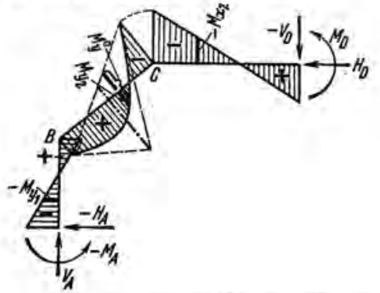
$$\mathfrak{B}_1=\gamma C_1\mathfrak{S}_r+\delta\mathfrak{L}k_2 \qquad X_1=-\mathfrak{B}_1 n_{11}-\mathfrak{B}_2 n_{21}+\mathfrak{B}_3 n_{31}$$
$$\mathfrak{B}_2=\gamma C_2\mathfrak{S}_r+(\gamma\mathfrak{L}+\mathfrak{R})k_2 \qquad X_2=+\mathfrak{B}_1 n_{12}+\mathfrak{B}_2 n_{22}-\mathfrak{B}_3 n_{32}$$
$$\mathfrak{B}_3=\gamma C_3\mathfrak{S}_r+\delta\mathfrak{L}k_2; \qquad X_3=-\mathfrak{B}_1 n_{13}-\mathfrak{B}_2 n_{23}+\mathfrak{B}_3 n_{33}.$$
$$M_A=X_1 \qquad M_C=\gamma(\mathfrak{S}_r-X_2)+\delta(X_1-X_3)$$
$$M_B=X_1-\alpha X_3 \qquad M_D=-X_2;$$
$$V_A=\frac{\mathfrak{S}_r-X_1-X_2+X_3}{l} \qquad V_D=S-V_A; \qquad H_A=H_D=\frac{X_3}{h}.$$

Formulas for M_y and M_x same as for case 36/5. For M_{x2} add M_x^0 to these.

See Appendix A, Load Terms, pp. 440-445.

Case 36/3: Inclined member loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_1 = \delta C_1 \mathfrak{S}_r - (\mathfrak{L} + \delta \mathfrak{R})$$
$$\mathfrak{B}_2 = \delta C_2 \mathfrak{S}_r - \gamma \mathfrak{R}$$
$$\mathfrak{B}_3 = \delta C_3 \mathfrak{S}_r - (\alpha \mathfrak{L} + \delta \mathfrak{R});$$

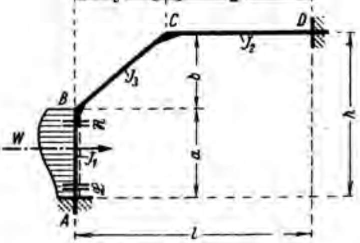
$$X_1 = -\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$
$$X_2 = +\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} - \mathfrak{B}_3 n_{32}$$
$$X_3 = -\mathfrak{B}_1 n_{13} - \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

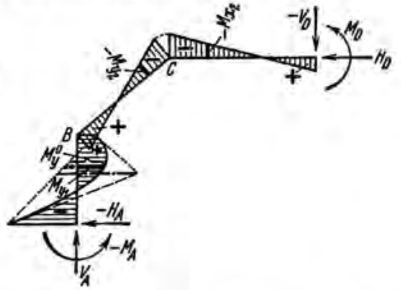
$$M_A = -X_1 \qquad M_C = -\delta(\mathfrak{S}_r + X_1 - X_3) + \gamma X_2$$
$$M_B = \alpha X_3 - X_1 \qquad M_D = X_2;$$

$$H_A = -\frac{X_3}{h} \qquad H_D = W + H_A; \qquad V_A = -V_D = \frac{\mathfrak{S}_r + X_1 + X_2 - X_3}{l}.$$

Formulas for M_y and M_z same as for case 36/5. For M_{y2} add M_y^0 to these.

Case 36/4: Leg loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_1 = (B_1 + \delta C_1)\mathfrak{S}_l + (\mathfrak{L} + \mathfrak{R}) k_1$$
$$\mathfrak{B}_2 = (\gamma + \delta C_2)\mathfrak{S}_l$$
$$\mathfrak{B}_3 = (B_3 + \delta C_3)\mathfrak{S}_l + \alpha \mathfrak{R} k_1;$$

$$X_1 = +\mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21} - \mathfrak{B}_3 n_{31}$$
$$X_2 = -\mathfrak{B}_1 n_{12} - \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$
$$X_3 = -\mathfrak{B}_1 n_{13} - \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

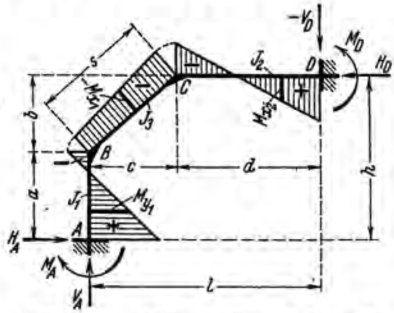
$$M_A = -X_1 \qquad M_C = \delta(\mathfrak{S}_l - X_1 - X_3) + \gamma X_2$$
$$M_B = \mathfrak{S}_l - X_1 - \alpha X_3 \qquad M_D = X_2;$$

$$V_A = -V_D = \frac{X_1 + X_2 + X_3 - \mathfrak{S}_l}{l}; \qquad H_D = \frac{X_3}{h} \qquad H_A = -(W - H_D).$$

Formulas for M_y and M_z same as for case 36/5. For M_{y1} add M_y^0 to these.

FRAME 36

Case 36/5: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$T=\frac{6EJ_3\varepsilon t h}{s l} \qquad \lambda=\frac{l^2+h^2}{h^2};$$

$$X_1=T(-n_{11}+n_{21}+\lambda n_{31})$$
$$X_2=T(-n_{12}+n_{22}+\lambda n_{32})$$
$$X_3=T(-n_{13}+n_{23}+\lambda n_{33}).$$

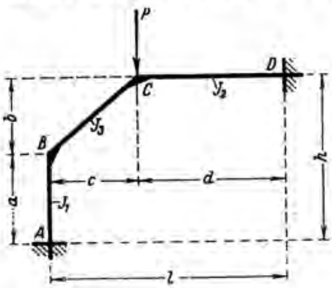
$$M_A=X_1 \qquad M_D=X_2 \qquad M_B=X_1-\alpha X_3 \qquad M_C=\delta(X_1-X_3)+\gamma X_2;$$

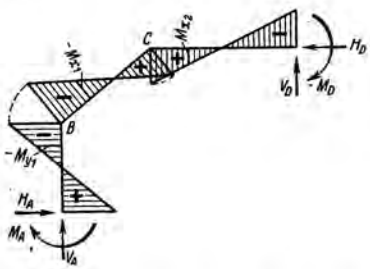
$$H_A=H_D=\frac{X_3}{h}; \qquad V_A=-V_D=\frac{X_3+X_2-X_1}{l};$$

$$M_{y1}=\frac{y_1'}{a}M_A+\frac{y_1}{a}M_B \qquad M_{x1}=\frac{x_1'}{c}M_B+\frac{x_1}{c}M_C \qquad M_{x2}=\frac{x_2'}{d}M_C+\frac{x_2}{d}M_D.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 36/6: Vertical concentrated load at joint C





Constants:

$$X_1=\frac{Pcd}{l}(-C_1n_{11}-C_2n_{21}+C_3n_{31}) \qquad M_A=X_1 \qquad M_D=-X_2$$

$$X_2=\frac{Pcd}{l}(+C_1n_{12}+C_2n_{22}-C_3n_{32}) \qquad M_B=X_1-\alpha X_3$$

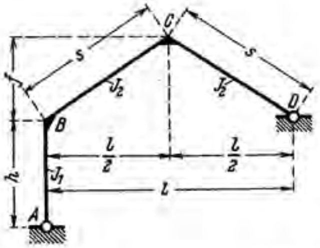
$$X_3=\frac{Pcd}{l}(-C_1n_{13}-C_2n_{23}+C_3n_{33}). \qquad M_C=\frac{Pcd}{l}+\delta(X_1-X_3)-\gamma X_2;$$

$$V_A=\delta P+\frac{X_3-X_2-X_1}{l} \qquad V_D=P-V_A; \qquad H_A=H_D=\frac{X_3}{h}$$

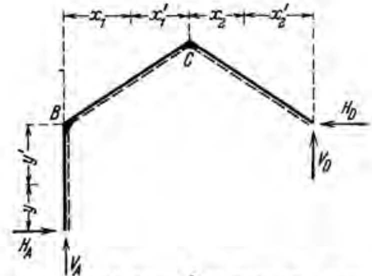
Formulas for M_y and M_x as above.

Frame 37

Single-leg, two-hinged gable frame.



Shape of Frame
Dimensions and Notations

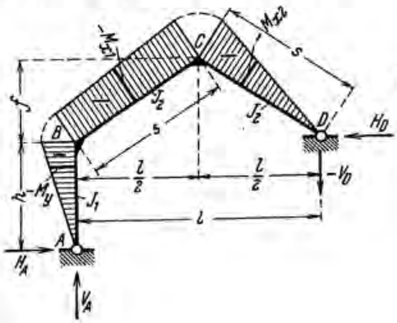


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{s}$; $\varphi = \frac{f}{h}$ $\gamma = \frac{1}{2} + \varphi$;

$B = 2k + \frac{5}{2} + \varphi$ $C = \frac{3}{2} + 2\varphi$; $N = B + 2\gamma C$.

Case 37/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant:

$$X = \frac{6 E J_2 \varepsilon t}{s N}\left(\frac{h}{l} + \frac{l}{h}\right).$$

$M_B = -X$ $M_C = -\gamma X$; $V_C = -V_D = \frac{X}{l}$; $H_A = H_D = \frac{X}{h}$;

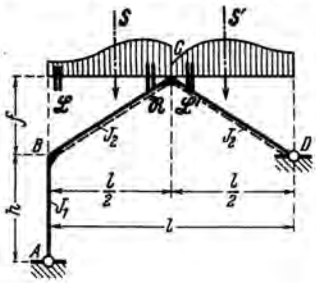
$M_y = \frac{y}{h} M_B$ $M_{x1} = \frac{2x_1'}{l} M_B + \frac{2x_1}{l} M_C$ $M_{x2} = \frac{2x_2'}{l} M_C$.

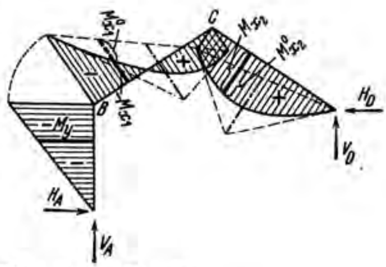
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 37

See Appendix A, Load Terms, pp. 440-445.

Case 37/2: Both halves of the girder loaded by any type of vertical load





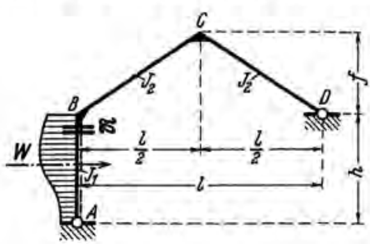
Constant: $$X=\frac{C\,(\mathfrak{S}_l+\mathfrak{S}_r')+\mathfrak{L}+\gamma\,(\mathfrak{R}+\mathfrak{L}')}{N}.$$

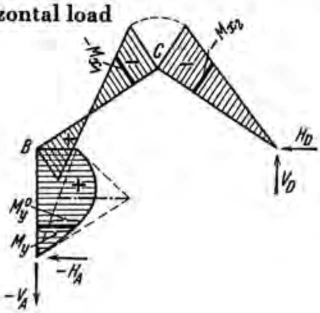
$$M_B=-X \qquad M_C=\frac{\mathfrak{S}_l+\mathfrak{S}_r'}{2}-\gamma X; \qquad H_A=H_D=\frac{X}{h};$$

$$V_A=\left(S-\frac{\mathfrak{S}_l}{l}\right)+\frac{\mathfrak{S}_r'}{l}+\frac{X}{l} \qquad V_D=\frac{\mathfrak{S}_l}{l}+\left(S'-\frac{\mathfrak{S}_r'}{l}\right)-\frac{X}{l};$$

$$M_y=\frac{y}{h}M_B \qquad M_{x1}=M_{x1}^0+\frac{2x_1'}{l}M_B+\frac{2x_1}{l}M_C \qquad M_{x2}=M_{x2}^0+\frac{2x_2'}{l}M_C.$$

Case 37/3: Leg loaded by any type of horizontal load





Constant: $$X=\frac{(B+C)\,\mathfrak{S}_l+\mathfrak{R}k}{N}.$$

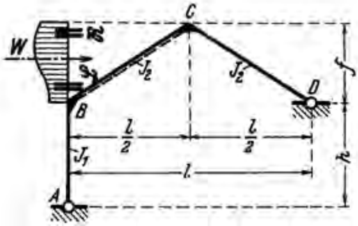
$$M_B=\mathfrak{S}_l-X \qquad M_C=\frac{\mathfrak{S}_l}{2}-\gamma X;$$

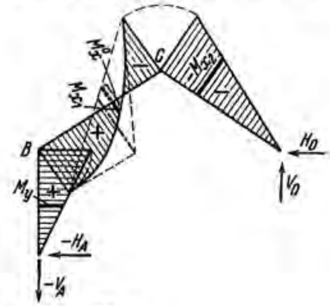
$$V_D=-V_A=\frac{\mathfrak{S}_l-X}{l}; \qquad H_D=\frac{X}{h} \qquad H_A=-(W-H_D);$$

$$M_y=M_y^0+\frac{y}{h}M_B \qquad M_{x1}=\frac{2x_1'}{l}M_B+\frac{2x_1}{l}M_C \qquad M_{x2}=\frac{2x_2'}{l}M_C.$$

See Appendix A, Load Terms, pp. 440-445.

Case 37/4: Left-half of the girder loaded by any type of horizontal load





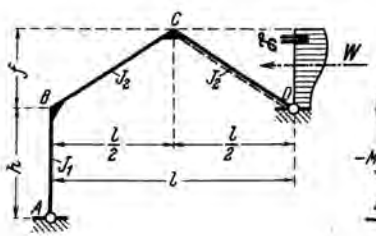
Constant:
$$X = \frac{C(\mathfrak{S}_l + 2\,\mathfrak{S}_r) - \mathfrak{L} - \gamma\,\mathfrak{R}}{N}.$$

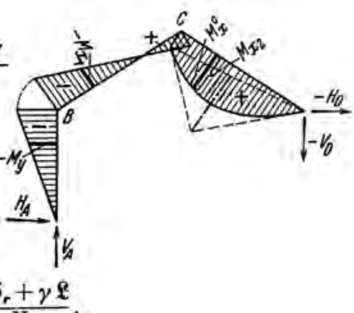
$$M_B = +X \qquad M_C = -\frac{\mathfrak{S}_l + 2\,\mathfrak{S}_r}{2} + \gamma X;$$

$$V_D = -V_A = \frac{\mathfrak{S}_l + X}{l}; \qquad H_A = -\frac{X}{h} \qquad H_D = W + H_A;$$

$$M_y = \frac{y}{h} M_B \qquad M_{x1} = M_x^0 + \frac{2\,x_1'}{l} M_B + \frac{2\,x_1}{l} M_C \qquad M_{x2} = \frac{2\,x_2'}{l} M_C.$$

Case 37/5: Right-half of the girder loaded by any type of horizontal load





Constant:
$$X = \frac{C\,\mathfrak{S}_r + \gamma\,\mathfrak{L}}{N}.$$

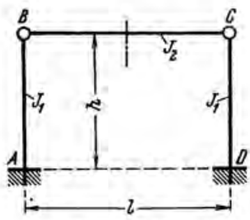
$$M_B = -X \qquad M_C = \frac{\mathfrak{S}_r}{2} - \gamma X;$$

$$V_A = -V_D = \frac{\mathfrak{S}_r + X}{l}; \qquad H_A = \frac{X}{h} \qquad H_D = -(W - H_A);$$

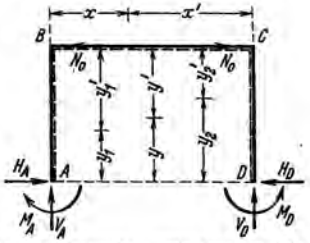
$$M_y = \frac{y}{h} M_B \qquad M_{x1} = \frac{2\,x_1'}{l} M_B + \frac{2\,x_1}{l} M_C \qquad M_{x2} = M_x^0 + \frac{2\,x_2'}{l} M_C.$$

FRAME 38

Symmetrical fixed rectangular frame with hinged girder

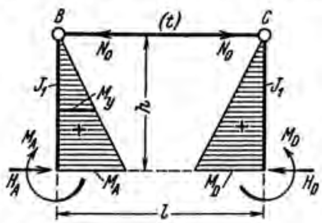


Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions, and the axial forces in the girder.[1]

Case 38/1: Uniform increase in temperature of the girder by t degrees[2]



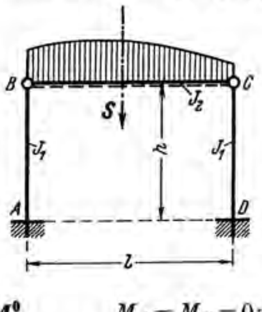
E = Modulus of elasticity
ε = Coefficient of thermal expansion

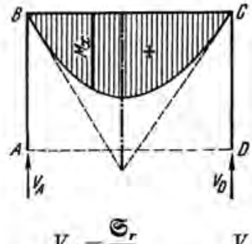
$$M_A = M_D = \frac{3 E J_1 l \varepsilon t}{2 h^2};$$

$$M_y = \frac{y'}{h} M_A; \quad H_A = H_D = N_0 = \frac{M_A}{h}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 38/2: Girder loaded by any type of load





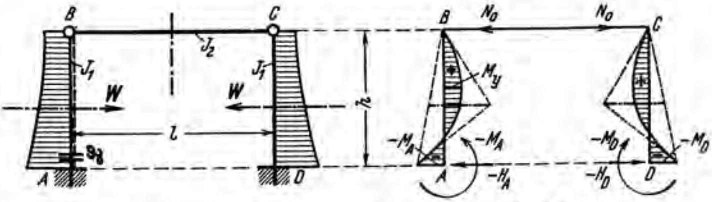
$$M_x = M_x^0 \qquad M_A = M_D = 0; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l}.$$

[1] Positive bending moments M cause tension at the face marked by a dashed line. Positive axial forces N produce compression.

[2] Temperature change in the members has no static influence.

See Appendix A, Load Terms, pp. 440-445.

Case 38/3: Both legs loaded by any type of external symmetrical load

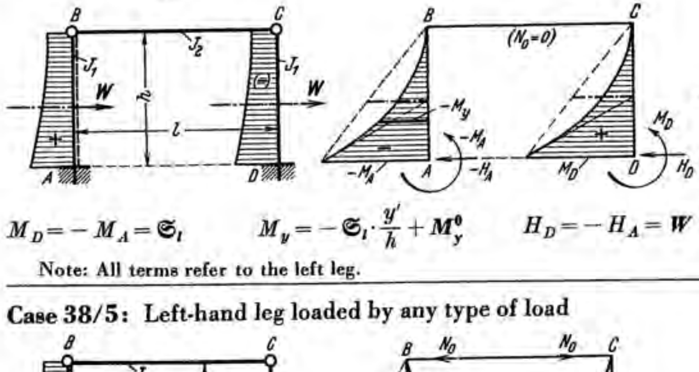


$$M_A = M_D = -\frac{\mathfrak{L}}{2} \qquad M_y = \mathbf{M}_y^0 - \frac{y'}{h}\cdot\frac{\mathfrak{L}}{2};$$

$$H_A = H_D = -\frac{\mathfrak{S}_r}{h} - \frac{\mathfrak{L}}{2h}; \qquad N_0 = W + H_A; \qquad V_A = V_D = 0.$$

Note: All terms refer to the left leg.

Case 38/4: Both legs loaded by any type of antisymmetrical load from the left

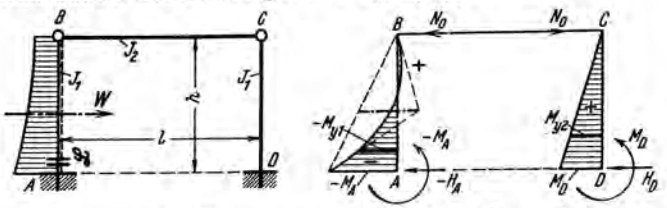


$$V_A = V_D = 0;$$

$$M_D = -M_A = \mathfrak{S}_l \qquad M_y = -\mathfrak{S}_l\cdot\frac{y'}{h} + \mathbf{M}_y^0 \qquad H_D = -H_A = W \qquad N_0 = 0.$$

Note: All terms refer to the left leg.

Case 38/5: Left-hand leg loaded by any type of load



$$V_A = V_D = 0;$$

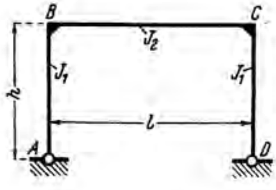
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -\frac{\mathfrak{L}}{4} \mp \frac{\mathfrak{S}_l}{2} \qquad H_D = N_0 = \frac{M_D}{h} = \frac{\mathfrak{S}_l}{2h} - \frac{\mathfrak{L}}{4h} \qquad H_A = -W + H_D.$$

Special case 38/5: Rectangular load $W = qh$

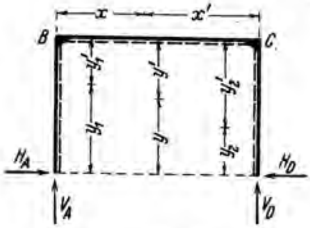
$$M_A = -\frac{5qh^2}{16} \qquad M_D = +\frac{3qh^2}{16}; \qquad H_A = -\frac{13qh}{16} \qquad H_D = N_0 = \frac{3qh}{16}.$$

Frame 39

Symmetrical rectangular two-hinged frame



Shape of Frame
Dimensions and Notations

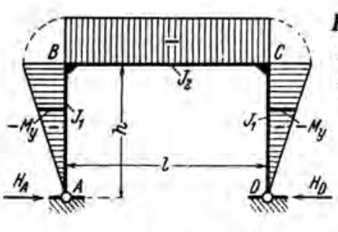


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{h}{l} \qquad N = 2k + 3.$$

Case 39/1: Uniform increase in temperature of the entire frame



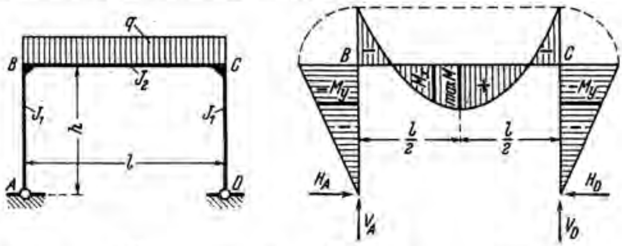
E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$M_B = M_C = -\frac{3 E J_2 \varepsilon t}{h N};$$

$$H_A = H_D = \frac{-M_B}{h}; \quad M_y = \frac{y}{h} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

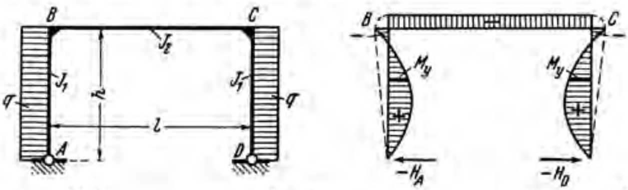
Case 39/2: Rectangular load on the girder



$$M_B = M_C = -\frac{q l^2}{4N} \qquad \max M = \frac{q l^2}{8} + M_B \qquad M_x = \frac{q x x'}{2} + M_B;$$

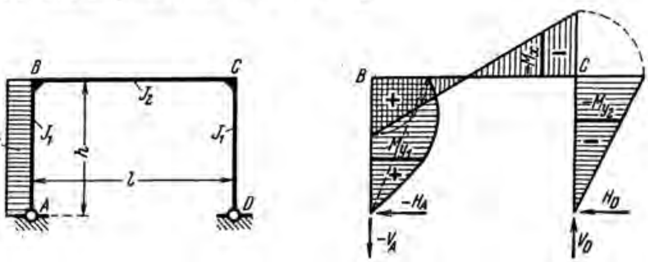
$$V_A = V_D = \frac{q l}{2} \qquad H_A = H_D = \frac{-M_B}{h}; \qquad M_y = \frac{y}{h} M_B.$$

Case 39/3: Rectangular load on both legs



$$M_B = M_C = -\frac{q h^2 k}{4N} \qquad M_y = \frac{q y y'}{2} + \frac{y}{h} M_B; \qquad H_A = H_D = -\left(\frac{q h}{2} + \frac{M_B}{h}\right).$$

Case 39/4: Rectangular load on the left leg



$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\} = \frac{q h^2}{4}\left[-\frac{k}{2N} \pm 1\right];$$

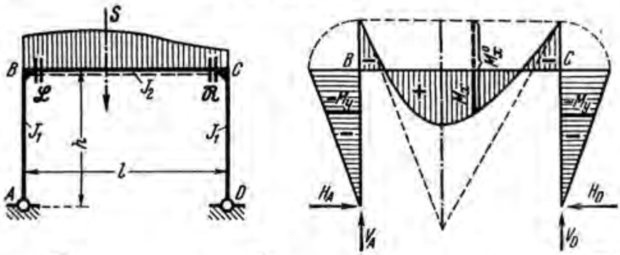
$$H_D = \frac{-M_C}{h} \qquad H_A = -(q h - H_D); \qquad V_D = -V_A = \frac{q h^2}{2 l};$$

$$M_{y1} = \frac{q y_1 y_1'}{2} + \frac{y_1}{h} M_B \qquad M_x = M_C + V_D x' \qquad M_{y2} = -H_D y_2.$$

FRAME 39

See Appendix A, Load Terms, pp. 440-445.

Case 39/5: Girder loaded by any type of vertical load



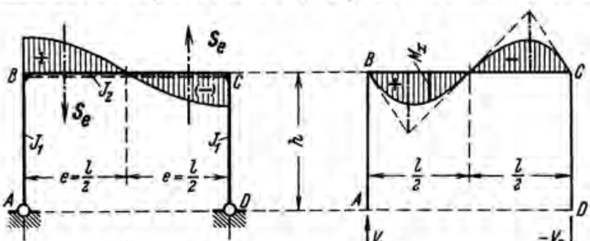
$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l}; \qquad M_B = M_C = -\frac{(\mathfrak{L} + \mathfrak{R})}{2N};$$

$$H_A = H_D = \frac{-M_B}{h}; \qquad M_x = M_x^0 + M_B \qquad M_y = \frac{y}{h} M_B .$$

Special case 39/5a: Symmetrical load ($\mathfrak{R} = \mathfrak{L}$)

$$V_A = V_D = S/2; \qquad M_B = M_C = -\mathfrak{L}/N .$$

Case 39/6: Girder loaded by any type of antisymmetrical load ($\mathfrak{R} = -\mathfrak{L}$)

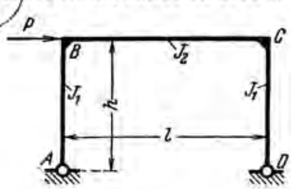


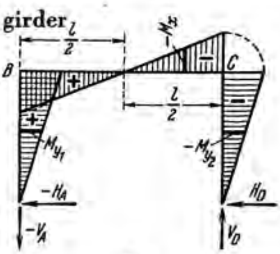
$$V_A = -V_D = \frac{\mathfrak{S}_r}{l};$$

$$M_B = M_C = 0;$$

$$H_A = H_D = 0 .$$

Case 39/7: Horizontal concentrated load at the girder

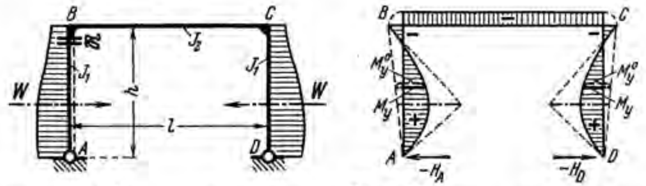




$$V_D = -V_A = \frac{Ph}{l}; \qquad H_D = -H_A = \frac{P}{2};$$

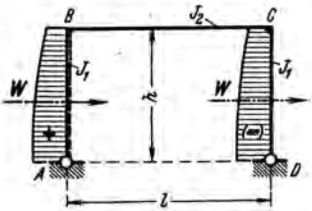
$$M_B = -M_C = +\frac{Ph}{2}; \qquad M_x = Ph\left(\frac{1}{2} - \frac{x}{l}\right) \qquad M_{y1} = -M_{y2} = \frac{P}{2} y .$$

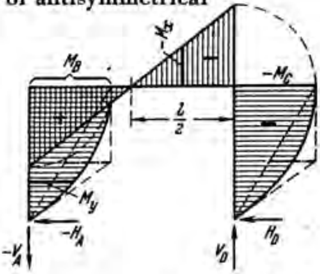
Case 39/8: Both legs loaded by any type of external symmetrical load*



$$M_B = M_C = -\frac{\mathfrak{R}'k}{N} \qquad H_A = H_D = -\frac{\mathfrak{S}_r + M_B}{h} \qquad M_y = \mathbf{M}_y^0 + \frac{y}{h} M_B .$$

Case 39/9: Both legs loaded by any type of antisymmetrical load from the left*

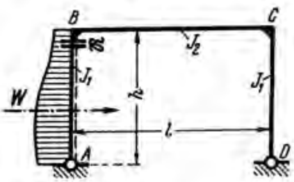


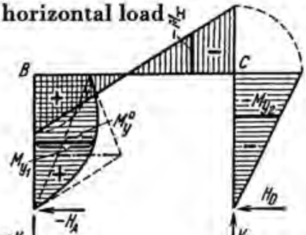


$$M_B = -M_C = +\mathfrak{S}_l ; \qquad M_y = \mathbf{M}_y^0 + \frac{y}{h}\mathfrak{S}_l \qquad M_x = \mathfrak{S}_l \cdot \frac{x' - x}{l} ;$$

$$V_D = -V_A = \frac{2\mathfrak{S}_l}{l} \qquad H_D = -H_A = W .$$

Case 39/10: Left leg loaded by any type of horizontal load*





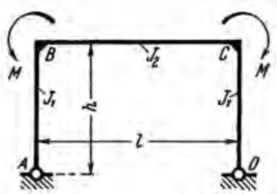
$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -\frac{\mathfrak{R}\,k}{2N} \pm \frac{\mathfrak{S}_l}{2} \qquad (M_B - M_C = \mathfrak{S}_l) ; \qquad V_D = -V_A = \frac{\mathfrak{S}_l}{l} ;$$

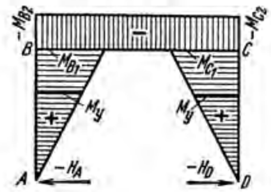
$$H_D = \frac{-M_C}{h} \qquad H_A = -(W - H_D) ;$$

$$M_x = M_B - \frac{x}{l}\mathfrak{S}_l \qquad M_{y1} = \mathbf{M}_y^0 + \frac{y_1}{h} M_B \qquad M_{y2} = \frac{y_2}{h} M_C .$$

*Note: All the load terms refer to the left leg.

Case 39/11: Symmetrical moments acting at the corners

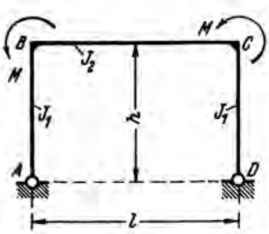


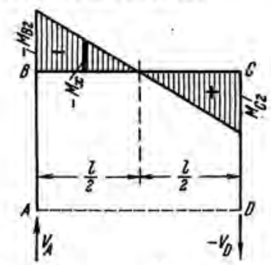


$$M_{B1} = M_{C1} = +\frac{3\,M}{N} \qquad M_{B2} = M_{C2} = -\frac{2\,M\,k}{N};$$

$$H_A = H_D = -\frac{M_{B1}}{h}; \qquad M_y = \frac{y}{h}\,M_{B1} \qquad (M_{B1} - M_{B2} = M).$$

Case 39/12: Antisymmetrical moments acting at the corners

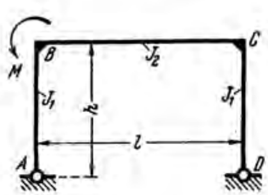


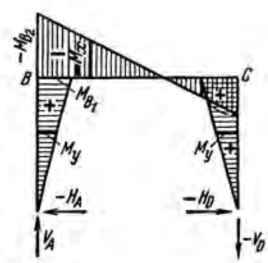


$$M_{C2} = -M_{B2} = M \qquad M_{B1} = M_{C1} = 0; \qquad H_A = H_D = 0;$$

$$M_x = M \cdot \frac{x - x'}{l} = 2\,M\left(\frac{x}{l} - \frac{1}{2}\right); \qquad V_A = -V_D = \frac{2\,M}{l}.$$

Case 39/13: The moment acts at joint *B*



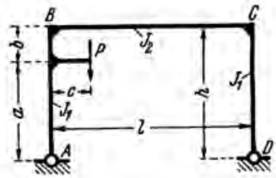


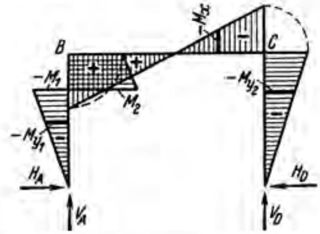
$$M_{B1} = M_C = \frac{3\,M}{2\,N}; \qquad V_A = -V_D = \frac{M}{l}; \qquad H_A = H_D = -\frac{M_{B1}}{h};$$

$$M_{B2} = -M + M_{B1}; \qquad M_x = \frac{x'}{l}\,M_{B2} + \frac{x}{l}\,M_C \qquad M_y = \frac{y}{h}\,M_{B1}.$$

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{l}$ $\quad N = 2k + 3.$

Case 39/14: Load on bracket on the left leg





$$\alpha = \frac{a}{h}; \qquad \begin{matrix} M_B \\ M_C \end{matrix} = \frac{Pc}{2}\left[\frac{(3\alpha^2 - 1)k}{N} \pm 1\right]$$

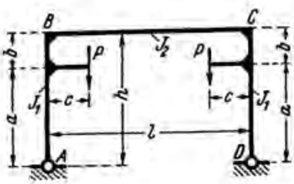
$$M_1 = -H_A a \qquad M_2 = Pc - H_A a;$$

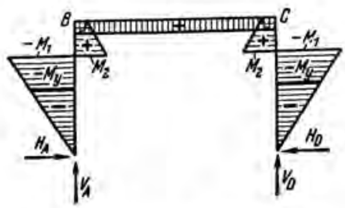
$$V_D = \frac{Pc}{l} \qquad V_A = P - V_D; \qquad H_A = H_D = \frac{-M_C}{h}.$$

Within the limits of a: $M_{y1} = -H_A y_1$

Within the limits of b: $M_{y1} = Pc - H_A y_1$ $\quad M_x = M_C + V_D x'$ $\quad M_{y2} = -H_D y_2.$

Case 39/15: Equal loads on bracket on the legs (Symmetrical load)





$$\alpha = \frac{a}{h}; \qquad M_B = M_C = \frac{Pc(3\alpha^2 - 1)k}{N};$$

$$H_A = H_D = \frac{Pc - M_B}{h}; \qquad V_A = V_D = P.$$

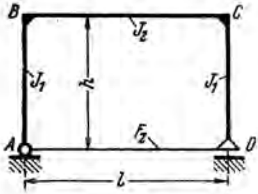
Within the limits of a: $M_y = -H_A y$

Within the limits of b: $M_y = Pc - H_A y.$

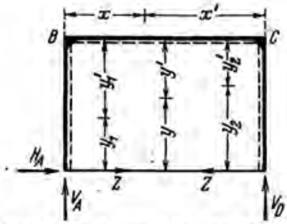
$$M_1 = -H_A a \qquad M_2 = Pc - H_A a;$$

Frame 40

Symmetrical rectangular frame with tie-rod, externally simply supported



Shape of Frame
Dimensions and Notations



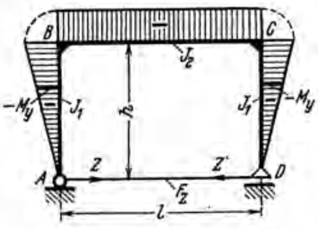
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{h}{l} \qquad L = \frac{3J_2}{h^2 F_Z} \cdot \frac{E}{E_Z} \qquad N = 2k + 3 \qquad N_Z = N + L$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Case 40/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z = \frac{3EJ_2 \varepsilon t}{h^2 N_Z};$$

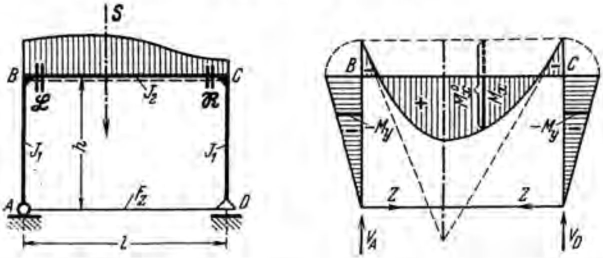
$$M_B = M_C = -Zh \qquad M_y = -Zy.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

*See footnote on page 146.

See Appendix A, Load Terms, pp. 440-445.

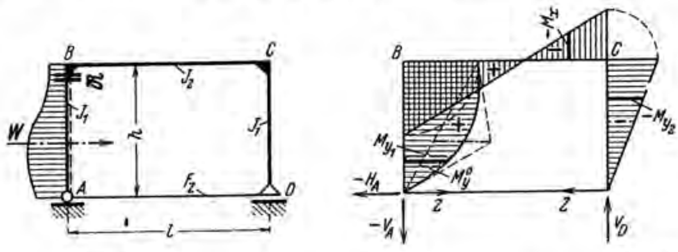
Case 40/2: Girder loaded by any type of vertical load*



$$Z = \frac{(\mathfrak{L} + \mathfrak{R})}{2\,h\,N_Z}; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l}; \qquad H_A = 0;$$

$$M_B = M_C = -Z\,h \qquad M_x = M_x^0 + M_B \qquad M_y = -Z\,y.$$

Case 40/3: Left-hand leg loaded by any type of horizontal load



$$Z = \frac{N\,\mathfrak{S}_l + \mathfrak{R}\,k}{2\,h\,N_Z}; \qquad H_A = -W \qquad V_D = -V_A = \frac{\mathfrak{S}_l}{l};$$

$$M_C = -Z\,h \qquad M_B = \mathfrak{S}_l + M_C$$

$$M_{y1} = M_y^0 + \frac{y_1}{h} M_B \qquad M_x = M_C + V_D\,x' \qquad M_{y2} = -Z\,y_2.$$

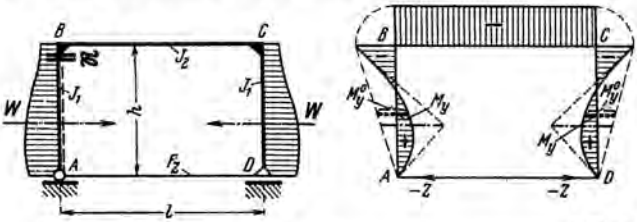
Special case 40/3a: Single concentrated horizontal load *P* at the girder

$$(W = P; \qquad \mathfrak{S}_l = P\,h; \qquad \mathfrak{R} = 0).$$

$$Z = \frac{P}{2} \cdot \frac{N}{N_Z}; \qquad V_D = -V_A = \frac{P\,h}{l}; \qquad M_B = (P - Z)\,h \qquad M_C = -Z\,h;$$

$$H_A = -P; \qquad M_{y1} = (P - Z)\,y_1 \qquad M_x = M_C + V_D\,x' \qquad M_{y2} = -Z\,y_2.$$

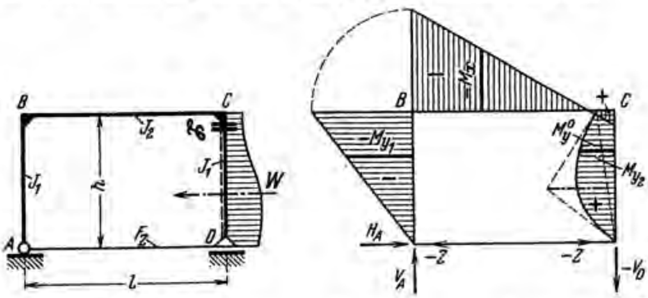
ase 40/4: Both legs loaded by any load, both carrying the same load



$$Z = -\frac{N\mathfrak{S}_r - \mathfrak{R}k}{hN_Z}*; \qquad M_B = M_C = -(\mathfrak{S}_r + Zh) = -\frac{L\mathfrak{S}_r + \mathfrak{R}k}{N_Z}$$

$$H_A = 0; \qquad M_y = M_y^0 + \frac{y}{h}M_B.$$

Case 40/5: Right-hand leg loaded by any type of horizontal load



$$Z = -\frac{(Wh + \mathfrak{S}_l)N - \mathfrak{L}k}{2hN_Z}* \qquad H_A = W \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l};$$

$$M_B = -(W + Z)h \qquad M_C = \mathfrak{S}_r + M_B$$

$$M_{y1} = \frac{y}{h}M_B \qquad M_x = M_B + V_A x \qquad M_{y2} = M_y^0 + \frac{y}{h}M_C.$$

Special case 40/5a: Single concentrated horizontal load P at the girder

$$(W = P; \qquad \mathfrak{S}_l = 0 \qquad \mathfrak{S}_r = Ph \qquad \mathfrak{L} = 0).$$

$$Z = -\frac{P}{2}\cdot\frac{N}{N_Z}* \qquad V_A = -V_D = \frac{Ph}{l}; \qquad H_A = P;$$

$$M_B = -(P + Z)h \qquad M_C = (-Z)h$$

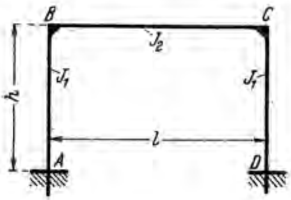
$$M_{y1} = -(P + Z)y_1 \qquad M_x = M_B + V_A x \qquad M_{y2} = (-Z)y_2.$$

---

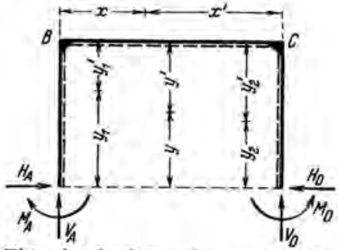
* For the above three loading conditions and for decrease in temperature (p. 144 bottom) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

Frame 41

Fully fixed symmetrical rectangular frame



Shape of Frame
Dimensions and Notations

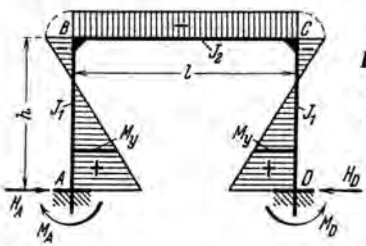


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{h}{l} \qquad N_1 = k + 2 \qquad N_2 = 6k + 1.$$

Case 41/1: Uniform increase in temperature of the entire frame*



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $$T = \frac{3 E J_2 \varepsilon t}{h N_1}.$$

$$M_A = M_D = +T \cdot \frac{k+1}{k} \qquad M_B = M_C = -T$$

$$M_y = M_A - H_A y; \qquad H_A = H_D = \frac{T}{h} \cdot \frac{2k+1}{k}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

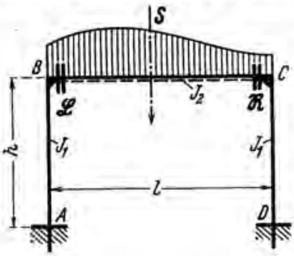
---

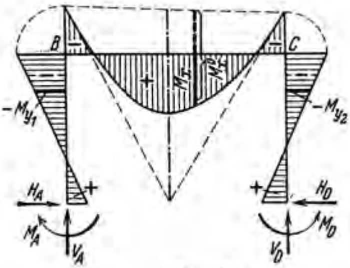
* Only the temperature change of the girder causes stress. For an antisymmetrical change in temperatur (left leg $+t$, right leg $-t$) substitute in the formula of the footnote on p. 148 the following: $\mathfrak{L} = 12 E J_2 h \varepsilon t/l$ and $\mathfrak{S}_r = 0$.

RAME 41

See Appendix A, Load Terms, pp. 440-445.

Case 41/2: Girder loaded by any type of vertical load*





Constant:

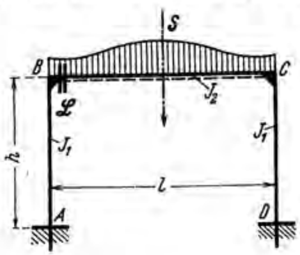
$$X_1 = \frac{(\mathfrak{L} + \mathfrak{R})}{6N_1} \qquad X_3 = \frac{(\mathfrak{L} - \mathfrak{R})}{2N_2}.$$

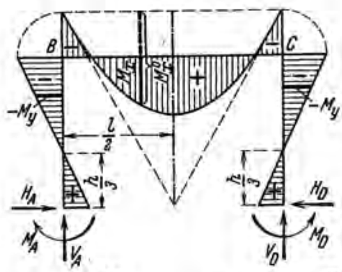
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = +X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -2X_1 \mp X_3;$$

$$V_A = \frac{\mathfrak{S}_r + 2X_3}{l} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{3X_1}{h};$$

$$M_{y1} = M_A - H_A y_1 \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = M_D - H_D y_2.$$

Case 41/3: Girder loaded by any type of vertical load, acting symmetrically





$$M_A = M_D = +\frac{\mathfrak{L}}{3N_1}; \qquad H_A = H_D = \frac{3M_A}{h} \qquad V_A = V_D = \frac{S}{2};$$

$$M_B = M_C = -2M_A \qquad M_x = M_x^0 + M_B \qquad M_y = M_A - H_A y.$$

Special case 41/3a: Uniformly distributed load $S = ql$

$$M_A = M_D = +\frac{ql^2}{12N_1} \qquad V_A = V_D = \frac{ql}{2} \qquad \max M_x = \frac{ql^2}{8} + M_B.$$

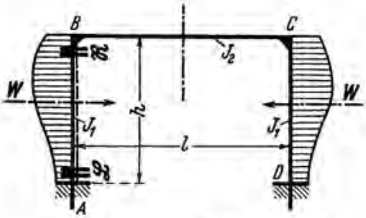
All other formulas as above.

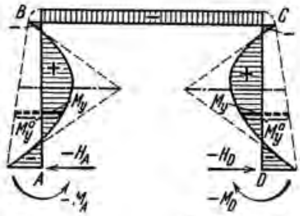
---

*For an antisymmetrical load ($\mathfrak{R} = -\mathfrak{L}$) $X_1 = 0$, $X_3 = \mathfrak{L}/N_2$; $M_D = M_C = -M_A = -M_B = \mathfrak{L}/N_2$ and $H_A = H_D = 0$.

See Appendix A, Load Terms, pp. 440-445.

Case 41/4: Both legs loaded by any type of external symmetrical load*





$$M_A = M_D = -\frac{\mathfrak{L}(2k+3) - \mathfrak{R}k}{3N_1} \qquad M_B = M_C = -\frac{(2\mathfrak{R} - \mathfrak{L})k}{3N_1};$$

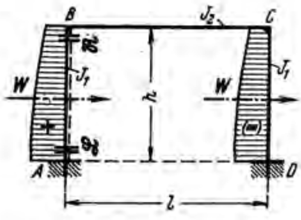
$$H_A = H_D = -\frac{\mathfrak{S}_r - M_A + M_B}{h}; \qquad M_y = \mathbf{M}_y^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B.$$

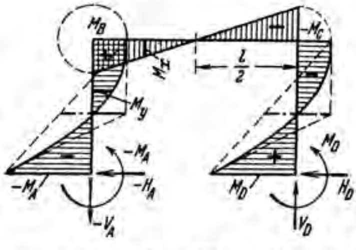
Special case 41/4a: Uniformly distributed loads $W = qh$

$$M_A = M_D = -\frac{qh^2}{12}\cdot\frac{k+3}{N_1} \qquad M_B = M_C = -\frac{qh^2}{12}\cdot\frac{k}{N_1};$$

$$H_A = H_D = -\frac{qh}{4}\cdot\frac{2k+5}{N_1}; \qquad M_y = \frac{qyy'}{2} + \frac{y'}{h}M_A + \frac{y}{h}M_B.$$

Case 41/5: Both legs loaded by any type of antisymmetrical load from the left*





$$M_B = -M_C = [3\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\frac{k}{N_2} \quad M_D = -M_A = \mathfrak{S}_l - M_B; \quad H_D = -H_A = W$$

$$V_D = -V_A = \frac{2M_B}{l} \qquad M_y = \mathbf{M}_y^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B \qquad M_x = \frac{x' - x}{l}\cdot M_B$$

Special case 41/5a: Uniformly distributed loads $W = qh$

$$M_B = -M_C = qh^2\cdot\frac{k}{N_2} \qquad M_D = -M_A = \frac{qh^2}{2}\cdot\frac{4k+1}{N_2} \qquad \mathbf{M}_y^0 = \frac{qyy'}{2}.$$

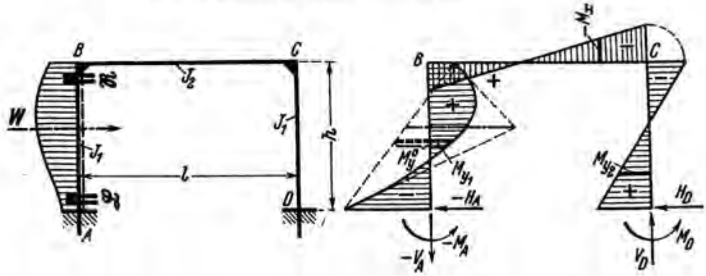
All other formulas as above.

* Note: All the load terms refer to the left leg.

FRAME 41

Case 41/6: Left-hand leg loaded by any type of horizontal load

See Appendix A, Load Terms, pp. 440-445.



Constants:

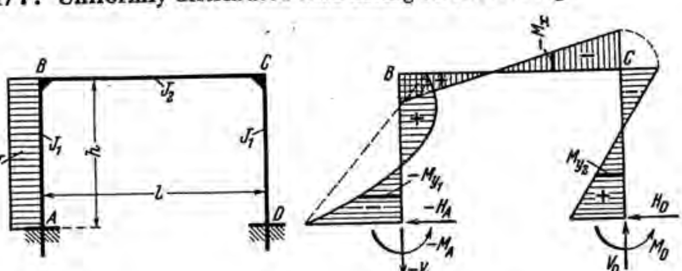
$$X_1 = \frac{\mathfrak{L}(2k+3) - \mathfrak{R}k}{6N_1} \qquad X_2 = \frac{(2\mathfrak{R} - \mathfrak{L})k}{6N_1} \qquad X_3 = \frac{[3\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]k}{2N_2}$$

$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -X_1 \mp \left(\frac{\mathfrak{S}_l}{2} - X_3\right) \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -X_2 \pm X_3;$$

$$H_D = \frac{\mathfrak{S}_l}{2h} - \frac{X_1 - X_2}{h} \qquad H_A = -(W - H_D) \qquad V_D = -V_A = \frac{2X_3}{l};$$

$$M_{y1} = M_y^0 + \frac{y_1'}{h}M_A + \frac{y_1}{h}M_B \qquad M_x = M_C + V_D x' \qquad M_{y2} = M_D - H_D y_2.$$

Case 41/7: Uniformly distributed load acting on the left leg



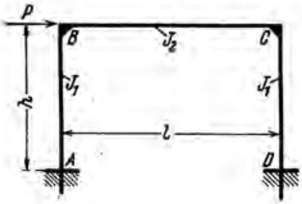
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = \frac{qh^2}{4}\left[-\frac{k+3}{6N_1} \mp \frac{4k+1}{N_2}\right] \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = \frac{qh^2}{4}\left[-\frac{k}{6N_1} \pm \frac{2k}{N_2}\right];$$

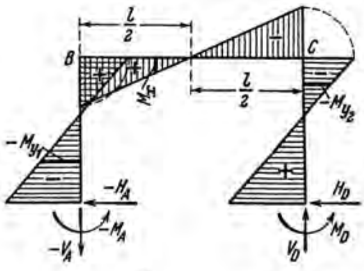
$$H_D = \frac{qh(2k+3)}{8N_1} \qquad H_A = -(qh - H_D); \qquad V_D = -V_A = \frac{qh^2k}{lN_2};$$

$$M_{y1} = \frac{q y_1 y_1'}{2} + \frac{y_1'}{h}M_A + \frac{y_1}{h}M_B$$

$$M_x = M_C + V_D x' \qquad M_{y2} = M_D - H_D y_2.$$

Case 41/8: Horizontal concentrated load at the girder



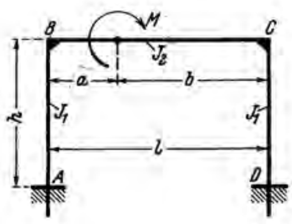


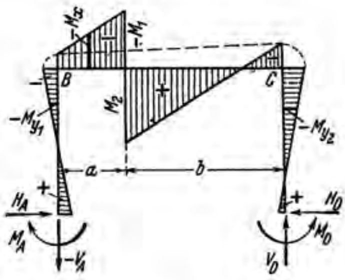
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = \mp \frac{Ph}{2} \cdot \frac{3k+1}{N_2} \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = \pm \frac{Ph}{2} \cdot \frac{3k}{N_2};$$

$$H_D = -H_A = \frac{P}{2}; \qquad V_D = -V_A = \frac{2M_B}{l};$$

$$M_{y1} = M_A + \frac{P}{2} y_1 \qquad M_x = \frac{x'-x}{l} M_B \qquad M_{y2} = M_D - \frac{P}{2} y_2.$$

Case 41/9: The moment acts at any point of the girder





$$\alpha = \frac{a}{l} \qquad \beta = \frac{b}{l} \qquad (\alpha + \beta = 1).$$

$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = M\left[+\frac{\beta-\alpha}{2N_1} \mp \frac{1-6\alpha\beta}{2N_2}\right] \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = M\left[-\frac{\beta-\alpha}{N_1} \mp \frac{1-6\alpha\beta}{2N_2}\right];$$

$$H_A = H_D = \frac{3M(\beta-\alpha)}{2hN_1}; \qquad V_D = -V_A = \frac{6M(k+\alpha\beta)}{lN_2}.$$

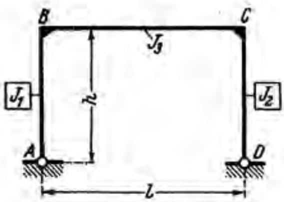
Within the limits of ***a***: $M_x = M_B + V_A x$ $\qquad M_1 = M_B + V_A a$

Within the limits of ***b***: $M_x = M_C + V_D x'$; $\qquad M_2 = M_C + V_D b$.

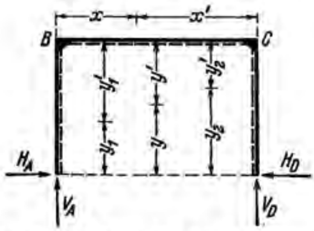
$(M_2 - M_1 = M)$ $\qquad M_{y1} = M_A - H_A y_1$ $\qquad M_{y2} = M_D - H_D y_2$.

Frame 42

Rectangular two-hinged frame with unequal moments of inertia of the legs



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For equal moments in both legs use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h}{l} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h}{l} ;$$

$$B = 2k_1 + 3 \qquad C = 2k_2 + 3 ; \qquad N = B + C .$$

Note: The moment diagrams are based on the assumption $J_2 > J_1$.

Formulas to case 42/3, p. 153.

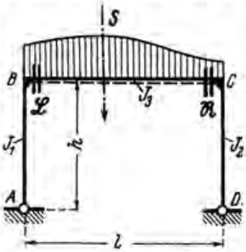
$$M_B = -\frac{\mathfrak{S}_r C + \mathfrak{L} k_2}{N} \qquad M_C = \mathfrak{S}_r + M_B ;$$

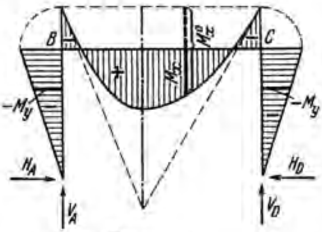
$$H_A = \frac{-M_B}{h} \qquad H_D = -(W - H_A) \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l};$$

$$M_{y1} = -H_A y_1 \qquad M_x = M_B + V_A x \qquad M_{y2} = M_y^0 + \frac{y_2}{h} M_C .$$

See Appendix A, Load Terms, pp. 440-445.

Case 42/1: Girder loaded by any type of vertical load



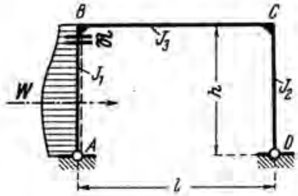


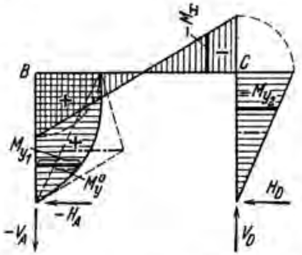
$$V_A = \frac{\mathfrak{S}_r}{l}$$

$$V_D = \frac{\mathfrak{S}_l}{l};$$

$$M_B = M_C = -\frac{(\mathfrak{L}+\mathfrak{R})}{N}; \quad H_A = H_D = \frac{-M_B}{h}; \quad M_x = M_x^0 + M_B \quad M_y = \frac{y}{h} M_B$$

Case 42/2: Left-hand leg loaded by any type of horizontal load



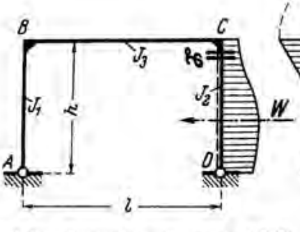


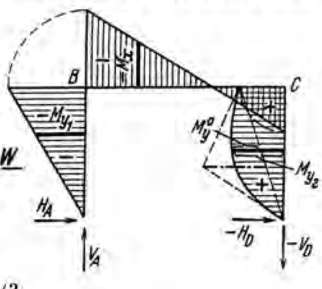
$$V_D = -V_A = \frac{\mathfrak{S}}{l}$$

$$M_C = -\frac{\mathfrak{S}_l B + \mathfrak{R} k_1}{N} \quad M_B = \mathfrak{S}_l + M_C; \quad H_D = \frac{-M_C}{h} \quad H_A = -(W - H_D);$$

$$M_{y1} = M_y^0 + \frac{y_1}{h} M_B \qquad M_x = M_C + V_D x' \qquad M_{y2} = -H_D y_2.$$

Case 42/3: Right-hand leg loaded by any type of horizontal load

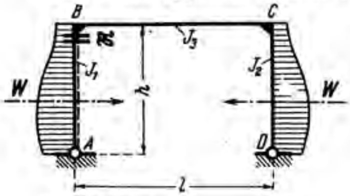


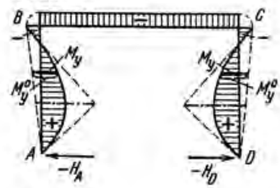


See p. 152 for formulas to case 42/3.

FRAME 42

Case 42/4: Both legs loaded by any load, both members carrying the same load * See Appendix A, Load Terms, pp. 440-445.

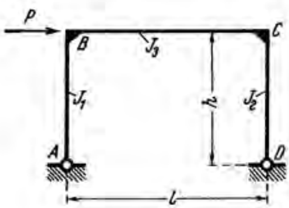


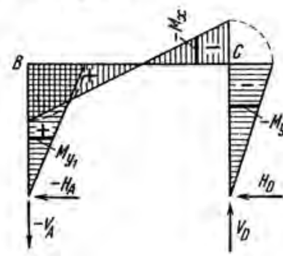


$$M_B = M_C = -\frac{\mathfrak{R}(k_1 + k_2)}{N} \qquad H_A = H_D = -\frac{\mathfrak{S}_r + M_B}{h} \qquad M_y = \mathbf{M}_y^0 + \frac{y}{h} M_B.$$

Note: All the load terms refer to the left leg.

Case 42/5: Horizontal concentrated load at the girder

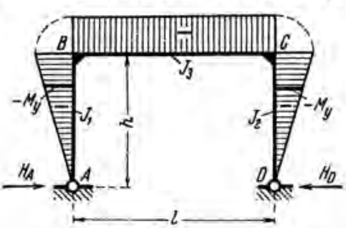




$$H_A = -P\frac{C}{N} \qquad H_D = +P\frac{B}{N}; \qquad M_B = (-H_A)h \qquad M_C = -H_D h;$$

$$V_D = -V_A = \frac{Ph}{l}; \quad M_{y1} = (-H_A)y_1 \quad M_x = M_C + V_D x' \quad M_{y2} = -H_D y_2.$$

Case 42/6: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

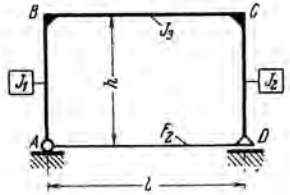
$$M_B = M_C = -\frac{6EJ_3\varepsilon t}{hN}; \qquad H_A = H_D = \frac{-M_B}{h}; \qquad M_y = -H_A y.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

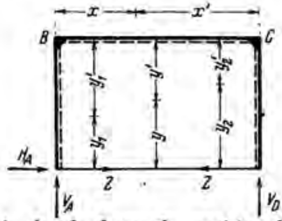
* Symmetrical loading condition. The moment diagram is symmetrical in spite of the unequal moments of inertia of the legs.

Frame 43

Rectangular frame with tie-rod and unequal moments of the legs, externally simply supported



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For equal moments in both legs use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

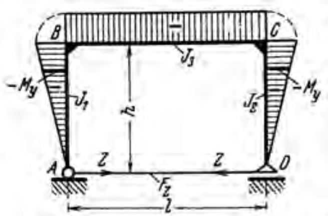
$$k_1 = \frac{J_3}{J_1} \cdot \frac{h}{l} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h}{l}; \qquad B = 2k_1 + 3 \qquad C = 3 + 2k_2;$$

$$N = B + C \qquad L = \frac{6J_3}{h^2 F_Z} \cdot \frac{E}{E_Z}; \qquad N_Z = N + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Note: The moment diagrams are based on the assumption $J_2 > J_1$.

Case 43/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z = \frac{6EJ_3\varepsilon t}{h^2 N_Z};$$

$$M_B = M_C = -Zh \qquad M_y = -Zy.$$

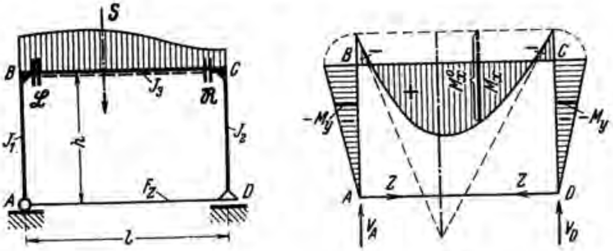
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

*See footnote on page 157.

FRAME 43

See Appendix A, Load Terms, pp. 440-445.

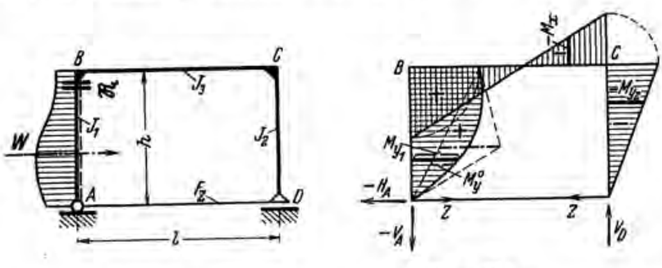
Case 43/2: Girder loaded by any type of vertical load



$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l}; \qquad Z = \frac{(\mathfrak{L} + \mathfrak{R})}{h N_Z}; \qquad M_B = M_C = -Zh$$

$$M_x = M_x^0 + M_B \qquad M_y = -Zy.$$

Case 43/3: Left-hand leg loaded by any type of horizontal load



$$V_D = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_A = -W; \qquad Z = \frac{B\mathfrak{S}_l + \mathfrak{R}k_1}{hN_Z}; \qquad M_C = -Zh$$

$$M_B = \mathfrak{S}_l - Zh \qquad M_{y1} = M_y^0 + \frac{y_1}{h} M_B \qquad M_x = M_C + V_D x' \qquad M_{y2} = -Z y_2.$$

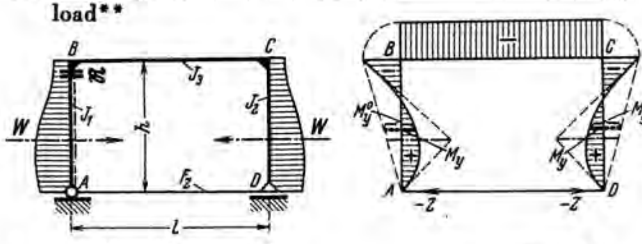
Special case 43/3a: Single concentrated horizontal load P at the girder

$$(W = P; \qquad \mathfrak{S}_l = Ph; \qquad \mathfrak{R} = 0; \qquad M_y^0 = 0).$$

$$Z = P\frac{B}{N_Z}; \qquad V_D = -V_A = \frac{Ph}{l}; \qquad M_B = (P - Z)h \qquad M_C = -Zh;$$

$$H_A = -P; \qquad M_{y1} = (P - Z)y_1 \qquad M_x = M_C + V_D x' \qquad M_{y2} = -Zy_2.$$

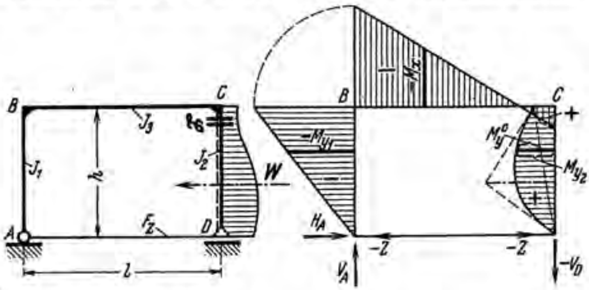
Case 43/4: Both legs loaded by any load, both members carrying the same load**



$$Z = -\frac{N_r \mathfrak{S}_r - \mathfrak{R}(k_1 + k_2)}{h N_Z}^{*}; \qquad M_y = M_y^0 + \frac{y}{h} M_B;$$

$$H_A = 0; \qquad M_B = M_C = -(\mathfrak{S}_r + Z h) = -\frac{L \mathfrak{S}_r + \mathfrak{R}(k_1 + k_2)}{N_Z}.$$

Case 43/5: Right-hand leg loaded by any type of horizontal load



$$Z = -\frac{W h B + C \mathfrak{S}_l - \mathfrak{L} k_2}{h N_Z}^{*}; \qquad H_A = W; \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l};$$

$$M_B = -(W + Z) h \qquad M_C = \mathfrak{S}_r + M_B;$$

$$M_{y1} = -(W + Z) y_1 \qquad M_x = M_B + V_A x \qquad M_{y2} = M_y^0 + \frac{y}{h} M_C.$$

Special case 43/5a: Single concentrated horizontal load P at the girder

$$(W = P; \quad \mathfrak{S}_l = 0; \quad \mathfrak{S}_r = P h; \quad \mathfrak{L} = 0; \quad M_y^0 = 0).$$

$$Z = -P \frac{B}{N_Z}^{*}; \qquad V_A = -V_D = \frac{P h}{l}; \qquad M_B = -(P + Z) h, \quad M_C = (-Z) h;$$

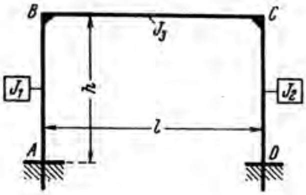
$$H_A = P; \qquad M_{y1} = -(P + Z) y_1 \qquad M_x = M_B + V_A x \qquad M_{y2} = (-Z) y_2.$$

* For the above three loading conditions and for decrease in temperature (p. 155 bottom) Z becomes negative i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

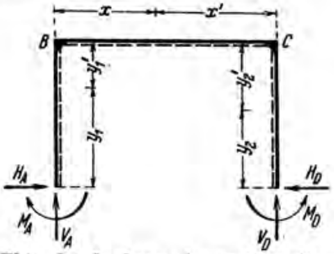
** See footnote p. 154.

Frame 44

Fully fixed rectangular frame with unequal moments of inertia of the legs



Shape of Frame
Dimensions and Notations

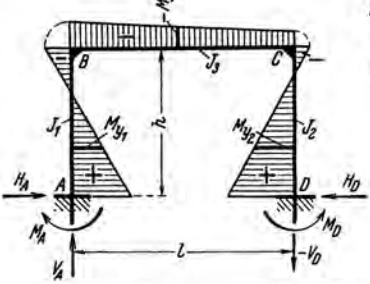


This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* same as for frame 48 (p. 168) with the following simplifications:

$$(h_1 = h_2) = h \qquad n = 1 \qquad (v = 0).$$

Case 44/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$T = \frac{6 E J_3 \varepsilon t}{h}; \qquad X_1 = T n_{31}$$

$$X_2 = T n_{32} \qquad X_3 = T n_{33}.$$

$$M_A = X_3 - X_1 \qquad M_B = -X_1$$

$$M_C = -X_2 \qquad M_D = X_3 - X_2;$$

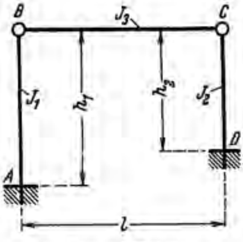
$$V_A = -V_D = \frac{X_1 - X_2}{l} \qquad H_A = H_D = \frac{X_3}{h};$$

$$M_{y1} = \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h} M_C + \frac{y_2'}{h} M_D.$$

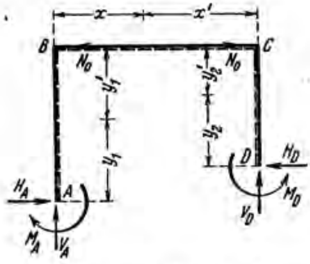
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 45

Fixed rectangular frame with hinged knees and unequal-length legs



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions. and the axial forces in the girder.[1]

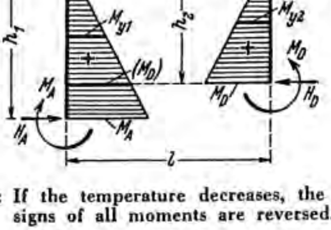
Coefficients:

$$K_1 = \frac{J_1}{h_1^3} \qquad K_2 = \frac{J_2}{h_2^3}\,; \qquad \alpha = \frac{K_1}{K_1 + K_2} \qquad \delta = \frac{K_2}{K_1 + K_2} = 1 - \alpha\,; \qquad n = \frac{h_2}{h_1}$$

Case 45/1: Girder loaded by any type of load

The girder behaves like a simple beam. Formulas same as for case 38/2, p. 1

Case 45/2: Uniform increase in temperature of the girder by t degrees[2]



E = Modulus of elasticity
ε = Coefficient of thermal expansion

$$H_A = N_0 = H_D = 3\,E\,l \cdot \varepsilon\,t \cdot \frac{K_1 K_2}{K_1 + K_2}$$

$$M_A = H_A \cdot h_1 \qquad M_D = H_D \cdot h_2$$

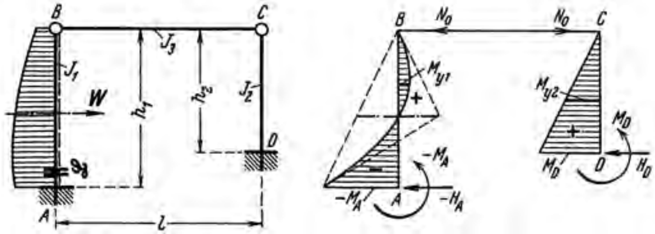
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

[1] Positive bending moments M cause tension at the face marked by a dashed line. Positive axial forces produce compression.

[2] Temperature change in the members has no static influence.

See Appendix A, Load Terms, pp. 440-445.

:se 45/3: Left-hand leg loaded by any type of load



$$M_A = -\mathfrak{S}_l \cdot \alpha - \frac{\mathfrak{L}}{2} \cdot \delta \qquad M_D = \left(\mathfrak{S}_l - \frac{\mathfrak{L}}{2}\right) n\,\delta\,;$$

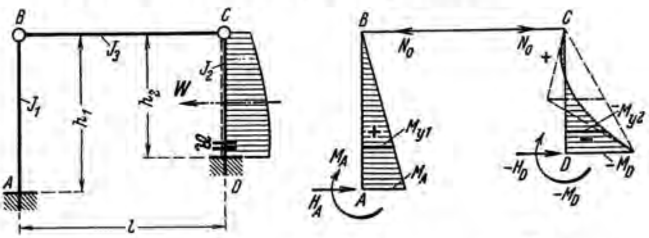
$$H_D = N_0 = \frac{M_D}{h_2} \qquad H_A = -W + H_D\,; \qquad M_{y1} = \mathbf{M}_y^0 + \frac{y_1'}{h_1} \cdot M_A\,.$$

;pecial case 45/3a: Single concentrated horizontal load P at ridge B

$$(W = P\,; \qquad \mathfrak{S}_l = P h_1\,; \qquad \mathfrak{L} = 0\,; \qquad \mathbf{M}_y^0 = 0)\,.$$

$$I_A = -P\alpha \qquad H_D = N_0 = P\delta\,; \qquad M_A = -P\alpha \cdot h_1 \qquad M_D = +P\delta \cdot h_2\,.$$

:ase 45/4: Right-hand leg loaded by any type of load



$$M_A = \left(\mathfrak{S}_r - \frac{\mathfrak{R}}{2}\right)\frac{\alpha}{n} \qquad M_D = -\mathfrak{S}_r \cdot \delta - \frac{\mathfrak{R}}{2} \cdot \alpha\,;$$

$$H_A = N_0 = \frac{M_A}{h_1} \qquad H_D = -W + H_A\,; \qquad M_{y2} = \mathbf{M}_y^0 + \frac{y_2'}{h_2} \cdot M_D\,.$$

Special case 45/4a: Single concentrated horizontal load P at ridge C

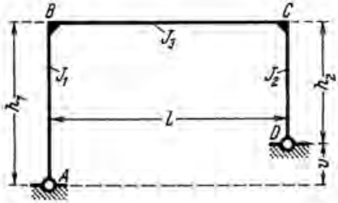
$$(W = P\,; \qquad \mathfrak{S}_r = P h_2\,; \qquad \mathfrak{R} = 0\,; \qquad \mathbf{M}_y^0 = 0)\,.$$

$$H_A = N_0 = P\alpha \qquad H_D = -P\delta\,; \qquad M_A = +P\alpha \cdot h_1 \qquad M_D = -P\delta \cdot h_2\,.$$

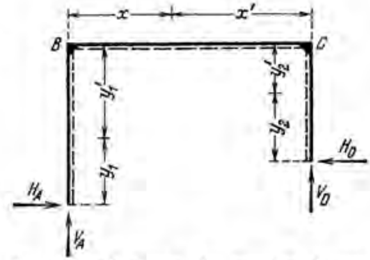
Note: With the exception of N_0 case 45/4a is the same as the negative case 45/3a.

Frame 46

Two-hinged bent with legs of unequal length.



Shape of Frame
Dimensions and Notations



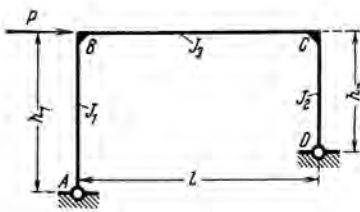
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

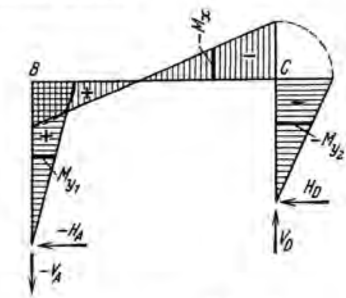
Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{l} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{l}; \qquad n = \frac{h_2}{h_1};$$

$$B = 2(k_1 + 1) + n \qquad C = 1 + 2n(1 + k_2); \qquad N = B + nC.$$

Case 46/1: Horizontal concentrated load at the girder





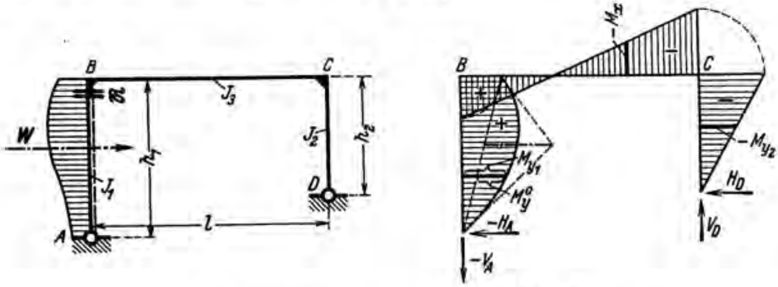
$$H_A = -P \cdot \frac{nC}{N} \qquad H_D = P \cdot \frac{B}{N}; \qquad V_D = -V_A = \frac{M_B - M_C}{l};$$

$$M_B = (-H_A)\, h_1 \qquad M_C = -H_D h_2;$$

$$M_{y1} = (-H_A)\, y_1 \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = -H_D y_2.$$

See Appendix A, Load Terms, pp. 440-445.

ase 46/2: Left-hand leg loaded by any type of horizontal load



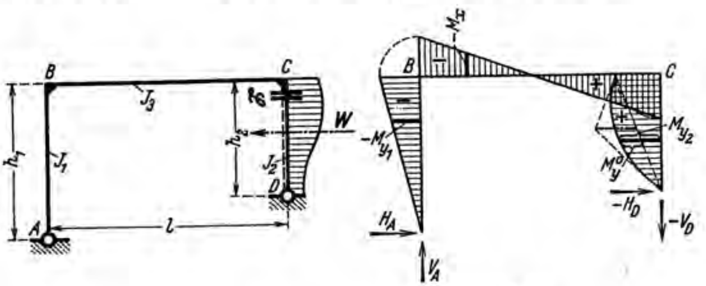
Constant: $X = \frac{B\mathfrak{S}_l + \mathfrak{R}k_1}{N}$.

$M_B = \mathfrak{S}_l - X \qquad M_C = -nX;$

$V_D = -V_A = \frac{M_B - M_C}{l}; \qquad H_D = \frac{X}{h_1} \qquad H_A = -(W - H_D);$

$M_{y1} = M_y^0 + \frac{y_1}{h_1}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = \frac{y_2}{h_2}M_C.$

Case 46/3: Right-hand leg loaded by any type of horizontal load



Constant: $X = \frac{C\mathfrak{S}_r + n\mathfrak{L}k_2}{N}$;

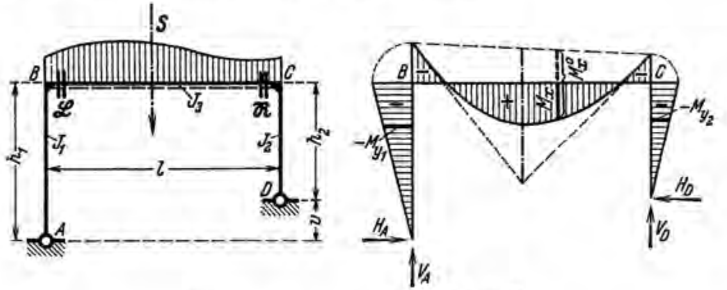
$M_B = -X \qquad M_C = \mathfrak{S}_r - nX;$

$V_A = -V_D = \frac{M_C - M_B}{l}; \qquad H_A = \frac{X}{h_1} \qquad H_D = -(W - H_A);$

$M_{y1} = \frac{y_1}{h_1}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h_2}M_C.$

(See Appendix A, Load Terms, pp. 440-445.)

Case 46/4: Girder loaded by any type of vertical load



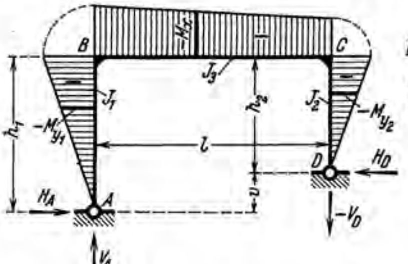
Constant: $X = \frac{\mathfrak{L} + n\,\mathfrak{R}}{N}$.

$M_B = -X \qquad M_C = -n\,X;$

$V_A = \frac{\mathfrak{S}_r}{l} + \frac{X v}{h_1 l} \qquad V_D = \frac{\mathfrak{S}_l}{l} - \frac{X v}{h_1 l}; \qquad H_A = H_D = \frac{X}{h_1};$

$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$

Case 46/5: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant:

$$X = \frac{6\,E\,J_3\,\varepsilon\,t\,(l^2 + v^2)}{h_1\,l^2\,N}.$$

$M_B = -X \qquad M_C = -n\,X;$

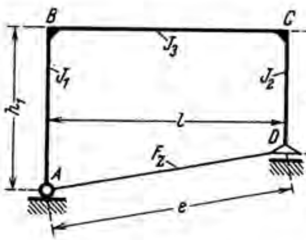
$V_A = -V_D = \frac{X v}{h_1 l}; \qquad H_A = H_D = \frac{X}{h_1};$

$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$

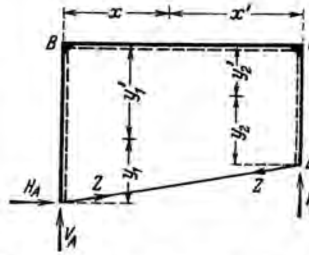
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 47

Tied bent with inclined tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{l} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{l}; \qquad n = \frac{h_2}{h_1};$$

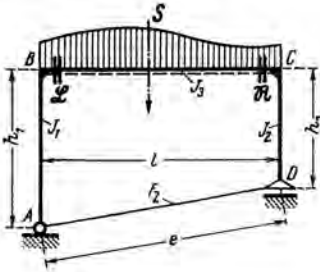
$$B = 2(k_1 + 1) + n \qquad C = 1 + 2n(1 + k_2);$$

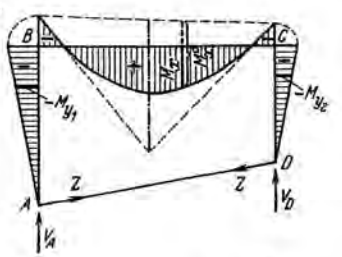
$$N = B + nC \qquad L = \frac{6J_3}{h_1^2 F_Z} \cdot \frac{E}{E_Z}; \qquad N_Z = N\frac{l}{e} + L\frac{e^2}{l^2}.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

See Appendix A, Load Terms, pp. 440-445.

Case 47/1: Girder loaded by any type of vertical load



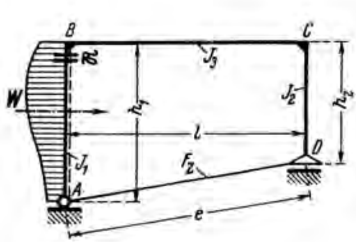


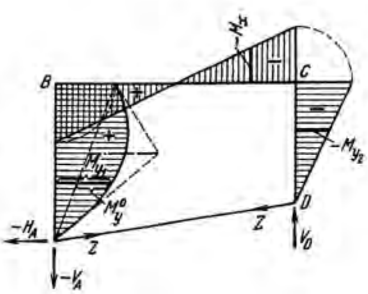
$$Z = \frac{\mathfrak{L} + n\,\mathfrak{R}}{h_1 N_Z}\;; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l}\;;$$

$$M_B = -Z\frac{l}{e}h_1 \qquad M_C = -Z\frac{l}{e}h_2\;;$$

$$M_{y1} = \frac{y_1}{h_1}M_B \qquad M_x = M_x^0 + \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = \frac{y_2}{h_2}M_C.$$

Case 47/2: Left-hand leg loaded by any type of horizontal load





$$Z = \frac{B\,\mathfrak{S}_l + \mathfrak{R}\,k_1}{h_1 N_Z} \qquad H_A = -W\;; \qquad V_D = -V_A = \frac{\mathfrak{S}_l}{l}\;;$$

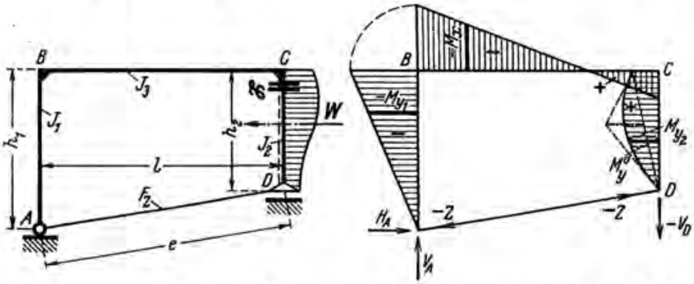
$$M_B = \mathfrak{S}_l - Z\frac{l}{e}h_1 \qquad M_C = -Z\frac{l}{e}h_2\;;$$

$$M_{y1} = M_y^0 + \frac{y_1}{h_1}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = \frac{y_2}{h_2}M_C.$$

RAME 47

See Appendix A, Load Terms, pp. 440-445.

Case 47/3: Right-hand leg loaded by any type of horizontal load

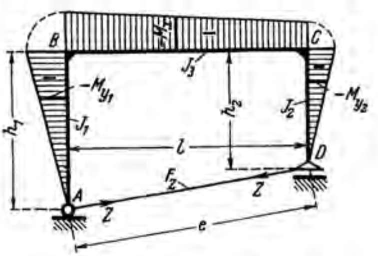


$$Z = -\left(W\frac{B}{N_Z} + \frac{C\mathfrak{S}_l - n\mathfrak{L}k_2}{h_1 N_Z}\right)^* ; \qquad V_A = -V_D = \frac{\mathfrak{S}_r + Wv}{l};$$

$$H_A = W; \qquad M_B = -\left(W + Z\frac{l}{e}\right)h_1 \qquad M_C = -Z\frac{l}{e}h_2 - \mathfrak{S}_l;$$

$$M_{y1} = \frac{y_1}{h_1}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h_2}M_C.$$

Case 47/4: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z = \frac{6EJ_3\varepsilon t}{h_1^2 N_Z}\cdot\frac{e^2}{l^2};$$

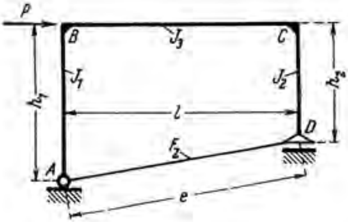
$$M_B = -Z\frac{l}{e}h_1 \qquad M_C = -Z\frac{l}{e}h_2;$$

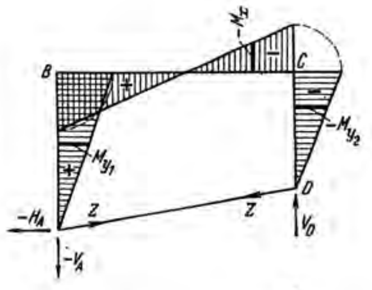
$$M_{y1} = \frac{y_1}{h_1}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = \frac{y_2}{h_2}M_C.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

* For the above loading condition, decrease in temperature, and case 47/6 (p. 167) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

Case 47/5: Horizontal concentrated load at the girder



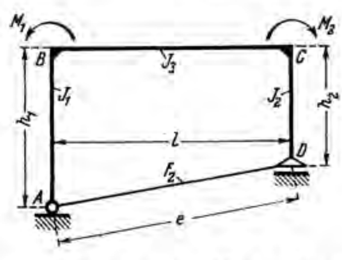


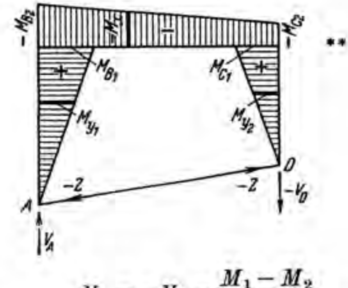
$$Z = P\frac{B}{N_Z} \qquad H_A = -P; \qquad V_D = -V_A = P\frac{h_1}{l};$$

$$M_B = \left(P - Z\frac{l}{e}\right)h_1 \qquad M_C = -Z\frac{l}{e}h_2;$$

$$M_{y1} = \frac{y_1}{h_1}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = \frac{y_2}{h_2}M_C.$$

Case 47/6: Moments of different magnitude acting at joints *B* and *C*





$$Z = -\frac{M_1(2+n) + M_2(1+2n)}{h_1 N_Z}* ; \qquad V_A = -V_D = \frac{M_1 - M_2}{l};$$

$$M_{B1} = (-Z)\frac{l}{e}h_1 \qquad M_{C1} = (-Z)\frac{l}{e}h_2$$

$$M_{B2} = -(M_1 - M_{B1}) \qquad M_{C2} = -(M_2 - M_{C1});$$

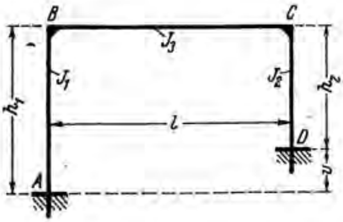
$$M_{y1} = \frac{y_1}{h_1}M_{B1} \qquad M_x = \frac{x'}{l}M_{B2} + \frac{x}{l}M_{C2} \qquad M_{y2} = \frac{y_2}{h_2}M_{C1}.$$

*See footnote on page 180

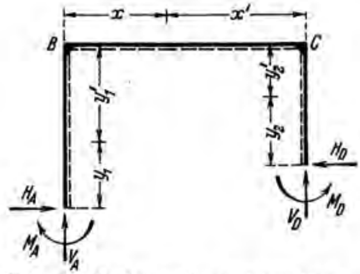
**The moment diagram is based on the assumption $M_1 > M_2$.

Frame 48

Hingeless bent with legs of unequal length.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{l} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{l}; \qquad n = \frac{h_2}{h_1};$$

$$R_1 = 2(3k_1 + 1) \qquad R_2 = 2(1 + 3k_2) \qquad R_3 = 2(k_1 + n^2 k_2);$$

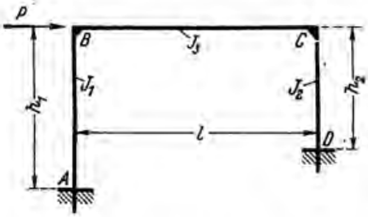
$$N = R_3(k_1 + 1 + k_2) + 6k_1 k_2 (k_1 + 1 + n + n^2 + n^2 k_2).$$

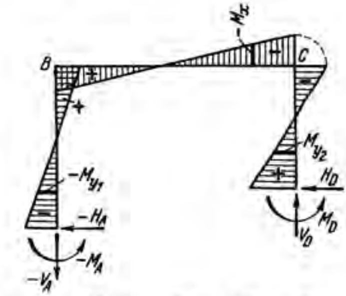
$$n_{11} = \frac{R_2 R_3 - 9n^2 k_2^2}{3N} \qquad n_{12} = n_{21} = \frac{9n k_1 k_2 - R_3}{3N}$$

$$n_{22} = \frac{R_1 R_3 - 9k_1^2}{3N} \qquad n_{13} = n_{31} = \frac{k_1 R_2 - n k_2}{N}$$

$$n_{33} = \frac{R_1 R_2 - 1}{3N} \qquad n_{23} = n_{32} = \frac{n k_2 R_1 - k_1}{N}.$$

Case 48/1: Horizontal concentrated load at the girder





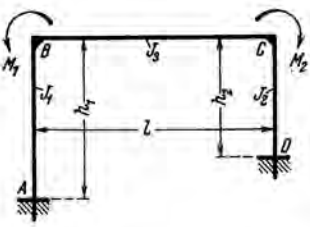
Constants:

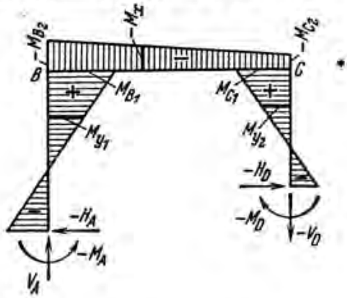
$$X_1 = P h_1 k_1 (+3 n_{11} - 2 n_{31}) \qquad M_A = -P h_1 + X_1 + X_3$$
$$X_2 = P h_1 k_1 (-3 n_{12} + 2 n_{32}) \qquad M_B = X_1 \qquad M_C = -X_2$$
$$X_3 = P h_1 k_1 (-3 n_{13} + 2 n_{33}) . \qquad M_D = n X_3 - X_2 ;$$

$$V_D = -V_A = \frac{X_1 + X_2}{l} ; \qquad H_D = \frac{X_3}{h_1} \qquad H_A = -(P - H_D) ;$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D .$$

Case 48/2: Moments of different magnitude acting at joints *B* and *C*





Constants:

$$X_1 = +M_1 (2 n_{11} + n_{21}) + M_2 (n_{11} + 2 n_{21}) \qquad M_A = X_1 + X_3$$
$$X_2 = +M_1 (2 n_{12} + n_{22}) + M_2 (n_{12} + 2 n_{22}) \qquad M_{B1} = X_1 \qquad M_{C1} = X_2$$
$$X_3 = -M_1 (2 n_{13} + n_{23}) - M_2 (n_{13} + 2 n_{23}) . \qquad M_{B2} = -(M_1 - X_1)$$
$$M_{C2} = -(M_2 - X_2)$$

$$V_A = -V_D = \frac{M_{C2} - M_{B2}}{l} ; \quad H_A = H_D = \frac{X_3}{h_1} ; \qquad M_D = X_2 + n X_3 ;$$

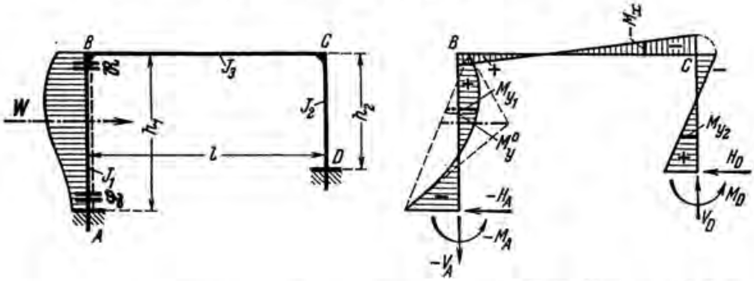
$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_{B1} \qquad M_x = \frac{x'}{l} M_{B2} + \frac{x}{l} M_{C2} \qquad M_{y2} = \frac{y_2}{h_2} M_{C1} + \frac{y_2'}{h_2} M_D .$$

*The moment diagram is based on the assumption $M_1 > M_2$.

RAME 48

See Appendix A, Load Terms, pp. 440-445.

Case 48/3: Left-hand leg loaded by any type of horizontal load



Constants: $\mathfrak{B}_1 = [3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\,k_1$ $\quad \mathfrak{B}_3 = [2\,\mathfrak{S}_l - \mathfrak{L}]\,k_1\,;$

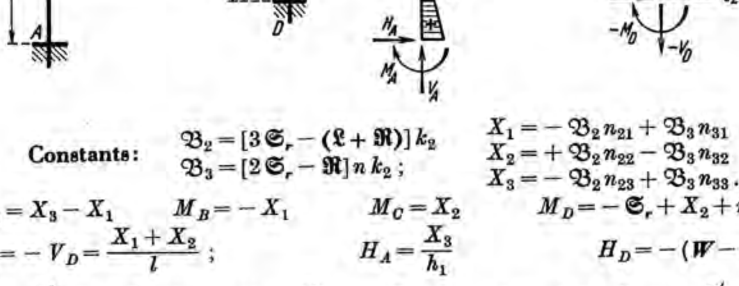
$$X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_3 n_{31}$$
$$X_2 = -\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_3 n_{32}$$
$$X_3 = -\,\mathfrak{B}_1 n_{13} + \mathfrak{B}_3 n_{33}\,.$$

$$M_A = -\mathfrak{S}_l + X_1 + X_3 \qquad M_B = X_1 \qquad M_C = -X_2 \qquad M_D = n\,X_3 - X_2\,;$$

$$V_D = -V_A = \frac{X_1 + X_2}{l}\,; \qquad H_D = \frac{X_3}{h_1} \qquad H_A = -(W - H_D)\,;$$

$$M_{y1} = \mathbf{M}_y^0 + \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D\,.$$

Case 48/4: Right-hand leg loaded by any type of horizontal load



Constants: $\mathfrak{B}_2 = [3\,\mathfrak{S}_r - (\mathfrak{L} + \mathfrak{R})]\,k_2$ $\quad \mathfrak{B}_3 = [2\,\mathfrak{S}_r - \mathfrak{R}]\,n\,k_2\,;$

$$X_1 = -\,\mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$
$$X_2 = +\,\mathfrak{B}_2 n_{22} - \mathfrak{B}_3 n_{32}$$
$$X_3 = -\,\mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}\,.$$

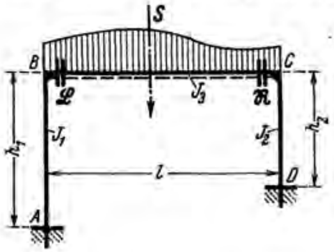
$$M_A = X_3 - X_1 \qquad M_B = -X_1 \qquad M_C = X_2 \qquad M_D = -\,\mathfrak{S}_r + X_2 + n\,X_3\,;$$

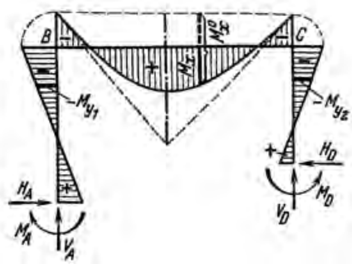
$$V_A = -V_D = \frac{X_1 + X_2}{l}\,; \qquad H_A = \frac{X_3}{h_1} \qquad H_D = -(W - H_A)\,;$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \mathbf{M}_y^0 + \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D\,.$$

See Appendix A, Load Terms, pp. 440-445.

Case 48/5: Girder loaded by any type of vertical load





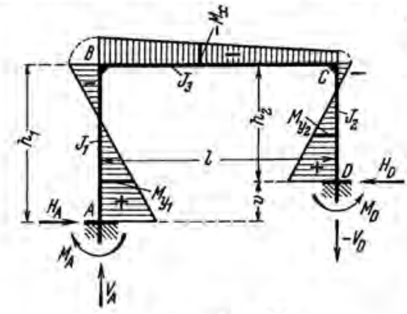
Constants:

$$X_1 = \mathfrak{L}\, n_{11} + \mathfrak{R}\, n_{21} \qquad M_A = X_3 - X_1$$
$$X_2 = \mathfrak{L}\, n_{12} + \mathfrak{R}\, n_{22} \qquad M_B = -X_1 \qquad M_C = -X_2$$
$$X_3 = \mathfrak{L}\, n_{13} + \mathfrak{R}\, n_{23}. \qquad M_D = n\, X_3 - X_2;$$

$$V_A = \frac{\mathfrak{S}_r + X_1 - X_2}{l} \qquad V_D = S - V_A; \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C$$

$$H_A = H_D = \frac{X_3}{h_1}; \qquad M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Case 48/6: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degree

Constants:

$$v = h_1 - h_2{}^* \ ; \qquad T = \frac{6 E J_3 \varepsilon t}{l};$$

$$X_1 = T\left[\frac{v}{l}(n_{11} - n_{21}) + \frac{l}{h_1} n_{31}\right]$$

$$X_2 = T\left[\frac{v}{l}(n_{12} - n_{22}) + \frac{l}{h_1} n_{32}\right]$$

$$X_3 = T\left[\frac{v}{l}(n_{13} - n_{23}) + \frac{l}{h_1} n_{33}\right]$$

$$V_A = -V_D = \frac{X_1 - X_2}{l};$$

$$H_A = H_D = \frac{X_3}{h_1}; \qquad M_A = X_3 - X_1 \qquad M_C = -X_2$$
$$M_B = -X_1 \qquad M_D = n\, X_3 - X_2;$$

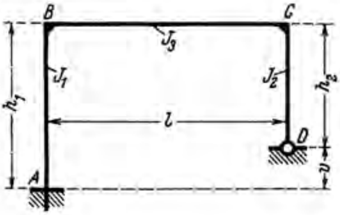
$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

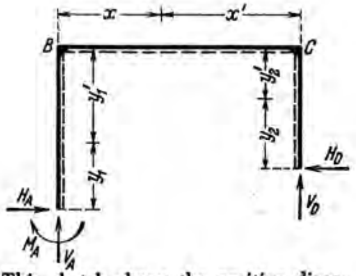
*When $h_2 > h_1$, v becomes negative.

Frame 49

Bent with legs of unequal length. One support fixed, one support hinged.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{l} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{l}; \qquad m = \frac{h_1}{h_2};$$

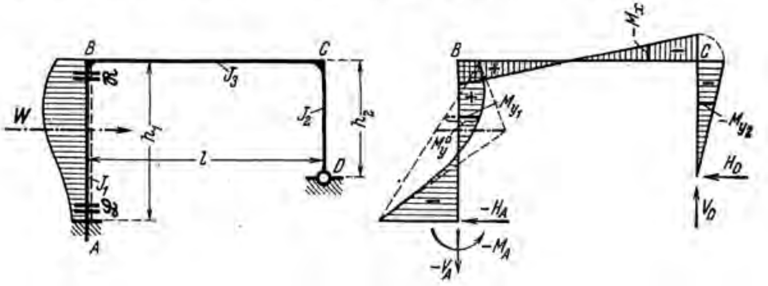
$$N = 3(m k_1 + 1)^2 + 4 k_1 (3 + m^2) + 4 k_2 (3 k_1 + 1);$$

$$n_{11} = \frac{2(m^2 k_1 + 1 + k_2)}{N} \qquad n_{22} = \frac{2(3 k_1 + 1)}{N}$$

$$n_{12} = n_{21} = \frac{3 m k_1 - 1}{N}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 49/1: Left-hand leg loaded by any type of horizontal load



Constants: $\mathfrak{B}_1 = [3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\,k_1 \qquad X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$

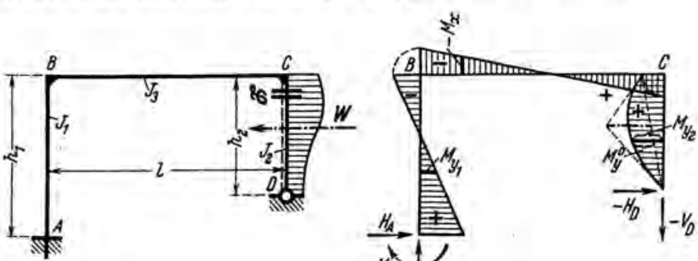
$\mathfrak{B}_2 = [2\,\mathfrak{S}_l - \mathfrak{L}]\,m\,k_1; \qquad X_2 = -\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$

$$M_A = -\,\mathfrak{S}_l + X_1 + m\,X_2 \qquad M_B = X_1 \qquad M_C = -\,X_2;$$

$$V_D = -\,V_A = \frac{X_1 + X_2}{l}; \qquad H_D = \frac{X_2}{h_2} \qquad H_A = -\,(W - H_D);$$

$$M_{y1} = M_y^0 + \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

Case 49/2: Right-hand leg loaded by any type of horizontal load



Constants: $\mathfrak{B}_1 = 3\,m\,\mathfrak{S}_r\,k_1 \qquad X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$

$\mathfrak{B}_2 = 2\,m^2\,\mathfrak{S}_r\,k_1 - \mathfrak{L}\,k_2; \qquad X_2 = -\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$

$$M_A = m\,(\mathfrak{S}_r - X_2) - X_1 \qquad M_B = -\,X_1 \qquad M_C = X_2;$$

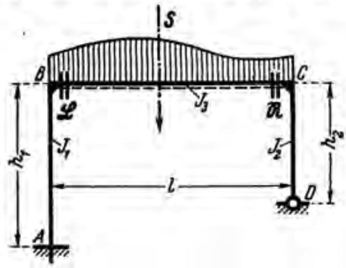
$$V_A = -\,V_D = \frac{X_1 + X_2}{l}; \qquad H_A = \frac{\mathfrak{S}_r - X_2}{h_2} \qquad H_D = -\,(W - H_A);$$

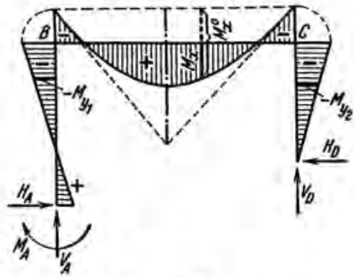
$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h_2} M_C.$$

FRAME 49

See Appendix A. Load Terms, pp. 440-445.

Case 49/3: Girder loaded by any type of vertical load





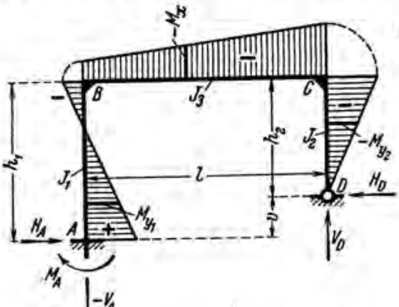
Constants: $X_1 = \mathfrak{L} n_{11} + \mathfrak{R} n_{21}$ $\quad X_2 = \mathfrak{L} n_{12} + \mathfrak{R} n_{22}$.

$$M_A = m X_2 - X_1 \qquad M_B = -X_1 \qquad M_C = -X_2;$$

$$V_A = \frac{\mathfrak{S}_r}{l} + \frac{X_1 - X_2}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l} - \frac{X_1 - X_2}{l}; \qquad H_A = H_D = \frac{X_2}{h_2};$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

Case 49/4: Uniform increase in temperature of the entire frame



E = Modulus of elasticity

ε = Coefficient of thermal expansion

t = Change of temperature in degrees

Constants: $T = \frac{6 E J_3 \varepsilon t}{l}$;

$$X_1 = T\left[\frac{v}{l}(n_{11} - n_{21}) + \frac{l}{h_2} n_{21}\right]$$

$$X_2 = T\left[\frac{v}{l}(n_{12} - n_{22}) + \frac{l}{h_2} n_{22}\right].$$

$$M_A = m X_2 - X_1 \qquad M_B = -X_1 \qquad M_C = -X_2;$$

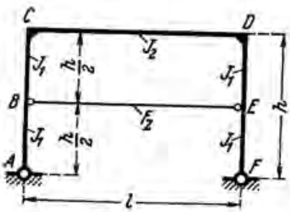
$$V_A = -V_D = \frac{X_1 - X_2}{l}; \qquad H_A = H_D = \frac{X_2}{h_2};$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

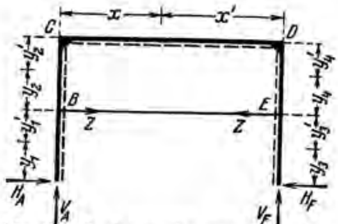
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 50

Symmetrical two-hinged bent with tie-rod at mid-height*



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$\left(v = \frac{h}{2}\right), \qquad k = \frac{J_2}{J_1} \cdot \frac{h}{l} \qquad L = \frac{6\,J_2}{v^2 F_Z} \cdot \frac{E}{E_Z}^{*} \quad ;$$

$$K_1 = 7\,k + 24 \qquad K_2 = 5\,k + 12 \qquad K_3 = 2\,k + 6\,;$$

$$N = K_1 + 8\,(2\,k + 3) \cdot \frac{L}{k}\,.$$

E = Modulus of elasticity of the material of the frame
E_z = Modulus of elasticity of the tie rod
F_z = Cross-sectional area of the tie rod

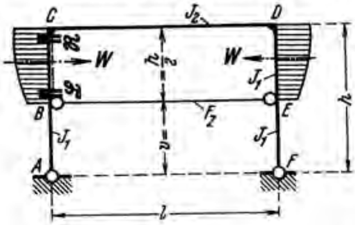
Note: The tie rod force becomes negative (compression) for cases 50/1, 3, 5, 5a, 7, 8, 12, and 13. This is only admissible if there exist simultaneous other loads which cause sufficiently large tensile forces to make the resultant tie rod force positive (tension).

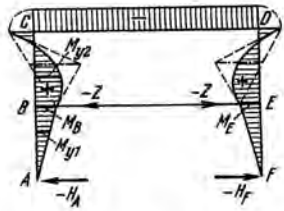
* All formulas for frame 50 are valid for a compression tie if L is set equal to zero (L = 0).

FRAME 50

See Appendix A, Load Terms, pp. 440-445.

