

Calculation of flow rate from differential pressure devices – orifice plates

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- Introduction - Why using dp-devices when other principles provide higher accuracy? Situation of very high temperatures.
- How is flow and pressure linked?
- Why do we need a discharge coefficient?
- What must be known to calculate flow rate correctly?
- What help does standards like ISO 5167 provide?
- Differences between different version (1991 to 2003).
- Which other information's are essential in a given application?
- What influences the flow measurement result and how important is it?
- What uncertainties must we expect?



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Introduction 1 – Flow is always measured indirectly

- **Flow rate** – one of the most complex and difficult quantity to measure and to calibrate.
- **No principle** for direct measurement exists.
- **Many different physical measurement principles** – all have advantages and drawbacks.
- **Roundabout way** via speed, rotation speed, frequency, phase difference, temperature difference, force, voltage etc. (thousands of patents exist!)
- **But:** All metering devices need a calibration in one form or another.
- **Question:** How should we calibrate meters for use in hot water (> 300 °C, 180 bar) or steam?



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Introduction 2 – Standards for dp-meters

- **“Primary devices”** - measure pressure difference Δp produced by flow passing an obstruction and “calculate” the mass flow.
- **Any obstruction** can be used if the relation between **q** and Δp is well known.
- Orifice plates – Nozzles – Venturi tubes are standardized obstructions. But others exist as well.
- The standards tell detailed how to use these devices and what the limitations are.
- ISO 5167 Part 1, 2, 3 and 4 from 2003 (revision from 1991)
- ISO/TR 15377 contains complementing information to ISO 5167
- ASME MFC-3M, 2004 (revision from 1989)
- ASME PTC 6-2004 Performance Test Codes (revision from 1996)



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Introduction 3 – The Situation for Power Plants

- In the late 60-ties and 70-ties no robust flow measurement technique was available for the high temperature/pressure applications.
- The nuclear power industry is conservative and focused on safety.
- The thermal effect is limited to a licensed value.
- A flow measurement accuracy of 2 % kept the output at 98 % of rated effect.
- dp-devices showed no stable behavior due to fouling.
- Over the years a lot of measuring technique has been exchanged.



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Introduction 4 – Thermal Power Increase

- “MUR” – Measurement Uncertainty Recapture – is a trial to lower measurement uncertainty (mainly in flow), which in turn can be used to increase the thermal output closer to the rated effect.
- How can measurement uncertainty be reduced?
 - By calibration!
 - And by sensing of eventual changes over time!
- Who can calibrate flow meters at 330 °C, 180 bar, $Re > 10^7$?
 - Nobody!
- What to use for sensing?
 - A second measurement principle!



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Introduction 5 - Difficulties in Meter Calibration

Gravimetric Method - (short traceability to kg, s, °C)

- The water must be kept at constant pressure and temperature
- Large amount of water – big vessels are needed
- Diverter and weighing tank in closed system to avoid steam evaporation

Volumetric Method - (longer traceability to m^3 , ρ_{water} , s, °C)

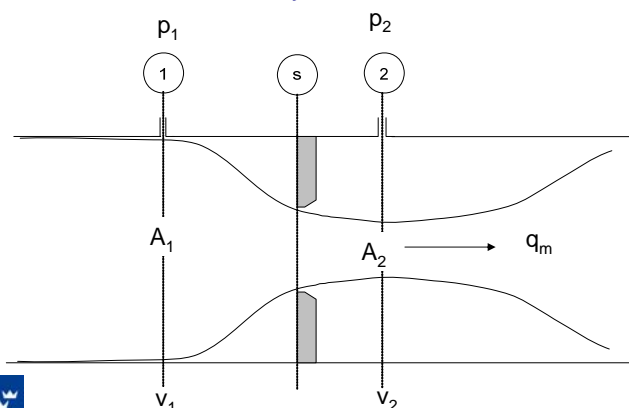
- Pipe- or Ball prover (long big pipes – working under high pressure)
- Suitable material for ball or piston to tighten at high temperature/pressure (lubrication)
- Large positive displacement meter as master meter



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Flow is forced through the orifice

We need a standard to clear up how pressure p , speed v , flow q_m and flow area are interrelated and used to calculated mass flow from the pressure difference between point 1 and 2 with satisfactory measurement uncertainty?