Case 50/1: **Upper halves of both legs loaded by any type of load from the outside (Symmetrical load)**





$$M_B = M_E = \frac{\mathfrak{S}_r \cdot K_2 L / k - \mathfrak{L}(K_3 + L) - \mathfrak{R}(2L - k)}{N} = X_1$$

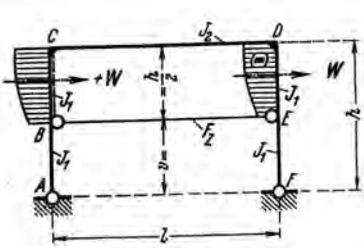
$$V_A = V_F = 0\,;$$

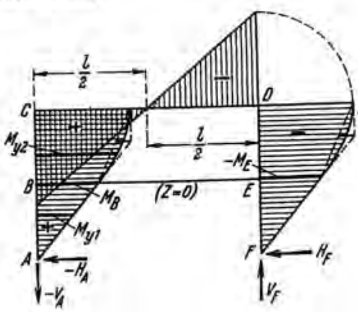
$$M_C = M_D = -\frac{\mathfrak{S}_r \cdot 6L + \mathfrak{L}(2L - k) + \mathfrak{R} \cdot 4(L + k)}{N} = X_2\,;$$

$$Z = -\frac{\mathfrak{S}_r - 2X_1 + X_2}{v} = -\frac{\mathfrak{S}_r \cdot K_1 + \mathfrak{L} \cdot K_2 - \mathfrak{R} \cdot 6k}{vN}{}^{*} \qquad H_A = H_F = -\frac{X_1}{v}\,;$$

$$M_{y1} = \frac{y_1}{v} X_1 \qquad M_{y2} = M_y^0 + \frac{y_2'}{v} M_B + \frac{y_2}{v} M_C \qquad M_x = X_2\,.$$

Case 50/2: **Upper halves of both legs loaded by any type of load, acting from the left (Antisymmetrical load)**





$$M_B = -M_E = Wv \qquad M_C = -M_D = Wv + \mathfrak{S}_l\,;$$

$$Z = 0\,; \qquad H_F = -H_A = W \qquad V_F = -V_A = \frac{Wh + 2\mathfrak{S}_l}{l}\,;$$

$$M_{y1} = W \cdot y_1 \qquad M_{y2} = M_y^0 + Wv + \frac{y_2}{v}\mathfrak{S}_l \qquad M_x = \frac{x' - x}{l} M_C\,.$$

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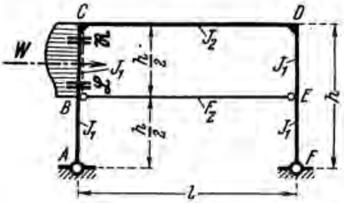
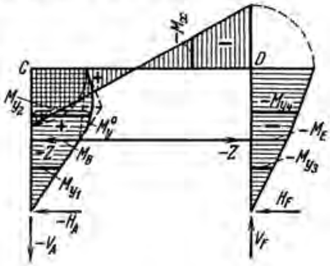
*See footnote on page 191

Case 50/3: Left-hand leg above the tie rod loaded by any type of horizontal load See Appendix A, Load Terms, pp. 440-445.





$v=\frac{h}{2}$

Constants X_1 and X_2 same as case 50/1, p. 176.

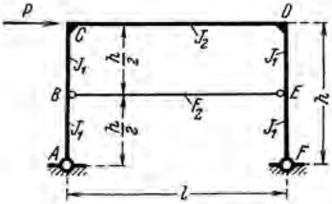
$$\left.\begin{matrix}M_B\\M_E\end{matrix}\right\rangle=\frac{X_1}{2}\pm\frac{Wv}{2}\qquad \left.\begin{matrix}M_C\\M_D\end{matrix}\right\rangle=\frac{X_2}{2}\pm\frac{Wv+\mathfrak{S}_l}{2};\qquad \left.\begin{matrix}H_A\\H_F\end{matrix}\right\rangle=-\frac{X_1}{h}\mp\frac{W}{2};$$

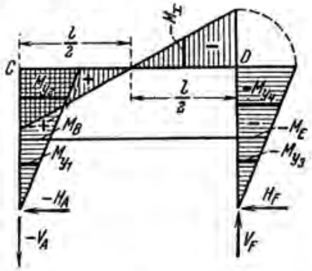
$$Z=-\frac{\mathfrak{S}_r-2X_1+X_2}{h}=-\frac{\mathfrak{S}_r\cdot K_1+\mathfrak{L}\cdot K_2-\mathfrak{R}\cdot 6k}{hN}\,^*;\qquad V_F=-V_A=\frac{Wv+\mathfrak{S}_l}{l};$$

$$M_{y1}=(-H_A)\,y_1\qquad M_{y3}=-H_F\cdot y_3\qquad M_{y4}=-H_F\cdot v-(H_F+Z)\,y_4$$

$$M_{y2}=M_y^0+\frac{y_2'}{v}M_B+\frac{y_2}{v}M_C\qquad\qquad M_x=\frac{x'}{l}M_C+\frac{x}{l}M_D\,.$$

Case 50/4: Horizontal concentrated load at the girder





$$H_F=-H_A=\frac{P}{2}\qquad\qquad Z=0\qquad\qquad V_F=-V_A=\frac{Ph}{l};$$

$$M_B=-M_E=\frac{Ph}{4}\qquad\qquad M_C=-M_D=\frac{Ph}{2};\qquad\qquad M_x=Ph\left(\frac{1}{2}-\frac{x}{l}\right)$$

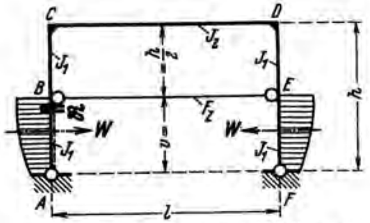
$$M_{y1}=\frac{P}{2}y_1\qquad M_{y2}=\frac{P}{2}(v+y_2)\qquad M_{y3}=-\frac{P}{2}y_3\qquad M_{y4}=-\frac{P}{2}(v+y_4).$$

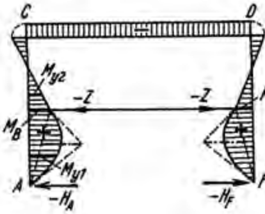
* For $Z=0$ see note p. 175. Z for case 50/3 equals one-half of Z for case 50/1.

FRAME 50

See Appendix A, Load Terms, pp. 440-445.

Case 50/5: Lower halves of both legs loaded by any type of load from the outside (Symmetrical load)





$$M_B = M_E = \frac{\mathfrak{S}_l \cdot K_2 L/k - \mathfrak{R}(K_3 + L)}{N} = X_1 \qquad H_A = H_F = -\frac{\mathfrak{S}_r + X_1}{v};$$

$$M_C = M_D = -\frac{\mathfrak{S}_l \cdot 6L + \mathfrak{R}(2L - k)}{N} = X_2;$$

$$Z = -\frac{\mathfrak{S}_l - 2X_1 + X_2}{v} = -\frac{\mathfrak{S}_l \cdot K_1 + \mathfrak{R} \cdot K_2}{vN}^{*}; \qquad V_A = V_F = 0;$$

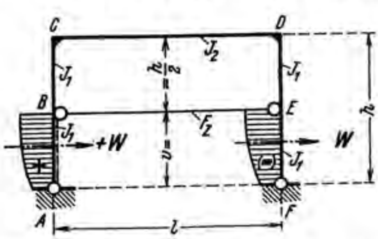
$$M_{y1} = M_y^0 + \frac{y_1}{v} M_B \qquad M_{y2} = \frac{y_2'}{v} M_B + \frac{y_2}{v} M_C \qquad M_x = X_2.$$

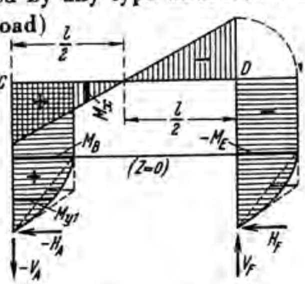
Special case 50/5a: Pair of concentrated loads at B and E acting from the outside $\quad (\mathfrak{S}_l = Wv;\ \mathfrak{S}_r = 0;\ \mathfrak{R} = 0;\ M_y^0 = 0)$

$$X_1 = \frac{Wv \cdot K_2 L}{kN} \qquad X_2 = -\frac{Wv \cdot 6L}{N}; \qquad Z = -W \cdot \frac{K_1}{N}{}^{*)} \qquad H_A = H_F = -\frac{X_1}{v}.$$

All other formulas as above.

Case 50/6: Lower halves of both legs loaded by any type of load, acting from the left (Antisymmetrical load)



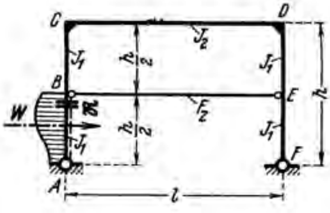


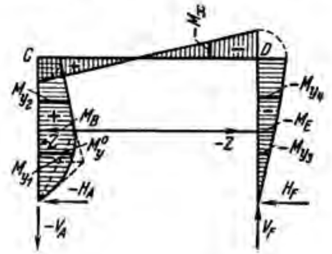
$$M_B = M_C = -M_D = -M_E = \mathfrak{S}_l; \qquad H_F = -H_A = W; \qquad V_F = -V_A = \frac{2\mathfrak{S}_l}{l};$$

$$Z = 0; \qquad M_{y1} = M_y^0 + \frac{y_1}{v}\mathfrak{S}_l \qquad M_{y2} = \mathfrak{S}_l \qquad M_x = \frac{x' - x}{l}\mathfrak{S}_l.$$

* See footnote p. 175 for Z negative.

Case 50/7: Left-hand leg below the tie rod loaded by any type of horizontal load See Appendix A, Load Terms, pp. 440-445.





$v = \frac{h}{2}$

Constants X_1 and X_2 same as case 50/5, p. 178.

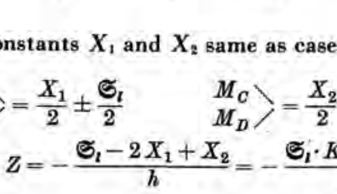
$$\left.\begin{matrix} M_B \\ M_E \end{matrix}\right\rangle = \frac{X_1}{2} \pm \frac{\mathfrak{S}_l}{2} \qquad \left.\begin{matrix} M_C \\ M_D \end{matrix}\right\rangle = \frac{X_2}{2} \pm \frac{\mathfrak{S}_l}{2}; \qquad \left.\begin{matrix} H_A \\ H_F \end{matrix}\right\rangle = -\frac{\mathfrak{S}_r + X_1}{h} \mp \frac{W}{2};$$

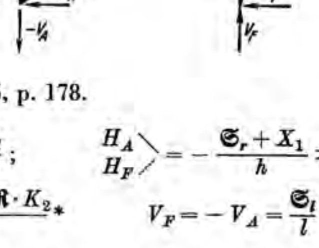
$$Z = -\frac{\mathfrak{S}_l - 2X_1 + X_2}{h} = -\frac{\mathfrak{S}_l \cdot K_1 + \mathfrak{R} \cdot K_2}{hN}{}^{*} \qquad V_F = -V_A = \frac{\mathfrak{S}_l}{l};$$

$$M_{y1} = M_y^0 + \frac{y_1}{v} M_B \qquad M_{y2} = \frac{y_2'}{v} M_B + \frac{y_2}{v} M_C \qquad M_x = \frac{x'}{l} M_C + \frac{x}{l} M_D$$

$$M_{y3} = -H_F \cdot y_3 \qquad M_{y4} = -H_F \cdot v - (H_F + Z)\, y_4 .$$

Case 50/8: Horizontal concentrated load acting from the left at the tie rod





$$\left.\begin{matrix} M_B \\ M_E \end{matrix}\right\rangle = \frac{Pv}{2}\left(\frac{L K_2}{kN} \pm 1\right) \qquad \left.\begin{matrix} M_C \\ M_D \end{matrix}\right\rangle = \frac{Pv}{2}\left(-\frac{6L}{N} \pm 1\right);$$

$$Z = -Pv \cdot \frac{K_1}{N}{}^{*}; \qquad H_A = -\frac{M_B}{v} \qquad H_F = \frac{-M_E}{v}; \qquad V_F = -V_A = \frac{Pv}{l}.$$

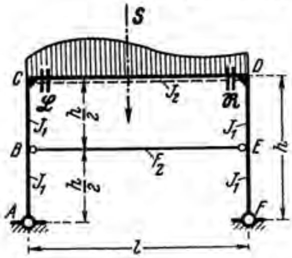
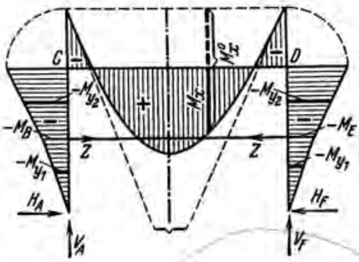
$M_{y1} = \frac{y_1}{v} M_B$ All other formulas same as case 50/7.

* For Z negative see note p. 175. Z for case 50/7 is one-half Z for case 50/5.

FRAME 50

Case 50/9: Girder loaded by any type of vertical load

See Appendix A, Load Terms, pp. 440-445.

$$v = \frac{h}{2}$$

$$M_B = M_E = -(\mathfrak{L}+\mathfrak{R})\cdot\frac{2L-k}{kN} \qquad M_C = M_D = -(\mathfrak{L}+\mathfrak{R})\cdot\frac{4(L+k)}{kN};$$

$$Z = \frac{6(\mathfrak{L}+\mathfrak{R})}{vN}; \qquad H_A = H_F = -\frac{M_B}{v}; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_F = \frac{\mathfrak{S}_l}{l};$$

$$M_{y1} = \frac{y_1}{v}M_B \qquad M_{y2} = \frac{y_2'}{v}M_B + \frac{y_2}{v}M_C \qquad M_x = M_x^0 + M_C.$$

Special case 50/9a: Symmetrical load

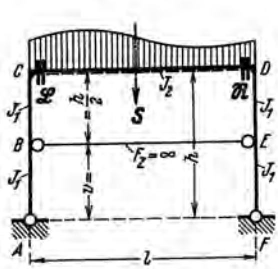
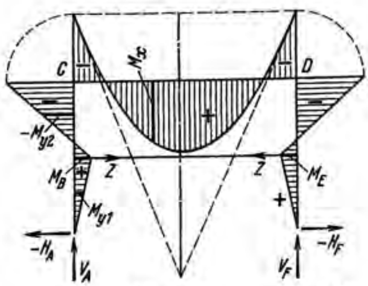
$\mathfrak{R} = \mathfrak{L}$ $\qquad (\mathfrak{L}+\mathfrak{R}) = 2\mathfrak{L}.$ $\qquad$ All other formulas same as above

Special case 50/9b: Antisymmetrical load

$\mathfrak{R} = -\mathfrak{L}$ $\qquad (\mathfrak{L}+\mathfrak{R}) = 0;$ $\qquad M_B = M_C = M_D = M_E = 0;$ $\qquad Z = 0.$

Note: This case is identical with case 39/6, p. 140.

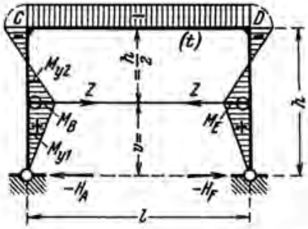
Case 50/10: Girder loaded by any type of vertical load—fully rigid tie ($L = 0$)

$$M_B = M_E = +\frac{(\mathfrak{L}+\mathfrak{R})}{K_1} \qquad M_C = M_D = -\frac{4(\mathfrak{L}+\mathfrak{R})}{K_1}; \qquad Z = \frac{6(\mathfrak{L}+\mathfrak{R})}{vK_1}.$$

All other formulas same as case 50/9.

Case 50/11: Uniform increase in temperature of the girder[1]



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

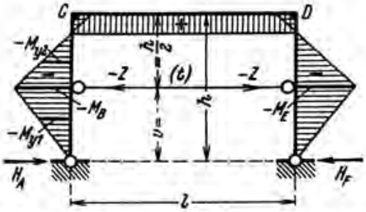
Constants: $T = \frac{3 E J_1 l \cdot \varepsilon t}{N v^2}$

$M_B = M_E = +T(3k + 6 - L)$
$M_C = M_D = -T(5k + 2L);$

$$Z = \frac{T}{v}(11k + 12) \qquad H_A = H_F = -\frac{M_B}{v} \qquad V_A = V_F = 0; \qquad v = \frac{h}{2};$$

$$M_{y1} = \frac{y_1}{v} M_B \qquad M_{y2} = \frac{y_2'}{v} M_B + \frac{y_2}{v} M_C \qquad M_x = M_C.$$

Case 50/12: Uniform increase in temperature of the tie rod[1]



E, ε, t and constant T same as case 50/11.

$M_B = M_E = -T \cdot K_2$
$M_C = M_D = +T \cdot 6k;$
$Z = -\frac{T}{v} \cdot 8(2k + 3).$*

All other formulas same as case 50/11.

Case 50/13: Uniform increase in temperature of the entire frame[1]
Superposition of cases 50/11 and 50/12.

$$M_B = M_C = -T(K_3 + L) \qquad M_C = M_D = -T(2L - k); \qquad Z = -\frac{T}{v} \cdot K_2^{*}$$

All other formulas same as case 50/11.

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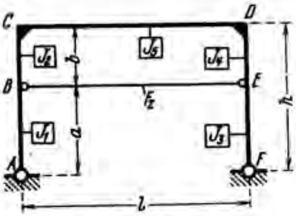
[1]A uniform temperature increase in one or both legs does not cause stress. All signs are to be reversed for a temperature decrease.

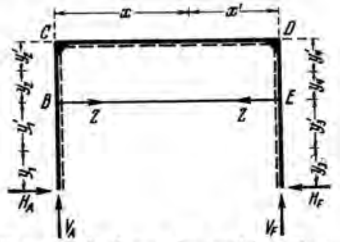
* See footnote p. 175 for Z negative.

Frame 51

Two-hinged bent with horizonal tie-rod at any elevation. Moments of inertia of the legs change discontinuously at tie-rod elevation*



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1=\frac{J_5}{J_1}\cdot\frac{a}{l}\qquad k_2=\frac{J_5}{J_2}\cdot\frac{b}{l}\qquad k_3=\frac{J_5}{J_3}\cdot\frac{a}{l}\qquad k_4=\frac{J_5}{J_4}\cdot\frac{b}{l};$$

$$\alpha=\frac{a}{h}\qquad \beta=\frac{b}{h}\qquad (\alpha+\beta=1);\qquad L=\frac{6J_5}{b^2F_Z}\cdot\frac{E_R}{E_Z}^{*};$$

$$B=2\alpha(k_1+k_2)+k_2\qquad D=3+(2+\alpha)k_4$$
$$C=(\alpha+2)k_2+3\qquad E=k_4+2\alpha(k_3+k_4)$$
$$R_1=2(k_2+3+k_4)+L\qquad R_2=\alpha(B+E)+(C+D)$$
$$K=C+D;\qquad N=R_1R_2-K^2=\alpha^2\cdot G+R_2\cdot L;$$
$$G=4(k_1+k_3)(k_2+3+k_4)+3(k_2+k_4)(k_2+4+k_4).$$

E_R = Modulus of elasticity of the material of the frame*
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

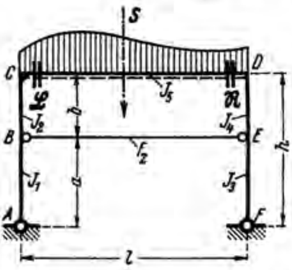
Note: The moment diagrams are based on the assumption $J_3, J_4 > J_1, J_2$.

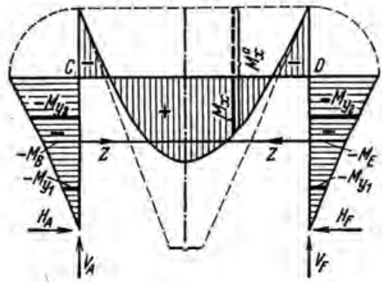
*To prevent confusion with the constant E, the subscript R was added to the modulus of elasticity E.

All formulas for Frame 51 are valid for a compression tie if L is set equal to zero ($L = 0$). See, for example, case 51/2, p. 183.

Case 51/1: Girder loaded by any type of vertical load

See Appendix A, Load Terms, pp. 440-445.





Constants: $X_1 = (\mathfrak{L} + \mathfrak{R}) \cdot \dfrac{R_1 - K}{N}$ $\qquad X_2 = (\mathfrak{L} + \mathfrak{R}) \cdot \dfrac{\alpha(B+E)}{N}$.

$$M_B = M_E = -\alpha X_1 \qquad M_C = M_D = -(X_1 + X_2);$$

$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_F = \frac{\mathfrak{S}_l}{l} \qquad H_A = H_F = \frac{X_1}{h} \qquad Z = \frac{X_2}{b};$$

$$M_{y1} = -H_A y_1 \qquad M_{y2} = M_B - (H_A + Z) y_2 \qquad M_x = M_x^0 + M_C.$$

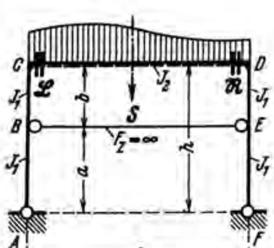
Special case 51/1a: Symmetrical load

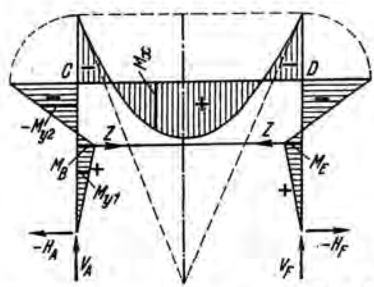
$\mathfrak{R} = \mathfrak{L} \qquad (\mathfrak{L} + \mathfrak{R}) = 2\mathfrak{L}.$ **All other formulas same as above**

Special case 51/1b: Antisymmetrical load

$\mathfrak{R} = -\mathfrak{L} \qquad (\mathfrak{L} + \mathfrak{R}) = 0; \qquad M_B = M_C = M_D = M_E = 0; \qquad Z = 0.$

Case 51/2: Girder loaded by any type of vertical load—fully rigid tie ($L = 0$)





$$M_B = M_E = +(\mathfrak{L} + \mathfrak{R}) \cdot \frac{k_2 + k_4}{G} \qquad M_C = M_D = -2(\mathfrak{L} + \mathfrak{R}) \cdot \frac{k_1 + k_2 + k_3 + k_4}{G};$$

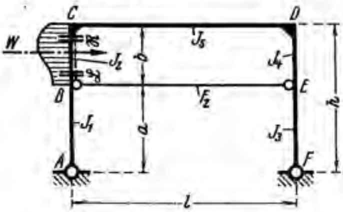
$$H_A = H_F = -\frac{M_B}{a}; \qquad Z = \frac{(\mathfrak{L} + \mathfrak{R})h}{ab} \cdot \frac{B+E}{G}; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_F = \frac{\mathfrak{S}_l}{l};$$

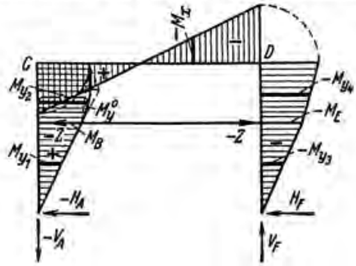
$$M_{y1} = (-H_A) y_1 \qquad M_{y2} = M_B - (H_A + Z) y_2 \qquad M_x = M_x^0 + M_C.$$

FRAME 51

See Appendix A, Load Terms, pp. 440-445.

Case 51/3: **Left-hand leg above the tie rod loaded by any type of horizontal load**





Constants:

$$\mathfrak{B}_1 = W a\,(B + C) + \mathfrak{S}_l C + (\alpha\,\mathfrak{L} + \mathfrak{R})\,k_2$$
$$\mathfrak{B}_2 = 3\,W a\,(k_2 + 1) + \mathfrak{S}_l (2\,k_2 + 3) + \mathfrak{R}\,k_2$$

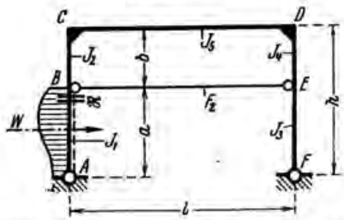
$$X_1 = \frac{+\mathfrak{B}_1 R_1 - \mathfrak{B}_2 K}{N} \qquad X_2 = \frac{-\mathfrak{B}_1 K + \mathfrak{B}_2 R_2}{N}.$$

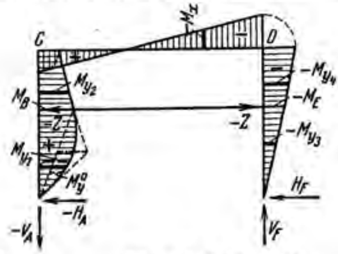
$$V_F = -V_A = \frac{W a + \mathfrak{S}_l}{l} \qquad H_F = \frac{X_1}{h} \qquad H_A = -(W - H_F) \qquad Z = \frac{X_2}{b}^{*};$$

$$M_B = W a - \alpha X_1 \qquad M_C = W a + \mathfrak{S}_l - (X_1 + X_2) \qquad M_D = -(X_1 + X_2)$$
$$M_E = -\alpha X_1 \qquad M_{y3} = -H_F\,y_3 \qquad M_{y4} = -H_F a - (H_F + Z)\,y_4$$

$$M_{y1} = (-H_A)\,y_1 \qquad M_{y2} = M_y^0 + \frac{y_2'}{b} M_B + \frac{y_2}{b} M_C \qquad M_x = \frac{x'}{l} M_C + \frac{x}{l} M_D.$$

Case 51/4: **Left-hand leg below the tie rod loaded by any type of horizontal load**





Constants: $\qquad \mathfrak{B}_1 = \mathfrak{S}_l (B + C) + \alpha\,\mathfrak{R}\,k_1 \qquad \mathfrak{B}_2 = 3\,\mathfrak{S}_l (k_2 + 1).$

The formulas for X_1 and X_2 same as above.

$$V_F = -V_A = \frac{\mathfrak{S}_l}{l} \qquad M_B = \mathfrak{S}_l - \alpha X_1 \qquad M_C = \mathfrak{S}_l - (X_1 + X_2)$$

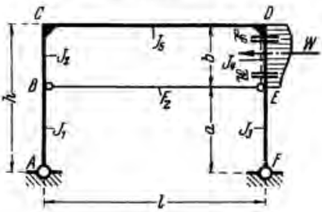
$$M_{y1} = M_y^0 + \frac{y_1}{a} M_B \qquad M_{y2} = \frac{y_2'}{b} M_B + \frac{y_2}{b} M_C.$$

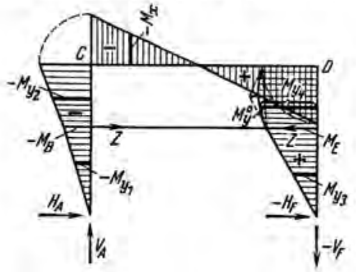
The formulas for H_F, H_A, Z^*, M_D, M_E, M_x, M_{y3} and M_{y4} are the same as above.

*See footnote on page 188.

See Appendix A, Load Terms, pp. 440-445.

Case 51/5: Right-hand leg above the tie rod loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_1 = W a\,(D + E) + \mathfrak{S}_r D + (\mathfrak{L} + \alpha\,\mathfrak{R})\,k_4 \qquad X_1 = \frac{+\mathfrak{B}_1 R_1 - \mathfrak{B}_2 K}{N}$$

$$\mathfrak{B}_2 = 3\,W a\,(1 + k_4) + \mathfrak{S}_r (3 + 2\,k_4) + \mathfrak{L}\,k_4 \qquad X_2 = \frac{-\mathfrak{B}_1 K + \mathfrak{B}_2 R_2}{N}.$$

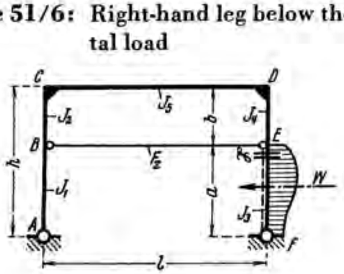
$$V_A = -V_F = \frac{W a + \mathfrak{S}_r}{l} \qquad H_A = \frac{X_1}{h} \qquad H_F = -(W - H_A) \qquad Z = \frac{X_2}{b}^{*};$$

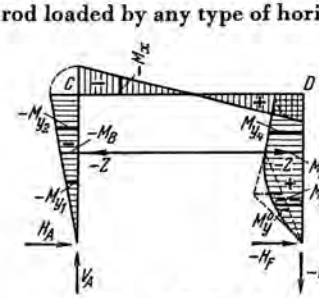
$$M_B = -\alpha X_1 \qquad M_C = -(X_1 + X_2) \qquad M_D = W a + \mathfrak{S}_r - (X_1 + X_2)$$

$$M_E = W a - \alpha X_1 \qquad M_{y1} = -H_A y_1 \qquad M_{y2} = -H_A a - (H_A + Z)\,y_2$$

$$M_x = \frac{x'}{l} M_C + \frac{x}{l} M_D \qquad M_{y3} = (-H_F)\,y_3 \qquad M_{y4} = M_y^0 + \frac{y_4}{b} M_D + \frac{y_4'}{b} M_E.$$

Case 51/6: Right-hand leg below the tie rod loaded by any type of horizontal load





Constants: $\mathfrak{B}_1 = \mathfrak{S}_r (D + E) + \alpha\,\mathfrak{L}\,k_3 \qquad \mathfrak{B}_2 = 3\,\mathfrak{S}_r (1 + k_4)$.

The formulas for X_1 and X_2 are the same as above.

$$V_A = -V_F = \frac{\mathfrak{S}_r}{l} \qquad M_D = \mathfrak{S}_r - (X_1 + X_2) \qquad M_E = \mathfrak{S}_r - \alpha X_1$$

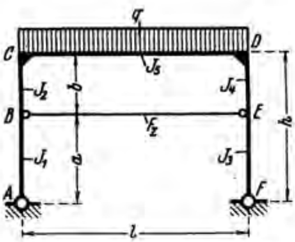
$$M_{y3} = M_y^0 + \frac{y_3}{a} M_E \qquad M_{y4} = \frac{y_4}{b} M_D + \frac{y_4'}{b} M_E.$$

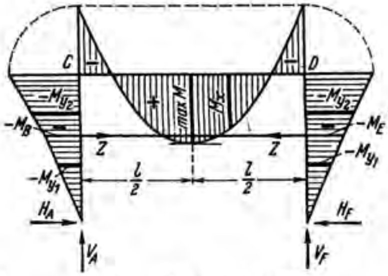
The formulas for H_A, H_F, Z^*, M_B, M_C, M_{y1}, M_{y2} and M_x are the same as above.

*See footnote on page 188.

FRAME 51

Case 51/7: Full uniform load acting at the girder





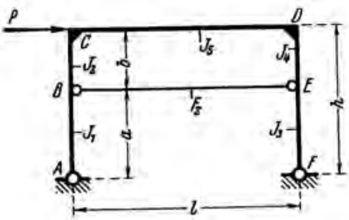
Constants: $X_1 = \dfrac{q\,l^2}{2} \cdot \dfrac{R_1 - K}{N}$ $\qquad X_2 = \dfrac{q\,l^2}{2} \cdot \dfrac{\alpha\,(B + E)}{N}.$

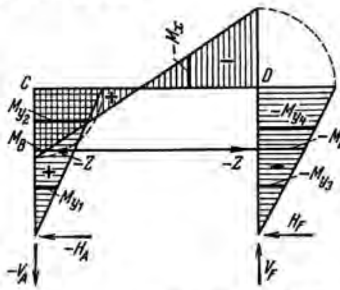
$$M_B = M_E = -\,\alpha\,X_1 \qquad M_C = M_D = -\,(X_1 + X_2) \qquad \max M = \frac{q\,l^2}{8} + M_C\,;$$

$$V_A = V_F = \frac{q\,l}{2} \qquad H_A = H_F = \frac{X_1}{h} \qquad Z = \frac{X_2}{b};$$

$$M_{y1} = -\,H_A\,y_1 \qquad M_{y2} = -\,(H_A + Z)\,y_2 + M_B \qquad M_x = \frac{q\,x\,x'}{2} + M_C.$$

Case 51/8: Horizontal concentrated load at the girder





$$H_A = -\,P \cdot \frac{(D + \alpha E)\,R_1 - D\,K}{N} \qquad H_F = P \cdot \frac{(\alpha\,B + C)\,R_1 - C\,K}{N}$$

$$(H_F - H_A = P) \qquad Z = \frac{P\,a}{b} \cdot \frac{C\,E - B\,D^*}{N}\;; \qquad V_F = -\,V_A = \frac{P\,h}{l};$$

$$M_B = (-\,H_A)\,a \qquad M_C = (-\,H_A)\,h - Z\,b$$

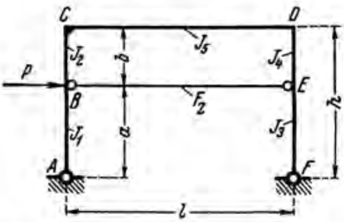
$$M_E = -\,H_F\,a \qquad M_D = -\,H_F\,h - Z\,b;$$

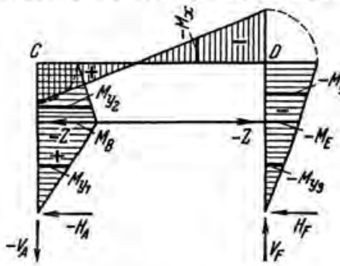
$$M_{y1} = (-\,H_A)\,y_1 \qquad M_x = \frac{x'}{l}\,M_C + \frac{x}{l}\,M_D \qquad M_{y3} = -\,H_F\,y_3$$

$$M_{y2} = (-\,H_A)\,(a + y_2) - Z\,y_2 \qquad M_{y4} = -\,H_F\,(a + y_4) - Z\,y_4.$$

* Z can also become negative. See footnote 2, p. 188.

Case 51/9: Horizontal concentrated load from the left acting at the tie rod





Constants:

$$X_1 = Pa\cdot\frac{(B+C)\,R_1 - 3\,(k_2+1)\,K}{N} \qquad X_2 = Pa\cdot\frac{3\,(k_2+1)\,R_2 - (B+C)\,K}{N}.$$

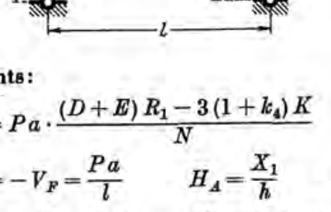
$$V_F = -V_A = \frac{Pa}{l} \qquad H_F = \frac{X_1}{h} \qquad Z = \frac{X_2}{b}{}^{*}\;; \qquad M_x = \frac{x'}{l}M_C + \frac{x}{l}M_D\,;$$

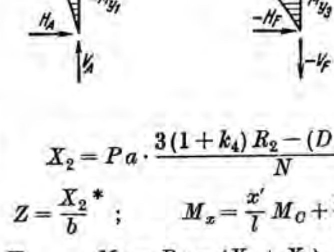
$$H_A = -(P - H_F); \qquad M_B = Pa - \alpha X_1 \qquad M_C = Pa - (X_1 + X_2)$$

$$M_D = -(X_1 + X_2) \qquad M_E = -\alpha X_1; \qquad M_{y1} = (-H_A)\,y_1 \qquad M_{y3} = -H_F\,y_3$$

$$M_{y2} = (P - H_F)\,a - (H_F + Z)\,y_2 \qquad M_{y4} = -H_F\,a - (H_F + Z)\,y_4.$$

Case 51/10: Horizontal concentrated load from the right acting at the tie rod





Constants:

$$X_1 = Pa\cdot\frac{(D+E)\,R_1 - 3\,(1+k_4)\,K}{N} \qquad X_2 = Pa\cdot\frac{3\,(1+k_4)\,R_2 - (D+E)\,K}{N}.$$

$$V_A = -V_F = \frac{Pa}{l} \qquad H_A = \frac{X_1}{h} \qquad Z = \frac{X_2}{b}{}^{*}\;; \qquad M_x = \frac{x'}{l}M_C + \frac{x}{l}M_D\,;$$

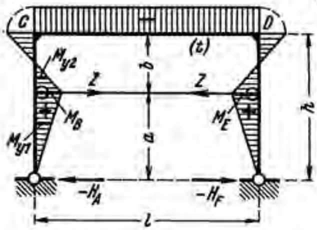
$$H_F = -(P - H_A); \qquad M_E = Pa - \alpha X_1 \qquad M_D = Pa - (X_1 + X_2)$$

$$M_B = -\alpha X_1 \qquad M_C = -(X_1 + X_2); \qquad M_{y1} = -H_A\,y_1 \qquad M_{y3} = (-H_F)\,y_3$$

$$M_{y2} = -H_A\,a - (H_A + Z)\,y_2 \qquad M_{y4} = (P - H_A)\,a - (H_A + Z)\,y_4.$$

*See footnote on page 188.

Case 51/11: Uniform increase in temperature of the girder[1]



E_R = Modulus of elasticity of the material
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants: $T = \frac{6\,E_R\,J_5\,\varepsilon\,t}{h\,N}$;

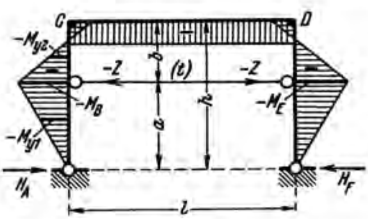
X [illegible] T [illegible] $R_1 - \frac{K}{\beta}\Big)$

[illegible] $\frac{R_2}{\beta} - K\Big)$.

$M_B = M_E = -\alpha X_1$ $\qquad M_C = M_D = -(X_1 + X_2)$; $\qquad H_A = H_F = \frac{X_1}{h}$;

$Z = \frac{X_2}{b}$; $\qquad M_{y1} = -H_A\,y_1$ $\qquad M_{y2} = -(H_A + Z)\,y_2 + M_B$.

Case 51/12: Uniform increase in temperature of the tie rod[1]



E_R, ε, t and constant T same as case 51/11.

$X_1 = +T\cdot\frac{K}{\beta}$ $\qquad X_2 = -T\cdot\frac{R_2}{\beta}$.

All other formulas same as case 51/11.[2]

Case 51/13: Uniform increase in temperature of the entire frame[1] (Superposition of the cases 51/11 and 51/12)

$X_1 = +T\cdot R_1$ $\qquad X_2 = -T\cdot K$.

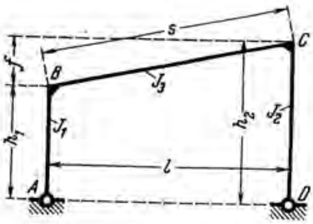
All other formulas same as case 50/11.

[1] Uniform temperature change in one or both legs produces no moments or forces. With a decrease in temperature all moments and forces reverse their directions.

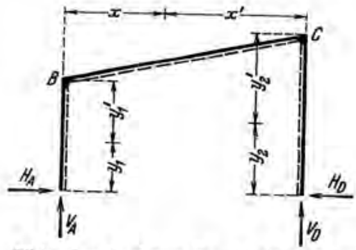
[2] For case 51/12, as well as case 51/3, 4, 5, 6, 9, and 10 Z becomes negative, i.e., the tie rod is stressed in compression, and can become negative in cases 51/8 and 13. If the tie rod (e.g. a slack structure) is not in a condition to take compression, then this condition is only valid if the collective compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

Frame 52

Two-hinged rigid frame shed. Hinges at same elevation.



Shape of Frame
Dimensions and Notations



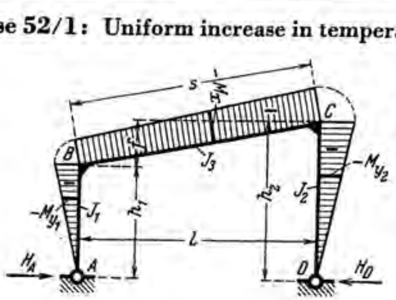
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{s} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{s}\,; \qquad n = \frac{h_2}{h_1};$$

$$B = 2\,(k_1 + 1) + n \qquad C = 1 + 2\,n\,(1 + k_2)\,; \qquad N = B + n\,C\,.$$

Case 52/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degre

Constant:

$$X = \frac{6\,E\,J_3\,\varepsilon\,t\,l}{s\,h_1\,N}\,.$$

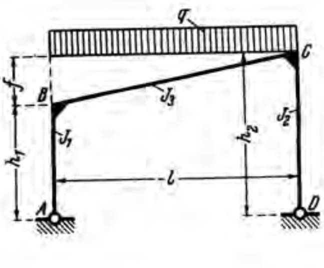
$$M_B = -X \qquad M_C = -n\,X\,; \qquad H_A = H_D = \frac{X}{h_1}\,;$$

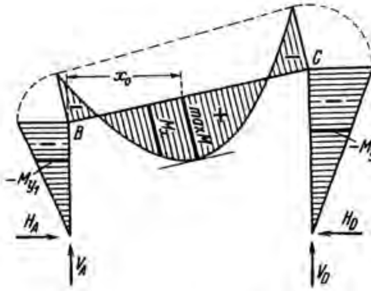
$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C\,.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 52

Case 52/2: Vertical rectangular load on the girder



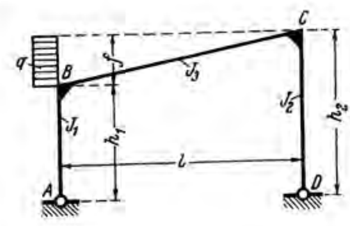


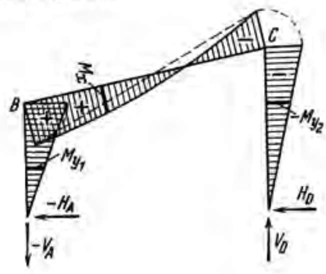
Constant: $X = \frac{q l^2}{4} \cdot \frac{1+n}{N}$. $M_B = -X$ $M_C = -nX$;

$$V_A = V_D = \frac{ql}{2}; \quad H_A = H_D = \frac{X}{h_1}; \quad x_0 = \frac{l}{2} - \frac{X(n-1)}{ql};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \quad M_x = \frac{q x x'}{2} + \frac{x'}{l} M_B + \frac{x}{l} M_C \quad M_{y2} = \frac{y_2}{h_2} M_C.$$

Case 52/3: Horizontal rectangular load on the girder





Constants:

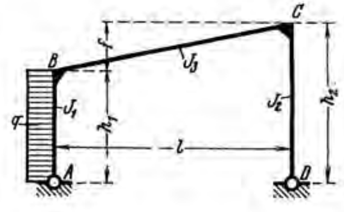
$$\varphi = \frac{h_1}{f}; \quad X = \frac{p f^2}{4} \cdot \frac{4B\varphi + 1 + n}{N}. \quad M_B = q f h_1 - X \quad M_C = -nX;$$

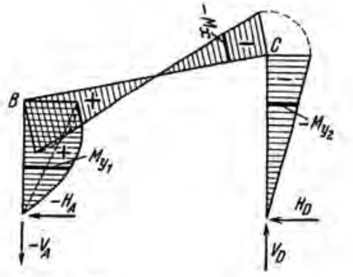
$$V_D = -V_A = \frac{q f^2 (2\varphi + 1)}{2l}; \quad H_D = \frac{X}{h_1} \quad H_A = -(qf - H_D);$$

$$M_{y1} = \frac{y_1}{h_1} M_B \quad M_x = \frac{q x x'}{2} \cdot \frac{f^2}{l^2} + \frac{x'}{l} M_B + \frac{x}{l} M_C \quad M_{y2} = \frac{y_2}{h_2} M_C.$$

Case 52/4: Inclined rectangular load qs acting normally to the girder over its entire length s (wind load). Superposition of cases 52/2 and 52/3 for the same load q

Case 52/5: Rectangular load on the left leg



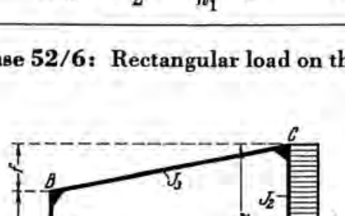


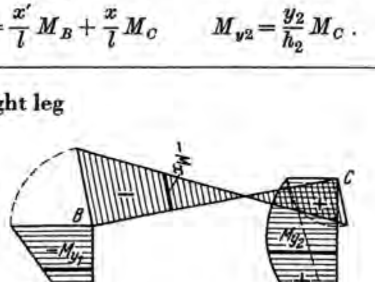
Constant: $X = \frac{q h_1^2}{4} \cdot \frac{2B + k_1}{N}$, $M_B = \frac{q h_1^2}{2} - X$ $M_C = -nX$;

$V_D = -V_A = \frac{q h_1^2}{2l}$; $H_D = \frac{X}{h_1}$ $H_A = -(q h_1 - H_D)$;

$M_{y1} = \frac{q y_1 y_1'}{2} + \frac{y_1}{h_1} M_B$ $M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C$ $M_{y2} = \frac{y_2}{h_2} M_C$.

Case 52/6: Rectangular load on the right leg





Constant: $X = \frac{q h_2^2}{4} \cdot \frac{2C + n k_2}{N}$. $M_B = -X$ $M_C = \frac{q h_2^2}{2} - nX$;

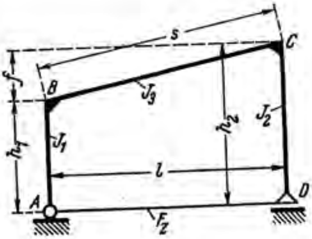
$V_A = -V_D = \frac{q h_2^2}{2l}$ $H_A = \frac{X}{h_1}$ $H_D = -(q h_2 - H_A)$;

$M_{y1} = \frac{y_1}{h_1} M_B$ $M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C$ $M_{y2} = \frac{q y_2 y_2'}{2} + \frac{y_2}{h_2} M_C$.

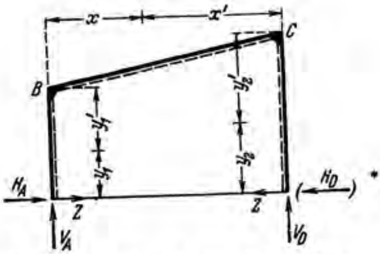
Cases 54/2 and 54/3, p. 198, as well as 54/4 and 54/5, p. 199, are valid for frame 52 with the simplification $r = 0$ (because of $v = 0$).

Frame 53

Rigid frame shed with horizontal tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1}\cdot\frac{h_1}{s} \qquad k_2 = \frac{J_3}{J_2}\cdot\frac{h_2}{s}; \qquad n = \frac{h_2}{h_1};$$

$$B = 2(k_1+1)+n \qquad C = 1+2n(1+k_2); \qquad N = B + nC;$$

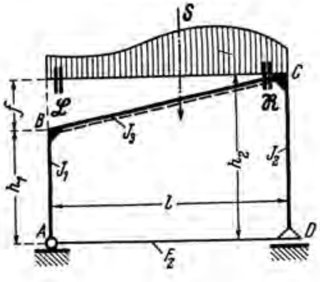
$$L = \frac{6J_3}{h_1^2 F_Z}\cdot\frac{E}{E_Z}\cdot\frac{l}{s}; \qquad N_Z = N + L.$$

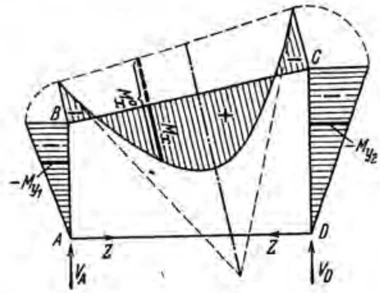
E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

*H_D occurs when the hinged support is at D.

See Appendix A, Load Terms, pp. 440-445.

Case 53/1: Girder loaded by any type of vertical load (Hinged support at A or D)



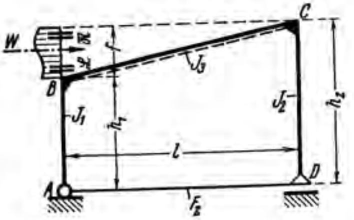


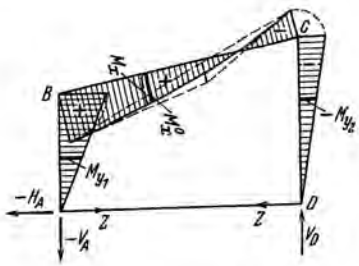
$$Z = \frac{\mathfrak{L} + n\,\mathfrak{R}}{h_1 N_Z}; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l};$$

$$M_B = -Z h_1 \qquad M_C = -Z h_2;$$

$$M_{y1} = -Z y_1 \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = -Z y_2 .$$

Case 53/2: Girder loaded by any type of horizontal load (Hinged support at A)





$$Z = W \frac{B}{N_Z} + \frac{\mathfrak{L} + n\,\mathfrak{R}}{h_1 N_Z}; \qquad V_D = -V_A = \frac{W h_1 + \mathfrak{S}_l}{l};$$

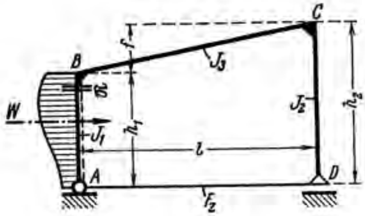
$$H_A = -W; \qquad M_B = (W - Z)\,h_1 \qquad M_C = -Z h_2;$$

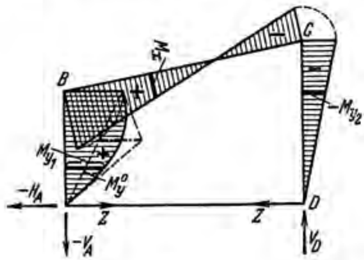
$$M_{y1} = (W - Z)\,y_1 \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = -Z y_2 .$$

FRAME 53

See Appendix A, Load Terms, pp. 440-445.

Case 53/3: Left-hand leg loaded by any type of horizontal load (Hinged support at A)



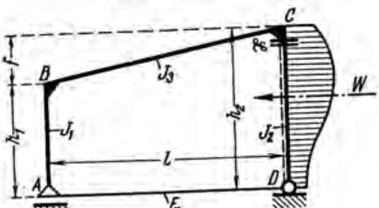


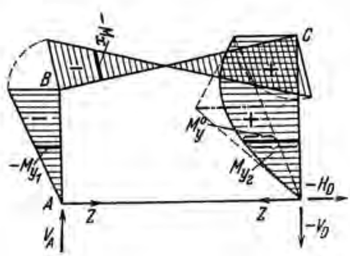
$$Z = \frac{B\,\mathfrak{S}_l + \mathfrak{R}\,k_1}{h_1 N_Z}; \qquad V_D = -V_A = \frac{\mathfrak{S}_l}{l};$$

$$H_A = -W; \qquad M_B = \mathfrak{S}_l - Z h_1 \qquad M_C = -Z y_2.$$

$$M_{y1} = \mathbf{M}_y^0 + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = -Z y_2.$$

Case 53/4: Right-hand leg loaded by any type of horizontal load (Hinged support at D)





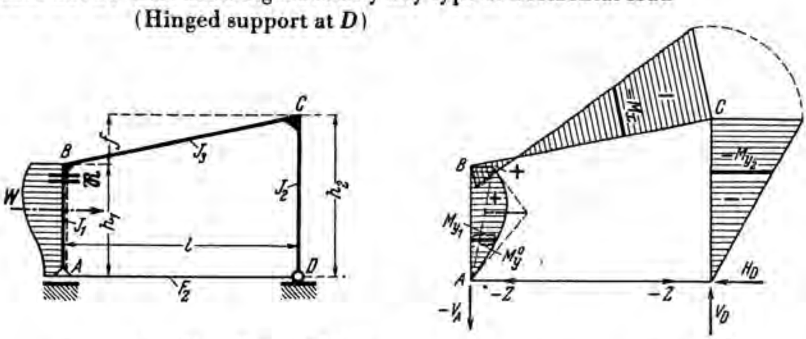
$$Z = \frac{C\,\mathfrak{S}_r + n\,\mathfrak{L}\,k_2}{h_1 N_Z}; \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l};$$

$$H_D = -W; \qquad M_B = -Z h_1 \qquad M_C = \mathfrak{S}_r - Z h_2;$$

$$M_{y1} = -Z y_1 \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \mathbf{M}_y^0 + \frac{y_2}{h_2} M_C.$$

See Appendix A, Load Terms, pp. 440-445.

Case 53/5: Left-hand leg loaded by any type of horizontal load (Hinged support at D)

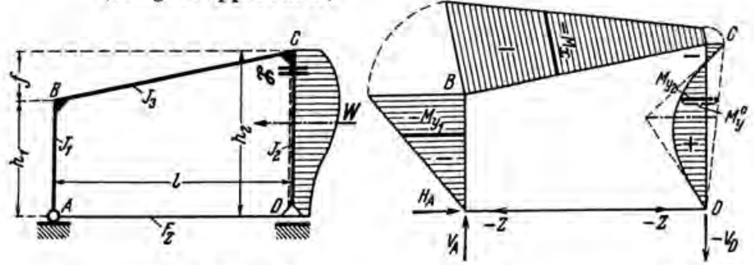


$$Z = -\left(W\frac{nC}{N_Z} + \frac{B\mathfrak{S}_r - \mathfrak{R}k_1}{h_1 N_Z}\right)^* ; \qquad V_D = -V_A = \frac{\mathfrak{S}_l}{l} ;$$

$$H_D = W ; \qquad M_B = -\mathfrak{S}_r - Z h_1 \qquad M_C = -(W + Z) h_2 ;$$

$$M_{y1} = M_y^0 + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C .$$

Case 53/6: Right-hand leg loaded by any type of horizontal load (Hinged support at A)



$$Z = -\left(W\frac{B}{N_Z} + \frac{C\mathfrak{S}_l - n\mathfrak{L}k_2}{h_1 N_Z}\right)^* ; \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l} ;$$

$$H_A = W ; \qquad M_B = -(W + Z) h_1 \qquad M_C = -\mathfrak{S}_l - Z h_2 ;$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h_2} M_C .$$

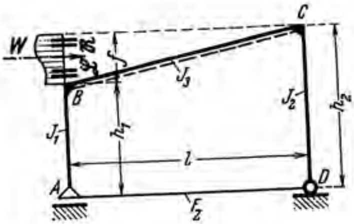
---

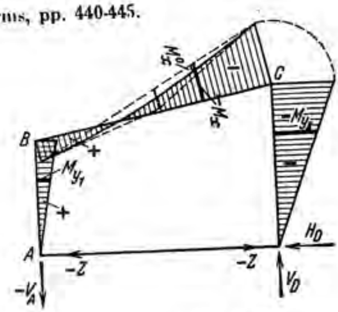
* For the above two loading conditions and case 53/7 (p. 196) and for decrease in temperature (p. 196 bottom) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

FRAME 53

Case 53/7: Girder loaded by any type of horizontal load (Hinged support at D)

See Appendix A, Load Terms, pp. 440-445.



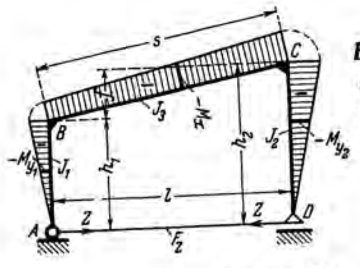


$$Z = -\left(W\frac{nC}{N_Z} - \frac{\mathfrak{L} + n\,\mathfrak{R}}{h_1 N_Z}\right)^* ; \qquad V_D = -V_A = \frac{W h_1 + \mathfrak{S}_l}{l} ;$$

$$H_D = W ; \qquad M_B = (-Z)\,h_1 ; \qquad M_C = -(W + Z)\,h_2 ;$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C .$$

Case 53/8: Uniform increase in temperature of the entire frame (Hinged support at A or D)



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z = \frac{6 E J_3 \varepsilon t}{h_1^2 N_Z} \cdot \frac{l}{s} ;$$

$$M_B = -Z h_1 \qquad M_C = -Z h_2 ;$$

$$M_{y1} = -Z y_1 \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = -Z y_2 .$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

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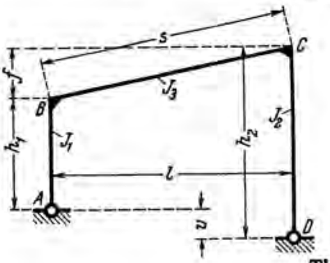
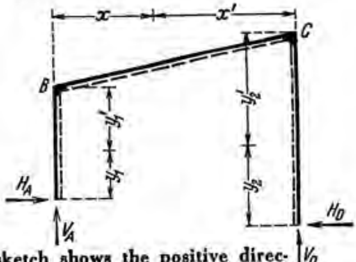
*See footnote on page 195.

Frame 54

Two-hinged rigid frame shed. Hinges at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

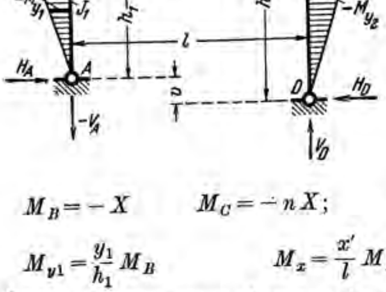
Coefficients:

$$v = h_2 - (h_1 + f)^* ;$$

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{s} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{s}; \qquad n = \frac{h_2}{h_1} \qquad r = \frac{v}{h_1}^* ;$$

$$B = 2(k_1 + 1) + n \qquad C = 1 + 2n(1 + k_2); \qquad N = B + nC.$$

Case 54/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degre

Constant:

$$X = \frac{6 E J_3 \varepsilon t (l^2 + v^2)}{s l h_1 N}.$$

$$M_B = -X \qquad M_C = -nX; \qquad V_D = -V_A = \frac{rX}{l}; \qquad H_A = H_D = \frac{X}{h_1};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

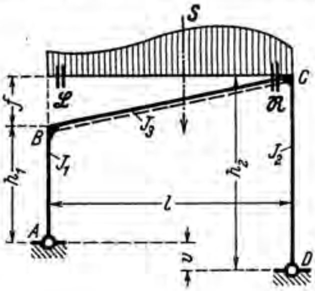
---

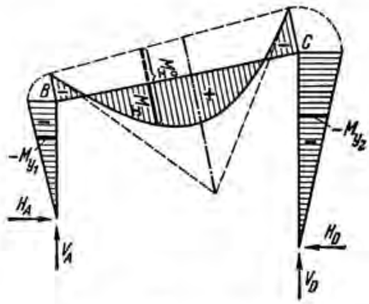
*When $(h_1 + f) > h_2$, v and r become negative.

FRAME 54

See Appendix A, Load Terms, pp. 440-445.

Case 54/2: Girder loaded by any type of vertical load



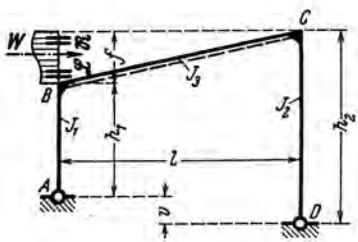


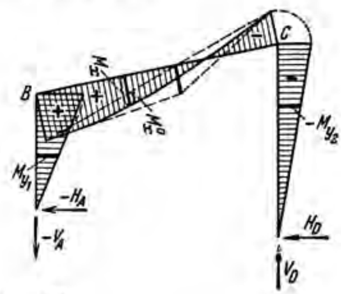
Constant: $X = \dfrac{\mathfrak{L} + n\,\mathfrak{R}}{N}$. $\quad M_B = -X \quad M_C = -n\,X;$

$$V_A = \frac{\mathfrak{S}_r - r\,X\,^*}{l} \qquad V_D = \frac{\mathfrak{S}_l + r\,X\,^*}{l}\;; \qquad H_A = H_D = \frac{X}{h_1};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C\,.$$

Case 54/3: Girder loaded by any type of horizontal load





Constant: $X = \dfrac{W h_1 B + \mathfrak{L} + n\,\mathfrak{R}}{N}$.

$$M_B = W h_1 - X \qquad M_C = -n\,X; \qquad H_A = -(W - H_D);$$

$$V_D = -V_A = \frac{W h_1 + \mathfrak{S}_l + r\,X\,^*}{l}\;; \qquad H_D = \frac{X}{h_1};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C\,.$$

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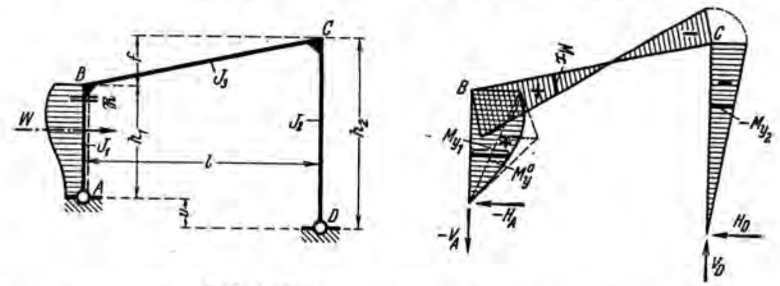
*See footnote on page 199.

See Appendix A, Load Terms, pp. 440-445.

Case 54/4: Left-hand leg loaded by any type of horizontal load

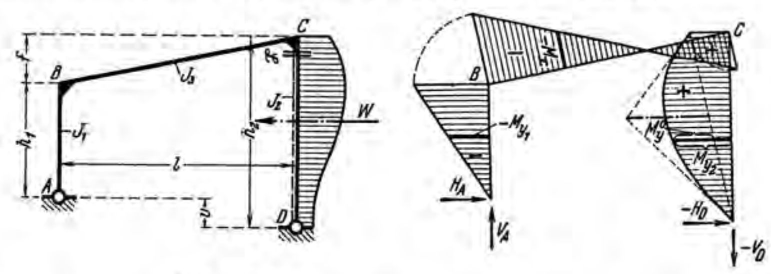


Constant: $X = \dfrac{B\mathfrak{S}_l + \mathfrak{R}\,k_1}{N}$. $\quad M_B = \mathfrak{S}_l - X \quad M_C = -n\,X;$

$$V_D = -V_A = \frac{\mathfrak{S}_l + r\,X\,*}{l} \cdot \qquad H_D = \frac{X}{h_1} \qquad H_A = -(W - H_D);$$

$$M_{y1} = M_y^0 + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

Case 54/5: Right-hand leg loaded by any type of horizontal load



Constant: $X = \dfrac{C\mathfrak{S}_r + n\,\mathfrak{L}\,k_2}{N}$. $\quad M_B = -X \quad M_C = \mathfrak{S}_r - n\,X;$

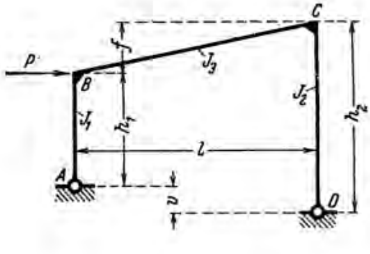
$$V_A = -V_D = \frac{\mathfrak{S}_r - r\,X\,*}{l}; \qquad H_A = \frac{X}{h_1} \qquad H_D = -(W - H_A);$$

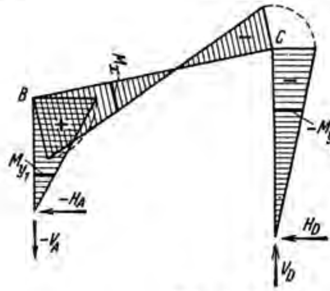
$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h_2} M_C.$$

---

* If A and D are at the same elevation set $v = 0$ and $r = 0$, hence the term containing X disappears in the expressions for V_A and V_D. See frame 52 and note, p. 191.

Case 54/6: Horizontal concentrated load at B





$$H_A = -P\frac{nC}{N} \qquad H_D = P\frac{B}{N}; \qquad V_D = -V_A = \frac{Ph_1 + H_D v}{l};$$

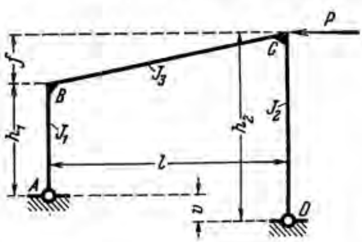
$$M_B = (-H_A)\,h_1 \qquad M_C = -H_D h_2;$$

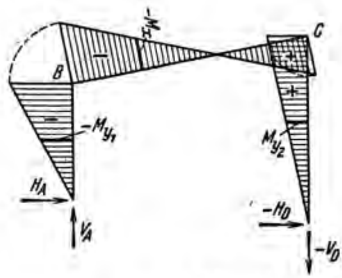
$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

Special case 54/6a: Supports at same elevation ($v = 0$; frame 52)

$V_D = -V_A = Ph_1/l.$ All other formulas as above.

Case 54/7: Horizontal concentrated load at C





$$H_A = P\frac{nC}{N} \qquad H_D = -P\frac{B}{N}; \qquad V_A = -V_D = \frac{Ph_2 - H_A v}{l};$$

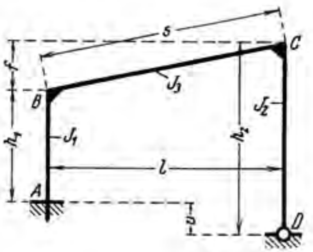
$$M_B = -H_A h_1 \qquad M_C = (-H_D)\,h_2.$$

Special case 54/7a: Supports at same elevation ($v = 0$; frame 52)

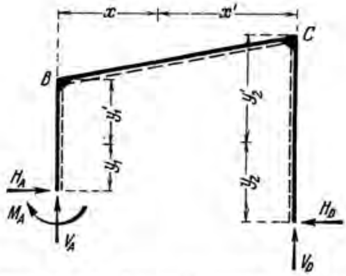
$V_A = -V_D = Ph_2/l.$ All other formulas as above.

Frame 55

Rigid frame shed. One support fixed, one support hinged; supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{s} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{s}; \qquad m = \frac{h_1}{h_2} \qquad \varphi = \frac{f}{h_2};$$

$$N = 3\,(m\,k_1 + 1)^2 + 4\,k_1\,(3 + m^2) + 4\,k_2\,(3\,k_1 + 1);$$

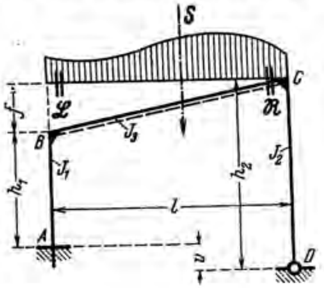
$$n_{11} = \frac{2\,(m^2\,k_1 + 1 + k_2)}{N} \qquad n_{22} = \frac{2\,(3\,k_1 + 1)}{N}$$

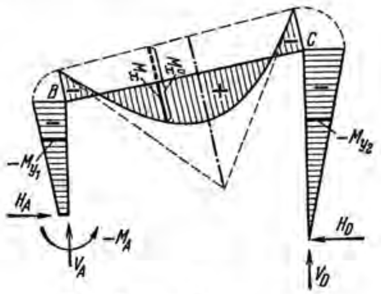
$$n_{12} = n_{21} = \frac{3\,m\,k_1 - 1}{N}.$$

FRAME 55

See Appendix A, Load Terms, pp. 440-445.

Case 55/1: Girder loaded by any type of vertical load





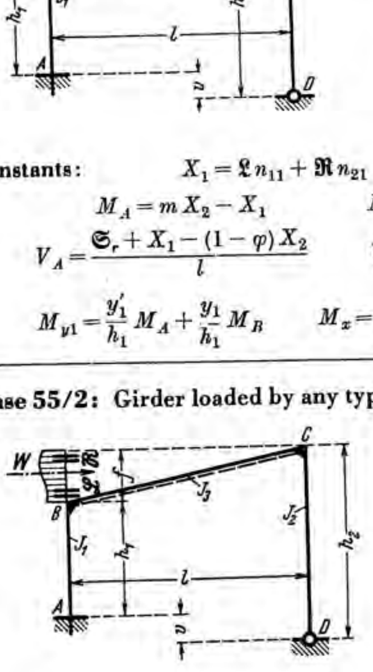
Constants: $X_1 = \mathfrak{L} n_{11} + \mathfrak{R} n_{21}$ $\qquad X_2 = \mathfrak{L} n_{12} + \mathfrak{R} n_{22}$.

$$M_A = m X_2 - X_1 \qquad M_B = -X_1 \qquad M_C = -X_2;$$

$$V_A = \frac{\mathfrak{S}_r + X_1 - (1-\varphi) X_2}{l} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{X_2}{h_2};$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

Case 55/2: Girder loaded by any type of horizontal load



Constants:

$$\mathfrak{B}_1 = 3 W h_1 k_1 - \mathfrak{L} \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

$$\mathfrak{B}_2 = 2 m W h_1 k_1 - \mathfrak{R}; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

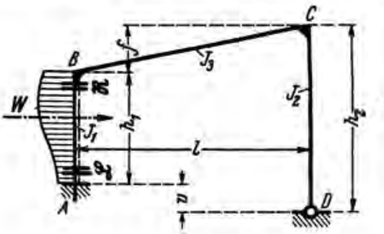
$$M_A = -W h_1 + X_1 + m X_2 \qquad M_B = X_1 \qquad M_C = -X_2;$$

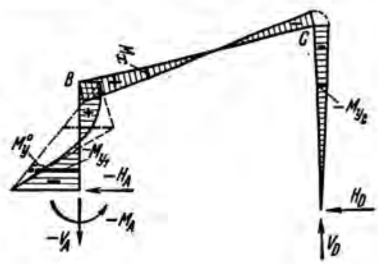
$$V_D = -V_A = \frac{\mathfrak{S}_l + X_1 + (1-\varphi) X_2}{l}; \qquad H_D = \frac{X_2}{h_2} \qquad H_A = -(W - H_D);$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

See Appendix A, Load Terms, pp. 440-445.

Case 55/3: Left-hand leg loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_1 = [3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\,k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

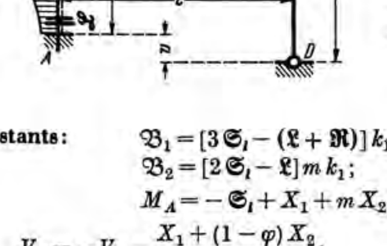
$$\mathfrak{B}_2 = [2\,\mathfrak{S}_l - \mathfrak{L}]\,m\,k_1; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

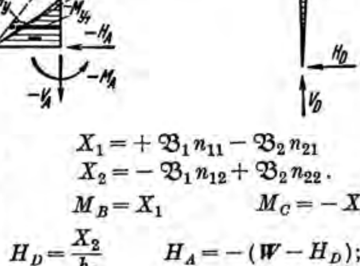
$$M_A = -\mathfrak{S}_l + X_1 + m X_2 \qquad M_B = X_1 \qquad M_C = -X_2;$$

$$V_D = -V_A = \frac{X_1 + (1-\varphi) X_2}{l}; \qquad H_D = \frac{X_2}{h_2} \qquad H_A = -(W - H_D);$$

$$M_{y1} = M_y^0 + \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

Case 55/4: Right-hand leg loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_1 = 3\,m\,\mathfrak{S}_r\,k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

$$\mathfrak{B}_2 = 2\,m^2\,\mathfrak{S}_r\,k_1 - \mathfrak{L}\,k_2; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

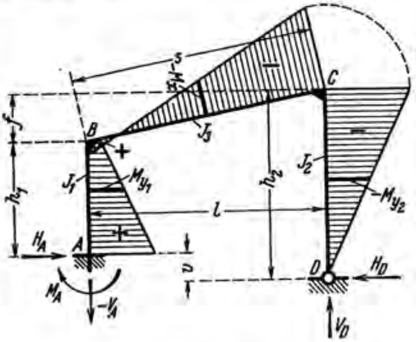
$$M_A = m\,(\mathfrak{S}_r - X_2) - X_1 \qquad M_B = -X_1 \qquad M_C = X_2;$$

$$V_A = -V_D = \frac{\varphi\,\mathfrak{S}_r + X_1 + (1-\varphi) X_2}{l}; \qquad H_A = \frac{\mathfrak{S}_r - X_2}{h_2}$$

$$H_D = -(W - H_A);$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h_2} M_C.$$

ase 55/5: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$v = h_2 - (h_1 + f)^*$;

$T = \frac{6 E J_3 \varepsilon t}{s}$;

$$X_1 = T\left[-\frac{v}{l} n_{11} + \left(\frac{l}{h_2} + \frac{(1-\varphi)v}{l}\right) n_{21}\right]$$

$$X_2 = T\left[-\frac{v}{l} n_{12} + \left(\frac{l}{h_2} + \frac{(1-\varphi)v}{l}\right) n_{22}\right].$$

$$M_A = m X_2 - X_1 \qquad M_B = -X_1 \qquad M_C = -X_2;$$

$$V_D = -V_A = \frac{(1-\varphi)X_2 - X_1}{l}; \qquad H_A = H_D = \frac{X_2}{h_2};$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Special case: Frame 56, see p. 205
$v = 0$ (supports at the same elevation)

Constants: $T' = \frac{6 E J_3 \varepsilon t}{s} \cdot \frac{l}{h_2}$;

$X_1 = T' n_{21} \qquad X_2 = T' n_{22}$.

All the other formulas are the same as above.

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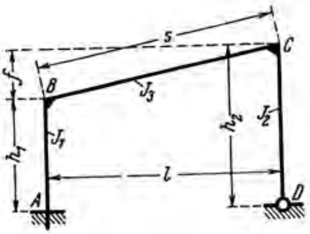
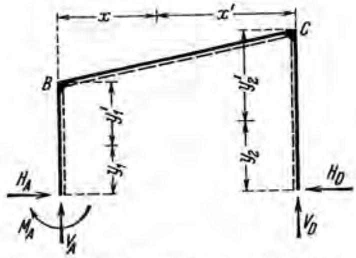
*When $(h_1 + f) > h_2$, v becomes negative.

Frame 56

Rigid frame shed. One support fixed, one support hinged; both supports at the same elevation.



Shape of Frame
Dimensions and Notations



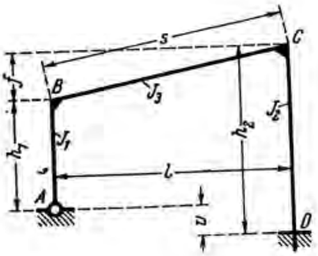
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* are the same as for Frame 55 (pp. 201-203)

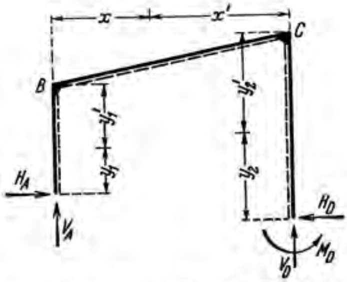
For the formulas for the *temperature change* see p. 204, special case.

Frame 57

Rigid frame shed. One support fixed, one support hinged; supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

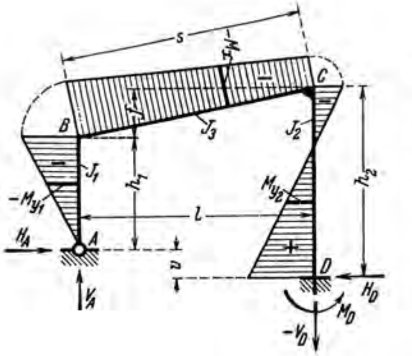
$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{s} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{s}; \qquad n = \frac{h_2}{h_1} \qquad \varphi = \frac{f}{h_1};$$

$$N = 3(1 + n\,k_2)^2 + 4\,k_1(1 + 3\,k_2) + 4\,k_2(3 + n^2);$$

$$n_{11} = \frac{2(1 + 3\,k_2)}{N} \qquad n_{22} = \frac{2(k_1 + 1 + n^2 k_2)}{N}$$

$$n_{12} = n_{21} = \frac{3\,n\,k_2 - 1}{N}.$$

Case 57/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansio
t = Change of temperature in deg

Constants:

$$v = h_2 - (h_1 + f)^* \; ;$$
$$T = \frac{6 E J_3 \varepsilon t}{s};$$

$$X_1 = T\left[\left(\frac{l}{h_1} - \frac{(1+\varphi)v}{l}\right) n_{11} + \frac{v}{l} n_{21}\right]$$

$$X_2 = T\left[\left(\frac{l}{h_1} - \frac{(1+\varphi)v}{l}\right) n_{12} + \frac{v}{l} n_{22}\right].$$

$$M_B = -X_1 \qquad M_C = -X_2 \qquad M_D = n X_1 - X_2;$$

$$V_A = -V_D = \frac{(1+\varphi) X_1 - X_2}{l}; \qquad H_A = H_D = \frac{X_1}{h_1};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Special case: Frame 58, see p. 210
$v = 0$ (supports at the same elevation)

Constants:

$$T' = \frac{6 E J_3 \varepsilon t}{s} \cdot \frac{l}{h_1}; \qquad X_1 = T' n_{11} \qquad X_2 = T' n_{12}.$$

All the other formulas are the same as above.

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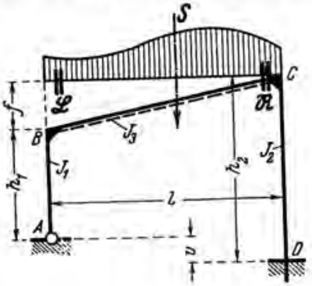
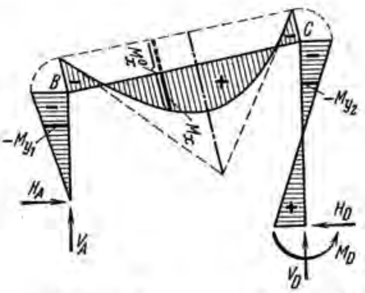
*When $(h_1 + f) > h_2$, v becomes negative.

FRAME 57

See Appendix A, Load Terms, pp. 440-445.

Case 57/2: Girder loaded by any type of vertical load





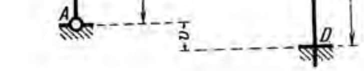
Constants: $X_1 = \mathfrak{L}\, n_{11} + \mathfrak{R}\, n_{21}$ $\quad M_B = -X_1$ $\quad M_C = -X_2$

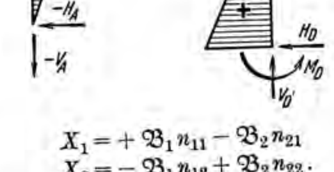
$X_2 = \mathfrak{L}\, n_{12} + \mathfrak{R}\, n_{22}$. $\quad M_D = n X_1 - X_2$;

$$V_A = \frac{\mathfrak{S}_r + (1+\varphi) X_1 - X_2}{l} \qquad V_D = \mathbf{S} - V_A; \qquad H_A = H_D = \frac{X_1}{h_1};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \mathbf{M}_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Case 57/3: Girder loaded by any type of horizontal load





Constants: $\mathfrak{B}_1 = 2n\, W h_2 k_2 - \mathfrak{L}$ $\quad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$

$\mathfrak{B}_2 = 3\, W h_2 k_2 + \mathfrak{R}$; $\quad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}$.

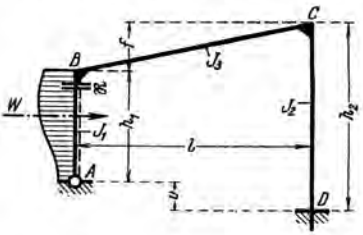
$M_B = X_1 \quad M_C = -X_2 \quad M_D = W h_2 - n X_1 - X_2$;

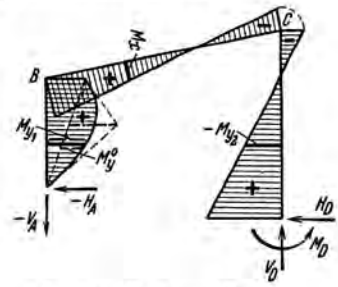
$$V_A = -V_D = \frac{\mathfrak{S}_r - (1+\varphi) X_1 - X_2}{l}; \qquad H_A = -\frac{X_1}{h_1} \qquad H_D = W - \frac{X_1}{h_1};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \mathbf{M}_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

See Appendix A, Load Terms, pp. 440-445.

Case 57/4: Left-hand leg loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_1 = 2n^2\mathfrak{S}_l k_2 - \mathfrak{R} k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

$$\mathfrak{B}_2 = 3n\mathfrak{S}_l k_2; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

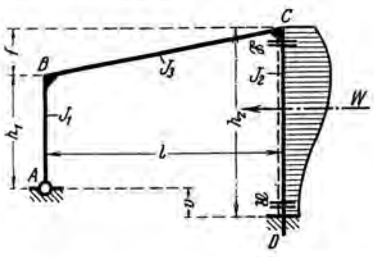
$$M_B = X_1 \qquad M_C = -X_2 \qquad M_D = n(\mathfrak{S}_l - X_1) - X_2;$$

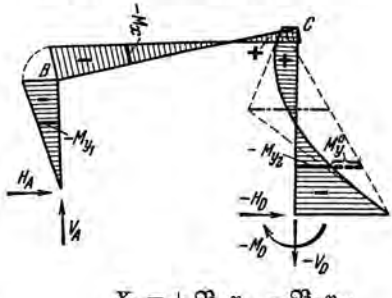
$$V_D = -V_A = \frac{(1+\varphi)X_1 + X_2 - \varphi\mathfrak{S}_l}{l}; \qquad H_D = \frac{\mathfrak{S}_l - X_1}{h_1}$$

$$H_A = -(W - H_D);$$

$$M_{y1} = M_y^0 + \frac{y_1}{h_1}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = \frac{y_2}{h_2}M_C + \frac{y_2'}{h_2}M_D.$$

Case 57/5: Right-hand leg loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_1 = [2\mathfrak{S}_r - \mathfrak{R}]\,n k_2 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

$$\mathfrak{B}_2 = [3\mathfrak{S}_r - (\mathfrak{L} + \mathfrak{R})]\,k_2; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

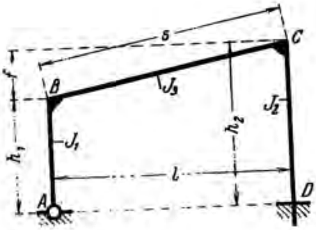
$$M_B = -X_1 \qquad M_C = X_2 \qquad M_D = -\mathfrak{S}_r + nX_1 + X_2;$$

$$V_A = -V_D = \frac{(1+\varphi)X_1 + X_2}{l}; \qquad H_A = \frac{X_1}{h_1} \qquad H_D = -(W - H_A);$$

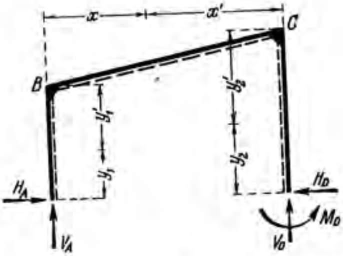
$$M_{y1} = \frac{y_1}{h_1}M_B \qquad M_x = \frac{x'}{l}M_B + \frac{x}{l}M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h_2}M_C + \frac{y_2'}{h_2}M_D.$$

Frame 58

Rigid frame shed. One support fixed, one support hinged; both supports at the same elevation.



Shape of Frame
Dimensions and Notations



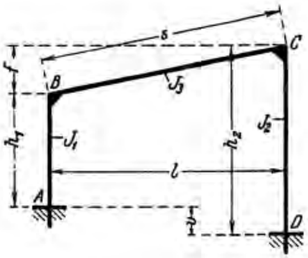
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* are the same as for Frame 57 (pp. 206, 208, and 209)

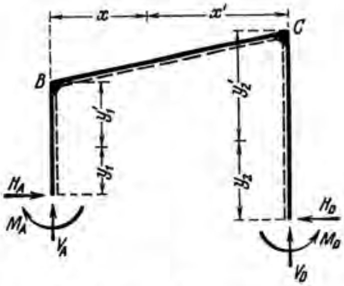
For the formulas for the *temperature change* see p. 207, special case.

Frame 59

Rigid frame shed with fixed supports at different elevation



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{s} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{s}; \qquad n = \frac{h_2}{h_1} \qquad \varphi = \frac{f}{h_1};$$

$$R_1 = 2(3k_1 + 1) \qquad R_2 = 2(1 + 3k_2) \qquad R_3 = 2(k_1 + n^2 k_2);$$

$$N = R_3(k_1 + 1 + k_2) + 6k_1k_2(k_1 + 1 + n + n^2 + n^2 k_2);$$

$$n_{11} = \frac{R_2 R_3 - 9n^2 k_2^2}{3N} \qquad n_{12} = n_{21} = \frac{9nk_1k_2 - R_3}{3N}$$

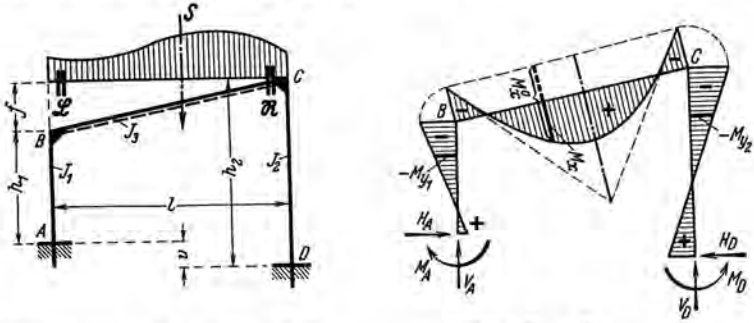
$$n_{22} = \frac{R_1 R_3 - 9k_1^2}{3N} \qquad n_{13} = n_{31} = \frac{k_1 R_2 - nk_2}{N}$$

$$n_{33} = \frac{R_1 R_2 - 1}{3N} \qquad n_{23} = n_{32} = \frac{nk_2 R_1 - k_1}{N}$$

FRAME 59

See Appendix A, Load Terms, pp. 440-445.

Case 59/1: Girder loaded by any type of vertical load



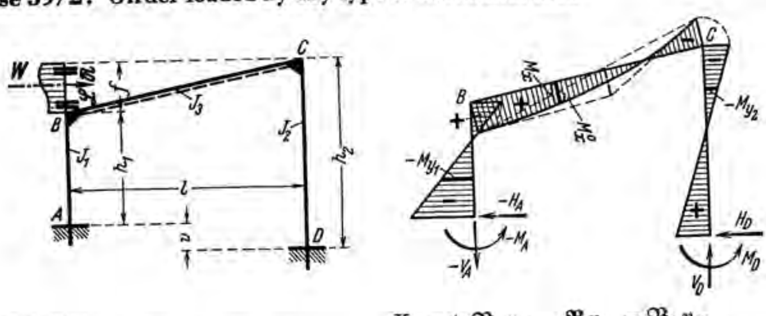
Constants:

$$X_1 = \mathfrak{L}\, n_{11} + \mathfrak{R}\, n_{21} \qquad M_A = X_3 - X_1$$
$$X_2 = \mathfrak{L}\, n_{12} + \mathfrak{R}\, n_{22} \qquad M_B = -X_1 \qquad M_C = -X_2$$
$$X_3 = \mathfrak{L}\, n_{13} + \mathfrak{R}\, n_{23}. \qquad M_D = n\, X_3 - X_2;$$

$$V_A = \frac{\mathfrak{S}_r + X_1 - X_2 + \varphi X_3}{l} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{X_3}{h_1};$$

$$M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Case 59/2: Girder loaded by any type of horizontal load



Constants:

$$X_1 = +\mathfrak{V}_1 n_{11} - \mathfrak{R}\, n_{21} - \mathfrak{V}_3 n_{31}$$
$$\mathfrak{V}_1 = 3\, W h_1 k_1 - \mathfrak{L} \qquad X_2 = -\mathfrak{V}_1 n_{12} + \mathfrak{R}\, n_{22} + \mathfrak{V}_3 n_{32}$$
$$\mathfrak{V}_3 = 2\, W h_1 k_1; \qquad X_3 = -\mathfrak{V}_1 n_{13} + \mathfrak{R}\, n_{23} + \mathfrak{V}_3 n_{33}.$$

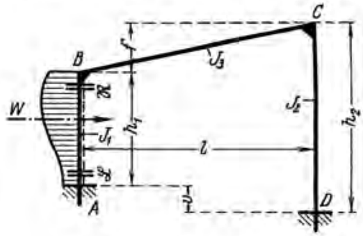
$$M_A = -W h_1 + X_1 + X_3 \qquad M_B = +X_1 \qquad M_C = -X_2 \qquad M_D = n\, X_3 - X_2;$$

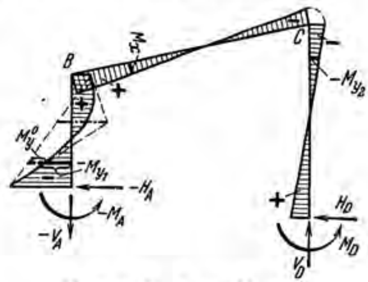
$$V_D = -V_A = \frac{\mathfrak{S}_l + X_1 + X_2 - \varphi X_3}{l}; \qquad H_D = \frac{X_3}{h_1} \qquad H_A = -(W - H_D);$$

$$M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

See Appendix A, Load Terms, pp. 440-445.

Case 59/3: Left-hand leg loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_1 = [3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\,k_1 \qquad \mathfrak{B}_3 = [2\,\mathfrak{S}_l - \mathfrak{L}]\,k_1;$$

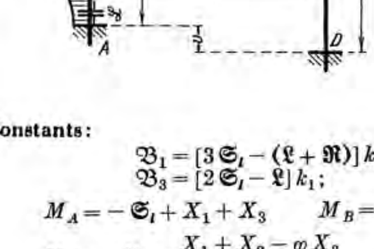
$$X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_3 n_{31} \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_3 n_{32} \qquad X_3 = -\mathfrak{B}_1 n_{13} + \mathfrak{B}_3 n_{33}.$$

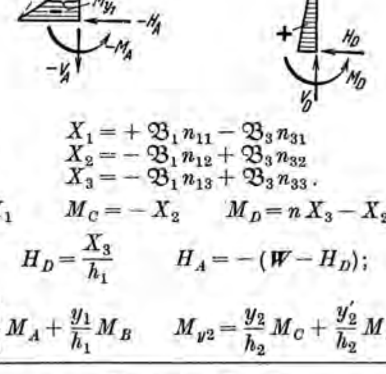
$$M_A = -\mathfrak{S}_l + X_1 + X_3 \qquad M_B = X_1 \qquad M_C = -X_2 \qquad M_D = n\,X_3 - X_2;$$

$$V_D = -V_A = \frac{X_1 + X_2 - \varphi X_3}{l}; \qquad H_D = \frac{X_3}{h_1} \qquad H_A = -(W - H_D);$$

$$M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y1} = \mathbf{M}_y^0 + \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Case 59/4: Right-hand leg loaded by any type of horizontal load





Constants:

$$\mathfrak{B}_2 = [3\,\mathfrak{S}_r - (\mathfrak{L} + \mathfrak{R})]\,k_2 \qquad \mathfrak{B}_3 = [2\,\mathfrak{S}_r - \mathfrak{R}]\,n\,k_2;$$

$$X_1 = -\mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31} \qquad X_2 = +\mathfrak{B}_2 n_{22} - \mathfrak{B}_3 n_{32} \qquad X_3 = -\mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

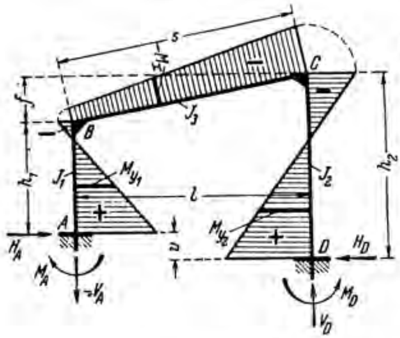
$$M_A = X_3 - X_1 \qquad M_B = -X_1 \qquad M_C = X_2 \qquad M_D = -\mathfrak{S}_r + X_2 + n\,X_3;$$

$$V_A = -V_D = \frac{X_1 + X_2 + \varphi X_3}{l}; \qquad H_A = \frac{X_3}{h_1} \qquad H_D = -(W - H_A);$$

$$M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C \qquad M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_{y2} = \mathbf{M}_y^0 + \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

FRAME 59

Case 59/5: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant:

$$v = h_2 - (h_1 + f)^* ;$$
$$T = \frac{6 E J_3 \varepsilon t}{s};$$

$$X_1 = T\left[\frac{v}{l}(-n_{11} + n_{21}) + \left(\frac{l}{h_1} - \frac{\varphi v}{l}\right) n_{31}\right]$$
$$X_2 = T\left[\frac{v}{l}(-n_{12} + n_{22}) + \left(\frac{l}{h_1} - \frac{\varphi v}{l}\right) n_{32}\right]$$
$$X_3 = T\left[\frac{v}{l}(-n_{13} + n_{23}) + \left(\frac{l}{h_1} - \frac{\varphi v}{l}\right) n_{33}\right]$$

$$M_A = X_3 - X_1 \qquad M_B = -X_1 \qquad M_C = -X_2 \qquad M_D = n X_3 - X_2;$$
$$V_A = -V_D = \frac{X_1 - X_2 + \varphi X_3}{l}; \qquad H_A = H_D = \frac{X_3}{h_1};$$
$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D$$
$$M_x = \frac{x'}{l} M_B + \frac{x}{l} M_C .$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Special case: Frame 60, see p. 215
$v = 0$ (supports at the same elevation)

Constants:

$$T' = \frac{6 E J_3 \varepsilon t}{s} \cdot \frac{l}{h_1}; \qquad X_1 = T' \cdot n_{31} \qquad X_2 = T' \cdot n_{32} \qquad X_3 = T' \cdot n_{33} .$$

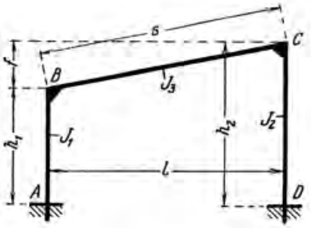
All the other formulas are the same as above.

---

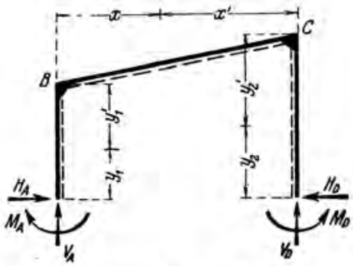
*When $(h_1 + f) > h_2$, v becomes negative.

Frame 60

Rigid frame shed with fixed supports at the same elevation.



Shape of Frame
Dimensions and Notations



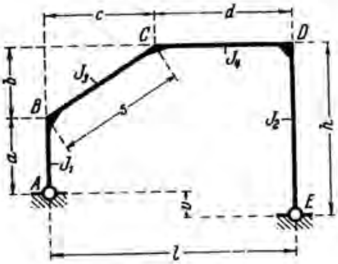
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* are the same as for Frame 59 (pp. 211-213)

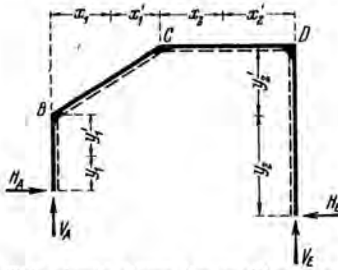
For the formulas for the *temperature change* see p. 214, special case.

Frame 61

Two-hinged bent with one skew corner. Hinges at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_4}{J_1} \cdot \frac{a}{d} \qquad k_2 = \frac{J_4}{J_2} \cdot \frac{h}{d} \qquad k_3 = \frac{J_4}{J_3} \cdot \frac{s}{d}\,;$$

$$\alpha = \frac{a}{h} \qquad \gamma = \frac{c}{l} \qquad \delta = \frac{d}{l} \qquad (\gamma + \delta = 1)\,;$$

$$v = h - (a+b)^{*} \qquad n = \frac{v}{h}^{*} \qquad m = 1 - \delta n\,;$$

$$B = 2\alpha(k_1 + k_3) + m k_3 \qquad C = \alpha k_3 + 2m(k_3 + 1) + 1$$

$$D = m + 2(1 + k_2)\,; \qquad N = \alpha B + mC + D\,.$$

Formulas for moments in all members which are not directly loaded; valid for all loading cases for Frame 61.

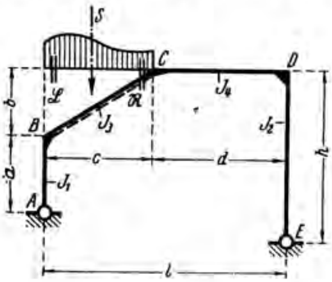
$$M_{x1} = \frac{x_1'}{c} \cdot M_B + \frac{x_1}{c} \cdot M_C \qquad M_{x2} = \frac{x_2'}{d} \cdot M_C + \frac{x_2}{d} \cdot M_D$$

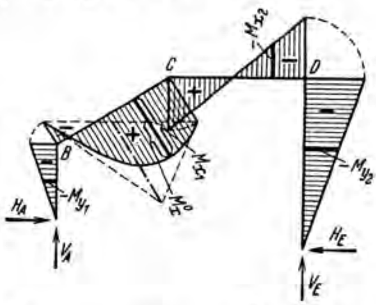
$$M_{y1} = \frac{y_1}{a} \cdot M_B \qquad M_{y2} = \frac{y_2}{h} \cdot M_D\,.$$

*When $(a + b) > h$, v and n become negative.

See Appendix A, Load Terms, pp. 440-445.

Case 61/1: Inclined member loaded by any type of vertical load



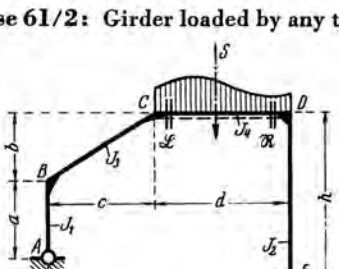


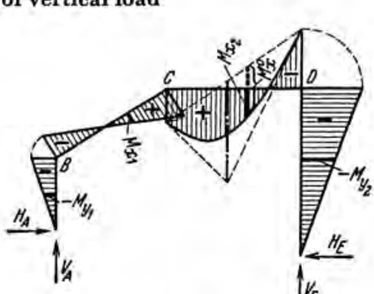
Constant: $\quad X = \dfrac{C\,\delta\,\mathfrak{S}_l + (\alpha\,\mathfrak{L} + m\,\mathfrak{R})\,k_3}{N}$. $\quad M_{x1} = M_x^0 + \dfrac{x_1'}{c} M_B + \dfrac{x_1}{c} M_C$;

$M_B = -\alpha X \qquad M_C = \delta\,\mathfrak{S}_l - m X \qquad M_D = -X$;

$V_E = \dfrac{\mathfrak{S}_l + n X}{l} \qquad V_A = S - V_E$; $\qquad H_A = H_E = \dfrac{X}{h}$.

Case 61/2: Girder loaded by any type of vertical load





Constant: $\quad X = \dfrac{C\,\gamma\,\mathfrak{S}_r + m\,\mathfrak{L} + \mathfrak{R}}{N}$. $\quad M_{x2} = M_x^0 + \dfrac{x_2'}{d} M_C + \dfrac{x_2}{d} M_D$;

$M_B = -\alpha X \qquad M_C = \gamma\,\mathfrak{S}_r - m X \qquad M_D = -X$;

$V_A = \dfrac{\mathfrak{S}_r - n X}{l} \qquad V_E = S - V_A$; $\qquad H_A = H_E = \dfrac{X}{h}$.

Case 61/3: Vertical concentrated load *P* at *C*

Substitute in case 61/1:

$S = P \qquad \mathfrak{S}_l = P c; \qquad \mathfrak{L} = \mathfrak{R} = 0 \qquad M_x^0 = 0$;

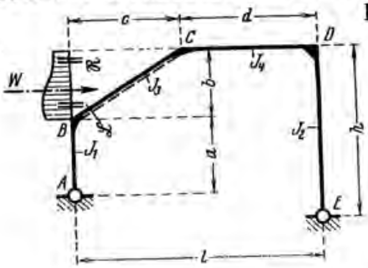
or substitute in case 61/2:

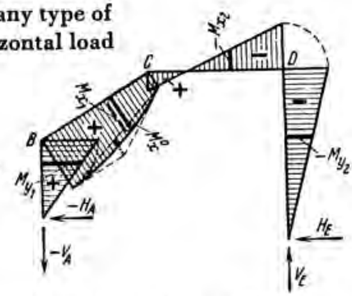
$S = P \qquad \mathfrak{S}_r = P d; \qquad \mathfrak{L} = \mathfrak{R} = 0 \qquad M_x^0 = 0$.

FRAME 61

See Appendix A, Load Terms, pp. 440-445.

Case 61/4: Inclined member loaded by any type of horizontal load





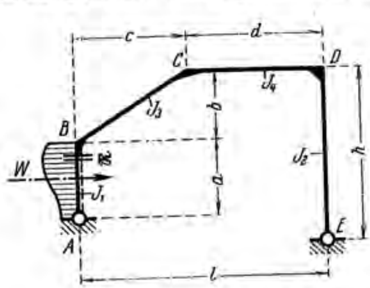
Constant: $X = \dfrac{W a (B + \delta C) + \delta C \mathfrak{S}_l + (\alpha \mathfrak{L} + m \mathfrak{R}) k_3}{N}$.

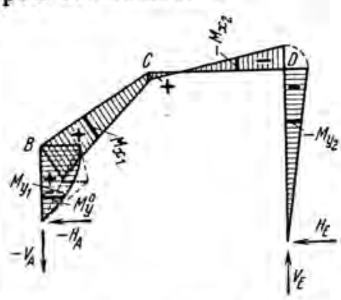
$M_B = W a - \alpha X \qquad M_C = (W a + \mathfrak{S}_l)\delta - m X \qquad M_D = -X$;

$V_E = -V_A = \dfrac{W a + \mathfrak{S}_l + n X}{l}$; $\qquad H_E = \dfrac{X}{h} \qquad H_A = -(W - H_E)$;

$$M_{x1} = M_x^0 + \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C .$$

Case 61/5: Left-hand leg loaded by any type of horizontal load





Constant: $X = \dfrac{\mathfrak{S}_l (B + \delta C) + \alpha \mathfrak{R} k_1}{N}$. $\qquad M_{y1} = M_y^0 + \dfrac{y_1}{a} M_B$;

$M_B = \mathfrak{S}_l - \alpha X \qquad M_C = \delta \mathfrak{S}_l - m X \qquad M_D = -X$;

$V_E = -V_A = \dfrac{\mathfrak{S}_l + n X}{l}$; $\qquad H_E = \dfrac{X}{h} \qquad H_A = -(W - H_E)$.

Case 61/6: Horizontal concentrated load P at B

Substitute in case 61/4:

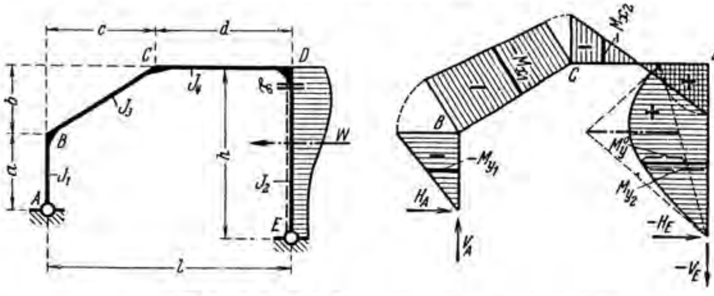
$W = P$; $\qquad \mathfrak{S}_l = 0 \qquad \mathfrak{L} = \mathfrak{R} = 0 \qquad M_x^0 = 0$;

or substitute in case 61/5:

$W = P \qquad \mathfrak{S}_l = P a$; $\qquad \mathfrak{R} = 0 \qquad M_y^0 = 0$.

Case 61/7: Right-hand leg loaded by any type of horizontal load

See Appendix A, Load Terms, pp. 440-445.



Constant:
$$X=\frac{\mathfrak{S}_r(\gamma C+D)+\mathfrak{L}k_2}{N}\qquad M_{y2}=M_y^0+\frac{y_2}{h}M_D;$$

$$M_B=-\alpha X\qquad M_C=\gamma\mathfrak{S}_r-mX\qquad M_D=\mathfrak{S}_r-X;$$

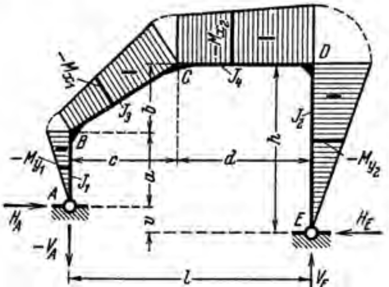
$$V_A=-V_E=\frac{\mathfrak{S}_r-nX}{l};\qquad H_A=\frac{X}{h}\qquad H_E=-(W-H_A).$$

Case 61/8: Horizontal concentrated load *P* at *D*

Substitute in case 61/7:

$$W=P\qquad \mathfrak{S}_r=Ph;\qquad \mathfrak{L}=0\qquad M_y^0=0.$$

Case 61/9: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degre

Constant:
$$X=\frac{6EJ_4\varepsilon t\,(l^2+v^2)}{dhlN}.$$

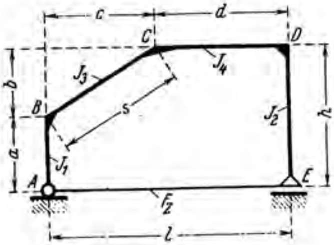
$$M_B=-\alpha X\qquad M_C=-mX\qquad M_D=-X;$$

$$V_E=-V_A=\frac{nX}{l}\qquad H_A=H_E=\frac{X}{h}.$$

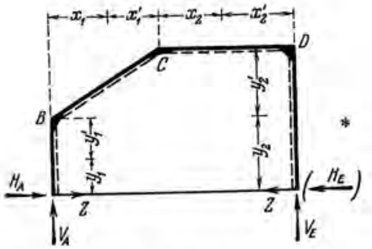
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 62

Tied bent with one skew corner and horizontal tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_4}{J_1} \cdot \frac{a}{d} \qquad k_2 = \frac{J_4}{J_2} \cdot \frac{h}{d} \qquad k_3 = \frac{J_4}{J_3} \cdot \frac{s}{d};$$

$$\alpha = \frac{a}{h} \qquad \beta = 1 - \alpha \qquad \delta = \frac{d}{l} \qquad \gamma = 1 - \delta;$$

$$B = 2\alpha(k_1 + k_3) + k_3 \qquad C = (\alpha + 2)k_3 + 3 \qquad D = 3 + 2k_2;$$

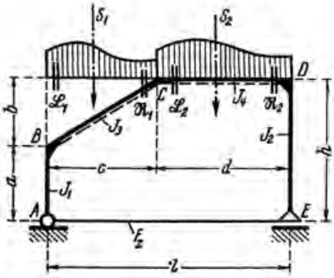
$$N = \alpha B + C + D \qquad L = \frac{6J_4}{h^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{l}{d}; \qquad N_Z = N + L.$$

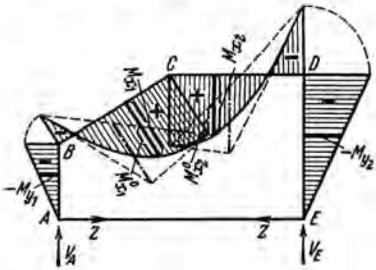
E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

*H_E occurs when the hinged support is at E.

Case 62/1: Inclined member and girder loaded by any type of vertical load (Hinged support at A or E)

See Appendix A, Load Terms, pp. 440-445.





$$Z = \frac{\delta C \mathfrak{S}_{l1} + (\alpha \mathfrak{L}_1 + \mathfrak{R}_1) k_3 + \gamma C \mathfrak{S}_{r2} + (\mathfrak{L}_2 + \mathfrak{R}_2)}{h N_Z};$$

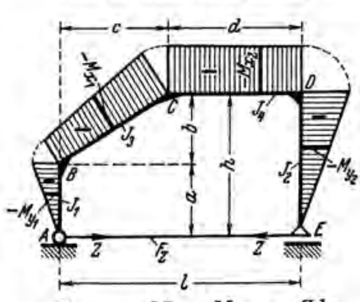
$$V_A = \frac{\mathfrak{S}_{r1} + S_1 d}{l} + \frac{\mathfrak{S}_{r2}}{l} \qquad V_E = \frac{\mathfrak{S}_{l1}}{l} + \frac{S_2 c + \mathfrak{S}_{l2}}{l};$$

$$M_B = -Za \qquad M_C = \delta \mathfrak{S}_{l1} + \gamma \mathfrak{S}_{r2} - Zh \qquad M_D = -Zh;$$

$$M_{y1} = -Z y_1 \qquad M_{y2} = -Z y_2$$

$$M_{x1} = M^0_{x1} + \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = M^0_{x2} + \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D.$$

Case 62/2: Uniform increase in temperature of the entire frame except for the tie rod (Hinged support at A or E)



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degree

$$Z = \frac{6 E J_4 \varepsilon t l}{d h^2 N_Z};$$

$$M_B = -Za \qquad M_C = M_D = -Zh; \qquad M_{y1} = -Z y_1 \qquad M_{y2} = -Z y_2$$

$$M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = M_C.$$

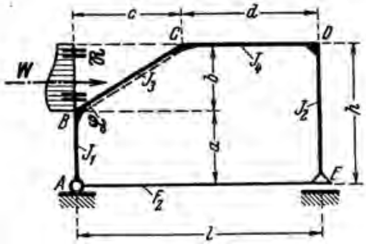
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

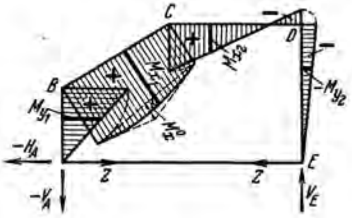
*See footnote on page 224.

FRAME 62

(See Appendix A, Load Terms, pp. 440-445.)

Case 62/3: Inclined member loaded by any type of horizontal load (Hinged support at A)





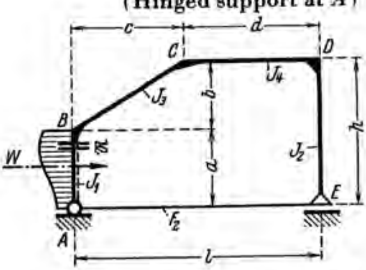
$$Z = \frac{W a (B + \delta C) + \delta C \mathfrak{S}_l + (\alpha \mathfrak{L} + \mathfrak{R}) k_3}{h N_Z};$$

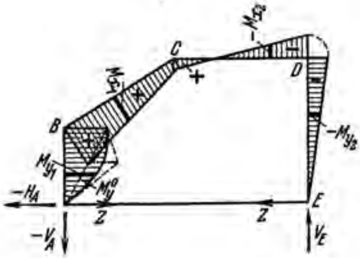
$$M_B = (W - Z) a \qquad M_C = (W a + \mathfrak{S}_l) \delta - Z h \qquad M_D = - Z h;$$

$$V_E = - V_A = \frac{W a + \mathfrak{S}_l}{l}; \qquad H_A = - W; \qquad M_{y1} = (W - Z) y_1$$

$$M_{x1} = M_x^0 + \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D \qquad M_{y2} = - Z y_2$$

Case 62/4: Left-hand leg loaded by any type of horizontal load (Hinged support at A)





$$Z = \frac{\mathfrak{S}_l (B + \delta C) + \alpha \mathfrak{R} k_1}{h N_Z};$$

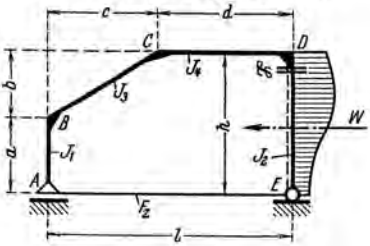
$$M_B = \mathfrak{S}_l - Z a \qquad M_C = \delta \mathfrak{S}_l - Z h \qquad M_D = - Z h;$$

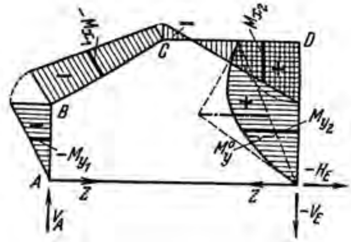
$$V_E = - V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_A = - W; \qquad M_{y2} = - Z y_2$$

$$M_{y1} = M_y^0 + \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D.$$

See Appendix A, Load Terms, pp. 440-445.

Case 62/5: Right-hand leg loaded by any type of horizontal load (Hinged support at E)





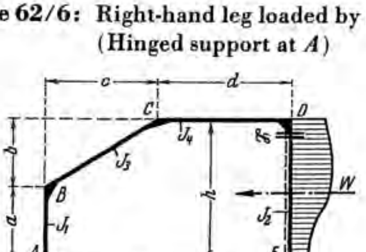
$$Z = \frac{\mathfrak{S}_r(\gamma C + D) + \mathfrak{L}k_2}{hN_Z};$$

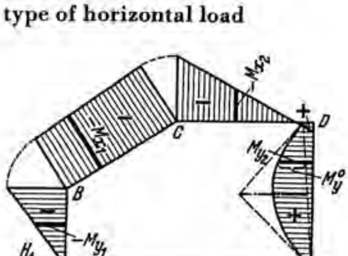
$M_B = -Za \qquad M_C = \gamma\mathfrak{S}_r - Zh \qquad M_D = \mathfrak{S}_r - Zh;$

$V_A = -V_E = \dfrac{\mathfrak{S}_r}{l}; \qquad H_E = -W; \qquad M_{y1} = -Zy_1$

$M_{x1} = \dfrac{x_1'}{c}M_B + \dfrac{x_1}{c}M_C \qquad M_{x2} = \dfrac{x_2'}{d}M_C + \dfrac{x_2}{d}M_D \qquad M_{y2} = M_y^0 + \dfrac{y_2}{h}M_D.$

Case 62/6: Right-hand leg loaded by any type of horizontal load (Hinged support at A)





$$Z = -\frac{WaB + \mathfrak{S}_l(C+D) + \delta C\mathfrak{S}_r - \mathfrak{L}k_2{}^*}{hN_Z};$$

$M_B = -(W+Z)a \qquad M_C = -(\mathfrak{S}_l + \delta\mathfrak{S}_r) - Zh \qquad M_D = -\mathfrak{S}_l - Zh;$

$V_A = -V_E = \dfrac{\mathfrak{S}_r}{l}; \qquad H_A = W; \qquad M_{y1} = -(W+Z)y_1.$

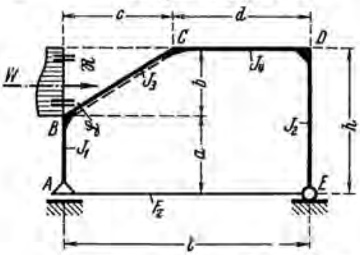
The formulas for M_{x1}, M_{x2} and M_{y2} are the same as above.

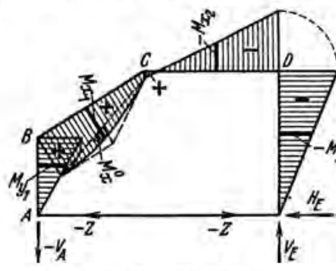
*See footnote on page 224.

FRAME 62

See Appendix A, Load Terms, pp. 440-445.

Case 62/7: Inclined member loaded by any type of horizontal load (Hinged support at E)





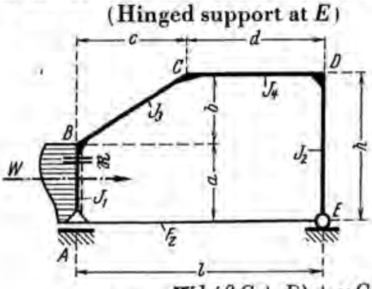
$$Z = -\frac{W h(\alpha\gamma C + D) + (\gamma \mathfrak{S}_l + \mathfrak{S}_r) C - (\alpha \mathfrak{L} + \mathfrak{R}) k_3}{h N_Z}\,{}^{*};$$

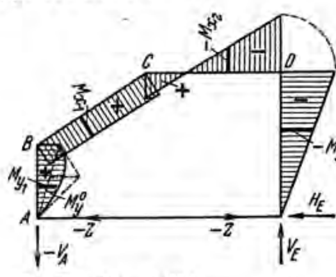
$M_B = (-Z) a \qquad M_C = -\gamma(W a + \mathfrak{S}_l) - \mathfrak{S}_r - Z h \qquad M_D = -(W + Z) h;$

$V_E = -V_A = \dfrac{W a + \mathfrak{S}_l}{l}; \qquad H_E = W; \qquad M_{y2} = -(W + Z) y_2$

$M_{y1} = (-Z) y_1 \qquad M_{x1} = M_x^0 + \dfrac{x_1'}{c} M_B + \dfrac{x_1}{c} M_C \qquad M_{x2} = \dfrac{x_2'}{d} M_C + \dfrac{x_2}{d} M_D.$

Case 62/8: Left-hand leg loaded by any type of horizontal load (Hinged support at E)





$$Z = -\frac{W h(\beta C + D) + \gamma C \mathfrak{S}_l + \mathfrak{S}_r (B + C) - \alpha \mathfrak{R} k_1}{h N_Z}\,{}^{*};$$

$M_B = -\mathfrak{S}_r - Z a \qquad M_C = -(W b + \gamma \mathfrak{S}_l + \mathfrak{S}_r) - Z h \qquad M_D = -(W + Z) h;$

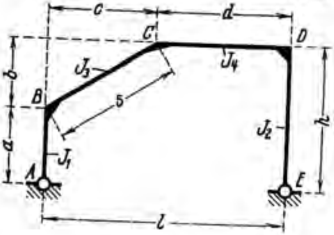
$V_E = -V_A = \dfrac{\mathfrak{S}_l}{l}; \qquad H_E = W; \qquad M_{y2} = -(W + Z) y_2$

$M_{y1} = M_x^0 + \dfrac{y_1}{a} M_B \qquad M_{x1} = \dfrac{x_1'}{c} M_B + \dfrac{x_1}{c} M_C \qquad M_{x2} = \dfrac{x_2'}{d} M_C + \dfrac{x_2}{d} M_D.$

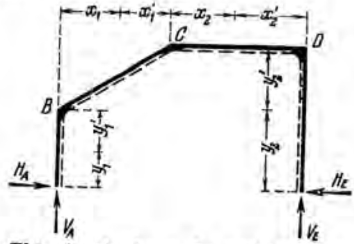
* For the above two loading conditions as well as case 62/6 (p. 223 bottom) and for decrease in temperature (p. 221 bottom) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

Frame 63

Two-hinged bent with one skew corner. Hinges at same elevation.



Shape of Frame
Dimensions and Notations



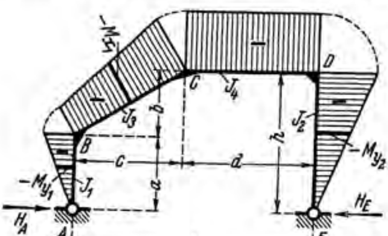
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* of frame 63 are the same as those for frame 61, with the simplifications:

$$v = 0 \qquad n = 0 \qquad m = 1 .$$

Note: The formulas for Frame 62 may be used as an alternative setting $L = 0$ (rigid tie). However, the expressions for H_A and H_E must then include the effect of Z.

Case 63/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant:

$$X = \frac{6 E J_4 \varepsilon t l}{d h N}.$$

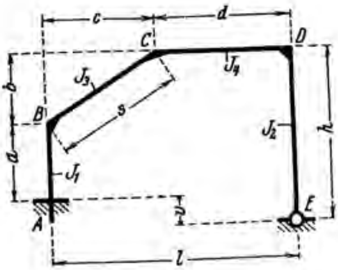
$$M_B = -\alpha X \qquad M_C = M_D = -X; \qquad H_A = H_E = \frac{X}{h};$$

$$M_{y1} = \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{y2} = \frac{y_2}{h} M_D.$$

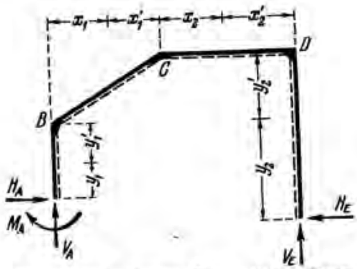
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 64

Bent with one skew corner. One support fixed, one support hinged; supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k_1 = \frac{J_4}{J_1} \cdot \frac{a}{d}$ $\qquad k_2 = \frac{J_4}{J_2} \cdot \frac{h}{d}$ $\qquad k_3 = \frac{J_4}{J_3} \cdot \frac{s}{d}$;

$\alpha = \frac{a}{h} \qquad \beta = \frac{b}{h} \qquad \gamma = \frac{c}{l} \qquad \delta = \frac{d}{l}; \qquad m = \gamma + \beta\,\delta;$

$C_1 = k_3 + 2\,\delta\,(k_3 + 1) \qquad R_1 = 6\,k_1 + (2 + \delta)\,k_3 + \delta\,C_1$

$C_2 = 2\,m\,(k_3 + 1) + 1 \qquad R_2 = 2\,(\alpha^2\,k_1 + 1 + k_2) + m\,(C_2 + 1)$

$K = m\,C_1 + \delta - 3\,\alpha\,k_1; \qquad N = R_1\,R_2 - K^2;$

$n_{11} = \frac{R_2}{N} \qquad n_{12} = n_{21} = \frac{K}{N} \qquad n_{22} = \frac{R_1}{N}.$

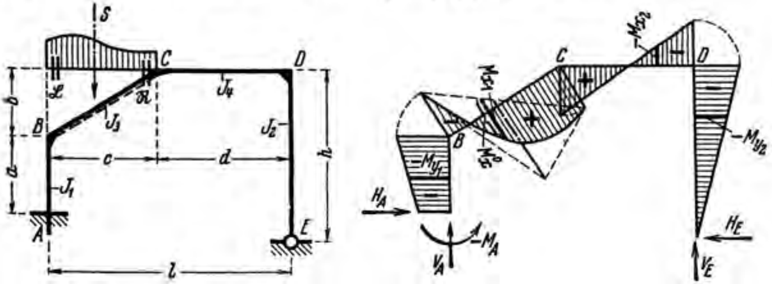
Equations for moments at any point of frame 64 for all loading conditions

$M_{x1} = \frac{x_1'}{c} \cdot M_B + \frac{x_1}{c} \cdot M_C \qquad M_{x2} = \frac{x_2'}{d} \cdot M_C + \frac{x_2}{d} \cdot M_D$

$M_{y1} = \frac{y_1'}{a} \cdot M_A + \frac{y_1}{a} \cdot M_B \qquad M_{y2} = \frac{y_2}{h} \cdot M_D.$

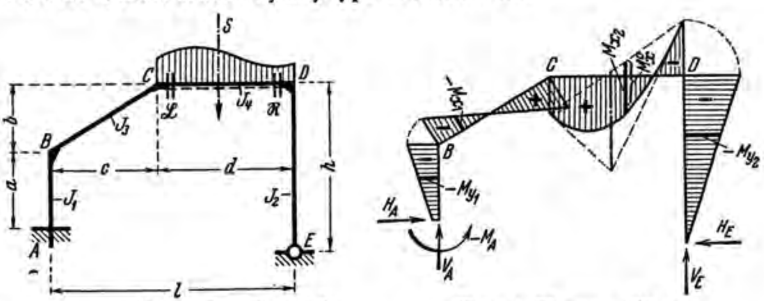
(See Appendix A. Load Terms, pp. 440-445.)

Case 64/1: Inclined member loaded by any type of vertical load



Constants:
$$\mathfrak{B}_1 = C_1\delta\,\mathfrak{S}_l + (\mathfrak{L} + \delta\,\mathfrak{R})\,k_3 \qquad X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = C_2\delta\,\mathfrak{S}_l + m\,\mathfrak{R}\,k_3; \qquad X_2 = -\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$
$$M_A = \alpha X_2 - X_1 \qquad M_B = -X_1 \qquad M_C = (\mathfrak{S}_l - X_1)\,\delta - m X_2;$$
$$V_E = \frac{\mathfrak{S}_l - X_1 + (1-\beta)\,X_2}{l} \qquad V_A = S - V_E; \qquad H_A = H_E = \frac{X_2}{h};$$
$$M_D = -X_2; \qquad M_{x1} = M_x^0 + \frac{x_1'}{c}M_B + \frac{x_1}{c}M_C.$$

Case 64/2: Girder loaded by any type of vertical load



Constants:
$$\mathfrak{B}_1 = C_1\gamma\,\mathfrak{S}_r + \delta\,\mathfrak{L} \qquad X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = C_2\gamma\,\mathfrak{S}_r + m\,\mathfrak{L} + \mathfrak{R}; \qquad X_2 = -\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$
$$M_A = \alpha X_2 - X_1 \qquad M_B = -X_1 \qquad M_C = \gamma\,\mathfrak{S}_r - \delta X_1 - m X_2;$$
$$V_A = \frac{\mathfrak{S}_r + X_1 - (1-\beta)\,X_2}{l} \qquad V_E = S - V_A; \qquad H_A = H_E = \frac{X_2}{h};$$
$$M_D = -X_2; \qquad M_{x2} = M_x^0 + \frac{x_2'}{d}M_C + \frac{x_2}{d}M_D.$$

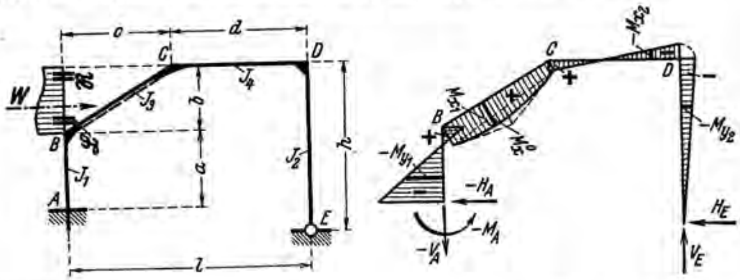
Case 64/3: Vertical concentrated load P at C

See case 61/3, p. 217.

FRAME 64

See Appendix A, Load Terms, pp. 440-445.

Case 64/4: Inclined member loaded by any type of horizontal load



Constants:

$$\mathfrak{B}_1 = 3\,W a\,k_1 - C_1 \delta\,\mathfrak{S}_l - (\mathfrak{L} + \delta\,\mathfrak{R})\,k_3 \qquad X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

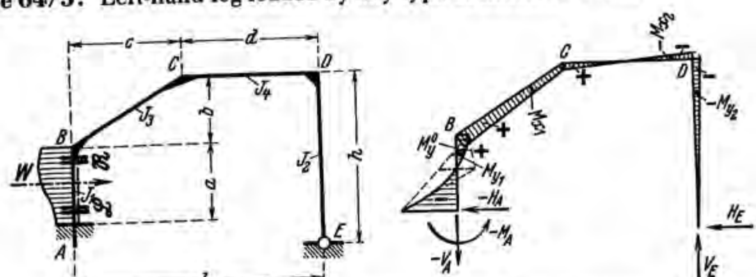
$$\mathfrak{B}_2 = 2\,\alpha\,W a\,k_1 + C_2 \delta\,\mathfrak{S}_l + m\,\mathfrak{R}\,k_3; \qquad X_2 = -\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

$$M_A = -\,W a + X_1 + \alpha X_2 \qquad M_B = X_1 \qquad H_E = \frac{X_2}{h} \qquad H_A = -(W - H_E);$$

$$M_C = (\mathfrak{S}_l + X_1)\,\delta - m X_2 \qquad M_D = -\,X_2;$$

$$V_E = -V_A = \frac{\mathfrak{S}_l + X_1 + (1-\beta) X_2}{l}; \qquad M_{x1} = M_x^0 + \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C.$$

Case 64/5: Left-hand leg loaded by any type of horizontal load



Constants:

$$\mathfrak{B}_1 = [3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\,k_1 \qquad X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

$$\mathfrak{B}_2 = [2\,\mathfrak{S}_l - \mathfrak{L}]\,\alpha\,k_1; \qquad X_2 = -\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

$$M_A = -\,\mathfrak{S}_l + X_1 + \alpha X_2 \qquad M_B = X_1 \qquad H_E = \frac{X_2}{h} \qquad H_A = -(W - H_E);$$

$$M_C = \delta X_1 - m X_2 \qquad M_D = -\,X_2;$$

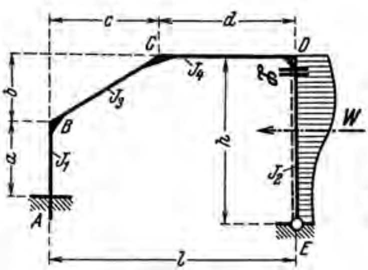
$$V_E = -V_A = \frac{X_1 + (1-\beta) X_2}{l}; \qquad M_{y1} = M_y^0 + \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B.$$

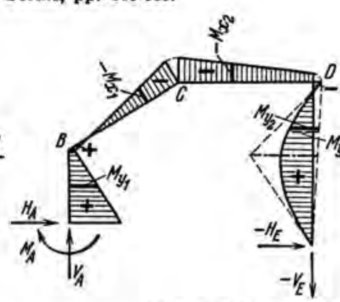
Case 64/6: Horizontal concentrated load P at B

See case 61/6, p. 218.

Case 64/7: Right-hand leg loaded by any type of horizontal load

See Appendix A, Load Terms, pp. 440-445.





Constants:

$$\mathfrak{B}_1 = \mathfrak{S}_r(3\alpha k_1 - C_1\beta\delta) \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = \mathfrak{S}_r(2\alpha^2 k_1 + C_2\beta\delta) - \mathfrak{L} k_2; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

$$M_A = \alpha(\mathfrak{S}_r - X_2) - X_1 \qquad M_B = -X_1$$
$$M_C = -\delta(\beta\mathfrak{S}_r + X_1) + m X_2 \qquad M_D = X_2; \qquad M_{y2} = M_y^0 + \frac{y_2}{h} M_D;$$

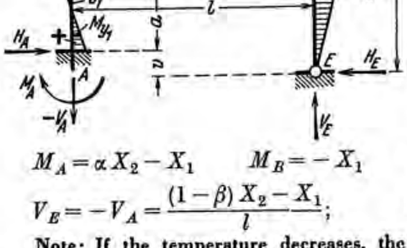
$$V_A = -V_E = \frac{\beta\mathfrak{S}_r + X_1 + (1-\beta) X_2}{l}; \qquad H_A = \frac{\mathfrak{S}_r - X_2}{h} \qquad H_E = -(W - H_A).$$

Case 64/8: Horizontal concentrated load P at D

Substitute in case 64/7

$$W = P \qquad \mathfrak{S}_r = P h; \qquad \mathfrak{L} = 0 \qquad M_y^0 = 0.$$

Case 64/9: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degree

Constants:

$$v = h - (a+b)^* ; \qquad T = \frac{6 E J_4 \varepsilon t}{d};$$

$$X_1 = T\left[-\frac{v}{l} n_{11} + \left(\frac{l}{h} + \frac{(1-\beta) v}{l}\right) n_{21}\right]$$

$$X_2 = T\left[-\frac{v}{l} n_{12} + \left(\frac{l}{h} + \frac{(1-\beta) v}{l}\right) n_{22}\right].$$

$$M_A = \alpha X_2 - X_1 \qquad M_B = -X_1 \qquad M_C = -\delta X_1 - m X_2 \qquad M_D = -X_2;$$

$$V_E = -V_A = \frac{(1-\beta) X_2 - X_1}{l}; \qquad H_A = H_E = \frac{X_2}{h}; \qquad M_{y2} = \frac{y_2}{h} M_D.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

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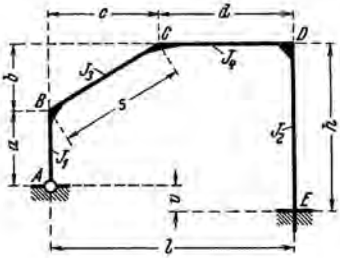
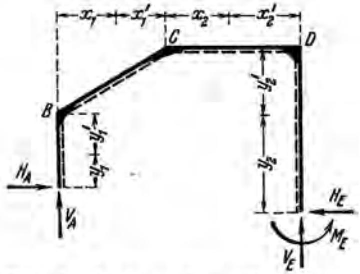
*When $(a + b) > h$, v becomes negative.

Frame 65

Bent with one skew corner. One support fixed, one support hinged; supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k_1 = \frac{J_4}{J_1} \cdot \frac{a}{d}$ $\quad k_2 = \frac{J_4}{J_2} \cdot \frac{h}{d}$ $\quad k_3 = \frac{J_4}{J_3} \cdot \frac{s}{d}$

$$\alpha = \frac{h}{a} \qquad \beta = \frac{b}{a} \qquad \gamma = \frac{c}{l} \qquad \delta = \frac{d}{l}; \qquad m = \delta(1+\beta);$$

$$C_1 = k_3 + 2m(k_3+1) \qquad R_1 = 2(k_1 + \alpha^2 k_2) + (2+m)k_3 + mC_1$$

$$C_2 = 2\gamma(k_3+1)+1 \qquad R_2 = \gamma(C_2+1) + 2(1+3k_2)$$

$$K = 3\alpha k_2 - m - \gamma C_1; \qquad N = R_1 R_2 - K^2;$$

$$n_{11} = \frac{R_2}{N} \qquad n_{12} = n_{21} = \frac{K}{N} \qquad n_{22} = \frac{R_1}{N}.$$

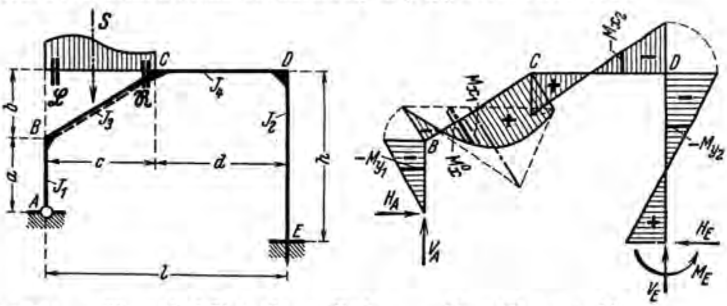
Formulas for the moments at any point of those members of Frame 65 which do not carry any external load

$$M_{x1} = \frac{x_1'}{c} \cdot M_B + \frac{x_1}{c} \cdot M_C \qquad M_{x2} = \frac{x_2'}{d} \cdot M_C + \frac{x_2}{d} \cdot M_D$$

$$M_{y1} = \frac{y_1}{a} \cdot M_B \qquad M_{y2} = \frac{y_2}{h} \cdot M_D + \frac{y_2'}{h} \cdot M_E.$$

See Appendix A, Load Terms, pp. 440-445.

Case 65/1: Inclined member loaded by any type of vertical load



Constants: $\mathfrak{B}_1 = C_1\,\delta\,\mathfrak{S}_l + (\mathfrak{L} + m\,\mathfrak{R})\,k_3$ $\qquad X_1 = \mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21}$

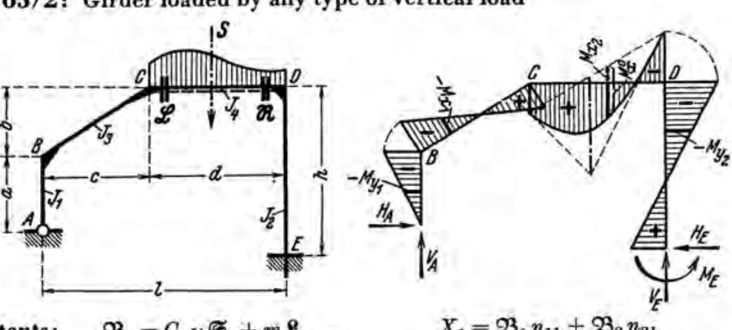
$\mathfrak{B}_2 = C_2\,\delta\,\mathfrak{S}_l + \gamma\,\mathfrak{R}\,k_3;$ $\qquad X_2 = \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$

$$M_B = -X_1 \qquad M_C = \delta\,\mathfrak{S}_l - m\,X_1 - \gamma\,X_2 \qquad M_D = -X_2;$$

$$V_E = \frac{\mathfrak{S}_l - (1+\beta)\,X_1 + X_2}{l} \qquad V_A = S - V_E; \qquad H_A = H_E = \frac{X_1}{a};$$

$$M_E = \alpha\,X_1 - X_2 \qquad M_{x1} = M_x^0 + \frac{x_1'}{c}\,M_B + \frac{x_1}{c}\,M_C.$$

Case 65/2: Girder loaded by any type of vertical load



Constants: $\mathfrak{B}_1 = C_1\,\gamma\,\mathfrak{S}_r + m\,\mathfrak{L}$ $\qquad X_1 = \mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21}$

$\mathfrak{B}_2 = C_2\,\gamma\,\mathfrak{S}_r + \gamma\,\mathfrak{L} + \mathfrak{R};$ $\qquad X_2 = \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$

$$M_B = -X_1 \qquad M_C = (\mathfrak{S}_r - X_2)\gamma - m\,X_1 \qquad M_D = -X_2;$$

$$V_A = \frac{\mathfrak{S}_r + (1+\beta)\,X_1 - X_2}{l} \qquad V_E = S - V_A; \qquad H_A = H_E = \frac{X_1}{a};$$

$$M_E = \alpha\,X_1 - X_2 \qquad M_{x2} = M_x^0 + \frac{x_2'}{d}\,M_C + \frac{x_2}{d}\,M_D.$$

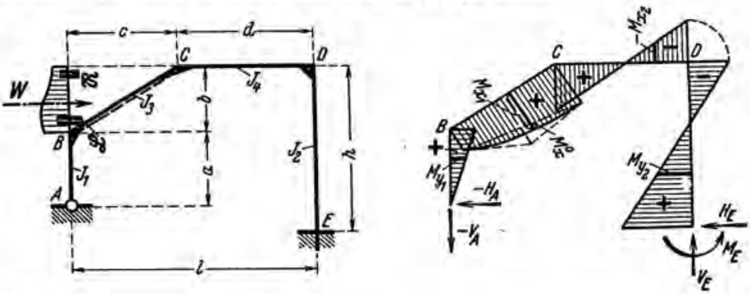
Case 65/3: Vertical concentrated load P at C

See case 61/3, p. 217.

FRAME 65

See Appendix A, Load Terms, pp. 440-445.

Case 65/4: Inclined member loaded by any type of horizontal load



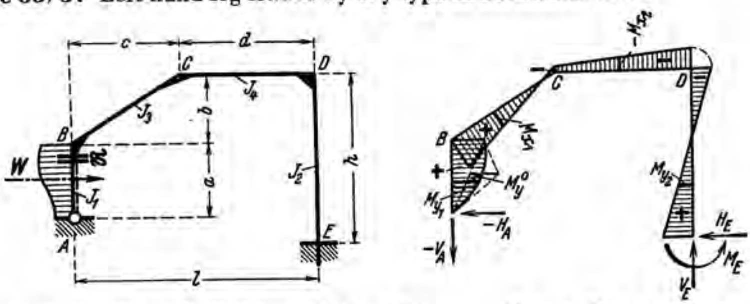
Constants:

$$\mathfrak{B}_1 = 2\alpha\, W h\, k_2 + C_1\,\delta\,\mathfrak{S}_r - (\mathfrak{L} + m\,\mathfrak{R})\,k_3 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = 3\, W h\, k_2 - C_2\,\delta\,\mathfrak{S}_r + \gamma\,\mathfrak{R}\,k_3; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

$$M_B = X_1 \qquad M_C = -\delta\,\mathfrak{S}_r + m\,X_1 - \gamma\,X_2 \qquad H_A = -\frac{X_1}{a} \qquad H_E = W + H_A;$$
$$M_D = -X_2 \qquad M_E = W h - \alpha\,X_1 - X_2;$$

$$V_A = -V_E = \frac{\mathfrak{S}_r - (1+\beta)\,X_1 - X_2}{l}; \qquad M_{x1} = M_x^0 + \frac{x_1'}{c}\,M_B + \frac{x_1}{c}\,M_C.$$

Case 65/5: Left-hand leg loaded by any type of horizontal load



Constants:

$$\mathfrak{B}_1 = \mathfrak{S}_l(2\alpha^2 k_2 + \beta\,\delta\,C_1) - \mathfrak{R}\,k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = \mathfrak{S}_l(3\alpha\,k_2 - \beta\,\delta\,C_2); \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

$$M_B = X_1 \qquad M_C = -\beta\,\delta\,\mathfrak{S}_l + m\,X_1 - \gamma\,X_2 \qquad M_{y1} = M_y^0 + \frac{y_1}{a}\,M_B;$$
$$M_D = -X_2 \qquad M_E = (\mathfrak{S}_l - X_1)\,\alpha - X_2;$$

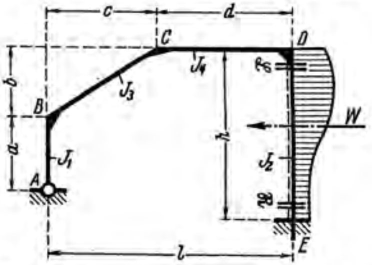
$$V_A = -V_E = \frac{\beta\,\mathfrak{S}_l - (1+\beta)\,X_1 - X_2}{l}; \qquad H_E = \frac{\mathfrak{S}_l - X_1}{a} \qquad H_A = -(W - H_E).$$

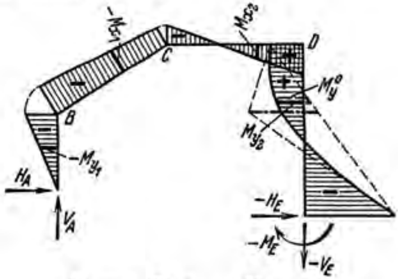
Case 65/6: Horizontal concentrated load at *B*

See case 61/6, p. 218.

Case 65/7: Right-hand leg loaded by any type of horizontal load

See Appendix A, Load Terms, pp. 440-445.





Constants:

$$\mathfrak{B}_1 = [2\,\mathfrak{S}_r - \mathfrak{R}]\,\alpha\, k_2 \qquad X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

$$\mathfrak{B}_2 = [3\,\mathfrak{S}_r - (\mathfrak{L} + \mathfrak{R})]\, k_2; \qquad X_2 = -\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}\,.$$

$$M_B = -X_1 \qquad M_C = -m\,X_1 + \gamma\,X_2 \qquad M_D = X_2$$

$$M_E = -\mathfrak{S}_r + \alpha\,X_1 + X_2 \qquad M_{y2} = M_y^0 + \frac{y_2}{h} M_D + \frac{y_2'}{h} M_E\,;$$

$$V_A = -V_E = \frac{(1+\beta)\,X_1 + X_2}{l}\,; \qquad H_A = \frac{X_1}{a} \qquad H_E = -(W - H_A)\,.$$

Case 65/8: Horizontal concentrated load at *D*

Substitute in case 65/7:

$$W = P \qquad \mathfrak{S}_r = P\,h\,; \qquad \mathfrak{L} = \mathfrak{R} = 0 \qquad M_y^0 = 0\,.$$

Case 65/9: Uniform increase in temperature of the entire frame

E = Modulus of elasticity

ε = Coefficient of thermal expansion

t = Change of temperature in degrees

Constants:

$$v = h - (a+b)^* \;; \qquad T = \frac{6\,E\,J_4\,\varepsilon\,t}{d}\,;$$

$$X_1 = T\left[\left(\frac{l}{a} - \frac{(1+\beta)\,v}{l}\right) n_{11} + \frac{v}{l}\, n_{21}\right]$$

$$X_2 = T\left[\left(\frac{l}{a} - \frac{(1+\beta)\,v}{l}\right) n_{12} + \frac{v}{l}\, n_{22}\right].$$

$$M_B = -X_1 \qquad M_C = -m\,X_1 - \gamma\,X_2 \qquad M_D = -X_2 \qquad M_E = \alpha\,X_1 - X_2\,;$$

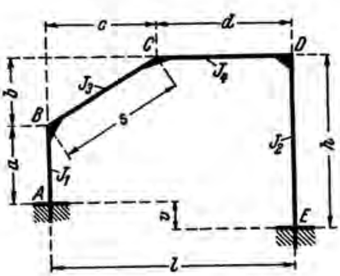
$$V_A = -V_E = \frac{(1+\beta)\,X_1 - X_2}{l} \qquad H_A = H_E = \frac{X_1}{a}\,.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

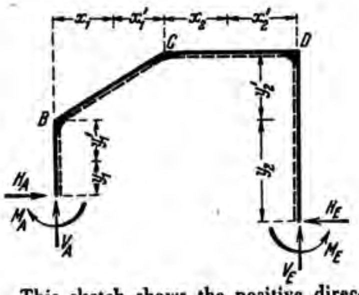
*When $h_2 > h_1$, v becomes negative.

Frame 66

Hingeless bent with one skew corner. Supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_4}{J_1} \cdot \frac{a}{d} \qquad k_2 = \frac{J_4}{J_2} \cdot \frac{h}{d} \qquad k_3 = \frac{J_4}{J_3} \cdot \frac{s}{d};$$

$$\alpha = \frac{a}{h} \qquad \beta = \frac{b}{h} \qquad \gamma = \frac{c}{l} \qquad \delta = \frac{d}{l} \qquad (\gamma + \delta = 1);$$

$$C_1 = k_3 + 2\delta(k_3 + 1) \qquad C_2 = 2\gamma(k_3 + 1) + 1 \qquad C_3 = 2\beta\delta(k_3 + 1);$$

$$R_1 = 6k_1 + (2 + \delta)k_3 + \delta C_1 \qquad K_1 = 3k_2 - \beta\delta C_2$$
$$R_2 = \gamma(C_2 + 1) + 2(1 + 3k_2) \qquad K_2 = 3\alpha k_1 - \beta\delta C_1$$
$$R_3 = 2(\alpha^2 k_1 + k_2) + \beta\delta C_3; \qquad K_3 = \gamma C_1 + \delta;$$

$$N = R_1 R_2 R_3 + 2K_1 K_2 K_3 - R_1 K_1^2 - R_2 K_2^2 - R_3 K_3^2;$$

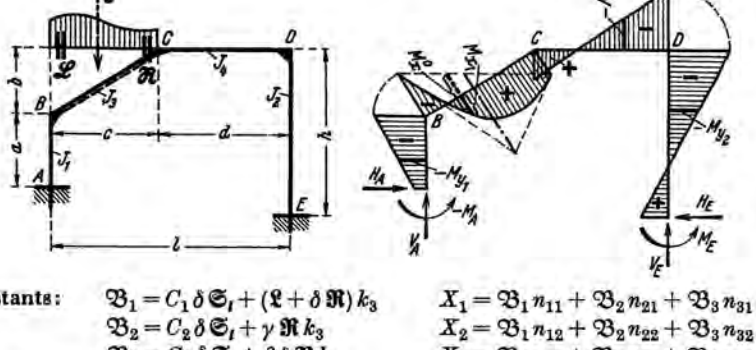
$$n_{11} = \frac{R_2 R_3 - K_1^2}{N} \qquad n_{12} = n_{21} = \frac{-R_3 K_3 + K_1 K_2}{N}$$

$$n_{22} = \frac{R_1 R_3 - K_2^2}{N} \qquad n_{13} = n_{31} = \frac{+R_2 K_2 - K_1 K_3}{N}$$

$$n_{33} = \frac{R_1 R_2 - K_3^2}{N} \qquad n_{23} = n_{32} = \frac{+R_1 K_1 - K_2 K_3}{N}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 66/1: Inclined member loaded by any type of vertical load*



Constants:

$$\mathfrak{B}_1 = C_1\delta\mathfrak{S}_l + (\mathfrak{L} + \delta\mathfrak{R})k_3 \qquad X_1 = \mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_2 = C_2\delta\mathfrak{S}_l + \gamma\mathfrak{R}k_3 \qquad X_2 = \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$

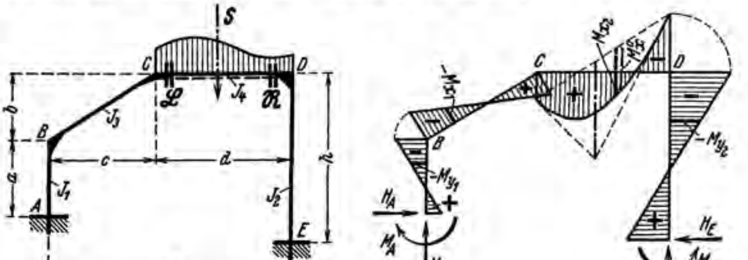
$$\mathfrak{B}_3 = C_3\delta\mathfrak{S}_l + \beta\delta\mathfrak{R}k_3 \qquad X_3 = \mathfrak{B}_1 n_{13} + \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

$$M_A = \alpha X_3 - X_1 \qquad M_B = -X_1 \qquad M_D = -X_2 \qquad M_E = X_3 - X_2$$

$$M_C = (\mathfrak{S}_l - X_1 - \beta X_3)\delta - \gamma X_2;$$

$$V_E = \frac{\mathfrak{S}_l - X_1 + X_2 - \beta X_3}{l} \qquad V_A = S - V_E; \qquad H_A = H_E = \frac{X_3}{h}.$$

Case 66/2: Girder loaded by any type of vertical load*



Constants:

$$\mathfrak{B}_1 = C_1\gamma\mathfrak{S}_r + \delta\mathfrak{L} \qquad X_1 = \mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_2 = C_2\gamma\mathfrak{S}_r + \gamma\mathfrak{L} + \mathfrak{R} \qquad X_2 = \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$

$$\mathfrak{B}_3 = C_3\gamma\mathfrak{S}_r + \beta\delta\mathfrak{L}; \qquad X_3 = \mathfrak{B}_1 n_{13} + \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

$$M_A = \alpha X_3 - X_1 \qquad M_B = -X_1 \qquad M_D = -X_2 \qquad M_E = X_3 - X_2$$

$$M_C = (\mathfrak{S}_r - X_2)\gamma - (X_1 + \beta X_3)\delta;$$

$$V_A = \frac{\mathfrak{S}_r + X_1 - X_2 + \beta X_3}{l} \qquad V_E = S - V_A; \qquad H_A = H_E = \frac{X_3}{h}.$$

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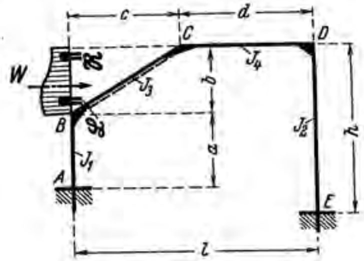
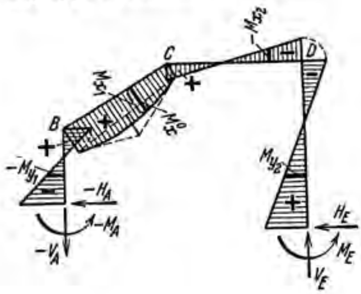
* See p. 239 for M_x and M_y.

FRAME 66

See Appendix A, Load Terms, pp. 440-445.

Case 66/3: Inclined member loaded by any type of horizontal load*





Constants:

$$\mathfrak{B}_1 = 3\,W a\,k_1 - C_1 \delta \mathfrak{S}_l - (\mathfrak{L} + \delta\,\mathfrak{R})\,k_3 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21} - \mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_2 = C_2 \delta \mathfrak{S}_l + \gamma\,\mathfrak{R}\,k_3 \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$

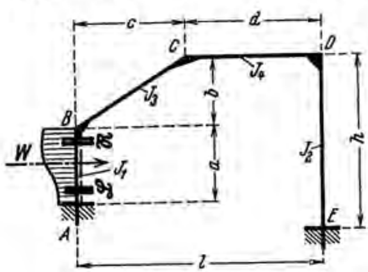
$$\mathfrak{B}_3 = 2\,W a \alpha\,k_1 + C_3 \delta \mathfrak{S}_l + \beta \delta\,\mathfrak{R}\,k_3; \qquad X_3 = -\mathfrak{B}_1 n_{13} + \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

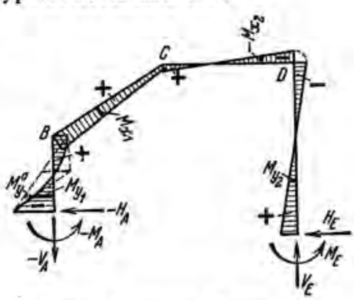
$$M_A = -W a + X_1 + \alpha X_3 \qquad M_B = X_1 \qquad M_D = -X_2$$

$$M_C = (\mathfrak{S}_l + X_1 - \beta X_3)\,\delta - \gamma X_2 \qquad M_E = X_3 - X_2;$$

$$V_E = -V_A = \frac{\mathfrak{S}_l + X_1 + X_2 - \beta X_3}{l}; \qquad H_E = \frac{X_3}{h} \qquad H_A = -(W - H_E).$$

Case 66/4: Left-hand leg loaded by any type of horizontal load*





Constants:

$$\mathfrak{B}_1 = [3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\,k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_3 = [2\,\mathfrak{S}_l - \mathfrak{L}]\,\alpha\,k_1; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_3 n_{32}$$

$$X_3 = -\mathfrak{B}_1 n_{13} + \mathfrak{B}_3 n_{33}.$$

$$M_A = -\mathfrak{S}_l + X_1 + \alpha X_3 \qquad M_B = X_1 \qquad M_D = -X_2$$

$$M_C = (X_1 - \beta X_3)\,\delta - \gamma X_2 \qquad M_E = X_3 - X_2;$$

$$V_E = -V_A = \frac{X_1 + X_2 - \beta X_3}{l}; \qquad H_E = \frac{X_3}{h} \qquad H_A = -(W - H_E).$$

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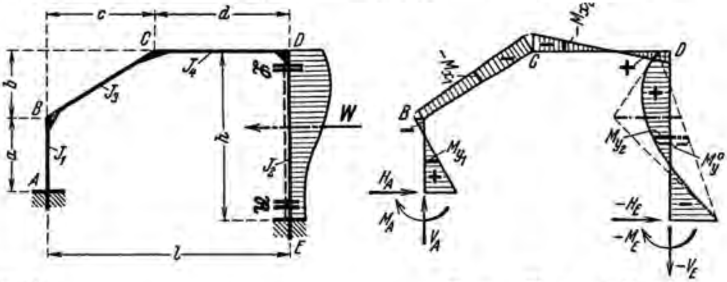
* See p. 239 for M_x and M_y.

Case 66/5: Right-hand leg loaded by any type of horizontal load*



Constants:

$$\mathfrak{B}_2=[3\,\mathfrak{S}_r-(\mathfrak{L}+\mathfrak{R})]\,k_2 \qquad X_1=-\mathfrak{B}_2\,n_{21}+\mathfrak{B}_3\,n_{31}$$

$$\mathfrak{B}_3=[2\,\mathfrak{S}_r-\mathfrak{R}]\,k_2; \qquad X_2=+\mathfrak{B}_2\,n_{22}-\mathfrak{B}_3\,n_{32}$$

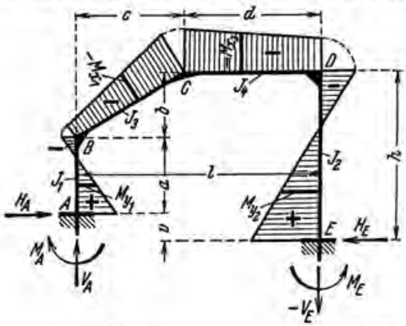
$$X_3=-\mathfrak{B}_2\,n_{23}+\mathfrak{B}_3\,n_{33}.$$

$$M_A=\alpha X_3-X_1 \qquad M_B=-X_1 \qquad M_E=-\mathfrak{S}_r+X_2+X$$

$$M_C=-(X_1+\beta X_3)\,\delta+\gamma X_2 \qquad M_D=X_2;$$

$$V_A=-V_E=\frac{X_1+X_2+\beta X_3}{l}; \qquad H_A=\frac{X_3}{h} \qquad H_E=-(W-H_A).$$

Case 66/6: Uniform increase in temperature of the entire frame*



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$v=h-(a+b)^{**};$$

$$T=\frac{6\,E\,J_4\,\varepsilon\,t}{d};$$

$$X_1=T\left[\frac{v}{l}(-n_{11}+n_{21})+\left(\frac{l}{h}-\frac{\beta v}{l}\right)n_{31}\right] \qquad M_A=\alpha X_3-X_1$$

$$X_2=T\left[\frac{v}{l}(-n_{12}+n_{22})+\left(\frac{l}{h}-\frac{\beta v}{l}\right)n_{32}\right] \qquad M_B=-X_1 \qquad M_D=-X_2$$

$$X_3=T\left[\frac{v}{l}(-n_{13}+n_{23})+\left(\frac{l}{h}-\frac{\beta v}{l}\right)n_{33}\right]. \qquad M_E=X_3-X_2$$

$$V_A=-V_E=\frac{X_1-X_2+\beta X_3}{l} \qquad H_A=H_E=\frac{X_3}{h} \qquad M_C=-(X_1+\beta X_3)\,\delta-\gamma X_2.$$

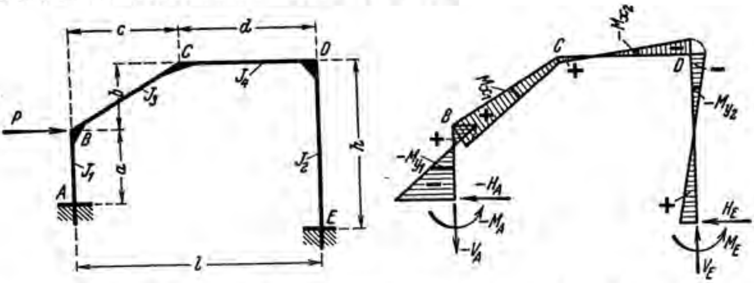
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

* See p. 239 for M_x and M_y.

**When $(a+b)>h$, v becomes negative.

FRAME 66

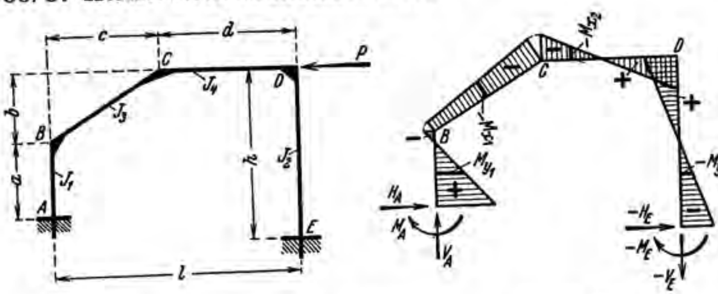
Case 66/7: Horizontal concentrated load at B*



Constants:

$$X_1 = P\,a\,k_1(+3\,n_{11} - 2\,\alpha\,n_{31}) \qquad M_B = X_1$$
$$X_2 = P\,a\,k_1(-3\,n_{12} + 2\,\alpha\,n_{32}) \qquad M_D = -X_2$$
$$X_3 = P\,a\,k_1(-3\,n_{13} + 2\,\alpha\,n_{33}). \qquad M_E = X_3 - X_2$$
$$M_A = -P\,a + X_1 + \alpha\,X_3 \qquad M_C = (X_1 - \beta\,X_3)\,\delta - \gamma\,X_2;$$
$$V_E = -V_A = \frac{X_1 + X_2 - \beta\,X_3}{l}; \qquad H_E = \frac{X_3}{h} \qquad H_A = -(P - H_E).$$

Case 66/8: Horizontal concentrated load at D*



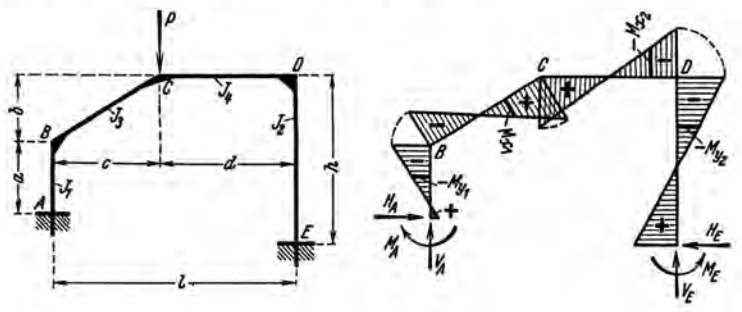
Constants:

$$X_1 = P\,h\,k_2(-3\,n_{21} + 2\,n_{31}) \qquad M_A = \alpha\,X_3 - X_1$$
$$X_2 = P\,h\,k_2(+3\,n_{22} - 2\,n_{32}) \qquad M_B = -X_1$$
$$X_3 = P\,h\,k_2(-3\,n_{23} + 2\,n_{33}). \qquad M_D = X_2$$
$$M_C = -(X_1 + \beta\,X_3)\,\delta + \gamma\,X_2 \qquad M_E = -P\,h + X_2 + X_3;$$
$$V_A = -V_E = \frac{X_1 + X_2 + \beta\,X_3}{l}; \qquad H_A = \frac{X_3}{h} \qquad H_E = -(P - H_A).$$

Note: If the horizontal load P acts at joint C from the left toward the right, the moments and forces in these formulas reverse their signs.

* See p. 239 for M_x and M_y.

Case 66/9: Vertical concentrated load at C



Constants:

$$X_1 = \frac{Pcd}{l}(C_1 n_{11} + C_2 n_{21} + C_3 n_{31})$$

$$X_2 = \frac{Pcd}{l}(C_1 n_{12} + C_2 n_{22} + C_3 n_{32})$$

$$X_3 = \frac{Pcd}{l}(C_1 n_{13} + C_2 n_{23} + C_3 n_{33}).$$

$$M_A = \alpha X_3 - X_1 \qquad M_B = -X_1 \qquad M_D = -X_2 \qquad M_E = X_3 - X_2$$

$$M_C = \frac{Pcd}{l} - (X_1 + \beta X_3)\delta - \gamma X_2;$$

$$V_A = \frac{Pd + X_1 - X_2 + \beta X_3}{l} \qquad V_E = P - V_A; \qquad H_A = H_E = \frac{X_3}{h}.$$

Formulas for the moments at any point of Frame 68 for any load

The moments at the joints and the fixed end moments contribute to the total moment:

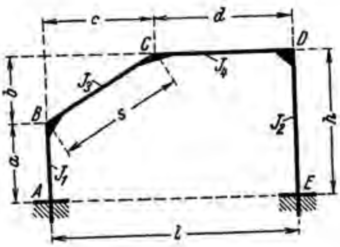
$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{y2} = \frac{y_2}{h} M_D + \frac{y_2'}{h} M_E$$

$$M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D.$$

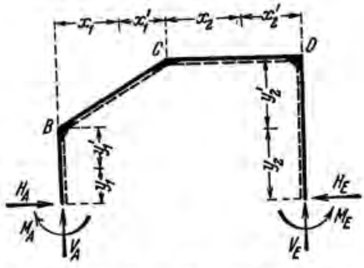
For the members that carry the load, add the value of $\boldsymbol{M_x}^0$ or $\boldsymbol{M_y}^0$ respectively.

Frame 67

Hingeless bent with one skew corner. Both supports at the same elevation.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for the external load* are the same as for Frame 66 (pp. 234-239)

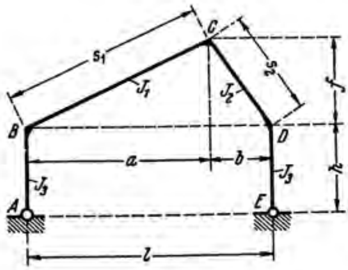
For a *uniform change of temperature* there will be $v = 0$, and the coefficients on p. 237 are reduced to:

$$T' = \frac{6 E J_4 \varepsilon t}{d} \cdot \frac{l}{h};$$

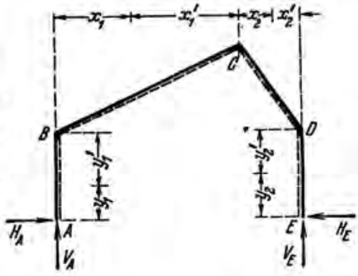
$$X_1 = T' \cdot n_{31} \qquad X_2 = T' \cdot n_{32} \qquad X_3 = T' \cdot n_{33}.$$

Frame 68

Two-hinged shed



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{s_1}{h} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{s_2}{h}; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l} \qquad (\alpha + \beta = 1);$$

$$\varphi = \frac{f}{h} \qquad m = 1 + \varphi; \qquad \begin{aligned} B &= 2 + (2 + m)\,k_1 \\ C &= (1 + 2\,m)(k_1 + k_2) \\ D &= 2 + (2 + m)\,k_2; \end{aligned}$$

$$N = B + m\,C + D = 4 + 2\,(1 + m + m^2)(k_1 + k_2)\,.$$

Formulas for the moments at any point of those members of Frame 68 which do not carry any external load

$$M_{x1} = \frac{x_1'}{a} \cdot M_B + \frac{x_1}{a} \cdot M_C \qquad M_{x2} = \frac{x_2'}{b} \cdot M_C + \frac{x_2}{b} \cdot M_D$$

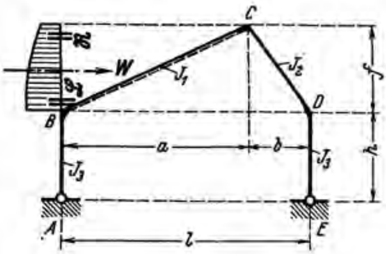
$$M_{y1} = \frac{y_1}{h} \cdot M_B \qquad M_{y2} = \frac{y_2}{h} \cdot M_D\,.$$

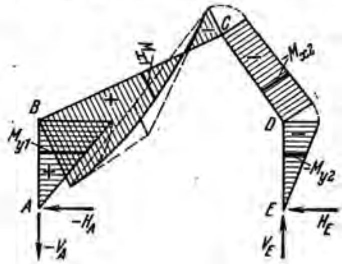
For the members that carry the load, add the value of $\boldsymbol{M}_y^0$ or $\boldsymbol{M}_x^0$ respectively.

FRAME 68

(See Appendix A, Load Terms, pp. 440-445.)

Case 68/1: Left inclined member loaded by any type of horizontal load



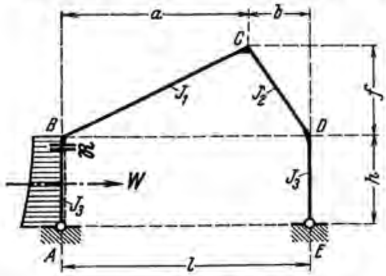


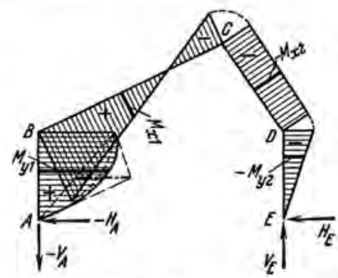
Constant: $$X = \frac{W h (B + \beta C) + \mathfrak{S}_l \cdot \beta C + (\mathfrak{L} + m\,\mathfrak{R}) k_1}{N}.$$

$$M_B = W h - X \qquad M_C = \beta (W h + \mathfrak{S}_l) - m X \qquad M_D = -X;$$

$$V_E = -V_A = \frac{W h + \mathfrak{S}_l}{l}; \qquad H_E = \frac{X}{h} \qquad H_A = -(W - H_E).$$

Case 68/2: Left-hand leg loaded by any type of horizontal load





Constant: $$X = \frac{\mathfrak{S}_l (B + \beta C) + \mathfrak{R}}{N}. \qquad M_{y1} = M_y^0 + \frac{y_1}{h} \cdot M_B;$$

$$M_B = \mathfrak{S}_l - X \qquad M_C = \beta\,\mathfrak{S}_l - m X \qquad M_D = -X;$$

$$V_E = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_E = \frac{X}{h} \qquad H_A = -(W - H_E).$$

Case 68/3: Horizontal concentrated load *P* at *B**

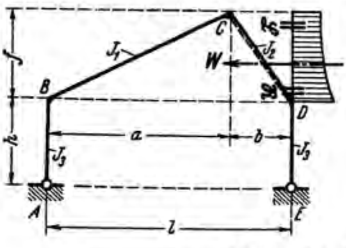
$$X = P h \cdot \frac{B + \beta C}{N}. \qquad M_B = P h - X \qquad M_C = P h \cdot \beta - m X \qquad M_D = -X;$$

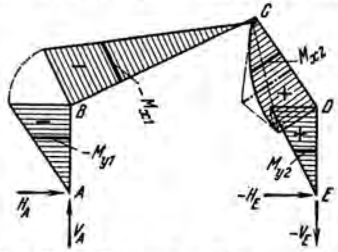
$$V_E = -V_A = \frac{P h}{l}; \qquad H_E = \frac{X}{h} \qquad H_A = -P + \frac{X}{h}.$$

* From 68/1 for $W = P$, or from 68/2 for $W = P$ and $\mathfrak{S}_l = P h$, with all other load terms equal to zero.

(See Appendix A, Load Terms, pp. 440-445.)

Case 68/4: Right inclined member loaded by any type of horizontal load



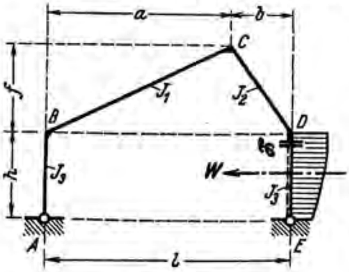


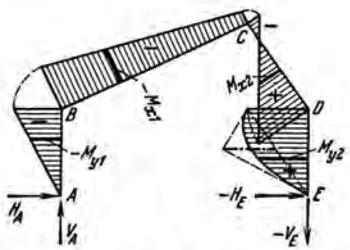
Constant:
$$X = \frac{Wh(\alpha C + D) + \mathfrak{S}_r \cdot \alpha C + (m\,\mathfrak{L} + \mathfrak{R})\,k_2}{N}.$$

$$M_B = -X \qquad M_C = \alpha(Wh + \mathfrak{S}_r) - mX \qquad M_D = Wh - X;$$

$$V_A = -V_E = \frac{Wh + \mathfrak{S}_r}{l}; \qquad H_A = \frac{X}{h} \qquad H_E = -(W - H_A).$$

Case 68/5: Right-hand leg loaded by any type of horizontal load





Constant: $X = \dfrac{\mathfrak{S}_r(\alpha C + D) + \mathfrak{L}}{N}$. $\qquad M_{y2} = M_y^0 + \dfrac{y_2}{h} \cdot M_D;$

$$M_B = -X \qquad M_C = \alpha\,\mathfrak{S}_r - mX \qquad M_D = \mathfrak{S}_r - X;$$

$$V_A = -V_E = \frac{\mathfrak{S}_r}{l}; \qquad H_A = \frac{X}{h} \qquad H_E = -(W - H_A).$$

Case 68/6: Horizontal concentrated load P at D*

$$X = Ph \cdot \frac{\alpha C + D}{N}. \qquad M_B = -X \qquad M_C = Ph \cdot \alpha - mX \qquad M_D = Ph - X;$$

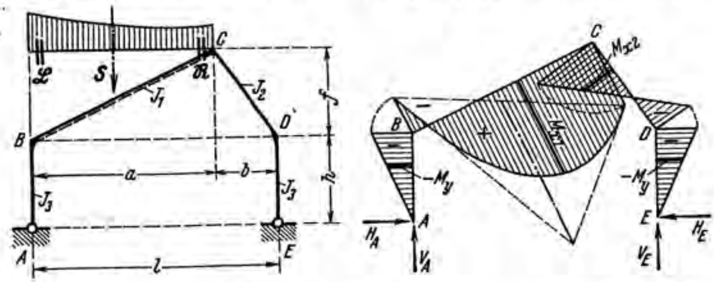
$$V_A = -V_E = \frac{Ph}{l}; \qquad H_A = \frac{X}{h} \qquad H_E = -P + \frac{X}{h}.$$

* From 68/4 for $W = P$, or from 68/5 for $W = P$, and $\mathfrak{S}_r = Ph$, with all other load terms equal to zero.

FRAME 68

See Appendix A, Load Terms, pp. 440-445.

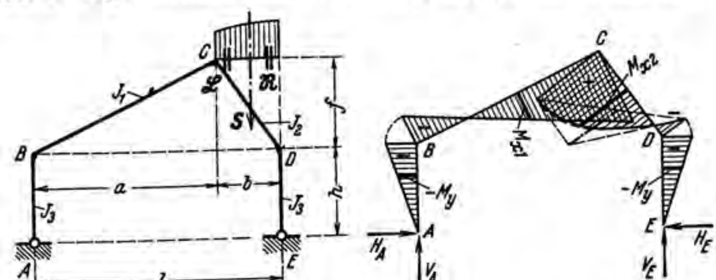
Case 68/7: Left inclined member loaded by any type of vertical load



Constant: $$X = \frac{\mathfrak{S}_l \cdot \beta C + (\mathfrak{L} + m\,\mathfrak{R})\,k_1}{N}. \qquad H_A = H_E = \frac{X}{h};$$

$$M_B = M_D = -X \qquad M_C = \beta\,\mathfrak{S}_l - m\,X; \qquad V_E = \frac{\mathfrak{S}_l}{l} \qquad V_A = S - V_E.$$

Case 68/8: Right inclined member loaded by any type of vertical load



Constant: $$X = \frac{\mathfrak{S}_r \cdot \alpha C + (m\,\mathfrak{L} + \mathfrak{R})\,k_2}{N}. \qquad H_A = H_E = \frac{X}{h};$$

$$M_B = M_D = -X \qquad M_C = \alpha\,\mathfrak{S}_r - m\,X; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_E = S - V_A.$$

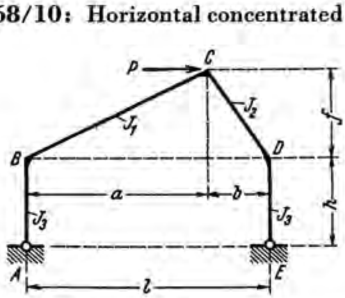
Case 68/9: Vertical concentrated load P at C*

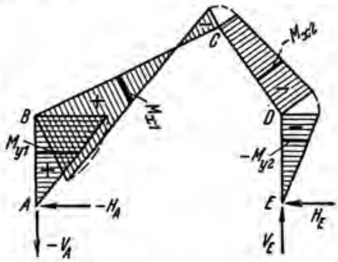
$$M_B = M_D = -\frac{P a b}{l} \cdot \frac{C}{N} \qquad M_C = +\frac{P a b}{l} \cdot \frac{B + D}{N};$$

$$V_A = \frac{P b}{l} \qquad V_E = \frac{P a}{l}; \qquad H_A = H_E = \frac{-M_B}{h}.$$

* From 68/7 for $S = P$ and $\mathfrak{S}_l = Pa$, or from case 68/8 for $S = P$ and $\mathfrak{S}_r = Pb$, with all other load terms equal to zero.

Case 68/10: Horizontal concentrated load at C



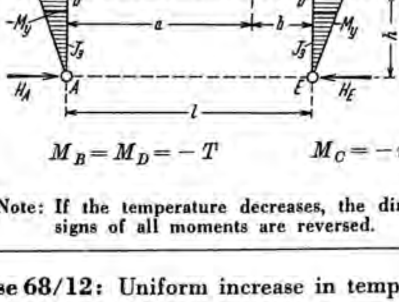


$$M_B = +Ph \cdot \frac{\alpha m C + D}{N} \qquad M_D = -Ph \cdot \frac{B + \beta m C}{N}; \qquad V_E = -V_A = P \cdot \frac{h+f}{l};$$

$$M_C = P(h+f) \cdot \frac{\beta D - \alpha B}{N}; \qquad H_A = -\frac{M_B}{h} \qquad H_E = \frac{-M_D}{h}.$$

Note: Case 68/10 follows from case 68/1 with $W = P$ and $\mathfrak{S}_l = Pf$, or from case 68/4 with $W = -P$ and $\mathfrak{S}_r = -Pf$, while all remaining load terms disappear.

Case 68/11: Uniform increase in temperature of the entire frame*



E = Modulus of elasticity

ε = Coefficient of thermal expansion

t = Change of temperature in degree

Constant:

$$T = \frac{6 E J_3 l \cdot \varepsilon t}{h^2 N}.$$

$$M_B = M_D = -T \qquad M_C = -mT; \qquad H_A = H_E = \frac{T}{h}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 68/12: Uniform increase in temperature of the tie BC only or CD only

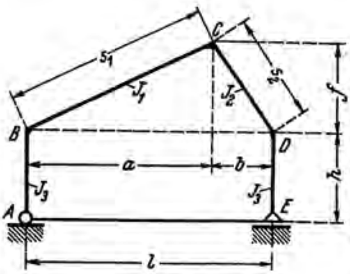
In case 68/11 in place of the constant T there appears

$$T_1 = \alpha \cdot T \qquad \text{or} \qquad T_2 = \beta \cdot T.$$

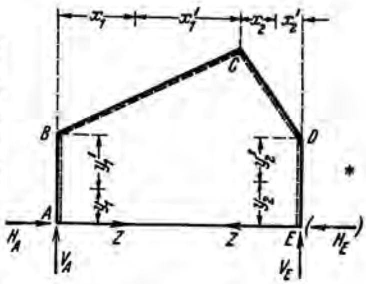
* Equal temperature changes in the vertical legs do not cause stress.

FRAME 69

Externally simply suppored shed with tie-rod



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{s_1}{h} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{s_2}{h}; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l} \qquad (\alpha + \beta = 1);$$

$$\varphi = \frac{f}{h} \qquad m = 1 + \varphi; \qquad \begin{array}{l} B = 2 + (2+m)\,k_1 \\ C = (1+2m)(k_1+k_2) \\ D = 2 + (2+m)\,k_2; \end{array}$$

$$N = B + mC + D = 4 + 2(1 + m + m^2)(k_1 + k_2);$$

$$L = \frac{6\,J_3\,l}{h^3 F_Z} \cdot \frac{E}{E_Z}; \qquad N_Z = N + L.$$

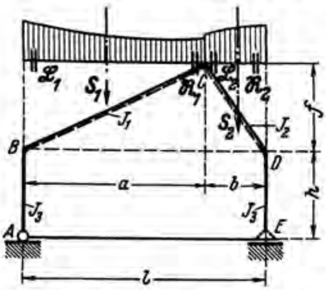
E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

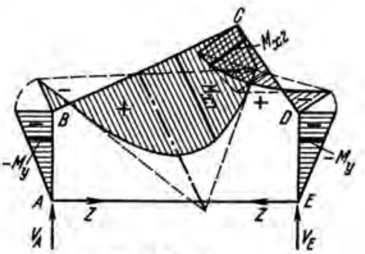
Note: The formulas for moments at arbitrary points of the frame are the same as for frame 68, p. 241.

*H_E occurs when the hinged support is at E.

Case 69/1: Both inclined members loaded by any type of vertical load (Hinged support at A or E)

See Appendix A, Load Terms, pp. 440-445.





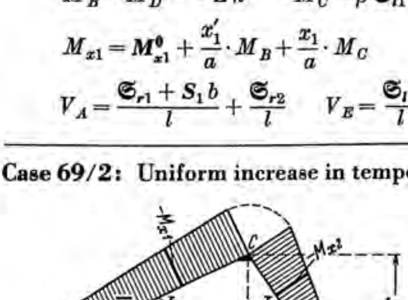
$$Z=\frac{\mathfrak{S}_{l1}\cdot\beta C+(\mathfrak{L}_1+m\,\mathfrak{R}_1)\,k_1+\mathfrak{S}_{r2}\cdot\alpha C+(m\,\mathfrak{L}_2+\mathfrak{R}_2)\,k_2}{h\,N_Z};$$

$$M_B=M_D=-Z\,h \qquad M_C=\beta\,\mathfrak{S}_{l1}+\alpha\,\mathfrak{S}_{r2}-Z\,(h+f) \qquad M_y=-Z\,y_1;$$

$$M_{x1}=\mathbf{M}^0_{x1}+\frac{x_1'}{a}\cdot M_B+\frac{x_1}{a}\cdot M_C \qquad M_{x2}=\mathbf{M}^0_{x2}+\frac{x_2'}{b}\cdot M_C+\frac{x_2}{b}\cdot M_D;$$

$$V_A=\frac{\mathfrak{S}_{r1}+S_1\,b}{l}+\frac{\mathfrak{S}_{r2}}{l} \qquad V_E=\frac{\mathfrak{S}_{l1}}{l}+\frac{S_2\,a+\mathfrak{S}_{l2}}{l} \qquad (V_A+V_E=S_1+S_2).$$

Case 69/2: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z=\frac{6\,E\,J_3\,l\cdot\varepsilon\,t}{h^3\,N_Z};$$

$$M_B=M_D=-Z\,h \qquad M_C=-Z\,(h+f);$$

$$M_{x1}=\frac{x_1'}{a}\cdot M_B+\frac{x_1}{a}\cdot M_C \qquad M_{x2}=\frac{x_2'}{b}\cdot M_C+\frac{x_2}{b}\cdot M_D \qquad M_y=-Z\,y_1.$$

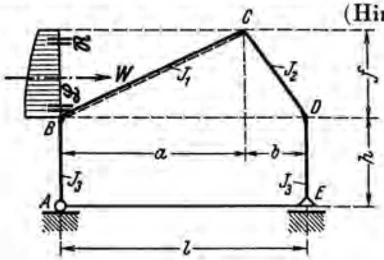
Notes: A uniform temperature increase in one or both legs does not cause stress. If s_1 only or s_2 only suffer temperature increases, replace l in the expression for Z by a or b, respectively. All signs are to be reversed for a temperature decrease.*

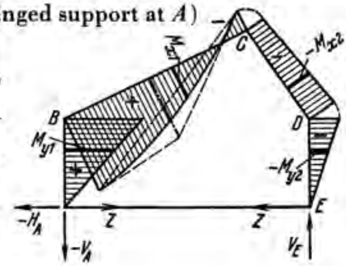
* With a decrease in temperature $Z=-Z'$, where Z' is a compressive force. See footnote p. 249.

FRAME 69

See Appendix A, Load Terms, pp. 440-445.

Case 69/3: Left inclined member loaded by any type of horizontal load (Hinged support at A)





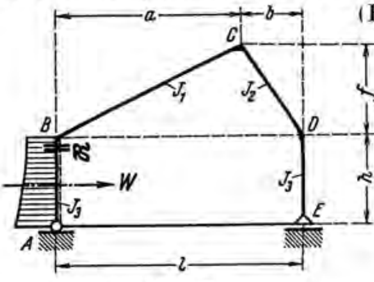
$$Z = \frac{W h (B + \beta C) + \mathfrak{S}_l \cdot \beta C + (\mathfrak{L} + m\,\mathfrak{N}) k_1}{h N_Z};$$

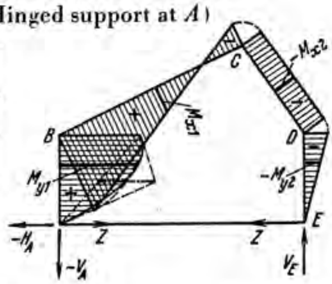
$$M_B = (W - Z) h \qquad M_C = \beta (W h + \mathfrak{S}_l) - Z (h + f) \qquad M_D = -Z h;$$

$$V_E = -V_A = \frac{W h + \mathfrak{S}_l}{l}; \qquad H_A = -W; \qquad M_{y1} = (W - Z) y_1$$

$$M_{x1} = M_x^0 + \frac{x_1'}{a} \cdot M_B + \frac{x_1}{a} \cdot M_C \qquad M_{x2} = \frac{x_2'}{b} \cdot M_C + \frac{x_2}{b} \cdot M_D \qquad M_{y2} = -Z y_2 .$$

Case 69/4: Left-hand leg loaded by any type of horizontal load (Hinged support at A)





$$Z = \frac{\mathfrak{S}_l (B + \beta C) + \mathfrak{N}}{h N_Z}; \qquad M_{y1} = M_y^0 + \frac{y_1}{h} \cdot M_B \qquad M_{y2} = -Z y_2;$$

$$M_B = \mathfrak{S}_l - Z h \qquad M_C = \beta\,\mathfrak{S}_l - Z (h + f) \qquad M_D = -Z h; \qquad H_A = -W;$$

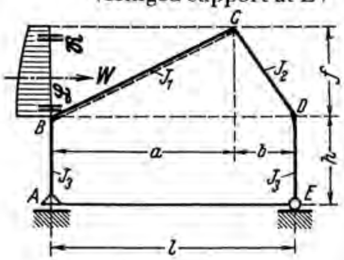
$$M_{x1} = \frac{x_1'}{a} \cdot M_B + \frac{x_1}{a} \cdot M_C \qquad M_{x2} = \frac{x_2'}{b} \cdot M_C + \frac{x_2}{b} \cdot M_D; \quad V_E = -V_A = \frac{\mathfrak{S}_l}{l}.$$

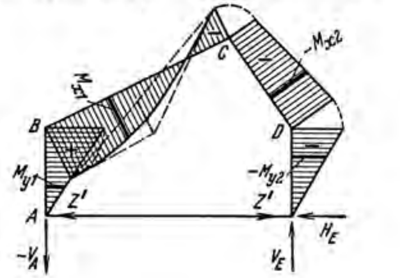
Case 69/5: Horizontal concentrated load P at ridge B (Hinged support at A)

Use case 69/3 and $W = P$; or use 69/4, $W = P$ and $\mathfrak{S}_l = Ph$, and all other load terms equal to zero.

See Appendix A, Load Terms, pp. 440-445.

Case 69/6: Left inclined member loaded by any type of horizontal load (Hinged support at E)





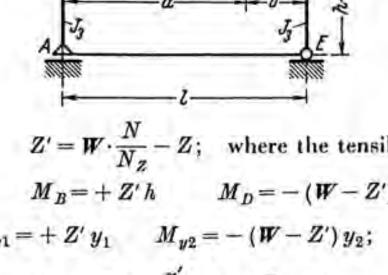
$$Z' = W \cdot \frac{N}{N_Z} - Z; \quad \text{where the tensile force is as in case 69/3*}$$

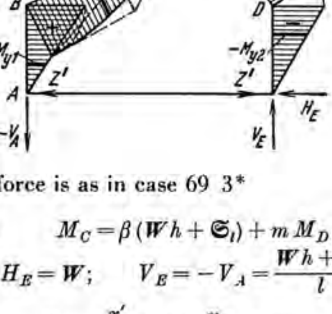
$$M_B = +Z'h \qquad M_D = -(W - Z')h \qquad M_C = \beta(Wh + \mathfrak{S}_l) + m M_D;$$

$$M_{y1} = +Z' y_1 \quad M_{y2} = -(W - Z') y_2; \quad H_E = W; \quad V_E = -V_A = \frac{Wh + \mathfrak{S}_l}{l};$$

$$M_{x1} = M_x^0 + \frac{x_1'}{a} \cdot M_B + \frac{x_1}{a} \cdot M_C \qquad M_{x2} = \frac{x_2'}{b} \cdot M_C + \frac{x_2}{b} \cdot M_D.$$

Case 69/7: Left-hand leg loaded by any type of horizontal load (Hinged support at E)





$$Z' = W \cdot \frac{N}{N_Z} - Z; \quad \text{where the tensile force is as in case 69/4*}$$

$$M_B = Z'h - \mathfrak{S}_r \qquad M_D = -(W - Z')h \qquad M_C = \beta \mathfrak{S}_l + m M_D;$$

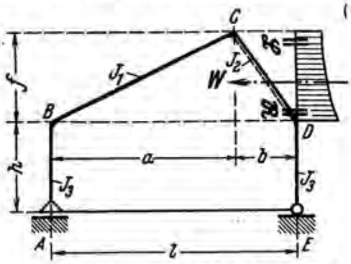
$$M_{y1} = M_y^0 + \frac{y_1}{h} \cdot M_B \quad M_{y2} = -(W - Z') y_2; \quad H_E = W; \quad V_E = -V_A = \frac{\mathfrak{S}_l}{l}.$$

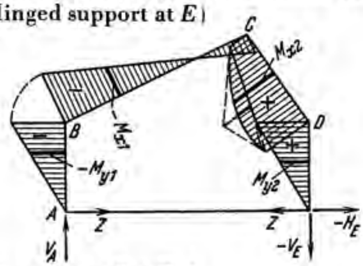
* The tension in the tie rod Z' is a compressive force in the above two cases. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod. The same applies to cases 69/11 and 12 (p. 251) and for decrease in temperature (p. 247).

FRAME 69

See Appendix A, Load Terms, pp. 440-445.

Case 69/8: Right inclined member loaded by any type of horizontal load (Hinged support at E)





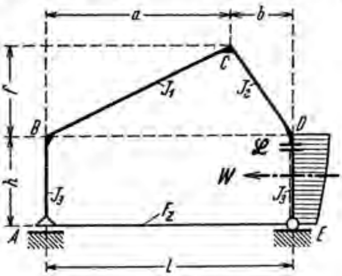
$$Z = \frac{W h (\alpha C + D) + \mathfrak{S}_r \cdot \alpha C + (m\mathfrak{L} + \mathfrak{R}) k_2}{h N_Z};$$

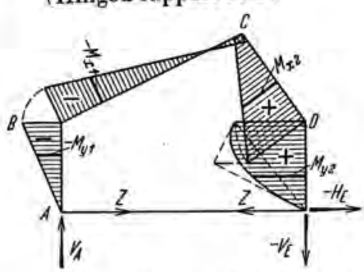
$$M_B = -Zh \qquad M_C = \alpha (Wh + \mathfrak{S}_r) - Z(h+f) \qquad M_D = (W - Z) h;$$

$$V_A = -V_E = \frac{Wh + \mathfrak{S}_r}{l}; \qquad H_E = -W; \qquad M_{y2} = (W - Z) y_2$$

$$M_{x1} = \frac{x_1'}{a} \cdot M_B + \frac{x_1}{a} \cdot M_C \qquad M_{x2} = M_x^0 + \frac{x_2'}{b} \cdot M_C + \frac{x_2}{b} \cdot M_D \qquad M_{y1} = -Z y_1.$$

Case 69/9: Right-hand leg loaded by any type of horizontal load (Hinged support at E)





$$Z = \frac{\mathfrak{S}_r (\alpha C + D) + \mathfrak{L}}{h N_Z}; \qquad M_{y1} = -Z y_1 \qquad M_{y2} = M_y^0 + \frac{y_2}{h} \cdot M_D;$$

$$M_B = -Zh \qquad M_C = \alpha \mathfrak{S}_r - Z(h+f) \qquad M_D = \mathfrak{S}_r - Zh; \qquad H_E = -W;$$

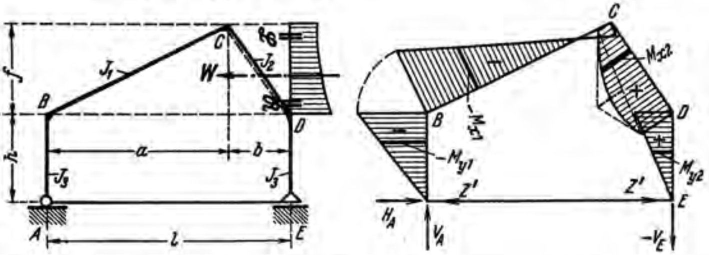
$$M_{x1} = \frac{x_1'}{a} \cdot M_B + \frac{x_1}{a} \cdot M_C \qquad M_{x2} = \frac{x_2'}{b} \cdot M_C + \frac{x_2}{b} \cdot M_D; \qquad V_A = -V_E = \frac{\mathfrak{S}_r}{l}.$$

Case 69/10: Horizontal concentrated load P at ridge D (Hinged support at E)

Use case 69/8 and $W = P$; or use 69/9, $W = P$ and $\mathfrak{S}_r = Ph$, and all other load terms equal to zero.

See Appendix A, Load Terms, pp. 440-445.

Case 69/11: Right inclined member loaded by any type of horizontal load (Hinged support at A)



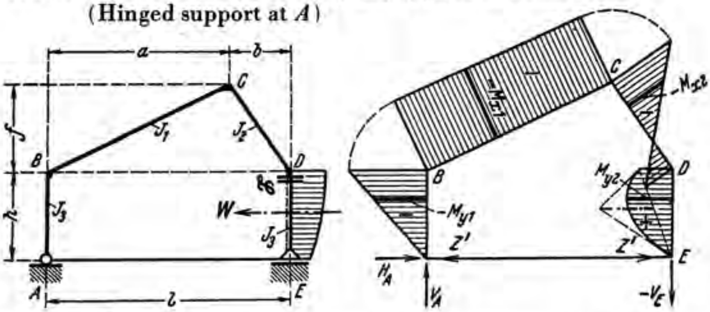
$$Z' = W \cdot \frac{N}{N_Z} - Z;\quad \text{where Z is obtained from case 69/8*.}$$

$$M_B = -(W - Z')h \qquad M_C = \alpha(Wh + \mathfrak{S}_r) + m M_B \qquad M_D = +Z'h;$$

$$M_{y1} = -(W - Z')y_1 \qquad M_{y2} = +Z'y_2; \qquad H_A = W; \qquad V_A = -V_E = \frac{Wh + \mathfrak{S}_r}{l};$$

$$M_{x1} = \frac{x_1'}{a} \cdot M_B + \frac{x_1}{a} \cdot M_C \qquad M_{x2} = M_x^0 + \frac{x_2'}{b} \cdot M_C + \frac{x_2}{b} \cdot M_D.$$

Case 69/12: Right-hand leg loaded by any type of horizontal load (Hinged support at A)



$$Z' = W \cdot \frac{N}{N_Z} - Z;\quad \text{where Z is obtained from case 69/9*.}$$

$$M_B = -(W - Z')h \qquad M_C = \alpha \mathfrak{S}_r + m M_B \qquad M_D = Z'h - \mathfrak{S}_l;$$

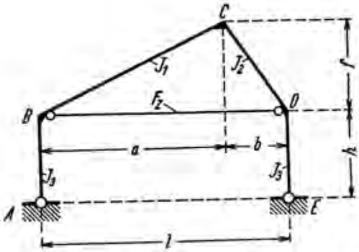
$$M_{y1} = -(W - Z')y_1 \qquad M_{y2} = M_y^0 + \frac{y_2}{h} \cdot M_D; \qquad V_A = -V_E = \frac{\mathfrak{S}_r}{l};$$

$$M_{x1} = \frac{x_1'}{a} \cdot M_B + \frac{x_1}{a} \cdot M_C \qquad M_{x2} = \frac{x_2'}{b} \cdot M_C + \frac{x_2}{b} \cdot M_D; \qquad H_A = W.$$

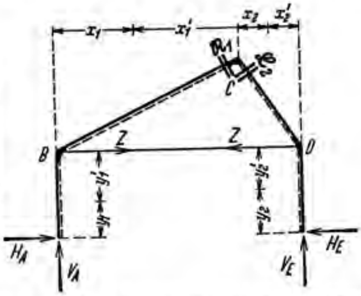
* The tension in the tie rod Z' is a compressive force in the above two cases. See footnote p. 249.

FRAME 70

Two-hinged shed with tie-rod at the eaves



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point.

General

Frame 70 with tie is best considered as a more general case of frame 68 without tie. The effect of the tie is easily shown as follows:

Steps in computing the stresses

First step: For each loading condition compute all corner moments M_B, M_C, M_D and the reactions H_A, H_E, V_A, V_E from frame 68 (see pp. 241-245)

Second step:

a) additional coefficients for frame 70

$$\gamma = \frac{B+D}{N} \qquad \delta = \frac{C}{N} \qquad (\gamma + m\,\delta = 1); \qquad G = \frac{[8+3(k_1+k_2)](k_1+k_2)}{N}$$

$$L = \frac{6J_3}{f^2 F_Z} \cdot \frac{l}{h} \cdot \frac{E}{E_Z} \qquad\qquad N_Z = G + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Note: For a rigid tie set $L = 0$, $N_Z = G$.

h) Figure the tension in the tie rod.

$$Z \cdot f = \frac{M_B k_1 + 2 M_C (k_1 + k_2) + M_D k_2 + \mathfrak{R}_1 k_1 + \mathfrak{L}_2 k_2}{N_Z}*$$

Note: The load terms $\mathfrak{R}_1$ and $\mathfrak{L}_2$ used in this formula are shown in the right-hand sketch on p. 252 and are to be used accordingly.**

Third step:

a) Moments at the joints and reactions for Frame 70.

$$\overline{M}_B = M_B + \delta \cdot Zf \qquad \overline{M}_C = M_C - \gamma \cdot Zf \qquad \overline{M}_D = M_D + \delta \cdot Zf;$$
$$\overline{H}_A = H_A - \varphi\delta \cdot Z \qquad \overline{H}_E = H_E - \varphi\delta \cdot Z; \qquad \overline{V}_A = V_A \quad \overline{V}_E = V_E.$$

Note: In order to distinguish the moments and reactions for Frame 70 the values are shown with a dash over the letter.

b) Moments at any point of Frame 70.

The formulas for $\overline{M}_x$ and $\overline{M}_y$ are the same as for Frame 68, except that the values $\overline{M}_B$, $\overline{M}_C$, $\overline{M}_D$ are to be used instead of M_B, M_C, M_D.

---

* For the case of various loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

** For use of the loading conditions of frame 68 substitute the following in the Zf formula for the load terms $\mathfrak{R}_1$ and $\mathfrak{L}_2$

Case 68/1: $\mathfrak{R}_1 = \mathfrak{R}$; $\mathfrak{L}_2 = 0$;

Case 68/4: $\mathfrak{R}_1 = 0$; $\mathfrak{L}_2 = \mathfrak{L}$;

Case 68/7: $\mathfrak{R}_1 = \mathfrak{R}$; $\mathfrak{L}_2 = 0$;

Case 68/8: $\mathfrak{R}_1 = 0$; $\mathfrak{L}_2 = \mathfrak{L}$;

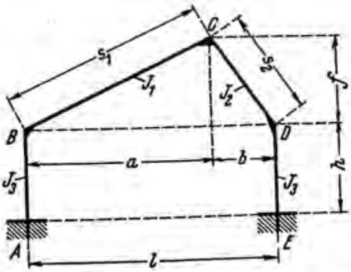
Case 68/11: $\mathfrak{R}_1 k_1 + \mathfrak{L}_2 k_2 = 6\,EJ_3 \cdot \varepsilon t \cdot l/hf$;

Case 68/12: $\mathfrak{R}_1 k_1 + \mathfrak{L}_2 k_2 = 6\,EJ_3 \cdot \varepsilon(a \cdot t_1 + b \cdot t_2)/hf$.

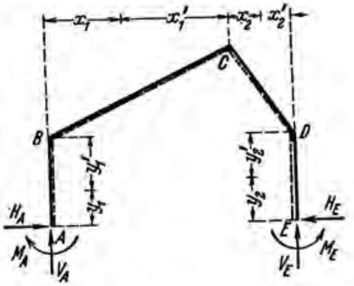
For all remaining load conditions, including the case of uniform temperature change in the entire frame including the tie rod, substitute $\mathfrak{R}_1 = \mathfrak{L}_2 = 0$ in the Zf formula.

Frame 71

Fully fixed shed



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{s_1}{h} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{s_2}{h}\,; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l}\,; \qquad \varphi = \frac{f}{h}\,;$$

$$C_1 = 2\beta(k_1 + k_2) + k_1 \qquad C_2 = 2\alpha(k_1 + k_2) + k_2 \qquad C_3 = 2\varphi(k_1 + k_2);$$

$$R_1 = 6 + \beta C_1 + (2 + \beta) k_1 \qquad K_1 = 3 - \varphi C_2$$

$$R_2 = 6 + \alpha C_2 + (2 + \alpha) k_2 \qquad K_2 = 3 - \varphi C_1$$

$$K_3 = \alpha C_1 + \beta k_2 = \beta C_2 + \alpha k_1\,;$$

$$R_3 = 4 + \varphi C_3\,;$$

$$\begin{aligned} N &= R_1 R_2 R_3 + 2 K_1 K_2 K_3 - R_1 K_1^2 - R_2 K_2^2 - R_3 K_3^2 = \\ &= 6\,[6 + 3(k_1 + k_2)(3 + 6\varphi + 4\varphi^2) + 2k_1(2\alpha^2 + 3\beta) + \\ &+ 2k_2(3\alpha + 2\beta^2) + k_1 k_2(8 + 9\varphi + 8\varphi^2) + 2(\alpha k_1 - \beta k_2)^2 + \\ &+ 3\varphi k_1^2(\alpha + \varphi) + 3\varphi k_2^2(\beta + \varphi) + \varphi^2 k_1 k_2(k_1 + k_2)]\,. \end{aligned}$$

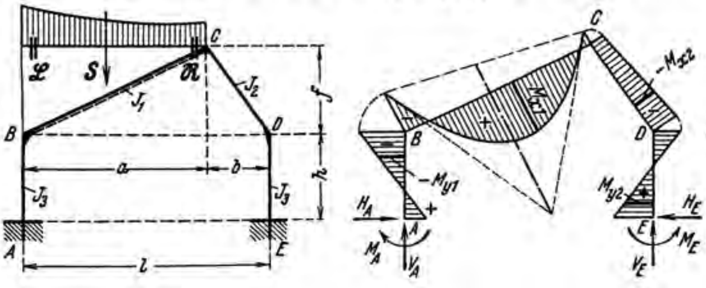
$$n_{11} = \frac{R_2 R_3 - K_1^2}{N} \qquad n_{12} = n_{21} = \frac{R_3 K_3 - K_1 K_2}{N}$$

$$n_{22} = \frac{R_1 R_3 - K_2^2}{N} \qquad n_{13} = n_{31} = \frac{R_2 K_2 - K_1 K_3}{N}$$

$$n_{33} = \frac{R_1 R_2 - K_3^2}{N} \qquad n_{23} = n_{32} = \frac{R_1 K_1 - K_2 K_3}{N}\,.$$

See Appendix A, Load Terms, pp. 440-445.

Case 71/1: Left inclined member loaded by any type of vertical load



Constants:

$$\mathfrak{B}_1=\beta C_1\mathfrak{S}_l+(\mathfrak{L}+\beta\mathfrak{R})k_1 \qquad X_1=+\mathfrak{B}_1 n_{11}-\mathfrak{B}_2 n_{21}+\mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_2=\beta C_2\mathfrak{S}_l+\alpha\mathfrak{R}k_1 \qquad X_2=-\mathfrak{B}_1 n_{12}+\mathfrak{B}_2 n_{22}+\mathfrak{B}_3 n_{32}$$

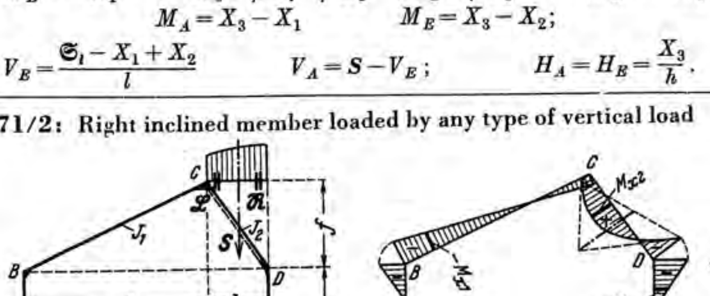
$$\mathfrak{B}_3=\beta C_3\mathfrak{S}_l+\varphi\mathfrak{R}k_1; \qquad X_3=+\mathfrak{B}_1 n_{13}+\mathfrak{B}_2 n_{23}+\mathfrak{B}_3 n_{33}.$$

$$M_B=-X_1 \qquad M_C=\beta\mathfrak{S}_l-\beta X_1-\alpha X_2-\varphi X_3 \qquad M_D=-X_2$$

$$M_A=X_3-X_1 \qquad M_E=X_3-X_2;$$

$$V_E=\frac{\mathfrak{S}_l-X_1+X_2}{l} \qquad V_A=S-V_E; \qquad H_A=H_E=\frac{X_3}{h}.$$

Case 71/2: Right inclined member loaded by any type of vertical load



Constants:

$$\mathfrak{B}_1=\alpha C_1\mathfrak{S}_r+\beta\mathfrak{L}k_2 \qquad X_1=+\mathfrak{B}_1 n_{11}-\mathfrak{B}_2 n_{21}+\mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_2=\alpha C_2\mathfrak{S}_r+(\alpha\mathfrak{L}+\mathfrak{R})k_2 \qquad X_2=-\mathfrak{B}_1 n_{12}+\mathfrak{B}_2 n_{22}+\mathfrak{B}_3 n_{32}$$

$$\mathfrak{B}_3=\alpha C_3\mathfrak{S}_r+\varphi\mathfrak{L}k_2; \qquad X_3=+\mathfrak{B}_1 n_{13}+\mathfrak{B}_2 n_{23}+\mathfrak{B}_3 n_{33}.$$

$$M_B=-X_1 \qquad M_C=\alpha\mathfrak{S}_r-\beta X_1-\alpha X_2-\varphi X_3 \qquad M_D=-X_2$$

$$M_A=X_3-X_1 \qquad M_E=X_3-X_2;$$

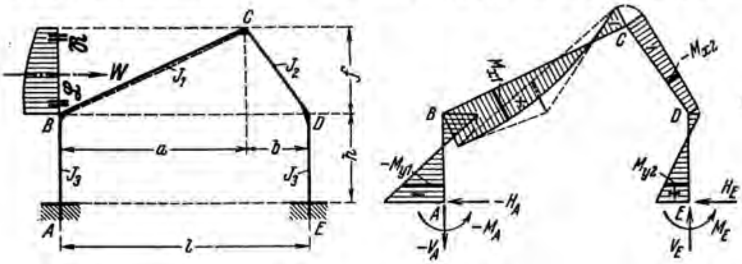
$$V_A=\frac{\mathfrak{S}_r+X_1-X_2}{l} \qquad V_E=S-V_A; \qquad H_A=H_E=\frac{X_3}{h}.$$

* See p. 259 for M_x and M_y

FRAME 71

See Appendix A, Load Terms, pp. 440-445.

Case 71/3: Left inclined member loaded by any type of horizontal load

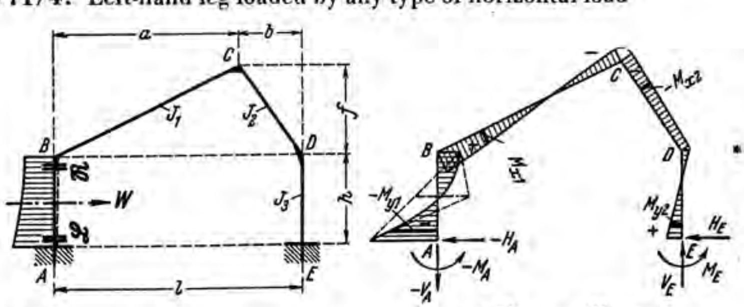


Constants:

$$\mathfrak{B}_1 = 3\,W h - \beta\, C_1\, \mathfrak{S}_l - (\mathfrak{L} + \beta\,\mathfrak{R})\, k_1 \qquad X_1 = +\,\mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21} - \mathfrak{B}_3 n_{31}$$
$$\mathfrak{B}_2 = \beta\, C_2\, \mathfrak{S}_l + \alpha\,\mathfrak{R}\, k_1 \qquad X_2 = +\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$
$$\mathfrak{B}_3 = 2\,W h + \beta\, C_3\, \mathfrak{S}_l + \varphi\,\mathfrak{R}\, k_1; \qquad X_3 = -\,\mathfrak{B}_1 n_{13} + \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

$$M_B = +\,X_1 \qquad M_C = \beta\,\mathfrak{S}_l + \beta\, X_1 - \alpha\, X_2 - \varphi\, X_3 \qquad M_D = -\,X_2$$
$$M_A = -\,W h + X_1 + X_3 \qquad M_E = X_3 - X_2;$$
$$V_E = -V_A = \frac{\mathfrak{S}_l + X_1 + X_2}{l}; \qquad H_E = \frac{X_3}{h} \qquad H_A = -\,(W - H_E).$$

Case 71/4: Left-hand leg loaded by any type of horizontal load



Constants:

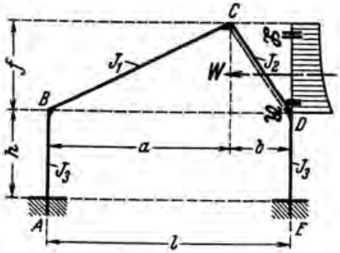
$$\mathfrak{B}_1 = 3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R}) \qquad X_1 = +\,\mathfrak{B}_1 n_{11} - \mathfrak{B}_3 n_{31}$$
$$\mathfrak{B}_3 = 2\,\mathfrak{S}_l - \mathfrak{L}; \qquad X_2 = +\,\mathfrak{B}_1 n_{12} + \mathfrak{B}_3 n_{32}$$
$$X_3 = -\,\mathfrak{B}_1 n_{13} + \mathfrak{B}_3 n_{33}.$$

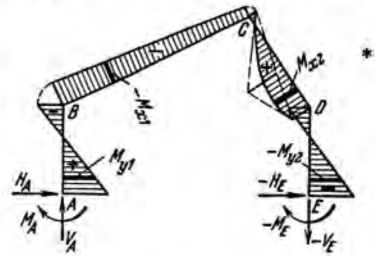
$$M_B = +\,X_1 \qquad M_C = \beta\, X_1 - \alpha\, X_2 - \varphi\, X_3 \qquad M_D = -\,X_2$$
$$M_A = -\,\mathfrak{S}_l + X_1 + X_3 \qquad M_E = X_3 - X_2;$$
$$V_E = -V_A = \frac{X_1 + X_2}{l}; \qquad H_E = \frac{X_3}{h} \qquad H_A = -\,(W - H_E).$$

* See p. 259 for M_x and M_y.

See Appendix A, Load Terms, pp. 440-445.

Case 71/5: Right inclined member loaded by any type of horizontal load



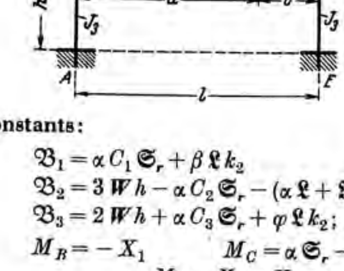


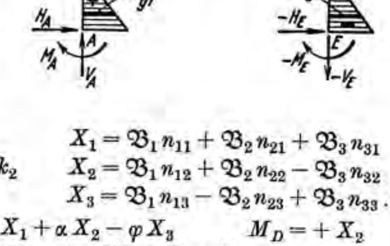
*

Constants:

$$\mathfrak{B}_1 = \alpha C_1 \mathfrak{S}_r + \beta \mathfrak{L} k_2 \qquad X_1 = \mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$
$$\mathfrak{B}_2 = 3 W h - \alpha C_2 \mathfrak{S}_r - (\alpha \mathfrak{L} + \mathfrak{R}) k_2 \qquad X_2 = \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} - \mathfrak{B}_3 n_{32}$$
$$\mathfrak{B}_3 = 2 W h + \alpha C_3 \mathfrak{S}_r + \varphi \mathfrak{L} k_2; \qquad X_3 = \mathfrak{B}_1 n_{13} - \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$
$$M_B = -X_1 \qquad M_C = \alpha \mathfrak{S}_r - \beta X_1 + \alpha X_2 - \varphi X_3 \qquad M_D = +X_2$$
$$M_A = X_3 - X_1 \qquad M_E = -W h + X_2 + X_3.$$
$$V_A = -V_E = \frac{\mathfrak{S}_r + X_1 + X_2}{l}; \qquad H_A = \frac{X_3}{h} \qquad H_E = -(W - H_A).$$

Case 71/6: Right-hand leg loaded by any type of horizontal load





*

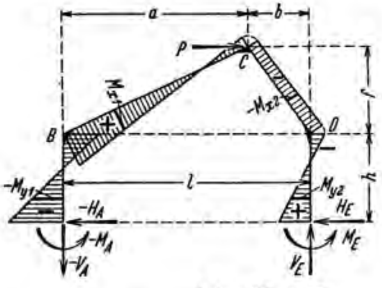
Constants:

$$X_1 = + \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$
$$\mathfrak{B}_2 = 3 \mathfrak{S}_r - (\mathfrak{L} + \mathfrak{R}); \qquad X_2 = + \mathfrak{B}_2 n_{22} - \mathfrak{B}_3 n_{32}$$
$$\mathfrak{B}_3 = 2 \mathfrak{S}_r - \mathfrak{R}; \qquad X_3 = - \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$
$$M_B = -X_1 \qquad M_C = -\beta X_1 + \alpha X_2 - \varphi X_3 \qquad M_D = +X_2$$
$$M_A = X_3 - X_1 \qquad M_E = -\mathfrak{S}_r + X_2 + X_3.$$
$$V_A = -V_E = \frac{X_1 + X_2}{l}; \qquad H_A = \frac{X_3}{h} \qquad H_E = -(W - H_A).$$

* See p. 239 for M_x and M_y.

FRAME 71

Case 71/7: Horizontal concentrated load at ridge C



Constants: $\mathfrak{B}_1 = Ph(3-\varphi\beta C_1)$

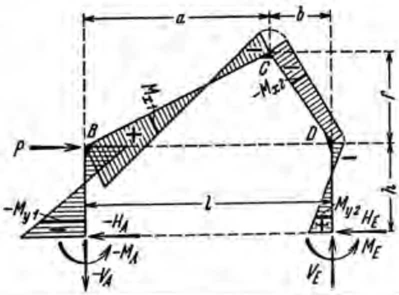
$\mathfrak{B}_2 = Pf\cdot\beta C_2 \quad \mathfrak{B}_3 = Ph(2+\varphi\beta C_3);$

$$X_1 = +\mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21} - \mathfrak{B}_3 n_{31}$$
$$X_2 = +\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$
$$X_3 = -\mathfrak{B}_1 n_{13} + \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

$$M_B = +X_1 \qquad M_D = -X_2$$
$$M_C = Pf\cdot\beta + \beta X_1 - \alpha X_2 - \varphi X_3$$
$$M_A = -Ph + X_1 + X_3$$
$$M_E = X_3 - X_2;$$

$$V_E = -V_A = \frac{Pf}{l} + \frac{X_1+X_2}{l}; \qquad H_E = \frac{X_3}{h} \qquad H_A = -P + \frac{X_3}{h}.$$

Case 71/8: Horizontal concentrated load at B



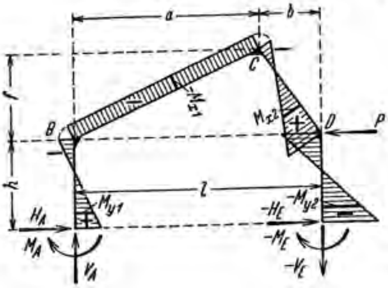
Constants: $X_1 = Ph(+3n_{11} - 2n_{31})$

$X_2 = Ph(+3n_{12} + 2n_{32})$

$X_3 = Ph(-3n_{13} + 2n_{33}).$

$$M_B = +X_1 \quad M_C = \beta X_1 - \alpha X_2 - \varphi X_3$$
$$M_D = -X_2 \quad M_A = -Ph + X_1 + X_3;$$
$$V_E = -V_A = \frac{X_1+X_2}{l}; \qquad H_E = \frac{X_3}{h}$$
$$H_A = -P + \frac{X_3}{h}; \qquad M_E = X_3 - X_2.$$

Case 71/9: Horizontal concentrated load at D



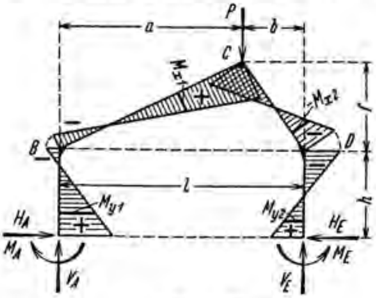
Constants: $X_1 = Ph(+3n_{21} + 2n_{31})$

$X_2 = Ph(+3n_{22} - 2n_{32})$

$X_3 = Ph(-3n_{23} + 2n_{33}).$

$$M_B = -X_1 \quad M_C = -\beta X_1 + \alpha X_2 - \varphi X_3$$
$$M_D = +X_2 \quad M_E = -Ph + X_2 + X_3;$$
$$V_A = -V_E = \frac{X_1+X_2}{l}; \qquad H_A = \frac{X_3}{h}$$
$$H_E = -P + \frac{X_3}{h}; \qquad M_A = X_3 - X_1.$$

Case 71/10: Vertical concentrated load at ridge C

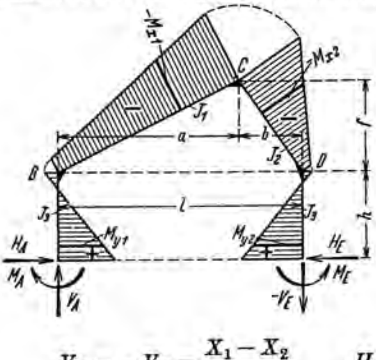


Constants: $M^0 = \frac{Pab}{l}$;

$$X_1 = M^0(+C_1 n_{11} - C_2 n_{21} + C_3 n_{31})$$
$$X_2 = M^0(-C_1 n_{12} + C_2 n_{22} + C_3 n_{32})$$
$$X_3 = M^0(+C_1 n_{13} + C_2 n_{23} + C_3 n_{33}).$$
$$M_A = X_3 - X_1 \qquad M_E = X_3 - X_2$$
$$M_B = -X_1$$
$$M_C = M^0 - \beta X_1 - \alpha X_2 - \varphi X_3$$
$$M_D = -X_2;$$

$$V_A = P\beta + \frac{X_1 - X_2}{l} \qquad V_E = P\alpha + \frac{X_2 - X_1}{l}; \qquad H_A = H_E = \frac{X_3}{h}.$$

Case 71/11: Uniform increase in temperature of the entire frame*



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants: $T = \frac{6EJ_3 l \cdot \varepsilon t}{h^2}$;

$$X_1 = T \cdot n_{31} \quad X_2 = T \cdot n_{32} \quad X_3 = T \cdot n_{33}.$$
$$M_B = -X_1$$
$$M_C = -\beta X_1 - \alpha X_2 - \varphi X_3$$
$$M_D = -X_2$$

$$V_A = -V_E = \frac{X_1 - X_2}{l} \qquad H_A = H_E = \frac{X_3}{h}; \qquad M_A = X_3 - X_1, \quad M_E = X_3 - X_2.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Equations for moments at any point of frame 71 for all loading conditions

The moments at the joints and the fixed end moments contribute to the total moment:

$$M_{y1} = \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_{y2} = \frac{y_2}{h} M_D + \frac{y_2'}{h} M_E$$

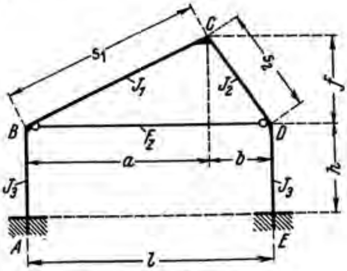
$$M_{x1} = \frac{x_1'}{a} M_B + \frac{x_1}{a} M_C \qquad M_{x2} = \frac{x_2'}{b} M_C + \frac{x_2}{b} M_D.$$

To these moments add the moments M_y^o and M_x^o resp. for directly loaded members only.