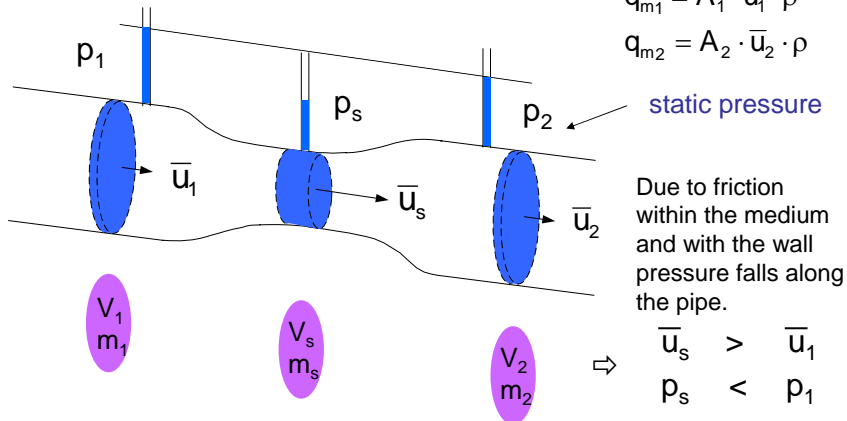
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Flow in pipe with change in cross area

Assume continuity and incompressibility

$$q_{m1} = A_1 \cdot \bar{u}_1 \cdot \rho$$

$$q_{m2} = A_2 \cdot \bar{u}_2 \cdot \rho$$



Mass m_1 and volume V_1 of a "flow element" does not change if area changes

Average speed increases, static pressure falls where the area is smaller.

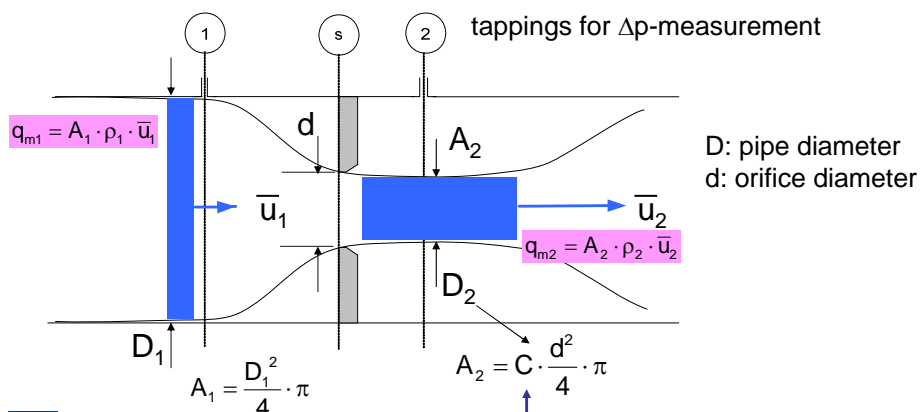


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Measurement principle

Incompressibility means density does not change $\rho_2 = \rho_1$

Continuity means the flow is the same every where in the pipe $q_{m1} = q_{m2}$



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Area at the smallest flow cross section not exactly known!



Bernoulli's equation on energy conservation

$$p_1 + \frac{1}{2} \rho_1 \bar{u}_1^2 = p_2 + \frac{1}{2} \rho_2 \bar{u}_2^2 = \text{const} = p_0 \quad \begin{array}{l} p_0: \text{total pressure in medium} \\ p = p_0 \text{ where } u = 0 \end{array}$$

The sum of static and dynamic pressure is the same everywhere in pipe.

$$\Downarrow$$

$$\Delta p = p_1 - p_2 = \frac{\rho}{2} (\bar{u}_2^2 - \bar{u}_1^2) \quad \frac{1}{2} \rho \bar{u}^2 : \text{"dynamic pressure"}$$

Replace the speed terms through geometric measures and insert.