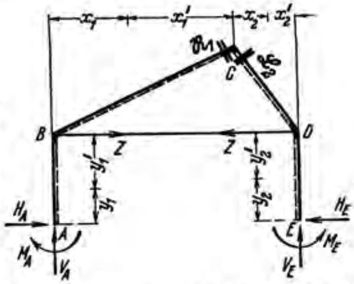
* Equal temperature changes in the vertical legs do not cause stress.

Frame 72

Fully fixed shed with tie-rod at the eaves



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point.

General notes

Frame 72 with tie is best considered as a more general case of frame 71 without tie. The effect of the tie is easily shown as follows:

Steps in computing the stresses

First step: For each loading condition compute all moments M_A, M_B, M_C, M_D, M_E and reactions H_A, H_E, V_A, V_E from frame 71.

Second step:

a) additional coefficients for frame 72

$$m_1 = +3n_{11} - 3n_{21} - 4n_{31} \qquad m_a = 1 - m_3 - m_1$$
$$m_2 = -3n_{12} + 3n_{22} - 4n_{32} \qquad m_e = 1 - m_3 - m_2$$
$$m_3 = -3n_{13} - 3n_{23} + 4n_{33}\,; \qquad m_c = \varphi m_3 - \beta m_1 - \alpha m_2\,.$$

$$L = \frac{6J_3}{h^2F_Z}\cdot\frac{l}{f}\cdot\frac{E}{E_Z} \qquad G = 2m_c(k_1+k_2) - m_1k_1 - m_2k_2 \qquad N_Z = G + L\,.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Note: For a rigid tie set $L = 0$, $N_Z = G$.

b) Figure the tension in the tie rod.

$$Z \cdot h = \frac{M_B k_1 + 2\,M_C (k_1 + k_2) + M_D k_2 + \mathfrak{R}_1 k_1 + \mathfrak{L}_2 k_2}{N_Z} *$$

Note: The load terms $\mathfrak{R}_1$ and $\mathfrak{L}_2$ used in this formula are shown in the right-hand sketch on p. 260 and are to be used accordingly.**

Third step:

a) Moments at the joints, moments at the supports and reactions for Frame 72

$$\overline{M}_B = M_B + Z\,h \cdot m_1 \qquad \overline{M}_C = M_C - Z\,h \cdot m_c \qquad \overline{M}_D = M_D + Z\,h \cdot m_2$$

$$\overline{M}_A = M_A - Z\,h \cdot m_a \qquad \overline{M}_E = M_E - Z\,h \cdot m_e;$$

$$\overline{H}_A = H_A - Z(1 - m_3) \qquad \overline{H}_E = H_E - Z(1 - m_3); \qquad \overline{V}_A = V_A \qquad \overline{V}_E = V_E.$$

Note: In order to distinguish the moments and reactions for Frame 72 the values are shown with a dash over the letter.

b) Moments at any point of Frame 72.

The formulas for $\overline{M}_x$ and $\overline{M}_y$ are the same as for Frame 71, except that the values $\overline{M}_A$, $\overline{M}_B$, $\overline{M}_C$, $\overline{M}_D$, $\overline{M}_E$ are to be used instead of M_A, M_B, M_C, M_D, M_E.

---

* For the case of various loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

** For use of the loading conditions of frame 71 substitute the following in the Zh formula for the load terms $\mathfrak{R}_1$ and $\mathfrak{L}_2$

Case 71/1: $\mathfrak{R}_1 = \mathfrak{R}$; $\mathfrak{L}_2 = 0$; Case 71/2: $\mathfrak{R}_1 = 0$; $\mathfrak{L}_2 = \mathfrak{L}$;

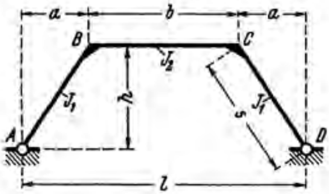
Case 71/3: $\mathfrak{R}_1 = \mathfrak{R}$; $\mathfrak{L}_2 = 0$; Case 71/5: $\mathfrak{R}_1 = 0$; $\mathfrak{L}_2 = \mathfrak{L}$;

Case 71/11: $\mathfrak{R}_1 k_1 + \mathfrak{L}_2 k_2 = 6\,E J_2 \cdot \varepsilon t \cdot l/h/$.

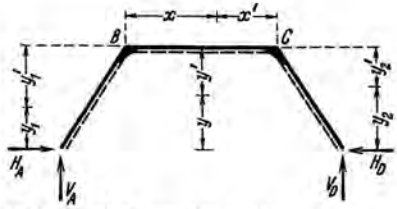
For all remaining load conditions, including the case of uniform temperature change in the entire frame including the tie rod, substitute $\mathfrak{R}_1 = \mathfrak{L}_2 = 0$ in the Zh formula.

Frame 73

Symmetrical two-hinged, trapezoidal rigid frame.



Shape of Frame
Dimensions and Notations



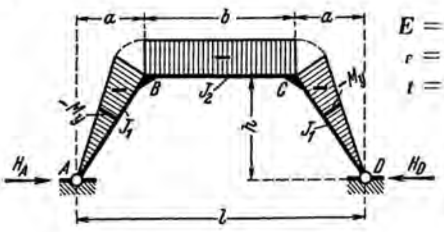
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{s}{b}; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l}; \qquad N = 2k + 3.$$

Note: Formulas for moments same as for frame 74, pp. 268-271, or frame 76, p. 278, using $h_1 = h_2$.

Case 73/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

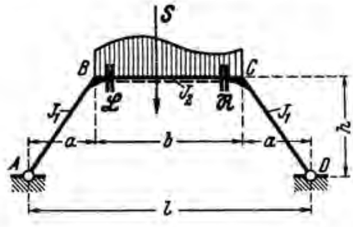
$$M_B = M_C = -\frac{3 E J_2 \varepsilon t l}{b h N};$$

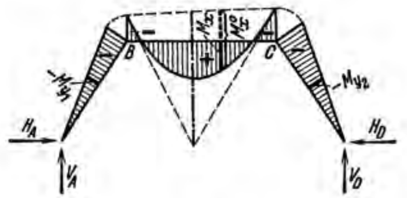
$$H_A = H_D = \frac{-M_B}{h}; \quad M_y = -H_A y.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

See Appendix A, Load Terms, pp. 440-445.

Case 73/2: Girder loaded by any type of vertical load





$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -\frac{(\mathfrak{L}+\mathfrak{R})}{2N} \pm \frac{\alpha(\mathfrak{S}_r - \mathfrak{S}_l)}{2};$$

$$V_A = \frac{\mathfrak{S}_r + Sa}{l} \qquad V_D = \frac{Sa + \mathfrak{S}_l}{l}; \qquad H_A = H_D = \frac{SaN + (\mathfrak{L}+\mathfrak{R})}{2hN};$$

$$M_{y1} = \frac{y_1}{h} M_B \qquad M_x = M_x^0 + \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h} M_C.$$

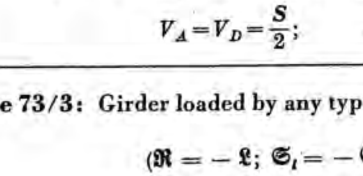
Special case 73/2a: Symmetrical girder load ($\mathfrak{R} = \mathfrak{L}$; $\mathfrak{S}_l = \mathfrak{S}_r$)

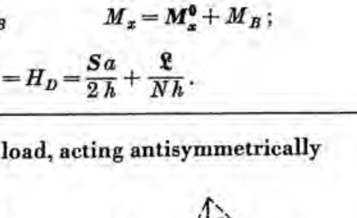
$$M_B = M_C = -\frac{\mathfrak{L}}{N} \qquad M_y = \frac{y}{h} M_B \qquad M_x = M_x^0 + M_B;$$

$$V_A = V_D = \frac{S}{2}; \qquad H_A = H_D = \frac{Sa}{2h} + \frac{\mathfrak{L}}{Nh}.$$

Case 73/3: Girder loaded by any type of load, acting antisymmetrically

($\mathfrak{R} = -\mathfrak{L}$; $\mathfrak{S}_l = -\mathfrak{S}_r$)





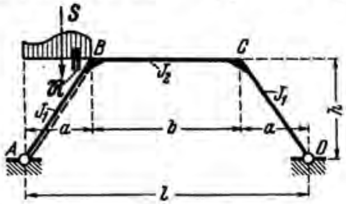
$$V_A = -V_D = \frac{\mathfrak{S}_r}{l} \qquad M_B = -M_C = \alpha\,\mathfrak{S}_r; \qquad H_A = H_D = 0;$$

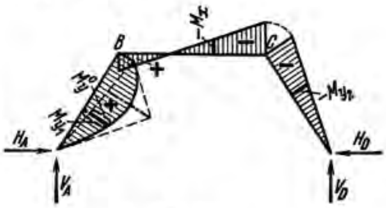
$$M_{y1} = -M_{y2} = \frac{y_1}{h} \cdot M_B \qquad M_x = M_x^0 + \left(1 - 2\frac{x}{b}\right) \cdot M_B.$$

FRAME 73

See Appendix A, Load Terms, pp. 440-445.

Case 73/4: Left-hand leg loaded by any type of vertical load

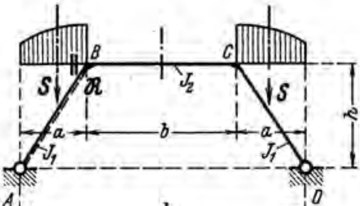


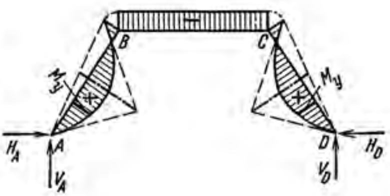


$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\} = \pm\frac{\beta\mathfrak{S}_l}{2} - \frac{\mathfrak{R}k}{2N}; \qquad M_{y1} = M_y^0 + \frac{y_1}{h}M_B;$$

$$V_D = \frac{\mathfrak{S}}{l} \qquad V_A = S - V_D; \qquad H_A = H_D = \frac{\alpha\mathfrak{S}_l - M_C}{h}.$$

Case 73/5: Both legs loaded by any type of symmetrical vertical load

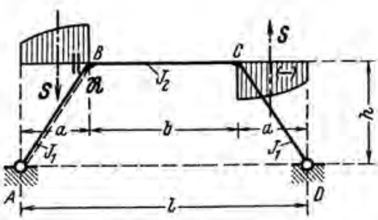


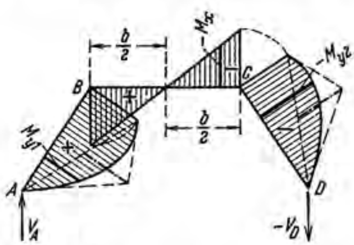


$$M_B = M_C = -\frac{\mathfrak{R}k}{N}; \qquad H_A = H_D = \frac{\mathfrak{S}_l - M_B}{h}; \qquad V_A = V_D = S.$$

Note: All terms refer to the left leg.

Case 73/6: Both legs loaded by any type of antisymmetrical vertical load



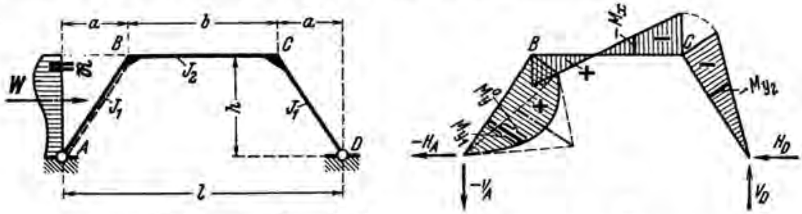


$$M_B = -M_C = \beta\mathfrak{S}_l; \qquad V_A = -V_D = S - \frac{2\mathfrak{S}_l}{l}; \qquad H_A = H_D = 0.$$

Note: All terms refer to the left leg.

See Appendix A, Load Terms, pp. 440-445.

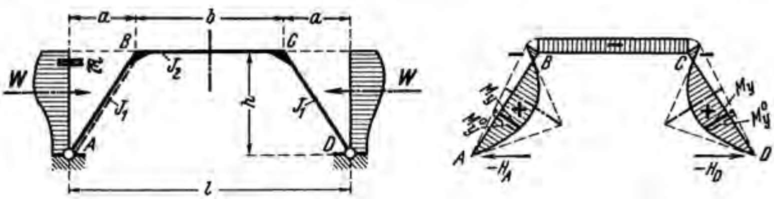
Case 73/7: Left-hand leg loaded by any type horizontal load



$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = \pm\frac{\beta\,\mathfrak{S}_l}{2} - \frac{\mathfrak{N}\,k}{2\,N}; \qquad M_{y1} = M_y^0 + \frac{y_1}{h} M_B;$$

$$V_D = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_D = \frac{\alpha\,\mathfrak{S}_l - M_C}{h} \qquad H_A = -(W - H_D).$$

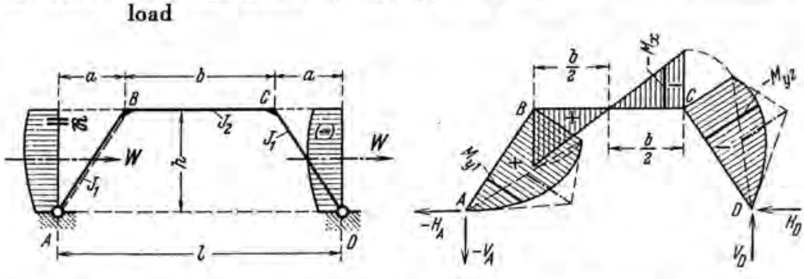
Case 73/8: Both legs loaded by any type of symmetrical horizontal load



$$M_B = M_C = -\frac{\mathfrak{N}\,k}{N}; \qquad H_A = H_D = -\frac{\mathfrak{S}_r + M_B}{h}; \qquad V_A = V_D = 0.$$

Note: All terms refer to the left leg.

Case 73/9: Both legs loaded by any type of antisymmetrical horizontal load

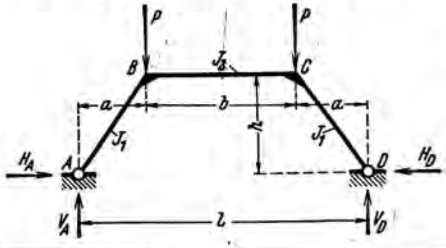


$$M_B = -M_C = \beta\,\mathfrak{S}_l; \qquad V_D = -V_A = \frac{2\,\mathfrak{S}_l}{l}; \qquad H_D = -H_A = W.$$

Note: All terms refer to the left leg.

FRAME 73

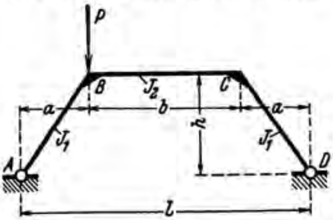
Case 73/10: Two equal vertical concentrated loads at B and C

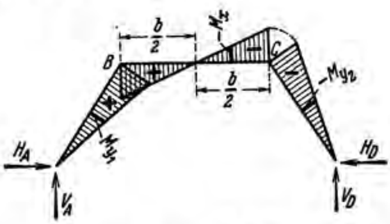


There are no bending moments

$$V_A = V_D = P$$
$$H_A = H_D = \frac{Pa}{h}.$$

Case 73/11: Vertical concentrated load at B



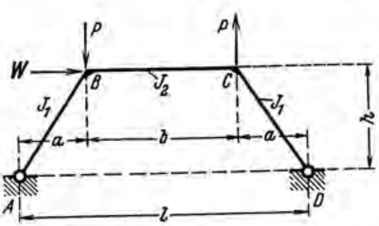


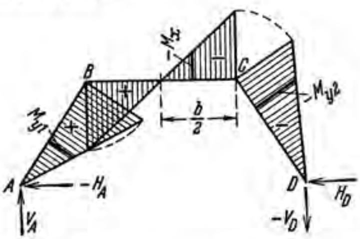
$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = \pm \frac{Pa\beta}{2} \qquad M_{y1} = -M_{y2} = \frac{y}{h} M_B \qquad M_x = \frac{x'-x}{b} M_B;$$

$$V_D = \alpha P \qquad V_A = (1-\alpha) P; \qquad H_A = H_D = \frac{Pa}{2h}.$$

Note: Moments are antisymmetrical.

Case 73/12 and 13: Vertical couple Pb at the corners B and C and additional horizontal concentrated load W, acting at the girders (antisymmetrical load)



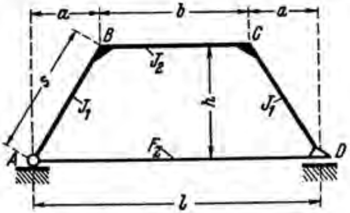


$$M_B = -M_C = \left(Pa + \frac{Wh}{2}\right)\beta \qquad M_{y1} = -M_{y2} = \frac{y}{h} M_B \qquad M_x = \frac{x'-x}{b} M_B;$$

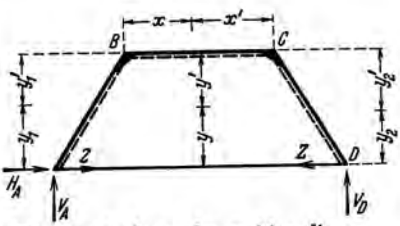
$$V_A = -V_D = P\beta - \frac{Wh}{l}; \qquad H_D = -H_A = \frac{W}{2}.$$

Frame 74

Symmetrical trapezoidal rigid frame with horizontal tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{s}{b}; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l};$$

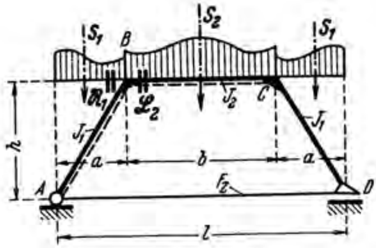
$$N = 2k + 3 \qquad L = \frac{3J_2}{h^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{l}{b}; \qquad N_Z = N + L.$$

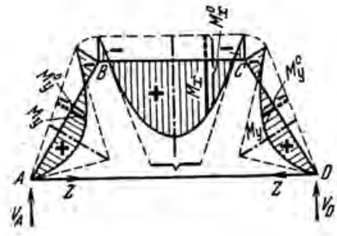
E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

FRAME 74

(See Appendix A, Load Terms, pp. 440-445.)

Case 74/1: Entire frame loaded by any type of vertical load, acting symmetrically



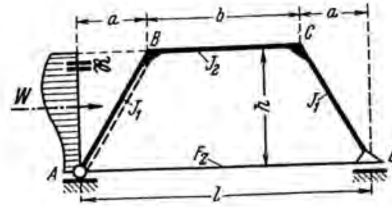


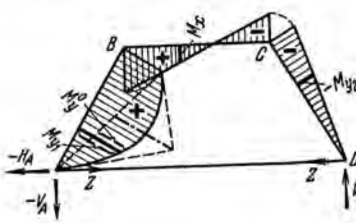
$$Z=\frac{N\,\mathfrak{S}_{l1}+\mathfrak{R}_1 k}{h\,N_Z}+\frac{N\,S_2 a+2\,\mathfrak{L}_2}{2h\,N_Z};\qquad V_A=V_D=S_1+\frac{S_2}{2};$$

$$M_B=M_C=\mathfrak{S}_{l1}+\frac{S_2 a}{2}-Z h\qquad M_y=M_y^0+\frac{y}{h}M_B\qquad M_x=M_x^0+M_B.$$

Note: All the load terms with the subscript 1 refer to the left leg.

Case 74/3: Left-hand leg loaded by any type of horizontal load





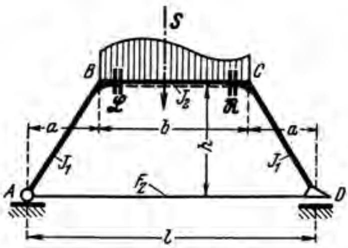
$$Z=\frac{N\,\mathfrak{S}_l+\mathfrak{R}\,k}{2h\,N_Z};\qquad V_D=-V_A=\frac{\mathfrak{S}_l}{l};\qquad H_A=-W;$$

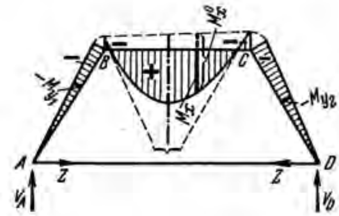
$$M_B=(1-\alpha)\,\mathfrak{S}_l-Z h\qquad M_C=\alpha\,\mathfrak{S}_l-Z h$$

$$M_{y1}=M_y^0+\frac{y_1}{h}M_B\qquad M_x=\frac{x'}{b}M_B+\frac{x}{b}M_C\qquad M_{y2}=\frac{y_2}{h}M_C.$$

See Appendix A, Load Terms, pp. 440-445.

Case 74/2: Girder loaded by any type of vertical load



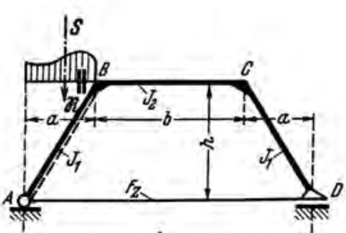


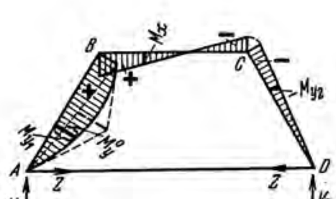
$$Z = \frac{N\,S a + (\mathfrak{L} + \mathfrak{R})}{2\,h\,N_Z}; \qquad V_A = \frac{\mathfrak{S}_r + S a}{l} \qquad V_D = \frac{S a + \mathfrak{S}_l}{l};$$

$$M_B = (\mathfrak{S}_r + S a)\,\alpha - Z h \qquad\qquad M_C = (S a + \mathfrak{S}_l)\,\alpha - Z h$$

$$M_{y1} = \frac{y_1}{h} M_B \qquad M_x = M_x^0 + \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h} M_C\,.$$

Case 74/4: Left-hand leg loaded by any type of vertical load





$$Z = \frac{N\,\mathfrak{S}_l + \mathfrak{R}\,k}{2\,h\,N_Z}; \qquad V_D = \frac{\mathfrak{S}_l}{l} \qquad V_A = S - V_D\,;$$

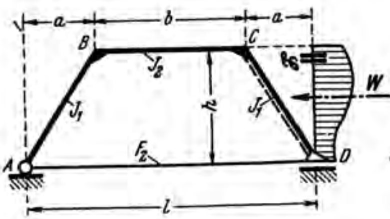
$$M_B = (1-\alpha)\,\mathfrak{S}_l - Z h \qquad\qquad M_C = \alpha\,\mathfrak{S}_l - Z h$$

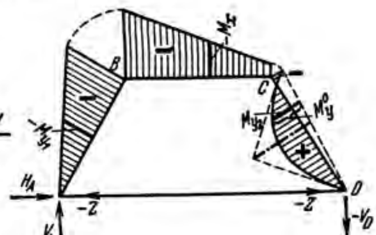
$$M_{y1} = M_y^0 + \frac{y_1}{h} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h} M_C\,.$$

FRAME 74

See Appendix A, Load Terms, pp. 440-445.

Case 74/5: Right-hand leg loaded by any type of horizontal load



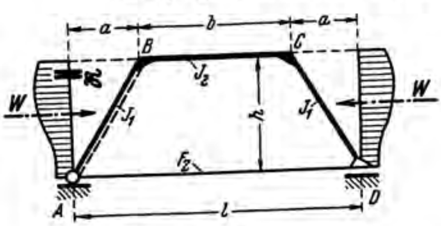


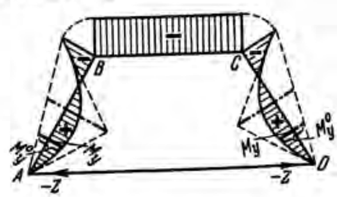
$$Z = -\left(W\frac{N}{N_Z} - \frac{N\,\mathfrak{S}_r + \mathfrak{L}k}{2hN_Z}\right)^* \;; \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l}\;; \qquad H_A = W\;;$$

$$M_B = -(W + Z)h + \alpha\,\mathfrak{S}_r \qquad M_C = -(W + Z)h + (1-\alpha)\,\mathfrak{S}_r$$

$$M_{y1} = \frac{y_1}{h}M_B \qquad M_x = \frac{x'}{b}M_B + \frac{x}{b}M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h}M_C\,.$$

Case 74/6: Both legs loaded by any type of horizontal load, both members carrying the same load (Symmetrical load)





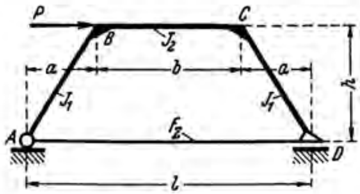
$$Z = -\frac{N\,\mathfrak{S}_r - \mathfrak{R}k}{hN_Z}* \;; \qquad M_B = M_C = -(\mathfrak{S}_r + Zh) = -\frac{L\,\mathfrak{S}_r + \mathfrak{R}k}{N_Z}$$

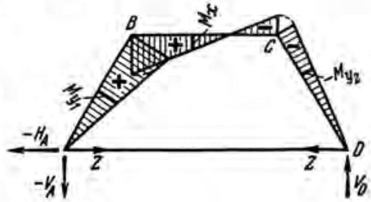
$$M_y = M_y^0 + \frac{y}{h}M_B\,.$$

Note: All the load terms refer to the left leg.

* For the above two loading conditions and for a decrease in temperature (p. 271 bottom) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

Case 74/7: Horizontal concentrated load acting at the girder



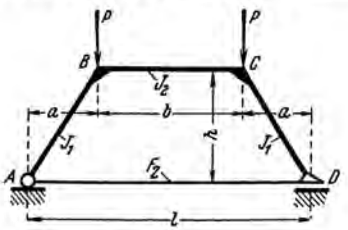


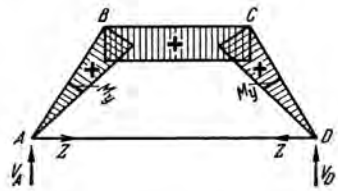
$$Z=\frac{P}{2}\cdot\frac{N}{N_Z};\qquad V_D=-V_A=\frac{Ph}{l};\qquad H_A=-P;$$

$$M_B=[(1-\alpha)P-Z]h\qquad M_C=(\alpha P-Z)h;$$

$$M_{y1}=\frac{y_1}{h}M_B\qquad M_x=\frac{x'}{b}M_B+\frac{x}{b}M_C\qquad M_{y2}=\frac{y_2}{h}M_C.$$

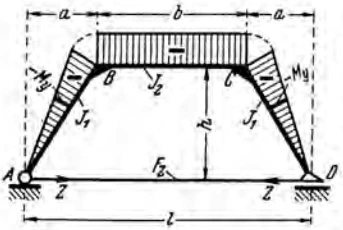
Case 74/8: Two equal vertical concentrated loads at *B* and *C*





$$Z=\frac{Pa}{h}\cdot\frac{N}{N_Z};\qquad V_A=V_D=P;\qquad M_B=M_C=Pa-Zh\qquad M_y=\frac{y}{h}M_B.$$

Case 74/9: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z=\frac{3EJ_2\varepsilon t l}{bh^2N_Z};$$

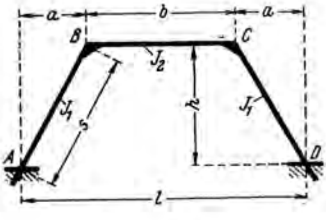
$$M_B=M_C=-Zh\qquad M_y=-Zy.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

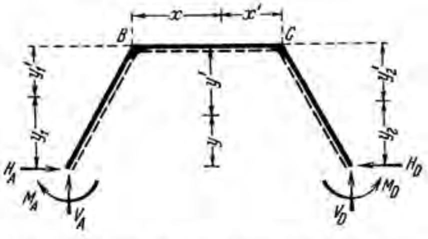
*See footnote on page 270.

Frame 75

Symmetrical hingeless, trapezoidal rigid frame.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

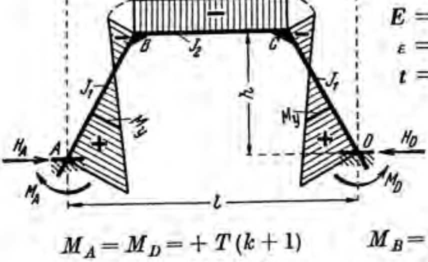
Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{s}{b}; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l};$$

$$K_1 = 2k + 3 \qquad K_2 = k(1+\beta) + \beta(1+k);$$

$$N_1 = k + 2 \qquad N_2 = 2(1+\beta+\beta^2)k + \beta^2.$$

Case 75/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $$T = \frac{3EJ_1 l \cdot \varepsilon t}{s h N_1}.$$

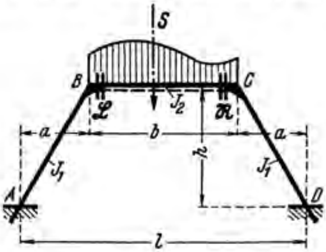
$$M_A = M_D = +T(k+1) \qquad M_B = M_C = -Tk; \qquad V_A = V_D = 0$$

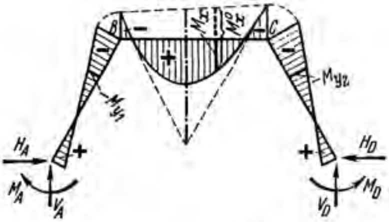
$$H_A = H_D = \frac{M_A - M_B}{h}; \qquad M_y = \frac{y'}{h} M_A + \frac{y}{h} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

See Appendix A, Load Terms, pp. 440-445.

Case 75/2: Girder loaded by any type of vertical load





Constants: $$X_1 = \frac{(\mathfrak{L} + \mathfrak{R})}{6\,N_1} \qquad X_3 = \frac{\alpha\,(\mathfrak{S}_r - \mathfrak{S}_l)\,K_2 + \beta\,(\mathfrak{L} - \mathfrak{R})}{2\,N_2}.$$

$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = +X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -2\,X_1 \pm \left[\frac{\alpha\,(\mathfrak{S}_r - \mathfrak{S}_l)}{2} - \beta\,X_3\right];$$

$$H_A = H_D = \frac{S\,a}{2\,h} + \frac{3\,X_1}{h}; \qquad \left.\begin{matrix} V_A \\ V_D \end{matrix}\right\rangle = \frac{S}{2} \pm \left[\frac{(\mathfrak{S}_r - \mathfrak{S}_l)}{2\,l} + \frac{2\,X_3}{l}\right];$$

$$M_x = \boldsymbol{M}_x^0 + \frac{x'}{b}\,M_B + \frac{x}{b}\,M_C.$$

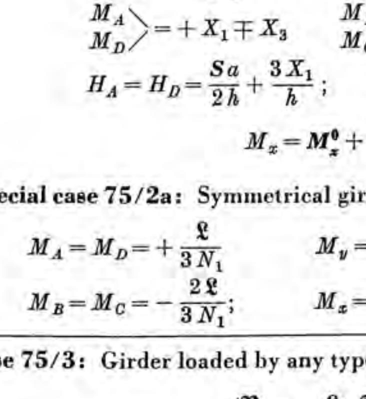
Special case 75/2a: Symmetrical girder load ($\mathfrak{R} = \mathfrak{L}$; $\mathfrak{S}_l = \mathfrak{S}_r$)

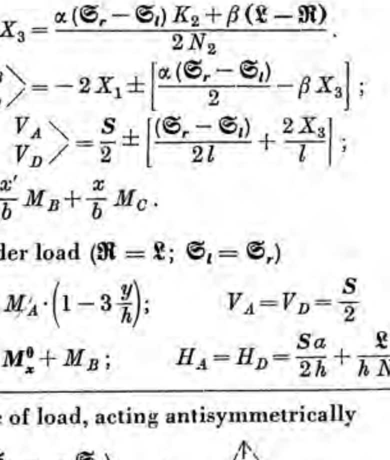
$$M_A = M_D = +\frac{\mathfrak{L}}{3\,N_1} \qquad M_y = M_A \cdot \left(1 - 3\,\frac{y}{h}\right); \qquad V_A = V_D = \frac{S}{2}$$

$$M_B = M_C = -\frac{2\,\mathfrak{L}}{3\,N_1}; \qquad M_x = \boldsymbol{M}_x^0 + M_B; \qquad H_A = H_D = \frac{S\,a}{2\,h} + \frac{\mathfrak{L}}{h\,N_1}.$$

Case 75/3: Girder loaded by any type of load, acting antisymmetrically

($\mathfrak{R} = -\mathfrak{L}$; $\mathfrak{S}_l = -\mathfrak{S}_r$)





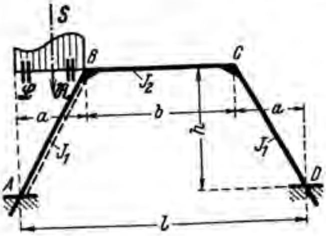
$$M_D = -M_A = \frac{\alpha\,\mathfrak{S}_r\,K_2 + \beta\,\mathfrak{L}}{N_2} \qquad M_B = -M_C = \alpha\,\mathfrak{S}_r - \beta\,M_D; \qquad H_A = H_D = 0;$$

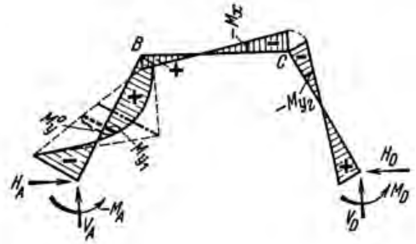
$$V_A = -V_D = \frac{\mathfrak{S}_r + 2\,M_D}{l} = \frac{\mathfrak{S}_r - 2\,M_B}{b}; \qquad M_x = \boldsymbol{M}_x^0 + \frac{x' - x}{b} \cdot M_B.$$

FRAME 75

(See Appendix A, Load Terms, pp. 440-445.)

Case 75/4: Left-hand leg loaded by any type of vertical load





Constants:

$$X_1 = \frac{\mathfrak{L} K_1 - \mathfrak{R} k}{6 N_1} \qquad X_2 = \frac{(2\,\mathfrak{R} - \mathfrak{L}) k}{6 N_1} \qquad X_3 = \frac{\beta\, \mathfrak{S}_l K_2 + (\mathfrak{L} + \beta\, \mathfrak{R}) k}{2 N_2}.$$

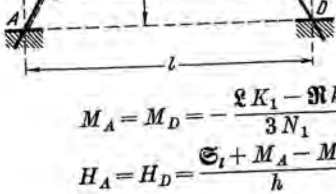
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -X_2 \pm \beta\left(\frac{\mathfrak{S}_l}{2} - X_3\right);$$

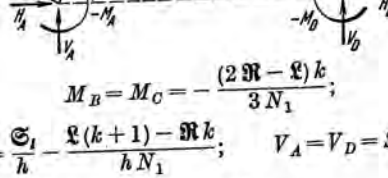
$$H_A = H_D = \frac{\mathfrak{S}_l}{2h} - \frac{X_1 - X_2}{h}; \qquad V_D = \frac{\mathfrak{S}_l - 2 X_3}{l} \qquad V_A = S - V_D;$$

$$M_{y1} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_{y2} = \frac{y_2}{h} M_C + \frac{y_2'}{h} M_D$$

$$M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C.$$

Case 75/5: Both legs loaded by any type of symmetrical vertical load





$$M_A = M_D = -\frac{\mathfrak{L} K_1 - \mathfrak{R} k}{3 N_1} \qquad M_B = M_C = -\frac{(2\,\mathfrak{R} - \mathfrak{L}) k}{3 N_1};$$

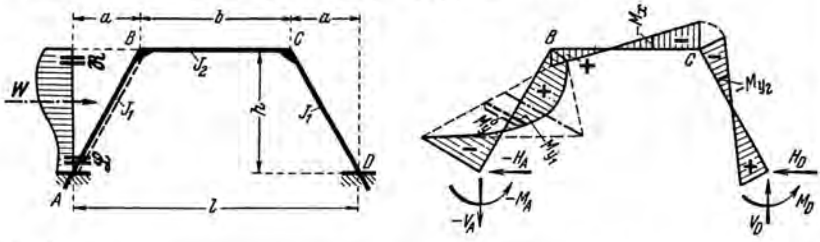
$$H_A = H_D = \frac{\mathfrak{S}_l + M_A - M_B}{h} = \frac{\mathfrak{S}_l}{h} - \frac{\mathfrak{L}(k+1) - \mathfrak{R} k}{h N_1}; \qquad V_A = V_D = S;$$

$$M_y = M_y^0 + \frac{y'}{h} M_A + \frac{y}{h} M_B \qquad M_x = M_B.$$

Note: All the load terms refer to the left half of the frame.

See Appendix A, Load Terms, pp. 440-445.

Case 75/6: Left-hand leg loaded by any type of horizontal load



Constants:

$$X_1 = \frac{\mathfrak{L} k_1 - \mathfrak{R} k}{6 N_1} \qquad X_2 = \frac{(2\mathfrak{R} - \mathfrak{L}) k}{6 N_1} \qquad X_3 = \frac{\beta \mathfrak{S}_l K_2 + (\mathfrak{L} + \beta \mathfrak{R}) k}{2 N_2}.$$

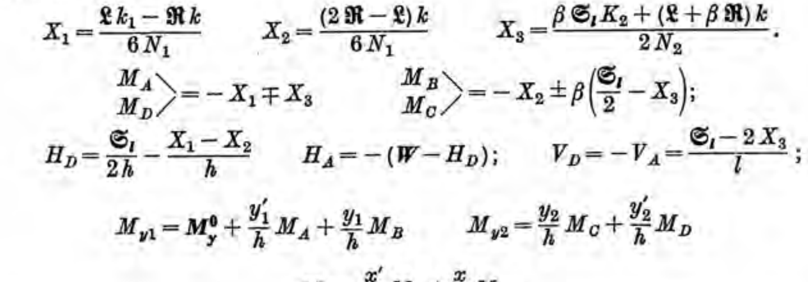
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -X_2 \pm \beta\left(\frac{\mathfrak{S}_l}{2} - X_3\right);$$

$$H_D = \frac{\mathfrak{S}_l}{2h} - \frac{X_1 - X_2}{h} \qquad H_A = -(W - H_D); \qquad V_D = -V_A = \frac{\mathfrak{S}_l - 2X_3}{l};$$

$$M_{y1} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_{y2} = \frac{y_2}{h} M_C + \frac{y_2'}{h} M_D$$

$$M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C.$$

Case 75/7: Both legs loaded by any type of symmetrical horizontal load



$$M_A = M_D = -\frac{\mathfrak{L} K_1 - \mathfrak{R} k}{3 N_1} \qquad M_B = M_C = -\frac{(2\mathfrak{R} - \mathfrak{L}) k}{3 N_1};$$

$$H_A = H_D = -\frac{\mathfrak{S}_r - M_A + M_B}{h}; \qquad M_y = M_y^0 + \frac{y'}{h} M_A + \frac{y}{h} M_B;$$

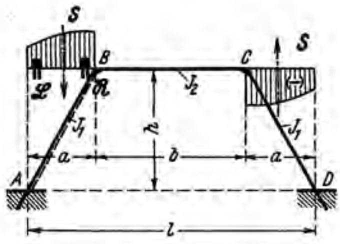
$$V_A = V_D = 0; \qquad M_x = M_B.$$

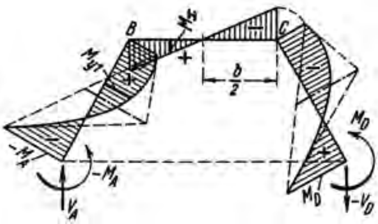
Note: All terms refer to the left leg.

FRAME 75

See Appendix A, Load Terms, pp. 440-445.

Case 75/8: Both legs loaded by any type of antisymmetrical vertical load





$$M_D = -M_A = \frac{\beta\,\mathfrak{S}_l K_2 + (\mathfrak{L} + \beta\,\mathfrak{R})\,k}{N_2} \qquad M_B = -M_C = \beta\,(\mathfrak{S}_l - M_D);$$

$$V_A = -V_D = S - \frac{2\,(\mathfrak{S}_l - M_D)}{l}; \qquad H_A = H_D = 0;$$

$$M_{y1} = -M_{y2} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_x = \frac{x' - x}{b} \cdot M_B .$$

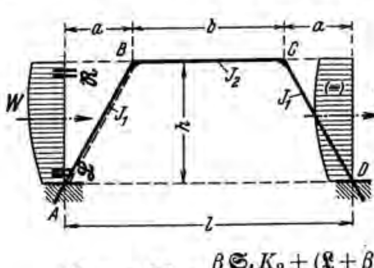
Note: All terms refer to the left leg.

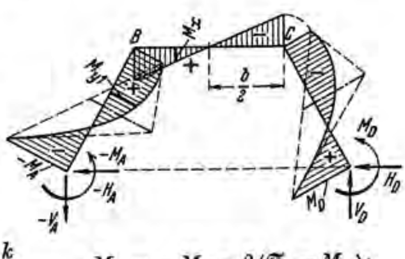
Case 75/9: Vertical couple Pb at the corners B and C (cf. case 73/12, page 266)

Substitute in case 75/8:

$$S = P \qquad \mathfrak{S}_l = Pa; \qquad \mathfrak{L} = \mathfrak{R} = 0 \qquad M_y^0 = 0 .$$

Case 75/10: Both legs loaded by any type of antisymmetrical load





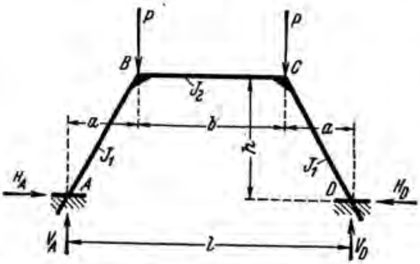
$$M_D = -M_A = \frac{\beta\,\mathfrak{S}_l K_2 + (\mathfrak{L} + \beta\,\mathfrak{R})\,k}{N_2} \qquad M_B = -M_C = \beta\,(\mathfrak{S}_l - M_D);$$

$$V_D = -V_A = \frac{2\,(\mathfrak{S}_l - M_D)}{l}; \qquad H_D = -H_A = W.$$

M_y and M_x same as case 75/8.

Note: All terms refer to the left leg.

Case 75/11: Two equal vertical concentrated loads at B and C

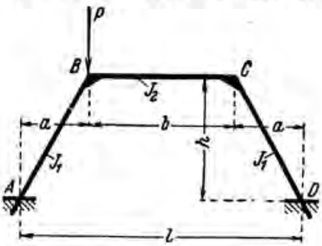


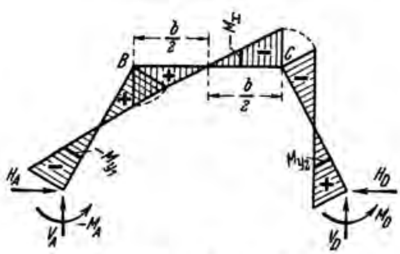
There are no bending moments

$$V_A = V_D = P;$$

$$H_A = H_D = \frac{Pa}{h}.$$

Case 75/12: Vertical concentrated load at B



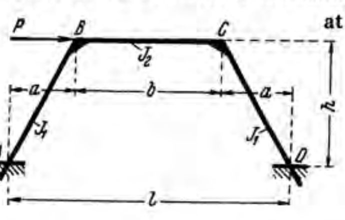


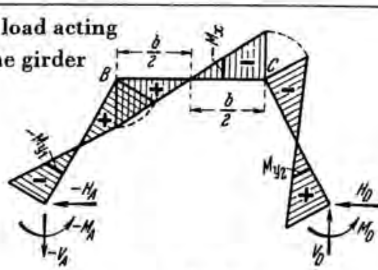
$$M_A = -M_D = -\frac{Pa\beta K_2}{2N_2} \qquad M_B = -M_C = \frac{Pa\beta k(2+\beta)}{2N_2};$$

$$H_A = H_D = \frac{Pa}{2h}; \qquad V_D = \frac{2M_B}{b} \qquad V_A = P - V_D.$$

Note: Moments are antisymmetrical.

Case 75/13: Horizontal concentrated load acting at the girder





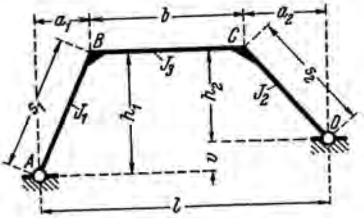
$$M_A = -M_D = -\frac{Ph\beta K_2}{2N_2} \qquad M_B = -M_C = \frac{Ph\beta k(2+\beta)}{2N_2};$$

$$H_D = -H_A = \frac{P}{2}; \qquad V_D = -V_A = \frac{2M_B}{b}.$$

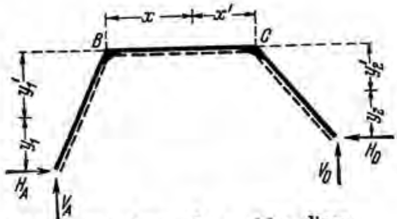
Note: The moment diagram is antisymmetrical and similar to case 75/12.

Frame 76

Two-hinged trapezoidal rigid frame with legs of different slopes and lengths.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1}\cdot\frac{s_1}{b} \qquad k_2 = \frac{J_3}{J_2}\cdot\frac{s_2}{b}; \qquad n = \frac{h_2}{h_1}; \qquad v = h_1 - h_2{}^{*});$$

$$\alpha_1 = \frac{a_1}{l} \qquad \beta_1 = 1 - \alpha_1 \qquad \alpha_2 = \frac{a_2}{l} \qquad \beta_2 = 1 - \alpha_2; \qquad r = \frac{v}{h_1}{}^{*});$$

$$m_1 = n\,\alpha_1 + \beta_1 \qquad B = 2\,m_1\,(k_1 + 1) + m_2 \qquad K_1 = \beta_1\,B + \alpha_2\,C$$

$$m_2 = \alpha_2 + n\,\beta_2; \qquad C = m_1 + 2\,m_2\,(1 + k_2) \qquad K_2 = \alpha_1\,B + \beta_2\,C;$$

$$N = m_1\,B + m_2\,C = K_1 + n\,K_2\,.$$

Equations for moments at any point of frame 76 for all loading conditions

Due to corner moments:

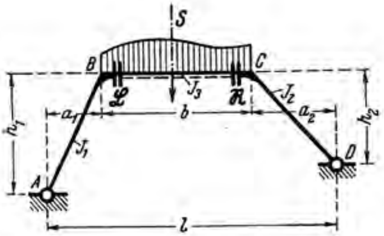
$$M_{y1} = \frac{y_1}{h_1}\,M_B \qquad M_x = \frac{x'}{b}\,M_B + \frac{x}{b}\,M_C \qquad M_{y2} = \frac{y_2}{h_2}\,M_C\,.$$

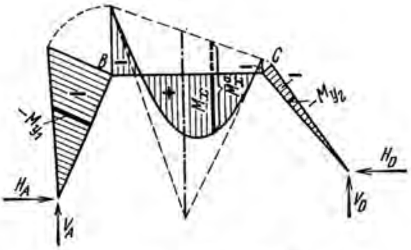
To these moments add the moments M_y^o and M_x^o resp. for directly loaded members only.

*When $h_2 > h_1$, v and r become negative.

Case 76/1: Girder loaded by any type of vertical load

See Appendix A, Load Terms, pp. 440-445.





Constant:

$$a=\frac{a_1 a_2}{l}; \qquad X=\frac{\mathfrak{L} m_1+\mathfrak{S}_r \alpha_1 B+S a(B+C)+\mathfrak{S}_l \alpha_2 C+\mathfrak{R} m_2}{N}.$$

$$M_B=\alpha_1 \mathfrak{S}_r+S a-m_1 X \qquad M_C=S a+\alpha_2 \mathfrak{S}_l-m_2 X;$$

$$V_A=\frac{\mathfrak{S}_r+S a_2+r X}{l} \qquad V_D=S-V_A; \qquad H_A=H_D=\frac{X}{h_1}.$$

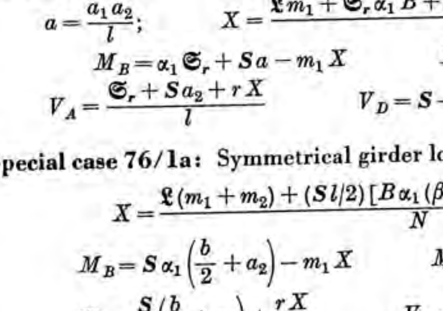
Special case 76/1a: Symmetrical girder load ($\mathfrak{R}=\mathfrak{L};\ \mathfrak{S}_l=\mathfrak{S}_r$)

$$X=\frac{\mathfrak{L}(m_1+m_2)+(S l/2)\left[B \alpha_1(\beta_1+\alpha_2)+C \alpha_2(\alpha_1+\beta_2)\right]}{N}.$$

$$M_B=S \alpha_1\left(\frac{b}{2}+a_2\right)-m_1 X \qquad M_C=S \alpha_2\left(a_1+\frac{b}{2}\right)-m_2 X;$$

$$V_A=\frac{S}{l}\left(\frac{b}{2}+a_2\right)+\frac{r X}{l} \qquad V_D=\frac{S}{l}\left(a_1+\frac{b}{2}\right)-\frac{r X}{l}=S-V_A.$$

Case 76/2: Uniform increase in temperature of the entire frame



E = Modulus of elasticity

ε = Coefficient of thermal expansion

t = Change of temperature in degrees

Constants:

$$X=\frac{6 E J_3 \varepsilon t(l^2+v^2)}{l b h_1 N}.$$

$$M_B=-m_1 X \qquad M_C=-m_2 X;$$

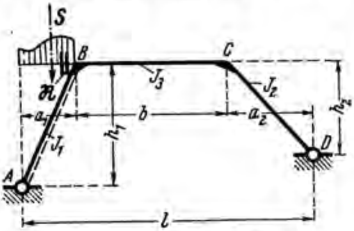
$$V_A=-V_D=\frac{r X}{l}; \qquad H_A=H_D=\frac{X}{h_1}.$$

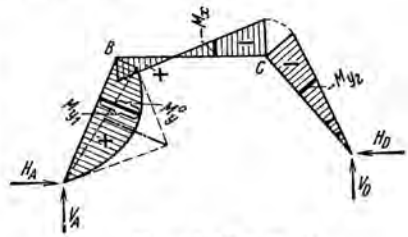
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

FRAME 76

See Appendix A, Load Terms, pp. 440-445.

Case 76/3: Left-hand leg loaded by any type of vertical load





Constant: $$X = \frac{\mathfrak{S}_l K_1 + \mathfrak{R} k_1 m_1}{N}.\qquad M_B = \beta_1 \mathfrak{S}_l - m_1 X \qquad M_C = \alpha_2 \mathfrak{S}_l - m_2 X;$$

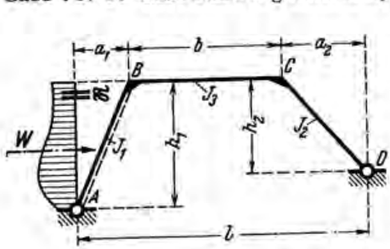
$$V_D = \frac{\mathfrak{S}_l - r X}{l} \qquad V_A = S - V_D; \qquad H_A = H_D = \frac{X}{h_1}.$$

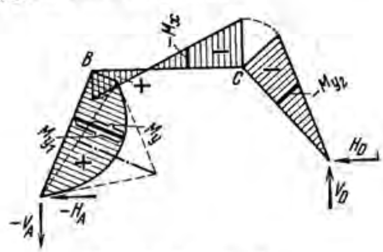
Special case 76/3a: Vertical concentrated load at ridge B

$$M_B = + \frac{P b \alpha_1 n C}{N} \qquad M_C = - \frac{P b \alpha_1 n B}{N};$$

$$V_D = \frac{M_B - M_C}{b} \qquad V_A = P - V_D; \qquad H_A = H_D = \frac{P a_1}{h_1} \frac{K_1}{N}$$

Case 76/4: Left-hand leg loaded by any type of horizontal load





Constant: $$X = \frac{\mathfrak{S}_l K_1 + \mathfrak{R} k_1 m_1}{N}.\qquad M_B = \beta_1 \mathfrak{S}_l - m_1 X \qquad M_C = \alpha_2 \mathfrak{S}_l - m_2 X;$$

$$V_D = -V_A = \frac{\mathfrak{S}_l - r X}{l}; \qquad H_D = \frac{X}{h_1} \qquad H_A = -(W - H_D).$$

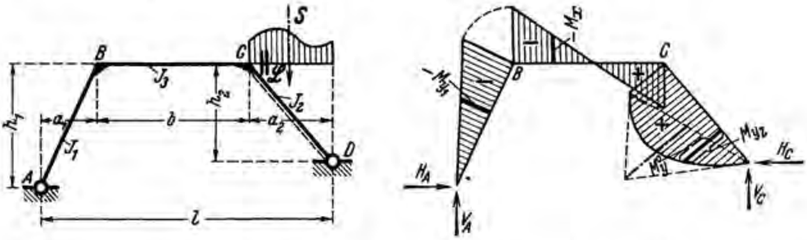
Special case 76/4a: Horizontal concentrated load at ridge B

$$M_B = + \frac{P h_2 b C}{l N} \qquad M_C = - \frac{P h_2 b B}{l N};$$

$$V_D = -V_A = \frac{M_B - M_C}{b}; \qquad H_A = - \frac{P n K_2}{N} \qquad H_D = \frac{P K_1}{N}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 76/5: Right-hand leg loaded by any type of vertical load



Constant: $X = \dfrac{\mathfrak{S}_r K_2 + \mathfrak{L} k_2 m_2}{N}$. $\quad M_B = \alpha_1 \mathfrak{S}_r - m_1 X$
$M_C = \beta_2 \mathfrak{S}_r - m_2 X;$

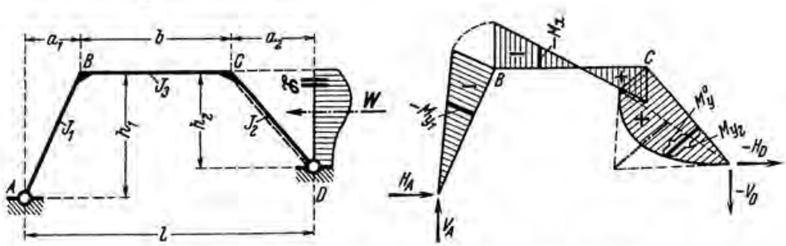
$$V_A = \frac{\mathfrak{S}_r + r X}{l} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{X}{h_1}.$$

Special case 76/5a: Vertical concentrated load at ridge *C*

$$M_B = -\frac{P b \alpha_2 C}{N} \qquad M_C = +\frac{P b \alpha_2 B}{N};$$

$$V_A = \frac{M_C - M_B}{b} \qquad V_D = P - V_A; \qquad H_A = H_D = \frac{P a_2 K_2}{h_1 N}.$$

Case 76/6: Right-hand leg loaded by any type of horizontal load



Constant: $X = \dfrac{\mathfrak{S}_r K_2 + \mathfrak{L} k_2 m_2}{N}$. $\quad M_B = \alpha_1 \mathfrak{S}_r - m_1 X$
$M_C = \beta_2 \mathfrak{S}_r - m_2 X;$

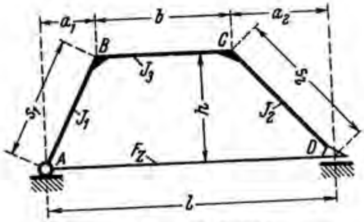
$$V_A = -V_D = \frac{\mathfrak{S}_r + r X}{l}; \qquad H_A = \frac{X}{h_1} \qquad H_D = -(W - H_A).$$

Special case 76/6a: Horizontal concentrated load at ridge *C*

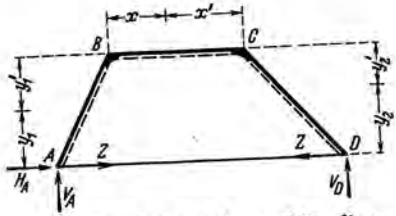
Formulas same as for special case 76/4a with all signs inversed.

Frame 77

Unsymmetrical trapezoidal rigid frame with horizontal tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k_1 = \frac{J_3}{J_1} \cdot \frac{s_1}{b}$ $\qquad k_2 = \frac{J_3}{J_2} \cdot \frac{s_2}{b}$;

$$\alpha_1 = \frac{a_1}{l} \qquad \beta_1 = 1 - \alpha_1 \qquad \alpha_2 = \frac{a_2}{l} \qquad \beta_2 = 1 - \alpha_2;$$

$$B = 2k_1 + 3 \qquad K_1 = \beta_1 B + \alpha_2 C \qquad L = \frac{6J_3}{h^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{l}{b};$$

$$C = 3 + 2k_2 \qquad K_2 = \alpha_1 B + \beta_2 C;$$

$$N = B + C = K_1 + K_2 \qquad N_Z = N + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Equations for moments at any point of frame 77 for all loading conditions

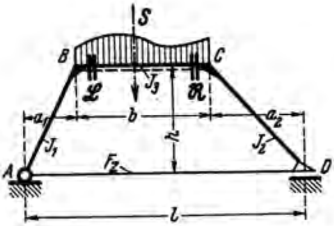
Due to corner moments:

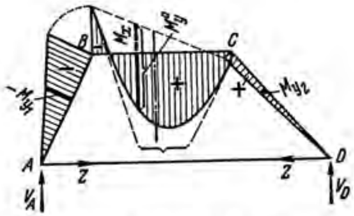
$$M_{y1} = \frac{y_1}{h} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h} M_C.$$

To these moments add the moments M_y° and M_x° resp. for directly loaded members only.

Case 77/1: Girder loaded by any type of vertical load

(See Appendix A, Load Terms, pp. 440-445.)





$$a=\frac{a_1 a_2}{l};\qquad Z=\frac{\mathfrak{S}_r\alpha_1 B+\mathfrak{S}_l\alpha_2 C+S a N+(\mathfrak{L}+\mathfrak{R})}{h N_Z};$$

$$V_A=\frac{\mathfrak{S}_r}{l}+S\alpha_2\qquad V_D=S\alpha_1+\frac{\mathfrak{S}_l}{l}\qquad (V_A+V_D=S);$$

$$M_B=\alpha_1\mathfrak{S}_r+Sa-Zh\qquad M_C=Sa+\alpha_2\mathfrak{S}_l-Zh.$$

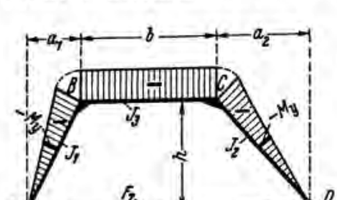
Special case 77/1a: Symmetrical girder load ($\mathfrak{R}=\mathfrak{L}$; $\mathfrak{S}_l=\mathfrak{S}_r$)

$$Z=\frac{2\mathfrak{L}+(Sl/2)\,[B\alpha_1(\beta_1+\alpha_2)+C\alpha_2(\alpha_1+\beta_2)}{h N_Z};$$

$$V_A=\frac{S}{l}\left(\frac{b}{2}+a_2\right)\qquad V_D=\frac{S}{l}\left(a_1+\frac{b}{2}\right)\qquad (V_A+V_D=S);$$

$$M_B=S\alpha_1\left(\frac{b}{2}+a_2\right)-Zh\qquad M_C=S\alpha_2\left(a_1+\frac{b}{2}\right)-Zh.$$

Case 77/2: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z=\frac{6EJ_3\varepsilon t l}{b h^2 N_Z};$$

$$M_B=M_C=-Zh\qquad M_y=-Zy_1.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

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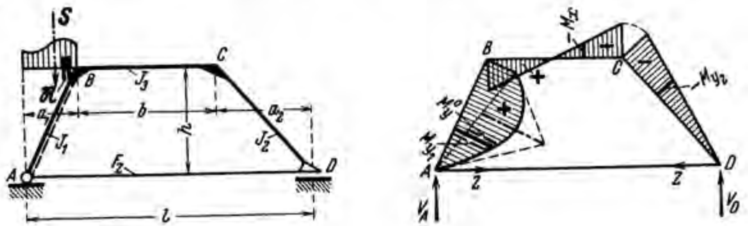
*See footnote on page 285.

FRAME 77

See Appendix A, Load Terms, pp. 440-445.

Case 77/3: Left-hand leg loaded by any type of vertical load

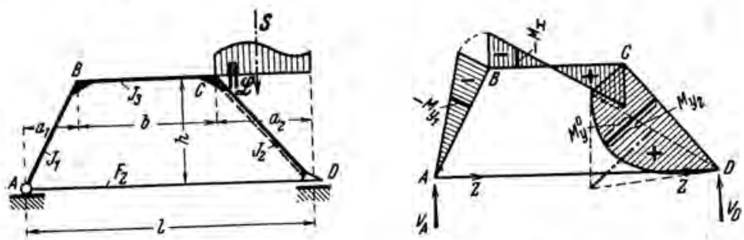


$$Z = \frac{\mathfrak{S}_l K_1 + \mathfrak{R} k_1}{h N_Z}; \qquad V_D = \frac{\mathfrak{S}_l}{l} \qquad V_A = S - V_D;$$
$$M_B = \beta_1 \mathfrak{S}_l - Zh \qquad M_C = \alpha_2 \mathfrak{S}_l - Zh.$$

Special case 77/3a: Vertical concentrated load at B

$$Z = \frac{P a_1}{h} \cdot \frac{K_1}{N_Z}; \qquad V_D = \alpha_1 \cdot P \qquad V_A = \beta_1 \cdot P = P - V_D;$$
$$M_B = P a_1 \cdot \beta_1 - Zh \qquad M_C = P a_1 \cdot \alpha_2 - Zh.$$

Case 77/4: Right-hand leg loaded by any type of vertical load



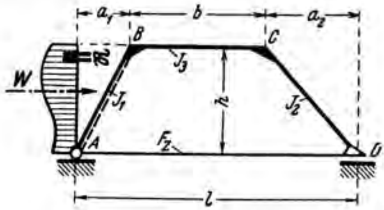
$$Z = \frac{\mathfrak{S}_r K_2 + \mathfrak{L} k_2}{h N_Z}; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = S - V_A;$$
$$M_B = \alpha_1 \mathfrak{S}_r - Zh \qquad M_C = \beta_2 \mathfrak{S}_r - Zh.$$

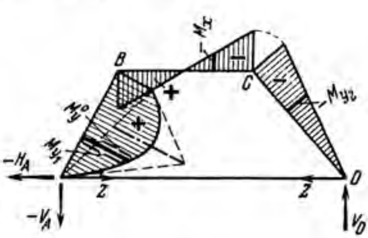
Special case 77/4a: Vertical concentrated load at C

$$Z = \frac{P a_2}{h} \cdot \frac{K_2}{N_Z}; \qquad V_A = \alpha_2 \cdot P \qquad V_D = \beta_2 \cdot P = P - V_A;$$
$$M_B = P a_2 \cdot \alpha_1 - Zh \qquad M_C\, P a_2 \cdot \beta_2 - Zh.$$

See Appendix A, Load Terms, pp. 440-445.

Case 77/5: Left-hand leg loaded by any type of horizontal load





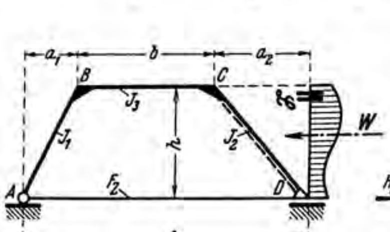
$$Z = \frac{\mathfrak{S}_l K_1 + \mathfrak{R} k_1}{h N_Z}; \qquad V_D = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_A = -W;$$

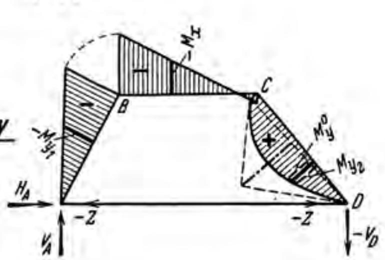
$$M_B = \beta_1 \mathfrak{S}_l - Z h \qquad M_C = \alpha_2 \mathfrak{S}_l - Z h .$$

Special case 77/5a: Horizontal concentrated load at *B*

$$Z = P \cdot \frac{K_1}{N_Z}; \qquad \begin{aligned} M_B &= (\beta_1 P - Z) h \\ M_C &= (\alpha_2 P - Z) h \end{aligned} \qquad V_D = -V_A = \frac{P h}{l}.$$

Case 77/6: Right-hand leg loaded by any type of horizontal load





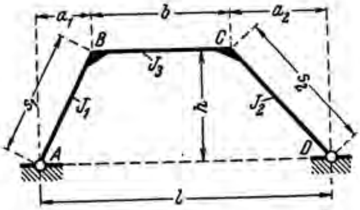
$$Z = -\left(W \frac{N}{N_Z} - \frac{\mathfrak{S}_r K_2 + \mathfrak{L} k_2}{h N_Z}\right)^{*)}; \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l}; \qquad H_A = W;$$

$$M_B = -(W + Z) h + \alpha_1 \mathfrak{S}_r \qquad M_C = -(W + Z) h + \beta_2 \mathfrak{S}_r .$$

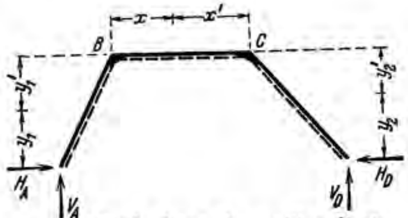
* For the above loading conditions and for a decrease in temperature (p. 283 bottom) Z becomes negative, i the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile fo due to dead load, so that a residual force remains in the tie rod.

Frame 78

Unsymmetrical two-hinged trapezoidal rigid frame. Hinges at same elevation.



Shape of Frame
Dimensions and Notations



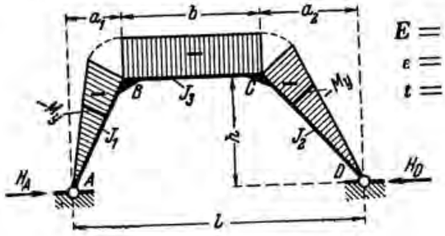
This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* of frame 78 are the same as those for frame 76, with the simplifications $(h_1 = h_2) = h$, $v = 0$, $n = 1$, $r = 0$, $(m_1 = m_2) = 1$, and

$$B = 2k_1 + 3 \qquad K_1 = \beta_1 B + \alpha_2 C \qquad N = B + C = K_1 + K_2 .$$
$$C = 3 + 2k_2 \qquad K_2 = \alpha_1 B + \beta_2 C$$

Note: The equations for frame 77 may also be used for frame 78 when $L = 0$ and $N_Z = N$ are substituted. It must be remembered, however, to include the effect of the tie-rod force Z in the reactions of H_A and H_D.

Case 78/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

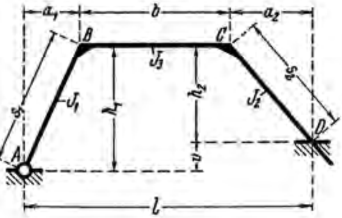
$$M_B = M_C = -\frac{6 E J_3 \varepsilon t l}{b h N}$$

$$H_A = H_D = \frac{-M_B}{h}; \qquad M_y = \frac{y_1}{h} M_B .$$

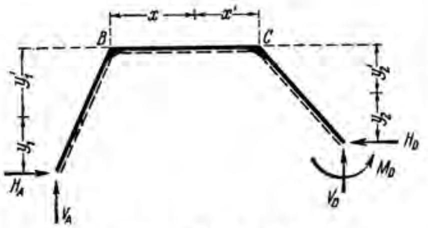
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 79

Trapezoidal rigid frame with legs of different slopes and lengths. One support fixed, one support hinged; supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{s_1}{b} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{s_2}{b}; \qquad n = \frac{h_2}{h_1}; \qquad \alpha_1 = \frac{a_1}{b} \qquad \alpha_2 = \frac{a_2}{b};$$

$$\beta = \delta n + \alpha_2 \qquad \gamma = n\alpha_1 + 1 + \alpha_2 \qquad \delta = \alpha_1 + 1;$$

$$D = (1 + 2\gamma) k_2 \qquad R_1 = 2(k_1 + 1 + \beta^2 k_2)$$

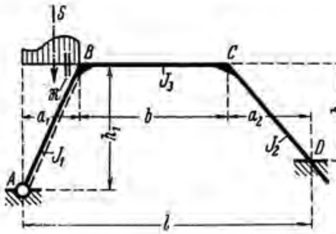
$$K = \beta D - 1 \qquad R_2 = 2(1 + k_2) + \gamma(k_2 + D);$$

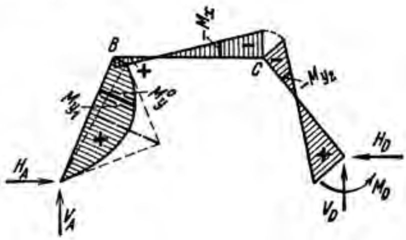
$$N = R_1 R_2 - K^2; \qquad n_{11} = \frac{R_2}{N} \qquad n_{12} = n_{21} = \frac{K}{N} \qquad n_{22} = \frac{R_1}{N}.$$

FRAME 79

See Appendix A, Load Terms, pp. 440-445.

Case 79/1: Left-hand leg loaded by any type of vertical load





Constants:

$$\mathfrak{B}_1 = 2n\mathfrak{S}_l\beta k_2 - \mathfrak{R}k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$

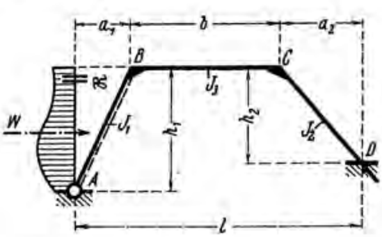
$$\mathfrak{B}_2 = n\mathfrak{S}_l D; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

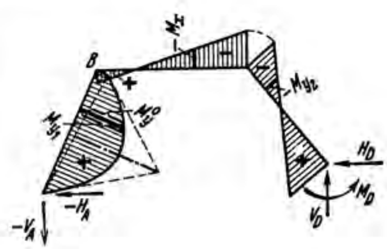
$$M_B = X_1 \qquad M_C = -X_2 \qquad M_D = n\mathfrak{S}_l - \beta X_1 - \gamma X_2;$$

$$V_D = \frac{X_1 + X_2}{b} \qquad V_A = S - V_D; \qquad H_A = H_D = \frac{\mathfrak{S}_l - \delta X_1 - \alpha_1 X_2}{h_1};$$

$$M_{y1} = M_y^0 + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Case 79/2: Left-hand leg loaded by any type of horizontal load



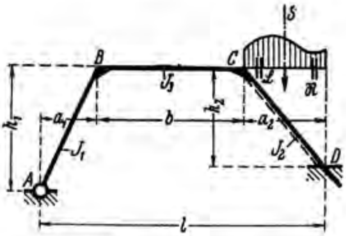


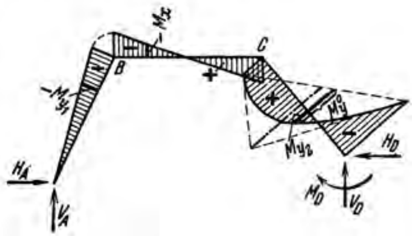
All the formulas are the same as above, except those for H and V-forces:

$$V_D = -V_A = \frac{X_1 + X_2}{b}; \qquad H_D = \frac{\mathfrak{S}_l - \delta X_1 - \alpha_1 X_2}{h_1} \qquad H_A = -(W - H_D).$$

(See Appendix A, Load Terms, pp. 440-445.)

Case 79/3: Right-hand leg loaded by any type of vertical load





Constants:

$$\mathfrak{B}_1=(2\,\mathfrak{S}_r-\mathfrak{R})\,\beta\,k_2 \qquad X_1=+\mathfrak{B}_1 n_{11}-\mathfrak{B}_2 n_{21}$$

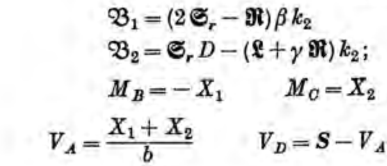
$$\mathfrak{B}_2=\mathfrak{S}_r D-(\mathfrak{L}+\gamma\,\mathfrak{R})\,k_2; \qquad X_2=-\mathfrak{B}_1 n_{12}+\mathfrak{B}_2 n_{22}.$$

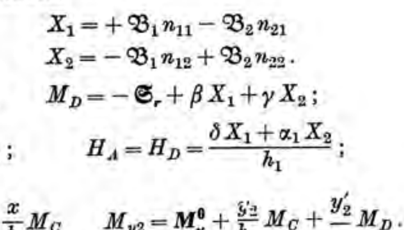
$$M_B=-X_1 \qquad M_C=X_2 \qquad M_D=-\mathfrak{S}_r+\beta X_1+\gamma X_2;$$

$$V_A=\frac{X_1+X_2}{b} \qquad V_D=S-V_A; \qquad H_A=H_D=\frac{\delta X_1+\alpha_1 X_2}{h_1};$$

$$M_{y1}=\frac{y_1}{h_1}M_B \qquad M_x=\frac{x'}{b}M_B+\frac{x}{b}M_C \qquad M_{y2}=M_y^0+\frac{y_2}{h_2}M_C+\frac{y_2'}{h_2}M_D.$$

Case 79/4: Right-hand leg loaded by any type of horizontal load





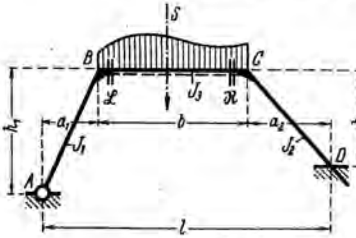
All the formulas are the same as above, except those for V- and H-forces

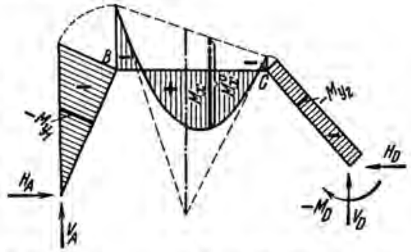
$$V_A=-V_D=\frac{X_1+X_2}{b}; \qquad H_A=\frac{\delta X_1+\alpha_1 X_2}{h_1} \qquad H_D=-(W-H_A).$$

FRAME 79

Case 79/5: Girder loaded by any type of vertical load

See Appendix A, Load Terms, pp. 440-445.





Constants: $\mathfrak{B}_1 = \mathfrak{L} + 2(\alpha_2 \mathfrak{S}_l - n\,\alpha_1 \mathfrak{S}_r)\beta k_2$ $\qquad X_1 = \mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21}$

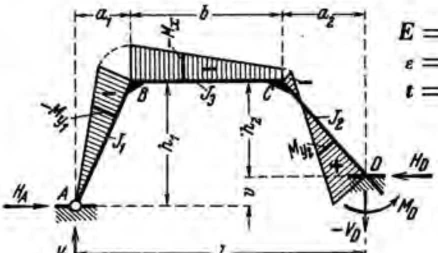
$\mathfrak{B}_2 = \mathfrak{R} - D(\alpha_2 \mathfrak{S}_l - n\,\alpha_1 \mathfrak{S}_r);$ $\qquad X_2 = \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$

$$M_B = -X_1 \qquad M_C = -X_2 \qquad M_D = -(\alpha_2 \mathfrak{S}_l - n\,\alpha_1 \mathfrak{S}_r) + \beta X_1 - \gamma X_2;$$

$$V_A = \frac{\mathfrak{S}_r + X_1 - X_2}{b} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{\alpha_1(\mathfrak{S}_r - X_2) + \delta X_1}{h_1};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Case 79/6: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$v = h_1 - h_2{}^* ; \qquad T = \frac{6 E J_3 \varepsilon t}{b};$$

$$\mathfrak{B}_1 = \frac{v}{b} + \frac{l\delta}{h_1} \qquad \mathfrak{B}_2 = \frac{v}{b} + \frac{l\alpha_1}{h_1}; \qquad X_1 = T(\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21})$$
$$X_2 = T(\mathfrak{B}_1 n_{12} - \mathfrak{B}_2 n_{22}).$$

$$M_B = -X_1 \qquad M_C = -X_2 \qquad M_D = \beta X_1 - \gamma X_2;$$

$$V_A = -V_D = \frac{X_1 - X_2}{b}; \qquad H_A = H_D = \frac{\delta X_1 - \alpha_1 X_2}{h_1}.$$

The formulas for M_{y1}, M_{y2} and M_x the same as above but with $M_x{}^0 = 0$.

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

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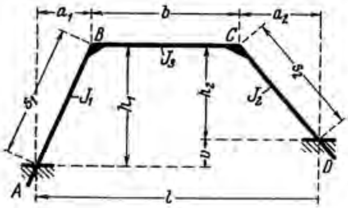
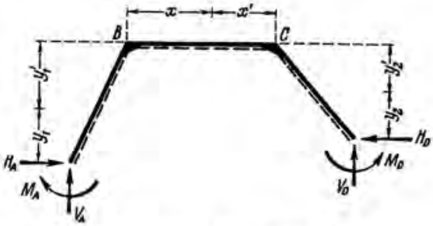
*When $h_2 > h_1$, v becomes negative.

Frame 80

Hingeless trapezoidal rigid frame with legs of different slopes and lengths. Supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{s_1}{b} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{s_2}{b}; \qquad n = \frac{h_2}{h_1}; \qquad \alpha_1 = \frac{a_1}{b} \qquad \alpha_2 = \frac{a_2}{b};$$

$$A = (2\alpha_1 + 3)\,k_1 \qquad D = (3 + 2\alpha_2)\,k_2; \qquad \beta_1 = \alpha_1 + 1 \qquad \beta_2 = 1 + \alpha_2;$$

$$R_1 = 2\,(A + \alpha_1 \beta_1 k_1 + 1 + \alpha_2^2 k_2) \qquad K_1 = n\,D - 2\,\alpha_1 k_1$$

$$R_2 = 2\,(\alpha_1^2 k_1 + 1 + \alpha_2 \beta_2 k_2 + D) \qquad K_2 = A - 2\,\alpha_2 n\,k_2$$

$$R_3 = 2\,(k_1 + n^2 k_2); \qquad K_3 = \alpha_1 A + \alpha_2 D - 1;$$

$$N = R_1 R_2 R_3 - 2\,K_1 K_2 K_3 - R_1 K_1^2 - R_2 K_2^2 - R_3 K_3^2;$$

$$n_{11} = \frac{R_2 R_3 - K_1^2}{N} \qquad n_{12} = n_{21} = \frac{K_1 K_2 + R_3 K_3}{N}$$

$$n_{22} = \frac{R_1 R_3 - K_2^2}{N} \qquad n_{13} = n_{31} = \frac{K_1 K_3 + R_2 K_2}{N}$$

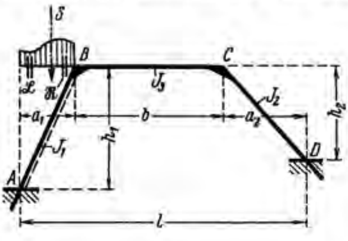
$$n_{33} = \frac{R_1 R_2 - K_3^2}{N} \qquad n_{23} = n_{32} = \frac{K_2 K_3 + R_1 K_1}{N}.$$

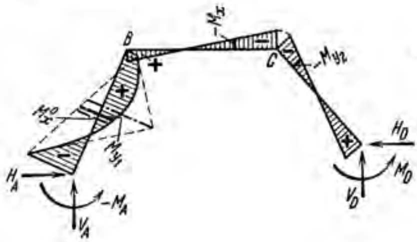
Note: For moments at arbitrary points due to all loading conditions for frame 80 see p. 2[illegible] bottom.

FRAME 80

See Appendix A, Load Terms, pp. 440-445.

Case 80/1: Left-hand leg loaded by any type of vertical load





*

Constants:

$$\mathfrak{B}_1 = \mathfrak{S}_l A - (\beta_1 \mathfrak{L} + \mathfrak{R})\, k_1 \qquad X_1 = + \mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21} - \mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_2 = (2\,\mathfrak{S}_l - \mathfrak{L})\, \alpha_1 k_1 \qquad X_2 = - \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$

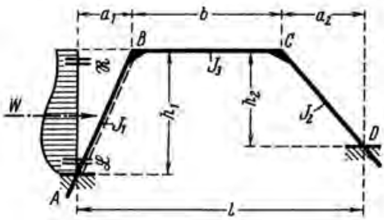
$$\mathfrak{B}_3 = (2\,\mathfrak{S}_l - \mathfrak{L})\, k_1\,; \qquad X_3 = - \mathfrak{B}_1 n_{13} + \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}\,.$$

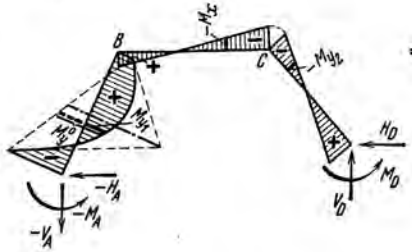
$$M_A = -\mathfrak{S}_l + \beta_1 X_1 + \alpha_1 X_2 + X_3 \qquad M_B = X_1$$

$$M_C = -X_2 \qquad M_D = -\alpha_2 X_1 - \beta_2 X_2 + n X_3\,;$$

$$V_D = \frac{X_1 + X_2}{b} \qquad V_A = \boldsymbol{S} - V_D\,; \qquad H_A = H_D = \frac{X_3}{h_1}\,.$$

Case 80/2: Left-hand leg loaded by any type of horizontal load





*

All the formulas are the same as above, except those for V- and H-forces:

$$V_D = -V_A = \frac{X_1 + X_2}{b}\,; \qquad H_D = \frac{X_3}{h_1} \qquad H_A = -(W - H_D)\,.$$

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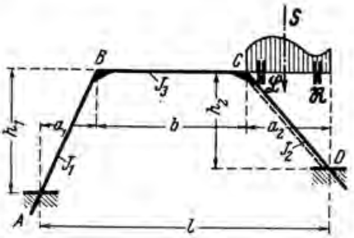
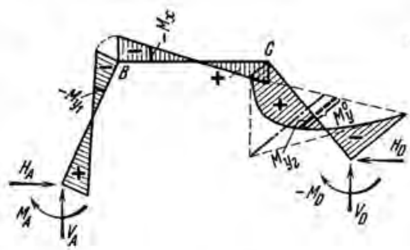
* See p. 295 bottom for M_y and M_x.

See Appendix A, Load Terms, pp. 440-445.

Case 80/3: Right-hand leg loaded by any type of vertical load





*

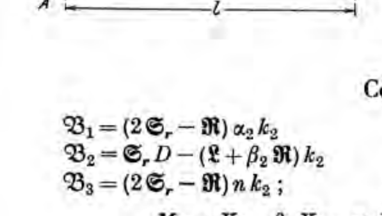
Constants:

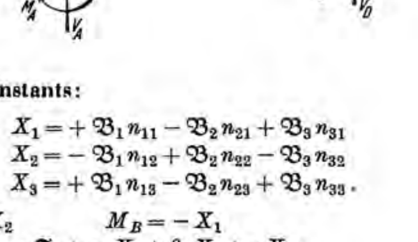
$$\mathfrak{B}_1 = (2\,\mathfrak{S}_r - \mathfrak{R})\,\alpha_2 k_2 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$
$$\mathfrak{B}_2 = \mathfrak{S}_r D - (\mathfrak{L} + \beta_2\,\mathfrak{R})\,k_2 \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} - \mathfrak{B}_3 n_{32}$$
$$\mathfrak{B}_3 = (2\,\mathfrak{S}_r - \mathfrak{R})\,n\,k_2\,; \qquad X_3 = +\mathfrak{B}_1 n_{13} - \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}\,.$$

$$M_A = X_3 - \beta_1 X_1 - \alpha_1 X_2 \qquad M_B = -X_1$$
$$M_C = X_2 \qquad M_D = -\mathfrak{S}_r + \alpha_2 X_1 + \beta_2 X_2 + n X_3\,;$$

$$V_A = \frac{X_1 + X_2}{b} \qquad V_D = S - V_A\,; \qquad H_A = H_D = \frac{X_3}{h_1}\,.$$

Case 80/4: Right-hand leg loaded by any type of horizontal load





*

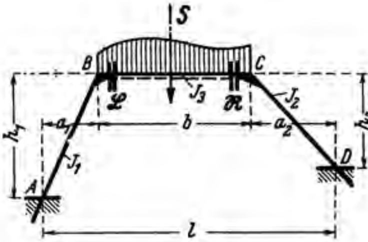
All the formulas are the same as above, except those for V- and H-forces:

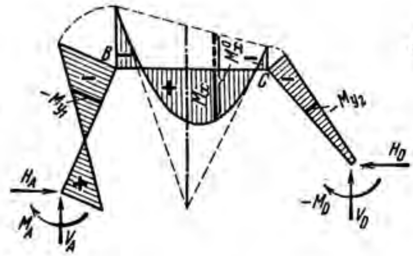
$$V_A = -V_D = \frac{X_1 + X_2}{b}\,; \qquad H_A = \frac{X_3}{h_1} \qquad H_D = -(W - H_A)\,.$$

* See p. 295 bottom for M_y and M_x.

Case 80/5: Girder loaded by any type of load

See Appendix A, Load Terms, pp. 440-445.





*

Constants:

$$\mathfrak{B}_1 = \mathfrak{S}_r \alpha_1 A - 2\mathfrak{S}_l \alpha_2^2 k_2 - \mathfrak{L} \qquad X_1 = -\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_2 = \mathfrak{S}_l \alpha_2 D - 2\mathfrak{S}_r \alpha_1^2 k_1 - \mathfrak{R} \qquad X_2 = -\mathfrak{B}_1 n_{12} - \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$

$$\mathfrak{B}_3 = 2(\mathfrak{S}_r \alpha_1 k_1 + n\,\mathfrak{S}_l \alpha_2 k_2); \qquad X_3 = -\mathfrak{B}_1 n_{13} - \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

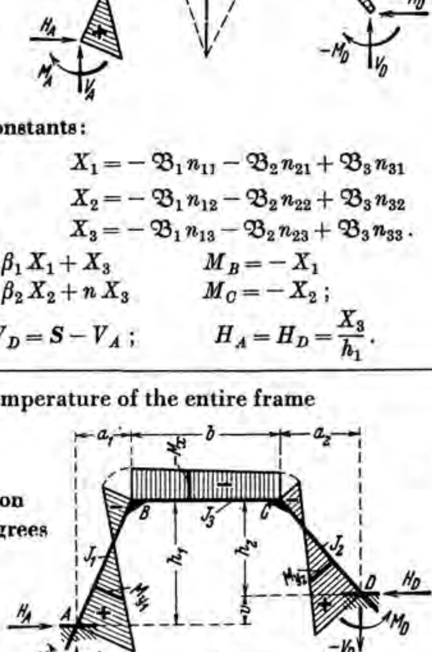
$$M_A = -\alpha_1(\mathfrak{S}_r - X_2) - \beta_1 X_1 + X_3 \qquad M_B = -X_1$$

$$M_D = -\alpha_2(\mathfrak{S}_l - X_1) - \beta_2 X_2 + n X_3 \qquad M_C = -X_2;$$

$$V_A = \frac{\mathfrak{S}_r + X_1 - X_2}{b} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{X_3}{h_1}.$$

Case 80/6: Uniform increase in temperature of the entire frame

E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees



*

Constants:

$$v = h_1 - h_2^{**}; \qquad T = \frac{6EJ_3\varepsilon t}{b};$$

$$X_1 = T\left[\frac{v}{b}(n_{11} - n_{21}) + \frac{l}{h_1} n_{31}\right] \qquad V_A = -V_D = \frac{X_1 - X_2}{b}; \qquad M_B = -X_1$$

$$X_2 = T\left[\frac{v}{b}(n_{12} - n_{22}) + \frac{l}{h_1} n_{32}\right] \qquad H_A = H_D = \frac{X_3}{h_1}; \qquad M_C = -X_2$$

$$X_3 = T\left[\frac{v}{b}(n_{13} - n_{23}) + \frac{l}{h_1} n_{33}\right]. \qquad M_A = -\beta_1 X_1 + \alpha_1 X_2 + X_3$$

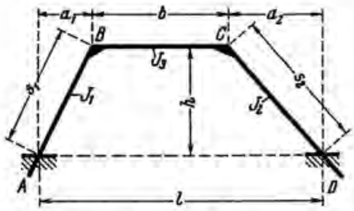
$$M_D = \alpha_2 X_1 - \beta_2 X_2 + n X_3.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

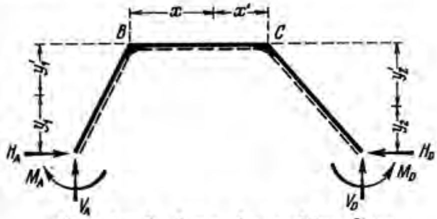
* See p. 295 bottom.
** When $h_2 > h_1$, v becomes negative.

Frame 81

Hingeless trapezoidal rigid frame with legs of different slopes and lengths. Supports at same elevation.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* are the same as for frame 80 with the exception that $n = 1$ (for $h_1 = h_2 = h$). See pp. 291–294.

For a *uniform change of temperature* there will be $v = 0$, and the coefficients on p. 294 bottom are reduced to:

$$T = \frac{6 E J_3 \varepsilon t}{b} \cdot \frac{l}{h};$$

$$X_1 = T n_{31}, \qquad X_2 = T n_{32}, \qquad X_3 = T n_{33}.$$

Equations for moments at any point of frame 80 (pp. 292-294) for all loading conditions

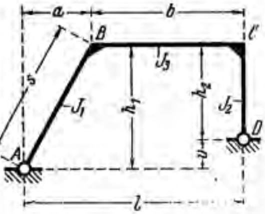
The moments at the joints and the fixed end moments contribute to the total moment:

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

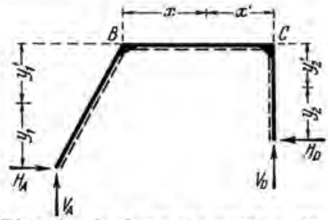
To these moments add the moments M_y^0 and M_x^0 resp. for directly loaded members only.

Frame 82

Two-hinged trapezoidal rigid frame with one vertical leg. Hinges at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

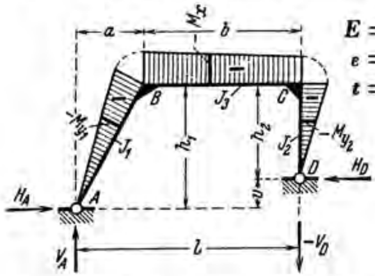
$$k_1 = \frac{J_3}{J_1} \cdot \frac{s}{b} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{b}; \qquad n = \frac{h_2}{h_1}; \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l};$$

$$m = \alpha n + \beta; \qquad B = 2m(k_1 + 1) + n \qquad C = m + 2n(1 + k_2);$$

$$K = \alpha B + C; \qquad N = mB + nC = \beta B + nK;$$

$$v = h_1 - h_2{}^* \qquad r = \frac{v}{h_1}{}^* .$$

Case 82/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $$X = \frac{6 E J_3 \varepsilon t (l^2 + v^2)}{l b h_1 N}.$$

$$M_B = -mX \qquad M_C = -nX;$$

$$V_A = -V_D = \frac{rX}{l}; \quad H_A = H_D = \frac{X}{h_1}.$$

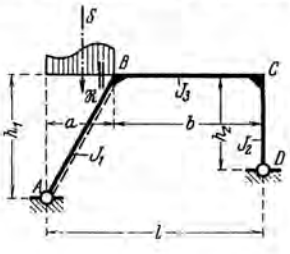
$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C .$$

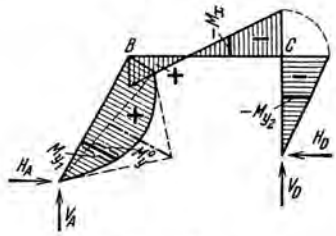
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

*When $h_2 > h_1$, v and r become negative.

See Appendix A, Load Terms, pp. 440-445.

Case 82/2: Left-hand leg loaded by any type of vertical load





Constant:
$$X = \frac{\beta\, B\, \mathfrak{S}_l + \mathfrak{R}\, k_1\, m}{N}. \qquad M_B = \beta\, \mathfrak{S}_l - m\, X \qquad M_C = -\, n\, X\,;$$

$$V_D = \frac{\mathfrak{S}_l - r\, X}{l} \qquad V_A = S - V_D\,; \qquad H_A = H_D = \frac{X}{h_1}.$$

M_y and M_x same as case 82/1, with M_y^o for M_{y1}.

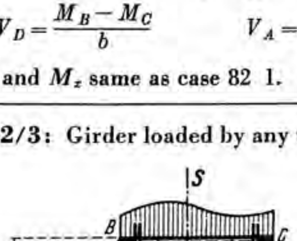
Special case 82/2a: Vertical concentrated load P at B

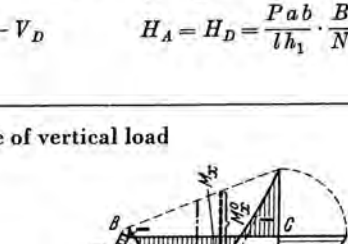
$$M_B = +\frac{P\,a\,b}{l} \cdot \frac{n\,C}{N} \qquad M_C = -\frac{P\,a\,b}{l} \cdot \frac{n\,B}{N};$$

$$V_D = \frac{M_B - M_C}{b} \qquad V_A = P - V_D \qquad H_A = H_D = \frac{P\,a\,b}{l\,h_1} \cdot \frac{B}{N}.$$

M_y and M_x same as case 82 1.

Case 82/3: Girder loaded by any type of vertical load





Constant:
$$X = \frac{\alpha\, B\, \mathfrak{S}_r + \mathfrak{L}\, m + \mathfrak{R}\, n}{N} \qquad M_B = \alpha\, \mathfrak{S}_r - m\, X \qquad M_C = -\, n\, X\,;$$

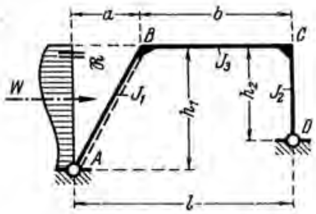
$$V_A = \frac{\mathfrak{S}_r + r\, X}{l} \qquad V_D = S - V_A\,; \qquad H_A = H_D = \frac{X}{h_1}.$$

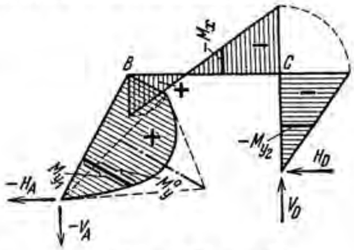
M_y and M_x same as case 82/1, with M_x^o for M_x.

FRAME 82

(See Appendix A, Load Terms, pp. 440-445.)

Case 82/4: Left-hand leg loaded by any type of horizontal load





Constant:
$$X = \frac{\beta B \mathfrak{S}_l + \mathfrak{R} k_1 m}{N}. \qquad M_B = \beta \mathfrak{S}_l - m X \qquad M_C = -n X;$$

$$V_D = -V_A = \frac{\mathfrak{S}_l - r X}{l}; \qquad H_D = \frac{X}{h_1} \qquad H_A = -(W - H_D).$$

M_y and M_x same as case 82/1, with M_y^0 for M_{y1}.

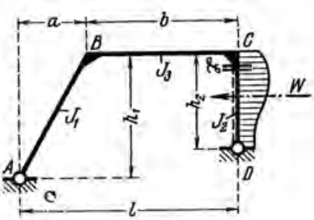
Special case 82/4a: Horizontal concentrated load P at B

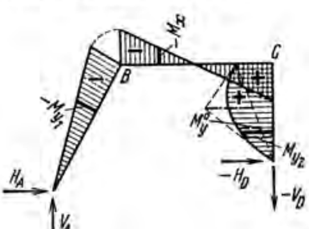
$$M_B = +\frac{P h_2 \beta C}{N} \qquad M_C = -\frac{P h_2 \beta B}{N};$$

$$V_D = -V_A = \frac{M_B - M_C}{b}; \qquad H_A = -\frac{P n K}{N} \qquad H_D = \frac{P \beta B}{N}.$$

M_y and M_x same as case 82/1.

Case 82/5: Right-hand leg loaded by any type of horizontal load





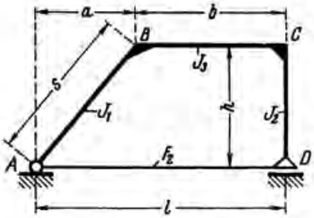
Constant:
$$X = \frac{\mathfrak{S}_r K + \mathfrak{L} k_2 n}{N}. \qquad M_B = \alpha \mathfrak{S}_r - m X \qquad M_C = \mathfrak{S}_r - n X;$$

$$V_A = -V_D = \frac{\mathfrak{S}_r + r X}{l}; \qquad H_A = \frac{X}{h_1} \qquad H_D = -(W - H_A).$$

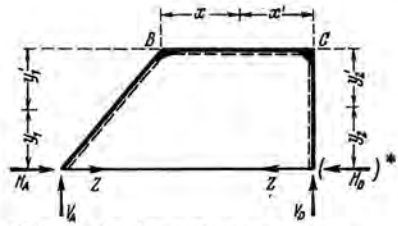
M_y and M_x same as case 82/1, with M_y^0 for M_{y2}.

Frame 83

Trapezoidal rigid frame with horizontal tie-rod and one vertical leg. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

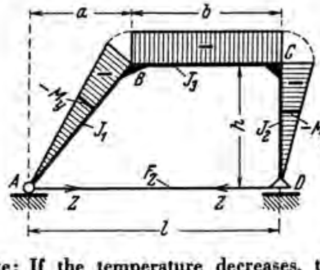
$$k_1 = \frac{J_3}{J_1} \cdot \frac{s}{b} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h}{b} \qquad \alpha = \frac{a}{l} \qquad \beta = \frac{b}{l}$$

$$B = 2k_1 + 3 \qquad C = 3 + 2k_2 \qquad K = \alpha B + C$$

$$N = B + C \qquad L = \frac{6J_3}{h^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{l}{b} \qquad N_Z = N + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Case 83/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

$$Z = \frac{6EJ_3 \varepsilon t l}{b h^2 N_Z};$$

$$M_B = M_C = -Zh \qquad M_y = -Z y_1$$

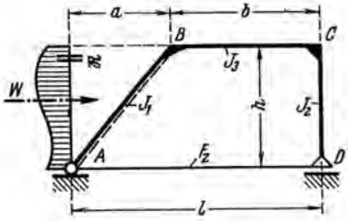
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed. See footnote on page 301

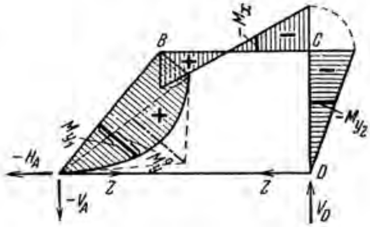
*H_D occurs when the hinged support is at D.

FRAME 83

See Appendix A, Load Terms, pp. 440-445.

Case 83/2: Left-hand leg loaded by any type of horizontal load (Hinged support at A)





$$Z = \frac{\beta B \mathfrak{S}_l + \mathfrak{R} k_1}{h N_Z}; \qquad V_D = -V_A = \frac{\mathfrak{S}}{l}; \qquad H_A = -W;$$

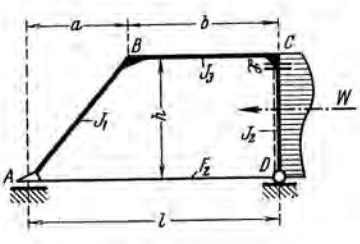
$$M_B = \beta \mathfrak{S}_l - Z h \qquad M_C = -Z h; \qquad M_{y1} = \mathbf{M}_y^0 + \frac{y_1}{h} M_B.$$

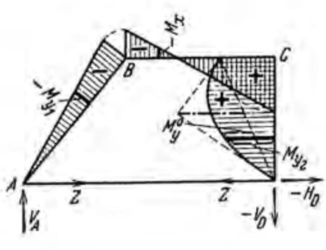
Special case 83/2a: Horizontal concentrated load P at B

$$Z = P \cdot \frac{\beta B}{N_Z}; \qquad V_D = -V_A = \frac{P h}{l}; \qquad H_A = -P;$$

$$M_B = (\beta P - Z) h \qquad M_C = -Z h. \qquad (\mathbf{M}_y^0 = 0).$$

Case 83/3: Right-hand leg loaded by any type of horizontal load (Hinged support at D)



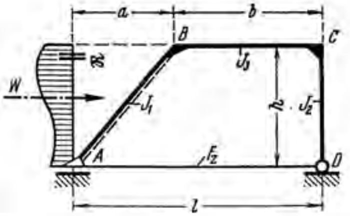


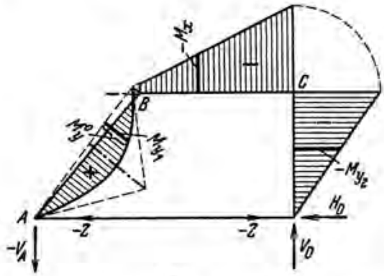
$$Z = \frac{\mathfrak{S}_r K + \mathfrak{L} k_2}{h N_Z} \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l} \qquad H_D = -W$$

$$M_B = \alpha \mathfrak{S}_r - Z h \qquad M_C = \mathfrak{S}_r - Z h \qquad M_{y2} = \mathbf{M}_y^0 + \frac{y_2}{h} M_C.$$

See Appendix A, Load Terms, pp. 440-445.

Case 83/4: Left-hand leg loaded by any type of horizontal load (Hinged support at D)



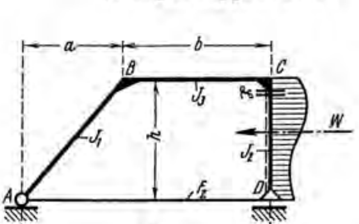


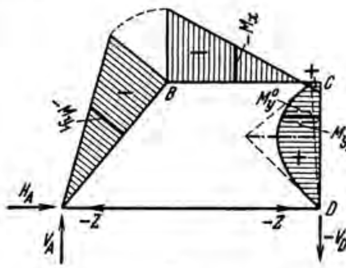
$$Z = -\left(W\frac{N}{N_Z} - \frac{\beta B \mathfrak{S}_l + \mathfrak{R} k_1}{h N_Z}\right)^* ; \qquad V_D = -V_A = \frac{\mathfrak{S}}{l}; \qquad H_D = W;$$

$$M_B = -(W+Z)h + \beta \mathfrak{S}_l \qquad M_C = -(W+Z)h;$$

$$M_{y1} = M_y^0 + \frac{y_1}{h} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h} M_C .$$

Case 83/5: Right-hand leg loaded by any type of horizontal load (Hinged support at A)





$$Z = -\left(W\frac{N}{N_Z} - \frac{\mathfrak{S}_r K + \mathfrak{L} k_2}{h N_Z}\right)^* ; \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l}; \qquad H_A = W;$$

$$M_B = -(W+Z)h + \alpha \mathfrak{S}_r \qquad M_C = -(W+Z)h + \mathfrak{S}_r ;$$

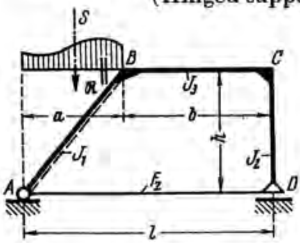
$$M_{y1} = \frac{y_1}{h} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = M_y^0 + \frac{y_2}{h} M_C .$$

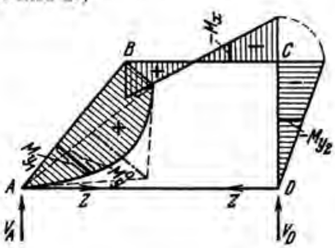
* For the above loading conditions and for a decrease in temperature (p. 299 bottom) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

FRAME 83

See Appendix A, Load Terms, pp. 440-445.

Case 83/6: Left-hand leg loaded by any type of vertical load (Hinged support at A and D)





$$Z = \frac{\beta B \mathfrak{S}_l + \mathfrak{N} k_1}{h N_Z}; \qquad V_D = \frac{\mathfrak{S}_l}{l} \qquad V_A = S - V_D;$$

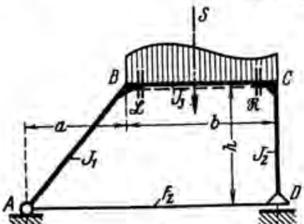
$$M_B = \beta \mathfrak{S}_l - Z h \qquad M_C = -Z h; \qquad M_{y1} = M_y^0 + \frac{y_1}{h} M_B.$$

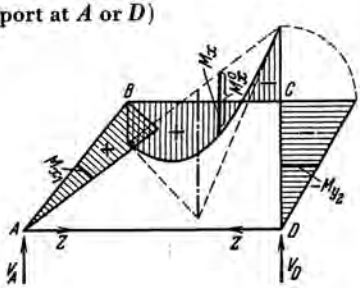
Special case 83/6a: Vertical concentrated load P at B

$$Z \cdot h = \frac{P a b}{l} \cdot \frac{\beta B}{N_Z}; \qquad V_A = \beta P \qquad V_D = \alpha P;$$

$$M_B = \frac{P a b}{l} - Z h \qquad M_C = -Z h. \qquad (M_y^0 = 0).$$

Case 83/7: Girder loaded by any type of vertical load (Hinged support at A or D)



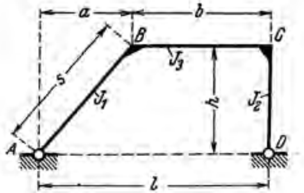


$$Z = \frac{\alpha B \mathfrak{S}_r + (\mathfrak{L} + \mathfrak{N})}{h N_Z}; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = S - V_A;$$

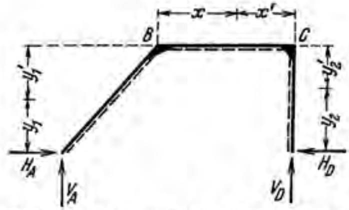
$$M_B = \alpha \mathfrak{S}_r - Z h \qquad M_C = -Z h; \qquad M_x = M_x^0 + \frac{x'}{b} M_B + \frac{x}{b} M_C.$$

Frame 84

Two-hinged trapezoidal rigid frame with one vertical leg. Hinges at same elevation.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

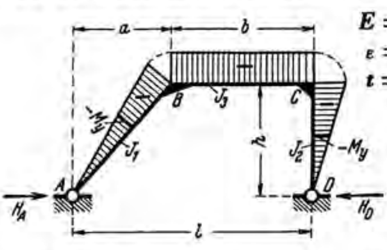
All *coefficients* and *formulas for external loads* of frame 84 are the same as those for frame 82, with the simplifications $(h_1 = h_2) = h$, $v = 0$, $n = m = 1$, $r = 0$, and

$$B = 2k_1 + 3 \qquad C = 3 + 2k_2 \qquad K = \alpha B + C$$

$$N = B + C = \beta B + K\,.$$

Note: The equations for frame 83 may also be used for frame 84 when $L = 0$ and $N_Z = N$ are substituted. It must be remembered, however, to include the effect of the tie-rod force Z in the reactions of H_A and H_D.

Case 84/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

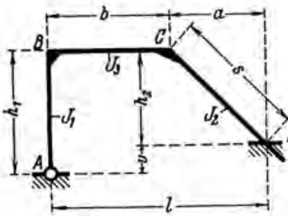
$$M_B = M_C = -\frac{6EJ_3\varepsilon t l}{bhN}\,;$$

$$H_A = H_D = \frac{-M_B}{h} \qquad M_y = \frac{y_1}{h} M_B\,.$$

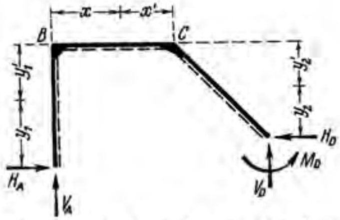
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Frame 85

Trapezoidal rigid frame with one vertical leg, hinged at bottom. Other leg fixed. Supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{h_1}{b} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{s}{b}\,; \qquad n = \frac{h_2}{h_1} \qquad \alpha = \frac{a}{b}$$

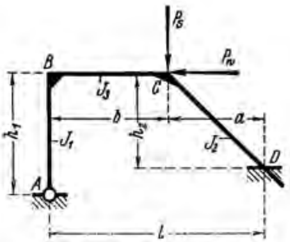
$$\beta = n + \alpha \qquad \lambda = {}^{l}\,; \qquad D = (1 + 2\lambda)\, k_2\,;$$

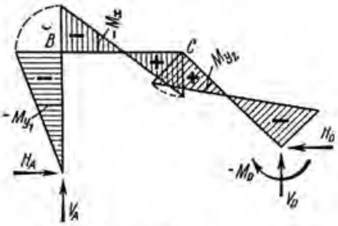
$$R_1 = 2\,(k_1 + 1 + \beta^2 k_2) \qquad K = \beta D - 1$$

$$R_2 = 2\,(1 + k_2) + \lambda\,(k_2 + D)\,; \qquad N = R_1 R_2 - K^2\,;$$

$$n_{11} = \frac{R_2}{N} \qquad n_{12} = n_{21} = \frac{K}{N} \qquad n_{22} = \frac{R_1}{N}\,.$$

Case 85/1: Vertical and horizontal loads at C



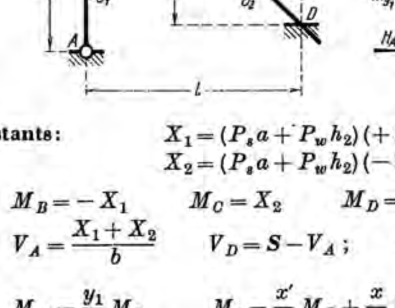


Constants:

$$X_1 = (P_s a + P_w h_2)(+2\beta k_2 n_{11} - D n_{21})$$
$$X_2 = (P_s a + P_w h_2)(-2\beta k_2 n_{12} + D n_{22})\,.$$

$$M_B = -X_1 \qquad M_C = X_2 \qquad M_D = -(P_s a + P_w h_2) + \beta X_1 + \lambda X_2;$$
$$V_A = \frac{X_1 + X_2}{b} \qquad V_D = S - V_A\,; \qquad H_A = \frac{X_1}{h_1} \qquad H_D = -(P_w - H_A)\,;$$
$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D\,.$$

Case 85/2: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants:

$$v = h_1 - h_2{}^* \,; \qquad T = \frac{6 E J_3 \varepsilon t}{b};$$

$$\mathfrak{B}_1 = \frac{v}{b} + \frac{l}{h_1} \qquad \mathfrak{B}_2 = \frac{v}{b}; \qquad X_1 = T(\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21})$$
$$X_2 = T(\mathfrak{B}_1 n_{12} - \mathfrak{B}_2 n_{22})\,.$$

$$M_B = -X_1 \qquad M_C = -X_2 \qquad M_D = \beta X_1 - \lambda X_2;$$
$$V_A = -V_D = \frac{X_1 - X_2}{b} \qquad H_A = H_D = \frac{X_1}{h_1}\,.$$

The formulas for M_{y1}, M_{y2} and M_x are the same as above.

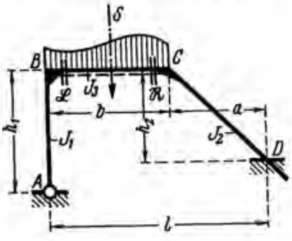
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

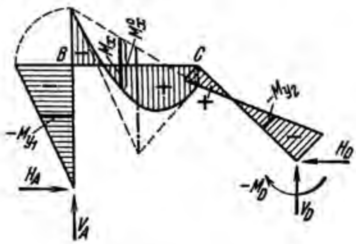
*When $h_2 > h_1$, v becomes negative.

FRAME 85

See Appendix A, Load Terms, pp. 440-445.

Case 85/3: Girder loaded by any type of vertical load

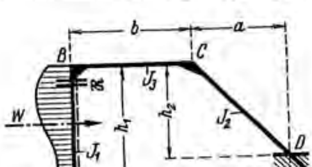


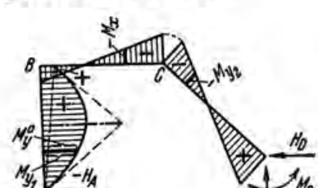


Constant:

$$\mathfrak{B}_1 = \mathfrak{L} + 2\alpha\beta k_2 \mathfrak{S}_l \qquad X_1 = \mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = \mathfrak{R} - \alpha D \mathfrak{S}_l; \qquad X_2 = \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$
$$M_B = -X_1 \qquad M_C = -X_2 \qquad M_D = -\alpha\mathfrak{S}_l + \beta X_1 - \lambda X_2;$$
$$V_A = \frac{\mathfrak{S}_r + X_1 - X_2}{b} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{X_1}{h_1};$$
$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

Case 85/4: Left-hand leg loaded by any type of horizontal load



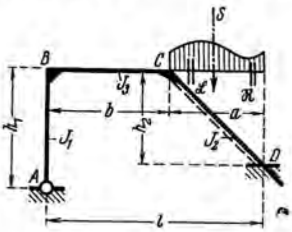


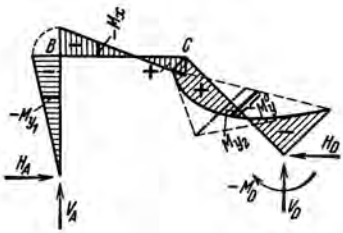
Constants:

$$\mathfrak{B}_1 = 2n\mathfrak{S}_l \beta k_2 - \mathfrak{R} k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21}$$
$$\mathfrak{B}_2 = n\mathfrak{S}_l D; \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$
$$M_B = X_1 \qquad M_C = -X_2 \qquad M_D = n\mathfrak{S}_l - \beta X_1 - \lambda X_2;$$
$$V_D = -V_A = \frac{X_1 + X_2}{b}; \qquad H_D = \frac{\mathfrak{S}_l - X_1}{h_1} \qquad H_A = -(W - H_D);$$
$$M_{y1} = M_y^0 + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

See Appendix A, Load Terms, pp. 440-445.

Case 85/5: Right-hand leg loaded by any type of vertical load





Constants:

$$\mathfrak{B}_1=(2\,\mathfrak{S}_r-\mathfrak{R})\,\beta\,k_2 \qquad X_1=+\,\mathfrak{B}_1\,n_{11}-\mathfrak{B}_2\,n_{21}$$

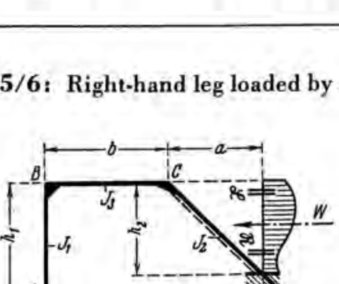
$$\mathfrak{B}_2=\mathfrak{S}_r\,D-(\mathfrak{L}+\lambda\,\mathfrak{R})\,k_2\,; \qquad X_2=-\,\mathfrak{B}_1\,n_{12}+\mathfrak{B}_2\,n_{22}\,.$$

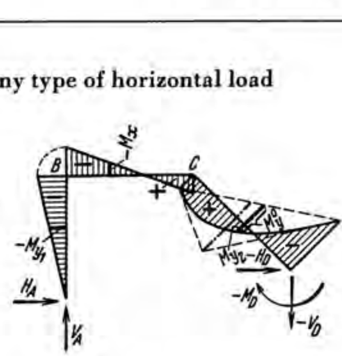
$$M_B=-X_1 \qquad M_C=X_2 \qquad M_D=-\,\mathfrak{S}_r+\beta\,X_1+\lambda\,X_2\,;$$

$$V_A=\frac{X_1+X_2}{b} \qquad V_D=S-V_A\,; \qquad H_A=H_D=\frac{X_1}{h_1}\,;$$

$$M_{y1}=\frac{y_1}{h_1}\,M_B \qquad M_x=\frac{x'}{b}\,M_B+\frac{x}{b}\,M_C \qquad M_{y2}=M_y^0+\frac{y_2}{h_2}\,M_C+\frac{y_2'}{h_2}\,M_D\,.$$

Case 85/6: Right-hand leg loaded by any type of horizontal load



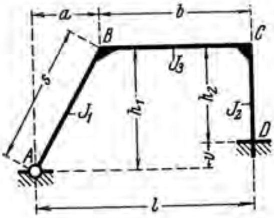


All the formulas are the same as above, except those for *V*- and *H*-forces:

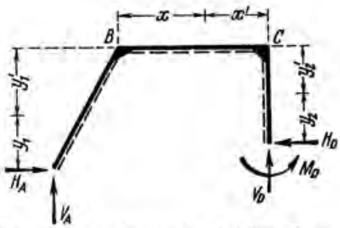
$$V_A=-\,V_D=\frac{X_1+X_2}{b}\,; \qquad H_A=\frac{X_1}{h_1} \qquad H_D=-\,(W-H_A)\,.$$

Frame 86

Trapezoidal rigid frame with one vertical leg, fixed at bottom. Other leg hinged. Supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{s}{b} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{b}\,; \qquad n = \frac{h_2}{h_1} \qquad \alpha = \frac{a}{b}$$

$$\gamma = 1 + \alpha n \qquad \lambda = \frac{l}{b}\,; \qquad D = (1 + 2\gamma)\,k_2\,;$$

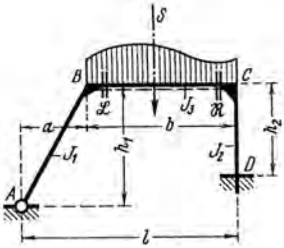
$$R_1 = 2\,(k_1 + 1 + n^2 \lambda^2 k_2) \qquad K = \lambda n D - 1$$

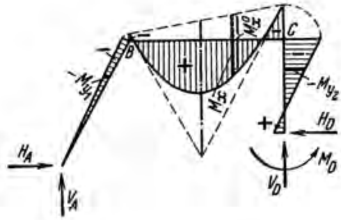
$$R_2 = 2\,(1 + k_2) + \gamma\,(k_2 + D)\,; \qquad N = R_1 R_2 - K^2\,;$$

$$n_{11} = \frac{R_2}{N} \qquad n_{12} = n_{21} = \frac{K}{N} \qquad n_{22} = \frac{R_1}{N}\,.$$

Case 86/1: Girder loaded by any type of vertical load

See Appendix A, Load Terms, pp. 440-445.





Constants:

$$\mathfrak{B}_1 = \mathfrak{L} - 2\alpha\lambda n^2 k_2 \mathfrak{S}_r \qquad X_1 = \mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21}$$

$$\mathfrak{B}_2 = \mathfrak{R} + \alpha n D \mathfrak{S}_r \qquad X_2 = \mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22}.$$

$$M_B = -X_1 \qquad M_C = -X_2 \qquad M_D = n(\alpha \mathfrak{S}_r + \lambda X_1) - \gamma X_2;$$

$$V_A = \frac{\mathfrak{S}_r + X_1 - X_2}{b} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{\alpha(\mathfrak{S}_r - X_2) + \lambda X_1}{h_1};$$

$$M_{y1} = \frac{y_1}{h_1} M_B \qquad M_x = M_x^0 + \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

All other loading conditions follow the equations of frame 79 if $\alpha_1 = \alpha$ and $\beta = \lambda n$ are substituted.

In particular

arbitrary vertical load on the left leg: see case 79/1, p. 288.

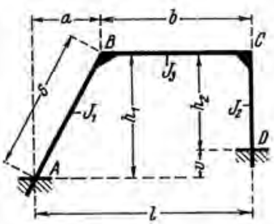
arbitrary horizontal load on the left leg: see case 79/2, p. 288.

arbitrary horizontal load on the right leg: see case 79/4, p. 289.

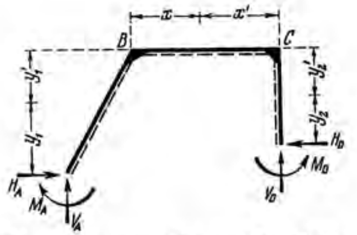
uniform increase in temperature of entire frame: see case 79/6, p. 290.

Frame 87

Hingeless trapezoidal rigid frame with one vertical leg. Supports at different elevations.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{s}{b} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h_2}{b}; \qquad n = \frac{h_2}{h_1} \qquad \alpha = \frac{a}{b} \qquad \lambda = \frac{l}{b};$$

$$K_1 = 3\,n\,k_2 - 2\,\alpha\,k_1 \qquad R_1 = 2\,(K_2 + \alpha\,\lambda\,k_1 + 1)$$

$$K_2 = (2\,\alpha + 3)\,k_1 \qquad R_2 = 2\,(\alpha^2 k_1 + 1 + 3\,k_2)$$

$$K_3 = \alpha\,K_2 - 1\,; \qquad R_3 = 2\,(k_1 + n^2 k_2)\,;$$

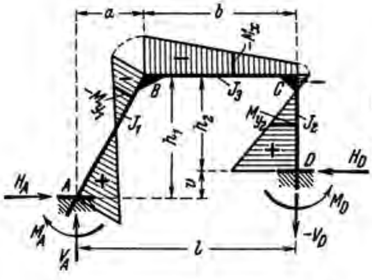
$$N = R_1\,R_2\,R_3 - 2\,K_1\,K_2\,K_3 - R_1\,K_1^2 - R_2\,K_2^2 - R_3\,K_3^2\,;$$

$$n_{11} = \frac{R_2\,R_3 - K_1^2}{N} \qquad n_{12} = n_{21} = \frac{K_1\,K_2 + R_3\,K_3}{N}$$

$$n_{22} = \frac{R_1\,R_3 - K_2^2}{N} \qquad n_{13} = n_{31} = \frac{K_1\,K_3 + R_2\,K_2}{N}$$

$$n_{33} = \frac{R_1\,R_2 - K_3^2}{N} \qquad n_{23} = n_{32} = \frac{K_2\,K_3 + R_1\,K_1}{N}.$$

Case 87/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degree

Constants:

$$X_1 = T\left[\frac{v}{b}(n_{11} - n_{21}) + \frac{l}{h_1} n_{31}\right]$$

$$v = h_1 - h_2^{*}; \qquad X_2 = T\left[\frac{v}{b}(n_{12} - n_{22}) + \frac{l}{h_1} n_{32}\right]$$

$$T = \frac{6 E J_3 \varepsilon t}{b}; \qquad X_3 = T\left[\frac{v}{b}(n_{13} - n_{23}) + \frac{l}{h_1} n_{33}\right]$$

$$M_A = \alpha X_2 - \lambda X_1 + X_3 \qquad M_B = -X_1$$

$$M_D = n X_3 - X_2 \qquad M_C = -X_2;$$

$$V_A = -V_D = \frac{X_1 - X_2}{b}; \qquad H_A = H_D = \frac{X_3}{h_1}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Equations for moments at any point of frame 87 for all loading conditions

The moments at the joints and the fixed end moments contribute to the total moment:

$$M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C$$

$$M_{y1} = \frac{y_1'}{h_1} M_A + \frac{y_1}{h_1} M_B \qquad M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2'}{h_2} M_D.$$

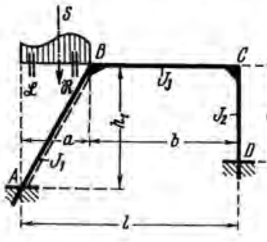
To these moments add the moments M_y^o and M_x^o resp. for directly loaded members only.

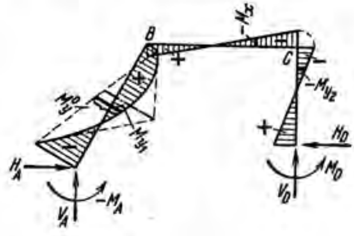
*When $h_2 > h_1$, v becomes negative.

FRAME 87

(See Appendix A, Load Terms, pp. 440-445.)

Case 87/2: Left-hand leg loaded by any type of vertical load





Constants:

$$\mathfrak{B}_1 = \mathfrak{S}_l K_2 - (\lambda\,\mathfrak{L} + \mathfrak{R})\,k_1 \qquad X_1 = +\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21} - \mathfrak{B}_3 n_{31}$$

$$\mathfrak{B}_2 = (2\,\mathfrak{S}_l - \mathfrak{L})\,\alpha\,k_1 \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$

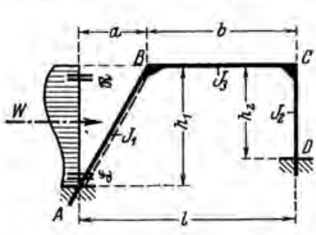
$$\mathfrak{B}_3 = (2\,\mathfrak{S}_l - \mathfrak{L})\,k_1; \qquad X_3 = -\mathfrak{B}_1 n_{13} + \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

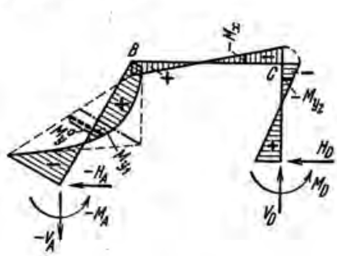
$$M_A = -\mathfrak{S}_l + \lambda X_1 + \alpha X_2 + X_3 \qquad M_B = X_1$$

$$M_C = -X_2 \qquad M_D = n X_3 - X_2;$$

$$V_D = \frac{X_1 + X_2}{b} \qquad V_A = S - V_D; \qquad H_A = H_D = \frac{X_3}{h_1}.$$

Case 87/3: Left-hand leg loaded by any type of horizontal load



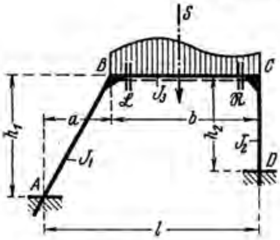


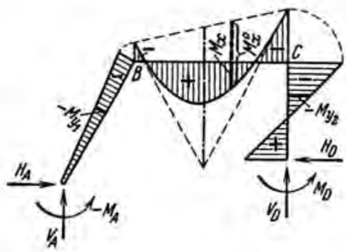
All the formulas are the same as above, except those for V- and H-forces:

$$V_D = -V_A = \frac{X_1 + X_2}{b}; \qquad H_D = \frac{X_3}{h_1} \qquad H_A = -(W - H_D).$$

Case 87/4: Girder loaded by any type of vertical load

(See Appendix A, Load Terms, pp. 440-445.)



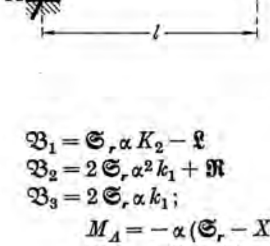


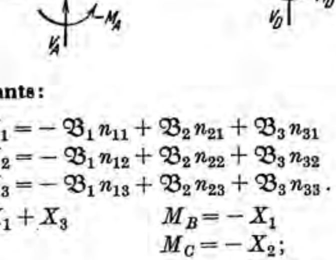
Constants:

$$\mathfrak{B}_1 = \mathfrak{S}_r \alpha K_2 - \mathfrak{L} \qquad X_1 = -\mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$
$$\mathfrak{B}_2 = 2\,\mathfrak{S}_r \alpha^2 k_1 + \mathfrak{R} \qquad X_2 = -\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$$
$$\mathfrak{B}_3 = 2\,\mathfrak{S}_r \alpha k_1; \qquad X_3 = -\mathfrak{B}_1 n_{13} + \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

$$M_A = -\alpha(\mathfrak{S}_r - X_2) - \lambda X_1 + X_3 \qquad M_B = -X_1$$
$$M_D = n X_3 - X_2 \qquad M_C = -X_2;$$
$$V_A = \frac{\mathfrak{S}_r + X_1 - X_2}{b} \qquad V_D = S - V_A; \qquad H_A = H_D = \frac{X_3}{h_1}.$$

Case 87/5: Vertical concentrated load at *B*





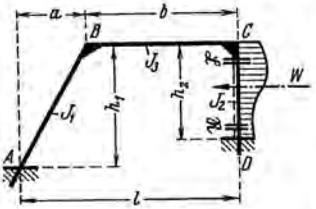
Constants:

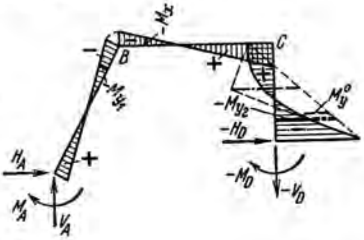
$$X_1 = P a k_1 [+ (2\alpha + 3) n_{11} - 2(\alpha n_{21} + n_{31})] \qquad M_B = X_1$$
$$X_2 = P a k_1 [- (2\alpha + 3) n_{12} + 2(\alpha n_{22} + n_{32})] \qquad M_C = -X_2$$
$$X_3 = P a k_1 [- (2\alpha + 3) n_{13} + 2(\alpha n_{23} + n_{33})].$$
$$M_A = -Pa + \lambda X_1 + \alpha X_2 + X_3 \qquad M_D = n X_3 - X_2;$$
$$V_D = \frac{X_1 + X_2}{b} \qquad V_A = P - V_D; \qquad H_A = H_D = \frac{X_3}{h_1}.$$

FRAME 87

Case 87/6: Right-hand leg loaded by any type of horizontal load

See Appendix A, Load Terms, pp. 440-445.





Constants:

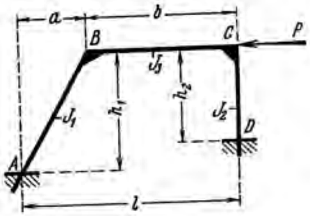
$$\mathfrak{B}_2 = [3\,\mathfrak{S}_r - (\mathfrak{L} + \mathfrak{R})]\,k_2$$
$$\mathfrak{B}_3 = (2\,\mathfrak{S}_r - \mathfrak{R})\,n\,k_2;$$

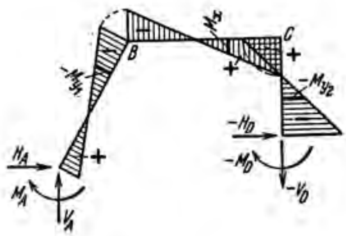
$$X_1 = -\,\mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$$
$$X_2 = +\,\mathfrak{B}_2 n_{22} - \mathfrak{B}_3 n_{32}$$
$$X_3 = -\,\mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$$

$$M_A = X_3 - \lambda X_1 - \alpha X_2 \qquad M_B = -X_1$$
$$M_C = X_2 \qquad M_D = -\mathfrak{S}_r + X_2 + n X_3;$$

$$H_A = \frac{X_3}{h_1} \qquad H_D = -(W - H_A); \qquad V_A = -V_D = \frac{X_1 + X_2}{b}.$$

Case 87/7: Horizontal concentrated load at C





Constants:

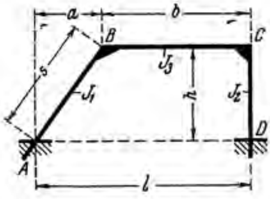
$$X_1 = P h_2 k_2 (-3\,n_{21} + 2\,n\,n_{31}) \qquad M_B = -X_1$$
$$X_2 = P h_2 k_2 (+3\,n_{22} - 2\,n\,n_{32}) \qquad M_C = X_2$$
$$X_3 = P h_2 k_2 (-3\,n_{23} + 2\,n\,n_{33}).$$

$$M_A = X_3 - \lambda X_1 - \alpha X_2 \qquad M_D = -P h_2 + X_2 + n X_3;$$

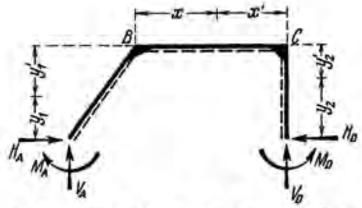
$$H_A = \frac{X_3}{h_1} \qquad H_D = -(P - H_A); \qquad V_A = -V_D = \frac{X_1 + X_2}{b}.$$

Frame 88

Hingeless trapezoidal rigid frame with one vertical leg. Supports at same elevation.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

All *coefficients* and *formulas for external loads* are the same as for Frame 87 (pp. 310-314) with the following changes

$$(h_1 = h_2) = h \qquad n = 1.$$

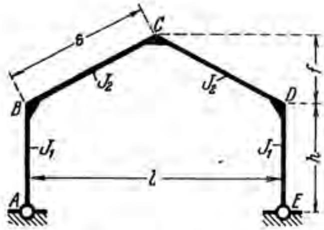
For a *uniform change of temperature* there will be $v = 0$, and the coefficients on p. 311 are reduced to:

$$T' = \frac{6 E J_3 \varepsilon t}{b} \cdot \frac{l}{h};$$

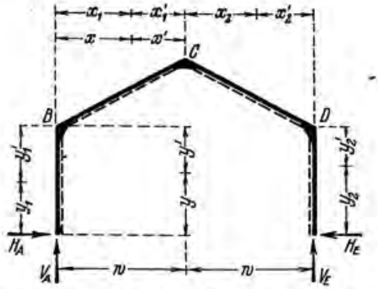
$$X_1 = T' n_{31} \qquad X_2 = T' n_{32} \qquad X_3 = T' n_{33}.$$

Frame 89

Symmetrical two-hinged gable frame with vertical legs.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and coordinates assigned to any point. For symmetrical types of loading use x, x' and y, y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: $k = \frac{J_2}{J_1} \cdot \frac{h}{s}$ $\qquad \varphi = \frac{f}{h}$ $\qquad m = 1 + \varphi;$

$B = 2(k+1) + m$ $\qquad C = 1 + 2m;$ $\qquad N = B + mC.$

Equations for moments at any point of frame 89 for all loading conditions

a) For unsymmetrical loading conditions:

$$M_{x1} = M_x^0 + \frac{x_1'}{w} M_B + \frac{x_1}{w} M_C \qquad M_{x2} = \frac{x_2'}{w} M_C + \frac{x_2}{w} M_D$$

$$M_{y1} = M_y^0 + \frac{y_1}{h} M_B \qquad M_{y2} = \frac{y_2}{h} M_D; \qquad \left(w = \frac{l}{2}\right).$$

b) For symmetrical loading conditions:

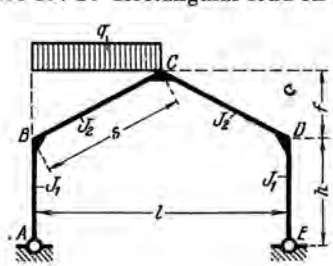
$$M_x = M_x^0 + \frac{x'}{w} M_B + \frac{x}{w} M_C \qquad M_y = M_y^0 + \frac{y}{h} M_B.$$

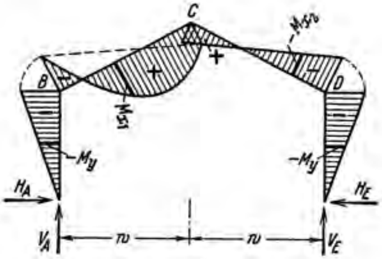
c) For antisymmetrical loading conditions:

$$M_{x2}' = -M_{x1}; \; M_{y2} = -M_{y1}.$$

For members that do not carry any load directly, cancel the values for M_x^0 or M_y^0, respectively.

Case 89/1: Rectangular load on the left girder





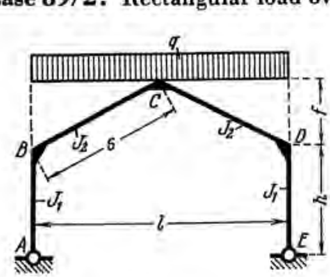
$$M_B = M_D = -\frac{q\,l^2(3+5m)}{32N} \qquad M_C = \frac{q\,l^2}{16} + m\,M_B;$$

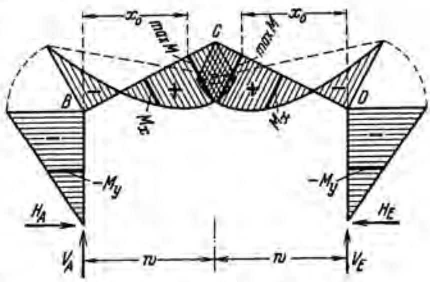
$$H_A = H_E = \frac{-M_B}{h}; \qquad V_A = \frac{3\,q\,l}{8} \qquad V_E = \frac{q\,l}{8};$$

$$M_{x1} = \frac{q\,x_1\,x_1'}{2} + \frac{x_1'}{w}M_B + \frac{x_1}{w}M_C \qquad M_{x2} = \frac{x_2'}{w}M_C + \frac{x_2}{w}M_D;$$

$$Q_{x1} = \frac{q\,l^2}{4\,s}\left(\frac{1}{2} - \frac{x_1}{w}\right) + \frac{M_C - M_B}{s} \qquad Q_{x2} = \frac{M_D - M_C}{s}.$$

Case 89/2: Rectangular load over both girders





$$M_B = M_D = -\frac{q\,l^2(3+5m)}{16N} \qquad M_C = \frac{q\,l^2}{8} + m\,M_B;$$

$$H_A = H_E = \frac{-M_B}{h}; \qquad V_A = V_E = \frac{q\,l}{2};$$

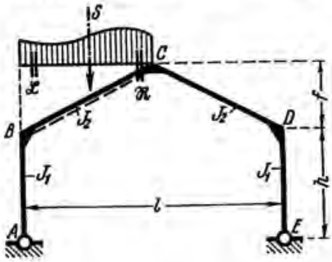
$$M_x = \frac{q\,x\,x'}{2} + \frac{x'}{w}M_B + \frac{x}{w}M_C; \qquad x_0 = \frac{l}{4} + \frac{M_C - M_B}{q\,w};$$

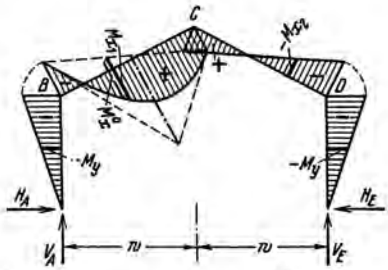
$$M_y = \frac{y}{h}M_B; \qquad Q_x = \frac{q\,l^2}{4\,s}\left(\frac{1}{2} - \frac{x}{w}\right) + \frac{M_C - M_B}{s}.$$

FRAME 89

(See Appendix A, Load Terms, pp. 440-445.)

Case 89/3: Left girder loaded by any type of vertical load

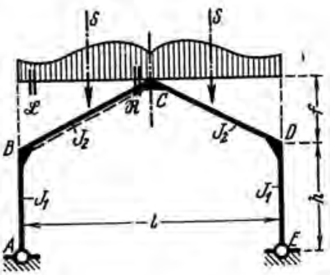


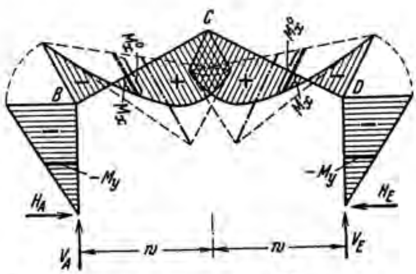


$$M_B = M_D = -\frac{C\mathfrak{S}_l + \mathfrak{L} + m\mathfrak{R}}{2N}; \qquad H_A = H_E = \frac{-M_B}{h};$$

$$M_C = \frac{\mathfrak{S}_l}{2} + m M_B = \frac{B\mathfrak{S}_l - m\mathfrak{L} - m^2\mathfrak{R}}{2N}; \qquad V_E = \frac{\mathfrak{S}_l}{l} \qquad V_A = S - V_E.$$

Case 89/4: Both girders loaded by any type of symmetrical vertical load





$$M_B = M_D = -\frac{C\mathfrak{S}_l + \mathfrak{L} + m\mathfrak{R}}{N} \qquad H_A = H_E = \frac{-M_B}{h};$$

$$M_C = \mathfrak{S}_l + m M_B = \frac{B\mathfrak{S}_l - m\mathfrak{L} - m^2\mathfrak{R}}{N}; \qquad V_A = V_E = S.$$

Note: All load terms refer to the left girder. All corner moments are double the values of case 89/3.

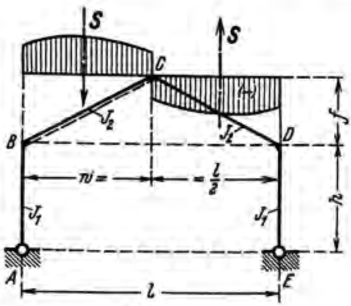
Special case 89/4a: Vertical concentrated load P at C

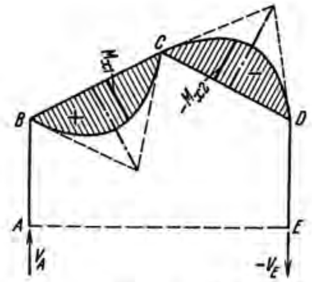
$$(\mathfrak{S}_l = Pw/2; \quad S = P/2).$$

$$M_B = M_D = -\frac{Pl}{4}\cdot\frac{C}{N} \qquad M_C = +\frac{Pl}{4}\cdot\frac{B}{N}; \qquad V_A = V_E = \frac{P}{2}; \quad H_A = H_E = \frac{-M_B}{h}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 89/5: Both girders loaded by any type of antisymmetrical vertical load

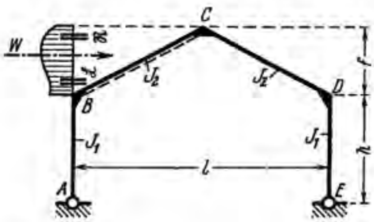


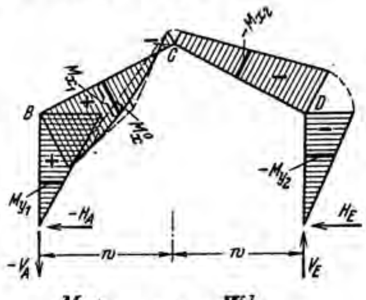


$$V_A = -V_E = \frac{\mathfrak{S}_r}{w} \qquad M_B = M_C = M_D = 0; \qquad M_{x1} = -M_{x2} = \mathbf{M}_x^0.$$

Note: All load terms $\left(\mathfrak{S}_r \text{ and } M_x^0\right)$ refer to the left girder.

Case 89/6: Left girder loaded by any type of horizontal load





Constant: $$X = \frac{C\,\mathfrak{S}_r - \mathfrak{L} - m\,\mathfrak{R}}{2N}. \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = +X \pm \frac{Wh}{2}$$

$$M_C = -\frac{\mathfrak{S}_r}{2} + mX; \qquad V_E = -V_A = \frac{Wh + \mathfrak{S}_l}{l}; \qquad \left.\begin{matrix} H_A \\ H_E \end{matrix}\right\rangle = -\frac{X}{h} \mp \frac{W}{2}.$$

Special case 89/6a: Horizontal concentrated load P at B

$$(W = P; \quad \mathfrak{S}_r = Pf; \quad \mathfrak{S}_l = 0; \quad \mathfrak{L} = \mathfrak{R} = 0).$$

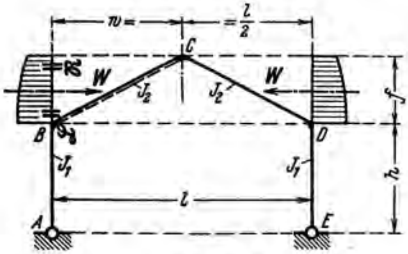
$$M_D = -\frac{Ph(B+C)}{2N} \qquad M_B = Ph + M_D \qquad M_C = \frac{Ph}{2} + m\,M_D;$$

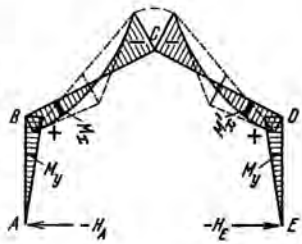
$$V_E = -V_A = \frac{Ph}{l}; \qquad H_E = \frac{-M_D}{h} \qquad H_A = -(P - H_E).$$

FRAME 89

See Appendix A, Load Terms, pp. 440-445.

Case 89/7: Both girders loaded by any type of symmetrical horizontal load



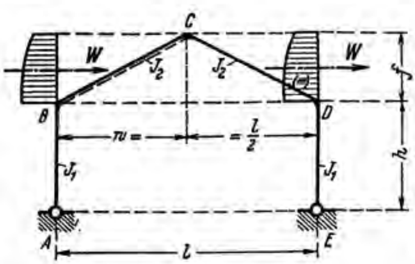


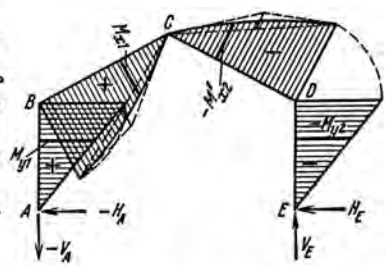
$$M_B = M_D = \frac{C\,\mathfrak{S}_r - \mathfrak{L} - m\,\mathfrak{R}}{N}; \qquad H_A = H_E = -\frac{M_B}{h};$$

$$M_C = -\mathfrak{S}_r + m\,M_B = -\frac{B\,\mathfrak{S}_r + m\,\mathfrak{L} + m^2\,\mathfrak{R}}{N}; \qquad V_A = V_E = 0.$$

Note: All the load terms refer to the left girder.

Case 89/8: Both girders loaded by any type of antisymmetrical horizontal load





$$M_B = -M_D = W h \qquad M_C = 0; \qquad V_E = -V_A = \frac{W h + \mathfrak{S}_l}{w}; \qquad H_E = -H_A = W.$$

Note: All load terms W and $\mathfrak{S}_l$ refer to the left girder.

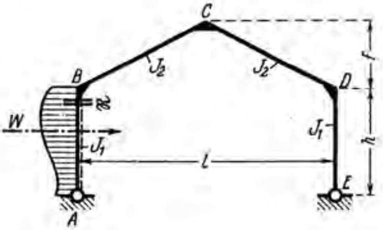
Special case 89/8a: Horizontal concentrated load P at C

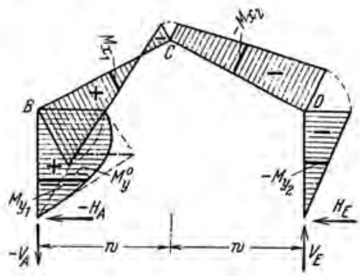
$$(W = P/2;\ \mathfrak{S}_l = P f/2).$$

$$M_B = -M_D = \frac{P h}{2} \qquad M_C = 0; \qquad V_E = -V_A = \frac{P h m}{l}; \qquad H_E = -H_A = \frac{P}{2}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 89/9: Left-hand leg loaded by any type of horizontal load



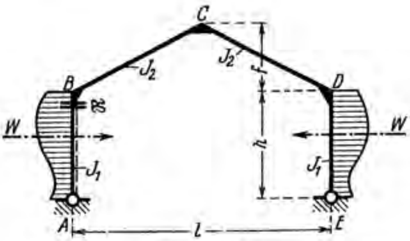


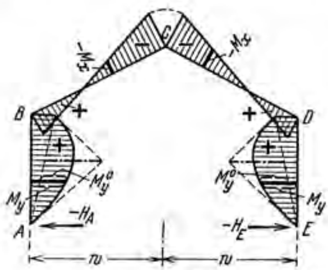
Constant:
$$X = \frac{\mathfrak{S}_l (B + C) + \mathfrak{R}\, k}{2N}.$$

$$M_B = \mathfrak{S}_l - X \qquad M_D = -X; \qquad M_C = \frac{\mathfrak{S}_l}{2} - m\,X;$$

$$V_E = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_E = \frac{X}{h} \qquad H_A = -(W - H_E).$$

Case 89/10: Both legs loaded by any type of symmetrical horizontal load





$$M_B = M_D = \frac{\varphi\, C\, \mathfrak{S}_l - \mathfrak{R}\, k}{N} \qquad M_C = -\varphi\, \mathfrak{S}_l + m\, M_B = \frac{\varphi\, B\, \mathfrak{S}_l + m\, \mathfrak{R}\, k}{N};$$

$$H_A = H_E = -\frac{\mathfrak{S}_r + M_B}{h}; \qquad V_A = V_E = 0.$$

Note: All the load terms refer to the left leg.

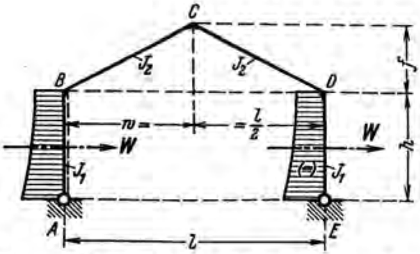
Special case 89/10a: Two equal horizontal concentrated loads P at corners B and D acting from outside

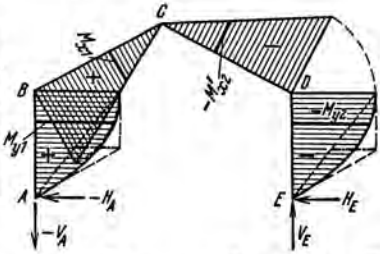
$$(\mathfrak{S}_l = P\,h; \qquad \mathfrak{S}_r = 0; \qquad \mathfrak{R} = 0).$$

$$M_B = M_D = +P f \cdot \frac{C}{N} \qquad M_C = -P f \cdot \frac{B}{N}; \qquad H_A = H_E = -\frac{M_B}{h} = -P \cdot \frac{\varphi\, C}{N}.$$

FRAME 89

Case 89/11: Both legs loaded by any type of antisymmetrical horizontal load See Appendix A, Load Terms, pp. 440-445.





$M_B = -M_D = \mathfrak{S}_l \qquad M_C = 0; \qquad V_E = -V_A = \mathfrak{S}_l/w; \qquad H_E = -H_A = W.$

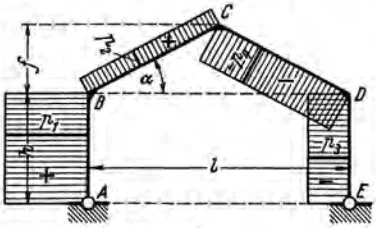
Note: The terms $\mathfrak{S}_l$ and W refer to the left leg.

Special case 89/11a: Two equal horizontal concentrated loads P at corners B and D from the left ($\mathfrak{S}_l = Ph$; $W = P$).

$M_B = -M_D = Ph \qquad M_C = 0; \qquad V_E = -V_A = Ph/w; \qquad H_E = -H_A = P.$

Case 89/12: Uniform increase in temperature of the entire frame–see p. 324.

Case 89/13: Uniformly distributed wind pressure (and suction) normal to all members. Use superposition at 89/14 and 89/15.



Note: p_2 becomes negative for flat roofs.

Formulas to case 89/15 from p. 323:

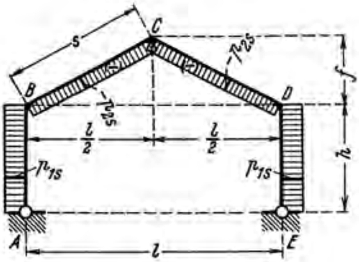
Referring to case 89/13: $\qquad p_{1a} = \frac{p_1 - p_3}{2} \qquad p_{2a} = \frac{p_2 - p_4}{2}.$

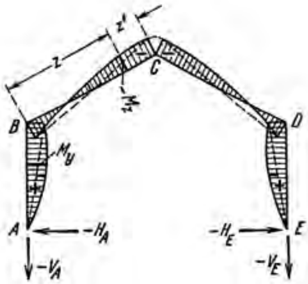
$$M_B = -M_D = \frac{p_{1a} h^2}{2} + p_{2a} f h \qquad M_C = 0; \qquad M_y = \frac{p_{1a} \cdot y y'}{2} + \frac{y}{h} \cdot M_B$$

$$M_z = \frac{p_{2a} \cdot z z'}{2} + \frac{z'}{s} \cdot M_B; \qquad V_E = -V_A = \frac{p_{1a} h^2}{l} + \frac{p_{2a}(2m \cdot fh - s^2)}{l};$$

$$H_E = -H_A = p_{1a} h + p_{2a} f; \qquad Q_z = p_{2a} s \left(\frac{1}{2} - \frac{z}{s}\right) - \frac{M_B}{s}.$$

Case 89/14: **Entire frame loaded by external pressure normal to all members. (Symmetrical load)**





Referring to 89/13 and 89/15: $p_{1s}=\frac{p_1+p_3}{2}$ $p_{2s}=\frac{p_2+p_4}{2}$.

$$M_B=M_D=\frac{p_{1s}h^2(2\varphi C-k)}{4N}-\frac{p_{2s}[l^2C-s^2(C+m)]}{4N};$$

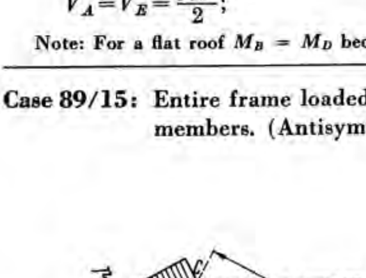
$$M_C=-\frac{p_{1s}hf}{2}+\frac{p_{2s}(w^2-f^2)}{2}+mM_B;\qquad H_A=H_E=-\frac{p_{1s}h}{2}-\frac{M_B}{h};$$

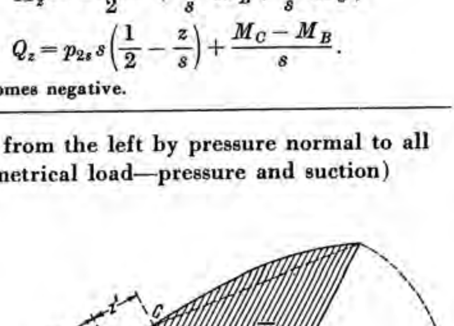
$$M_y=\frac{p_{1s}\cdot yy'}{2}+\frac{y}{h}\cdot M_B\qquad M_z=\frac{p_{2s}\cdot zz'}{2}+\frac{z'}{s}\cdot M_B+\frac{z}{s}\cdot M_C;$$

$$V_A=V_E=\frac{p_{2s}l}{2};\qquad Q_z=p_{2s}s\left(\frac{1}{2}-\frac{z}{s}\right)+\frac{M_C-M_B}{s}.$$

Note: For a flat roof $M_B = M_D$ becomes negative.

Case 89/15: **Entire frame loaded from the left by pressure normal to all members. (Antisymmetrical load—pressure and suction)**

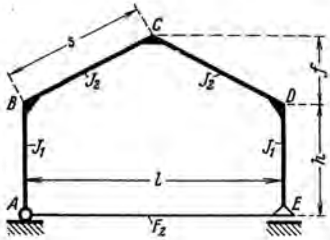




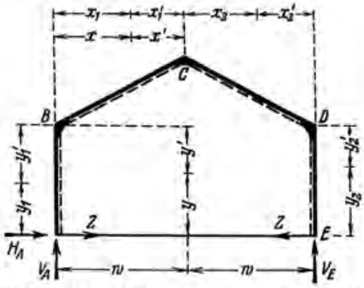
Formulas to case 89/15 see p. 322 bottom.

Frame 90

Symmetrical gable frame with vertical legs and horizontal tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and coordinates assigned to any point. For symmetrical types of loading use x, x' and y, y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k=\frac{J_2}{J_1}\cdot\frac{h}{s};\qquad \varphi=\frac{f}{h};\qquad L=\frac{3J_2}{h^2F_Z}\cdot\frac{E}{E_Z}\cdot\frac{l}{s};\qquad w=\frac{l}{2};$$

$$m=1+\varphi;\qquad B=2(k+1)+m\qquad C=1+2m;$$

$$N=B+mC;\qquad N_Z=N+L.$$

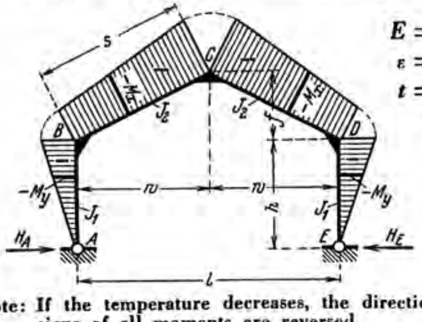
E = Modulus of elasticity of the material of the frame

E_Z = Modulus of elasticity of the tie rod

F_Z = Cross-sectional area of the tie rod

Frame 89 continued:

Frame 89/12: Uniform increase in temperature of the entire frame



E = Modulus of elasticity

ε = Coefficient of thermal expansion

t = Change of temperature in degree

$$M_B=M_D=-\frac{3EJ_2l\cdot\varepsilon t}{shN}$$

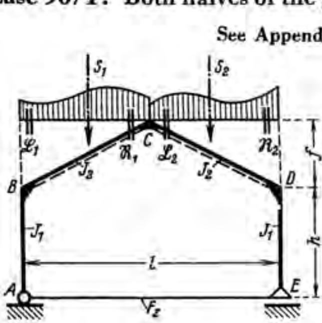
$$M_C=mM_B;$$

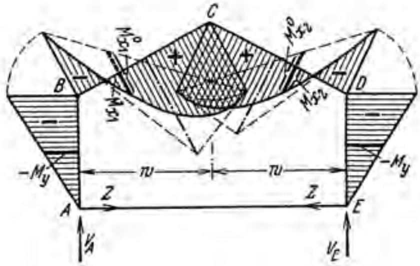
$$H_A=H_E=\frac{-M_B}{h}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 90/1: Both halves of the girder loaded by any type of vertical load

See Appendix A, Load Terms, pp. 440-445.





$$Z = \frac{C\,\mathfrak{S}_{l1} + \mathfrak{L}_1 + m\,\mathfrak{R}_1 + C\,\mathfrak{S}_{r2} + \mathfrak{R}_2 + m\,\mathfrak{L}_2}{2\,h\,N_Z};$$

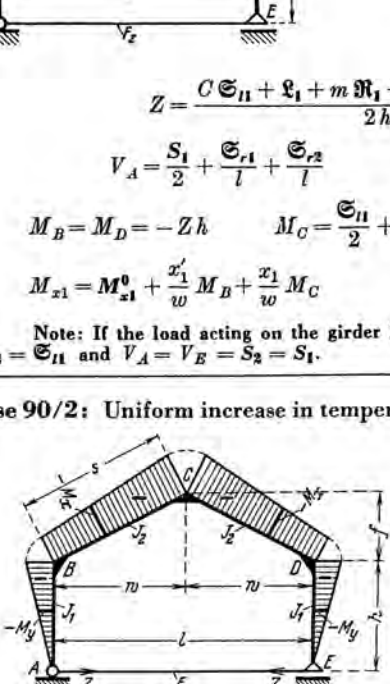
$$V_A = \frac{S_1}{2} + \frac{\mathfrak{S}_{r1}}{l} + \frac{\mathfrak{S}_{r2}}{l} \qquad V_E = \frac{\mathfrak{S}_{l1}}{l} + \frac{\mathfrak{S}_{l2}}{l} + \frac{S_2}{2};$$

$$M_B = M_D = -Z\,h \qquad M_C = \frac{\mathfrak{S}_{l1}}{2} + \frac{\mathfrak{S}_{r2}}{2} - Z\,(h+f); \qquad M_y = -Z\,y$$

$$M_{x1} = M^0_{x1} + \frac{x_1'}{w}\,M_B + \frac{x_1}{w}\,M_C \qquad M_{x2} = M^0_{x2} + \frac{x_2'}{w}\,M_C + \frac{x_2}{w}\,M_D.$$

Note: If the load acting on the girder is symmetrical about C, $\mathfrak{R}_2 = \mathfrak{L}_1$, $\mathfrak{L}_2 = \mathfrak{R}_1$, $\mathfrak{S}_{r2} = \mathfrak{S}_{l1}$ and $V_A = V_E = S_2 = S_1$.

Case 90/2: Uniform increase in temperature of the entire frame



E = Modulus of elasticity

ε = Coefficient of thermal expansion

t = Change of temperature in degree

$$Z = \frac{3\,E\,J_2\,\varepsilon\,t\,l}{s\,h^2\,N_Z}; \qquad M_B = -Z\,h \qquad M_y = -Z\,y$$

$$M_C = -Z\,(h+f) \qquad M_x = -Z\,h\left(1 + \varphi\,\frac{x}{w}\right).$$

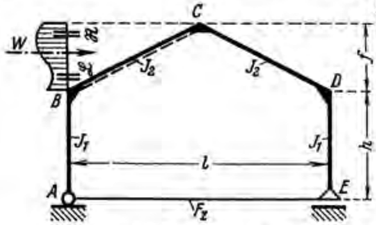
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.*

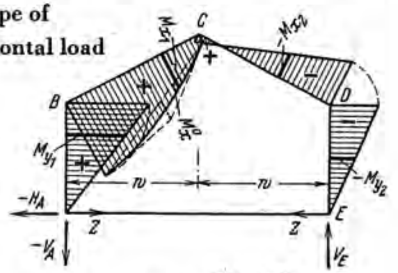
*See footnote on page 327.

FRAME 90

See Appendix A, Load Terms, pp. 440-445.

Case 90/3: Left girder loaded by any type of horizontal load





$$Z = \frac{W h (B + C) + \mathfrak{S}_l C + \mathfrak{L} + m \mathfrak{R}}{2 h N_Z}; \qquad V_E = -V_A = \frac{W h + \mathfrak{S}_l}{l};$$

$$M_B = (W - Z) h \qquad M_C = \frac{W h + \mathfrak{S}_l}{2} - Z (h + f) \qquad M_D = -Z h;$$

$$H_A = -W; \qquad M_{y1} = (W - Z) y_1 \qquad M_{y2} = -Z y_2$$

$$M_{x1} = M_x^0 + \frac{x_1'}{w} M_B + \frac{x_1}{w} M_C \qquad M_{x2} = \frac{x_2'}{w} M_C + \frac{x_2}{w} M_D.$$

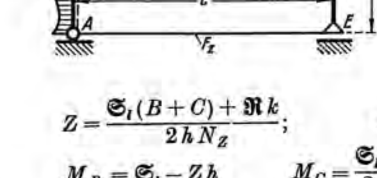
Special case 90/3a: Horizontal concentrated load *P* at *C*

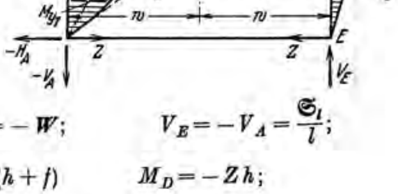
$$(W = P; \qquad \mathfrak{S}_l = P f; \qquad M_x^0 = 0).$$

$$Z = \frac{P}{2} \cdot \frac{N}{N_Z}; \qquad V_E = -V_A = \frac{P (h + f)}{l}; \qquad M_C = \frac{P L (h + f)}{2 N_Z}$$

$$M_B = (P - Z) h \qquad M_D = -Z h \qquad M_{y1} = (P - Z) y_1; \qquad H_A = -P.$$

Case 90/4: Left-hand leg loaded by any type of horizontal load





$$Z = \frac{\mathfrak{S}_l (B + C) + \mathfrak{R} k}{2 h N_Z}; \qquad H_A = -W; \qquad V_E = -V_A = \frac{\mathfrak{S}_l}{l};$$

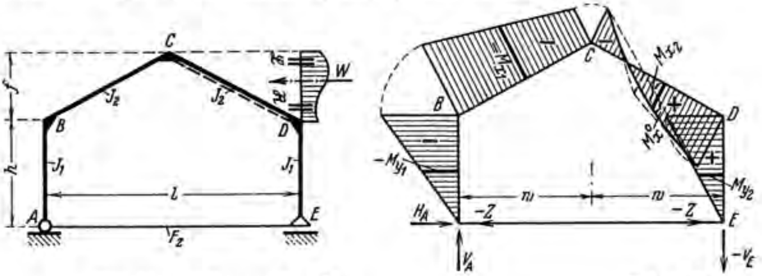
$$M_B = \mathfrak{S}_l - Z h \qquad M_C = \frac{\mathfrak{S}_l}{2} - Z (h + f) \qquad M_D = -Z h;$$

$$M_{y1} = M_y^0 + \frac{y_1}{h} M_B \qquad M_{y2} = -Z y_2$$

$$M_{x1} = \frac{x_1'}{w} M_B + \frac{x_1}{w} M_C \qquad M_{x2} = \frac{x_2'}{w} M_C + \frac{x_2}{w} M_D.$$

(See Appendix A, Load Terms, pp. 440-445.)

Case 90/5: Right girder loaded by any type of horizontal load



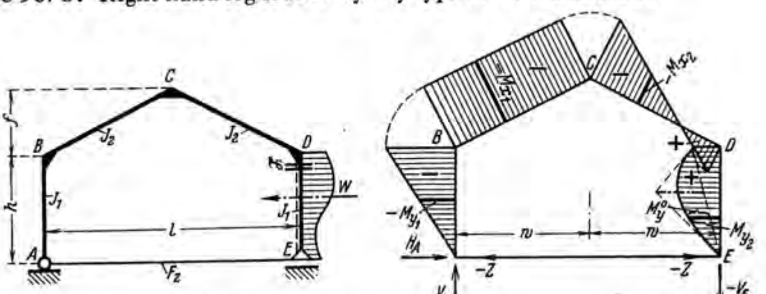
$$Z = -\frac{W h N + C \mathfrak{S}_l - m \mathfrak{L} - \mathfrak{R}}{2 h N_Z}{}^{*}; \qquad V_A = -V_E = \frac{W h + \mathfrak{S}_r}{l};$$

$$M_B = -(W + Z) h \qquad M_C = -\frac{\mathfrak{S}_l}{2} - \left(\frac{W}{2} + Z\right)(h + f) \qquad M_D = (-Z) h;$$

$$H_A = W; \qquad M_{y1} = -(W + Z) y_1 \qquad M_{y2} = (-Z) y_2$$

$$M_{x1} = \frac{x_1'}{w} M_B + \frac{x_1}{w} M_C \qquad M_{x2} = M_x^0 + \frac{x_2'}{w} M_C + \frac{x_2}{w} M_D.$$

Case 90/6: Right-hand leg loaded by any type of horizontal load



$$Z = -\frac{(W h + \mathfrak{S}_l) N + \varphi C \mathfrak{S}_r - \mathfrak{L} k}{2 h N_Z}{}^{*}; \qquad V_A = -V_E = \frac{\mathfrak{S}_r}{l};$$

$$M_B = -(W + Z) h \qquad M_C = \frac{\mathfrak{S}_r}{2} - (W + Z)(h + f) \qquad M_D = -\mathfrak{S}_l - Z h;$$

$$H_A = W; \qquad M_{y1} = -(W + Z) h \qquad M_{y2} = M_y^0 + \frac{y_2}{h} M_D$$

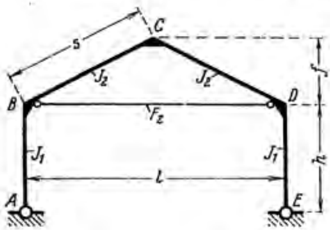
$$M_{x1} = \frac{x_1'}{w} M_B + \frac{x_1}{w} M_C \qquad M_{x2} = \frac{x_2'}{w} M_C + \frac{x_2}{w} M_D.$$

---

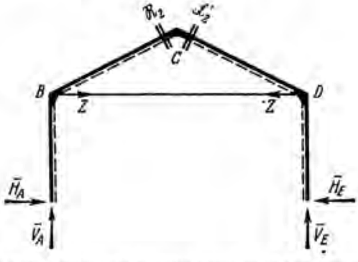
* For the above two loading conditions and for decrease in temperature (p. 325) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

Frame 91

Symmetrical two-hinged gable frame with vertical legs and horizontal tie-rod at bottom of gable.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point exactly as frame 89 (see p. 316). Positive bending moments cause tension at the face marked by a dashed line.

General notes

In order to compute Frame 91 (with tie rod) we can start by using Frame 89 (the same frame without tie rod). The effect of the tie is easily shown as follows:

Steps in computing the stresses

First step: Figure the moments at the joints M_B, M_C, M_D and the reactions H_A, H_E, V_A, V_E by using the formulas for Frame 89 (pp. 316–323)

Second step:

a) Figure the additional coefficients for Frame 91.

$$\beta = \frac{B}{N} \qquad \gamma = \frac{C}{N}; \qquad L = \frac{3J_2}{f^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{l}{s}; \qquad N_Z = \frac{4k+3}{N} + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Note: For a rigid tie set $L = 0$.

b) Figure the tension in the tie rod.

$$Z = \frac{M_B + M_D + 4\,M_C + \mathfrak{R}_2 + \mathfrak{L}_2'}{2\,f\,N_Z}\,*$$

Note: The load terms $\mathfrak{R}_2$ and $\mathfrak{L}_2'$ used in this formula are shown in the right-hand sketch on p. 328 and are to be used accordingly.**

Third step:

a) Moments at the joints and reactions for Frame 91.

$\overline{M}_B = M_B + \gamma Z f$ $\qquad \overline{M}_C = M_C - \beta Z f$ $\qquad \overline{M}_D = M_D + \gamma Z f$

$\overline{H}_A = H_A - \varphi\gamma Z$ $\qquad \overline{H}_E = H_E - \varphi\gamma Z$ $\qquad \overline{V}_A = V_A$ $\qquad \overline{V}_E = V_E$.

Note: In order to distinguish the moments and reactions for Frame 91 from those of Frame 89, the values for Frame 91 are shown with a dash over the letter.

b) Moments at any point of Frame 91.

The formulas for $\overline{M}_x$ and $\overline{M}_y$ are the same as for Frame 89 except that the values $\overline{M}_B$, $\overline{M}_C$, $\overline{M}_D$ are to be used instead of M_B, M_C, M_D.

* For the case of various loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

** For use of the loading conditions of frame 89 substitute the following in the Z formula for the load terms $\mathfrak{R}$ and $\mathfrak{L}_2'$

Case 89/1: $\mathfrak{R}_2 = \frac{q\,l^2}{16}$; $\quad \mathfrak{L}_2' = 0$; $\qquad$ Case 89/2: $\mathfrak{R}_2 + \mathfrak{L}_2' = \frac{q\,l^2}{8}$;

Case 89/3: $\mathfrak{R}_2 = \mathfrak{R}$; $\quad \mathfrak{L}_2' = 0$; $\qquad$ Case 89/4: $\mathfrak{R}_2 + \mathfrak{L}_2' = 2\,\mathfrak{R}$;

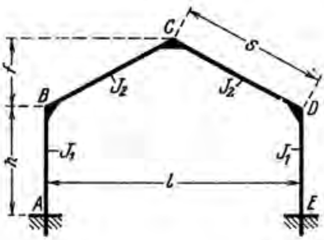
Case 89/6: $\mathfrak{R}_2 = \mathfrak{R}$; $\quad \mathfrak{L}_2' = 0$; $\qquad$ Case 89/7: $\mathfrak{R}_2 + \mathfrak{L}_2' = 2\,\mathfrak{R}$;

Case 89/12: $\mathfrak{R}_2 + \mathfrak{L}_2' = \frac{6\,E\,J_2\,l \cdot \varepsilon t}{s\,f}$; $\qquad$ Case 89/14: $\mathfrak{R}_2 + \mathfrak{L}_2' = \frac{p_{2l} \cdot s^2}{2}$.

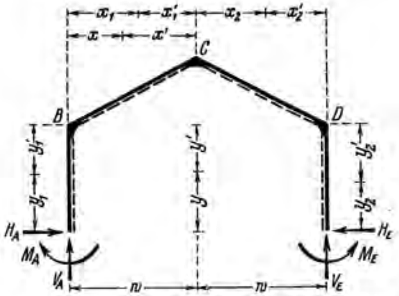
For all remaining load conditions, including the case of uniform temperature change in the entire frame including tie rod, set $\mathfrak{R}_2 = \mathfrak{L}_2' = 0$. All antisymmetrical loading conditions of frame 89 (cases 89/5, 8, 11, and 15) apply to frame 91, since $Z = 0$.

Frame 92

Symmetrical hingeless gable frame with vertical legs.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and coordinates assigned to any point. For symmetrical types of loading use x, x' and y, y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k=\frac{J_2}{J_1}\cdot\frac{h}{s} \qquad \varphi=\frac{f}{h} \qquad m=1+\varphi \qquad B=3k+2 \qquad C=1+2m$$

$$K_1=2(k+1+m+m^2) \qquad K_2=2(k+\varphi^2) \qquad R=\varphi C-k$$

$$N_1=K_1K_2-R^2 \qquad N_2=6k+2.$$

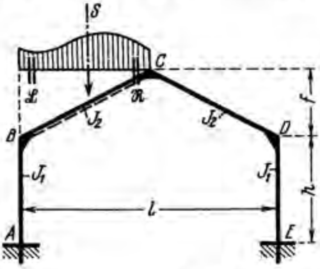
Formulas for the moments at any point of those members of Frame 92 which do not carry any external load

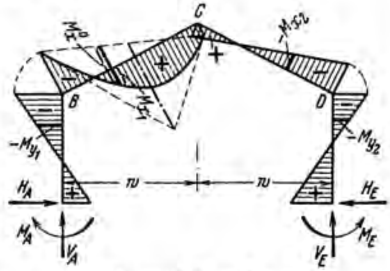
$$M_{y1}=\frac{y_1'}{h}M_A+\frac{y_1}{h}M_B \qquad M_{y2}=\frac{y_2}{h}M_D+\frac{y_2'}{h}M_E$$

$$M_{x1}=\frac{x_1'}{w}M_B+\frac{x_1}{w}M_C \qquad M_{x2}=\frac{x_2'}{w}M_C+\frac{x_2}{w}M_D.$$

(See Appendix A, Load Terms, pp. 440-445.)

Case 92/1: Left girder loaded by any type of vertical load





Constant: $\mathfrak{B}_1 = \varphi\,(2\,\mathfrak{S}_l + \mathfrak{R})$ $\qquad \mathfrak{B}_2 = C\,\mathfrak{S}_l + \mathfrak{L} + m\,\mathfrak{R};$

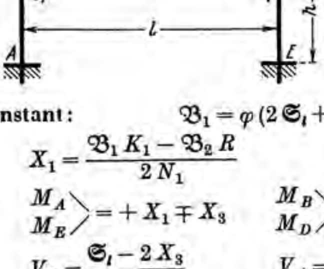
$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2\,N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2\,N_1} \qquad X_3 = \frac{\mathfrak{L}}{2\,N_2}.$$

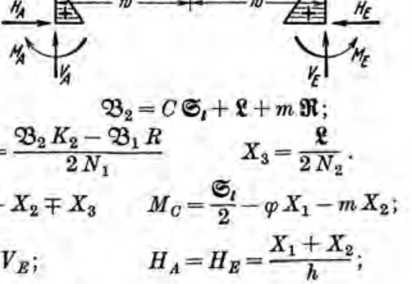
$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = +X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = -X_2 \mp X_3 \qquad M_C = \frac{\mathfrak{S}_l}{2} - \varphi X_1 - m X_2;$$

$$V_E = \frac{\mathfrak{S}_l - 2\,X_3}{l} \qquad V_A = \mathbf{S} - V_E; \qquad H_A = H_E = \frac{X_1 + X_2}{h};$$

$$M_{x1} = \mathbf{M}_x^0 + \frac{x_1'}{w} M_B + \frac{x_1}{w} M_C.$$

Case 92/2: Both girders loaded by any type of symmetrical vertical load





Constant: $\mathfrak{B}_1 = \varphi\,(2\,\mathfrak{S}_l + \mathfrak{R})$ $\qquad \mathfrak{B}_2 = C\,\mathfrak{S}_l + \mathfrak{L} + m\,\mathfrak{R}.$

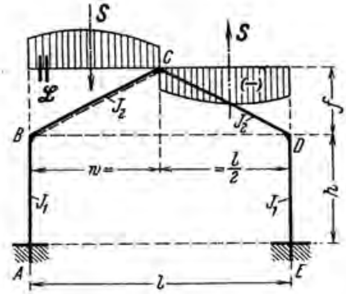
$$M_A = M_E = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{N_1} \qquad M_B = M_D = -\frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{N_1}$$

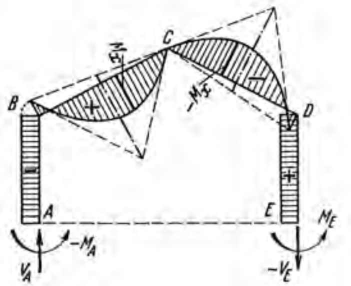
$$M_C = \mathfrak{S}_l - \varphi\,M_A + m\,M_B \qquad M_x = \mathbf{M}_x^0 + \frac{x'}{w} M_B + \frac{x}{w} M_C;$$

$$V_A = V_E = \mathbf{S}; \qquad H_A = H_E = \frac{M_A - M_B}{h}.$$

Note: All the load terms refer to the left girder.

Case 92/3: Both girders loaded by any type of antisymmetrical vertical load



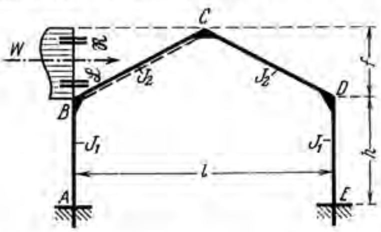


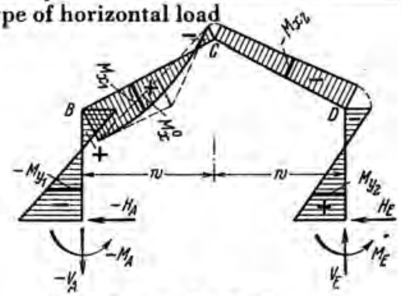
$$M_E = M_D = -M_A = -M_B = \frac{\mathfrak{L}}{N_2} \qquad M_x = M_x^0 + \frac{x'}{w} \cdot M_B$$

$$M_C = 0; \qquad H_A = H_E = 0; \qquad V_A = -V_E = \frac{\mathfrak{S}_r + M_D}{w}.$$

Note: All the load terms refer to the left girder.

Case 92/4: Left girder loaded by any type of horizontal load





Constants: $\mathfrak{B}_1 = \varphi(2\,\mathfrak{S}_r - \mathfrak{R}) \qquad \mathfrak{B}_2 = C\,\mathfrak{S}_r - (\mathfrak{L} + m\,R);$

$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2 N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2 N_1} \qquad X_3 = \frac{W h B + \mathfrak{L}}{2 N_2}.$$

$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = +X_2 \pm \left(\frac{Wh}{2} - X_3\right) \qquad M_C = -\frac{\mathfrak{S}_r}{2} + \varphi X_1 + m X_2;$$

$$V_E = -V_A = \frac{Wh + \mathfrak{S}_l - 2X_3}{l} \qquad H_E = \frac{W}{2} - \frac{X_1 + X_2}{h} \qquad H_A = -(W - H_E).$$

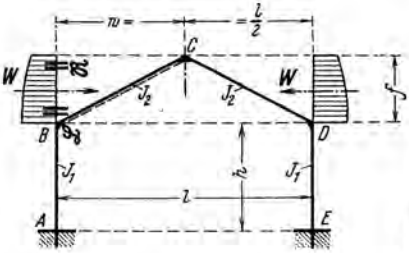
Special case 92/4a: Horizontal concentrated load P at C

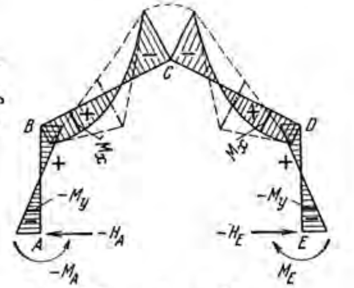
$$M_A = -M_E = -\frac{P h B}{2 N_2} \qquad M_B = -M_D = +\frac{3 P h k}{2 N_2} \qquad M_C = 0;$$

$$V_E = -V_A = \frac{P(h+f) + 2 M_A}{l} \qquad H_E = -H_A = \frac{P}{2}.$$

(See Appendix A, Load Terms, pp. 440-445.)

Case 92/5: Both girders loaded by any type of symmetrical horizontal load





Constants: $\mathfrak{B}_1 = \varphi\,(2\,\mathfrak{S}_r - \mathfrak{N})$ $\qquad \mathfrak{B}_2 = C\,\mathfrak{S}_r - (\mathfrak{L} + m\,\mathfrak{N})$.

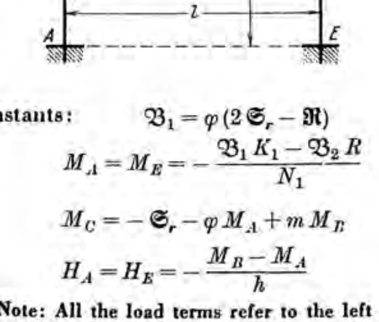
$$M_A = M_E = -\frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{N_1} \qquad M_B = M_D = +\frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{N_1}$$

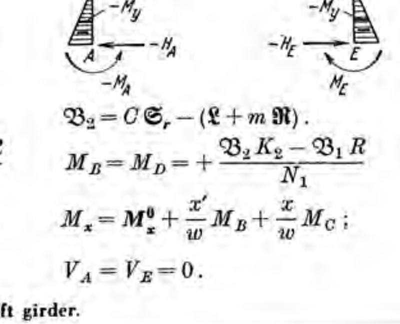
$$M_C = -\mathfrak{S}_r - \varphi\, M_A + m\, M_B \qquad M_x = M_x^0 + \frac{x'}{w} M_B + \frac{x}{w} M_C\,;$$

$$H_A = H_E = -\frac{M_B - M_A}{h} \qquad V_A = V_E = 0\,.$$

Note: All the load terms refer to the left girder.

Case 92/6: Both girders loaded by any type of antisymmetrical horizontal loads





$$M_E = -M_A = \frac{B \cdot Wh + \mathfrak{L}}{N_2} \qquad M_B = -M_D = \frac{3k \cdot Wh - \mathfrak{L}}{N_2} \qquad M_C = 0$$

$$(M_B - M_A = M_E - M_D = Wh) \qquad M_x = M_x^0 + \frac{x'}{w} M_B\,;$$

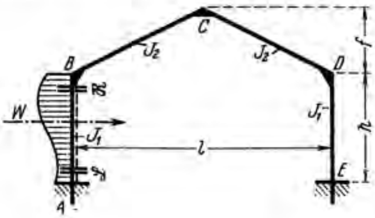
$$V_E = -V_A = \frac{\mathfrak{S}_l + M_B}{w} \qquad H_E = -H_A = W\,.$$

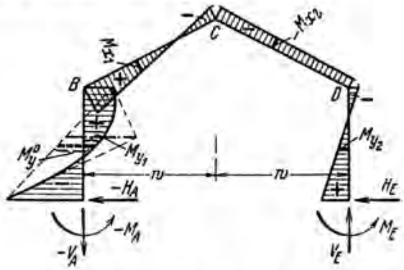
Note: All load terms refer to the left girder.

FRAME 92

(See Appendix A, Load Terms, pp. 440-445.)

Case 92/7: Left leg loaded by any type of horizontal load





Constants: $\mathfrak{B}_1 = \mathfrak{L}k + 2\varphi^2 \mathfrak{S}_l \qquad \mathfrak{B}_2 = \varphi \mathfrak{S}_l C - \mathfrak{R}k;$

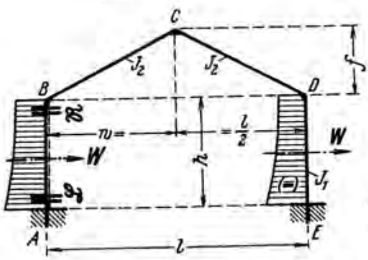
$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2N_1} \qquad X_3 = \frac{B\mathfrak{S}_l + (\mathfrak{L}+\mathfrak{R})k}{2N_2}.$$

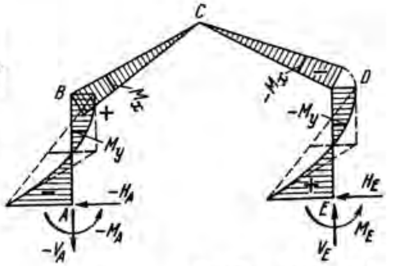
$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = +X_2 \pm \left(\frac{\mathfrak{S}_l}{2} - X_3\right)$$

$$M_C = -\frac{\varphi\mathfrak{S}_l}{2} + \varphi X_1 + m X_2 \qquad M_{y1} = \mathbf{M}_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B;$$

$$V_E = -V_A = \frac{\mathfrak{S}_l - 2X_3}{l}; \qquad H_E = \frac{\mathfrak{S}_l}{2h} - \frac{X_1 + X_2}{h} \qquad H_A = -(\mathbf{W} - H_E).$$

Case 92/8: Both legs loaded by any type of antisymmetrical horizontal load





$$M_E = -M_A = \frac{B\mathfrak{S}_l + (\mathfrak{L}+\mathfrak{R})k}{N_2} \qquad M_B = -M_D = \frac{3k\mathfrak{S}_l - (\mathfrak{L}+\mathfrak{R})k}{N_2}$$

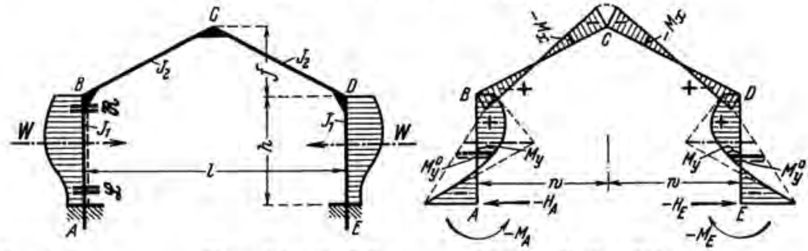
$$(M_B - M_A = M_E - M_D = \mathfrak{S}_l) \qquad M_{y1} = \mathbf{M}_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B$$

$$M_C = 0; \qquad V_E = -V_A = \frac{M_B}{w}; \qquad H_E = -H_A = \mathbf{W}.$$

Note: All the load terms refer to the left leg.

Case 92/9: Both legs loaded by any type of symmetrical horizontal load

See Appendix A, Load Terms, pp. 440-445.



Constants: $\mathfrak{B}_1 = \mathfrak{L}k + 2\varphi^2\mathfrak{S}_l$ $\qquad \mathfrak{B}_2 = \varphi\mathfrak{S}_l C - \mathfrak{R}k$.

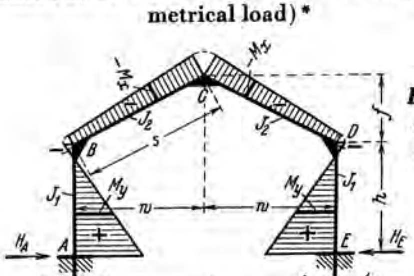
$$M_A = M_E = -\frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{N_1} \qquad M_B = M_D = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{N_1}$$

$$M_C = -\varphi(\mathfrak{S}_l + M_A) + m M_B \qquad M_y = M_y^0 + \frac{y'}{h} M_A + \frac{y}{h} M_B;$$

$$H_A = H_E = -\frac{\mathfrak{S}_r - M_A + M_B}{h}; \qquad V_A = V_E = 0.$$

Note: All terms refer to the left leg.

Case 92/10: Uniform increase in temperature of the entire frame (symmetrical load)*



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $\quad T = \frac{9EJ_2\varepsilon t l}{hsN_1}$

$$M_A = M_E = +T(k + 2 + \varphi) \qquad M_B = M_D = -T(k - \varphi)$$

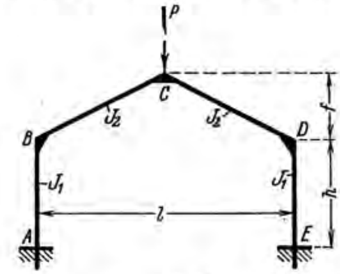
$$M_C = -\varphi M_A + m M_B; \qquad H_A = H_E = \frac{M_A - M_B}{h}; \quad V_A = V_E = 0.$$

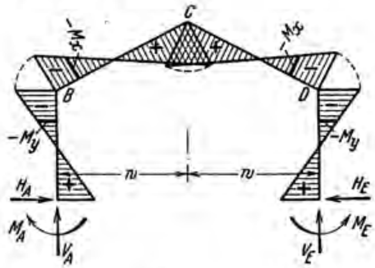
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

* Only the temperature change of the diagonals causes stress; equal temperature changes in both legs have no effect. For an antisymmetrical change in temperature (left half $+t_1$ and $+t_2$, right half $-t_1$ and $-t_2$) substitute $\mathfrak{L} = 12EJ_2\varepsilon\cdot(ht_1 + ft_2)/sl$ in the formulas for case 92/3 and set all other load terms equal to zero. ($\mathfrak{S}_r = 0$; $M_x^0 = 0$).

FRAME 92

Case 92/11: Vertical concentrated load at ridge C

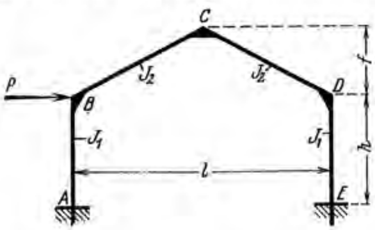


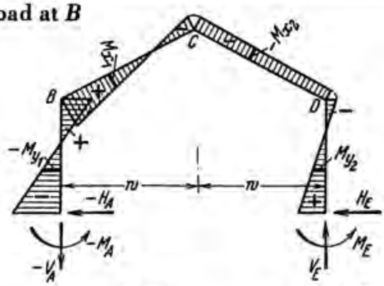


$$M_A = M_E = \frac{3\,P\,l\,(k + 2\,k\,\varphi + \varphi)}{4\,N_1} \qquad M_B = M_D = -\frac{3\,P\,l\,k\,m}{2\,N_1}$$

$$M_C = \frac{P\,l}{4} - \varphi\,M_A + m\,M_B\,; \qquad V_A = V_E = \frac{P}{2}; \qquad H_A = H_E = \frac{M_A - M_B}{h}.$$

Case 92/12: Horizontal concentrated load at B





Constants: $X_1 = \dfrac{3\,P\,f\,(k + 2\,\varphi\,k + \varphi)}{2\,N_1} \qquad X_2 = \dfrac{3\,P\,f\,m\,k}{N_1} \qquad X_3 = \dfrac{P\,h\,B}{2\,N_2}.$

$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = X_2 \pm \left(\frac{P\,h}{2} - X_3\right) \qquad M_C = -\frac{P\,f}{2} + \varphi\,X_1 + m\,X_2;$$

$$V_E = -V_A = \frac{P\,h - 2\,X_3}{l}; \qquad H_E = \frac{P}{2} - \frac{X_1 + X_2}{h} \qquad H_A = -(P - H_E).$$

Formulas to case 92/15 from p. 337

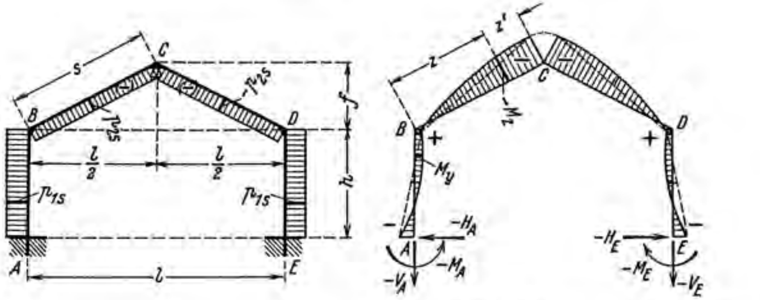
Referring to 92/13: $\qquad p_{1a} = \dfrac{p_1 - p_3}{2} \qquad p_{2a} = \dfrac{p_2 - p_4}{2}.$

$$M_B = -M_D = \frac{p_{1a}\,h^2 \cdot k}{N_2} + \frac{p_{2a}\,(12\,k \cdot f\,h - s^2)}{4\,N_2} \qquad M_C = 0$$

$$M_E = -M_A = \frac{p_{1a}\,h^2\,(2\,k + 1)}{N_2} + \frac{p_{2a}\,(4\,B \cdot f\,h + s^2)}{4\,N_2};$$

$$V_E = -V_A = \frac{p_{1a}\,h^2}{l} + \frac{p_{2a}\,(2\,m \cdot f\,h - s^2)}{l} - \frac{M_E}{w}; \qquad H_E = -H_A = p_{1a}\,h + p_{2a}\,f.$$

Case 92/14: Entire frame loaded by external pressure normal to all members. (Symmetrical load)



Referring to 92/13: $\quad p_{1s} = \dfrac{p_1 + p_3}{2} \qquad p_{2s} = \dfrac{p_2 + p_4}{2}.$

Constants:

$$\mathfrak{B}_1 = \frac{p_{1s} h^2}{4}(k + 4\varphi^2) + \frac{p_{2s} f^2}{4} \cdot 3\varphi - \frac{p_{2s} w^2}{4} \cdot 5\varphi$$

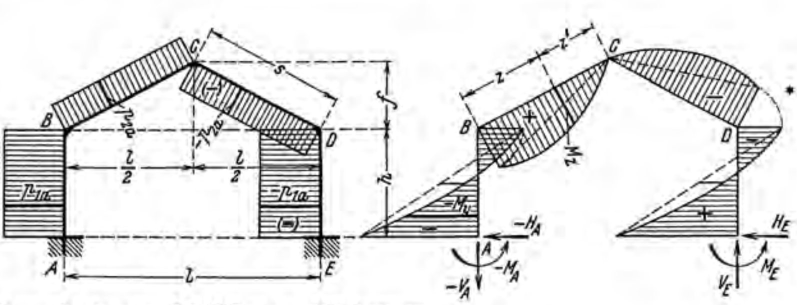
$$\mathfrak{B}_2 = \frac{p_{1s} h^2}{4}(2\varphi C - k) + \frac{p_{2s} f^2}{4}(1 + 3m) - \frac{p_{2s} w^2}{4}(3 + 5m).$$

$$M_A = M_E = \frac{-\mathfrak{B}_1 K_1 + \mathfrak{B}_2 R}{N_1} \qquad M_B = M_D = \frac{-\mathfrak{B}_1 R + \mathfrak{B}_2 K_2}{N_1}; \quad V_A = V_E = \frac{p_{2s} l}{2};$$

$$M_C = -\frac{p_{1s} h f}{2} + \frac{p_{2s}(w^2 - f^2)}{2} - \varphi M_A + m M_B; \quad H_A = H_E = -\frac{p_{1s} h}{2} + \frac{M_A - M_B}{h}.$$

Note: For a flat roof $M_B = M_D$ becomes negative.

Case 92/15: Entire frame loaded from the left by pressure normal to all members. (Antisymmetrical load—pressure and suction)

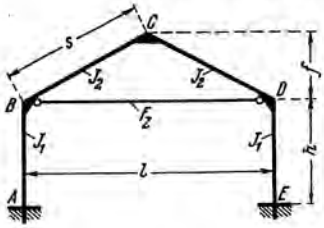


Formulas to case **92/15** see p. 336 bottom

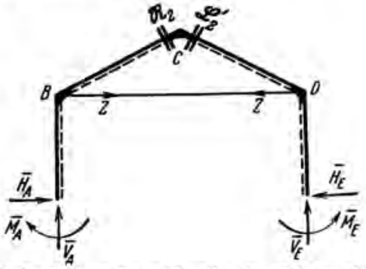
* M_z and Q_z for cases 92/14 and 92/15 are identical with those values for cases 89/14 and 89/15 respectively.

Frame 93

Symmetrical hingeless gable frame with vertical legs and horizontal tie-rod at bottom of gable.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point exactly as frame 92 (see p. 330). Positive bending moments cause tension at the face marked by a dashed line.

General notes

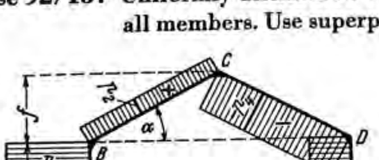
In order to compute Frame 93 (with tie rod) we can start by using Frame 92 (the same frame without tie rod). The effect of the tie is easily shown as follows:

Steps in computing the stresses

First step: Figure the moments at the joints M_A, M_B, M_C, M_D, M_E and the reactions H_A, H_E, V_A, V_E by using the formulas for Frame 92 (pp. 330–337). Frame 93 continued on p. 339.

Frame 92 continued:

Case 92/13: Uniformly distributed wind pressure (and suction) normal to all members. Use superposition of 92/14 and 92/15.



Note: p_2 becomes negative for flat roofs.

This general wind load can be obtained by superposition of a symmetrical load (case 92/14) and an antisymmetrical load (case 92/15).

Second step:

a) Figure the additional coefficients for Frame 93.

$$\alpha = \frac{3(mk + \varphi k + \varphi)}{N_1} \qquad \beta = \frac{6mk}{N_1} \qquad \gamma = \frac{3k(k+1+m)}{N_1}$$

$$L = \frac{3J_2}{f^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{l}{s} \qquad N_Z = 2\gamma - \beta + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Note: For a rigid tie set $L = 0$.

b) Figure the tension in the tie rod.

$$Z = \frac{M_B + M_D + 4M_C + \mathfrak{R}_2 + \mathfrak{L}_2'}{2fN_Z} *$$

Note: The load terms $\mathfrak{R}_2$ and $\mathfrak{L}_2'$ used in this formula are shown in the right-hand sketch on p. 338 and are to be used accordingly.**

Third step:

a) Moments at the joints, moments at the supports and reactions for Frame 93.

$$\overline{M}_B = M_B + \beta Z f \qquad \overline{M}_C = M_C - \gamma Z f \qquad \overline{M}_D = M_D + \beta Z f$$

$$\overline{M}_A = M_A - \alpha Z f \qquad \overline{M}_E = M_E - \alpha Z f$$

$$\overline{H}_A = H_A - \varphi(\alpha + \beta) Z \quad \overline{H}_E = H_E - \varphi(\alpha + \beta) Z \quad \overline{V}_A = V_A \quad \overline{V}_E = V_E.$$

Note: In order to distinguish the moments and reactions for Frame 93 from those of Frame 92, the values for Frame 93 are shown with a dash over the letter.

b) Moments at any point of Frame 93.

The formulas for $\overline{M}_x$ and $\overline{M}_y$ are the same as for Frame 92 except that the values $\overline{M}_A$, $\overline{M}_B$, $\overline{M}_C$, $\overline{M}_D$, $\overline{M}_E$ are to be used instead of M_A, M_B, M_C, M_D, M_E.

---

* For the case of various loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

** For use of the loading conditions of frame 92 substitute the following in the Z formula for the load terms $\mathfrak{R}_2$ and $\mathfrak{L}_2'$

Case 92/1: $\mathfrak{R}_2 = \mathfrak{R}$; $\mathfrak{L}_2' = 0$; Case 92/2: $\mathfrak{R}_2 + \mathfrak{L}_2' = 2\mathfrak{R}$;

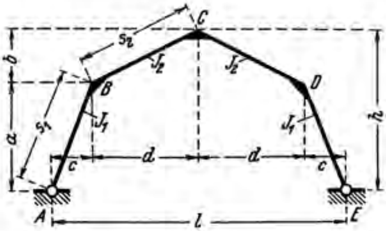
Case 92/4: $\mathfrak{R}_2 = \mathfrak{R}$; $\mathfrak{L}_2' = 0$; Case 92/5: $\mathfrak{R}_2 + \mathfrak{L}_2' = 2\mathfrak{R}$;

Case 92/10: $\mathfrak{R}_2 + \mathfrak{L}_2' = \frac{6EJ_2 l \cdot \varepsilon t}{s f}$; Case 92/14: $\mathfrak{R}_2 + \mathfrak{L}_2' = \frac{p_{2s} \cdot s^2}{2}$.

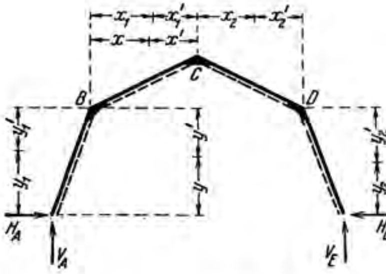
For all remaining load conditions, including the case of uniform temperature change in the entire frame including tie rod, set $\mathfrak{R}_2 = \mathfrak{L}_2' = 0$. All antisymmetrical loading conditions of frame 92 (cases 92/3, 6, 8, and 15) apply to frame 93, since $Z = 0$.

Frame 94

Symmetrical two-hinged gable frame with inclined legs.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and coordinates assigned to any point. For symmetrical types of loading use x, x' and y, y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k=\frac{J_2}{J_1}\cdot\frac{s_1}{s_2}; \qquad \varphi=\frac{b}{a} \qquad m=\frac{h}{a}; \qquad \gamma=\frac{c}{l} \qquad \delta=\frac{d}{l};$$

$$B=2\,(k+1)+m \qquad C=1+2\,m; \qquad N=B+m\,C.$$

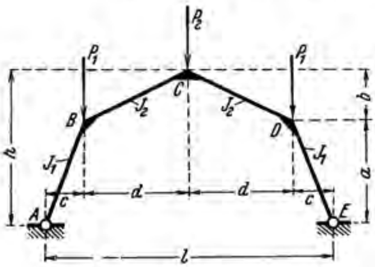
Formulas for the moments at any point of those members of Frame 94 which do not carry any external load

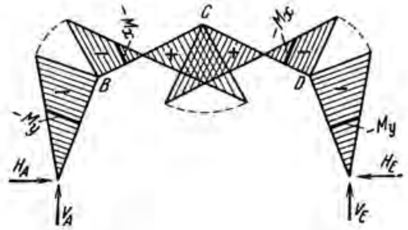
$$M_{y1}=\frac{y_1}{a}\,M_B \qquad M_{x1}=\frac{x_1'}{d}\,M_B+\frac{x_1}{d}\,M_C$$

$$M_{y2}=\frac{y_2}{a}\,M_D \qquad M_{x2}=\frac{x_2'}{d}\,M_C+\frac{x_2}{d}\,M_D.$$

Note: The formulas in terms of z (cases 94/13 and 94/14, pp. 346-347) may be used instead of the above formulas in terms of x and y.

Case 94/1: Vertical concentrated loads acting at B, C, D, acting symmetrically about the center line of the frame*





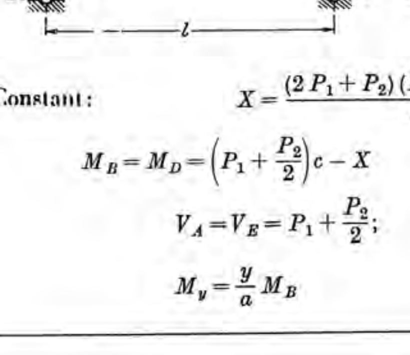
Constant:
$$X=\frac{(2\,P_1+P_2)\,(B+C)\,c+P_2\,C\,d}{2\,N}.$$

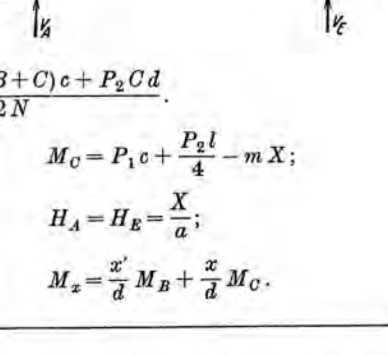
$$M_B=M_D=\left(P_1+\frac{P_2}{2}\right)c-X \qquad M_C=P_1\,c+\frac{P_2\,l}{4}-m\,X;$$

$$V_A=V_E=P_1+\frac{P_2}{2}; \qquad H_A=H_E=\frac{X}{a};$$

$$M_y=\frac{y}{a}\,M_B \qquad M_x=\frac{x'}{d}\,M_B+\frac{x}{d}\,M_C.$$

Case 94/2: Horizontal concentrated load P at ridge C (Antisymmetrical loa





$$M_B=-M_D=\frac{P\,(a\,d-b\,c)}{l} \qquad M_C=0; \qquad H_E=-H_A=\frac{P}{2};$$

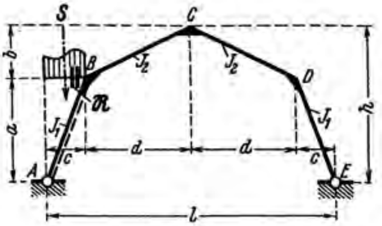
$$V_E=-V_A=\frac{P\,h}{l}; \qquad M_y=\frac{y}{a}\,M_B \qquad M_x=\frac{x'}{d}\,M_B.$$

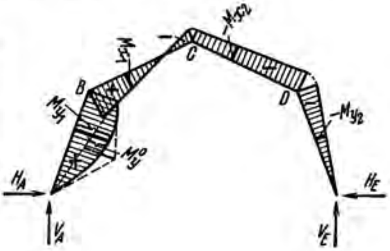
*The moment diagram is based on the assumption $P_2 > P_1$.

FRAME 94

See Appendix A, Load Terms, pp. 440-445.

Case 94/3: Left-hand leg loaded by any type of vertical load





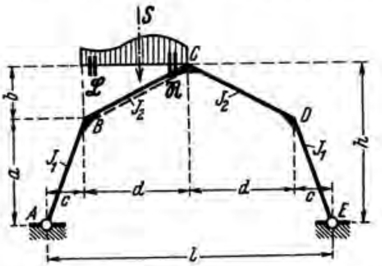
Constant:
$$X = \frac{\mathfrak{S}_l(B+C) + \mathfrak{R}k}{2N}.$$

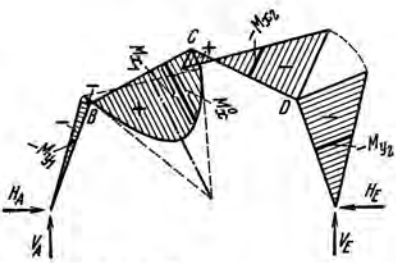
$$M_B = (1-\gamma)\mathfrak{S}_l - X \qquad M_D = -X + \gamma\,\mathfrak{S}_l$$

$$M_C = -mX + \frac{\mathfrak{S}_l}{2}; \qquad M_{y1} = \mathbf{M}_y^0 + \frac{y_1}{a}M_B;$$

$$V_E = \frac{\mathfrak{S}_l}{l} \qquad V_A = \mathbf{S} - V_E; \qquad H_A = H_E = \frac{X}{a}.$$

Case 94/4: Left girder loaded by any type of vertical load





Constant:
$$X = \frac{\mathbf{S}c(B+C) + \mathfrak{S}_l C + \mathfrak{L} + m\,\mathfrak{R}}{2N}.$$

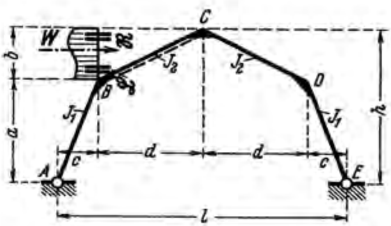
$$M_B = \gamma\,\mathfrak{S}_r + \frac{\mathbf{S}c}{2} - X \qquad M_D = -X + \gamma(\mathbf{S}c + \mathfrak{S}_l)$$

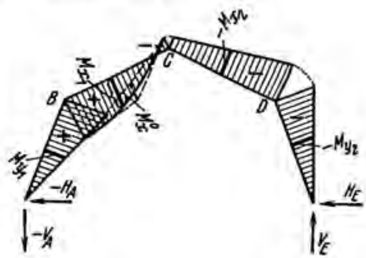
$$M_C = \frac{\mathbf{S}c + \mathfrak{S}_l}{2} - mX; \qquad M_{x1} = \mathbf{M}_x^0 + \frac{x_1'}{d}M_B + \frac{x_1}{d}M_C;$$

$$V_E = \frac{\mathbf{S}c + \mathfrak{S}_l}{l} \qquad V_A = \mathbf{S} - V_E; \qquad H_A = H_E = \frac{X}{a}.$$

(See Appendix A, Load Terms, pp. 440-445.)

Case 94/5: Left girder loaded by any type of horizontal load





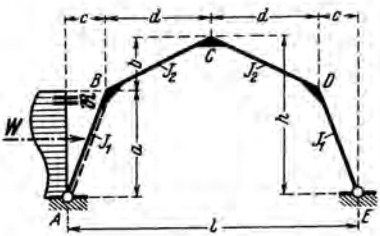
Constant: $$X = \frac{W a N - \mathfrak{S}_r C + \mathfrak{L} + m\,\mathfrak{R}}{2N}.$$

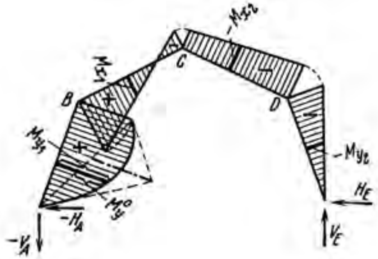
$$M_B = (1-\gamma)\,W a - \gamma\,\mathfrak{S}_l - X \qquad M_D = -X + \gamma\,(W a + \mathfrak{S}_l)$$

$$M_C = -m X + \frac{W a + \mathfrak{S}_l}{2}; \qquad M_{x1} = M_x^0 + \frac{x_1'}{d} M_B + \frac{x_1}{d} M_C;$$

$$V_E = -V_A = \frac{W a + \mathfrak{S}_l}{l}; \qquad H_E = \frac{X}{a} \qquad H_A = -(W - H_E).$$

Case 94/6: Left-hand leg loaded by any type of horizontal load





Constant: $$X = \frac{\mathfrak{S}_l\,(B + C) + \mathfrak{R}\,k}{2N}.$$

$$M_B = (1-\gamma)\,\mathfrak{S}_l - X \qquad M_D = -X + \gamma\,\mathfrak{S}_l$$

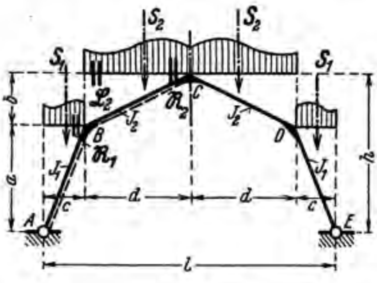
$$M_C = -m X + \frac{\mathfrak{S}_l}{2}; \qquad M_{y1} = M_y^0 + \frac{y_1}{a} M_B;$$

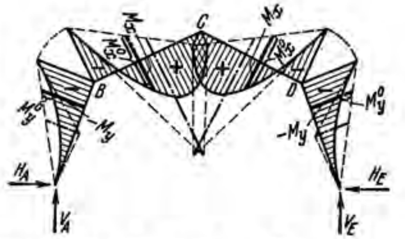
$$V_E = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_E = \frac{X}{a} \qquad H_A = -(W - H_E).$$

FRAME 94

See Appendix A, Load Terms, pp. 440-445.

Case 94/7: Entire frame loaded by any type of symmetrical vertical load





Constant: $$X = \frac{(\mathfrak{S}_{l1} + S_2 c)(B + C) + \mathfrak{S}_{l2} C + \mathfrak{R}_1 k + \mathfrak{L}_2 + m\,\mathfrak{R}_2}{N}.$$

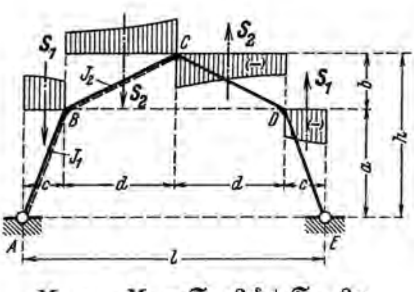
$$M_B = M_D = \mathfrak{S}_{l1} + S_2 c - X \qquad M_C = \mathfrak{S}_{l1} + S_2 c + \mathfrak{S}_{l2} - m X;$$

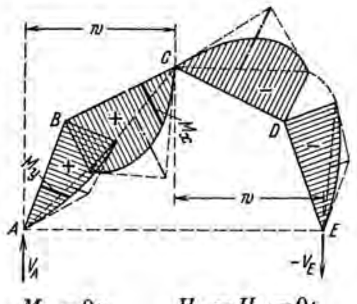
$$V_A = V_E = S_1 + S_2; \qquad H_A = H_E = \frac{X}{a};$$

$$M_y = M_y^0 + \frac{y}{a} M_B \qquad M_x = M_x^0 + \frac{x'}{d} M_B + \frac{x}{d} M_C.$$

Note: All the load terms refer to the left half of the frame.

Case 94/8: Entire frame loaded by any type of antisymmetrical vertical load





$$M_B = -M_D = \mathfrak{S}_{l1} \cdot 2\delta + \mathfrak{S}_{r2} \cdot 2\gamma \qquad M_C = 0; \qquad H_A = H_E = 0;$$

$$M_y = M_y^0 + \frac{y}{a} M_B \qquad M_x = M_x^0 + \frac{x'}{d} M_B; \qquad V_A = -V_E = \frac{\mathfrak{S}_{r1} + S_1 d + \mathfrak{S}_{r2}}{w}.$$

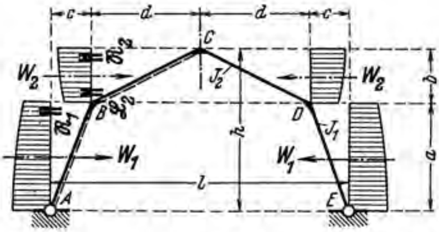
Note: All the load terms refer to the left half of the frame.

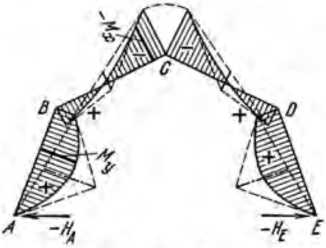
Special case 94/8a: Vertical couple P at the corners B and D

All load terms vanish except $S_1 = P$ and $\mathfrak{S}_{l1} = Pc$.

See Appendix A, Load Terms, pp. 440-445.

Case 94/9: Entire frame loaded by any type of symmetrical horizontal load from the outside*





Constant: $$X = \frac{(\mathfrak{S}_{l1} + W_2 a)(B + C) + \mathfrak{S}_{l2} C + \mathfrak{R}_1 k + \mathfrak{L}_2 + m\,\mathfrak{R}_2}{N}.$$

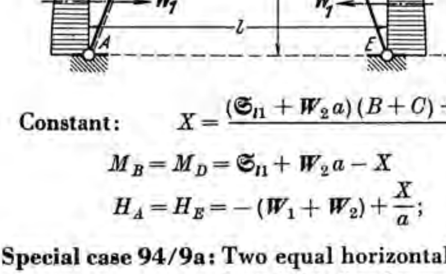
$$M_B = M_D = \mathfrak{S}_{l1} + W_2 a - X \qquad M_C = \mathfrak{S}_{l1} + W_2 a + \mathfrak{S}_{l2} - m X;$$

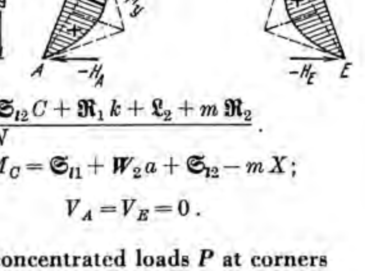
$$H_A = H_E = -(W_1 + W_2) + \frac{X}{a}; \qquad V_A = V_E = 0.$$

Special case 94/9a: Two equal horizontal concentrated loads P at corners B and D acting from outside

All load terms vanish except $W_1 = P$; $\mathfrak{S}_{l1} = Pa$.

Case 94/10: Entire frame loaded by any type of antisymmetrical horizontal load from the left*





$$M_B = -M_D = (\mathfrak{S}_{l1} + W_2 a)\cdot 2\delta - \mathfrak{S}_{l2}\cdot 2\gamma \qquad M_C = 0;$$

$$H_E = -H_A = W_1 + W_2; \qquad V_E = -V_A = \frac{\mathfrak{S}_{l1} + W_2 a + \mathfrak{S}_{l2}}{w}.$$

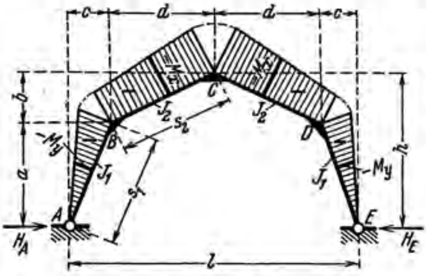
Special case 94/10a: Two equal horizontal concentrated loads P at corners B and D from the left

All load terms vanish except $W_1 = P$ and $\mathfrak{S}_{l1} = Pa$.

* All load terms refer to the left half of the frame. Formulas for M_x and M_y same as case 94/7 and 94/8.

FRAME 94

Case 94/11: Uniform increase in temperature of the entire frame



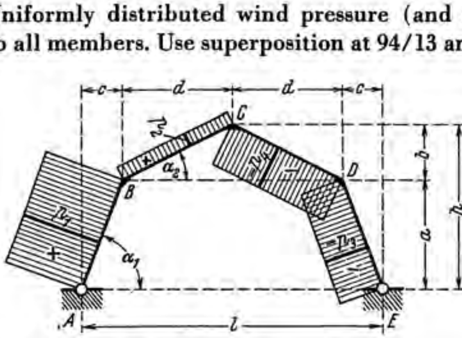
E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $T = \frac{3 E J_2' \varepsilon t l}{s_2 a N}$.*

$$M_B = M_D = -T \qquad M_C = -m\,T \qquad H_A = H_E = \frac{T}{a}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 94/12: Uniformly distributed wind pressure (and suction) normal to all members. Use superposition at 94/13 and 94/15.



Note: p_2 becomes negative for flat roofs.

Moments and shearing forces at any point of the left half of the frame in cases 94/13 and 14, p. 347.

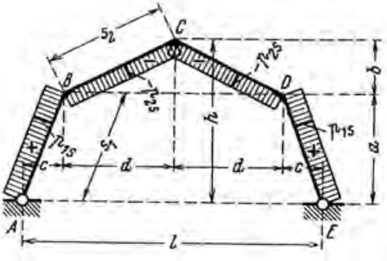
$$M_{z1} = \frac{p_{1s} \cdot z_1 z_1'}{2} + \frac{z_1}{s_1} \cdot M_B \qquad M_{z2} = \frac{p_{2s} \cdot z_2 z_2'}{2} + \frac{z_2'}{s_2} \cdot M_B + \frac{z_2}{s_2} \cdot M_C\,;$$

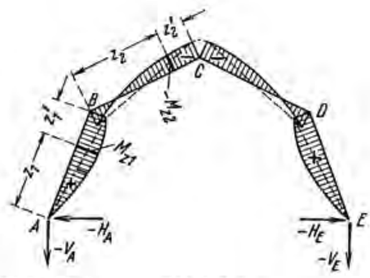
$$Q_{z1} = p_{1s} s_1 \left(\frac{1}{2} - \frac{z_1}{s_1}\right) + \frac{M_B}{s_1} \qquad Q_{z2} = p_{2s} s_2 \left(\frac{1}{2} - \frac{z_2}{s_2}\right) + \frac{M_C - M_B}{s_2}.$$

Note: In case 94/14 substitute $p_{1_a} = p_{2_a} = M_C = 0$.

* The constant T may be split as follows: $T = \frac{3 E J_2 \varepsilon (2 c \cdot t_1 + 2 d \cdot t_2)}{s_2 a N}$, where t_1 pertains to the members s_1, and t_2 to the members s_2. If only one half of the frame (or one diagonal alone) suffers a temperature change, the value of T is halved.

Case 94/13: Entire frame loaded by external pressure normal to all members. (Symmetrical load)





Referring to 94/12:
$$p_{1s} = \frac{p_1 + p_3}{2} \quad p_{2s} = \frac{p_2 + p_4}{2}.$$

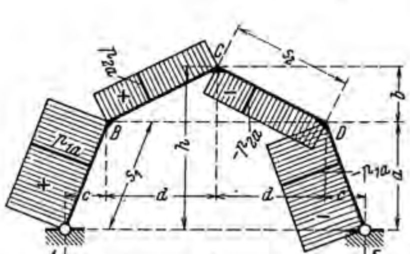
$$M_B = M_D = \frac{p_{1s}\, s_1^2 (2\varphi C - k)}{4N} + \frac{p_{2s}[(a\,b + c\,d)\cdot 4\varphi C - s_2^2(3 + 5m)]}{4N}$$

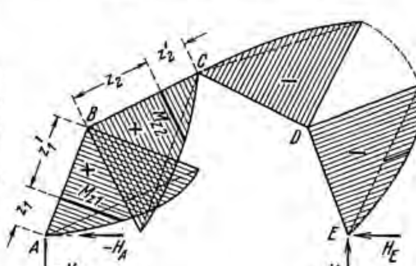
$$M_C = -\frac{p_{1s}\, s_1^2 \cdot \varphi}{2} + p_{2s}\left[\frac{s_2^2}{2} - (a\,b + c\,d)\,\varphi\right] + m\,M_B\,;$$

$$V_A = V_E = p_{1s}\,c + p_{2s}\,d\,; \qquad H_A = H_E = -\frac{p_{1s}(a^2 - c^2)}{2a} + \frac{p_{2s}\,c\,d}{a} - \frac{M_B}{a}.$$

Formulas for M_z and Q_z see p. 346 bottom.

Case 94/14: Entire frame loaded from the left by pressure normal to all members. (Antisymmetrical load—pressure and suction)





Referring to case 94/12:
$$p_{1a} = \frac{p_1 - p_3}{2} \quad p_{2a} = \frac{p_2 - p_4}{2}.$$

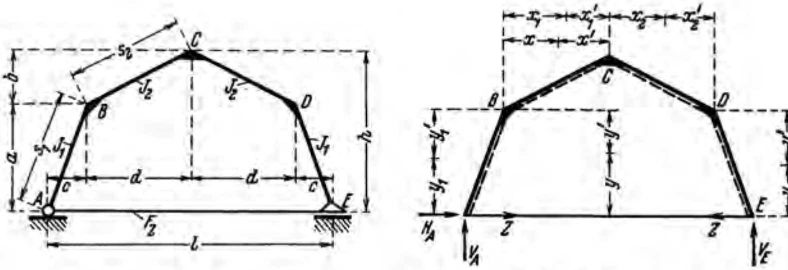
$$M_B = -M_D = p_{1a}\, s_1^2 \cdot \delta + p_{2a}[2\,\delta \cdot a\,b + \gamma(d^2 - b^2)] \quad M_C = 0\,;$$

$$H_E = -H_A = p_{1a}\,a + p_{2a}\,b\,; \quad V_E = -V_A = \frac{p_{1a}(s_1^2 - l\,c)}{w} + \frac{p_{2a}(2\,h\,b - s_2^2)}{w}.$$

Formulas for M_z and Q_z see p. 346 bottom.

Frame 95

Symmetrical gable frame with inclined legs and horizontal tie-rod. Externally simply supported.



Shape of Frame
Dimensions and Notations

This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: same as frame 94, p. 340.

Additional coefficients:

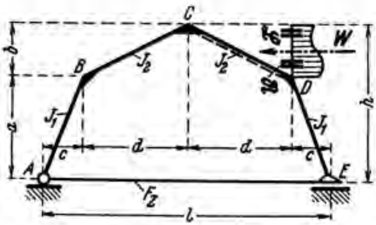
$$L = \frac{3\,J_2}{a^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{l}{s_2} \qquad N_Z = N + L\,.$$

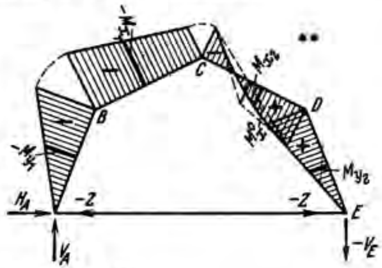
E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Cases 94/1, 3, 4, 5, 6, 7, and 11 may be used for frame 95 if N is replaced by N_Z. Use $H_A = H_E = Z$ for cases 94/5 and 6. The other cases of frame 94 cannot be directly transposed to frame 95. However, by the use of the following cases 95/1 and 95/2 all loading conditions can be obtained by superposition.

See Appendix A, Load Terms, pp. 440-445.

Case 95/1: Right girder loaded by any type of horizontal load





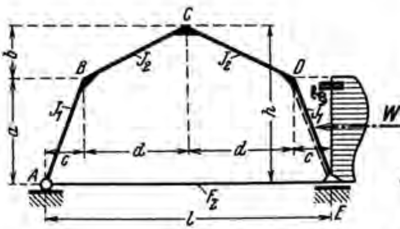
Constant:
$$X = \frac{W a N + \mathfrak{S}_l C - m \mathfrak{L} - \mathfrak{R}}{2 N_Z}.$$

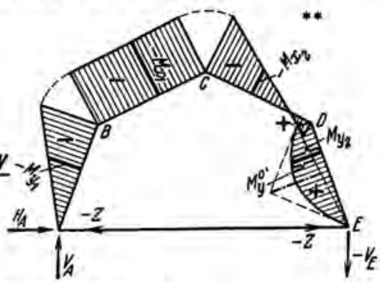
$$M_B = -W a (1-\gamma) + \gamma \mathfrak{S}_r + X \qquad M_D = -\gamma (W a + \mathfrak{S}_r) + X$$

$$M_C = -\frac{W h + \mathfrak{S}_l}{2} + m X; \qquad M_{x2} = M_x^0 + \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D;$$

$$Z = -\frac{X}{a}^* ; \qquad H_A = W; \qquad V_A = -V_E = \frac{W a + \mathfrak{S}_r}{l}.$$

Case 95/2: Right-hand leg loaded by any type of horizontal load





Constant:
$$X = \frac{W a (N + m C) + \mathfrak{S}_l B - \mathfrak{S}_r C - \mathfrak{L} k}{2 N_Z}.$$

$$M_B = -W a + \gamma \mathfrak{S}_r + X \qquad M_D = -\mathfrak{S}_l - \gamma \mathfrak{S}_r + X$$

$$M_C = -W h + \frac{\mathfrak{S}_r}{2} + m X; \qquad M_{y2} = M_y^0 + \frac{y_2}{a} M_D;$$

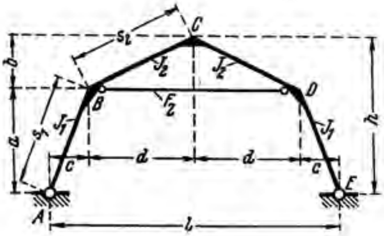
$$Z = -\frac{X}{a}^* ; \qquad H_A = W; \qquad V_A = -V_E = \frac{\mathfrak{S}_r}{l}.$$

*For the case of the above loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

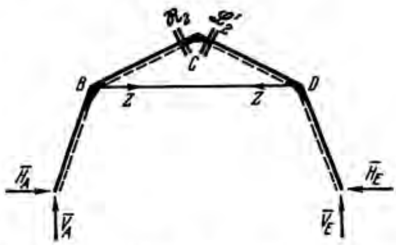
**See p. 340 for M_x and M_y for members that do not carry any exterior load.

Frame 96

Symmetrical two-hinged gable frame with inclined legs and horizontal tie-rod at bottom of gable.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point exactly as frame 94 (see p. 340). Positive bending moments cause tension at the face marked by a dashed line.

General notes

In order to compute Frame 96 (with tie rod) we can start by using Frame 94 (the same frame without tie rod). The effect of the tie is easily shown as follows:

Steps in computing the stresses

First step: Figure the moments at the joints M_B, M_C, M_D and the reactions H_A, H_E, V_A, V_E by using the formulas for Frame 94 (pp. 340-347).

Second step:

a) Figure the additional coefficients for Frame 96.

$$\beta_1 = \frac{B}{N} \qquad \gamma_1 = \frac{C}{N}; \qquad L = \frac{6 J_2}{b^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{d}{s_2}; \qquad N_Z = \frac{4k+3}{N} + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

Note: For a rigid tie set $L = 0$.

b) Figure the tension in the tie rod.

$$Z = \frac{M_B + M_D + 4\,M_C + \mathfrak{R}_2 + \mathfrak{L}_2'}{2\,b\,N_Z}\,*$$

Note: The load terms $\mathfrak{R}_2$ and $\mathfrak{L}_2'$ used in this formula are shown in the right-han sketch on p. 350 and are to be used accordingly.**

Third step:

a) Moments at the joints and reactions for Frame 96.

$$\overline{M}_B = M_B + \gamma_1 Z b \qquad \overline{M}_C = M_C - \beta_1 Z b \qquad \overline{M}_D = M_D + \gamma_1 Z b$$
$$\overline{H}_A = H_A - \varphi\gamma_1 Z \qquad \overline{H}_E = H_E - \varphi\gamma_1 Z \qquad \overline{V}_A = V_A \quad \overline{V}_E = V_E.$$

Note: In order to distinguish the moments and reactions for Frame 96 from those o Frame 94, the values for Frame 96 are shown with a dash over the letter.

b) Moments at any point of Frame 96.

The formulas for $\overline{M}_x$ and $\overline{M}_y$ are the same as for Frame 94 except that the values $\overline{M}_B$, $\overline{M}_C$, $\overline{M}_D$ are to be used instead of M_B, M_C, M_D

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* For the case of various loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. Thi only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile fo remains in the tie rod.

** For use of the loading conditions of frame 94 substitute the following in the Z formula for the load te $\mathfrak{R}_2$ and $\mathfrak{L}_2'$.

Case 94/4: $\mathfrak{R}_2 = \mathfrak{R}$; $\mathfrak{L}_2' = 0$; Case 94/5: $\mathfrak{R}_2 = \mathfrak{R}$; $\mathfrak{L}_2' = 0$;

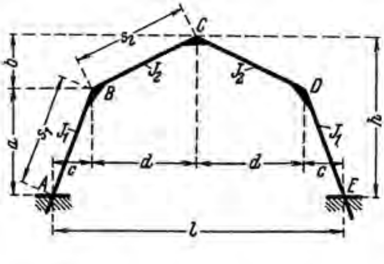
Case 94/7: $\mathfrak{R}_2 + \mathfrak{L}_2' = 2\,\mathfrak{R}_2$; Case 94/9: $\mathfrak{R}_2 + \mathfrak{L}_2' = 2\,\mathfrak{R}_2$;

Case 94/11: $\mathfrak{R}_2 + \mathfrak{L}_2' = \frac{12\,E\,J_2\,d \cdot \varepsilon\,t}{s_2\,b}$; Case 94/13: $\mathfrak{R}_2 + \mathfrak{L}_2' = \frac{p_{2s} \cdot s_2^2}{2}$.

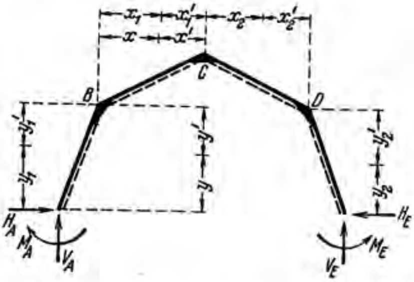
For all remaining load conditions, including the case of uniform temperature change in the entire fr including tie rod, set $\mathfrak{R}_2 = \mathfrak{L}_2' = 0$. All antisymmetrical loading conditions of frame 94 (cases 94/2, 8, 10, and apply to frame 96, since $Z = 0$.

Frame 97

Symmetrical hingeless gable frame with inclined legs.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and coordinates assigned to any point. For symmetrical types of loading use x, x' and y, y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{s_1}{s_2}\,; \qquad \varphi = \frac{b}{a} \qquad m = \frac{h}{a} = 1 + \varphi\,; \qquad \gamma = \frac{2c}{l} \qquad \delta = \frac{2d}{l}\,;$$

$$B = k + 2\delta(k+1) \qquad C = 1 + 2m$$

$$K_1 = 2(k + 1 + m + m^2) \qquad K_2 = 2(k + \varphi^2) \qquad R = \varphi C - k\,;$$

$$N_1 = K_1 K_2 - R^2 \qquad N_2 = k(2+\delta) + \delta B\,.$$

Formulas for the moments at any point of Frame 97 for any load

The moments at the joints and the fixed end moments contribute to the total moment*:

$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{y2} = \frac{y_2}{a} M_D + \frac{y_2'}{a} M_E$$

$$M_{x1} = \frac{x_1'}{d} M_B + \frac{x_1}{d} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D\,.$$

For the members that carry the load, add the value of $\boldsymbol{M_y^0}$ or $\boldsymbol{M_x^0}$ respectively.

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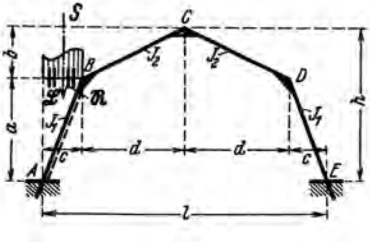
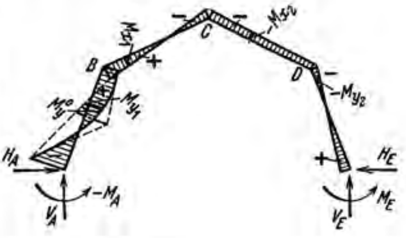
* Instead of the following forms with y and x, the forms with z may be used. See cases 97/13 and 14 (pp. 358-359).

See Appendix A, Load Terms, pp. 440-445.

Case 97/1: Left-hand leg loaded by any type of vertical load





Constants:

$$\mathfrak{B}_1 = 2\varphi^2 \mathfrak{S}_l + \mathfrak{L} k \qquad \mathfrak{B}_2 = \varphi \mathfrak{S}_l C - \mathfrak{R} k \qquad \mathfrak{B}_3 = \delta \mathfrak{S}_l B + (\mathfrak{L} + \delta \mathfrak{R}) k;$$

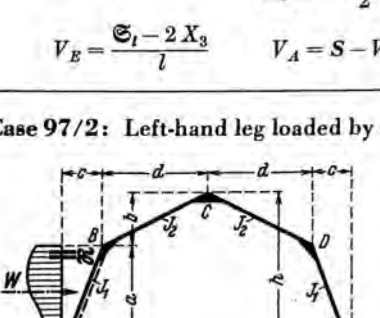
$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2 N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2 N_1} \qquad X_3 = \frac{\mathfrak{B}_3}{2 N_2}.$$

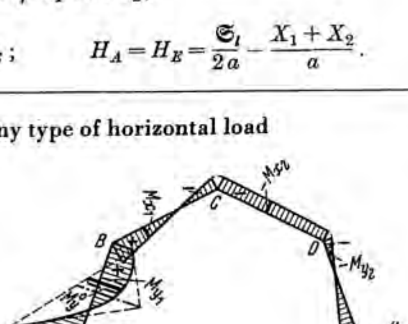
$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = +X_2 \pm \delta\left(\frac{\mathfrak{S}_l}{2} - X_3\right)$$

$$M_C = -\frac{\varphi \mathfrak{S}_l}{2} + \varphi X_1 + m X_2;$$

$$V_E = \frac{\mathfrak{S}_l - 2 X_3}{l} \qquad V_A = S - V_E; \qquad H_A = H_E = \frac{\mathfrak{S}_l}{2a} - \frac{X_1 + X_2}{a}.$$

Case 97/2: Left-hand leg loaded by any type of horizontal load





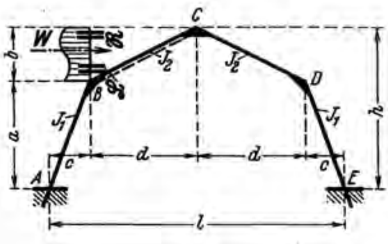
All the formulas are the same as above, except those for V- and H-forces:

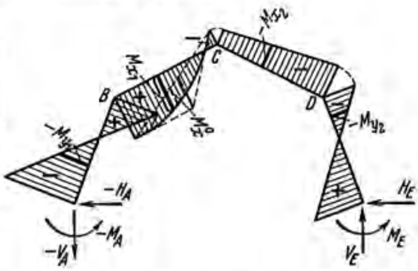
$$V_E = -V_A = \frac{\mathfrak{S}_l - 2 X_3}{l}; \qquad H_E = \frac{\mathfrak{S}_l}{2a} - \frac{X_1 + X_2}{a} \qquad H_A = -(W - H_E).$$

FRAME 97

Case 97/3: Left girder loaded by any type of horizontal load

(See Appendix A, Load Terms, pp. 440-445.)





Constants:

$$\mathfrak{B}_1 = \varphi\,(2\,\mathfrak{S}_r - \mathfrak{M}) \qquad \mathfrak{B}_2 = C\,\mathfrak{S}_r - (\mathfrak{L} + m\,\mathfrak{R}) \qquad \mathfrak{B}_3 = (\delta\,W a - \gamma\,\mathfrak{S}_l)\,B + \delta\,\mathfrak{L};$$

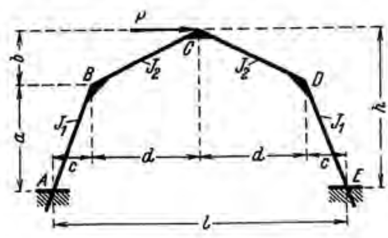
$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2\,N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2\,N_1} \qquad X_3 = \frac{\mathfrak{B}_3}{2\,N_2}.$$

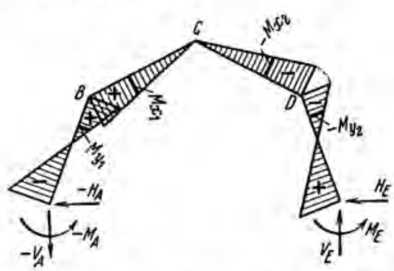
$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = +X_2 \pm \left(\frac{\delta\,W a - \gamma\,\mathfrak{S}_l}{2} - \delta\,X_3\right)$$

$$M_C = -\frac{\mathfrak{S}_r}{2} + \varphi X_1 + m X_2; \qquad V_E = -V_A = \frac{W a + \mathfrak{S}_l - 2\,X_3}{l};$$

$$H_E = \frac{W}{2} - \frac{X_1 + X_2}{a} \qquad H_A = -(W - H_E).$$

Case 97/4: Horizontal concentrated load at ridge C



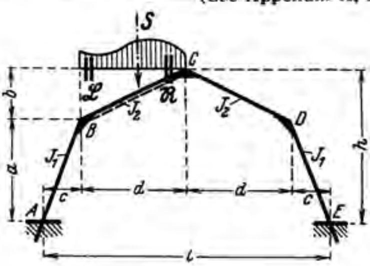


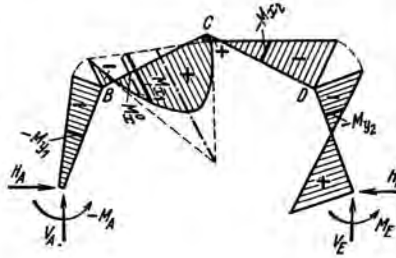
$$M_A = -M_E = -\frac{P\,(a - \gamma\,h)}{2\,N_2} \cdot B \qquad M_B = -M_D = +\frac{P\,(a - \gamma\,h)}{2\,N_2} \cdot (2 + \delta)\,k$$

$$M_C = 0; \qquad V_E = -V_A = \frac{P h - 2\,M_E}{l}; \qquad H_E = -H_A = \frac{P}{2}.$$

Case 97/5: Left girder loaded by any type of vertical load

(See Appendix A, Load Terms, pp. 440-445.)





Constants:

$$\mathfrak{B}_1 = \varphi\,[2(\mathfrak{S}_l - \varphi S c) + \mathfrak{R}] \qquad \mathfrak{B}_2 = C(\mathfrak{S}_l - \varphi S c) + \mathfrak{L} + m\,\mathfrak{R};$$

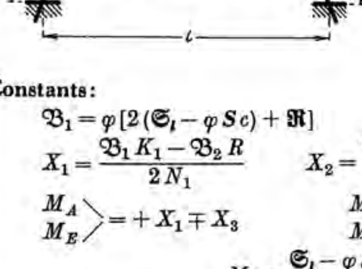
$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2 N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2 N_1} \qquad X_3 = \frac{\gamma\,\mathfrak{S}_r B + \delta\,\mathfrak{L}}{2 N_2}.$$

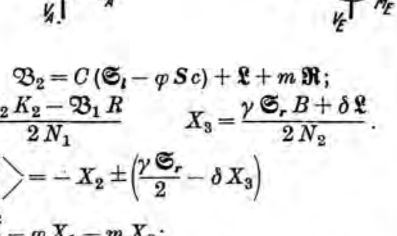
$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = + X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = - X_2 \pm \left(\frac{\gamma\,\mathfrak{S}_r}{2} - \delta X_3\right)$$

$$M_C = \frac{\mathfrak{S}_l - \varphi S c}{2} - \varphi X_1 - m X_2;$$

$$V_E = \frac{S c + \mathfrak{S}_l - 2 X_3}{l} \qquad V_A = S - V_E; \qquad H_A = H_E = \frac{S c}{2a} + \frac{X_1 + X_2}{a}.$$

Case 97/6: Vertical concentrated loads at *B*, *C*, *D*, acting symmetrically about the center line of the frame





Constant:

$$\mathfrak{B} = -P_1 \varphi c + \frac{P_2}{2}(d - \varphi c).$$

$$M_A = M_E = \frac{3\,\mathfrak{B}\,(k + 2 k \varphi + \varphi)}{N_1} \qquad M_B = M_D = -\frac{6\,\mathfrak{B}\,k m}{N_1}$$

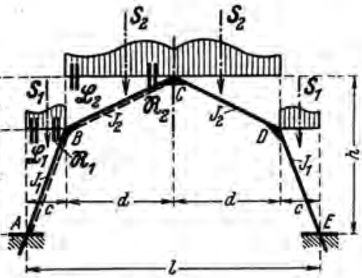
$$M_C = \mathfrak{B} - \varphi M_A + m M_B;$$

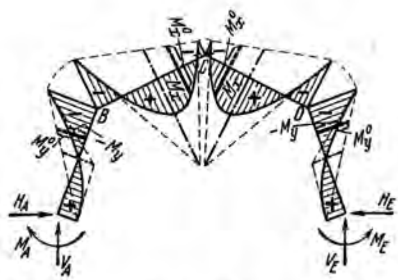
$$V_A = V_E = P_1 + \frac{P_2}{2}; \qquad H_A = H_E = \frac{V_A c + M_A - M_B}{a}$$

FRAME 97

See Appendix A, Load Terms, pp. 440-445.

Case 97/7: Entire frame loaded by any type of symmetrical vertical load





Constants:

$$\mathfrak{B}_1 = -[2\varphi^2 \mathfrak{S}_{l1} + \mathfrak{L}_1 k] + \varphi[2(\mathfrak{S}_{l2} - \varphi S_2 c) + \mathfrak{R}_2]$$

$$\mathfrak{B}_2 = [\varphi \mathfrak{S}_{l1} C - \mathfrak{R}_1 k] - [C(\mathfrak{S}_{l2} - \varphi S_2 c) + \mathfrak{L}_2 + m\,\mathfrak{R}_2].$$

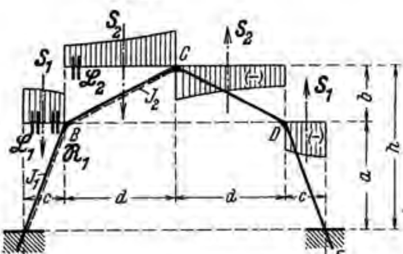
$$M_A = M_E = \frac{\mathfrak{B}_1 K_1 + \mathfrak{B}_2 R}{N_1} \qquad M_B = M_D = \frac{\mathfrak{B}_2 K_2 + \mathfrak{B}_2 R}{N_1}$$

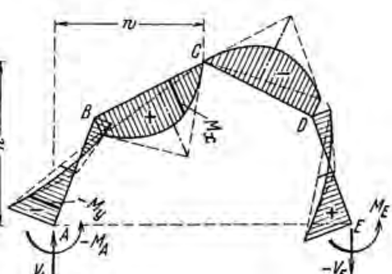
$$M_C = -\varphi \mathfrak{S}_{l1} + (\mathfrak{S}_{l2} - \varphi S_2 c) - \varphi M_A + m M_B;$$

$$V_A = V_E = S_1 + S_2; \qquad H_A = H_E = \frac{\mathfrak{S}_{l1} + S_2 c + M_A - M_B}{a}.$$

Note: All the load terms refer to the left half of the frame.

Case 97/8: Entire frame loaded by any type of antisymmetrical vertical load





$$M_E = -M_A = \frac{(\delta \mathfrak{S}_{l1} + \gamma \mathfrak{S}_{r2}) B + (\mathfrak{L}_1 + \delta \mathfrak{R}_1) k + \delta \mathfrak{L}_2}{N_2}$$

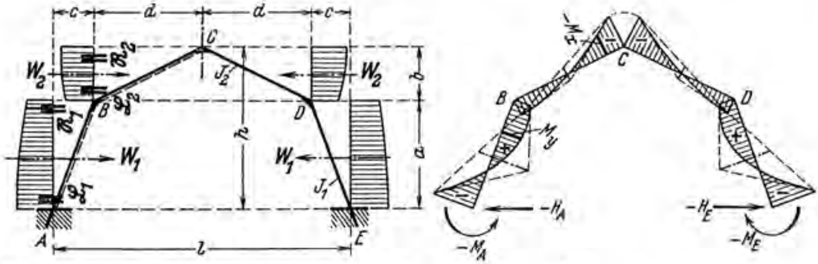
$$M_B = -M_D = \delta \mathfrak{S}_{l1} + \gamma \mathfrak{S}_{r2} - \delta M_E \qquad M_C = 0;$$

$$V_A = -V_E = \frac{\mathfrak{S}_{r1} + S_1 d + \mathfrak{S}_{r2} + M_E}{w} \qquad H_A = H_E = 0.$$

Note: All the load terms refer to the left half of the frame.

(See Appendix A, Load Terms, pp. 440-445.)

Case 97/9: Entire frame loaded by any type of symmetrical horizontal load



Constants:

$$\mathfrak{B}_1 = [2\varphi^2 \mathfrak{S}_{l1} + \mathfrak{L}_1 k] + \varphi[2\mathfrak{S}_{r2} - \mathfrak{R}_2]$$
$$\mathfrak{B}_2 = [\varphi C \mathfrak{S}_{l1} - \mathfrak{R}_1 k] + [C\mathfrak{S}_{r2} - (\mathfrak{L}_2 + m\,\mathfrak{R}_2)].$$

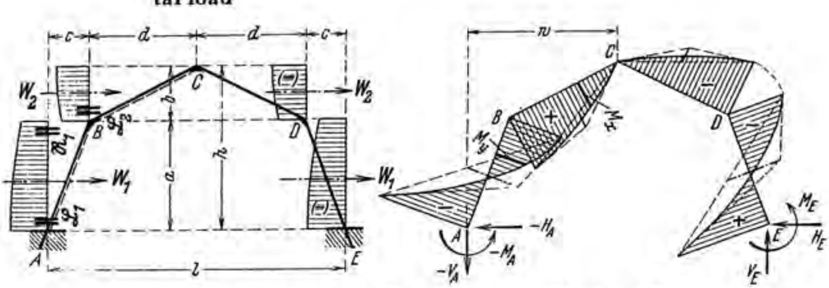
$$M_A = M_E = \frac{\mathfrak{B}_2 R - \mathfrak{B}_1 K_1}{N_1} \qquad M_B = M_D = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{N_1}$$

$$M_C = -\varphi\mathfrak{S}_{l1} - \mathfrak{S}_{r2} - \varphi M_A + m\,M_B;$$

$$H_A = H_E = -\frac{\mathfrak{S}_{r1}}{a} + \frac{M_A - M_B}{a} \qquad V_A = V_E = 0.$$

Note: All the load terms refer to the left half of the frame.

Case 97/10: Entire frame loaded by any type of antisymmetrical horizontal load



$$M_E = -M_A = \frac{(\delta\mathfrak{S}_{l1} + \delta W_2 a - \gamma\mathfrak{S}_{l2})\,B + (\mathfrak{L}_1 + \delta\mathfrak{R}_1)\,k + \delta\mathfrak{L}_2}{N_2}$$

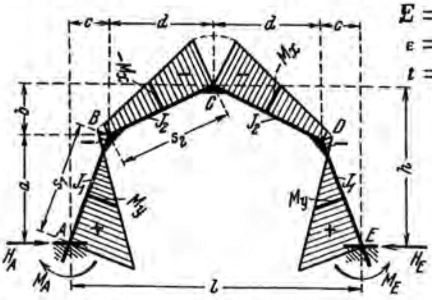
$$M_B = -M_D = \delta\mathfrak{S}_{l1} + \delta W_2 a - \gamma\mathfrak{S}_{l2} - \delta M_E \qquad M_C = 0;$$

$$V_E = -V_A = \frac{\mathfrak{S}_{l1} + W_2 a + \mathfrak{S}_{l2} - M_E}{w} \qquad H_E = -H_A = W_1 + W_2.$$

Note: All the load terms refer to the left half of the frame.

FRAME 97

Case 97/11: Uniform increase in temperature of the entire frame



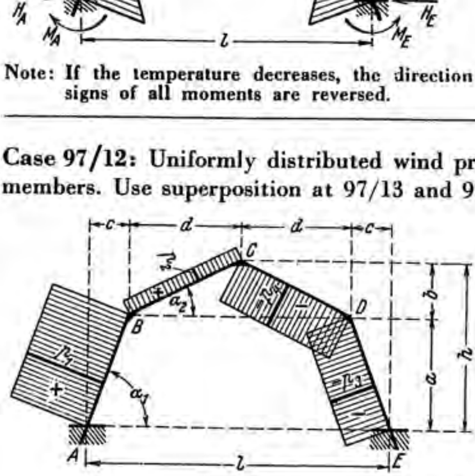
E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constant: $T = \dfrac{3 E J_2 \varepsilon t l}{s_2 a N_1}$.

$$M_A = M_E = + T(K_1 - R)$$
$$M_B = M_D = - T(K_2 - R)$$
$$M_C = -\varphi M_A + m M_B;$$
$$H_A = H_E = \frac{M_A - M_B}{a}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

Case 97/12: Uniformly distributed wind pressure (and suction) normal to all members. Use superposition at 97/13 and 97/14. Moments and shears for the left half of the frame for cases 97/13 and 97/14, p. 359.



Note: For a flat roof p_2 becomes negative.

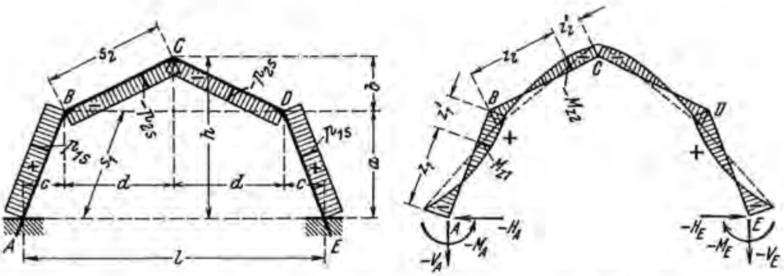
Moments and shearing forces at any point of the left half of the frame in cases 97/13 and 14, p. 359.

$$M_{z1} = \frac{p_{1s} \cdot z_1 z_1'}{2} + \frac{z_1'}{s_1} M_A + \frac{z_1}{s_1} M_B \qquad M_{z2} = \frac{p_{2s} \cdot z_2 z_2'}{2} + \frac{z_2'}{s_2} M_B + \frac{z_2}{s_2} M_C.$$

$$Q_{z1} = p_{1s} s_1 \left(\frac{1}{2} - \frac{z_1}{s_1}\right) + \frac{M_B - M_A}{s_1} \qquad Q_{z2} = p_{2s} s_2 \left(\frac{1}{2} - \frac{z_2}{s_2}\right) + \frac{M_C - M_B}{s_2}.$$

Note: In case 97/14 substitute $p_{1_a} = p_{2_a} = M_C = 0$.

Case 97/13: **Entire frame loaded by external pressure normal to all members. (Symmetrical load)***



Referring to case 97/12: $p_{1s} = \dfrac{p_1 + p_3}{2} \qquad p_{2s} = \dfrac{p_2 + p_4}{2}.$

Constants:

$$\mathfrak{B}_1 = p_{1s} s_1^2 (4\varphi^2 + k) + \varphi\, p_{2s} \left[8\varphi(ab + cd) - 5 s_2^2\right]$$

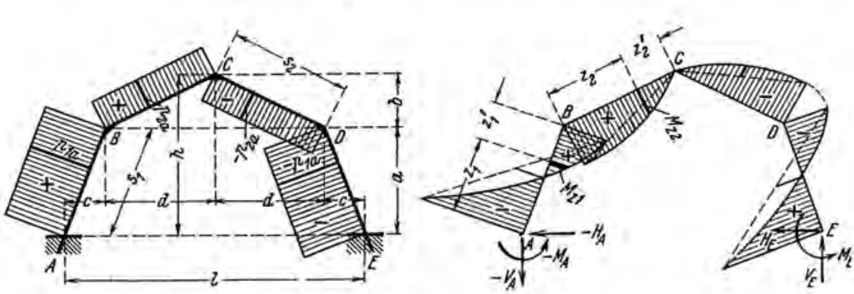
$$\mathfrak{B}_2 = p_{1s} s_1^2 (2\varphi C - k) + p_{2s}[4\varphi C(ab + cd) - s_2^2 (3 + 5m)].$$

$$M_A = M_E = \frac{-\mathfrak{B}_1 K_1 + \mathfrak{B}_2 R}{4 N_1} \qquad M_B = M_D = \frac{-\mathfrak{B}_1 R + \mathfrak{B}_2 K_2}{4 N_1}$$

$$M_C = -\frac{p_{1s} s_1^2}{2} \cdot \varphi + p_{2s} \left[\frac{s_2}{2} - \varphi(ab + cd)\right] - \varphi M_A + m M_B;$$

$$V_A = V_E = p_{1s} c + p_{2s} d; \qquad H_A = H_E = -\frac{p_{1s}(a^2 - c^2)}{2a} + \frac{p_{2s} cd}{a} + \frac{M_A - M_B}{a}.$$

Case 97/14: **Entire frame loaded from the left by pressure normal to all members. (Antisymmetrical load—pressure and suction)***

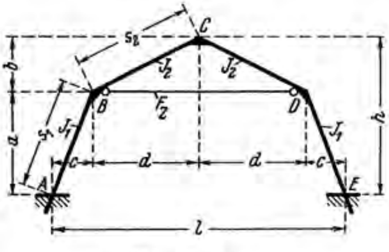


For formulas to case 97/14 see p. 360 bottom.

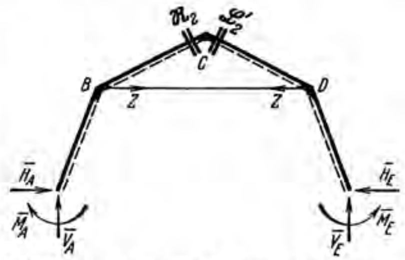
* Formulas for M_z and Q_z see p. 358.

Frame 98

Symmetrical hingeless gable frame with inclined legs and horizontal tie-rod at bottom of gable.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point exactly as frame 97 (see p. 352). Positive bending moments cause tension at the face marked by a dashed line.

General notes

In order to compute Frame 98 (with tie rod) we can start by using Frame 97 (the same frame without tie rod). The effect of the tie is easily shown as follows:

Steps in computing the stresses

First step: Figure the moments at the joints M_A, M_B, M_C, M_D, M_E and the reactions H_A, H_E, V_A, V_E by using the formulas for Frame 97 (pp. 352-359)

(Frame 98 continued on p. 361)

Frame 97 continued. Formulas to case 97/12, p. 359.

Referring to case 97/12: $p_{1a} = \frac{p_1 - p_3}{2}$ $p_{2a} = \frac{p_2 - p_4}{2}$.

$$M_E = -M_A = \frac{p_{1a} s_1^2}{4 N_2}[2\delta B + (1+\delta)k] + \frac{p_{2a}}{4 N_2}[\delta s_2^2 + 2\gamma B(d^2 - b^2) + 4\delta B \cdot a b]$$

$$M_B = -M_D = \frac{p_{1a} s_1^2}{2} \cdot \delta + \frac{p_{2a}}{2}[\gamma(d^2 - b^2) + 2\delta \cdot a b] - \delta M_E \qquad M_C = 0;$$

$$V_E = -V_A = \frac{p_{1a}(s_1^2 - l c)}{l} + \frac{p_{2a}(2 h b - s_2^2)}{l} - \frac{M_E}{w}; \quad H_E = -H_A = p_{1a} a + p_{2a} b.$$

Second step:

a) Figure the additional coefficients for Frame 98.

$$\alpha_1 = \frac{3(mk + \varphi k + \varphi)}{N_1} \qquad \beta_1 = \frac{6mk}{N_1} \qquad \gamma_1 = \frac{3k(k+1+m)}{N_1};$$

$$L = \frac{6J_2}{b^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{d}{s_2} \qquad N_Z = 2\gamma_1 - \beta_1 + L.$$

E_R = Modulus of elasticity of the material of the frame

E_Z = Modulus of elasticity of the tie rod

F_Z = Cross-sectional area of the tie rod

Note: For a rigid tie set $L = 0$.

b) Figure the tension in the tie rod.

$$Z = \frac{M_B + M_D + 4M_C + \mathfrak{R}_2 + \mathfrak{L}_2'}{2bN_Z}\,*$$

Note: The load terms $\mathfrak{R}_2$ and $\mathfrak{L}_2'$ used in this formula are shown in the right-hand sketch on p. 360 and are to be used accordingly.**

Third step:

a) Moments at the joints and reactions for Frame 98.

$$\overline{M}_B = M_B + \beta_1 Zb \qquad \overline{M}_C = M_C - \gamma_1 Zb \qquad \overline{M}_D = M_D + \beta_1 Zb$$

$$\overline{M}_A = M_A - \alpha_1 Zb \qquad \overline{M}_E = M_E - \alpha_1 Zb$$

$$\overline{H}_A = H_A - \varphi(\alpha_1 + \beta_1)Z \quad \overline{H}_E = H_E - \varphi(\alpha_1 + \beta_1)Z \quad \overline{V}_A = V_A \quad \overline{V}_E = V_E.$$

Note: In order to distinguish the moments and reactions for Frame 98 from those of Frame 97, the values for Frame 98 are shown with a dash over the letter.

b) Moments at any point of Frame 98.

The formulas for $\overline{M}_x$ and $\overline{M}_y$ are the same as for Frame 97 except that the values $\overline{M}_A$, $\overline{M}_B$, $\overline{M}_C$, $\overline{M}_D$, $\overline{M}_E$ are to be used instead of M_A, M_B, M_C, M_D, M_E.

---

* For the case of various loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

** For use of the loading conditions of frame 97 substitute the following in the Z formula for the load terms $\mathfrak{R}_2$ and $\mathfrak{L}_2'$

Case 97/3: $\mathfrak{R}_2 = \mathfrak{R};\ \mathfrak{L}_2' = 0;$ Case 97/5: $\mathfrak{R}_2 = \mathfrak{R};\ \mathfrak{L}_2' = 0;$

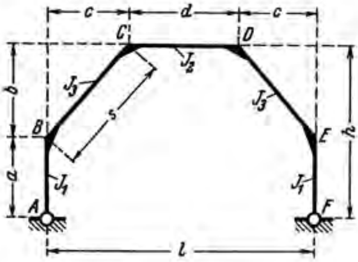
Case 97/7: $\mathfrak{R}_2 + \mathfrak{L}_2' = 2\mathfrak{R}_2;$ Case 97/9: $\mathfrak{R}_2 + \mathfrak{L}_2' = 2\mathfrak{R}_2;$

Case 97/11: $\mathfrak{R}_2 + \mathfrak{L}_2' = \dfrac{12EJ_2 d \cdot \varepsilon t}{s_2 b};$ Case 97/13: $\mathfrak{R}_2 + \mathfrak{L}_2' = \dfrac{p_{2s} \cdot s_2^2}{2}.$

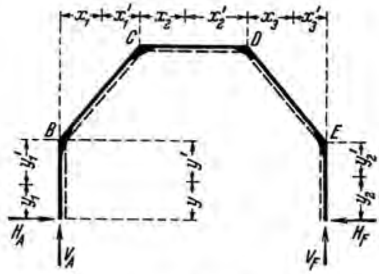
For all remaining load conditions, including the case of uniform temperature change in the entire frame including tie rod, set $\mathfrak{R}_2 = \mathfrak{L}_2' = 0$. All antisymmetrical loading conditions of frame 97 (cases 97/4, 8, 10, and 14) apply to frame 98, since $Z = 0$.

Frame 99

Symmetrical two-hinged bent with skew corners.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{a}{s} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{d}{s}; \qquad \alpha = \frac{a}{h} \qquad \gamma = \frac{c}{l} \qquad \delta = \frac{d}{l};$$

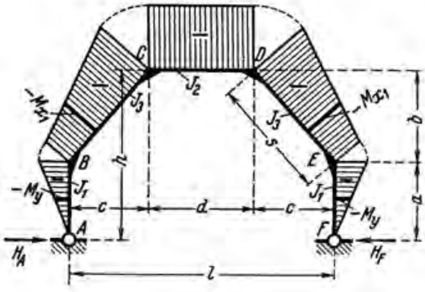
$$B = 2\alpha(k_1 + 1) + 1 \qquad C = \alpha + 2 + 3k_2; \qquad N = \alpha B + C.$$

Formulas for the moments at any point of those members of Frame 99 which do not carry any external load

$$M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D$$

$$M_{x3} = \frac{x_3'}{c} M_D + \frac{x_3}{c} M_E \qquad M_{y1} = \frac{y_1}{a} M_B \qquad M_{y2} = \frac{y_2}{a} M_E.$$

Case 99/1: Uniform increase in temperature of the entire frame



E = Modulus of elasticity
ε = Coefficient of thermal expansic
t = Change of temperature in deg

Constant: $T = \dfrac{3\,E J_3\, \varepsilon t l}{s h N}$.

$$M_B = M_E = -\alpha T \qquad M_C = M_D = -T; \qquad H_A = H_F = \frac{T}{h}.$$

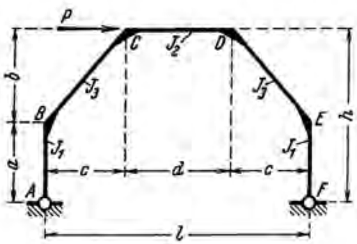
Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

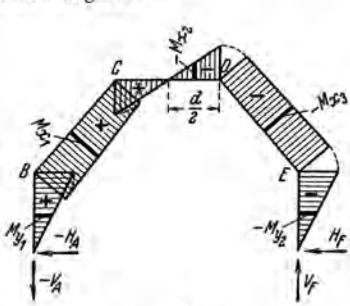
General case 99/1a: The value of T becomes equal to

$$T = \frac{3\,E J_3\, \varepsilon}{s h N}\,(c \cdot t_1 + d \cdot t_2 + c \cdot t_3),$$

where t_1, t_2 and t_3 denote the temperature increase in bars BC, CD, and DE, respectively. Temperature changes in the legs do not cause stresses in the frame.

Case 99/2: Horizontal concentrated load at the girder





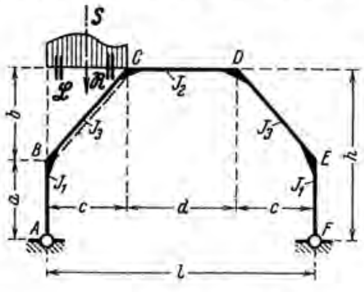
$$M_B = -M_E = \frac{P a}{2} \qquad M_C = -M_D = \frac{P h d}{2 l};$$

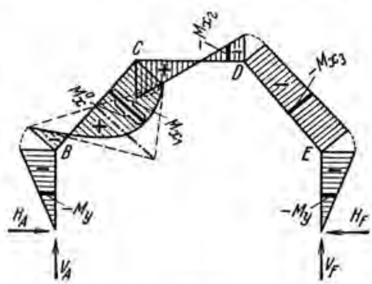
$$H_F = -H_A = \frac{P}{2} \qquad V_F = -V_A = \frac{P h}{l}.$$

FRAME 99

(See Appendix A, Load Terms, pp. 440-445.)

Case 99/3: Left-hand inclined member loaded by any type of vertical load





Constant: $X = \dfrac{C\mathfrak{S}_l + \alpha\mathfrak{L} + \mathfrak{R}}{2N}$. $\qquad M_{x1} = M_x^0 + \dfrac{x_1'}{c} M_B + \dfrac{x_1}{c} M_C$;

$M_B = M_E = -\alpha X \qquad M_C = (1-\gamma)\mathfrak{S}_l - X \qquad M_D = \gamma\mathfrak{S}_l - X$;

$V_F = \dfrac{\mathfrak{S}_l}{l} \qquad V_A = S - V_F$; $\qquad H_A = H_F = \dfrac{X}{h}$.

Special case 99/3a: Vertical concentrated load P at C

Substitute $\mathfrak{S}_l = Pc$, $\mathfrak{L} = \mathfrak{R} = 0$ and $M_x^0 = 0$.

Case 99/4: Girder loaded by any type of vertical load

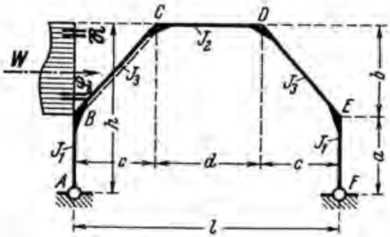
Constant: $X = \dfrac{ScC + (\mathfrak{L} + \mathfrak{R})k_2}{2N}$. $\qquad M_{x2} = M_x^0 + \dfrac{x_2'}{d} M_C + \dfrac{x_2}{d} M_D$;

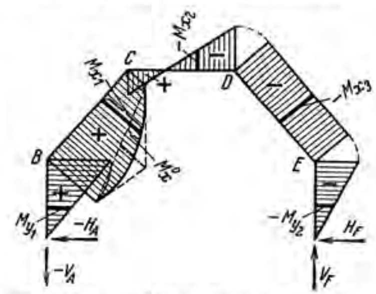
$M_B = M_E = -\alpha X \qquad M_C = \gamma(\mathfrak{S}_r + Sc) - X \qquad M_D = \gamma(Sc + \mathfrak{S}_l) - X$;

$V_A = \dfrac{\mathfrak{S}_r + Sc}{l} \qquad V_F = \dfrac{Sc + \mathfrak{S}_l}{l}$; $\qquad H_A = H_F = \dfrac{X}{h}$.

See Appendix A, Load Terms, pp. 440-445.

Case 99/5: Left-hand inclined member loaded by any type of horizontal load



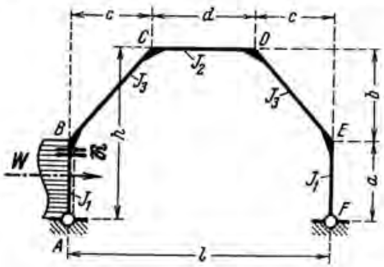


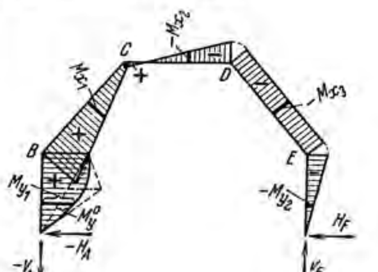
Constant: $$X = \frac{Wa(B+C) + \mathfrak{S}_l C + \alpha \mathfrak{L} + \mathfrak{R}}{2N}. \qquad M_B = Wa - \alpha X \qquad M_E = -\alpha X$$

$$M_C = (1-\gamma)(Wa + \mathfrak{S}_l) - X \qquad M_D = \gamma(Wa + \mathfrak{S}_l) - X; \qquad M_{x1} = M_x^0 + \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C;$$

$$V_F = -V_A = \frac{Wa + \mathfrak{S}_l}{l}; \qquad H_F = \frac{X}{h} \qquad H_A = -(W - H_F).$$

Case 99/6: Left-hand leg loaded by any type of horizontal load





Constant: $$X = \frac{\mathfrak{S}_l(B+C) + \alpha \mathfrak{R} k_1}{2N}.$$

$$M_B = \mathfrak{S}_l - \alpha X \qquad M_E = -\alpha X$$
$$M_C = (1-\gamma)\mathfrak{S}_l - X \qquad M_D = \gamma \mathfrak{S}_l - X; \qquad M_{y1} = M_y^0 + \frac{y_1}{a} M_B;$$

$$V_F = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_F = \frac{X}{h} \qquad H_A = -(W - H_F).$$

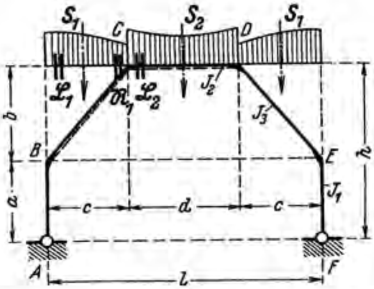
Special case 99/6a: Horizontal concentrated load P at B

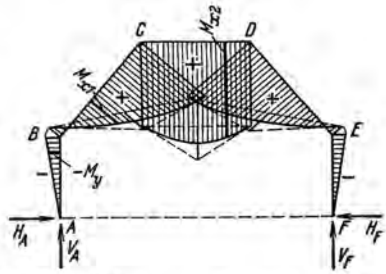
Substitute $\mathfrak{S}_l = Pa$ and $W = P$; with $\mathfrak{R} = 0$ and $M_y^0 = 0$.

FRAME 99

See Appendix A, Load Terms, pp. 440-445.

Case 99/7: Entire frame loaded by any type of symmetrical vertical load





Constant: $X = \dfrac{(\mathfrak{S}_{l1} + S_2 c/2)\,C + \alpha\,\mathfrak{L}_1 + \mathfrak{R}_1 + \mathfrak{L}_2 k_2}{N}$. $\qquad H_A = H_F = \dfrac{X}{h}$;

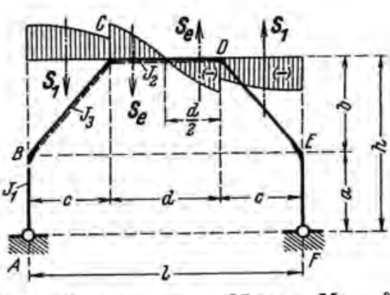
$$M_B = M_E = -\alpha X \qquad M_C = M_D = (\mathfrak{S}_{l1} + S_2 c/2) - X;$$

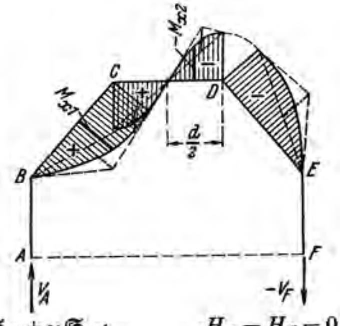
$$M_{x1} = M_x^0 + \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = M_x^0 + M_C; \qquad V_A = V_F = S_1 + \frac{S_2}{2}.$$

Note: All the load terms refer to the left half of the frame.

Special case 99/7a: Two equal horizontal concentrated loads P over C and D. Substitute $S_1 = P$ and $\mathfrak{S}_{l1} = Pc$, all other load terms are zero.

Case 99/8: Entire frame loaded by any type of antisymmetrical vertical load





$$M_B = M_E = 0 \qquad M_C = -M_D = \delta\,\mathfrak{S}_{l1} + \gamma\,\mathfrak{S}_{r2}; \qquad H_A = H_F = 0;$$

$$V_A = -V_F = \frac{2\,\mathfrak{S}_{r1} + S_1 d + \mathfrak{S}_{r2}}{l}; \quad M_{x1} = M_x^0 + \frac{x_1}{c} M_C \quad M_{x2} = M_x^0 + \frac{x_2' - x_2}{d} M_C.$$

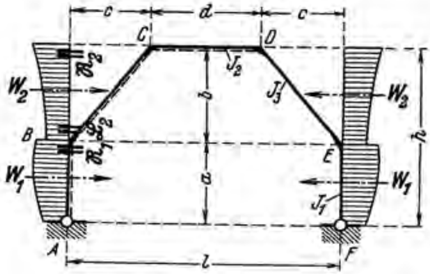
Note: All the load terms refer to the left half of the frame.

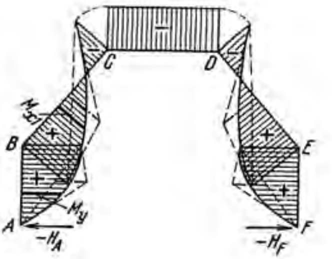
Special case 99/8a: Vertical couple P at the corners C and D

$$M_B = M_E = 0 \qquad M_C = -M_D = \delta\,Pc; \qquad V_A = -V_F = \delta\,P; \qquad M_x^0 = 0.$$

See Appendix A, Load Terms, pp. 440-445.

Case 99/9: **Entire frame loaded by any type of symmetrical external horizontal load***



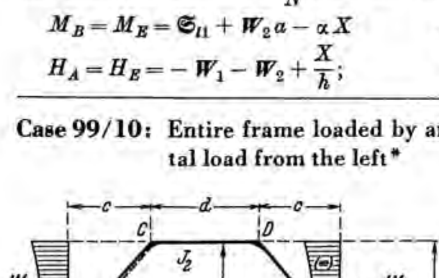


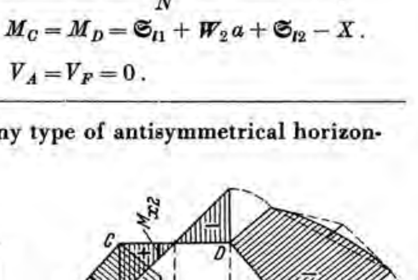
Constant: $X = \dfrac{\mathfrak{S}_{l1}(B+C) + \alpha\,\mathfrak{R}_1 k_1}{N} + \dfrac{W_2 a (B+C) + \mathfrak{S}_{l2} C + \alpha\,\mathfrak{L}_2 + \mathfrak{R}_2}{N}$.

$M_B = M_E = \mathfrak{S}_{l1} + W_2 a - \alpha X \qquad M_C = M_D = \mathfrak{S}_{l1} + W_2 a + \mathfrak{S}_{l2} - X$.

$H_A = H_E = -W_1 - W_2 + \dfrac{X}{h}; \qquad V_A = V_F = 0$.

Case 99/10: **Entire frame loaded by any type of antisymmetrical horizontal load from the left***





$M_B = -M_E = \mathfrak{S}_{l1} + W_2 a \qquad M_C = -M_D = \delta(\mathfrak{S}_{l1} + W_2 a + \mathfrak{S}_{l2})$;

$H_F = -H_A = W_1 + W_2 \qquad V_F = -V_A = \dfrac{2(\mathfrak{S}_{l1} + W_2 a + \mathfrak{S}_{l2})}{l}$.

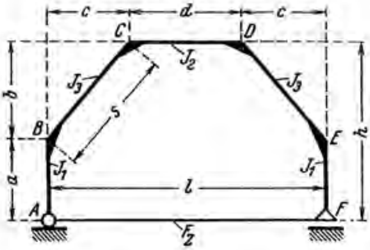
Special cases 99/9a and 99/10a: Two equal concentrated loads P acting from the left at B and E.

Substitute $W_1 = P$ and $\mathfrak{S}_{l1} = Pa$, all other load terms are zero.

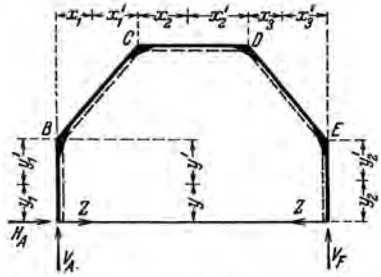
* All load terms refer to the left half of the frame. M_{y_1} and M_{x_1} are the same as 99/6 and 99/5 respectively.

Frame 100

Symmetrical tied bent with skew corners. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients: same as frame 99, p. 362.

Additional coefficients:

$$L = \frac{3J_3}{h^2 F_Z} \cdot \frac{E}{E_Z} \cdot \frac{l}{s} \qquad N_Z = N + L.$$

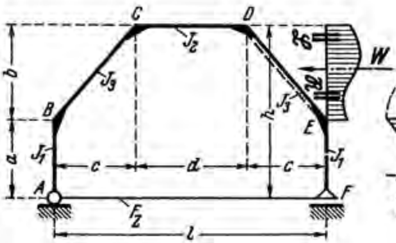
E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

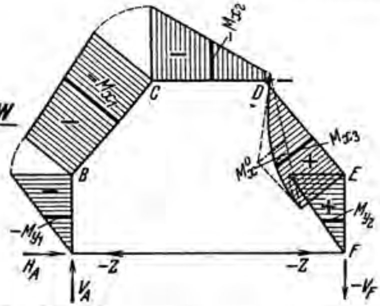
For frame 100 use the same formulas as for cases 99/1, 3, 4, 5, 6, and 7 (see pp. 363–366) and substitute $N_Z = N$. For cases 99/1, 3, 4, and 7 $(H_A = H_F) = Z$, and for cases 99/5 and 6, $H_F = Z$ and $H_A = -W$. For a single concentrated load at the girder (see case 99/2, p. 363) of frame 100 use the following values:

$$Z = \frac{P}{2} \cdot \frac{N}{N_Z}; \qquad M_B = (P - Z)\,a \qquad M_C = (1-\gamma)\,Ph - Zh$$
$$M_E = -Za \qquad M_D = \gamma\, Ph - Zh.$$

(See Appendix A, Load Terms, pp. 440-445.)

Case 100/1: Right-hand inclined member loaded by any type of horizontal load

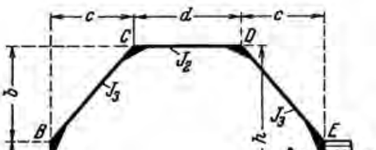


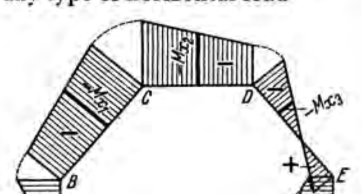


Constant: $X = \frac{W(a\,B + h\,C) + \mathfrak{S}_l C - \mathfrak{L} - \alpha\,\mathfrak{R}}{2\,N_Z}$. $\qquad Z = -\frac{X}{h}$* ;

$$M_B = -W a + \alpha X \qquad M_C = -W(h - a\gamma) + \gamma\,\mathfrak{S}_r + X$$
$$M_E = +\alpha X \qquad M_D = -W a\gamma - \mathfrak{S}_l - \gamma\,\mathfrak{S}_r + X;$$
$$M_{x3} = M_x^0 + \frac{x_3'}{c} M_D + \frac{x_3}{c} M_E; \qquad H_A = W; \qquad V_A = -V_F = \frac{W a + \mathfrak{S}_r}{l}.$$

Case 100/2: Right-hand leg loaded by any type of horizontal load





Constant: $X = \frac{(W a + \mathfrak{S}_l)\,B + (2\,W h - \mathfrak{S}_r)\,C - \alpha\,\mathfrak{L}\,k_1}{2\,N_Z}$.

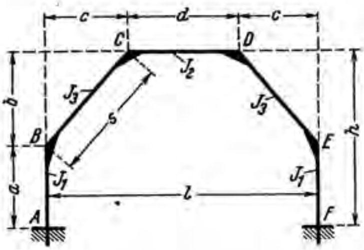
$$M_B = -W a + \alpha X \qquad M_C = -W h + \gamma\,\mathfrak{S}_r + X; \qquad Z = -\frac{X}{h}\text{*};$$
$$M_E = -\mathfrak{S}_l + \alpha X \qquad M_D = -W h + (1-\gamma)\,\mathfrak{S}_r + X;$$
$$M_{y2} = M_y^0 + \frac{y_2}{h} M_E; \qquad H_A = W; \qquad V_A = -V_F = \frac{\mathfrak{S}_r}{l}.$$

*For the case of the above loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

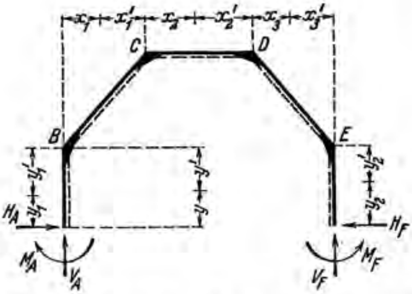
See p. 362 for M_x and M_y for members that do not carry a direct load.

Frame 101

Symmetrical hingeless bent with skew corners.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \cdot \frac{a}{s} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{d}{s}; \qquad \gamma = \frac{c}{l} \qquad \delta = \frac{d}{l};$$

$$\varphi = \frac{b}{a} \qquad m = \frac{h}{a} = 1 + \varphi; \qquad (2\gamma + \delta = 1);$$

$$C_1 = \varphi(2 + 3k_2) \qquad K_1 = 2(k_1 + 1) + m(1 + C_2)$$

$$C_2 = 1 + m(2 + 3k_2) \qquad K_2 = 2k_1 + \varphi C_1$$

$$R = \varphi C_2 - k_1; \qquad N_1 = K_1 K_2 - R^2;$$

$$B = 3k_1 + 2 + \delta \qquad C_3 = 1 + \delta(2 + k_2); \qquad N_2 = 3k_1 + B + \delta C_3.$$

Formulas for the moments at any point of Frame 101 for any load

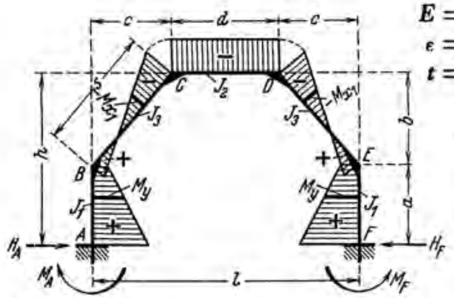
The moments at the joints and the fixed end moments contribute to the total moment:

$$M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D \qquad M_{x3} = \frac{x_3'}{c} M_D + \frac{x_3}{c} M_E$$

$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{y2} = \frac{y_2}{a} M_E + \frac{y_2'}{a} M_F.$$

For the members that carry the load, add the value of $\boldsymbol{M_x^0}$ or $\boldsymbol{M_y^0}$ respectively.

Case 101/1: Uniform increase in temperature of the entire frame (Symmetrical load)



E = Modulus of elasticity

ε = Coefficient of thermal expansion

t = Change of temperature in degrees

Constant: $T = \frac{3\,E\,J_3\,\varepsilon\,t\,l}{a\,s\,N_1}$.

$$M_A = M_F = T\,(K_1 - R)$$
$$M_B = M_E = T\,(R - K_2)$$
$$M_C = M_D = -\,\varphi\,M_A + m\,M_B\,;$$
$$H_A = H_F = \frac{M_A - M_B}{a}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

General case 101/1a: The value of T becomes equal to

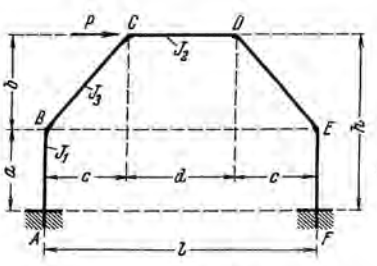
$$T = \frac{3\,E\,J_3\,\varepsilon}{a\,s\,N_1}\,(2\,c\cdot t_3 + d\cdot t_2)\,*$$

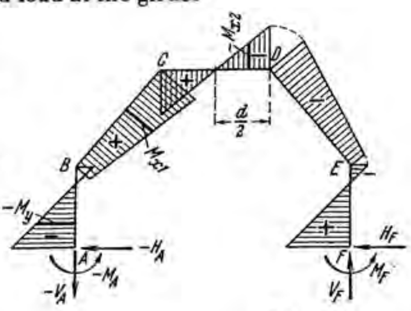
where t_3 refers to the diagonals s and t_2 to the girder d

Antisymmetrical change in temperature 101/1b: Left leg $+\,t_1$, right leg $-\,t_1$, left diagonal $+\,t_3$, right diagonal $-\,t_3$. **.

$$M_F = M_E = -\,M_B = -\,M_A = \frac{12\,E\,J_3\,\varepsilon}{s\,l\,N_2}\,(a\cdot t_1 + b\cdot t_3) \qquad M_D = -\,M_C = \delta\,M_F.$$

Case 101/2: Horizontal concentrated load at the girder





$$M_F = -\,M_A = \frac{P\,a}{2}\cdot\frac{B + \delta\,m\,C_3}{N_2} \qquad M_C = -\,M_D = \delta\left(\frac{P\,h}{2} - M_F\right)$$

$$M_B = -\,M_E = \frac{P\,a}{2} - M_F\,; \qquad V_F = -\,V_A = \frac{P\,h - 2\,M_F}{l}; \qquad H_F = -\,H_A = \frac{P}{2}.$$

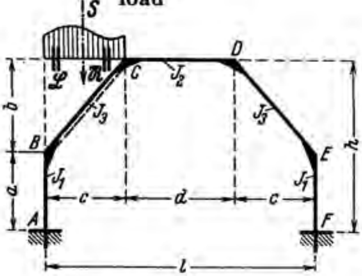
* Equal temperature changes in the vertical legs do not cause stress.

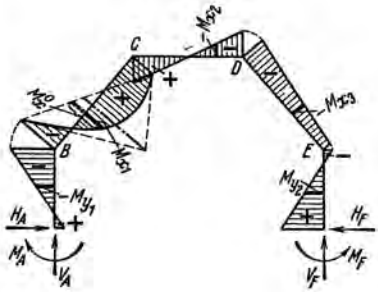
** Antisymmetrical temperature changes in the girder do not cause stress.

FRAME 101

See Appendix A, Load Terms, pp. 440-445.

Case 101/3: Left-hand inclined member loaded by any type of vertical load





Constants: $\mathfrak{B}_1 = C_1\mathfrak{S}_l + \varphi\,\mathfrak{R}$ $\qquad \mathfrak{B}_2 = C_2\mathfrak{S}_l + \mathfrak{L} + m\,\mathfrak{R}$

$$\mathfrak{B}_3 = \delta C_3\mathfrak{S}_l + \mathfrak{L} + \delta\,\mathfrak{R};$$

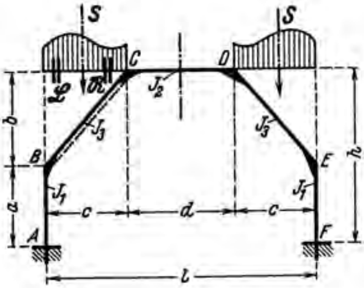
$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2N_1} \qquad X_3 = \frac{\mathfrak{B}_3}{2N_2}.$$

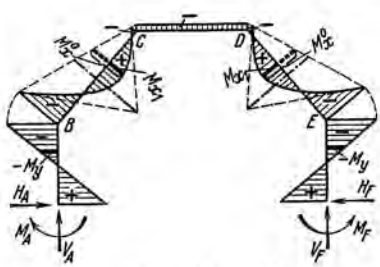
$$\left.\begin{matrix} M_A \\ M_F \end{matrix}\right\rangle = +X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_E \end{matrix}\right\rangle = -X_2 \mp X_3$$

$$\left.\begin{matrix} M_C \\ M_D \end{matrix}\right\rangle = +\frac{\mathfrak{S}_l}{2} - \varphi X_1 - m X_2 \pm \frac{\delta}{2}(\mathfrak{S}_l - 2X_3);$$

$$V_F = \frac{\mathfrak{S}_l - 2X_3}{l} \qquad V_A = S - V_F \qquad H_A = H_F = \frac{X_1 + X_2}{a}.$$

Case 101/4: Both inclined members loaded by any type of symmetrical vertical load





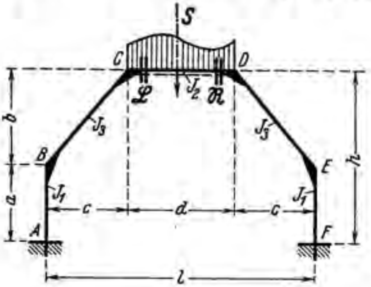
Constants: $\mathfrak{B}_1 = C_1\mathfrak{S}_l + \varphi\,\mathfrak{R}$ $\qquad \mathfrak{B}_2 = C_2\mathfrak{S}_l + \mathfrak{L} + m\,\mathfrak{R}.$

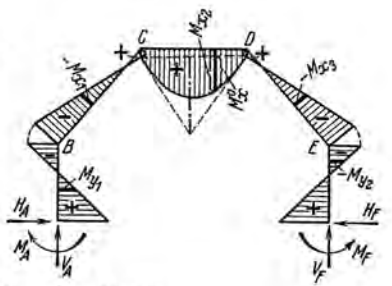
$$M_A = M_F = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{N_1} \qquad M_B = M_E = -\frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{N_1}$$

$$M_C = M_D = \mathfrak{S}_l - \varphi M_A + m M_B; \qquad V_A = V_F = S \qquad H_A = H_F = \frac{M_A - M_B}{a}.$$

Note: All the load terms refer to the left inclined member.

Case 101/5: Girder loaded by any type of vertical load





Constants:

$$\mathfrak{B}_3 = (\mathfrak{S}_r - \mathfrak{S}_l)\,\gamma\, C_3 + (\mathfrak{L} - \mathfrak{R})\,\delta\, k_2$$

$$\mathfrak{B}_1 = S\,c\,C_1 + (\mathfrak{L} + \mathfrak{R})\,\varphi\, k_2 \qquad \mathfrak{B}_2 = S\,c\,C_2 + (\mathfrak{L} + \mathfrak{R})\, m\, k_2;$$

$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2 N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2 N_1} \qquad X_3 = \frac{\mathfrak{B}_3}{2 N_2}.$$

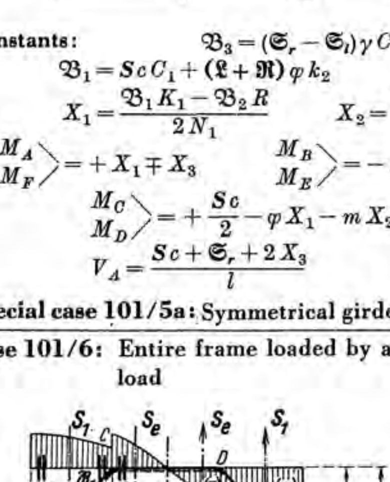
$$\left.\begin{matrix} M_A \\ M_F \end{matrix}\right\rangle = + X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_E \end{matrix}\right\rangle = - X_2 \mp X_3; \qquad H_A = H_F = \frac{X_1 + X_2}{a}$$

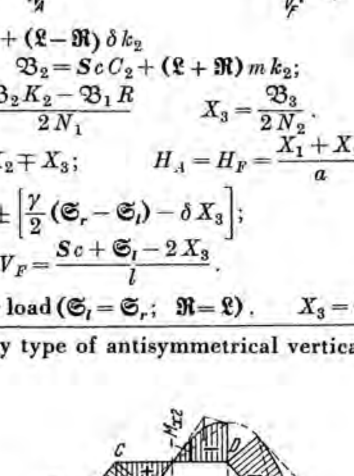
$$\left.\begin{matrix} M_C \\ M_D \end{matrix}\right\rangle = + \frac{S\,c}{2} - \varphi X_1 - m X_2 \pm \left[\frac{\gamma}{2}(\mathfrak{S}_r - \mathfrak{S}_l) - \delta X_3\right];$$

$$V_A = \frac{S\,c + \mathfrak{S}_r + 2 X_3}{l} \qquad V_F = \frac{S\,c + \mathfrak{S}_l - 2 X_3}{l}.$$

Special case 101/5a: Symmetrical girder load ($\mathfrak{S}_l = \mathfrak{S}_r$; $\mathfrak{R} = \mathfrak{L}$). $\quad X_3 = 0!$

Case 101/6: Entire frame loaded by any type of antisymmetrical vertical load





$$M_F = M_E = -M_B = -M_A = \frac{(\delta\,\mathfrak{S}_{l1} + \gamma\,\mathfrak{S}_{r2})\, C_3 + \mathfrak{L}_1 + \delta\,\mathfrak{R}_1 + \delta\,\mathfrak{L}_2 k_2}{N_2}$$

$$M_C = -M_D = (\delta\,\mathfrak{S}_{l1} + \gamma\,\mathfrak{S}_{r2}) - \delta\, M_F;$$

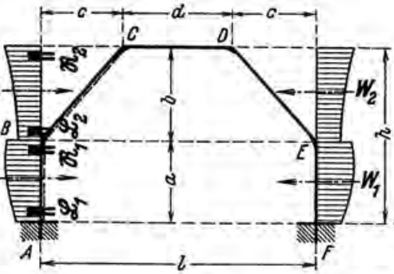
$$V_A = -V_F = \frac{2\,\mathfrak{S}_{r1} + S_1 d + \mathfrak{S}_{r2} + 2 M_F}{l}; \qquad H_A = H_F = 0.$$

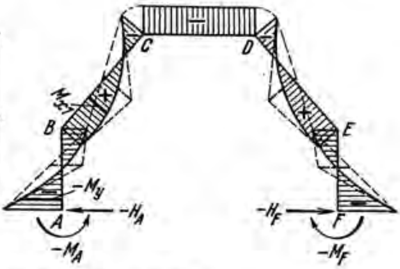
Note: All the load terms refer to the left half of the frame.

FRAME 101

See Appendix A, Load Terms, pp. 440-445.

Case 101/7: Entire frame loaded by any type of symmetrical external horizontal load





Constants:

$$\mathfrak{B}_1 = \varphi C_1 \mathfrak{S}_{l1} + \mathfrak{L}_1 k_1 + C_1 \mathfrak{S}_{r2} - \varphi \mathfrak{R}_2$$

$$\mathfrak{B}_2 = \varphi C_2 \mathfrak{S}_{l1} - \mathfrak{R}_1 k_1 + C_2 \mathfrak{S}_{r2} - \mathfrak{L}_2 - m \mathfrak{R}_2 .$$

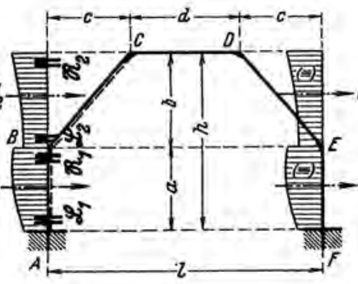
$$M_A = M_F = -\frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{N_1} \qquad M_B = M_E = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{N_1};$$

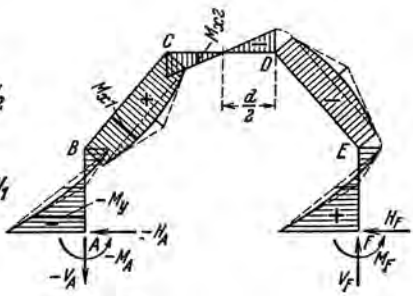
$$M_C = M_D = -\varphi \mathfrak{S}_{l1} - \mathfrak{S}_{r2} - \varphi M_A + m M_B ;$$

$$H_A = H_F = -\frac{\mathfrak{S}_{l1}}{a} + \frac{M_A - M_B}{a}; \qquad V_A = V_F = 0 .$$

Note: All the load terms refer to the left half of the frame.

Case 101/8: Entire frame loaded by any type of antisymmetrical horizontal load from the left





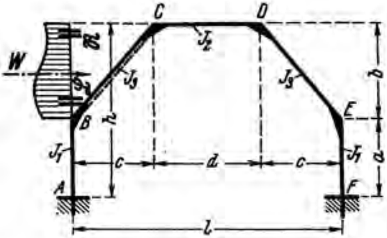
$$M_F = -M_A = \frac{(\mathfrak{S}_{l1} + W_2 a)(B + \delta C_3) + \delta C_3 \mathfrak{S}_{l2} + (\mathfrak{L}_1 + \mathfrak{R}_1) k_1 + \mathfrak{L}_2 + \delta \mathfrak{R}_2}{N_2}$$

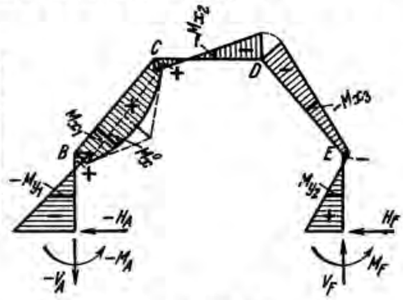
$$M_B = -M_E = \mathfrak{S}_{l1} + W_2 a + M_A \qquad M_C = -M_D = \delta (\mathfrak{S}_{l1} + W_2 a + \mathfrak{S}_{l2} + M_A);$$

$$V_F = -V_A = \frac{\mathfrak{S}_{l1} + W_2 a + \mathfrak{S}_{l2} + M_A}{l/2}; \qquad H_F = -H_A = W_1 + W_2 .$$

Note: All the load terms refer to the left half of the frame.

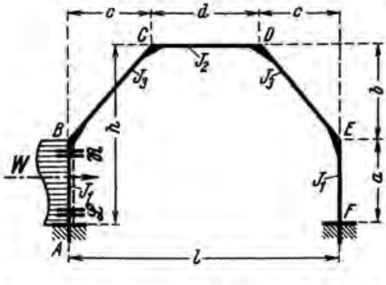
Case 101/9: Left-hand inclined member loaded by any type of horizontal load

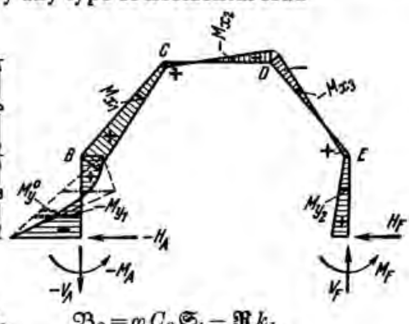




Constants:
$$\mathfrak{B}_1 = C_1 \mathfrak{S}_r - \varphi \mathfrak{R} \qquad \mathfrak{B}_2 = C_2 \mathfrak{S}_r - (\mathfrak{L} + m\,\mathfrak{R})$$
$$\mathfrak{B}_3 = W a (B + \delta C_3) + \delta C_3 \mathfrak{S}_l + \mathfrak{L} + \delta \mathfrak{R};$$
$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2 N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2 N_1} \qquad X_3 = \frac{\mathfrak{B}_3}{2 N_2}.$$
$$\left.\begin{matrix} M_A \\ M_F \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_E \end{matrix}\right\rangle = +X_2 \pm \left(\frac{W a}{2} - X_3\right)$$
$$\left.\begin{matrix} M_C \\ M_D \end{matrix}\right\rangle = -\frac{\mathfrak{S}_r}{2} + \varphi X_1 + m X_2 \pm \frac{\delta}{2}(W a + \mathfrak{S}_l - 2 X_3);$$
$$V_F = -V_A = \frac{W a + \mathfrak{S}_l - 2 X_3}{l}; \qquad H_F = \frac{W}{2} - \frac{X_1 + X_2}{a} \qquad H_A = -(W - H_F).$$

Case 101/10: Left-hand leg loaded by any type of horizontal load





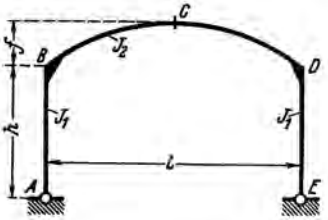
Constants:
$$\mathfrak{B}_1 = \varphi C_1 \mathfrak{S}_l + \mathfrak{L} k_1 \qquad \mathfrak{B}_2 = \varphi C_2 \mathfrak{S}_l - \mathfrak{R} k_1$$
$$\mathfrak{B}_3 = \mathfrak{S}_l (B + \delta C_3) + (\mathfrak{L} + \mathfrak{R}) k_1;$$

The formulas for X_1, X_2 and X_3 are the same as above.

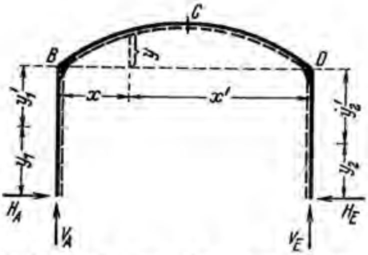
$$\left.\begin{matrix} M_A \\ M_F \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_E \end{matrix}\right\rangle = +X_2 \pm \left(\frac{\mathfrak{S}_l}{2} - X_3\right)$$
$$\left.\begin{matrix} M_C \\ M_D \end{matrix}\right\rangle = -\frac{\varphi \mathfrak{S}_l}{2} + \varphi X_1 + m X_2 \pm \delta\left(\frac{\mathfrak{S}_l}{2} - X_3\right);$$
$$V_F = -V_A = \frac{\mathfrak{S}_l - 2 X_3}{l}; \qquad H_F = \frac{\mathfrak{S}_l}{2a} - \frac{X_1 + X_2}{a} \qquad H_A = -(W - H_F).$$

Frame 102

Symmetrical two-hinged bent with parabolic girder.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{h}{l}; \qquad \varphi = \frac{f}{h};$$

$$B = 2k + 3 + 2\varphi \qquad C = 2\varphi\left(1 + \frac{4}{5}\varphi\right); \qquad N = B + C.$$

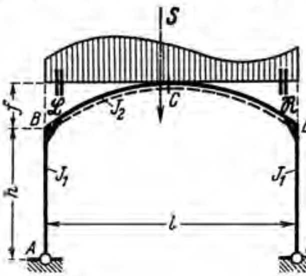
Equation of the parabolic girder: $y = \frac{4f}{l^2} x x' = 4f \cdot \omega_R$

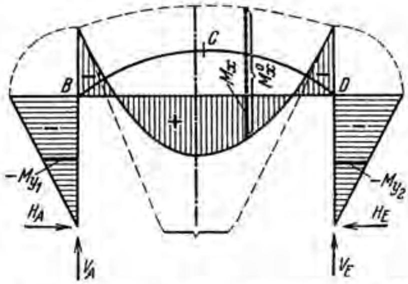
In deriving the formula for the girder, the substitution $ds = dx$ was used. Therefore the formulas for Frame 102 are theoretically exact only for parabolas with large radii of curvature.

The moment area for the girder is drawn with the chord (instead of the parabola) as axis. The ratio $f : l$ is usually so small that there is no appreciable difference between a parabolic and a circular girder. Therefore for all practical purposes the formulas for Frame 102 may be used also for Frames with a circularly curved girder.

See Appendix A, Load Terms, pp. 440-445.

Case 102/1: Girder loaded by any type of vertical load





$$M_B = M_D = -\frac{(\mathfrak{L}+\mathfrak{R})+\varphi\,\mathfrak{P}}{2N} \qquad M_C = \mathbf{M}_C^0 + (1+\varphi)\,M_B; \qquad H_A = H_E = \frac{-M_B}{h};$$

$$M_x = \mathbf{M}_x^0 + M_B\left(1+\frac{y}{h}\right) \qquad M_{y1} = M_{y2} = \frac{y_1}{h}\,M_B; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_E = \frac{\mathfrak{S}_l}{l}.$$

Note: The load terms $\mathfrak{P}$ which are valid for the parabolic girder only are tabulated p. 382. $\mathbf{M}_C^0$ is the moment at the center C of the simply supported beam BD.

Special case 102/1a: Symmetrical girder load ($\mathfrak{R} = \mathfrak{L}$; $\mathfrak{S}_l = \mathfrak{S}_r$) .

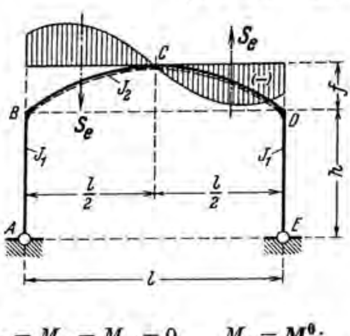
$$M_B = M_D = -\frac{2\,\mathfrak{L}+\varphi\,\mathfrak{P}}{2N}; \qquad V_A = V_E = \frac{S}{2}.$$

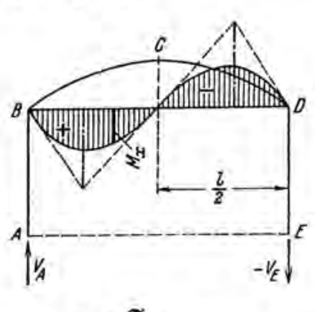
Special case 102/1b: Vertical concentrated load P at C

$$M_B = M_D = -\frac{Pl}{16}\cdot\frac{6+5\varphi}{N}; \qquad \mathbf{M}_C^0 = \frac{Pl}{4}; \qquad V_A = V_E = \frac{P}{2}.$$

Case 102/2: Girder loaded by any type of antisymmetrical vertical load

$$(\mathfrak{R} = -\mathfrak{L};\quad \mathfrak{S}_l = -\mathfrak{S}_r;\quad \mathfrak{P} = 0).$$





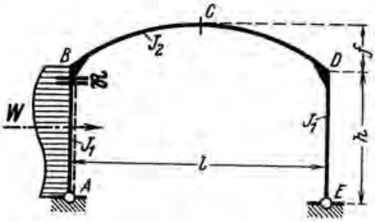
$$M_B = M_C = M_D = 0 \qquad M_x = \mathbf{M}_x^0; \qquad V_A = -V_E = \frac{\mathfrak{S}_r}{l}; \qquad H_A = H_E = 0.$$

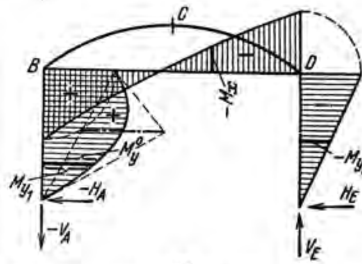
Note: For this load the girder becomes a statically determinate, simply supported bear

FRAME 102

See Appendix A, Load Terms, pp. 440-445.

Case 102/3: Left-hand leg loaded by any type of horizontal load





$$M_D = -\frac{\mathfrak{S}_l B + \mathfrak{R} k}{2N} \qquad M_B = \mathfrak{S}_l + M_D \qquad M_C = \frac{\mathfrak{S}_l}{2} + (1+\varphi) M_D;$$

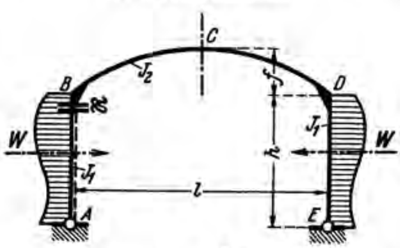
$$V_E = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad H_E = \frac{-M_D}{h} \qquad H_A = -(W - H_E);$$

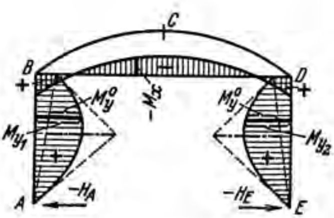
$$M_{y1} = M_y^0 + \frac{y_1}{h} M_B \qquad M_x = M_D\left(1 + \frac{y}{h}\right) \qquad M_{y2} = \frac{y_2}{h} M_D.$$

Special case 102/3a: Horizontal concentrated load P at B

Substitute $W = P \qquad \mathfrak{S}_l = Ph; \qquad \mathfrak{R} = 0 \qquad M_y^0 = 0.$

Case 102/4: Both legs loaded by any type of symmetrical horizontal load from the outside





$$M_B = M_D = \frac{\mathfrak{S}_l C - \mathfrak{R} k}{N} \qquad M_C = -\varphi \mathfrak{S}_l + (1+\varphi) M_B;$$

$$H_A = H_E = -\frac{\mathfrak{S}_r + M_B}{h}; \qquad V_A = V_E = 0;$$

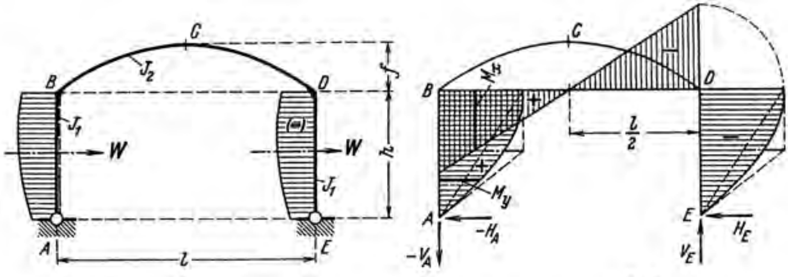
$$M_{y1} = M_{y2} = M_y^0 + \frac{y_1}{h} M_B \qquad M_x = M_B\left(1 + \frac{y}{h}\right) - \frac{y}{h}\mathfrak{S}_l.$$

Note: All terms refer to the left leg.

Special case 102/4a: Two horizontal concentrated loads P at corners B and D acting from outside

Substitute $\mathfrak{S}_l = Ph; \qquad \mathfrak{S}_r = 0 \qquad \mathfrak{R} = 0 \qquad M_y^0 = 0.$

Case 102/5: Both legs loaded by any type of antisymmetrical horizontal load from the left

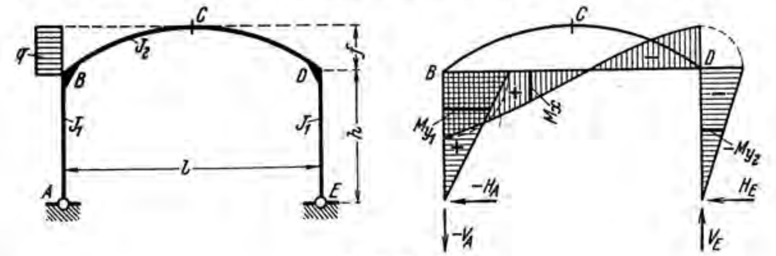


$$M_B = -M_D = +\mathfrak{S}_l \qquad M_C = 0; \qquad V_E = -V_A = \frac{2\,\mathfrak{S}_l}{l};$$

$$H_E = -H_A = W; \qquad M_y = M_y^0 + \frac{y_1}{h} M_B \qquad M_x = \frac{x' - x}{l} M_B.$$

Note: All the load terms refer to the left leg

Case 102/6: Horizontal rectangular load acting at the girder from the left



Constant: $$X = \frac{2\,q\,f^2\,(7 + 6\,\varphi)}{35\,N}.$$

$$M_B = +\frac{q\,f\,h}{2} + X \qquad M_D = -\frac{q\,f\,h}{2} + X \qquad M_C = -\frac{q\,f^2}{4} + (1 + \varphi)\,X;$$

$$V_E = -V_A = \frac{q\,f\,h\,(2 + \varphi)}{2\,l}; \qquad H_A = -\frac{M_B}{h} \qquad H_E = \frac{-M_D}{h};$$

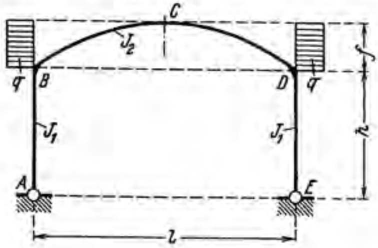
Within the limits of BC. $$M_x = M_B\left(1 + \frac{y}{h}\right) - V_E \cdot x - \frac{q\,y^2}{2}$$

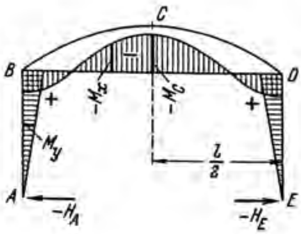
Within the limits of DC: $$M'_x = M_D\left(1 + \frac{y}{h}\right) + V_E \cdot x';$$

$$M_{y1} = \frac{y_1}{h} M_B = (-H_A) \cdot y_1 \qquad M_{y2} = \frac{y_2}{h} M_D = -H_E \cdot y_2.$$

FRAME 102

Case 102/7: **Two equal horizontal rectangular loads acting at the girder from outside (Symmetrical load)**



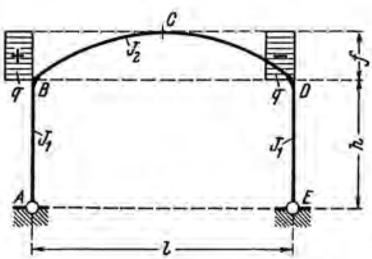


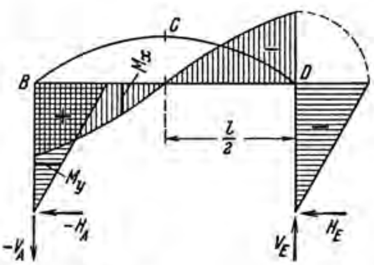
$$M_B = M_D = \frac{4qf^2(7+6\varphi)}{35N} \qquad M_C = -\frac{qf^2}{2} + (1+\varphi)M_B;$$

$$M_x = M'_x = M_B\left(1+\frac{y}{h}\right) - \frac{qy^2}{2} \qquad M_y = \frac{y_1}{h}M_B;$$

$$H_A = H_E = -\frac{M_B}{h} \qquad V_A = V_E = 0.$$

Case 102/8: **Two equal horizontal rectangular loads acting at the girder from the left (Pressure and suction; antisymmetrical load)**





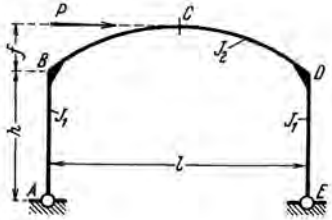
$$H_E = -H_A = qf; \qquad V_E = -V_A = \frac{qfh(2+\varphi)}{l};$$

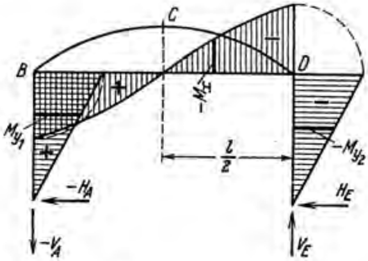
$$M_B = -M_D = qfh \qquad M_C = 0 \qquad M_y = \frac{y_1}{h}M_B.$$

Within the limits of BC: $\qquad M_x = M_B\left(1+\frac{y}{h}\right) - V_E\cdot x - \frac{qy^2}{2}$

Within the limits of DC: $\qquad M'_x = M_D\left(1+\frac{y}{h}\right) + V_E\cdot x' + \frac{qy^2}{2}.$

Case 102/9: Horizontal concentrated load at C





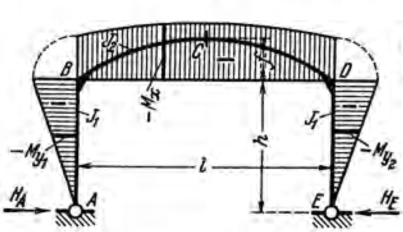
$$H_E = -H_A = \frac{P}{2}; \qquad V_E = -V_A = \frac{P(h+f)}{l};$$

$$M_B = -M_D = \frac{Ph}{2} \qquad M_C = 0 \qquad M_{y1} = -M_{y2} = \frac{P}{2} y_1$$

Within the limits of BC: $\quad M_x = +\frac{P}{2}(h+y) - V_E \cdot x$

Within the limits of DC: $\quad M'_x = -\frac{P}{2}(h+y) + V_E \cdot x'$.

Case 102/10: Uniform increase in temperature of the entire frame*



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degr

$$M_B = M_D = -\frac{3 E J_2 \varepsilon t}{h N}$$

$$M_C = (1+\varphi) M_B \qquad M_x = M_B\left(1 + \frac{y}{h}\right)$$

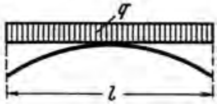
$$H_A = H_E = \frac{-M_B}{h}; \qquad M_{y1} = M_{y2} = \frac{y_1}{h} M_B.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

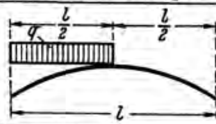
* Only temperature changes of the girder cause stress; temperature changes of the legs have no effect.

Appendix to Frames 102-105

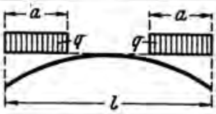
Load terms $\mathfrak{P}$ for parabolic members subjected to the more important types of loads



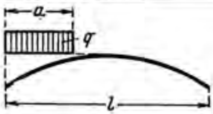
$$\mathfrak{P} = \frac{2}{5} q l^2$$



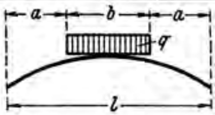
$$\mathfrak{P} = \frac{1}{5} q l^2$$



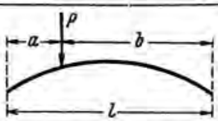
$\alpha = \frac{a}{l}$ $\qquad$ $\mathfrak{P} = \frac{2}{5} q a^2 (5 - 5\alpha^2 + 2\alpha^3)$



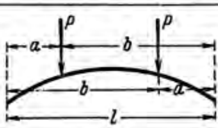
$\alpha = \frac{a}{l}$ $\qquad$ $\mathfrak{P} = \frac{1}{5} q a^2 (5 - 5\alpha^2 + 2\alpha^3)$



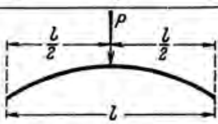
$\beta = \frac{b}{l}$ $\qquad$ $\mathfrak{P} = \frac{1}{40} q b l (5 - \beta^2)^2$



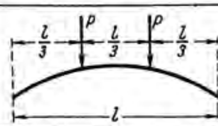
$\alpha = \frac{a}{l}$ $\qquad$ $\beta = \frac{b}{l}$ $\qquad$ $\mathfrak{P} = 2 \frac{P a b}{l} (1 + \alpha\beta)$



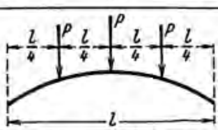
$\alpha = \frac{a}{l}$ $\qquad$ $\beta = \frac{b}{l}$ $\qquad$ $\mathfrak{P} = 4 \frac{P a b}{l} (1 + \alpha\beta)$



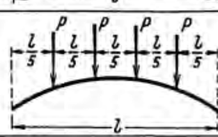
$$\mathfrak{P} = \frac{5}{8} P l$$



$$\mathfrak{P} = \frac{88}{81} P l$$



$$\mathfrak{P} = \frac{97}{64} P l$$



$$\mathfrak{P} = \frac{1208}{625} P l$$

The general formula for $\mathfrak{P}$ is:

$$\mathfrak{P} = \frac{24}{l^3} \int_0^l M_x^0 \cdot x\, x'\, dx = (\mathfrak{L} + \mathfrak{R}) - \frac{24}{l^3} T.$$

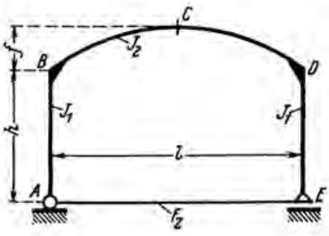
In this formula T is the moment of inertia of the moment diagram of the simple beam l, about the vertical axis of gravity of the moment diagram.

(For the load terms $\mathfrak{L}$ and $\mathfrak{R}$ see the chapter *Beam Formulas*.)

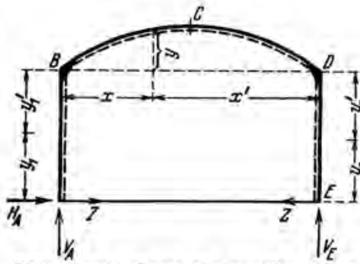
Note: For antisymmetrical loads $\mathfrak{P} = 0$.

Frame 103

Symmetrical tied bent with parabolic girder. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients and equations of the parabolic girder same as frame 102, p. 376*.

Additional coefficients:

$$L = \frac{3J_2}{h^2 F_Z} \cdot \frac{E}{E_Z} \qquad N_Z = N + L\,.$$

$E =$ Modulus of elasticity of the material of the frame
$E_Z =$ Modulus of elasticity of the tie rod
$F_Z =$ Cross-sectional area of the tie rod

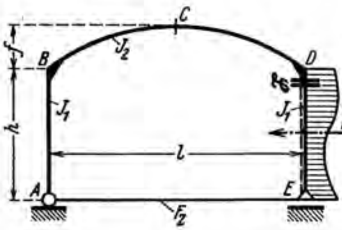
Cases 102/1, 3, 6, and 10 (see p. 377-381) may be used for frame 103 if N is replaced by N_Z. For cases 102/1 and 10 $H_A = H_E = Z$, for cases 102/3 and 6 $H_E = Z$ and $H_A = -W$, $H_A = -qf$. The other loading conditions of frame 102 cannot be used directly for frame 103. Use the following cases 103/1 through 4 instead.

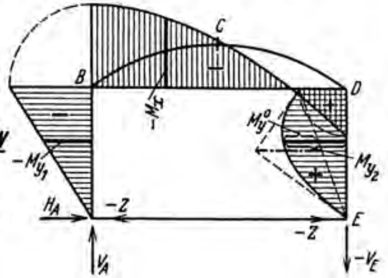
* See p. 376 for remarks on girder curvature.

FRAME 103

See Appendix A, Load Terms, pp. 440-445.

Case 103/1: Right-hand leg loaded by any type of horizontal load





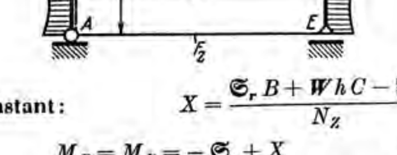
Constant: $X = \dfrac{Wh(N+C) + \mathfrak{S}_l B - \mathfrak{L}k}{2N_Z}$. $\qquad M_B = -Wh + X$

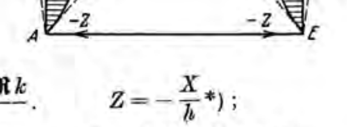
$$M_C = -W(h+f) + \frac{\mathfrak{S}_r}{2} + (1+\varphi)X \qquad M_D = -\mathfrak{S}_l + X$$

$$M_x = \frac{x'}{l}M_B + \frac{x}{l}M_D - \left(W - \frac{X}{h}\right)y \qquad M_{y2} = \mathbf{M}_y^0 + \frac{y_2}{h}M_D$$

$$M_{y1} = \frac{y_1}{h}M_B; \qquad V_A = -V_E = \frac{\mathfrak{S}_r}{l}; \qquad Z = -\frac{X}{h}\text{*)} \qquad H_A = +W.$$

Case 103/2: Both legs loaded by any type of symmetrical horizontal load





Constant: $X = \dfrac{\mathfrak{S}_r B + WhC - \mathfrak{R}k}{N_Z}$. $\qquad Z = -\dfrac{X}{h}$*);

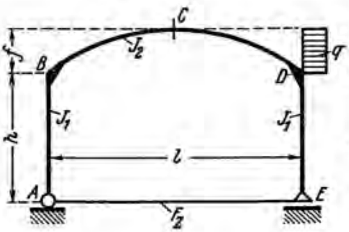
$$M_B = M_D = -\mathfrak{S}_r + X \qquad M_C = -\mathfrak{S}_r - Wf + (1+\varphi)X$$

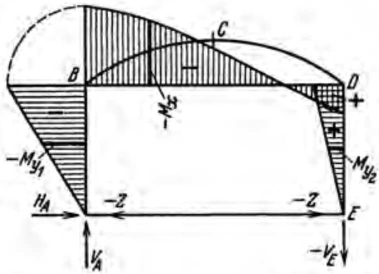
$$M_{y1} = M_{y2} = \mathbf{M}_y^0 + \frac{y_1}{h}M_B \qquad M_x = -\mathfrak{S}_r - Wy + \left(1 + \frac{y}{h}\right)X.$$

Note: All the load terms refer to the left leg.

* For the case of the above two loading conditions as well as case 103/3 (p. 385 top) Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual force remains in the tie rod.

Case 103/3: Horizontal rectangular load acting at the girder from the right





$$Z = -\frac{q f}{70} \cdot \frac{35(2k+3) + 8\varphi(21 + 10\varphi)}{N_Z} * \; ; \qquad V_A = -V_E = \frac{q f (2h+f)}{2l};$$

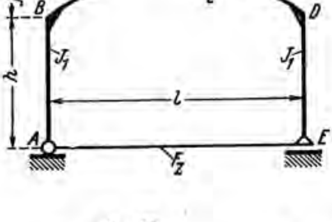
$$M_B = -(H_A + Z)h \qquad M_C = -(H_A + Z)(h+f) + V_A \frac{l}{2} \qquad M_D = (-Z)h$$

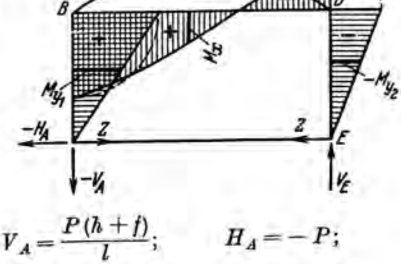
$$H_A = +qf \qquad M_{y1} = -(H_A + Z)y_1 \qquad M_{y2} = (-Z)y_2.$$

Within the limits of BC: $\quad M_x = M_B - (H_A + Z)y + V_A \cdot x$

Within the limits of DC: $\quad M_x' = M_D - Zy - V_A \cdot x' - \frac{q y^2}{2}.$

Case 103/4: Horizontal concentrated load at C





$$Z = \frac{P}{2} \cdot \frac{N}{N_Z}; \qquad V_E = -V_A = \frac{P(h+f)}{l}; \qquad H_A = -P;$$

$$M_B = (P - Z)h \qquad M_C = \left(\frac{P}{2} - Z\right)(h+f) \qquad M_D = -Zh.$$

$$M_{y1} = (P - Z)y_1 \qquad M_{y2} = -Zy_2$$

Within the limits of BC: $\quad M_x = M_B + (P - Z)y - V_E \cdot x$

Within the limits of DC: $\quad M_x' = M_D - Zy + V_E \cdot x'.$

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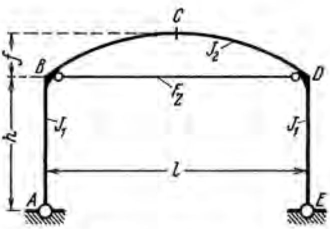
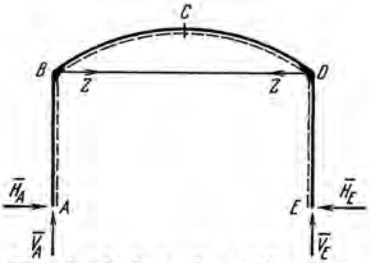
*See footnote on page 384.

Frame 104

Symmetrical two-hinged bent with parabolic girder and tie-rod under roof.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point exactly as Frame 102 (see p. 388). Positive bending moments cause tension at the face marked by a dashed line.

General notes

In order to compute Frame 104 (with tie rod) we can start by using Frame 102 (the same frame without tie rod). The effect of the tie is easily shown as follows:

Steps in computing the stresses

First step: Figure the moments at the joints M_B, M_C, M_D and the reactions H_A, H_E, V_A, V_E by using the formulas for Frame 102 (pp. 376–381).

Second step:

a) Figure the additional coefficients for Frame 104.

$$\beta = \frac{C}{N} \qquad \gamma = \frac{\varphi B - C}{N} \qquad L = \frac{15 J_2}{2 f^2 F_Z} \cdot \frac{E}{E_Z} \qquad N_Z = \frac{2(4k+1)}{N} + L.$$

E = Modulus of elasticity of the material of the frame
E_Z = Modulus of elasticity of the tie rod
F_Z = Cross-sectional area of the tie rod

b) Figure the tension in the tie rod.

$$Z = \frac{\frac{M_B + M_D}{2} + 4\,(M_C - M_C^0) + \frac{5}{4}\mathfrak{P}}{f N_Z} * .$$

Note: The load terms $M_C{}^0$ and $\mathfrak{P}$ are the same as those on p. 377.

Third step:

a) Moments at the joints and reactions for Frame 104.

$$\overline{M}_B = M_B + \beta Z h \qquad \overline{M}_C = M_C - \gamma Z h \qquad \overline{M}_D = M_D + \beta Z h$$

$$\overline{H}_A = H_A - \beta Z \qquad \overline{H}_E = H_E - \beta Z \qquad \overline{V}_A = V_A \qquad \overline{V}_E = V_E .$$

Note: For better distinction the moments and reactions for Frame 104 are shown with a dash over the letter.

b) Moments at any point of Frame 104.

$$\overline{M}_x = M_x + \beta Z h\left(1 + \frac{y}{h}\right) - Z h$$

$$\overline{M}_{y1} = M_{y1} + \beta Z y_1 \qquad \overline{M}_{y2} = M_{y2} + \beta Z y_2 .$$

Final Remarks

The formulas given above can be used for cases 102/1, 3, 4, and 10 (pp. 377, 378, and 381).**

The antisymmetric cases 102/2, 5, 8, and 9 apply unchanged to frame 104, since $Z = 0$.

For cases 102/6 and 7 (pp. 379–380) no formulas are given. The load qf can be replaced with good approximation however by two horizontal single loads $P = qf/2$, which act in case 102/6 at the points B and C and in case 102/7 at the points B and D.

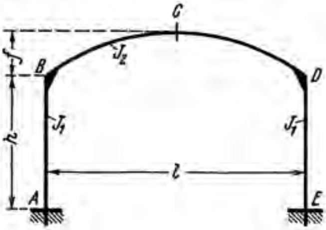
---

*For the case of various loading conditions Z becomes negative, i.e., the tie rod is stressed in compression. This is only valid if the compressive force is smaller than the tensile force due to dead load, so that a residual tensile force remains in the tie rod.

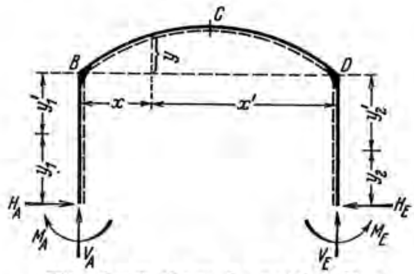
**For the case of a uniform increase in temperature of the entire frame with the exception of the tie rod, set $\mathfrak{P} = 6\,E\,J_2\,\varepsilon\,t/f$. For the case of a change in temperature of the entire frame including the tie rod, set $\mathfrak{P} = 0$.

Frame 105

Symmetrical hingeless bent with parabolic girder.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{h}{l}; \qquad \varphi = \frac{f}{h}; \qquad K_1 = 2k + \frac{8}{5}\varphi^2 \qquad K_2 = 3(2k+1)$$

$$R = 3k - 2\varphi; \qquad N_1 = K_1 K_2 - R^2 \qquad N_2 = 6k + 1.$$

Equation of the parabolic girder: $y = \frac{4f}{l^2} x x' = 4f \cdot \omega_R$*

In deriving the formula for the girder, the substitution $ds = dx$ was used. Therefore the formulas for Frame 105 are theoretically exact only for parabolas with large radii of curvature.

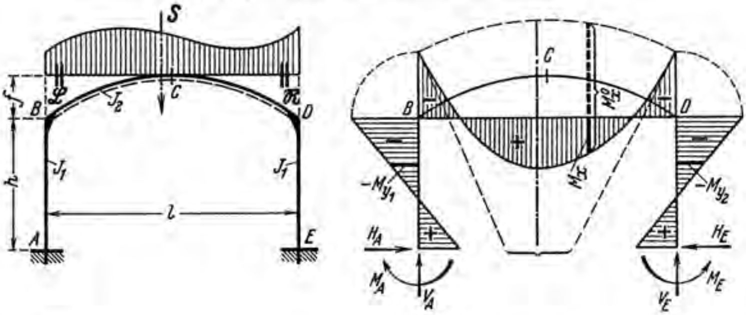
The moment area for the girder is drawn with the chord (instead of the parabola) as axis. The ratio $f : l$ is usually so small that there is no appreciable difference between a parabolic and a circular girder. Therefore for all practical purposes the formulas for Frame 105 may be used also for Frames with a circularly curved girder.

---

* For numerical tables for ω_R see "*Beam Formulas*" by A. Kleinlogel, American edition translated and adapted to American conditions by Harold G. Lorsch, Frederick Ungar Publishing Co. New York, p. 15.

Case 105/1: Girder loaded by any type of vertical load

See Appendix A, Load Terms, pp. 440-445.



Constants:

$$X_1 = \frac{(\mathfrak{L} + \mathfrak{R}) K_1 + \mathfrak{P} \varphi R}{2 N_1} \qquad X_2 = \frac{(\mathfrak{L} + \mathfrak{R}) R + \mathfrak{P} \varphi K_2}{2 N_1} \qquad X_3 = \frac{(\mathfrak{L} - \mathfrak{R})}{2 N_2}.$$

$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = X_2 - X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = -X_1 \mp X_3$$

$$M_C = M_C^0 - X_1 - \varphi X_2{}^* \; ; \qquad M_x = M_x^0 + \frac{x'}{l} M_B + \frac{x}{l} M_D - \frac{y}{h} X_2$$

$$M_{y1} = M_A - \frac{y_1}{h} X_2 \qquad M_{y2} = M_E - \frac{y_2}{h} X_2 ;$$

$$V_A = \frac{\mathfrak{S}_r + 2 X_3}{l} \qquad V_E = S - V_A ; \qquad H_A = H_E = \frac{X_2}{h}.$$

Note: The load terms which are valid for the parabolic girder only are tabulated on p. 382.

Special case 105/1a: Symmetrical girder load ($\mathfrak{R} = \mathfrak{L}$; $\mathfrak{S}_l = \mathfrak{S}_r$).

$$M_A = M_E = X_2 - X_1 \qquad M_B = M_D = -X_1 \qquad M_C = M_C^0 - X_1 - \varphi X_2 ;^*$$

$$V_A = V_E = \frac{S}{2}; \qquad H_A = H_E = \frac{X_2}{h}; \qquad M_x = M_x^0 - X_1 - \frac{y}{h} X_2 .$$

Special case 105/1b: Antisymmetrical girder load ($\mathfrak{R} = -\mathfrak{L}$; $\mathfrak{S}_l = -\mathfrak{S}_r$).

$$M_E = M_D = -M_B = -M_A = \frac{\mathfrak{L}}{N_2} \qquad M_x = M_x - \frac{\mathfrak{L}}{N_2} \cdot \frac{x' - x}{l}$$

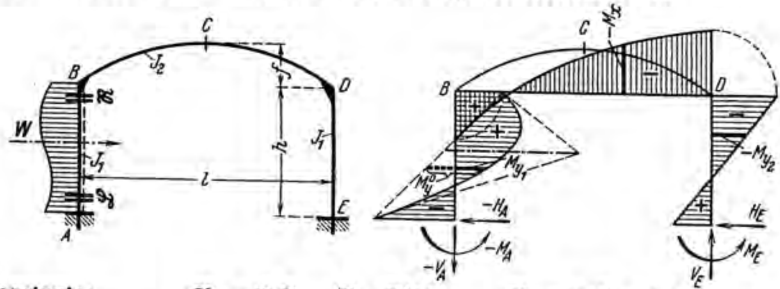
$$M_C = 0; \qquad V_A = -V_E = \frac{\mathfrak{S}_r + 2 M_D}{l}; \qquad H_A = H_E = 0 .$$

* M_C^0 is the moment at the center C of the simply supported beam BD.

FRAME 105

(See Appendix A, Load Terms, pp. 440-445.)

Case 105/2: Left-hand leg loaded by any type of horizontal load



Constants: $\mathfrak{B}_1 = [3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\,k \qquad \mathfrak{B}_2 = [2\,\mathfrak{S}_l - \mathfrak{L}]\,k;$

$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{2\,N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{2\,N_1} \qquad X_3 = \frac{\mathfrak{B}_1}{2\,N_2}.$$

$$M_B = +X_1 + X_3 \qquad M_D = +X_1 - X_3 \qquad M_C = +X_1 - \varphi X_2$$

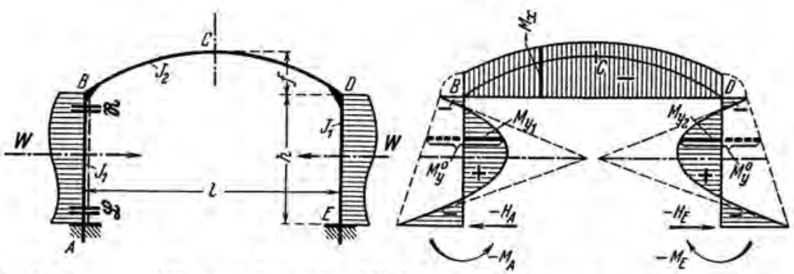
$$M_A = -\mathfrak{S}_l + X_1 + X_2 + X_3 \qquad M_E = +X_1 + X_2 - X_3;$$

$$V_E = -V_A = \frac{2\,X_3}{l}; \qquad H_E = +\frac{X_2}{h} \qquad H_A = -(W - H_E);$$

$$M_{y1} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_{y2} = \frac{y_2}{h} M_D + \frac{y_2'}{h} M_E$$

$$M_x = \frac{x'}{l} M_B + \frac{x}{l} M_D - \frac{y}{h} X_2.$$

Case 105/3: Both legs loaded by any type of symmetrical external horizontal load



Constants: $\mathfrak{B}_1 = [3\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\,k \qquad \mathfrak{B}_2 = [2\,\mathfrak{S}_l - \mathfrak{L}]\,k;$

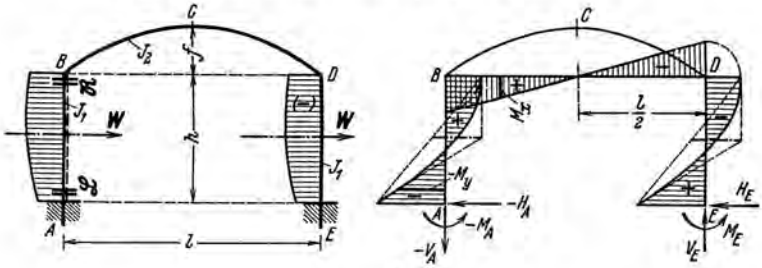
$$X_1 = \frac{\mathfrak{B}_1 K_1 - \mathfrak{B}_2 R}{N_1} \qquad X_2 = \frac{\mathfrak{B}_2 K_2 - \mathfrak{B}_1 R}{N_1}. \qquad H_A = H_E = -W + \frac{X_2}{h};$$

$$M_B = M_D = +X_1 \qquad M_C = +X_1 - \varphi X_2 \qquad M_A = M_E = -\mathfrak{S}_l + X_1 + X_2$$

$$M_{y1} = M_{y2} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_x = M_B - \frac{y}{h} X_2.$$

Note: All the load terms refer to the left leg.

Case 105/4: Both legs loaded by any type of antisymmetrical horizontal load from the left, both carrying the same load

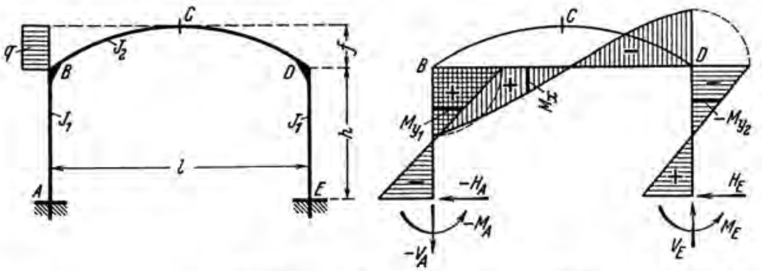


$$M_B = -M_D = [3\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})]\frac{k}{N_2} \qquad M_E = -M_A = \mathfrak{S}_l - M_B$$

$$M_C = 0; \qquad H_E = -H_A = W; \qquad V_E = -V_A = \frac{2\,M_B}{l}.$$

Note: All terms refer to the left leg.

Case 105/5: Horizontal rectangular load acting at the girder from the left



Constants: $\quad X_1 = \frac{2qf^2}{5N_1}\left(K_1 + \frac{6}{7}\varphi R\right) \qquad X_2 = \frac{2qf^2}{5N_1}\left(R + \frac{6}{7}\varphi K_2\right)$

$$X_3 = \frac{qfh\,(12k - \varphi)}{8N_2}. \qquad M_C = -\frac{qf^2}{4} + X_1 + \varphi X_2$$

$$\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = -(X_2 - X_1) \mp \left(\frac{qfh}{2} - X_3\right) \qquad \left.\begin{matrix} M_B \\ M_D \end{matrix}\right\rangle = +X_1 \pm X_3;$$

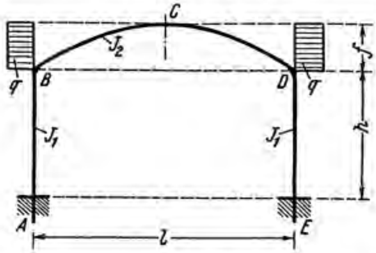
$$M_{y1} = \frac{y_1'}{h}M_A + \frac{y_1}{h}M_B \qquad M_{y2} = \frac{y_2}{h}M_D + \frac{y_2'}{h}M_E;$$

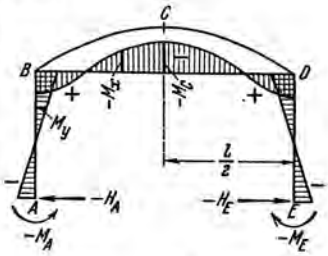
$$V_E = -V_A = \frac{qf^2}{2l} + \frac{2X_3}{l} \qquad \left.\begin{matrix} H_A \\ H_E \end{matrix}\right\rangle = -\frac{X_2}{h} \mp \frac{qf}{2}.$$

Within the limits of BC: $\quad M_x = M_B + (-H_A)\,y - V_E \cdot x - \frac{qy^2}{2}$

Within the limits of DC: $\quad M_x' = M_D - H_E \cdot y + V_E \cdot x'.$

Case 105/6: Two equal horizontal rectangular loads acting at the girder from outside (Symmetrical load)





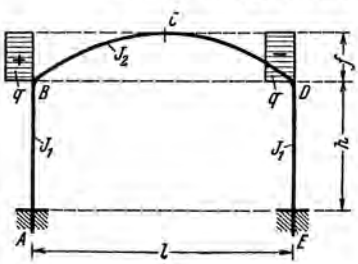
Constants: $X = \frac{4qf^2}{5N_1}\left(R + \frac{6}{7}\varphi K_2\right).$ $\qquad M_C = -\frac{qf^2}{2} + M_B + \varphi X$

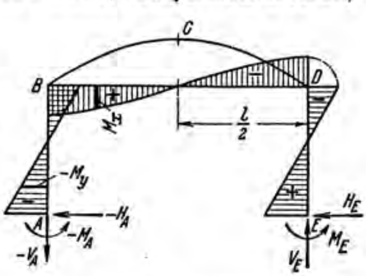
$$M_B = M_D = \frac{4qf^2}{5N_1}\left(K_1 + \frac{6}{7}\varphi R\right) \qquad M_A = M_E = -X + M_B;$$

$$M_x = M'_x = M_B + \frac{y}{h}X - \frac{qy^2}{2} \qquad M_y = \frac{y'_1}{h}M_A + \frac{y_1}{h}M_B;$$

$$H_A = H_E = -\frac{X}{h} \qquad V_A = V_E = 0.$$

Case 105/7: Two equal horizontal rectangular loads acting at the girder from the left (Pressure and suction; antisymmetrical load)





$$M_B = -M_D = \frac{qfh(12k - \varphi)}{4N_2} \qquad M_E = -M_A = qfh - M_B$$

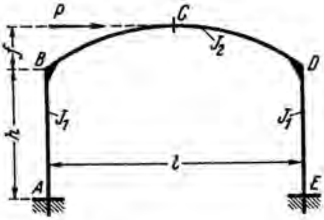
$$M_C = 0; \qquad H_E = -H_A = qf; \qquad V_E = -V_A = \frac{qf^2}{l} + \frac{2M_B}{l}.$$

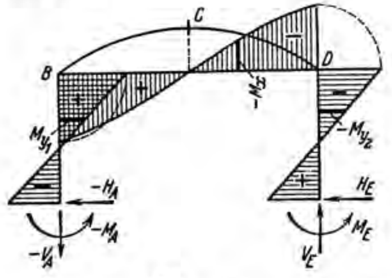
Within the limits of BC: $\quad M_x = +qf\cdot y - \frac{qy^2}{2} + M_B - V_E\cdot x$

Within the limits of DC: $\quad M'_x = -qf\cdot y + \frac{qy^2}{2} - M_B + V_E\cdot x'$

$$M_y = \frac{y'_1}{h}M_A + \frac{y_1}{h}M_B.$$

Case 105/8: Horizontal concentrated load at C





$$M_B = -M_D = \frac{Ph(12k-\varphi)}{8N_2} \qquad M_A = -M_E = -\frac{Ph}{2} + M_B \qquad M_C = 0;$$

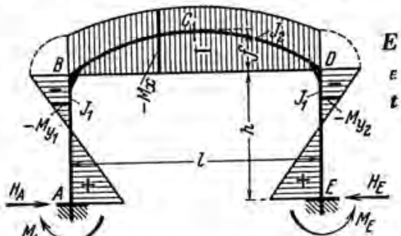
$$V_E = -V_A = \frac{Pf + 2M_B}{l}; \qquad H_E = -H_A = \frac{P}{2}.$$

Within the limits of BC: $\quad M_x = +M_B + \frac{P}{2}y - V_E \cdot x$

Within the limits of DC: $\quad M'_x = -M_B - \frac{P}{2}y + V_E \cdot x'.$

$$M_{y1} = -M_{y2} = M_A + \frac{P}{2}y_1.$$

Case 105/9: Uniform increase in temperature of the entire frame*



E = Modulus of elasticity
ε = Coefficient of thermal expansion
t = Change of temperature in degrees

Constants: $\quad T = \frac{3EJ_2\varepsilon t}{hN_1}.$

$$M_A = M_E = +T(K_2 - R) \qquad M_B = M_D = -TR$$

$$M_C = M_B - TK_2\varphi; \qquad M_{y1} = M_{y2} = M_A - H_A y_1;$$

$$H_A = H_E = \frac{TK_2}{h}; \qquad M_x = M_B - TK_2\frac{y}{h}.$$

Note: If the temperature decreases, the direction of all forces is reversed, and the signs of all moments are reversed.

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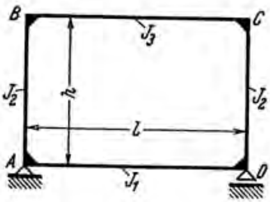
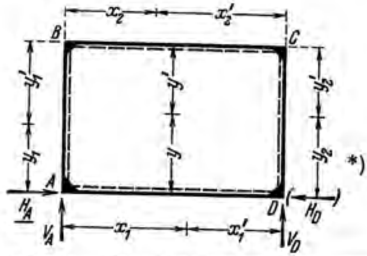
* Only the temperature change of the girder causes stress. Uniform and simultaneous temperature change in both legs produces no moments or forces. For an antisymmetrical change in temperature (left leg $+t$, right leg $-t$) substitute in the formulas of the special case 105/Ib (p. 389) the following: $\mathfrak{L} = 12EJ_2 h \cdot \varepsilon t/l^2$, as well as $\mathfrak{S}_r = 0$ and $M_x^0 = 0$.

Frame 106

Symmetrical Vierendeel frame. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h}{l};$$

$$K_1 = 2k_2 + 3 \qquad K_2 = 3k_1 + 2k_2 \qquad R_1 = 3k_2 + 1 \qquad R_2 = k_1 + 3k_2;$$

$$F_1 = K_1 K_2 - k_2^2 \qquad F_2 = 1 + k_1 + 6k_2.$$

Notations for the axial forces:

in bottom girder N_1	in left leg N_2
in top girder N_3	in right leg N_2'.

Note: Axial compression is called positive; tension is called negative.

Formulas for the moments at any point of those members of Frame 106 which do not carry any external load

$$M_{x1} = \frac{x_1'}{l} M_A + \frac{x_1}{l} M_D \qquad M_{x2} = \frac{x_2'}{l} M_B + \frac{x_2}{l} M_C$$

$$M_{y1} = \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_{y2} = \frac{y_2}{h} M_C + \frac{y_2'}{h} M_D.$$

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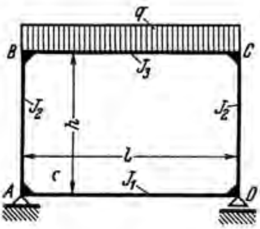
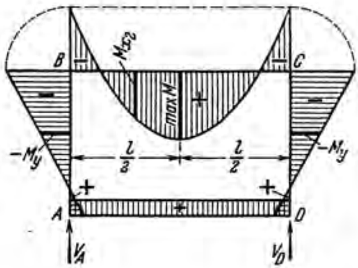
*H_D occurs when the hinged support is at D.

Case 106/1: Rectangular load at the top girder



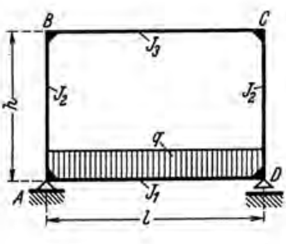


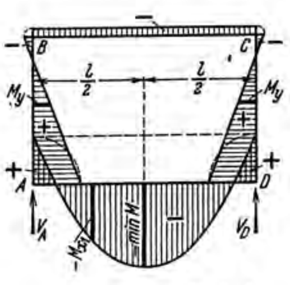
$$M_A = M_D = +\frac{q\,l^2}{4}\cdot\frac{k_2}{F_1} \qquad M_B = M_C = -\frac{q\,l^2}{4}\cdot\frac{K_2}{F_1};$$

$$M_{x2} = \frac{q\,x_2\,x_2'}{2} + M_B \qquad \max M = \frac{q\,l^2}{8} + M_B\,;$$

$$V_A = V_D = \frac{q\,l}{2}; \qquad N_3 = -N_1 = \frac{M_A - M_B}{h} \qquad N_2 = N_2' = \frac{q\,l}{2}.$$

Case 106/2: Rectangular load at the bottom girder





$$M_A = M_D = +\frac{q\,l^2}{4}\cdot\frac{k_1 K_1}{F_1} \qquad M_B = M_C = -\frac{q\,l^2}{4}\cdot\frac{k_1 k_2}{F_1};$$

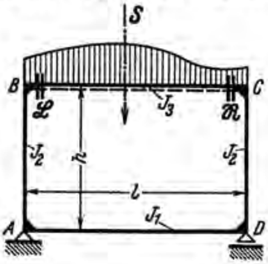
$$M_{x1} = -\frac{q\,x_1\,x_1'}{2} + M_A \qquad \min M = -\frac{q\,l^2}{8} + M_A\,;$$

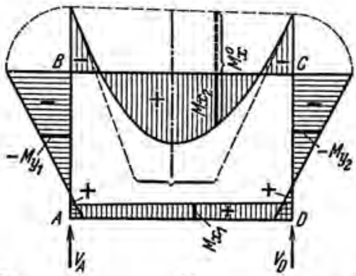
$$V_A = V_D = \frac{q\,l}{2}; \qquad N_3 = -N_1 = \frac{M_A - M_B}{h} \qquad N_2 = N_2' = 0.$$

FRAME 106

See Appendix A, Load Terms, pp. 440-445.

Case 106/3: Top girder loaded by any type of vertical load





$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = +\frac{(\mathfrak{L}+\mathfrak{R})\,k_2}{2F_1} \mp \frac{(\mathfrak{L}-\mathfrak{R})}{2F_2}; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l};$$

$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -\frac{(\mathfrak{L}+\mathfrak{R})\,K_2}{2F_1} \mp \frac{(\mathfrak{L}-\mathfrak{R})}{2F_2} \qquad M_{x2} = M_x^0 + \frac{x_2'}{l} M_B + \frac{x_2}{l} M_C;$$

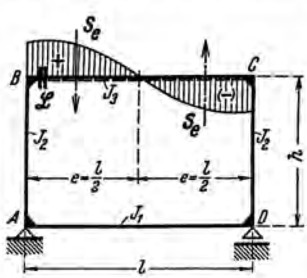
$$N_3 = -N_1 = \frac{M_A - M_B}{h} \qquad \left.\begin{matrix} N_2 = V_A \\ N_2' = V_D \end{matrix}\right\rangle \pm \frac{(\mathfrak{L}-\mathfrak{R})}{l F_2}.$$

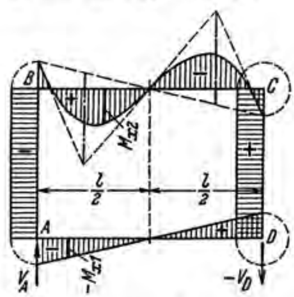
Special case 106/3a: Symmetrical girder load ($\mathfrak{R} = \mathfrak{L}$; $\mathfrak{S}_l = \mathfrak{S}_r$).

$$M_A = M_D = +\mathfrak{L}\cdot\frac{k_2}{F_1} \qquad M_B = M_C = -\mathfrak{L}\cdot\frac{K_2}{F_1} \qquad M_{x2} = M_x^0 + M_B;$$

$$V_A = V_D = \frac{S}{2}; \qquad N_3 = -N_1 = \frac{M_A - M_B}{h} \qquad N_2 = N_2' = \frac{S}{2}.$$

Case 106/4: Top girder loaded by any type of antisymmetrical load
(Special case to case 106/3 with $\mathfrak{R} = -\mathfrak{L}$; $\mathfrak{S}_l = -\mathfrak{S}_r$).



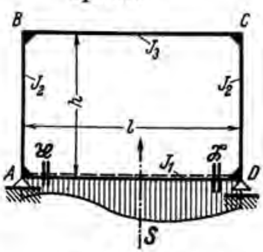


$$M_D = M_C = -M_B = -M_A = \frac{\mathfrak{L}}{F_2} \qquad M_{x2} = M_x^0 + \frac{x_2' - x_2}{l}\cdot M_B;$$

$$V_A = -V_D = \frac{\mathfrak{S}_r}{l}; \qquad N_1 = N_3 = 0 \qquad N_2 = -N_2' = \frac{\mathfrak{S}_r + 2M_C}{l}.$$

See Appendix A, Load Terms, pp. 440-445.

Case 106/5: Bottom girder loaded by any type of vertical load, acting upward*



$$\left.\begin{matrix}M_A\\M_D\end{matrix}\right\rangle = -\frac{(\mathfrak{L}+\mathfrak{R})k_1K_1}{2F_1} \pm \frac{(\mathfrak{L}-\mathfrak{R})k_1}{2F_2} \qquad M_{x1} = M_x^0 + \frac{x_1'}{l}M_A + \frac{x_1}{l}M_D;$$

$$\left.\begin{matrix}M_B\\M_C\end{matrix}\right\rangle = +\frac{(\mathfrak{L}+\mathfrak{R})k_1k_2}{2F_1} \pm \frac{(\mathfrak{L}-\mathfrak{R})k_1}{2F_2}; \qquad V_A = -\frac{\mathfrak{S}_l}{l} \qquad V_D = -\frac{\mathfrak{S}_r}{l};$$

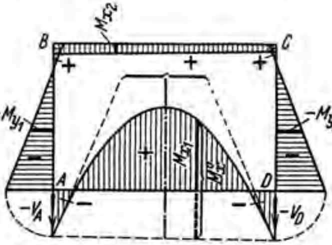
$$N_1 = -N_3 = \frac{M_B - M_A}{h} \qquad N_2' = -N_2 = \frac{(\mathfrak{L}-\mathfrak{R})k_1}{lF_2}.$$

Special case 106/5a: Symmetrical girder load ($\mathfrak{R}=\mathfrak{L};\ \mathfrak{S}_l=\mathfrak{S}_r$)

$$M_A = M_D = -\mathfrak{L}\cdot\frac{k_1K_1}{F_1} \qquad M_B = M_C = +\mathfrak{L}\cdot\frac{k_1k_2}{F_1} \qquad M_{x1} = M_x^0 + M_A;$$

$$V_A = V_D = -\frac{S}{2}; \qquad N_1 = -N_3 = \frac{M_B - M_A}{h} \qquad N_2 = N_2' = 0.$$

Case 106/6: Bottom girder loaded by any type of antisymmetrical load (Special case to case 106/5 with $\mathfrak{R} = -\mathfrak{L};\ \mathfrak{S}_l = -\mathfrak{S}_r$).



$$M_A = M_B = -M_C = -M_D = \frac{\mathfrak{L}k_1}{F_2} \qquad M_{x1} = M_x^0 + \frac{x_1' - x_1}{l}\cdot M_A;$$

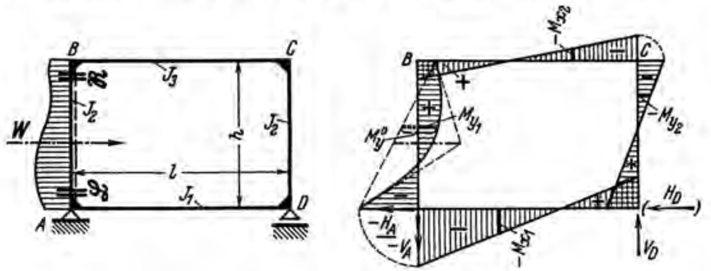
$$V_A = -V_D = \frac{\mathfrak{S}_r}{l}. \qquad N_1 = N_3 = 0 \qquad N_2 = -N_2 = \frac{2M_B}{l}.$$

* Corresponding to the position of the dashed line (throughout on the inside of the frame) a load on the lower girder working upwards is positive. With opposite direction of the load $\mathfrak{L}\ \mathfrak{R}\ \mathfrak{S}_r\ \mathfrak{S}_l$ are to be set in the formulas with negative signs.

FRAME 106

See Appendix A, Load Terms, pp. 440-445.

Case 106/7: Left-hand leg loaded by any type of horizontal load



$$\left.\begin{matrix}M_A\\M_D\end{matrix}\right\rangle = -k_2\frac{\mathfrak{L}K_1-\mathfrak{R}k_2}{2F_1} \mp \frac{\mathfrak{S}_l R_1+(\mathfrak{L}+\mathfrak{R})k_2}{2F_2} \qquad \underline{H_A = -W}$$

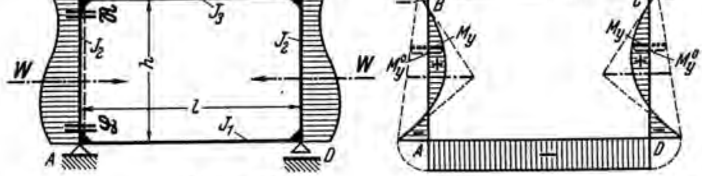
$$\left.\begin{matrix}M_B\\M_C\end{matrix}\right\rangle = -k_2\frac{\mathfrak{R}K_2-\mathfrak{L}k_2}{2F_1} \pm \frac{\mathfrak{S}_l R_2-(\mathfrak{L}+\mathfrak{R})k_2}{2F_2}\,; \qquad (H_D = +W)\,;$$

$$V_D = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad M_{y1} = M_y^0 + \frac{y_1'}{h}M_A + \frac{y_1}{h}M_B\,;$$

$$\left.\begin{matrix}N_3\\\underline{N_1}\end{matrix}\right\rangle = \pm\frac{M_D-M_C}{h} \qquad \left(N_1 = W - \frac{M_D-M_C}{h}\right) \qquad \left.\begin{matrix}N_2\\N_2'\end{matrix}\right\rangle = \mp\frac{M_B-M_C}{l}.$$

Note: If the hinged support is at D, use the values in parentheses instead of the underlined values.

Case 106/8: Both legs loaded by any type of symmetrical external horizontal load



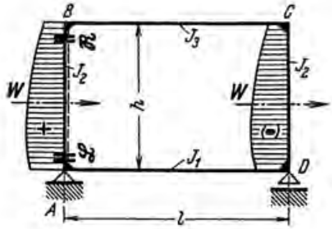
$$M_A = M_D = -k_2\frac{\mathfrak{L}K_1-\mathfrak{R}k_2}{F_1} \qquad M_B = M_C = -k_2\frac{\mathfrak{R}K_2-\mathfrak{L}k_2}{F_1}$$

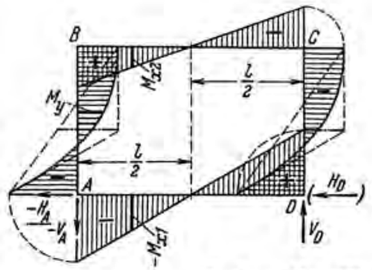
$$M_y = M_y^0 + \frac{y_1'}{h}M_A + \frac{y_1}{h}M_B\,; \qquad V_A = V_D = 0\,;$$

$$N_1 = \frac{\mathfrak{S}_r}{h} + \frac{M_B-M_A}{h} \qquad N_3 = \frac{\mathfrak{S}_l}{h} + \frac{M_A-M_B}{h} \qquad N_2 = N_2' = 0\,.$$

Note: All terms refer to the left leg.

Case 106/9: Both legs loaded by any type of antisymmetrical horizontal load from the left





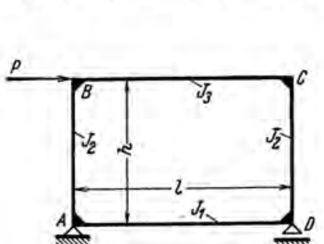
$$M_D = -M_A = \frac{\mathfrak{S}_l R_1 + (\mathfrak{L}+\mathfrak{R})k_2}{F_2} \qquad M_B = -M_C = \frac{\mathfrak{S}_l R_2 - (\mathfrak{L}+\mathfrak{R})k_2}{F_2};$$

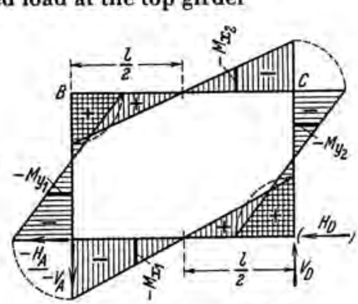
$$V_D = -V_A = \frac{2\mathfrak{S}_l}{l}; \qquad N_2' = -N_2 = \frac{2M_B}{l} \qquad N_3 = 0;$$

$$\underline{H_A} = -2W \qquad (H_D = +2W); \qquad \underline{N_1} = -W \qquad (N_1 = +W).$$

Note: If the hinged support is at D, use the values in parentheses instead of the underlined values. All the load terms refer to the left leg.

Case 106/10: Horizontal concentrated load at the top girder





$$M_B = -M_C = +\frac{PhR_2}{2F_2} \qquad M_D = -M_A = +\frac{PhR_1}{2F_2};$$

$$V_D = -V_A = +\frac{Ph}{l}; \qquad N_3 = -\underline{N_1} = +\frac{P}{2} \qquad N_2' = -N_2 = \frac{2M_B}{l}$$

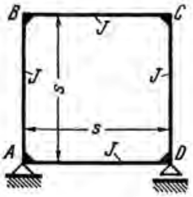
$$\underline{H_A} = -P \qquad (H_D = +P); \qquad \left(N_1 = +\frac{P}{2}\right).$$

Note: If the hinged support is at D, use the values in parentheses instead of the underlined values.

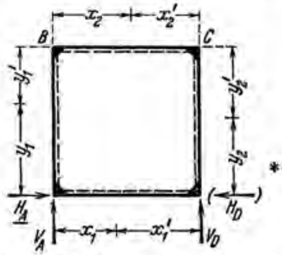
Case 106/11: Uniform change in temperature of the entire frame. No moments or forces occur.

Frame 107

Symmetrical square Vierendeel frame. Externally simply supported. All members having equal moments of inertia.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Notations for the axial loads acting at the

lower girder N_1	left leg N_2
upper girder N_3	right leg N_2'.

Note: Axial compression is called positive; tension is called negative.

Formulas for the moments at any point of Frame 107 for any load

The moments at the joints contribute to the total moment:

$$M_{x1}=\frac{x_1'}{s}M_A+\frac{x_1}{s}M_D \qquad M_{x2}=\frac{x_2'}{s}M_B+\frac{x_2}{s}M_C$$

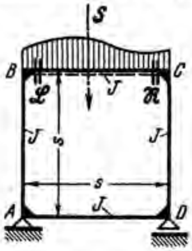
$$M_{y1}=\frac{y_1'}{s}M_A+\frac{y_1}{s}M_B \qquad M_{y2}=\frac{y_2}{s}M_C+\frac{y_2'}{s}M_D.$$

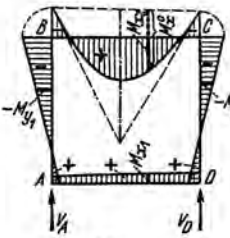
For the members that carry the load, add the value of $M_x{}^0$ or $M_y{}^0$ respectively.

* H_D occurs when the hinged support is at D.

See Appendix A, Load Terms, pp. 440-445.

Case 107/1: Top girder loaded by any type of vertical load





$$V_A = \frac{\mathfrak{S}_r}{s} \qquad V_D = \frac{\mathfrak{S}_l}{s};$$

$$N_3 = -N_1 = +\frac{(\mathfrak{L}+\mathfrak{R})}{8s}$$

$$\left.\begin{matrix} N_2 = V_A \\ N_2' = V_D \end{matrix}\right\rangle \pm \frac{(\mathfrak{L}-\mathfrak{R})}{8s};$$

$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = +\frac{(\mathfrak{L}+\mathfrak{R})}{48} \mp \frac{(\mathfrak{L}-\mathfrak{R})}{16} \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -\frac{5(\mathfrak{L}+\mathfrak{R})}{48} \mp \frac{(\mathfrak{L}-\mathfrak{R})}{16}.$$

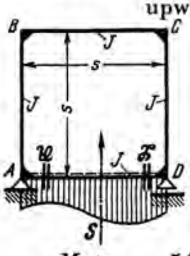
Special case 107/1a: Symmetrical girder load ($\mathfrak{R} = \mathfrak{L}$; $\mathfrak{S}_l = \mathfrak{S}_r$).

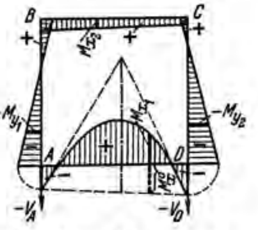
$$M_A = M_D = +\frac{\mathfrak{L}}{24} \qquad M_B = M_C = -\frac{5\mathfrak{L}}{24}; \qquad V_A = V_D = N_2 = N_2' = \frac{S}{2}.$$

Special case 107/1b: Antisymmetrical girder load ($\mathfrak{R} = -\mathfrak{L}$; $\mathfrak{S}_l = -\mathfrak{S}_r$).

$$M_D = M_C = -M_B = -M_A = \frac{\mathfrak{L}}{8}; \quad V_A = -V_D = \frac{\mathfrak{S}_r}{s}; \quad N_2 = -N_2' = \frac{\mathfrak{S}_r}{s} + \frac{\mathfrak{L}}{4s}.$$

Case 107/2: Bottom girder loaded by any type of vertical load, acting upward*





$$V_A = -\frac{\mathfrak{S}_l}{s} \qquad V_D = -\frac{\mathfrak{S}_r}{s};$$

$$N_1 = -N_3 = +\frac{(\mathfrak{L}+\mathfrak{R})}{8s}$$

$$N_2' = -N_2 = +\frac{(\mathfrak{L}-\mathfrak{R})}{8s};$$

$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -\frac{5(\mathfrak{L}+\mathfrak{R})}{48} \pm \frac{(\mathfrak{L}-\mathfrak{R})}{16} \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = +\frac{(\mathfrak{L}+\mathfrak{R})}{48} \pm \frac{(\mathfrak{L}-\mathfrak{R})}{16}.$$

Special case 107/2a: Symmetrical girder load ($\mathfrak{R} = \mathfrak{L}$; $\mathfrak{S}_l = \mathfrak{S}_r$).

$$M_A = M_D = -\frac{5\mathfrak{L}}{24} \qquad M_B = M_C = +\frac{\mathfrak{L}}{24}; \qquad V_A = V_D = -\frac{S}{2}.$$

Special case 107/2b: Antisymmetrical girder load ($\mathfrak{R} = -\mathfrak{L}$; $\mathfrak{S}_l = -\mathfrak{S}_r$).

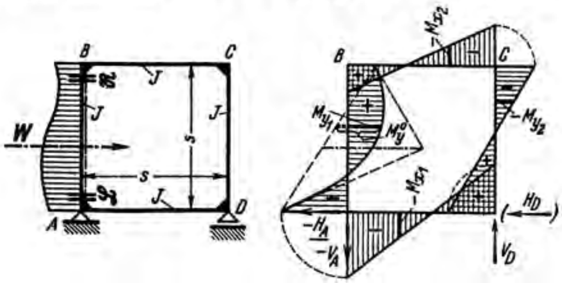
$$M_A = M_B = -M_C = -M_D = \frac{\mathfrak{L}}{8} \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l} \qquad N_2' = -N_2 = \frac{\mathfrak{L}}{4s}.$$

*See footnote on page 397.

FRAME 107

See Appendix A, Load Terms, pp. 440-445.

Case 107/3: Left-hand leg loaded by any type of horizontal load



$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\} = -\frac{5\mathfrak{L}-\mathfrak{R}}{48} \mp \frac{4\mathfrak{S}_l+(\mathfrak{L}+\mathfrak{R})}{16} \qquad \underline{H_A} = -W$$

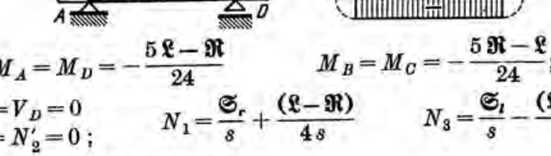
$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\} = -\frac{5\mathfrak{R}-\mathfrak{L}}{48} \pm \frac{4\mathfrak{S}_l-(\mathfrak{L}+\mathfrak{R})}{16}; \qquad (H_D = +W);$$

$$V_D = -V_A = \frac{\mathfrak{S}_l}{s}; \qquad N_3 = -\underline{N_1} = \frac{M_D - M_C}{s}$$

$$N_2' = -N_2 = \frac{4\mathfrak{S}_l - (\mathfrak{L}+\mathfrak{R})}{8s} \qquad \left(N_1 = W - \frac{M_D - M_C}{s}\right).$$

Note: If the hinged support is at D, use the values in parentheses instead of the underlined values.

Case 107/4: Both legs loaded by any type of symmetrical external horizontal load



$$M_A = M_D = -\frac{5\mathfrak{L}-\mathfrak{R}}{24} \qquad M_B = M_C = -\frac{5\mathfrak{R}-\mathfrak{L}}{24};$$

$$V_A = V_D = 0 \qquad N_1 = \frac{\mathfrak{S}_r}{s} + \frac{(\mathfrak{L}-\mathfrak{R})}{4s} \qquad N_3 = \frac{\mathfrak{S}_l}{s} - \frac{(\mathfrak{L}-\mathfrak{R})}{4s}$$

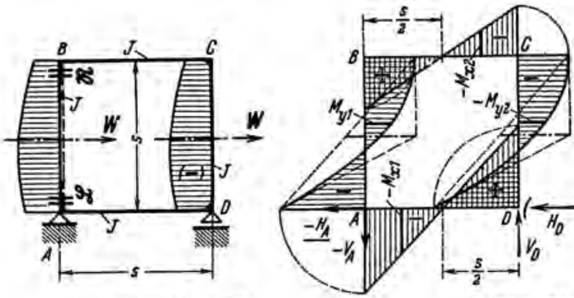
$$N_2 = N_2' = 0;$$

Note: All terms refer to the left leg.

Special case 107/4a: Loads symmetrical about a horizontal axis ($\mathfrak{R} = \mathfrak{L}$)

$$M_A = M_B = M_C = M_D = -\frac{\mathfrak{L}}{6}; \qquad N_1 = N_3 = \frac{W}{2}.$$

Case 107/5: Both legs loaded by any type of antisymmetrical horizontal load, acting from the left



$$M_D = -M_A = \frac{4\,\mathfrak{S}_l + (\mathfrak{L} + \mathfrak{R})}{8} \qquad M_B = -M_C = \frac{4\,\mathfrak{S}_l - (\mathfrak{L} + \mathfrak{R})}{8};$$

$$V_D = -V_A = \frac{2\,\mathfrak{S}_l}{s}; \qquad N_2' = -N_2 = \frac{2M_B}{s} \qquad N_3 = 0;$$

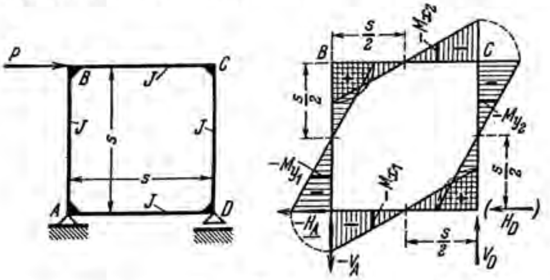
$$\underline{H_A} = -2W \quad (H_D = +2W); \qquad \underline{N_1} = -W \quad (N_1 = +W).$$

Note: All load terms refer to the left member. If the hinged support is at D, use the values in parentheses instead of the underlined values.

Special case 107/5a: Loads symmetrical about a horizontal axis $(\mathfrak{R} = \mathfrak{L})$.

$$M_D = -M_A = \frac{Ws + \mathfrak{L}}{4} \qquad M_B = -M_C = \frac{Ws - \mathfrak{L}}{4}; \qquad V_D = -V_A = W.$$

Case 107/6: Horizontal concentrated load at the top girder



$$M_B = M_D = -M_A = -M_C = \frac{Ps}{4}; \qquad V_D = -V_A = P;$$

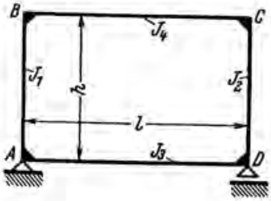
$$N_3 = N_2' = +\frac{P}{2} \qquad \underline{N_1} = N_2 = -\frac{P}{2} \qquad \left(N_1 = +\frac{P}{2}\right);$$

$$\underline{H_A} = -W \qquad (H_D = +W).$$

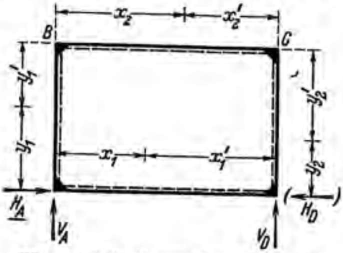
Note: If the hinged support is at D, use the values in parentheses instead of the underlined values.

Frame 108

Vierendeel frame. Externally simply supported. All members having different moments of inertia.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_4}{J_1} \cdot \frac{h}{l} \qquad k = \frac{J_4}{J_3} \qquad k_2 = \frac{J_4}{J_2} \cdot \frac{h}{l}$$

$$r_1 = k_1 + k \qquad r = 1 + k \qquad r_2 = k + k_2$$

$$R_1 = 2(3k_1 + r) \qquad R = 2(r_1 + k + r_2) \qquad R_2 = 2(r + 3k_2);$$

$$F = R(R_1 R_2 - r^2) - 9(R_1 r_2^2 - 2 r r_1 r_2 + R_2 r_1^2).$$

$$n_{11} = \frac{R R_2 - 9 r_2^2}{F} \qquad n_{12} = n_{21} = \frac{9 r_1 r_2 - R r}{F}$$

$$n_{22} = \frac{R R_1 - 9 r_1^2}{F} \qquad n_{13} = n_{31} = \frac{3(r_1 R_2 - r r_2)}{F}$$

$$n_{33} = \frac{R_1 R_2 - r^2}{F} \qquad n_{23} = n_{32} = \frac{3(R_1 r_2 - r_1 r)}{F}.$$

Notations for the axial loads acting at the

left leg N_1	lower girder N_3
right leg N_2	upper girder N_4.

Note: Axial compression is called positive; tension is called negative.

Formulas for the moments at any point of those members of Frame 108 which do not carry any external load

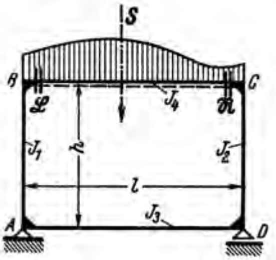
$$M_{x1} = \frac{x_1'}{l} M_A + \frac{x_1}{l} M_D \qquad M_{x2} = \frac{x_2'}{l} M_B + \frac{x_2}{l} M_C$$

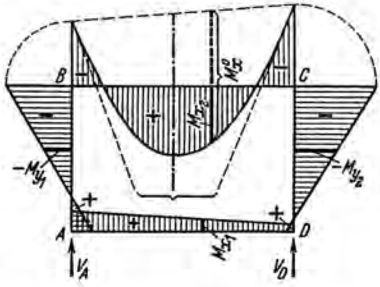
$$M_{y1} = \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_{y2} = \frac{y_2}{h} M_C + \frac{y_2'}{h} M_D.$$

* H_D occurs when the hinged support is at D.

See Appendix A, Load Terms, pp. 440-445.

Case 108/1: Top girder loaded by any type of vertical load





Constants:

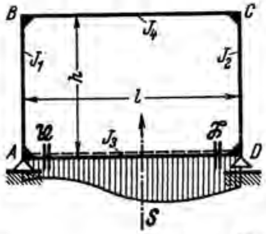
$$X_1 = \mathfrak{L} n_{11} + \mathfrak{R} n_{21} \qquad X_2 = \mathfrak{L} n_{12} + \mathfrak{R} n_{22} \qquad X_3 = \mathfrak{L} n_{13} + \mathfrak{R} n_{23}\,.$$

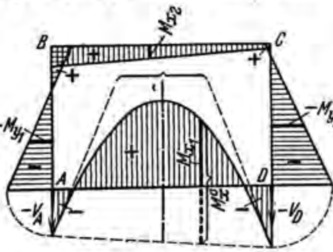
$$M_B = -X_1 \qquad M_C = -X_2 \qquad M_A = X_3 - X_1 \qquad M_D = X_3 - X_2;$$

$$M_{x2} = M_x^0 + \frac{x_2'}{l} M_B + \frac{x_2}{l} M_C\,; \qquad V_A = \frac{\mathfrak{S}_r}{l} \qquad V_D = \frac{\mathfrak{S}_l}{l}\,;$$

$$N_1 = V_A + \frac{X_1 - X_2}{l} \qquad N_2 = V_D - \frac{X_1 - X_2}{l} \qquad N_4 = -N_3 = \frac{X_3}{h}\,.$$

Case 108/2: Bottom girder loaded by any type of vertical load, acting upward*





Constants:

$$X_1 = k\,[-\mathfrak{R} n_{11} - \mathfrak{L} n_{21} + (\mathfrak{L} + \mathfrak{R})\, n_{31}]$$
$$X_2 = k\,[-\mathfrak{R} n_{12} - \mathfrak{L} n_{22} + (\mathfrak{L} + \mathfrak{R})\, n_{32}]$$
$$X_3 = k\,[-\mathfrak{R} n_{13} - \mathfrak{L} n_{23} + (\mathfrak{L} + \mathfrak{R})\, n_{33}]$$

$$M_B = +X_1 \qquad M_C = +X_2 \qquad M_A = -X_3 + X_1 \qquad M_D = -X_3 + X_2;$$

$$M_{x1} = M_x^0 + \frac{x_1'}{l} M_A + \frac{x_1}{l} M_D\,; \qquad V_A = -\frac{\mathfrak{S}_l}{l} \qquad V_D = -\frac{\mathfrak{S}_r}{l};$$

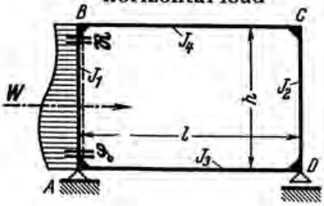
$$N_1 = -N_2 = \frac{X_2 - X_1}{l} \qquad N_3 = -N_4 = \frac{X_3}{h}\,.$$

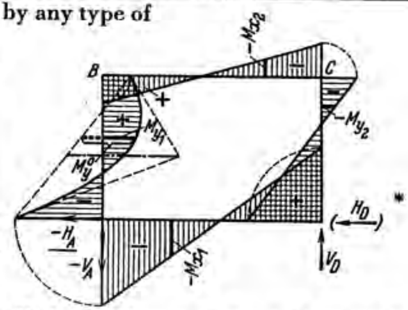
*See footnote on page 397.

FRAME 108

See Appendix A, Load Terms, pp. 440-445.

Case 108/3: Left-hand leg loaded by any type of horizontal load





Constants: $\mathfrak{B}_1 = \mathfrak{S}_l(k_1 + 2r_1) - (\mathfrak{L} + \mathfrak{R})\,k_1$ $\qquad X_1 = +\mathfrak{B}_1 n_{11} + \mathfrak{B}_2 n_{21} - \mathfrak{B}_3 n_{31}$

$\mathfrak{B}_2 = \mathfrak{S}_l k$ $\qquad X_2 = -\mathfrak{B}_1 n_{12} - \mathfrak{B}_2 n_{22} + \mathfrak{B}_3 n_{32}$

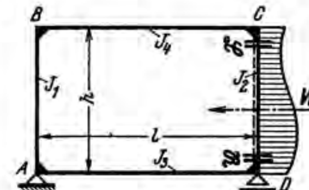
$\mathfrak{B}_3 = \mathfrak{S}_l(2r_1 + k) - \mathfrak{L} k_1;$ $\qquad X_3 = -\mathfrak{B}_1 n_{13} - \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$

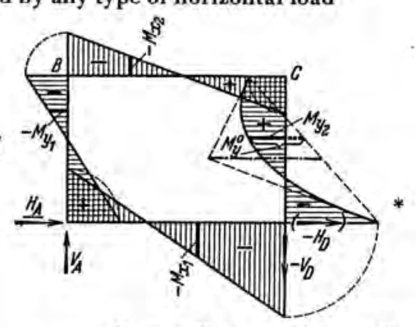
$M_B = +X_1 \qquad M_C = -X_2 \qquad M_A = -\mathfrak{S}_l + X_1 + X_3 \qquad M_D = +X_3 - X_2;$

$M_{y1} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B; \qquad V_D = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad \underline{H_A} = -W \quad (H_D = +W);$

$$N_2 = -N_1 = \frac{X_1 + X_2}{l} \qquad N_4 = -\underline{N_3} = \frac{X_3}{h} \qquad \left(N_3 = W - \frac{X_3}{h}\right).$$

Case 108/4: Right-hand leg loaded by any type of horizontal load





Constants: $\mathfrak{B}_1 = \mathfrak{S}_r k$ $\qquad X_1 = -\mathfrak{B}_1 n_{11} - \mathfrak{B}_2 n_{21} + \mathfrak{B}_3 n_{31}$

$\mathfrak{B}_2 = \mathfrak{S}_r(2r_2 + k_2) - (\mathfrak{L} + \mathfrak{R})\,k_2$ $\qquad X_2 = +\mathfrak{B}_1 n_{12} + \mathfrak{B}_2 n_{22} - \mathfrak{B}_3 n_{32}$

$\mathfrak{B}_3 = \mathfrak{S}_r(k + 2r_2) - \mathfrak{R} k_2;$ $\qquad X_3 = -\mathfrak{B}_1 n_{13} - \mathfrak{B}_2 n_{23} + \mathfrak{B}_3 n_{33}.$

$M_B = -X_1 \qquad M_C = +X_2 \qquad M_A = +X_3 - X_1 \qquad M_D = -\mathfrak{S}_r + X_2 + X_3;$

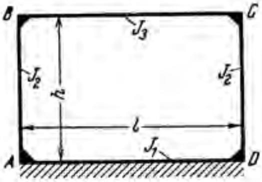
$M_{y2} = M_y^0 + \frac{y_2}{h} M_C + \frac{y_2'}{h} M_D; \qquad V_A = -V_D = \frac{\mathfrak{S}_r}{l}; \qquad \underline{H_A} = +W \quad (H_D = -W);$

$$N_1 = -N_2 = \frac{X_1 + X_2}{l} \qquad N_4 = \frac{X_3}{h} \qquad \underline{N_3} = W - \frac{X_3}{h} \qquad \left(N_3 = -\frac{X_3}{h}\right).$$

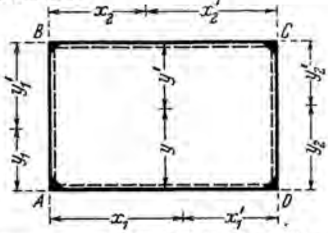
* If the hinged support is at D, use the values in parentheses instead of the underlined values.

Frame 109

Symmetrical Vierendeel frame on continuous elastic foundation.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1 = \frac{J_3}{J_1} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{h}{l}; \qquad K_1 = 2k_2 + 3 \qquad K_2 = 3k_1 + 2k_2$$

$$K_3 = 3k_2 + 1 - \frac{k_1}{5} \qquad K_4 = \frac{6k_1}{5} + 3k_2; \qquad F_1 = K_1 K_2 - k_2^2 \qquad F_2 = 1 + k_1 + 6k_2.$$

Notations for the axial loads acting at the

lower girder N_1	left leg N_2
upper girder N_3	right leg N_2'

Note: Axial compression is called positive; tension is called negative.

Note:

All formulas for Frame 109 are based on a straight line distribution of the soil pressure.**
The computations for unsymmetrical loading show a negative pressure, which is possible only if it is balanced by or smaller than the positive soil pressure caused by other loads.

Formulas for the moments at any point of those members of Frame 109 which do not carry any external load

$$M_{y1} = \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B \qquad M_{x2} = \frac{x_2'}{l} M_B + \frac{x_2}{l} M_C \qquad M_{y2} = \frac{y_2}{h} M_C + \frac{y_2'}{h} M_D.$$

Constants for the computation of M_{x1}:

$$\omega_D' = \frac{x_1'}{l} - \left(\frac{x_1'}{l}\right)^3, \qquad \omega_D = \frac{x_1}{l} - \left(\frac{x_1}{l}\right)^3, \qquad \omega_V = \frac{x_1 x_1'}{l^2} \cdot \frac{x_1' - x_1}{l}.*$$

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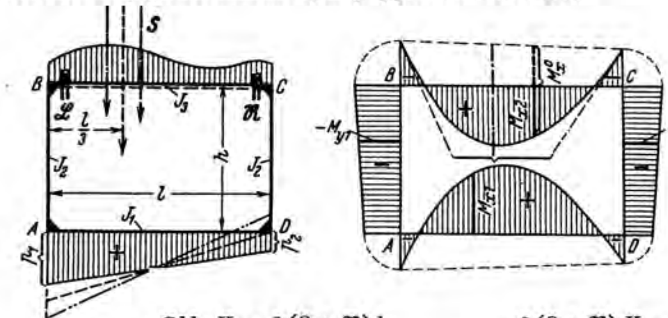
* For numerical tables for ω see "*Beam Formulas*" by A. Kleinlogel, American edition translated and adapted to American conditions by Harold G. Lorsch, Frederick Ungar Publishing Co. New York, p. 15.

** For non-linear earth pressures use frame 106 and omit the concentrated reaction forces.

FRAME 109

See Appendix A, Load Terms, pp. 440-445.

Case 109/1: Top girder loaded by any type of vertical load



Constants: $X_1 = \frac{S l k_1 K_1 - 2(\mathfrak{L} + \mathfrak{R}) k_2}{4 F_1}$ $\qquad X_2 = \frac{2(\mathfrak{L} + \mathfrak{R}) K_2 - S l k_1 k_2}{4 F_1}$

$X_3 = \frac{10(\mathfrak{L} - \mathfrak{R}) + (\mathfrak{S}_r - \mathfrak{S}_l) k_1}{20 F_2}$. $\qquad p_1 = \frac{2(2\mathfrak{S}_r - \mathfrak{S}_l)}{l^2}$

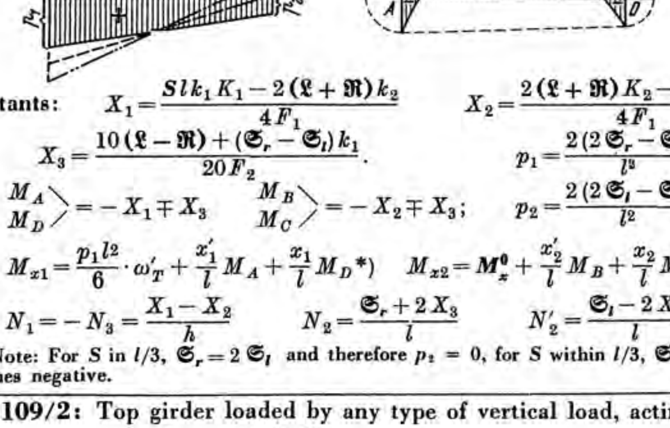
$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -X_1 \mp X_3 \qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -X_2 \mp X_3; \qquad p_2 = \frac{2(2\mathfrak{S}_l - \mathfrak{S}_r)}{l^2};$

$M_{x1} = \frac{p_1 l^2}{6} \cdot \omega'_T + \frac{x'_1}{l} M_A + \frac{x_1}{l} M_D$ *) $\qquad M_{x2} = \boldsymbol{M}^0_x + \frac{x'_2}{l} M_B + \frac{x_2}{l} M_C$;

$N_1 = -N_3 = \frac{X_1 - X_2}{h} \qquad N_2 = \frac{\mathfrak{S}_r + 2X_3}{l} \qquad N'_2 = \frac{\mathfrak{S}_l - 2X_3}{l}$.

Note: For S in $l/3$, $\mathfrak{S}_r = 2\mathfrak{S}_l$ and therefore $p_2 = 0$, for S within $l/3$, $\mathfrak{S}_r > 2\mathfrak{S}_l$, p_2 becomes negative.

Case 109/2: Top girder loaded by any type of vertical load, acting symmetrically



$M_A = M_D = -\frac{S l k_1 K_1 - 4\mathfrak{L} k_2}{4 F_1} \qquad M_{x1} = \frac{p x_1 x'_1}{2} + M_A$

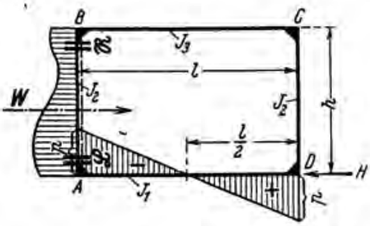
$M_B = M_C = -\frac{4\mathfrak{L} K_2 - S l k_1 k_2}{4 F_1} \qquad M_{x2} = \boldsymbol{M}^0_x + M_B$;

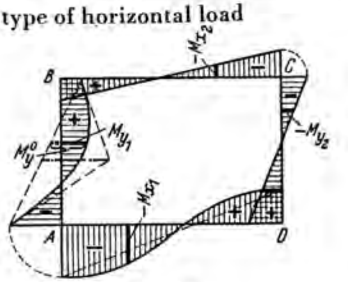
$p = \frac{S}{l}; \qquad N_3 = -N_1 = \frac{M_A - M_B}{h} \qquad N_2 = N'_2 = \frac{S}{2}$.

* $\omega'_T = \omega'_D + i\,\omega_D$ with $i = p_2/p_1$. Numerical tables for the Omega function may be found in the volume cited in the footnote on p. 407.

See Appendix A, Load Terms, pp. 440-445.

Case 109/3: Left-hand leg loaded by any type of horizontal load





$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -k_2 \frac{\mathfrak{L} K_1 - \mathfrak{R} k_2}{2F_1} \mp \frac{\mathfrak{S}_l K_3 + (\mathfrak{L} + \mathfrak{R}) k_2}{2F_2}$$

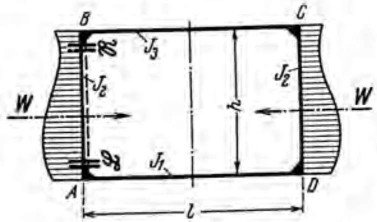
$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -k_2 \frac{\mathfrak{R} K_2 - \mathfrak{L} k_2}{2F_1} \pm \frac{\mathfrak{S}_l K_4 - (\mathfrak{L} + \mathfrak{R}) k_2}{2F_2};$$

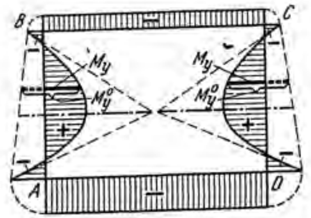
$$M_{x1} = -\mathfrak{S}_l \cdot \omega_V + \frac{x_1'}{l} M_A + \frac{x_1}{l} M_D \qquad M_{y1} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B;$$

$$N_3 = \frac{M_D - M_C}{h} \qquad N_2' = -N_2 = \frac{M_B - M_C}{l}; \qquad p = \frac{6\mathfrak{S}_l}{l^2};$$

$$H = W; \qquad (N_1 = -N_3 \text{ bzw. } N_1 = H - N_3)^* .$$

Case 109/4: Both legs loaded by any type of symmetrical external horizontal load





$$M_A = M_D = -k_2 \frac{\mathfrak{L} K_1 - \mathfrak{R} k_2}{F_1} \qquad M_B = M_C = -k_2 \frac{\mathfrak{R} K_2 - \mathfrak{L} k_2}{F_1};$$

$$M_y = M_y^0 + \frac{y'}{h} M_A + \frac{y}{h} M_B; \qquad N_2 = N_2' = 0$$

$$N_1 = \frac{\mathfrak{S}_r}{h} + \frac{M_B - M_A}{h} \qquad N_3 = \frac{\mathfrak{S}_l}{h} + \frac{M_A - M_B}{h}.$$

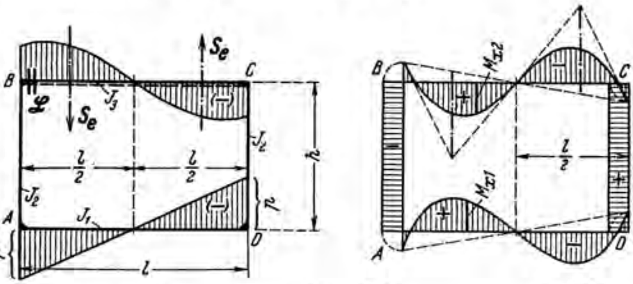
Note: All load terms refer to the left leg. There is no soil pressure.

* The values for N_1 are limit values. The actual magnitude and distribution of N_1 depend on the distribution of the shear force H (e.g., friction at the bottom).

FRAME 109

(See Appendix A, Load Terms, pp. 440-445.)

Case 109/5: Top girder loaded by any type of antisymmetrical load (Special case to case 109/1 with $\mathfrak{R} = -\mathfrak{L}$ and $\mathfrak{S}_l = -\mathfrak{S}_r$)

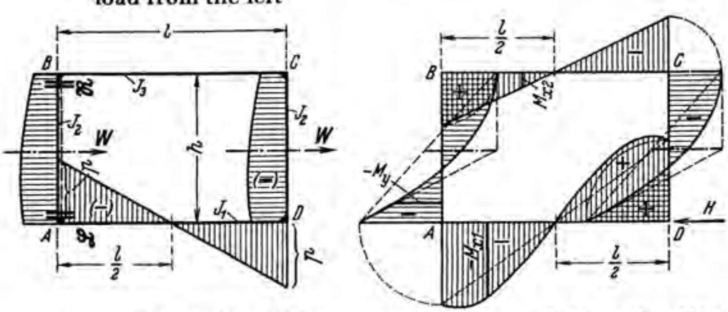


$$M_D = M_C = -M_B = -M_A = \frac{\mathfrak{L}}{F_2} + \frac{\mathfrak{S}_r k_1}{10 F_2}; \qquad p = \frac{6\,\mathfrak{S}_r}{l^2};$$

$$M_{x1} = \mathfrak{S}_r \cdot \omega_V + M_D \cdot \frac{x_1 - x_1'}{l} \qquad M_{x2} = \mathbf{M}_x^0 + M_C \cdot \frac{x_2 - x_2'}{l};$$

$$N_1 = N_3 = 0 \qquad N_2 = -N_2' = \frac{\mathfrak{S}_r + 2\,M_C}{l}.$$

Case 109/6: Both legs loaded by any type of antisymmetrical horizontal load from the left



$$M_D = -M_A = \frac{\mathfrak{S}_l K_3 + (\mathfrak{L} + \mathfrak{R})\,k_2}{F_2} \qquad M_B = -M_C = \frac{\mathfrak{S}_l K_4 - (\mathfrak{L} + \mathfrak{R})\,k_2}{F_2}$$

$$M_{x1} = -2\,\mathfrak{S}_l \cdot \omega_V + M_D \frac{x_1 - x_1'}{l} \qquad M_y = \mathbf{M}_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B;$$

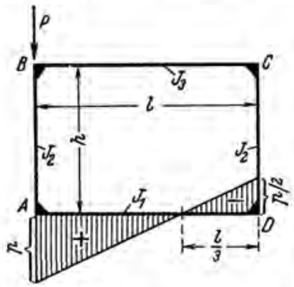
$$p = \frac{12\,\mathfrak{S}_l}{l^2}; \qquad H = 2\,W; \qquad N_2' = -N_2 = \frac{2\,M_B}{l} \qquad N_3 = 0$$

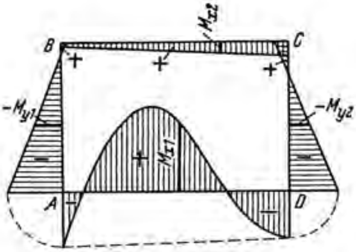
$$(N_1 = -W \;\text{ bzw. }\; N_1 = +W)^* .$$

Note: All the load terms refer to the left leg.

*See footnote on page 409.

Case 109/7: Vertical concentrated load at B



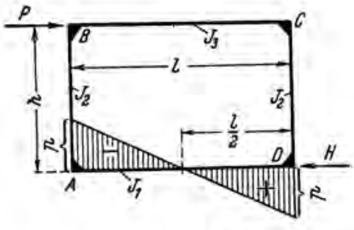


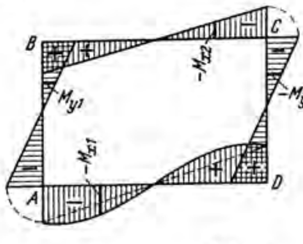
$$\begin{matrix} M_A \\ M_D \end{matrix} \Big\rangle = \frac{Plk_1}{4F_1}\left[-K_1 \mp \frac{F_1}{5F_2}\right] \qquad \begin{matrix} M_B \\ M_C \end{matrix} \Big\rangle = \frac{Plk_1}{4F_1}\left[+k_2 \mp \frac{F_1}{5F_2}\right];$$

$$M_{x1} = \frac{2Pl}{3}\cdot\omega'_T + \frac{x'_1}{l}M_A + \frac{x_1}{l}M_D{}^* ; \qquad p = \frac{4P}{l};$$

$$N_1 = -N_3 = \frac{3Plk_1(1+k_2)}{4hF_1} \qquad N'_2 = -\frac{Pk_1}{10F_2} \qquad N_2 = P - N'_2.$$

Case 109/8: Horizontal concentrated load at B





$$M_D = -M_A = +\frac{Ph}{2F_2}K_3 \qquad M_B = -M_C = +\frac{Ph}{2F_2}K_4;$$

$$M_{x1} = -Ph\cdot\omega_V + \frac{x'_1 - x_1}{l}M_A; \qquad p = \frac{6Ph}{l^2};$$

$$M_{x2} = \frac{x'_2 - x_2}{l}M_B \qquad M_{y1} = -M_{y2} = \frac{y'_1}{h}M_A + \frac{y_1}{h}M_B;$$

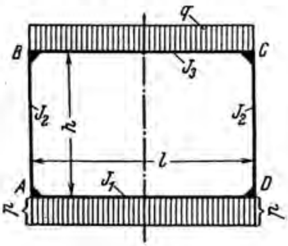
$$N'_2 = -N_2 = \frac{PhK_4}{lF_2} \qquad N_3 = \frac{P}{2} \qquad \left(N_1 = +\frac{P}{2} \text{ bzw. } N_1 = -\frac{P}{2}\right)^{**}.$$

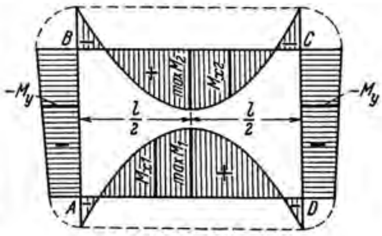
* $\omega'_T = \omega'_D - i\,\omega_D$ with $i = 1/2$. See footnote p. 408.

** See footnote p. 409.

FRAME 109

Case 109/9: Top girder with load uniformly distributed





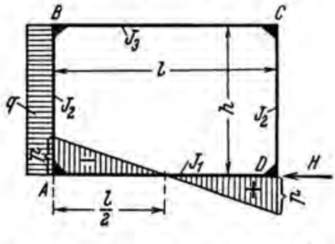
$$M_A = M_D = -\frac{q\,l^2}{4\,F_1}(k_1 K_1 - k_2) \qquad M_B = M_C = -\frac{q\,l^2}{4\,F_1}(K_2 - k_1 k_2);$$

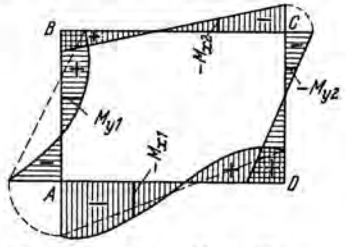
$$M_{x1} = \frac{q\,x_1 x_1'}{2} + M_A \quad M_{x2} = \frac{q\,x_2 x_2'}{2} + M_B \quad M_y = \frac{y'}{h} M_A + \frac{y}{h} M_B;$$

$$\max M_1 = \frac{q\,l^2}{8} + M_A \qquad \max M_2 = \frac{q\,l^2}{8} + M_B;$$

$$p = q; \quad N_1 = -N_3 = \frac{M_B - M_A}{h} \qquad N_2 = N_2' = \frac{q\,l}{2}.$$

Case 109/10: Left-hand leg with load uniformly distributed





$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = \frac{q\,h^2}{4}\left[-\frac{k_2(k_2+3)}{2F_1} \mp \frac{K_3+k_2}{F_2}\right] \qquad N_3 = \frac{M_D - M_C}{h}$$

$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = \frac{q\,h^2}{4}\left[-\frac{k_2(3\,k_1+k_2)}{2F_1} \pm \frac{K_4-k_2}{F_2}\right]; \qquad N_2' = -N_2 = \frac{M_B - M_C}{l};$$

$$M_{x1} = -\frac{q\,h^2}{2}\cdot\omega_V + \frac{x_1'}{l} M_A + \frac{x_1}{l} M_D \qquad M_{y1} = \frac{q\,y_1 y_1'}{2} + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B;$$

$$p = \frac{3\,q\,h^2}{l^2}; \qquad H = q\,h; \qquad (N_1 = -N_3 \text{ bzw. } N_1 = H - N_3)^*.$$

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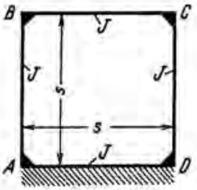
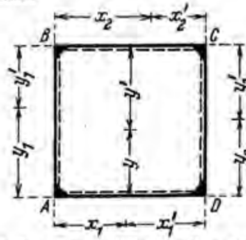
*See footnote on page 400.

Frame 110

Symmetrical square Vierendeel frame on continuous elastic foundation. All members having equal moments of inertia.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. For symmetrical loading of the frame use y and y'. Positive bending moments cause tension at the face marked by a dashed line.

Notations for the axial loads acting at the

lower girder N_1	left leg N_2
upper girder N_3	right leg N'_2.

Note: Axial compression is called positive; tension is called negative.

Note:

All formulas for Frame 110 are based on a straight line distribution of the soil pressure.**
The computations for unsymmetrical loading show a negative pressure, which is possible only if it is balanced by or smaller than the positive soil pressure caused by other loads.

Formulas for the moments at any point of those members of Frame 110 which do not carry any external load

$$M_{x2} = \frac{x_2'}{s} M_B + \frac{x_2}{s} M_C$$

$$M_{y1} = \frac{y_1'}{s} M_A + \frac{y_1}{s} M_B \qquad M_{y2} = \frac{y_2}{s} M_C + \frac{y_2'}{s} M_D .$$

Constants for the computation of M_{x1}:

$$\omega_D' = \frac{x_1'}{s} - \left(\frac{x_1'}{s}\right)^3, \qquad \omega_D = \frac{x_1}{s} - \left(\frac{x_1}{s}\right)^3, \qquad \omega_V = \frac{x_1 x_1'}{s^2} \cdot \frac{x_1' - x_1}{s} *$$

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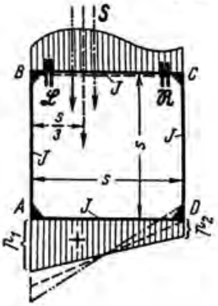
* For numerical tables see footnote p. 407.

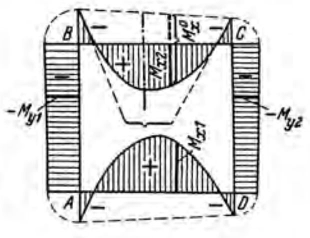
** For curvilinear soil pressure diagrams see footnote p. 407.

FRAME 110

See Appendix A, Load Terms, pp. 440-445.

Case 110/1: Top girder loaded by any type of vertical load





$$p_1 = \frac{2(2\mathfrak{S}_r - \mathfrak{S}_l)}{s^2}$$
$$p_2 = \frac{2(2\mathfrak{S}_l - \mathfrak{S}_r)}{s^2}.$$

Constants: $X_1 = \frac{5Ss - 2(\mathfrak{L}+\mathfrak{R})}{96}$ $\quad X_2 = \frac{10(\mathfrak{L}+\mathfrak{R}) - Ss}{96}$

$$X_3 = \frac{(\mathfrak{L}-\mathfrak{R})}{16} + \frac{(\mathfrak{S}_r - \mathfrak{S}_l)}{160}.$$

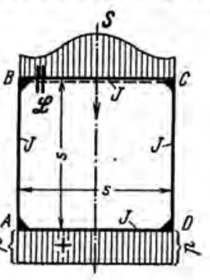
$$\begin{matrix} M_A \\ M_D \end{matrix} \Big\rangle = -X_1 \mp X_3 \qquad M_{x1} = \frac{p_1 s^2}{6}\cdot\omega'_T + \frac{x'_1}{s}M_A + \frac{x_1}{s}M_D\,{}^{*)}$$

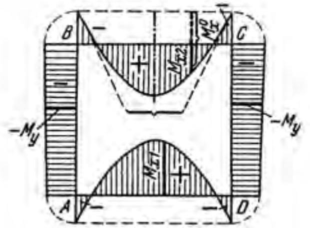
$$\begin{matrix} M_B \\ M_C \end{matrix} \Big\rangle = -X_2 \mp X_3 \qquad M_{x2} = M^0_x + \frac{x'_2}{s}M_B + \frac{x_2}{s}M_C\,;$$

$$N_1 = -N_3 = \frac{X_1 - X_2}{s} \qquad N_2 = \frac{\mathfrak{S}_r + 2X_3}{s} \qquad N'_2 = \frac{\mathfrak{S}_l - 2X_3}{s}.$$

Note: For S in $s/3$ $\mathfrak{S}_r = 2\,\mathfrak{S}_l$ and hence $p_2 = 0$; for S within $s/3$, i.e., $\mathfrak{S}_r > 2\,\mathfrak{S}_l$, p_2 becomes negative.

Case 110/2: Top girder loaded by any type of symmetrical vertical load





$$M_{x1} = \frac{p\,x_1 x'_1}{2} + M_A$$
$$M_{x2} = M^0_x + M_B\,;$$

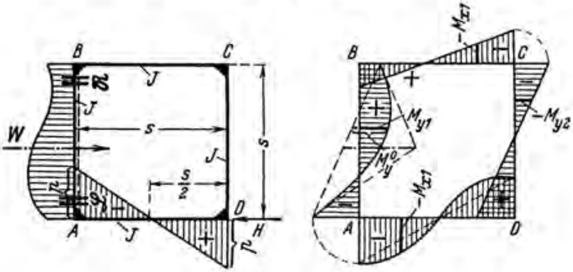
$$M_A = M_D = -\frac{5Ss - 4\mathfrak{L}}{96} \qquad M_B = M_C = -\frac{20\mathfrak{L} - Ss}{96};$$

$$p = \frac{S}{s}; \qquad N_3 = -N_1 = \frac{M_A - M_B}{s} \qquad N_2 = N'_2 = \frac{S}{2}.$$

* $\omega'_T = \omega'_D + i\,\omega_D$ with $i = p_2/p_1$. See footnote p. 408.

See Appendix A, Load Terms, pp. 440-445.

Case 110/3: Left-hand leg loaded by any type of horizontal load



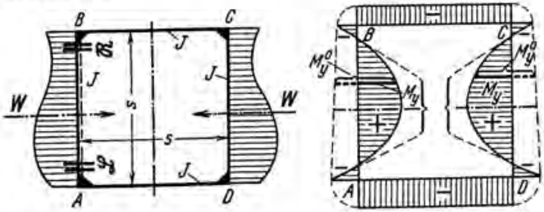
$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = -\frac{5\mathfrak{L}-\mathfrak{R}}{48} \mp \frac{19\,\mathfrak{S}_l + 5\,(\mathfrak{L}+\mathfrak{R})}{80} \qquad N_2' = -N_2 = \frac{M_B - M_C}{s}$$

$$\left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = -\frac{5\mathfrak{R}-\mathfrak{L}}{48} \pm \frac{21\,\mathfrak{S}_l - 5\,(\mathfrak{L}+\mathfrak{R})}{80}; \qquad N_3 = \frac{M_D - M_C}{s};$$

$$H = W; \qquad p = \frac{6\,\mathfrak{S}_l}{s^2}; \qquad (N_1 = -N_3 \text{ bzw. } N_1 = H - N_3)^{*};$$

$$M_{x1} = -\mathfrak{S}_l \cdot \omega_V + \frac{x_1'}{s} M_A + \frac{x_1}{s} M_D \qquad M_{y1} = \mathbf{M}_y^0 + \frac{y_1'}{s} M_A + \frac{y_1}{s} M_B .$$

Case 110/4: Both legs loaded by any type of symmetrical external horizontal load



$$M_A = M_D = -\frac{5\mathfrak{L}-\mathfrak{R}}{24} \qquad M_B = M_C = -\frac{5\mathfrak{R}-\mathfrak{L}}{24};$$

$$M_y = \mathbf{M}_y^0 + \frac{y'}{s} M_A + \frac{y}{s} M_B; \qquad N_2 = N_2' = 0$$

$$N_1 = \frac{\mathfrak{S}_r}{s} + \frac{(\mathfrak{L}-\mathfrak{R})}{4} \qquad N_3 = \frac{\mathfrak{S}_l}{s} - \frac{(\mathfrak{L}-\mathfrak{R})}{4}.$$

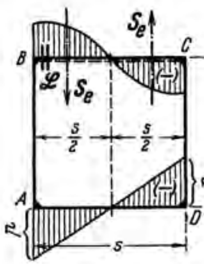
Note: All load terms refer to the left leg. There is no soil pressure.

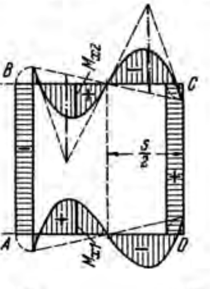
* The values for N_1 are limit values. The actual magnitude and distribution of N_1 depend on the distribution of the shear force H (*e.g.*, friction at the bottom).

FRAME 110

See Appendix A, Load Terms, pp. 440-445.

Case 110/5: Top girder loaded by any type of antisymmetrical load (Special case to case 110/1 with $\mathfrak{R} = -\mathfrak{L}$ and $\mathfrak{S}_l = -\mathfrak{S}_r$)



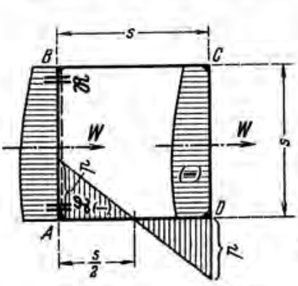


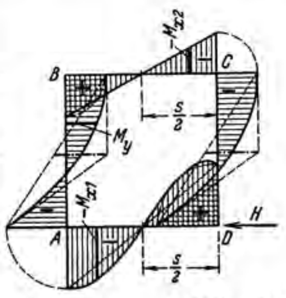
$$M_D = M_C = -M_B = -M_A = \frac{\mathfrak{L}}{8} + \frac{\mathfrak{S}_r}{80}; \qquad p = \frac{6\,\mathfrak{S}_r}{s^2};$$

$$M_{x1} = \mathfrak{S}_r \cdot \omega_V + M_D \cdot \frac{x_1 - x_1'}{s} \qquad M_{x2} = M_x^0 + M_C \cdot \frac{x_2 - x_2'}{s}$$

$$N_1 = N_3 = 0 \qquad N_2 = -N_2' = \frac{\mathfrak{S}_r + 2\,M_C}{s}.$$

Case 110/6: Both legs loaded by any type of antisymmetrical horizontal load, acting from the left





$$M_D = -M_A = \frac{19\,\mathfrak{S}_l + 5\,(\mathfrak{L} + \mathfrak{R})}{40} \qquad M_B = -M_C = \frac{21\,\mathfrak{S}_l - 5\,(\mathfrak{L} + \mathfrak{R})}{40};$$

$$M_{x1} = -2\,\mathfrak{S}_l \cdot \omega_V + M_D \cdot \frac{x_1 - x_1'}{s} \qquad M_y = M_y^0 + \frac{y_1'}{s} M_A + \frac{y_1}{s} M_B;$$

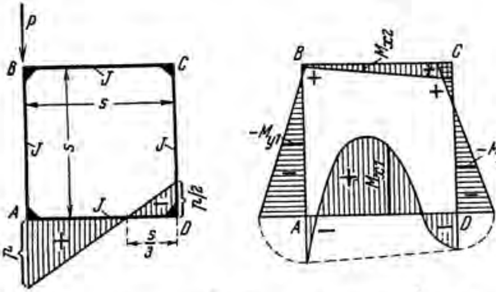
$$p = \frac{12\,\mathfrak{S}_l}{s^2}; \qquad H = 2\,W; \qquad N_2' = -N_2 = \frac{2\,M_B}{s} \qquad N_3 = 0$$

$$(N_1 = -W \text{ bzw. } N_1 = +W)^* .$$

Note: All the load terms refer to the left leg.

* See footnote p. 415.

Case 110/7: Vertical concentrated load at *B*

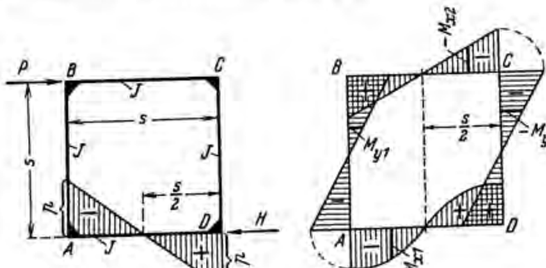


$$M_A = -\frac{14}{240} Ps \qquad M_B = +\frac{1}{240} Ps \qquad M_C = +\frac{4}{240} Ps \qquad M_D = -\frac{11}{240} Ps$$

$$M_{x1} = \frac{2\,Ps}{3} \cdot \omega_T' + \frac{x_1'}{s} M_A + \frac{x_1}{s} M_D \text{ with } \omega_T' = \omega_D' - \frac{1}{2}\omega_D \;;$$

$$p = \frac{4\,P}{s}\;; \qquad N_1 = -N_3 = \frac{P}{16} \qquad N_2' = -\frac{P}{80} \qquad N_2 = \frac{81}{80} P\,.$$

Case 110/8: Horizontal concentrated load at *B*



$$M_D = -M_A = \frac{19}{80} Ps \qquad M_B = -M_C = \frac{21}{80} Ps\;;$$

$$M_{x1} = -Ps \cdot \omega_V + \frac{x_1' - x_1}{s} M_A\;; \qquad p = \frac{6\,P}{s}\;;$$

$$M_{x2} = \frac{x_2' - x_2}{s} M_B \qquad M_{y1} = -M_{y2} = \frac{y_1'}{s} M_A + \frac{y_1}{s} M_B\;;$$

$$N_2' = -N_2 = \frac{21}{40} P \qquad N_3 = \frac{P}{2} \qquad \left(N_1 = +\frac{P}{2} \quad \text{bzw.} \quad N_1 = -\frac{P}{2}\right).^{*}$$

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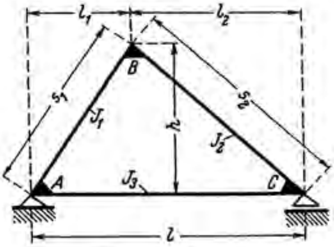
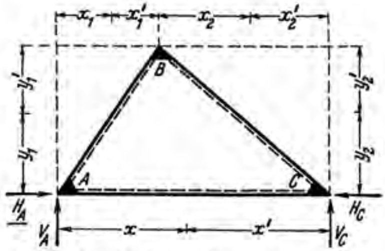
* See footnote p. 415.

Frame 111

Unsymmetrical closed triangular rigid frame. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k_1=\frac{J_3}{J_1}\cdot\frac{s_1}{l} \qquad k_2=\frac{J_3}{J_2}\cdot\frac{s_2}{l}; \qquad K=k_1+k_1k_2+k_2; \qquad F=6K(k_1+1+k_2);$$

$$n_{11}=\frac{4K+3k_2^2}{F} \qquad n_{12}=n_{21}=\frac{2K-3k_2}{F}$$

$$n_{22}=\frac{4K+3}{F} \qquad n_{13}=n_{31}=\frac{2K-3k_1k_2}{F}$$

$$n_{33}=\frac{4K+3k_1^2}{F} \qquad n_{23}=n_{32}=\frac{2K-3k_1}{F}.$$

Formulas for the moments at any point of those members of Frame 111 which do not carry any external load

$$M_{x1}=\frac{x_1'}{l_1}M_A+\frac{x_1}{l_1}M_B \qquad M_{x2}=\frac{x_2'}{l_2}M_B+\frac{x_2}{l_2}M_C$$

$$M_x=\frac{x'}{l}M_A+\frac{x}{l}M_C.$$

Notation for Axial Forces**

N_1 in the left diagonal, N_2 in the right diagonal, N_3 in the horizontal member.

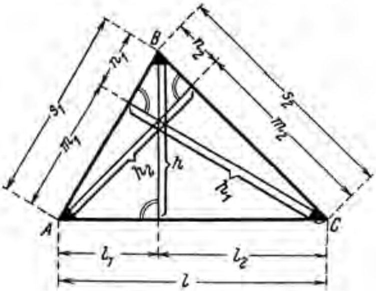
Note: Axial forces are positive for compression, negative for tension.

* H_C occurs when the hinge is at C and the roller at A, whereby H_A vanishes.

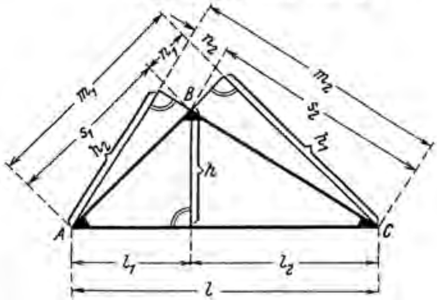
** The second index o denotes the upper end of the member, the index u the lower end.

The moments at the joints contribute to the axial force:

Angle at B less than 90°



Angle at B greater than 90°



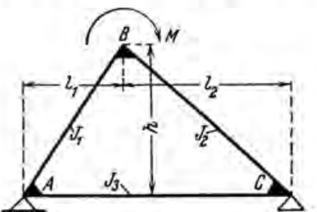
$$\nu_1 = \frac{n_1}{s_1} \qquad \mu_1 = \frac{m_1}{s_1} \qquad T_1 = \frac{-\nu_1 M_A - \mu_1 M_B + M_C}{h_1}$$

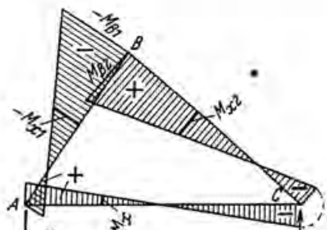
$$\nu_2 = \frac{n_2}{s_2} \qquad \mu_2 = \frac{m_2}{s_2} \qquad T_2 = \frac{+M_A - \mu_2 M_B - \nu_2 M_C}{h_2}$$

$$\lambda_1 = \frac{l_1}{l} \qquad \lambda_2 = \frac{l_2}{l}; \qquad T_3 = \frac{-\lambda_2 M_A + M_B - \lambda_1 M_C}{h}.$$

If the angle at B is greater than 90°, n_1, n_2, ν_1 and ν_2 are negative values.

Case 111/1: Moment M acting at ridge B





$$M_A = +M k_1(+n_{11} - 2n_{21}) \qquad M_{B1} = -M + M_{B2};$$

$$M_{B2} = +M k_1(-n_{12} + 2n_{22}) \qquad V_C = -V_A = \frac{M}{l}.$$

$$M_C = -M k_1(+n_{13} + 2n_{23});$$

$$N_1 = -\frac{\nu_1 M}{h_1} + T_1 \qquad N_2 = \frac{M}{h_2} + T_2 \qquad N_3 = -\frac{\lambda_2 M}{h} + T_3;$$

substitute in T_1, T_2 and T_3 : $M_B \mathrel{\hat=} M_{B2}$.

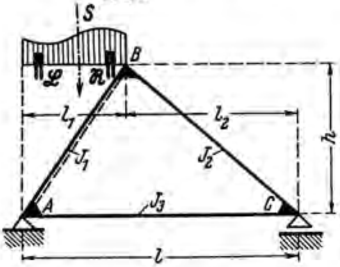
---

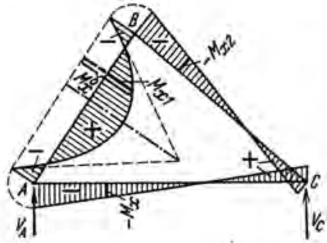
*See p. 418 for M_x. Substitute M_{B1} in M_{x1}, M_{B2} in M_{x2}.

FRAME 111

See Appendix A, Load Terms, pp. 440-445.

Case 111/2: Left-hand inclined member loaded by any type of vertical load



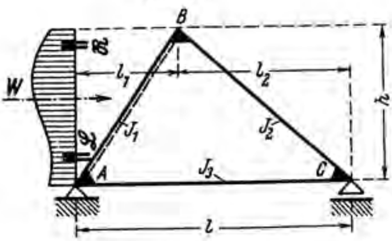


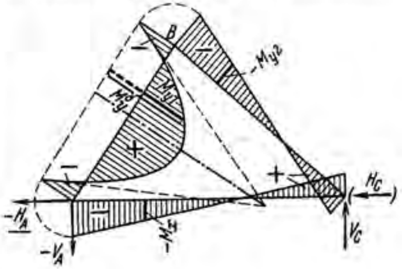
$$M_A = (-\mathfrak{L}\, n_{11} + \mathfrak{R}\, n_{21})\, k_1$$
$$M_B = (+\mathfrak{L}\, n_{12} - \mathfrak{R}\, n_{22})\, k_1 \qquad M_{x1} = M_x^0 + \frac{x_1'}{l_1} M_A + \frac{x_1}{l_1} M_B\,;$$
$$M_C = (+\mathfrak{L}\, n_{13} + \mathfrak{R}\, n_{23})\, k_1\,;$$

$$V_C = \frac{\mathfrak{S}_l}{l} \qquad V_A = S - V_C\,. \qquad N_2 = \frac{\mathfrak{S}_l}{h_2} + T_2 \qquad N_3 = -\frac{\lambda_2 \mathfrak{S}_l}{h} + T_3$$

$$N_{1u} = \frac{\nu_1 \mathfrak{S}_r + S\, l_2}{h_1} + T_1 \qquad N_{1o} = -\frac{\nu_1 \mathfrak{S}_l}{h_1} + T_1\,.$$

Case 111/3: Left-hand inclined member loaded by any type of horizontal load





$$M_A = (-\mathfrak{L}\, n_{11} + \mathfrak{R}\, n_{21})\, k_1$$
$$M_B = (+\mathfrak{L}\, n_{12} - \mathfrak{R}\, n_{22})\, k_1 \qquad M_{y1} = M_y^0 + \frac{y_1'}{h} M_A + \frac{y_1}{h} M_B\,;$$
$$M_C = (+\mathfrak{L}\, n_{13} + \mathfrak{R}\, n_{23})\, k_1\,;$$

$$\underline{H_A} = -W \qquad (H_C = +W)\,; \qquad V_C = -V_A = \frac{\mathfrak{S}_l}{l}\,.$$

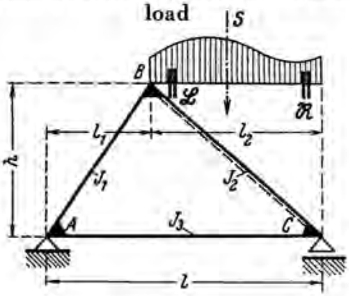
$$N_{1u} = -\frac{\mathfrak{S}_l + \mu_1 \mathfrak{S}_r}{h_1} + T_1 \qquad N_{1o} = -\frac{\nu_1 \mathfrak{S}_l}{h_1} + T_1$$

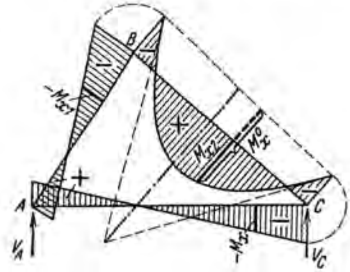
$$N_2 = \frac{\mathfrak{S}_l}{h_2} + T_2 \qquad \underline{N_3} = -\frac{\lambda_2 \mathfrak{S}_l}{h} + T_3 \qquad \left(N_3 = \frac{\lambda_1 \mathfrak{S}_l + \mathfrak{S}_r}{h} + T_3\right).$$

Note: If the hinged support is at *C*, use the values in parentheses instead of the underlined values.

(See Appendix A. Load Terms, pp. 440-445.)

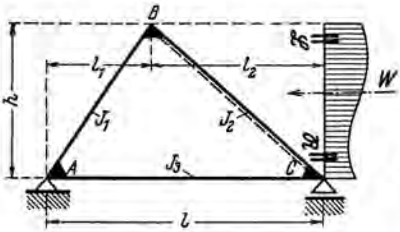
Case 111/4: Right-hand inclined member loaded by any type of vertical load

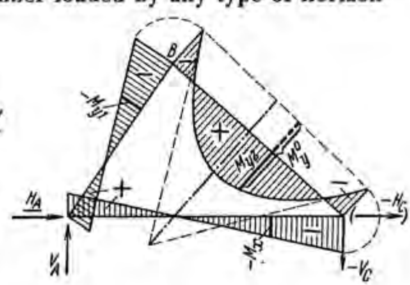




$$M_A = (+\mathfrak{L} n_{21} + \mathfrak{R} n_{31})\, k_2$$
$$M_B = (-\mathfrak{L} n_{22} + \mathfrak{R} n_{32})\, k_2 \qquad M_{x2} = M_x^0 + \frac{x_2'}{l_2} M_B + \frac{x_2}{l_2} M_C\,;$$
$$M_C = (+\mathfrak{L} n_{23} - \mathfrak{R} n_{33})\, k_2\,;$$
$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_C = S - V_A\,. \qquad N_1 = \frac{\mathfrak{S}_r}{h_1} + T_1 \qquad N_3 = -\frac{\lambda_1 \mathfrak{S}_r}{h} + T_3$$
$$N_{2o} = -\frac{\nu_2 \mathfrak{S}_r}{h_2} + T_2 \qquad N_{2u} = \frac{\nu_2 \mathfrak{S}_l + S l_1}{h_2} + T_2\,.$$

Case 111/5: Right-hand inclined member loaded by any type of horizontal load





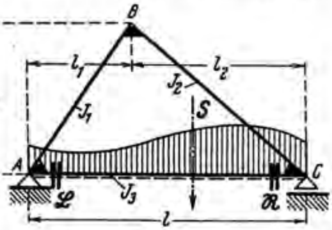
$$M_A = (+\mathfrak{L} n_{21} + \mathfrak{R} n_{31})\, k_2$$
$$M_B = (-\mathfrak{L} n_{22} + \mathfrak{R} n_{32})\, k_2 \qquad M_{y2} = M_y^0 + \frac{y_2}{h} M_B + \frac{y_2'}{h} M_C\,;$$
$$M_C = (+\mathfrak{L} n_{23} - \mathfrak{R} n_{33})\, k_2\,;$$
$$\underline{H_A} = +W \qquad (H_C = -W)\,; \qquad V_A = -V_C = \frac{\mathfrak{S}_r}{l}\,.$$
$$N_{2o} = -\frac{\nu_2 \mathfrak{S}_r}{h_2} + T_2 \qquad N_{2u} = -\frac{\mu_2 \mathfrak{S}_l + \mathfrak{S}_r}{h_2} + T_2$$
$$N_1 = \frac{\mathfrak{S}_r}{h_1} + T_1 \qquad \underline{N_3} = -\frac{\lambda_1 \mathfrak{S}_r}{h} + T_3 \qquad \left(N_3 = \frac{\mathfrak{S}_l + \lambda_2 \mathfrak{S}_r}{h} + T_3\right).$$

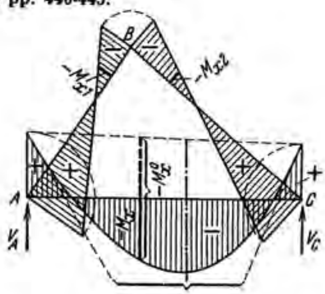
Note: If the hinged support is at C, use the values in parentheses instead of the underlined values.

FRAME 111

Case 111/6: Horizontal member loaded by any type of vertical load, acting downward

See Appendix A, Load Terms, pp. 440-445.

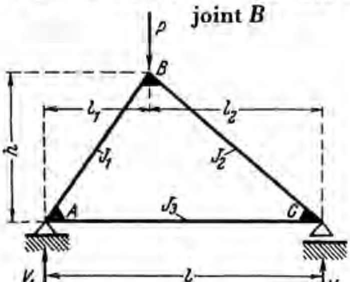




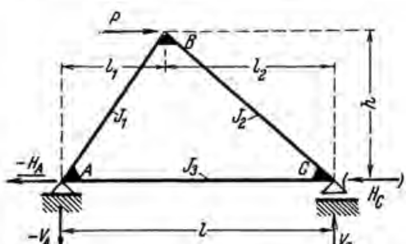
$$M_A = +\mathfrak{L} n_{11} - \mathfrak{R} n_{31}$$
$$M_B = -\mathfrak{L} n_{12} - \mathfrak{R} n_{32} \qquad M_x = M_x^0 + \frac{x'}{l} M_A + \frac{x}{l} M_C\,;$$
$$M_C = -\mathfrak{L} n_{13} + \mathfrak{R} n_{33}\,;$$
$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_C = \frac{\mathfrak{S}_l}{l}. \qquad N_1 = T_1 \qquad N_2 = T_2 \qquad N_3 = T_3.$$

Note: The lower face of member *AC* has been indicated by a dashed line in order to show the sign convention for the load terms $\mathfrak{L}$, $\mathfrak{R}$, $\mathfrak{S}_r$, $\mathfrak{S}_l$. Bending moment signs, however, follow the convention indicated by the title figure on p. 418.

Case 111/7 and 8: Vertical and horizontal concentrated load acting at joint *B*



$$V_A = \frac{P l_2}{l} \qquad V_C = \frac{P l_1}{l}.$$
$$N_1 = \frac{P l_2}{h_1} \qquad N_2 = \frac{P l_1}{h_2}$$
$$N_3 = -\frac{P l_1 l_2}{l h}.$$

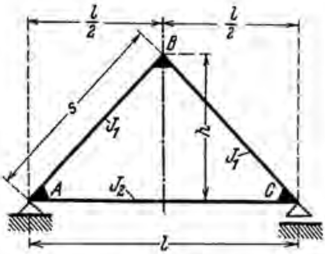


$$V_C = -V_A = \frac{P h}{l}; \qquad \underline{H_A} = -P$$
$$(H_C = +P).$$
$$N_1 = -\frac{P h}{h_1} \qquad N_2 = \frac{P h}{h_2}$$
$$\underline{N_3} = -\frac{P l_2}{l} \qquad \left(N_3 = \frac{P l_1}{l}\right).$$

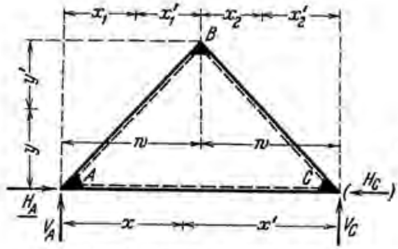
Note: There are no bending moments. For case 111/8 use the values in parentheses instead of the underlined values if the hinged support is at *C*.

Frame 112

Symmetrical closed triangular rigid frame. Externally simply supported.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Coefficients:

$$k = \frac{J_2}{J_1} \cdot \frac{s}{l}; \qquad F_1 = 2 + k \qquad F_2 = 1 + 2k.$$

Formulas for the moments at any point of those members of Frame 112 which do not carry any external load

$$M_{x1} = \frac{x_1'}{w} M_A + \frac{x_1}{w} M_B \qquad M_{x2} = \frac{x_2'}{w} M_B + \frac{x_2}{w} M_C$$

$$M_x = \frac{x'}{l} M_A + \frac{x}{l} M_C.$$

Notation for Axial Forces**

N_1 in the left diagonal, N_2 in the right diagonal, N_3 in the horizontal member.

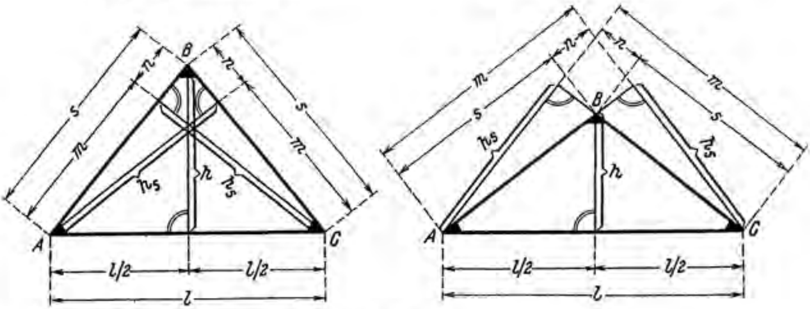
* H_C occurs when the hinge is at C and the roller at A, whereby H_A vanishes.

** The second index o denotes the upper end of the member, the index u the lower end.

FRAME 112

The moments at the joints contribute to the axial force:

Angle at B less than 90° | Angle at B greater than 90°



a) For arbitrary unsymmetrical loads

$$\mu = \frac{m}{s} \qquad \nu = \frac{n}{s} = 1 - \mu\,; \qquad T = \frac{-M_A + 2\,M_B - M_C}{2\,h}$$

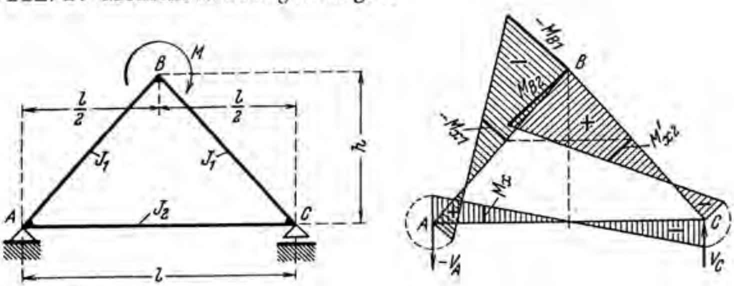
$$T_1 = \frac{-\nu\,M_A - \mu\,M_B + M_C}{h_s} \qquad T_2 = \frac{+M_A - \mu\,M_B - \nu\,M_C}{h_s}.$$

Note: n and v become negative for obtuse angles at B. For a right angle at B $(m = h_s) = s$, $\mu = 1$, $v = 0$.

b) For arbitrary symmetrical loads

$$T' = \frac{M_B - M_A}{h} \qquad T'_1 = T'_2 = -T' \cdot \frac{w}{s}.$$

Case 112/1: Moment M acting at ridge B

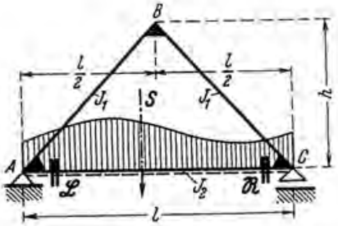


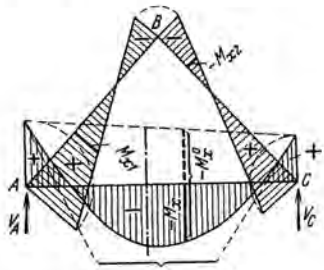
$$M_{B2} = -M_{B1} = \frac{M}{2}\,; \qquad M_A = -M_C = \frac{M}{2} \cdot \frac{k}{F_2}\,;$$

$$V_C = -V_A = \frac{M}{l}\,; \qquad N_2 = -N_1 = \frac{M + 2\,M_A}{l} \cdot \frac{h}{s} \qquad N = 0\,;$$

$$M_{x1} = -M'_{x2} = \frac{x'_1}{w}\,M_A + \frac{x_1}{w}\,M_{B1} \qquad M_x = \frac{x' - x}{l}\,M_A.$$

Case 112/2: Horizontal member loaded by any type of vertical load, acting downward





$$\left.\begin{matrix} M_A \\ M_C \end{matrix}\right\rangle = + \frac{(\mathfrak{L}+\mathfrak{R})}{3F_1} \pm \frac{(\mathfrak{L}-\mathfrak{R})}{2F_2} \qquad M_B = -\frac{(\mathfrak{L}+\mathfrak{R})}{6F_1};$$

$$V_A = \frac{\mathfrak{S}_r}{l} \qquad V_C = \frac{\mathfrak{S}_l}{l}; \qquad M_x = M_x^0 + \frac{x'}{l} M_A + \frac{x}{l} M_C;$$

$$N_1 = T_1 \qquad N_2 = T_2 \qquad N = T.$$

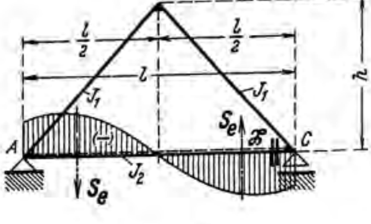
Note: The lower face of member AC has been indicated by a dashed line in order to show the sign convention for the load terms $\mathfrak{L}$, $\mathfrak{R}$, $\mathfrak{S}_r$, $\mathfrak{S}_l$. Bending moment signs, however, follow the convention indicated by the title figure on p. 423.

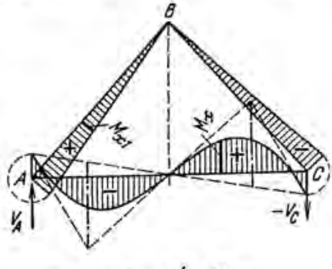
Special case 112/2a: Symmetrical load $(\mathfrak{R} = \mathfrak{L};\ \mathfrak{S}_l = \mathfrak{S}_r)$.

$$M_A = M_C = + \frac{2\mathfrak{L}}{3F_1} \qquad M_B = -\frac{\mathfrak{L}}{3F_1}; \qquad M_x = M_x^0 + M_A;$$

$$V_A = V_C = \frac{S}{2}; \qquad N_1 = N_2 = T_1' \qquad N = T'.$$

Case 112/3: Horizontal member loaded by any type of antisymmetrical load (Special case to case 112/2 with $\mathfrak{R} = -\mathfrak{L}$ and $\mathfrak{S}_l = -\mathfrak{S}_r$)





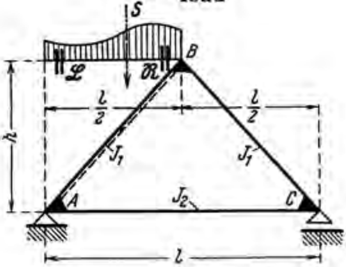
$$M_A = -M_C = \frac{\mathfrak{L}}{F_2} \qquad M_B = 0; \qquad M_x = M_x^0 + \frac{x'-x}{l} M_A;$$

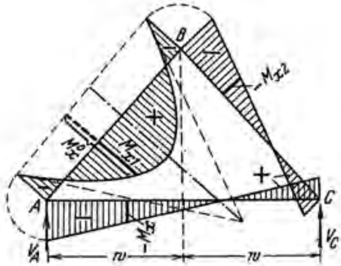
$$V_A = -V_C = \frac{\mathfrak{S}_r}{l}; \qquad N_2 = -N_1 = \frac{M_A}{w} \cdot \frac{h}{s} \qquad N = 0.$$

FRAME 112

See Appendix A, Load Terms, pp. 440-445.

Case 112/4: Left-hand inclined member loaded by any type of vertical load





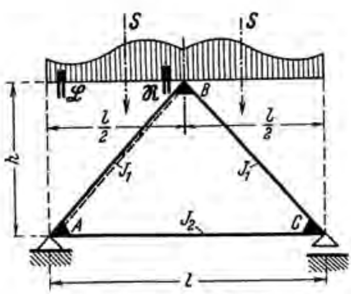
$$M_A \atop M_C \Big\rangle = -\frac{(2\mathfrak{L}-\mathfrak{R})\,k}{6F_1} \mp \frac{\mathfrak{L}\,k}{2F_2} \qquad M_B = -\frac{\mathfrak{R}(3+2k)-\mathfrak{L}k}{6F_1};$$

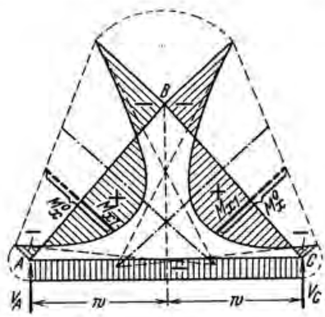
$$M_{x1} = M_x^0 + \frac{x_1'}{w}M_A + \frac{x_1}{w}M_B; \qquad V_C = \frac{\mathfrak{S}_l}{l} \qquad V_A = S - V_C;$$

$$N_{1o} = -\frac{\nu\,\mathfrak{S}_l}{h_s} + T_1 \qquad N_2 = \frac{\mathfrak{S}_l}{h_s} + T_2$$

$$N_{1u} = \frac{\nu\,\mathfrak{S}_r + S\,w}{h_s} + T_1 \qquad N = -\frac{\mathfrak{S}}{2h} + T.$$

Case 112/5: Both inclined members loaded by any type of symmetrical vertical load





$$M_A = M_C = -\frac{(2\mathfrak{L}-\mathfrak{R})\,k}{3F_1} \qquad M_B = -\frac{\mathfrak{R}(3+2k)-\mathfrak{L}k}{3F_1};$$

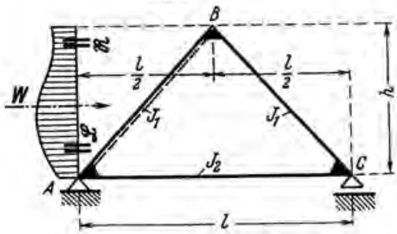
$$M_{x1} = M_x^0 + \frac{x_1'}{w}M_A + \frac{x_1}{w}M_B; \qquad V_A = V_C = S;$$

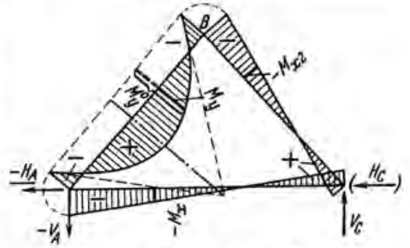
$$N_{1o} = N_{2o} = \frac{\mu\,\mathfrak{S}_l}{h_s} + T_1' \qquad N_{1u} = N_{2u} = \frac{S\,l - \mu\,\mathfrak{S}_r}{h_s} + T_1' \qquad N = -\frac{\mathfrak{S}_l}{h} + T'.$$

Note: All the load terms refer to the left inclined member.

See Appendix A, Load Terms, pp. 440-445.

Case 112/6: Left-hand inclined member loaded by any type of horizontal load





$$\left.\begin{matrix} M_A \\ M_C \end{matrix}\right\rangle = -\frac{(2\mathfrak{L}-\mathfrak{R})k}{6F_1} \mp \frac{\mathfrak{L}k}{2F_2} \qquad M_B = -\frac{\mathfrak{R}(3+2k)-\mathfrak{L}k}{6F_1};$$

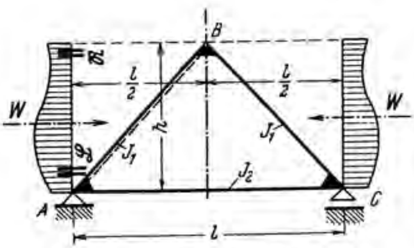
$$M_y = M_y^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B; \qquad V_C = -V_A = \frac{\mathfrak{S}_l}{l}; \qquad \begin{matrix} \underline{H_A = -W} \\ (H_C = +W) \end{matrix};$$

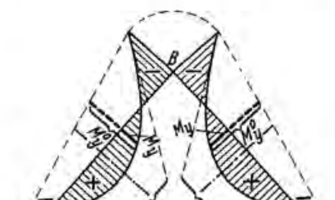
$$N_{1o} = -\frac{\nu\mathfrak{S}}{h_s} + T_1 \qquad N_{1u} = -\frac{\mathfrak{S}_l + \mu\mathfrak{S}_r}{h_s} + T_1$$

$$N_2 = \frac{\mathfrak{S}_l}{h_s} + T_2 \qquad \underline{N = -\frac{\mathfrak{S}_l}{2h} + T} \qquad \left(N = \frac{Wh + \mathfrak{S}_r}{2h} + T\right).$$

Note: If the hinged support is at C, use the values in parentheses instead of the underlined values.

Case 112/7: Both inclined members loaded by any type of symmetrical horizontal load





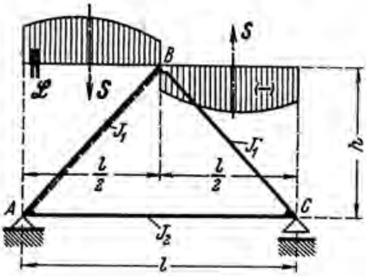
$$M_A = M_C = -\frac{(2\mathfrak{L}-\mathfrak{R})k}{3F_1} \qquad N_{1o} = N_{2o} = \frac{\mu\mathfrak{S}_l}{h_s} + T_1'$$

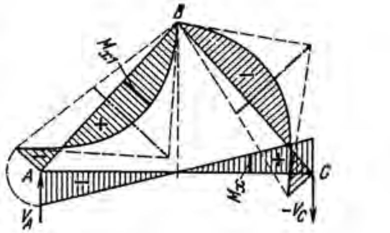
$$M_B = -\frac{\mathfrak{R}(3+2k)-\mathfrak{L}k}{3F_1}; \qquad N_{1u} = N_{2u} = -\frac{\mu\mathfrak{S}_r}{h_s} + T_1'$$

$$M_y = M_y^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B; \qquad N = \frac{\mathfrak{S}_r}{h} + T'.$$

Note: All the load terms refer to the left inclined member.

Case 112/8: Both inclined members loaded by any type of antisymmetrical vertical load



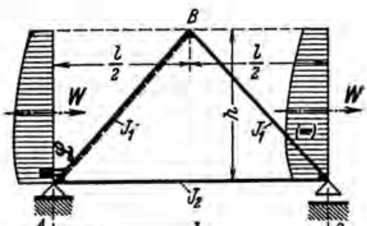


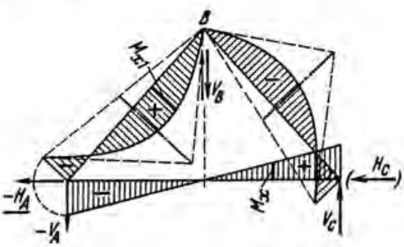
$$M_C = -M_A = \frac{\mathfrak{L} k}{F_2} \qquad M_B = 0; \qquad M_{x1} = M_x^0 + \frac{x_1'}{w} M_A; \qquad V_A = -V_C = \frac{\mathfrak{S}_r}{w};$$

$$N_{2o} = -N_{1o} = \frac{\mathfrak{S}_l + M_A}{w} \cdot \frac{h}{s} \qquad N_{1u} = -N_{2u} = \frac{\mathfrak{S}_r - M_A}{w} \cdot \frac{h}{s} \qquad N = 0.$$

Note: All the load terms refer to the left inclined member.

Case 112/9: Both inclined members loaded by any type of antisymmetrical horizontal load, acting from the left





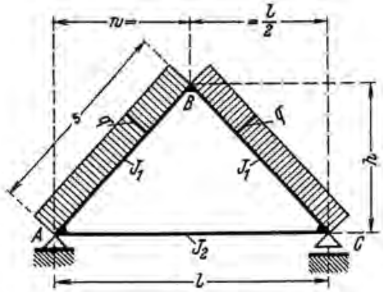
$$M_C = -M_A = \frac{\mathfrak{L} k}{F_2} \qquad M_B = 0; \qquad M_y = M_y^0 + \frac{y'}{h} M_A;$$

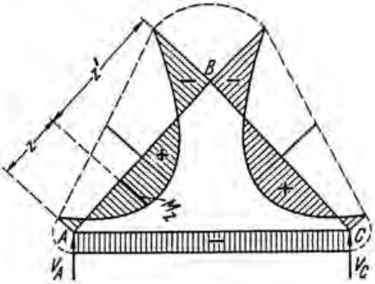
$$V_C = -V_A = \frac{\mathfrak{S}_l}{w} \qquad V_B = \frac{\mathfrak{S}_l + M_A}{w}; \qquad \underline{H_A} = -2W \qquad (H_C = +2W);$$

$$N_{2o} = -N_{1o} = V_B \cdot \frac{h}{s} \qquad N_{2u} = -N_{1u} = V_B \cdot \frac{h}{s} + W \cdot \frac{w}{s} \qquad \underline{N} = -W \;\; (N = +W).$$

Note: All load terms refer to the left member. If the hinged support is at C, use the values in parentheses instead of the underlined values.

Case 112/10: **Uniformly distributed symmetrical load, acting normally to the inclined members**



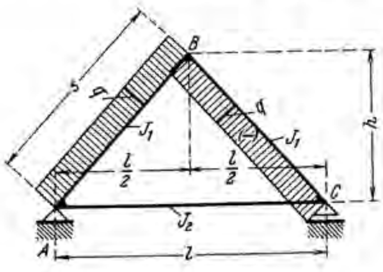


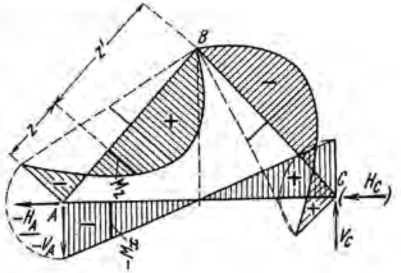
$$M_A = M_C = -\frac{q s^2}{12} \cdot \frac{k}{F_1} \qquad M_B = -\frac{q s^2}{12} \cdot \frac{3+k}{F_1};$$

$$M_z = \frac{q z z'}{2} + \frac{z'}{s} M_A + \frac{z}{s} M_B; \qquad V_A = V_C = q w; \qquad H_A = 0;$$

$$N_1 = N_2 = \frac{q s w}{2h} + T_1' \qquad N = \frac{q(h^2 - w^2)}{2h} + T'.$$

Case 112/11: **Uniformly distributed antisymmetrical load, acting normally to the inclined members (Pressure and suction)**





$$M_C = -M_A = \frac{q s^2}{4} \cdot \frac{k}{F_2} \qquad M_B = 0; \qquad M_z = \frac{q z z'}{2} + \frac{z'}{s} M_A;$$

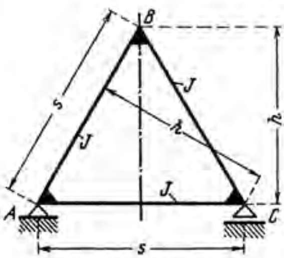
$$V_C = -V_A = \frac{q(h^2 - w^2)}{l}; \qquad \underline{H_A} = -2qh \qquad (H_C = +2qh);$$

$$N_2 = -N_1 = \left(\frac{q s^2}{l} + \frac{M_A}{w}\right)\frac{h}{s} \qquad \underline{N} = -qh \qquad (N = +qh).$$

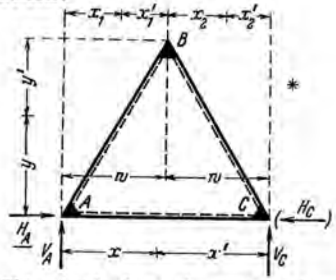
Note: If the hinged support is at C, use the values in parentheses instead of the underlined values.

Frame 113

Equilateral closed triangular rigid frame. Externally simply supported. All members having equal moments of inertia.



Shape of Frame
Dimensions and Notations



This sketch shows the positive direction of the reactions and the coordinates assigned to any point. Positive bending moments cause tension at the face marked by a dashed line.

Relations between frame dimensions

$$h = \frac{s\sqrt{3}}{2} \approx 0{,}8660\, s \qquad s = \frac{2h}{\sqrt{3}} \approx 1{,}1547\, h \qquad w = \frac{s}{2}.$$

Formulas for the moments at any point of not directly loaded members for all loading conditions.

$$M_{x1} = \frac{x_1'}{w} M_A + \frac{x_1}{w} M_B \qquad M_{x2} = \frac{x_2'}{w} M_B + \frac{x_2}{w} M_C$$

$$M_x = \frac{x'}{s} M_A + \frac{x}{s} M_C.$$

Notation for Axial Forces**

N_1 in the left diagonal, N_2 in the right diagonal, N in the horizontal member.

* $w = s/2$ is introduced for a simpler representation of the moments M_x of the inclined members as well as the xial forces produced by symmetrical and antisymmetrical loads. H_C occurs when the hinge is at C.

** The second index o denotes the upper end of the member, u the lower end.

Axial Forces due to Corner Moments alone

a) For arbitrary unsymmetrical loads

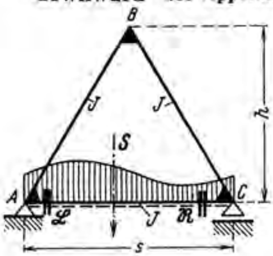
$$T_1 = \frac{2\,M_C - M_A - M_B}{2\,h} \qquad\qquad T_2 = \frac{2\,M_A - M_B - M_C}{2\,h}$$

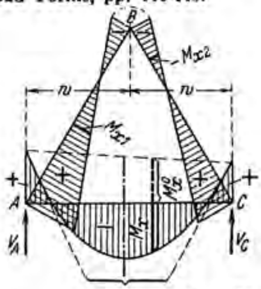
$$T = \frac{2\,M_B - M_A - M_C}{2\,h}.$$

b) For arbitrary symmetrical loads

$$T_1' = T_2' = \frac{M_A - M_B}{2\,h} \qquad\qquad T' = \frac{M_B - M_A}{h}.$$

Case 113/1: Horizontal member loaded by any type of vertical load acting downward See Appendix A, Load Terms, pp. 440-445.





$$\left.\begin{matrix} M_A \\ M_C \end{matrix}\right\} = +\frac{(\mathfrak{L}+\mathfrak{R})}{9} \pm \frac{(\mathfrak{L}-\mathfrak{R})}{6} \qquad\qquad M_B = -\frac{(\mathfrak{L}+\mathfrak{R})}{18};$$

$$V_A = \frac{\mathfrak{S}_r}{s} \qquad V_C = \frac{\mathfrak{S}_l}{s}; \qquad M_x = M_x^0 + \frac{x'}{s} M_A + \frac{x}{s} M_C;$$

$$N_1 = T_1 \qquad\qquad N_2 = T_2 \qquad\qquad N = T.$$

Note: The dashed line must be shown at the bottom of the face of the member to make $\mathfrak{L}$ $\mathfrak{R}$, $\mathfrak{S}_r$, $\mathfrak{S}_l$ agree with the definition given in the introductory chapter. For the positive direction of the moment see the sketch on p. 430.

Special case 113/1a: Symmetrical load $(\mathfrak{R} = \mathfrak{L};\ \mathfrak{S}_l = \mathfrak{S}_r)$.

$$M_A = M_C = +\frac{2\,\mathfrak{L}}{9} \qquad M_B = -\frac{\mathfrak{L}}{9}; \qquad M_x = M_x^0 + M_A;$$

$$V_A = V_C = \frac{S}{2}; \qquad N_1 = N_2 = \frac{\mathfrak{L}}{6\,h} \qquad N = -\frac{\mathfrak{L}}{3\,h}.$$

Special case 113/1b: Antisymmetrical load $(\mathfrak{R} = -\mathfrak{L};\ \mathfrak{S}_l = -\mathfrak{S}_r)$.

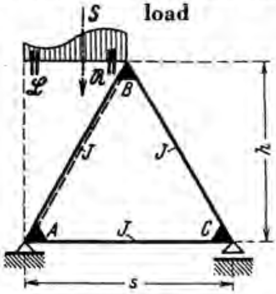
$$M_A = -M_C = \frac{\mathfrak{L}}{3} \qquad M_B = 0; \qquad M_x = M_x^0 + \frac{x'-x}{s} M_A;$$

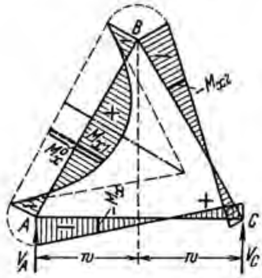
$$V_A = -V_C = \frac{\mathfrak{S}_r}{s} \qquad N_2 = -N_1 = \frac{\mathfrak{L}}{2h} \qquad N = 0.$$

Note: Load and moment diagrams same as for case 112/3, p. 425.

See Appendix A, Load Terms, pp. 440-445.

113/2: Left-hand inclined member loaded by any type of vertical load





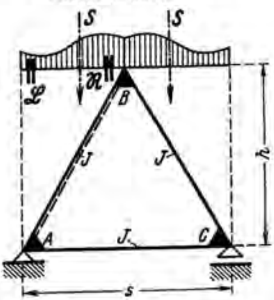
$$\left.\begin{matrix} M_A \\ M_C \end{matrix}\right\rangle = -\frac{2\mathfrak{L}-\mathfrak{R}}{18} \mp \frac{\mathfrak{L}}{6}\,; \qquad M_{x1} = M_x^0 + \frac{x_1'}{w} M_A + \frac{x_1}{w} M_B\,;$$

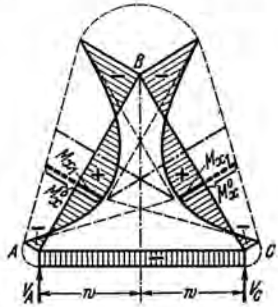
$$M_B = -\frac{5\mathfrak{R}-\mathfrak{L}}{18}\,; \qquad V_C = \frac{\mathfrak{S}_l}{s} \qquad V_A = S - V_C\,;$$

$$N_{1o} = -\frac{\mathfrak{S}_l}{2h} + T_1 \qquad N_2 = \frac{\mathfrak{S}_l}{h} + T_2$$

$$N_{1u} = \frac{Ss + \mathfrak{S}_r}{2h} + T_1 \qquad N = -\frac{\mathfrak{S}_l}{2h} + T\,.$$

e 113/3: Both inclined members loaded by any type of symmetrical vertical load





$$M_A = M_C = -\frac{2\mathfrak{L}-\mathfrak{R}}{9}\,; \qquad M_{x1} = M_x^0 + \frac{x_1'}{w} M_A + \frac{x_1}{w} M_B\,;$$

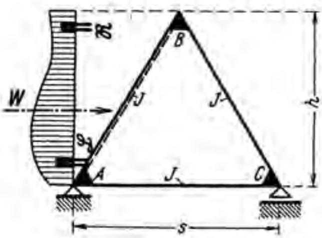
$$M_B = -\frac{5\mathfrak{R}-\mathfrak{L}}{9}\,; \qquad V_A = V_C = S; \qquad N = -\frac{\mathfrak{S}_l}{h} + T$$

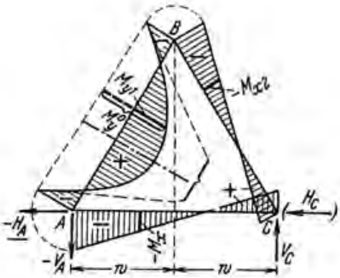
$$N_{1o} = N_{2o} = \frac{\mathfrak{S}_l}{2h} + T_1 \qquad N_{1u} = N_{2u} = \frac{2Ss - \mathfrak{S}_r}{2h} + T_1\,.$$

Note: All the load terms refer to the left inclined member.

See Appendix A, Load Terms, pp. 440-445.

Case 113/4: Left-hand inclined member loaded by any type of horizontal load





$$\left.\begin{matrix}M_A\\M_C\end{matrix}\right\rangle = -\frac{2\mathfrak{L}-\mathfrak{R}}{18} \mp \frac{\mathfrak{L}}{6} \qquad M_{y1} = M_y^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B;$$

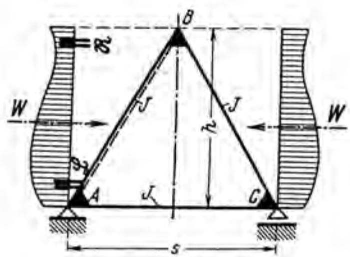
$$M_B = -\frac{5\mathfrak{R}-\mathfrak{L}}{18}; \qquad V_C = -V_A = \frac{\mathfrak{S}_l}{s}; \qquad \underline{H_A = -W} \quad (H_C = +W);$$

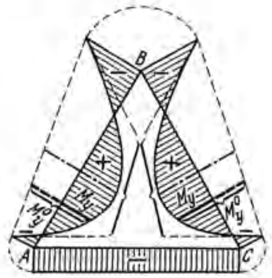
$$N_{1o} = -\frac{\mathfrak{S}_l}{2h} + T_1 \qquad N_{1u} = -\frac{Wh+\mathfrak{S}_l}{2h} + T_1$$

$$N_2 = \frac{\mathfrak{S}_l}{h} + T_2 \qquad \underline{N} = -\frac{\mathfrak{S}_l}{2h} + T \qquad \left(N = \frac{Wh+\mathfrak{S}_r}{2h} + T\right).$$

Note: If the hinged support is at C, use the values in parentheses instead of the underlined values.

Case 113/5: Both inclined members loaded by any type of symmetrical horizontal load





$$M_A = M_C = -\frac{2\mathfrak{L}-\mathfrak{R}}{9} \qquad N_{1o} = N_{2o} = \frac{\mathfrak{S}_l}{2h} + T_1$$

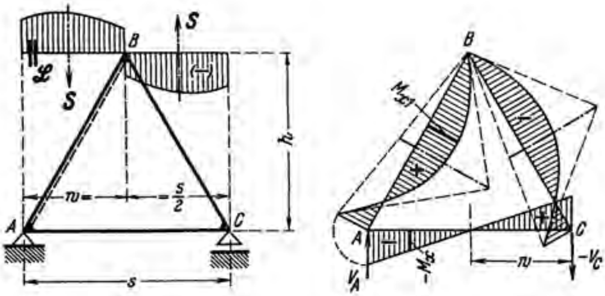
$$M_B = -\frac{5\mathfrak{R}-\mathfrak{L}}{9} \qquad N_{1u} = N_{2u} = -\frac{\mathfrak{S}_r}{2h} + T_1$$

$$M_y = M_y^0 + \frac{y'}{h}M_A + \frac{y}{h}M_B; \qquad N = \frac{\mathfrak{S}_r}{h} + T.$$

Note: All load terms refer to the left diagonal.

FRAME 113

Case 113/6: Both inclined members loaded by any type of antisymmetrical vertical load

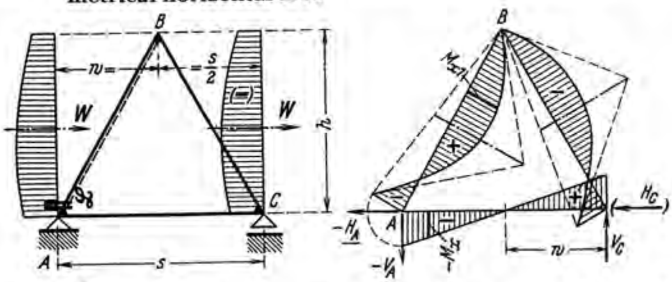


$$M_C = -M_A = \frac{\mathfrak{L}}{3} \qquad M_B = 0; \qquad M_{x1} = M_x^0 + \frac{x_1'}{w} M_A; \qquad V_A = -V_C = \frac{\mathfrak{S}_r}{w};$$

$$N_{2o} = -N_{1o} = \frac{3\mathfrak{S}_l - \mathfrak{L}}{2h} \qquad N_{1u} = -N_{2u} = \frac{3\mathfrak{S}_r + \mathfrak{L}}{2h} \qquad N = 0.$$

Note: All the load terms refer to the left member.

Case 113/7: Both inclined members loaded by any type of antisymmetrical horizontal load



$$M_C = -M_A = \frac{\mathfrak{L}}{3} \qquad M_B = 0; \qquad M_y = M_y^0 + \frac{y'}{h} M_A; \qquad V_C = -V_A = \frac{\mathfrak{S}_l}{w};$$

$$\underline{H_A} = -2W \qquad (H_C = +2W); \qquad \underline{N} = -W \qquad (N = +W);$$

$$N_{2o} = -N_{1o} = \frac{3\mathfrak{S}_l - \mathfrak{L}}{2h} \qquad N_{2u} = -N_{1u} = \frac{3\mathfrak{S}_l - \mathfrak{L}}{2h} + \frac{W}{2}.$$

Note: All load terms refer to the left member. If the hinged support is at C, use the values in parentheses instead of the underlined values.

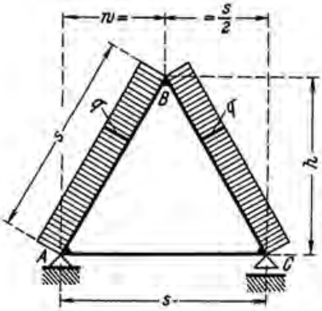
Special case 113/7a: Horizontal concentrated load P at B

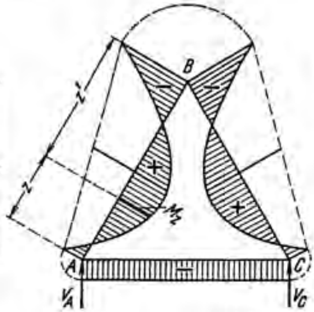
No bending moments occur

$$V_C = -V_A = \frac{P\sqrt{3}}{2} \approx 0{,}8660 \cdot P; \qquad \underline{H_A} = -P \qquad (H_C = +P);$$

$$\underline{N} = -\frac{P}{2} \qquad \left(N = +\frac{P}{2}\right) \qquad N_2 = -N_1 = P.$$

Case 113/8: Uniformly distributed symmetrical load, acting normally to the inclined members





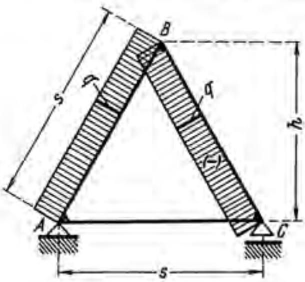
$$V_A = V_C = q\,w$$

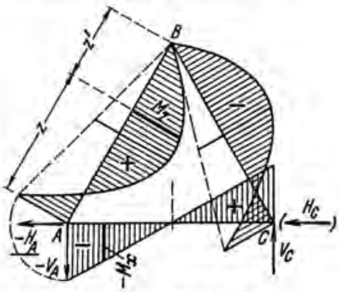
$$N_1 = N_2 = \frac{7\,q\,h}{18}$$

$$N = \frac{2\,q\,h}{9}.$$

$$M_A = M_C = -\frac{q\,s^2}{36} \qquad M_B = -\frac{q\,s^2}{9}; \qquad M_z = \frac{q\,z\,z'}{2} + \frac{z'}{s} M_A + \frac{z}{s} M_B .$$

Case 113/9: Uniformly distributed antisymmetrical load, acting normally to the inclined members (Pressure and suction)





$$M_C = -M_A = \frac{q\,s^2}{12} \qquad M_B = 0; \qquad M_z = \frac{q\,z\,z'}{2} + \frac{z'}{s} M_A\,;$$

$$V_C = -V_A = \frac{q\,s}{2}; \qquad \underline{H_A} = -2\,q\,h \qquad (H_C = +2\,q\,h)\;;$$

$$N_2 = -N_1 = \frac{5\,q\,h}{6} \qquad \underline{N} = -q\,h \qquad (N = +q\,h).$$

Note: If the hinged support is at *C*, use the values in parentheses instead of the und lined values.

Case 113/10: Clockwise moment *M* acting at ridge *B*

$$M_A = -M_C = \frac{M}{6} \qquad M_{B2} = -M_{B1} = \frac{M}{2}; \qquad V_C = -V_A = \frac{M}{s};$$

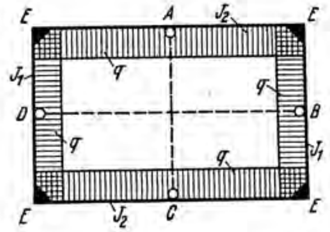
$$N_1 = -\frac{M}{h} \qquad N_2 = +\frac{M}{h} \qquad N = 0.$$

Note: Load and moment diagrams same as for case 112/1, p. 424.

Frame 114

Vierendeel frames with two axes of symmetry (cells), with or without non-yielding tie-rods, and subject to uniform internal pressure only.

(For tanks, silos, etc.)



The dimensions and coefficients are given for each case.

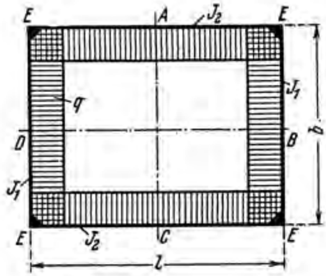
Notations for the axial forces:

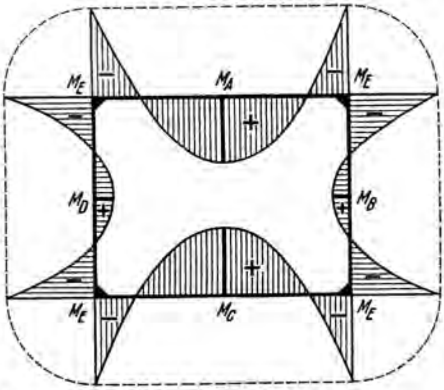
In the vertical member (with J_1) N_1

In the horizontal member (with J_2) N_2

Note: For this frame, the moment diagram is shown on the compression side, therefore the + sign means tension on the outside, the — sign tension on the inside of the frame. The axial forces for this frame are called positive if they cause tension.

Case 114/1: Rectangular frame without tie rod



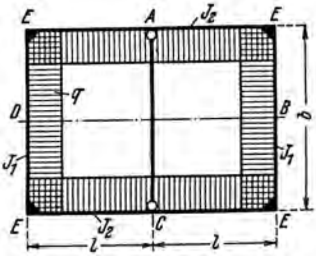


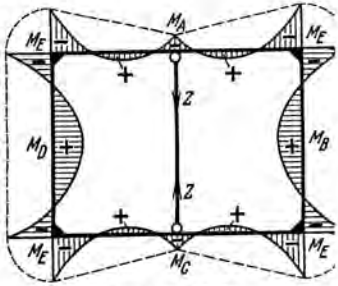
$$k = \frac{J_2}{J_1} \cdot \frac{b}{l} \qquad \beta = \frac{b}{l}\,; \qquad M_E = -\frac{q l^2}{12} \cdot \frac{1 + \beta^2 k}{1 + k}$$

$$M_A = M_C = \frac{q l^2}{8} + M_E \qquad M_B = M_D = \frac{q b^2}{8} + M_E.$$

Axial forces: $N_1 = \frac{q\,l}{2}$ $\qquad N_2 = \frac{q\,b}{2}.$

Case 114/2: Rectangular frame with rigid tie rod





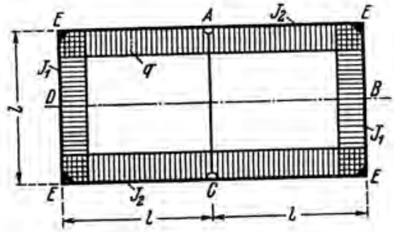
$$k = \frac{J_2}{J_1} \cdot \frac{b}{l} \qquad \beta = \frac{b}{l}\,; \qquad M_E = -\frac{q l^2}{12} \cdot \frac{1 + 2\beta^2 k}{1 + 2k}$$

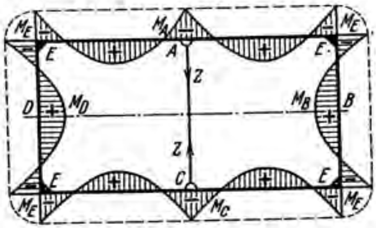
$$M_A = M_C = -\frac{q\,l^2}{12} \cdot \frac{1 + (3 - \beta^2)\,k}{1 + 2k} \qquad M_B = M_D = \frac{q\,b^2}{8} + M_E.$$

Tension in the tie rods: $\quad Z = \frac{q\,l}{2} \cdot \frac{2 + (5 - \beta^2)\,k}{1 + 2k}.$

Axial forces: $N_1 = q\,l - \frac{Z}{2}$ $\qquad N_2 = \frac{q\,b}{2}.$

ıse 114/3: Rectangular frame, ratio of the side dimensions 1:2, with equal moments of inertia for the sides and with one rigid tie rod between the longer sides





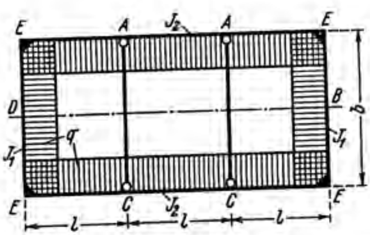
$$J_2 = J_1;\qquad M_E = M_A = M_C = -\frac{q\,l^2}{12}$$

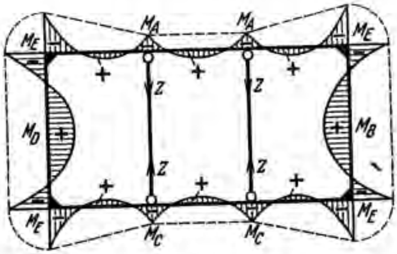
$$M_B = M_D = +\frac{q\,l^2}{24};\qquad \text{Axial forces:}\quad N_1 = N_2 = \frac{q\,l}{2}.$$

nsion in the tie rod: $Z = q\,l$.

---

ase 114/4: Rectangular frame with two rigid tie rods between the longer sides





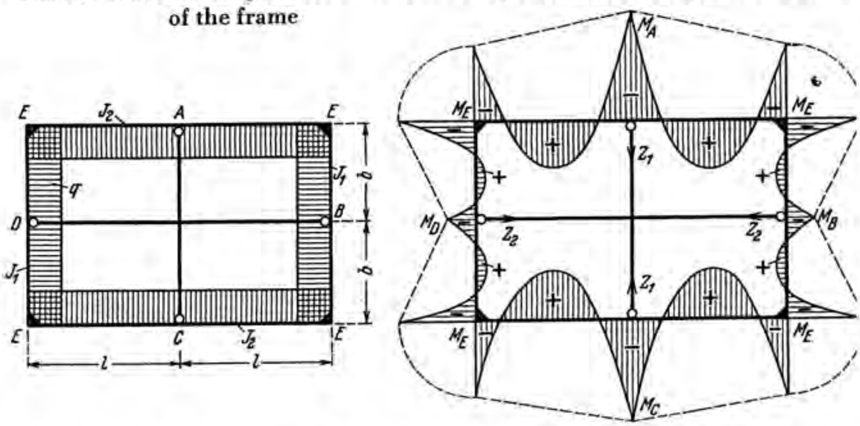
$$k = \frac{J_2}{J_1}\cdot\frac{b}{l}\qquad \beta = \frac{b}{l};\qquad M_E = -\frac{q\,l^2}{12}\cdot\frac{3+5\beta^2 k}{3+5k}$$

$$M_A = M_C = -\frac{q\,l^2}{12}\cdot\frac{3+(6-\beta^2)\,k}{3+5k}\qquad M_B = M_D = \frac{q\,b^2}{8} + M_E.$$

Tension in the tie rods: $$Z = \frac{q\,l}{2}\cdot\frac{6+(11-\beta^2)\,k}{3+5k}.$$

Axial forces: $$N_1 = \frac{3\,q\,l}{2} - Z\qquad N_2 = \frac{q\,b}{2}.$$

Case 114/5: **Rectangular frame with two rigid tie rods through the center of the frame**



$$k = \frac{J_2}{J_1} \cdot \frac{b}{l} \qquad \beta = \frac{b}{l}; \qquad M_A = M_C = -\frac{q\,l^2}{24} \cdot \frac{(2+3k) - \beta^2 k}{1+k}$$

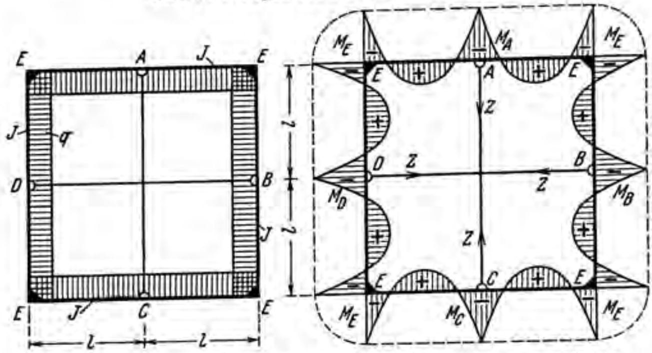
$$M_E = -\frac{q\,l^2}{12} \cdot \frac{1+\beta^2 k}{1+k} \qquad M_B = M_D = -\frac{q\,l^2}{24} \cdot \frac{(3+2k)\beta^2 - 1}{1+k}.$$

Tension in the tie rods:

$$AC: \quad Z_1 = \frac{q\,l}{4} \cdot \frac{(4+5k) - \beta^2 k}{1+k} \qquad BD: \quad Z_2 = \frac{q\,l}{4\beta} \cdot \frac{(5+4k)\beta^2 - 1}{1+k}.$$

Axial forces: $N_1 = q\,l - \dfrac{Z_1}{2}$ $\qquad N_2 = q\,b - \dfrac{Z_2}{2}.$

Case 114/6: **Quadratic frame with equal moments of inertia of the sides and two rigid tie rods through the center of the frame**



$$Z = q\,l$$

$$N = \frac{q\,l}{2}$$

$$M_A = M_B = M_C = M_D = M_E = -q\,l^2/12.$$

APPENDIX

A. Load Terms

(a) General Notations:

In the formulas the following notations printed in bold type are used:

$$\mathfrak{L}, \mathfrak{R}; \quad \mathfrak{S}_r, \mathfrak{S}_l; \quad \boldsymbol{S}, \boldsymbol{W}; \quad \boldsymbol{M}_x^0, \boldsymbol{M}_y^0.$$

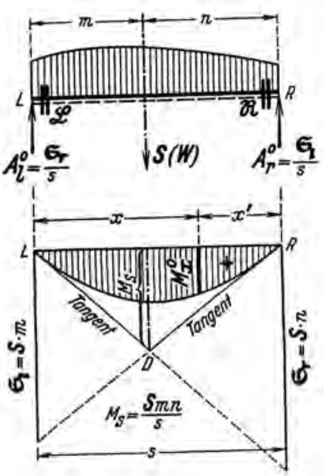
When several members are loaded these quantities are shown with an index $(\mathfrak{L}_1, \mathfrak{R}_1)$.

These quantities are called "load terms." They depend only on type, magnitude and point of application of the external load, but they do not depend on the form and dimensions of the frame.

In using these load terms each member of the frame should be considered as a simple beam, isolated from the frame.

The meaning of the load terms $\mathfrak{L}$ and $\mathfrak{R}$ is explained in *Beam Formulas* (see footnote p. 441). They are indicated in the sketches by a double line || at the end of the member which carries the load.

$\boldsymbol{S}$ in general is the resultant of the external loads acting on a member. For horizontal loads the notation $\boldsymbol{W}$ is used instead of S. $\mathfrak{S}_r$ is the statical moment of the resultant S or $\boldsymbol{W}$, about the right end of the beam, $\mathfrak{S}_l$ about the left end. Draw the moment diagram of the simple beam and its tangents at the supports. $\mathfrak{S}_r$ and $\mathfrak{S}_l$ are the distances cut off by these tangents at the vertical through the supports (cross line distances).



The moment of the simple beam at any point is denoted by $\boldsymbol{M}_x^0$ for vertical loads and by $\boldsymbol{M}_y^0$ for horizontal loads. The sketch on this page illustrates the meaning of these notations.

(b) Formulas for the Load Terms:

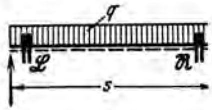
The following pages contain a summary of the most important loading conditions in abbreviated form. The reader is referred to *Beam Formulas* for a total of 72 loading conditions, their shear and moment diagrams, fixed end moments, end slopes, and equations for the elastic curves.

In the nineteen load cases to follow the numbers in brackets refer to the numbers of the loading conditions in *Beam Formulas*. This latter ... numerical tables for the load cases marked with an asterisk*.

For symmetrical loading:

$$\mathfrak{L} = \mathfrak{R} \quad (\mathfrak{L} + \mathfrak{R}) = 2\,\mathfrak{L} \quad (\mathfrak{L} - \mathfrak{R}) = 0 \quad \mathfrak{S}_r = \mathfrak{S}_l \quad (\mathfrak{S}_r - \mathfrak{S}_l) = 0.$$

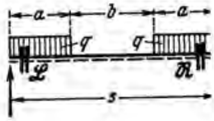
Case 1: [17] **Uniform load over the entire beam**



$$\mathfrak{L} = \mathfrak{R} = \frac{q s^2}{4} \qquad \mathfrak{S}_r = \mathfrak{S}_l = \frac{q s^2}{2}$$

$$S = q s \qquad M_x^0 = \frac{q x x'}{2}$$

Case 2: [23*] **Two uniform loads, one at each end of the beam**



$$\alpha = \frac{a}{s} \qquad \beta = \frac{b}{s} \qquad \mathfrak{L} = \mathfrak{R} = \frac{q a^2 (2 + \beta)}{2}$$

$$\mathfrak{S}_r = \mathfrak{S}_l = q a s \qquad S = 2 q a.$$

For the left-hand region a:

$$M_x^0 = q x \left(a - \frac{x}{2}\right)$$

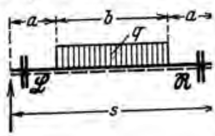
For the region b:

$$M_x^0 = \frac{q a^2}{2}$$

For the right-hand region a:

$$M_x^0 = q x' \left(a - \frac{x'}{2}\right).$$

Case 3: [22*] **Symmetrical uniform load in the center part of the beam**



$$\alpha = \frac{a}{s} \qquad \beta = \frac{b}{s} \qquad \mathfrak{L} = \mathfrak{R} = \frac{q b s (3 - \beta^2)}{8}$$

$$\mathfrak{S}_r = \mathfrak{S}_l = \frac{q b s}{2} \qquad S = q b.$$

For the left-hand region a:

$$M_x^0 = \frac{q b}{2} x$$

For the region b:

$$M_x^0 = \frac{q}{2} [b x - (x - a)^2]$$

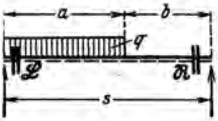
For the right-hand region

$$M_x^0 = \frac{q b}{2} x'$$

Beam Formulas by A. Kleinlogel. American edition translated and adapted to America conditions by Harold G. Lorsch. Frederick Ungar Publishing Co., New York.

ase 4: [19*] Uniform load near the left end of the beam

$$\alpha = \frac{a}{s} \qquad \beta = \frac{b}{s}. \qquad S = qa;$$



$$\mathfrak{L} = \frac{q a^2 (1+\beta)^2}{4} \qquad (\mathfrak{L} + \mathfrak{R}) = \frac{q a^2 (1 + 2\beta)}{2}$$

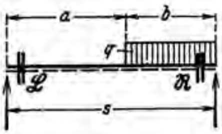
$$\mathfrak{R} = \frac{q a^2 (2 - \alpha^2)}{4} \qquad (\mathfrak{L} - \mathfrak{R}) = \frac{q a^2 \beta^2}{2};$$

$$\mathfrak{S}_r = \frac{q a (s + b)}{2} \qquad \mathfrak{S}_l = \frac{q a^2}{2}.$$

For the region a: $M_x^0 = \left(\frac{\mathfrak{S}_r}{s} - \frac{q x}{2}\right) x$ For the region b: $M_x^0 = \frac{\mathfrak{S}_l}{s} x'$.

:ase 5: Uniform load near the right end of the beam

$$\alpha = \frac{a}{s} \qquad \beta = \frac{b}{s}. \qquad S = qb;$$



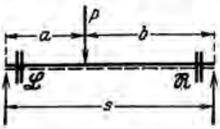
$$\mathfrak{L} = \frac{q b^2 (2 - \beta^2)}{4} \qquad (\mathfrak{L} + \mathfrak{R}) = \frac{q b^2 (1 + 2\alpha)}{2}$$

$$\mathfrak{R} = \frac{q b^2 (1 + \alpha)^2}{4} \qquad (\mathfrak{L} - \mathfrak{R}) = -\frac{q b^2 \alpha^2}{2};$$

$$\mathfrak{S}_r = \frac{q b^2}{2} \qquad \mathfrak{S}_l = \frac{q b (s + a)}{2}.$$

For the region a: $M_x^0 = \frac{\mathfrak{S}_r}{s} x$ For the region b: $M_x^0 = \left(\frac{\mathfrak{S}_l}{s} - \frac{q x'}{2}\right) x'$.

Case 6: [2*] Single concentrated load at any point of the beam



$$\alpha = \frac{a}{s} \qquad \beta = \frac{b}{s}. \qquad \begin{aligned} \mathfrak{L} &= P a \beta (1 + \beta) \\ \mathfrak{R} &= P b \alpha (1 + \alpha) \end{aligned}$$

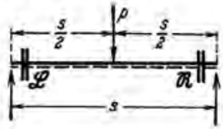
$$(\mathfrak{L} + \mathfrak{R}) = \frac{3 P a b}{s} \qquad (\mathfrak{L} - \mathfrak{R}) = P (b - a) \alpha \beta;$$

$$S = P \qquad \mathfrak{S}_r = P b \qquad \mathfrak{S}_l = P a.$$

For the region a: $M_x^0 = P \beta x$ For the region b: $M_x^0 = P \alpha x'$.

Case 7: [1] Single concentrated load at the center of the beam

$$\mathfrak{L} = \mathfrak{R} = \frac{3}{8} P s \qquad \mathfrak{S}_r = \mathfrak{S}_l = \frac{P s}{2} \qquad S = P.$$

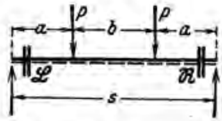


For the left half of the beam: $M_x^0 = \frac{P}{2} x$.

Case 8: [3*] Two equal concentrated loads symmetrical about the center of the beam

$$\alpha = \frac{a}{s}. \qquad S = 2P$$

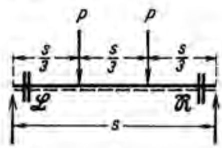
$$\mathfrak{L} = \mathfrak{R} = 3 P a (1 - \alpha) \qquad \mathfrak{S}_r = \mathfrak{S}_l = P s.$$



For the left-hand region a: $M_x^0 = P x$ For the region b: $M_x^0 = P a$.

Case 9: [4] Two equal concentrated loads at the third points

$$\mathfrak{L} = \mathfrak{R} = \frac{2}{3} P s \qquad \mathfrak{S}_r = \mathfrak{S}_l = P s \qquad S = 2P.$$

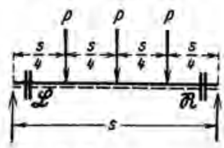


For the left third of the beam: $M_x^0 = P x$

For the middle third of the beam: $M_x^0 = \frac{P s}{3}$.

Case 10: [9] Three equal concentrated loads at the quarter points of the beam

$$\mathfrak{L} = \mathfrak{R} = \frac{15}{16} P s \qquad \mathfrak{S}_r = \mathfrak{S}_l = \frac{3}{2} P s.$$

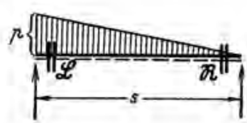


For the left quarter of the beam: $M_x^0 = \frac{3}{2} P x$;

For the second quarter of the beam: $M_x^0 = P\left(\frac{s}{4} + \frac{x}{2}\right)$. $S = 3P$.

Case 11: Triangular load starting at the right end

$$\mathfrak{L} = \frac{8 p s^2}{60} = \frac{2 p s^2}{15} \qquad \mathfrak{R} = \frac{7 p s^2}{60}$$



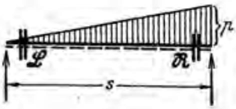
$$(\mathfrak{L} + \mathfrak{R}) = \frac{p s^2}{4} \qquad (\mathfrak{L} - \mathfrak{R}) = \frac{p s^2}{60}.$$

$$\mathfrak{S}_r = \frac{p s^2}{3} \qquad \mathfrak{S}_l = \frac{p s^2}{6} \qquad S = \frac{p s}{2}.$$

$$M_x^0 = \frac{p s^2}{6} \cdot \omega_D' \quad \text{where} \quad \omega_D' = \frac{x'}{s} - \left(\frac{x'}{s}\right)^3 *.$$

See footnote on p. 444.

ase 12: [28] **Triangular load starting at the left end**



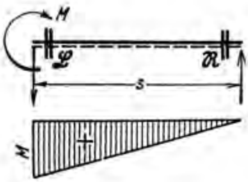
$$\mathfrak{L} = \frac{7\,p\,s^2}{60} \qquad \mathfrak{R} = \frac{8\,p\,s^2}{60} = \frac{2\,p\,s^2}{15}$$

$$(\mathfrak{L} + \mathfrak{R}) = \frac{p\,s^2}{4} \qquad (\mathfrak{L} - \mathfrak{R}) = -\frac{p\,s^2}{60};$$

$$\mathfrak{S}_r - \frac{p\,s^2}{6} \qquad \mathfrak{S}_l = \frac{p\,s^2}{3} \qquad S = \frac{p\,s}{2}.$$

$$M_x^0 = \frac{p\,s^2}{6} \cdot \omega_D \quad \text{where} \quad \omega_D = \frac{x}{s} - \left(\frac{x}{s}\right)^3 * .$$

Case 13: [53]
Moment acting at the left end of the beam**



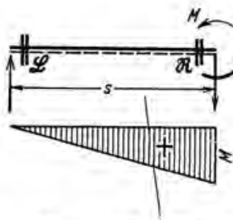
$$\mathfrak{L} = 2\,M \qquad (\mathfrak{L} + \mathfrak{R}) = 3\,M$$

$$\mathfrak{R} = M \qquad (\mathfrak{L} - \mathfrak{R}) = M;$$

$$\mathfrak{S}_r = -M \qquad \mathfrak{S} = +M.$$

$$M_x^0 = \frac{x'}{s} M.$$

Case 14: [54]
Moment acting at the right end of the beam**



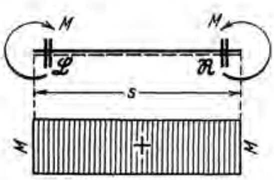
$$\mathfrak{L} = M \qquad (\mathfrak{L} + \mathfrak{R}) = 3\,M$$

$$\mathfrak{R} = 2\,M \qquad (\mathfrak{L} - \mathfrak{R}) = -M;$$

$$\mathfrak{S}_r = +M \qquad \mathfrak{S}_l = -M.$$

$$M_x^0 = \frac{x}{s} M.$$

Case 15: [56] **Equal moments acting at the ends of the beam****



$$\mathfrak{L} = \mathfrak{R} = 3\,M \qquad \mathfrak{S}_r = \mathfrak{S}_l = 0$$

$$M_x^0 = M.$$

---

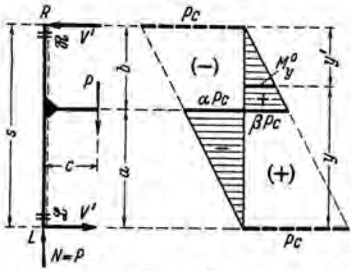
*Tables of $\omega_{D'}$- and ω_D-numbers are given in *Beam Formulas*, see footnote p. 441.

** $\mathfrak{S} = 0$ when external moments are the only loads on a beam.

Case 16-19: Single concentrated load acting on cantilever bracket of leg

$$\text{Generally: } \alpha = \frac{a}{s} \qquad \beta = \frac{b}{s} \qquad (\alpha + \beta = 1); \qquad W = 0.$$

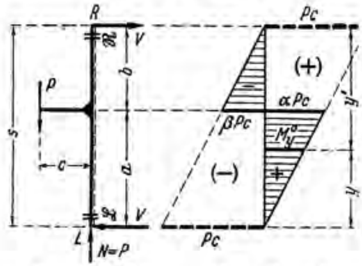
Case 16 [63*]



$$\mathfrak{L} = Pc(3\beta^2 - 1) \qquad \mathfrak{R} = Pc(1 - 3\alpha^2)$$
$$(\mathfrak{L} + \mathfrak{R}) = 3\,Pc(\beta - \alpha)$$
$$(\mathfrak{L} - \mathfrak{R}) = Pc(1 - 6\alpha\beta);$$
$$\mathfrak{S}_r = -Pc \qquad \mathfrak{S}_l = +Pc.$$

For the region ***a***: $M_y^0 = -\frac{y}{s}\,Pc$ For the region ***b***: $M_y^0 = +\frac{y'}{s}\,Pc.$

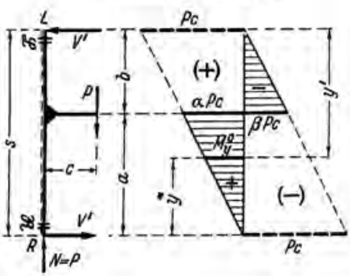
Case 17 [65*]



$$\mathfrak{L} = Pc(1 - 3\beta^2) \qquad \mathfrak{R} = Pc(3\alpha^2 -$$
$$(\mathfrak{L} + \mathfrak{R}) = 3\,Pc(\alpha - \beta)$$
$$(\mathfrak{L} - \mathfrak{R}) = Pc(6\alpha\beta - 1);$$
$$\mathfrak{S}_r = +Pc \qquad \mathfrak{S}_l = -Pc.$$

For the region ***a***: $M_y^0 = +\frac{y}{s}\,Pc$ For the region ***l***: $M_y^0 = -\frac{y'}{s}\,Pc.$

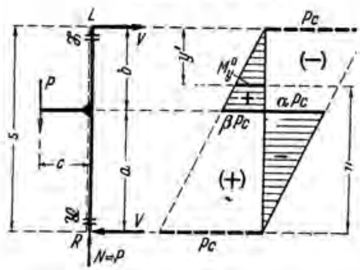
Case 18 [64*]



$$\mathfrak{L} = Pc(3\alpha^2 - 1) \qquad \mathfrak{R} = Pc(1 - 3\beta^2)$$
$$(\mathfrak{L} + \mathfrak{R}) = 3\,Pc(\alpha - \beta)$$
$$(\mathfrak{L} - \mathfrak{R}) = Pc(1 - 6\alpha\beta);$$
$$\mathfrak{S}_r = -Pc \qquad \mathfrak{S}_l = +Pc.$$

For the region ***a***: $M_y^0 = +\frac{y}{s}\,Pc$ For the region ***b***: $M_y^0 = -\frac{y'}{s}\,Pc.$

Case 19 [66*]



$$\mathfrak{L} = Pc(1 - 3\alpha^2) \qquad \mathfrak{R} = Pc(3\beta^2$$
$$(\mathfrak{L} + \mathfrak{R}) = 3\,Pc(\beta - \alpha)$$
$$(\mathfrak{L} - \mathfrak{R}) = Pc(6\alpha\beta - 1);$$
$$\mathfrak{S}_r = +Pc \qquad \mathfrak{S}_l = -Pc.$$

For the region ***a***: $M_y^0 = -\frac{y}{s}\,Pc$ For the region $M_y^0 = +\frac{y'}{s}\,Pc$

B. Moments and Cantilever Loads

(a) General Explanation:

In this book only a few formulas for the more frequent types of loads are given without using the load terms. All other types of loading use the load terms. It is important that the load terms are computed with their proper sign as explained in the Preface. A few illustrative examples are given in order to facilitate the use of the load terms.

A simple type of rigid frame has been used for these examples. The fundamental principles remain unchanged when applied to more complicated types.

(b) Examples: Moments and Cantilever Loads acting on Frame 49

The notations and the positive direction of the forces and loads are shown in the sketches on p. 172.

Six illustrative examples are computed using the six types of loads shown in fig. 1, p. 446.

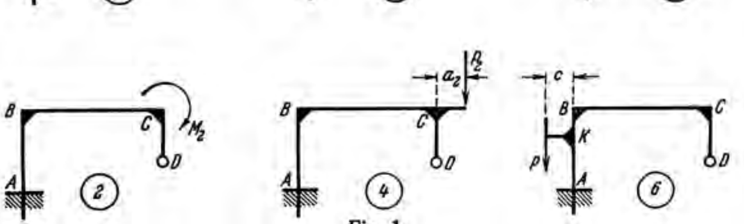


Fig. 1.

The dimensions:

$l = 10$ ft. $h_1 = 6$ ft. $h_2 = 4$ ft.

To simplify the computation we assume $k_1 = k_2 = 1$.

With these figures we obtain the coefficients (p. 186) as follows:

$$m=\frac{6,0}{4,0}=1,5$$

$$N=3\,(1,5\cdot1+1)^2+4\cdot1\,(3+1,5^2)+4\cdot1\,(3\cdot1+1)=55,75$$

$$n_{11}=\frac{2\,(1,5^2\cdot1+1+1)}{55,75}=0,1525 \qquad n_{22}=\frac{2\,(3\cdot1+1)}{55,75}=0,1435$$

$$n_{12}=n_{21}=\frac{3\cdot1,5\cdot1-1}{55,75}=0,0628.$$

Case 1: M_1 acting at the joint B

First Method of Analysis: Consider M_1 to act on the girder. Therefore use p. 174 top, "General vertical load on girder." The load term is given on p. 444, case 13. Fig. 2 shows the girder as a simple beam.

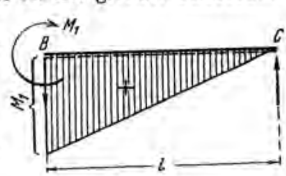


Fig. 2

From $M = M_1$ and $s = l$:

$$\mathfrak{L}=2\,M_1,\ \mathfrak{R}=M_1,\ \mathfrak{S}_r=-M,\ \mathfrak{S}_l=+M.$$

Substituting in the formulas on p. 174 top:

$$X_1=\mathfrak{L}\,n_{11}+\mathfrak{R}\,n_{21}=2\,M_1\cdot0,1525+M_1\cdot0,0628=0,3678\,M_1$$

$$X_2=\mathfrak{L}\,n_{12}+\mathfrak{R}\,n_{22}=2\,M_1\cdot0,0628+M_1\cdot0,1435=0,2691\,M_1.$$

Furthermore the moments become

$$M_A=1,5\cdot0,2691\,M_1-0,3678\,M_1=+\,0,0359\,M_1$$

$$M_B=-\,0,3678\,M_1 \qquad M_C=-\,0,2691\,M_1.$$

These moments result in the moment diagram 1-2-3-4-5-6 shown in fig. 3. The legs have no external load, therefore 1-2 and 5-6 are final moment curves. The moment curve 3-4 of the girder has to be combined with the moment curve for the external load (fig. 2), thus resulting in the final moment curve 3′-4. The final moment M_{BG} of the girder at the joint B is therefore

$$M_{BG}=M_B+M_1=-\,0.3678\,M_1+M_1=+\,0.6322\,M_1.$$

For completeness the moment M_B which pertains to the leg only may be denoted by M_{BL}.

From p. 174 top we finally compute

$$V_A=\frac{-\,M_1}{10,0}+\frac{0,3678\,M_1-0,2691\,M_1}{10,0}=-\,0,0901\,M_1$$

$$V_D=-\,V_A=+\,0,0901\,M_1 \qquad H_A=H_D=\frac{0,2691\,M_1}{4,0}=0,0673\,M_1.$$

Fig. 3 shows the direction of the reactions.

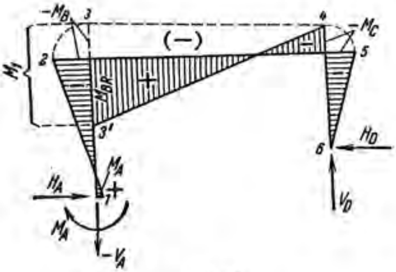


Fig. 3

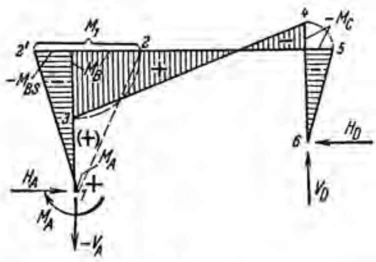


Fig. 5

Second Method of Analysis: Consider M_1 to act on the leg. In this case the formulas on p. 173 top, "General horizontal load on left leg" and the load term p. 444, case 14 apply.

The direction of M_1 is opposite to the direction of the moment shown in case 14, therefore all the coefficients of loading condition 14 should have their signs reversed. Fig. 4 shows the leg as a simple beam and its M_0-diagram.

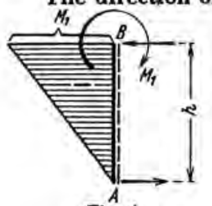


Fig. 4.

From $M = -M_1$ and $s = h_1$ we obtain

$$\mathfrak{L} = -M_1 \qquad \mathfrak{R} = -2\,M_1 \qquad (\mathfrak{L} + \mathfrak{R}) = -3\,M_1$$
$$\mathfrak{S}_r = -M_1 \qquad \mathfrak{S}_l = -(-M_1) = +M_1 \quad (W = 0).$$

Substituting in the formulas p. 173 top the auxiliary quantities:

$$\mathfrak{B}_1 = [3\,M_1 - (-3\,M_1)]\,1 \qquad = 6{,}0\,M_1$$
$$\mathfrak{B}_2 = [2\,M_1 - (-M_1)]\,1{,}5 \cdot 1 = 4{,}5\,M_1$$
$$X_1 = M_1\,(+6{,}0 \cdot 0{,}1525 - 4{,}5 \cdot 0{,}0628) = +0{,}6324\,M_1$$
$$X_2 = M_1\,(-6{,}0 \cdot 0{,}0628 + 4{,}5 \cdot 0{,}1435) = +0{,}2690\,M_1.$$

Furthermore the moments become

$$M_A = M_1\,(-1 + 0{,}6324 + 1{,}5 \times 0{,}2690) = +0{,}0359\,M_1$$
$$M_B = +0{,}6324\,M_1 \qquad M_C = -0{,}2690\,M_1.$$

In fig. 5 the moment diagram for these moments is shown as 1-2-3-4-5-6. This diagram is correct for the girder and the right leg. For the left leg it has to be corrected by the M_0-area (fig. 4), thus resulting in the final curve 1-2'. The final moment M_{BL} of the leg at the joint B is

$$M_{BL} = M_B - M_1 = +0{,}6324\,M_1 - M_1 = -0{,}3676\,M_1.$$

The reactions are (p. 173 top)

$$V_A = -V_D = -\frac{(0{,}6324 + 0{,}2690)\,M_1}{10{,}0} = -0{,}0901\,M_1$$

and for $W = 0$,

$$H_D = H_A = \frac{0{,}2690\ M_1}{4{,}0} = 0{,}0673\ M_1.$$

Case 2: Moment M_2 acting at joint C

Referring to the detailed example 1 (case 1) we have:

First Method of Analysis: M_2 acts on the girder. The formulas at top of p. 174 and the load terms of case 14 on p. 444 apply. Fig. 6 shows the girder as a simple beam l.

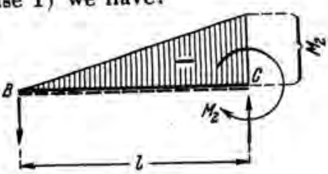


Fig. 6

From $M = -M_2$ and $s = l$ we obtain

$$\mathfrak{L} = -M_2 \qquad \mathfrak{R} = -2\,M_2 \qquad \mathfrak{S}_r = -M_2 \qquad \mathfrak{S}_l = +M_2.$$

Substituted in the formulas on p. 174 top:

$$X_1 = -M_2\,(0{,}1525 + 2\cdot 0{,}0628) = -0{,}2781\,M_2$$
$$X_2 = -M_2\,(0{,}0628 + 2\cdot 0{,}1435) = -0{,}3498\,M_2$$
$$M_A = M_2\,(-1.5\cdot 0{,}3498 + 0{,}2781) = -0{,}2466\,M_2$$
$$M_B = +0{,}2781\,M_2 \qquad M_C = M_{CL} = +0{,}3498\,M_2$$
$$M_{CO} = M_{CL} - M_2 = M_2\,(+0{,}3498 - 1) = -0{,}6502\,M_2$$
$$V_A = \frac{M_2\,(-1 - 0{,}2781 + 0{,}3498)}{10{,}0} = -0{,}0928\,M_2 = -V_D$$
$$H_A = H_D = \frac{-0{,}3498\,M_2}{4{,}0} = -0{,}0875\,M_2.$$

Fig. 7 shows the final moment diagram and reactions.

Second Method of Analysis: M_2 acts on the right leg. The formulas at the bottom of p. 173 and the load terms of case 13 on p. 444 apply. For $M = +M_2$

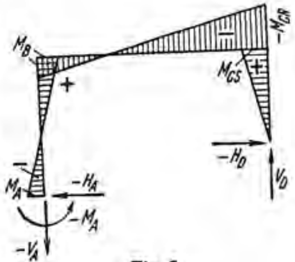


Fig. 7

$$\mathfrak{L} = +2\,M_2 \qquad \mathfrak{S}_r = -M_2 \qquad W = 0.$$

By substitution:

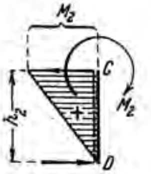


Fig. 8

$$\mathfrak{B}_1 = 3 \cdot 1{,}5\,(-M_2)\,1 = -4{,}5\,M_2$$

$$\mathfrak{B}_2 = 2 \cdot 1{,}5^2\,(-M_2)\,1 - 2\,M_2 \cdot 1 = -6{,}5\,M_2$$

$$X_1 = M_2\,(-4{,}5 \cdot 0{,}1525 + 6{,}5 \cdot 0{,}0628) = -0{,}2781\,M_2$$

$$X_2 = M_2\,(+4{,}5 \cdot 0{,}0628 - 6{,}5 \cdot 0{,}1435) = -0{,}6502\,M_2$$

$$M_A = M_2\,[1{,}5\,(-1 + 0{,}6502) + 0{,}2781] = -0{,}2466\,M_2$$

$$M_B = +0{,}2781\,M_2 \qquad M_C = M_{CR} = -0{,}6502\,M_2$$

$$M_{CL} = M_{CR} + M_2 = M_2\,(-0{,}6502 + 1) = +0{,}3498\,M_2$$

$$V_A = -V_D = \frac{M_2\,(-0{,}2781 - 0{,}6502)}{\cdot\,10{,}0} = -0{,}0928\,M_2$$

$$H_A = H_D = \frac{M_2\,(-1 + 0{,}6502)}{4{,}0} = -0{,}0875\,M_2.$$

Both methods yield identical results.

Case 3: Horizontal load P_1 acting on a cantilever on top of the left leg

This problem can be solved as a combination of a horizontal load (3a in fig. 9) and a moment (3b in fig. 9).

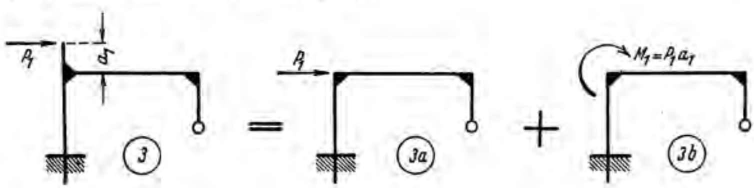


Fig. 9

For load 3a, fig. 9, the formulas at top of p. 173 and the load terms of case 6 on p. 442 apply. P_1 is assumed to be an external load on the left leg.

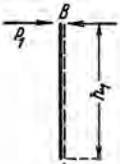


Fig. 10

From loading condition 6, p. 442, according to fig. 10 with $P = P_1$, $s = a = h_1$ and $b = 0$

$$\mathfrak{L} = \mathfrak{R} = 0 \quad \mathfrak{S}_l = +P_1 h_1 = 6{,}0\,P_1 \quad \mathfrak{S}_r = 0 \quad S = W = P_1.$$

Substituted in the formulas at top of p. 173:

$$\mathfrak{B}_1 = [3 \cdot 6{,}0\,P_1 - 0]\,1 = 18{,}0\,P_1$$

$$\mathfrak{B}_2 = [2 \cdot 6{,}0\,P_1 - 0]\,1{,}5 \cdot 1 = 18{,}0\,P_1.$$

As $\mathfrak{B}_1$ happens to equal $\mathfrak{B}_2$

$$X_1 = 18{,}0\,P_1\,(+\,0{,}1525 - 0{,}0628) = 1{,}615\,P_1$$
$$X_2 = 18{,}0\,P_1\,(-\,0{,}0628 + 0{,}1435) = 1{,}453\,P_1$$
$$M_A = P_1\,(-\,6{,}0 + 1{,}615 + 1{,}5 \cdot 1{,}453) = -\,2{,}206\,P_1$$
$$M_B = +\,1{,}615\,P_1 \qquad M_C = -\,1{,}453\,P_1$$
$$V_A = -\,V_D = -\,\frac{(1{,}615 + 1{,}453)\,P_1}{10{,}0} = -\,0{,}307\,P_1$$
$$H_D = \frac{1{,}453\,P_1}{4{,}0} = 0{,}363\,P_1 \qquad H_A = -\,(P_1 - 0{,}363\,P_1) = -\,0{,}637\,P_1.$$

Load 3b is the same as "case 1" on p. 447 except that $M_1 = P_1\,a_1$. Using the results of "case 1" on pp. 447 and 448 we obtain:

$$M_A = +\,0{,}0359\,P_1\,a_1 \qquad M_C = -\,0{,}2691\,P_1\,a_1$$
$$M_{BL} = -\,0{,}3678\,P_1\,a_1 \qquad M_{BG} = +\,0{,}6322\,P_1\,a_1$$
$$V_A = -\,V_D = -\,0{,}0901\,P_1\,a_1 \qquad H_A = H_D = 0{,}0673\,P_1\,a_1.$$

The combination of load 3a and load 3b yields the final result:

$$M_A = (-\,2{,}206 + 0{,}0359\,a_1)\,P_1 \qquad M_C = -\,(1{,}453 + 0{,}2691\,a_1)\,P_1$$
$$M_{BL} = (+\,1{,}615 - 0{,}3678\,a_1)\,P_1 \qquad M_{BG} = +\,(1{,}615 + 0{,}6322\,a_1)\,P_1$$
$$V_A = -\,V_D = -\,(0{,}307 + 0{,}0901\,a_1)\,P_1$$
$$H_A = (-\,0{,}637 + 0{,}0673\,a_1)\,P_1 \qquad H_D = (0{,}363 + 0{,}0673\,a_1)\,P_1.$$

Example: For $P_1 = 1k$ and $a_1 = 2$ feet, the moments and forces are:

$M_A = -\,2{,}134$ ft.k. $M_C = -\,1{,}991$ ft. k.

$M_{BL} = +\,0{,}879$ ft.k.

$M_{BG} = +\,2{,}879$ ft.k.

$V_A = -\,V_D = -\,0{,}487$ k.

$H_A = -\,0{,}502$ k. $H_D = 0{,}498$ k.

Fig. 11 shows the moment diagram.

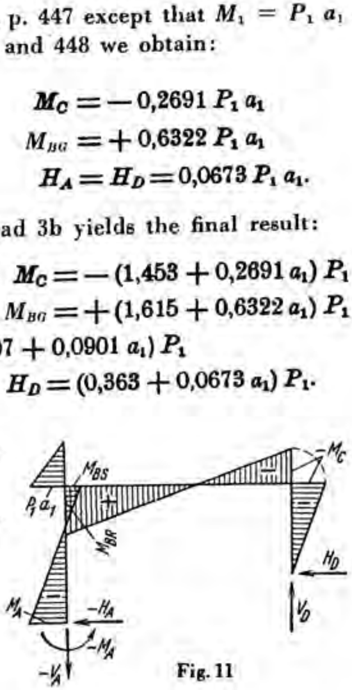


Fig. 11

Case 4: Vertical concentrated load P_2 on a cantilever at the right end of the girder

This problem, too, can be solved as a combination of load 4a, fig. 12 and 4b, fig. 12.

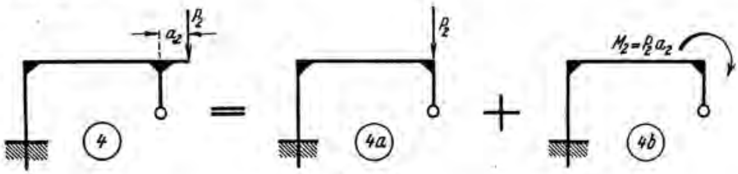


Fig. 12

Load 4a: The load P_2 causes axial stresses in the right leg and reaction $V_D = P_2$.

Load 4b: Is the same as case 2 p. 449, except that $M_2 = P_2\, a_2$.

Case 5: Cantilever load acting near the inside of the left leg (see fig. 1, p. 446)

This problem is again a combination of two simple loading conditions:

Load 5a: Concentrated load P acting at K along the axis of the leg.

Load 5b: Moment $M = Pc$ acting at K.

This load is a very common case (such as a crane load). Therefore the load terms for this load are given on p. 445.

For load 5b the dashed line and the cantilever are to the right of the axis of the leg. Thus the load terms of case 16, p. 445 apply.

Assume $a = 4{,}80$ ft. $b = 1{,}20$ ft. Then $s = h_1 = 6{,}0$ ft

$$\alpha = \frac{4{,}80}{6{,}0} = 0{,}8 \qquad \beta = 1 - 0{,}8 = 0{,}2$$

$$\mathfrak{L} = Pc\,(3 \cdot 0{,}2^2 - 1) = -\,0{,}88\,Pc \qquad \mathfrak{S}_r = -\,Pc$$

$$\mathfrak{R} = Pc\,(1 - 3 \cdot 0{,}8^2) = -\,0.92\,Pc \qquad \mathfrak{S}_l = +\,Pc$$

$$(\mathfrak{L} + \mathfrak{R}) = -\,1{,}80\,Pc \qquad \mathfrak{W} = 0.$$

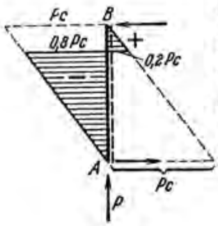


Fig. 13

The M_0 diagram is shown in fig. 13.

For the computation of stresses, the formulas on p. 173, "General horizontal load on left leg," apply. The fact that there is no horizontal force but only a moment is reflected in $\mathfrak{W} = 0$.

$$\mathfrak{B}_1 = Pc\,[3 \cdot 1 - (-1{,}80)]\,1 \qquad = 4{,}80\,Pc$$
$$\mathfrak{B}_2 = Pc\,[2 \cdot 1 - (-0{,}88)]\,1{,}5 \cdot 1 = 2{,}88\,Pc$$
$$X_1 = Pc\,(+4{,}80 \cdot 0{,}1525 - 2{,}88 \cdot 0{,}0628) = 0{,}551\,Pc$$
$$X_2 = Pc\,(-4{,}80 \cdot 0{,}0628 + 2{,}88 \cdot 0{,}1435) = 0{,}112\,Pc$$
$$M_A = Pc\,[-1 + 0{,}551 + 1{,}5 \cdot 0{,}112] = -0{,}281\,Pc$$
$$M_B = +0{,}551\,Pc \qquad M_C = -0{,}112\,Pc.$$

For the summation of reactive forces it should be kept in mind that the formula for $V_A = -V_D$ p. 173 top is valid for case 5b only, i.e., for the external moment Pc. Case 5a, single axial load, causes $V_A = P$ and $V_D = 0$. Keeping this in mind we obtain

$$V_A = P - \frac{0{,}551\,Pc + 0{,}112\,Pc}{10{,}0} = (1 - 0{,}066\,c)\,P$$

$$V_D = +0{,}036\,Pc \qquad H_D = H_A = \frac{0{,}112\,Pc}{4{,}0} = 0{,}028\,Pc.$$

Fig. 14 shows the moment diagram. The left leg was isolated from the frame for greater clarity. The M_0-area of fig. 13 must be plotted from the dashed closing line. The final moments at K are figured from the formulas for M_{yl} on p. 173, top, as follows:

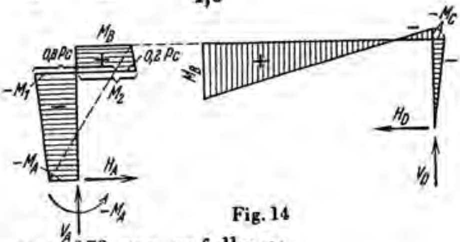


Fig. 14

$$M_1 = -0{,}8\,Pc + 0{,}2\,(-0{,}281\,Pc) + 0{,}8 \cdot 0{,}551\,Pc = -0{,}415\,Pc$$
$$M_2 = M_1 + Pc = +0{,}585\,Pc.$$

Case 6: Cantilever load acting on the outside of the left leg

Using the same dimensions as for case 5, the moment and the moment curve are the same except that directions and signs are reversed. However, the influence of the single load at K (load 5a) is the same, therefore

$$V_A = (1 + 0{,}066\,c)\,P \qquad V_D = -0{,}066\,Pc.$$

If case 5 had not yet been computed, the load terms would have to be computed by means of case 17, p. 445. It is apparent that they are the load terms of case 16, p. 445, multiplied by minus 1.

C. Influence Lines

(a) General Notations:

For all practical purposes, influence lines are used only for frames with girders that are horizontal or slightly sloped, such as frames of the types 1 – 14, **38 – 60**, 73 – 88 and **106 – 110**.

The equation for the influence line of a single load moving over the girder has the basic form

(1) $$y = e' \cdot \omega_D' + e \cdot \omega_D.$$

This equation represents the influence of the statically indeterminate moments at the joints (restraint at the end of the girder). The equation is correct for the moments at the joints. For the moment at any other point of the girder, for shear and reactions, another value representing the contribution of the girder as a simple beam has to be added. (See below.)

The values e and e' are coefficients that can be either positive or negative.

The ω-figures are functions of the ratios

(2) $$\xi = \frac{x}{l} \quad \text{and} \quad \xi' = \frac{x'}{l}.$$

For we have

(3) $$\omega_D' = \xi' - \xi'^3 \quad \text{and} \quad \omega_D = \xi - \xi^3.$$

Fig. 15 shows the basic shape of the influence line. t and t' are the intercepts cut off by the tangents at one support on the vertical through the other support. From the same author's *Beam Formulas* (see* p. 441) the following formulas have been developed:

(4) $$\begin{cases} t = e' - 2\,e \text{ and} \\ t' = 2\,e' - e. \end{cases}$$

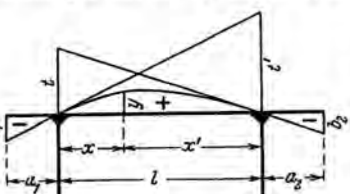


Fig. 15

If the girder extends beyond the leg as a cantilever the influence line for the cantilever is a straight line represented by the tangent at the support. The ordinates b_1 and b_2 at the end of the cantilever, as shown in fig. 15, are

(5) $$b_1 = -t'\,\alpha_1 \quad \text{and} \quad b_2 = -t\,\alpha_2,$$

where

(6) $$\alpha_1 = \frac{a_1}{l} \quad \text{and} \quad \alpha_2 = \frac{a_2}{l}.$$

In figuring the equation for the influence line, we always use the "general vertical load on girder" and substitute

$$(7)\qquad \begin{cases} \mathfrak{L} = l \cdot \omega_D' & \mathfrak{R} = l \cdot \omega_D \\ \mathfrak{S}_r = l \cdot \xi' & \mathfrak{S}_l = l \cdot \xi \qquad S = 1. \end{cases}$$

The following example shows how to use these formulas.

(b) Illustrative Example for Determining Influence Line Equations

Compute and draw the influence lines for moments, horizontal and vertical reactions for Frame 44 shown in fig. 16, for a single concentrated load $P = 1$, moving over the girder extending beyond the legs as a cantilever.

The dimensions are:

$l = 8{,}40$ ft. $\quad h = 4{,}80$ ft.
$a_1 = 1{,}35$ ft. $\quad a_2 = 1{,}80$ ft.

The moments of inertia are:

$J_1 = 0{,}0072$ ft.^{4}
$J_2 = 0{,}0216$ ft.^{4}
$J_3 = 0{,}0114$ ft.^{4}

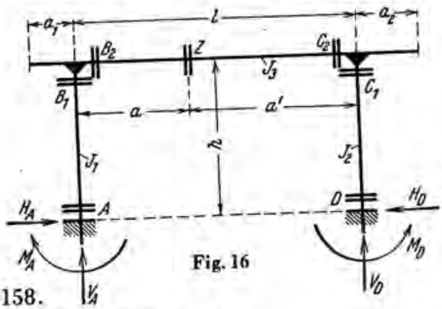


Fig. 16

First compute the coefficients on p. 158.

$$k_1 = \frac{114}{72} \cdot \frac{4{,}80}{8{,}40} = 0{,}905 \qquad k_2 = \frac{114}{216} \cdot \frac{4{,}80}{8{,}40} = 0{,}302$$

$$R_1 = 2\,(3 \cdot 0{,}905 + 1) = 7{,}430 \qquad k_1^2 = 0{,}819$$

$$R_2 = 2\,(1 + 3 \cdot 0{,}302) = 3{,}810 \qquad k_1\,k_2 = 0{,}273$$

$$R_3 = 2\,(0{,}905 + 0{,}302) = 2{,}415 \qquad k_2^2 = 0{,}091$$

$$N = (6 \cdot 0{,}273 + 2{,}415)\,(0{,}905 + 1 + 0{,}302) + 12 \cdot 0{,}273 = 12{,}22$$

$$n_{11} = \frac{3{,}810 \cdot 2{,}415 - 9 \cdot 0{,}091}{3 \cdot 12{,}22} = 0{,}2286$$

$$n_{22} = \frac{7{,}430 \cdot 2{,}415 - 9 \cdot 0{,}819}{3 \cdot 12{,}22} = 0{,}2884$$

$$n_{33} = \frac{7{,}430 \cdot 3{,}810 - 1}{3 \cdot 12{,}22} = 0{,}7450$$

$$n_{12} = n_{21} = \frac{9 \cdot 0{,}273 - 2{,}415}{3 \cdot 12{,}22} = 0{,}0011$$

$$n_{13} = n_{31} = \frac{0{,}905 \cdot 3{,}810 - 0{,}302}{12{,}22} = 0{,}2574$$

$$n_{23} = n_{32} = \frac{7{,}430 \cdot 0{,}302 - 0{,}905}{12{,}22} = 0{,}1096.$$

For Frame 44 no formulas for a single concentrated load acting on the girder are given. According to the note on p. 158, the formulas for Frame 48 may be used by substituting $h_1 = h_2 = h$ and $n = 1$. Therefore the top of p. 171, "general vertical load on girder," applies.

From formula 7, p. 455 follows:

$$\mathfrak{L} = 8{,}40\,\omega_D' \qquad \mathfrak{R} = 8{,}40\,\omega_D \qquad \mathfrak{S}_r = 8{,}40\,\xi' \qquad S = 1.$$

Therefore the constants X are:

$$X_1 = 8{,}40\,(0{,}2286\,\omega_D' + 0{,}0011\,\omega_D) = 1{,}920\,\omega_D' + 0{,}009\,\omega_D$$
$$X_2 = 8{,}40\,(0{,}0011\,\omega_D' + 0{,}2884\,\omega_D) = 0{,}009\,\omega_D' + 2{,}423\,\omega_D$$
$$X_3 = 8{,}40\,(0{,}2574\,\omega_D' + 0{,}1096\,\omega_D) = 2{,}162\,\omega_D' + 0{,}921\,\omega_D.$$

Influence Line for the End Moment M_A

From p. 171, top: $M_A = X_3 - X_1$, therefore

$$y = (2{,}162 - 1{,}920)\,\omega_D' + (0{,}921 - 0{,}009)\,\omega_D = 0{,}242\,\omega_D' + 0{,}912\,\omega_D.$$

The t-values (see equation 4, p. 454)

$$t = 0{,}242 + 2 \cdot 0{,}912 = 2{,}066 \text{ ft.} \quad t' = 2 \cdot 0{,}242 + 0{,}912 = 1{,}396 \text{ ft.}$$

With $\alpha_1 = \dfrac{1{,}35}{8{,}40} = 0{,}161$ and $\alpha_2 = \dfrac{1{,}80}{8{,}40} = 0{,}214$

(see equations 5 and 6, p. 454) the end ordinates of the cantilevers are:

$$b_1 = -\,1{,}396 \cdot 0{,}161 = -\,0{,}224 \text{ ft.} \quad b_2 = -\,2{,}066 \cdot 0{,}214 = -\,0{,}443 \text{ ft.}$$

The ordinates y are best compiled in a table (see below). In this example the influence ordinates at the tenth points were computed. ω_D'- and ω_D- figures from the book *Beam Formulas* (see footnote* p. 441)

ξ	ω_D'	ω_D	$0{,}242\,\omega_D'$	$0{,}912\,\omega_D$	y (in ft)
0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,171	0,099	0,042	0,090	0,132
0,2	0,288	0,192	0,070	0,175	0,245
0,3	0,357	0,273	0,086	0,249	0,335
0,4	0,384	0,336	0,093	0,307	0,400
0,5	0,375	0,375	0,090	0,342	0,432
0,6	0,336	0,384	0,081	0,350	0,431
0,7	0,273	0,357	0,066	0,326	0,392
0,8	0,192	0,288	0,046	0,262	0,308
0,9	0,099	0,171	0,024	0,156	0,180
1,0	0,0	0,0	0,0	0,0	0,0

The influence line is drawn as in fig. 17, p. 460.

Influence Line for the Moment M_{B1} at the Top of the Left Leg

From p. 171, top $M_B = - X_1$; therefore

$$y = -1{,}920\,\omega_D' - 0{,}009\,\omega_D.$$

Furthermore

$$t = -1{,}920 - 2 \cdot 0{,}009 = -1{,}938 \text{ ft}, \; t' = -2 \cdot 1{,}920 - 0{,}009 = -3{,}849 \text{ ft}.$$

$$b_1 = 3{,}849 \cdot 0{,}161 = +0{,}615 \text{ ft}. \quad b_2 = 1{,}938 \cdot 0{,}214 = +0{,}416 \text{ ft}.$$

The ordinates y are figured similarly as shown for M_A. Fig. 17, p. 460 shows the influence line.

Influence Line for the Moment M_{B2}

With the exception of the cantilever a_1 the influence line is the same as for M_{B1}.

$$b_1 = +0{,}615 - a_1 = +0{,}615 - 1{,}35 = -0{,}735 \text{ ft}.$$

Fig. 17, p. 460, shows the influence lines for M_{B2} and M_{B1} together. They differ only at the left cantilever as shown by the dashed line.

Influence Line for the Moment M_{C1} at the Top of the Right Leg

From p. 185, top, $M_C = - X_2$; therefore

$$y = -0{,}009\,\omega_D' - 2{,}423\,\omega_D.$$

$$t = -0{,}009 - 2 \cdot 2{,}423 = -4{,}855 \text{ ft}. \quad t' = -2 \cdot 0{,}009 - 2{,}423 = -2{,}441 \text{ ft}.$$

$$b_1 = 2{,}441 \cdot 0{,}161 = +0{,}392 \text{ ft}. \quad b_2 = 4{,}855 \cdot 0{,}214 = +1{,}037 \text{ ft}.$$

The influence line is shown in fig. 17.

Influence Line for the Moment M_{C2}

Except for the cantilever a_2 this influence line is the same as for M_{C1}. We find

$$b_2 = +1{,}037 - 1{,}80 = -0{,}763 \text{ ft}.$$

See fig. 17 for diagram.

Influence Line for the Moment M_D

From p. 185, top, $M_D = n\,X_3 - X_2$, therefore for $n = 1$

$$y = (2{,}162 - 0{,}009)\,\omega_D' + (0{,}921 - 2{,}423)\,\omega_D = 2{,}153\,\omega_D' - 1{,}502\,\omega_D$$

$$t = 2{,}153 - 2 \cdot 1{,}502 = -0{,}851 \text{ ft}. \qquad t' = 2 \cdot 2{,}153 - 1{,}502 = +2{,}804 \text{ ft}.$$

$$b_1 = -2{,}804 \cdot 0{,}161 = -0{,}453 \text{ ft}. \quad b_2 = +0{,}851 \cdot 0{,}214 = +0{,}182 \text{ ft}.$$

The influence line is shown in fig. 17, p. 460.

Influence Line for the Moment M_Z at Any Point Z of the Girder

We start from the equation on top, p. 171

$$M_z = M_x{}^0 + \frac{x'}{l} M_B + \frac{x}{l} M_C.$$

If the influence line for point Z (a and a') fig. 16, p. 455 is wanted, replace x' by a' and x by a in the above equation.

(8) $$\alpha' = \frac{a'}{l} \quad \text{and} \quad \alpha = \frac{a}{l}.$$

Consider the moment M_{ω}^{0}, which is the moment in the simple beam. If the load $P = 1$ moves within the limits of a or a', then

$$M_Z{}^0 = \frac{1\,x}{l}\,a' = a'\,\xi \quad \text{or} \quad M_Z{}^0 = \frac{1\,x'}{l}\,a = a\,\xi'.$$

Therefore the equation of the influence line is

(9) $$\begin{cases} y = a'\,\xi + \alpha'\,y_B + \alpha\,y_C & \text{(within the limits of a)} \\ y' = a\,\xi' + \alpha'\,y_B + \alpha\,y_C & \text{(within the limits of a')}. \end{cases}$$

In equation (9) y_B and y_C are the equations of the influence line for M_B and M_C.

As an example let us write the equation for $\alpha = 0{,}4$, $\alpha' = 0{,}6$
From equation 8 follows

$$a = 0{,}4 \cdot 8{,}40 = 3{,}36 \text{ ft.} \qquad a' = 8{,}40 - 3{,}36 = 5{,}04 \text{ ft.}$$

From p. 457

$$y_B = -1{,}920\,\omega_D' - 0{,}009\,\omega_D \qquad y_C = -0{,}009\,\omega_D' - 2{,}423\,\omega_D.$$

Therefore it follows from equation 9

$$y = 5{,}04\,\xi - 0{,}6\,(1{,}920\,\omega_D' + 0{,}009\,\omega_D) - 0{,}4\,(0{,}009\,\omega_D' + 2{,}423\,\omega_D)$$

$$y = 5{,}04\,\xi - 1{,}156\,\omega_D' - 0{,}975\,\omega_D$$

$$y' = 3{,}36\,\xi' - 1{,}156\,\omega_D' - 0{,}975\,\omega_D.$$

The tangent intercepts from equation 4 need an additional term

(10) $$t = a + e' + 2\,e \text{ and } t' = a' + 2\,e' + e.$$

Using numbers

$$t = 3{,}36 - 1{,}156 - 2 \cdot 0{,}975 = +\,0{,}254 \text{ ft.}$$
$$t' = 5{,}04 - 2 \cdot 1{,}156 - 0{,}975 = +\,1{,}753 \text{ ft.}$$

The expression for the end ordinates of the cantilevers from equation 5 are valid here:

$$b_1 = -1{,}753 \cdot 0{,}161 = -0{,}283 \text{ ft.}$$

$$b_2 = -0{,}254 \cdot 0{,}214 = -0{,}056 \text{ ft.}$$

It is best to figure y and y' values again by using a table. The influence line is shown in fig. 17.

The same procedure was used in determining the influence line for point Z ($\alpha = 0{,}5$ and $0{,}6$), which is shown in fig. 17.

Influence Line for the Horizontal Thrust H

From p. 171, top $H_A = H_D = H = \frac{X_2}{h}$, therefore

$$y = \frac{2{,}162\, \omega_D' + 0{,}921\, \omega_D}{4{,}80} = 0{,}451\, \omega_D' + 0{,}192\, \omega_D.$$

The H-line is shown in fig. 17.

Influence Line for the Reaction V_A

From p. 185, top $V_A = \frac{\mathfrak{S}_r + X_1 - X_2}{l}$.

Using the computations on p. 456.

$$y = \xi' + (0{,}2286 - 0{,}0011)\, \omega_D' + (0{,}0011 - 0{,}2884)\, \omega_D$$

$$y = \xi' + 0{,}227\, \omega_D' - 0{,}287\, \omega_D.$$

The tangent intercepts t and the ordinate b_1 become at the end of the cantilever

(11) $\quad t = 1 + e' + 2\,e \qquad b_1 = (1 + \alpha_1) - t'\,\alpha_1.$

therefore

$t = 1 + 0{,}227 - 2 \cdot 0{,}287 = +\,0{,}653 \qquad t' = 2 \cdot 0{,}227 - 0{,}287 = +\,0{,}167$

$b_1 = 1{,}161 - 0{,}167 \cdot 0{,}161 = +\,1{,}134 \qquad b_2 = -\,0{,}653 \cdot 0{,}214 = -\,0{,}140.$

The V_A line is shown in fig. 17.

Influence Line for the Reaction V_D

From p. 171, top $V_D = S - V_A$; hence for $S = 1$ using the equation for V_A

$$y = \xi - 0{,}227\, \omega_D' + 0{,}287\, \omega_D.$$

The t'-values and the b_2-values become:

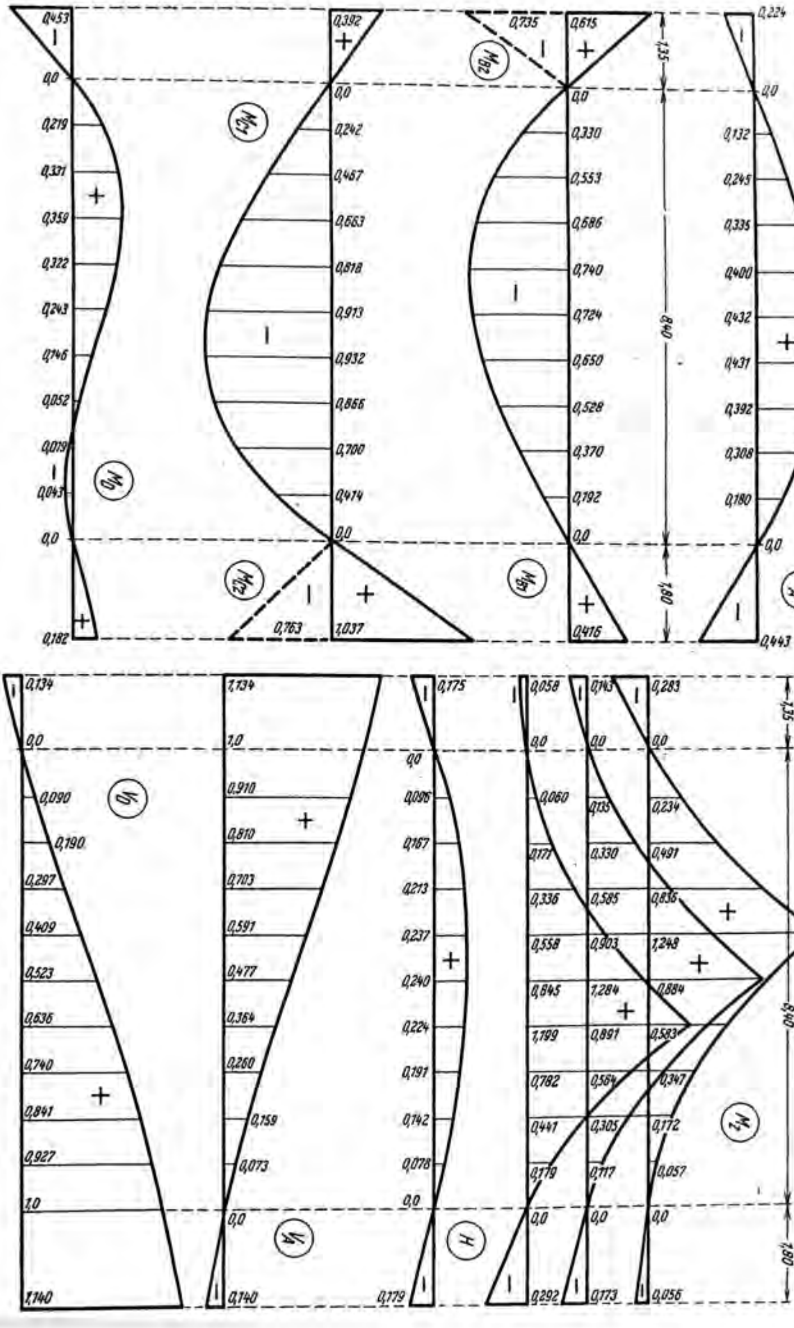
(12) $\quad t' = 1 + 2\,e' + e \qquad b_2 = (1 + \alpha_2) - t\,\alpha_2.$

Therefore

$t = -\,0{,}227 + 2 \cdot 0{,}287 = +\,0{,}347 \quad t' = 1 - 2 \cdot 0{,}227 + 0{,}287 = +\,0{,}833$

$b_1 = -\,0{,}833 \cdot 0{,}161 = -\,0{,}134 \qquad b_2 = 1{,}214 - 0{,}347 \cdot 0{,}214 = +\,1{,}140$

The V_D-line is shown in fig. 17.



0,453
0,0
0,219
0,331
0,359
0,322
0,243
0,146
0,052
0,019
0,043
0,0
0,182
M_D
0,392
0,0
0,242
0,467
0,663
0,818
0,913
0,932
0,866
0,700
0,414
0,0
0,763
1,037
M_{C1}
M_{C2}
0,735
0,615
0,0
0,330
0,553
0,686
0,740
0,724
0,650
0,528
0,370
0,192
0,0
0,416
M_{B2}
M_{B1}
1,35
8,40
1,80
0,224
0,0
0,132
0,245
0,335
0,400
0,432
0,431
0,392
0,308
0,180
0,0
0,443
M_A
0,134
0,0
0,090
0,190
0,297
0,409
0,523
0,636
0,740
0,841
0,927
1,0
1,140
V_D
1,134
1,0
0,910
0,810
0,703
0,591
0,477
0,364
0,260
0,159
0,073
0,0
0,140
V_A
0,175
0,0
0,096
0,167
0,213
0,237
0,240
0,224
0,191
0,142
0,078
0,0
0,179
H
0,058
0,0
0,060
0,177
0,336
0,558
0,845
1,199
0,782
0,441
0,179
0,0
0,292
0,143
0,0
0,135
0,330
0,585
0,903
1,284
0,891
0,564
0,305
0,117
0,0
0,173
0,283
0,0
0,234
0,491
0,836
1,248
0,884
0,583
0,347
0,172
0,057
0,0
0,056
M_z
1,35
8,40
1,80

Fig. 17