$$\begin{aligned} q_m &= A \cdot \rho \cdot \bar{u} \\ \bar{u}_1^2 &= \frac{4^2 \cdot q_m^2}{D_1^4 \cdot \pi^2 \cdot \rho^2} \\ \bar{u}_2^2 &= \frac{4^2 \cdot q_m^2}{D_2^4 \cdot \pi^2 \cdot \rho^2} \end{aligned}$$

$$\begin{aligned} \Delta p &= \frac{\rho}{2} \cdot \frac{4^2 q_m^2}{\pi^2 \cdot \rho^2} \left(\frac{1}{D_2^4} - \frac{1}{D_1^4} \right) & \Delta p \cdot D_2^4 &= \frac{\rho}{2} \cdot \frac{4^2 q_m^2}{\pi^2 \cdot \rho^2} \left(\frac{D_2^4}{D_2^4} - \frac{D_2^4}{D_1^4} \right) \\ 2 \cdot \Delta p \cdot \rho \cdot \pi^2 \cdot D_2^4 &= 4^2 q_m^2 \left(1 - \frac{D_2^4}{D_1^4} \right) \end{aligned}$$



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"Replace" D_2 with d , introduce β , solve for q_m

Final equation for calculating q_m

$$q_m^2 = \frac{1}{\left(1 - \frac{D_2^4}{D_1^4} \right)} \frac{\pi^2}{4^2} \cdot D_2^4 \cdot 2 \cdot \Delta p \cdot \rho \quad \begin{array}{l} \text{Exchange unknown } D_2 \\ \text{with known } d \end{array}$$

$$\beta = \frac{d}{D}$$

Introduce a **discharge coefficient C** relating the unknown diameter D_2 to the known orifice diameter d !

Introduce an **expansion coefficient ε** for compressible media (gas, steam).

$$q_m = \frac{C}{\sqrt{1 - \beta^4}} \cdot \varepsilon \cdot \frac{\pi}{4} \cdot d^2 \cdot \sqrt{2 \Delta p \cdot \rho_1}$$

To "measure flow" means to determine Δp and use it in the equation with all other "constants" and calculate q_m .



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Why a discharge coefficient?

C determines the relation between the real flow through a "primary device" and the theoretical possible flow.

For a given "primary device" this dimensionless parameter is determined using an incompressible fluid. With fixed geometry C only depends on the actual Reynolds number. In a way C can be regarded as a "calibration constant" for a "primary device".

Generally, different installations, that are geometrically equivalent and sense the same flow conditions and are characterized by the same Reynolds number, renders the same value for C .

But: Re varies with pressure, temperature, viscosity and flow. And so does C .

It is not easy to analyse the effect of different parameters on the flow.



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Why an expansibility factor

ϵ expresses the different compressibility of different fluids (gases, steam), i.e. molecules are, depending on their form more or less compressed when passing the orifice. $\epsilon \leq 1$

Water and other liquids are considered incompressible

$$\Rightarrow \epsilon = 1.$$

This makes calculations much simpler.

The size of ϵ depends on the pressure relation and the isentropic coefficient κ , i.e. the relation between a relative change in pressure and the corresponding relative change in density. κ is a property that is different for different media and varies also with the pressure and temperature of the medium.

For many gases there are no published data for κ . The standard recommends to use c_p/c_v .



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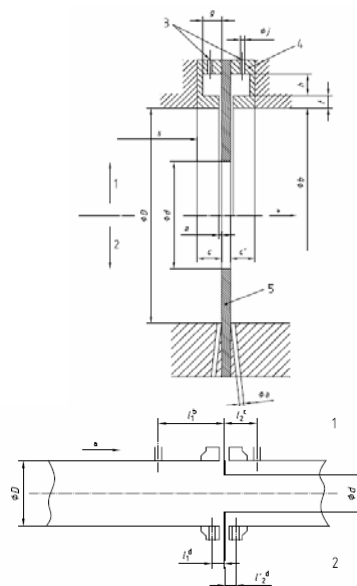
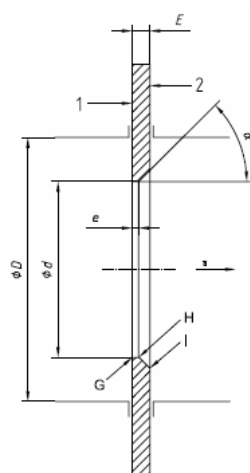
What does the standards treatise on?

- Allowed variations for different measures (pipe diameter, orifice diameter, distance to upstream disturbances).
- Tolerances for all measures.
- How orifices must be installed.
- How thick the orifice can be and how much it may buckle.
- Which tolerances in angle are allowed
- How sharp or round the edges have to be.
- Which upstream surface roughness the pipe must underpass.
- How the pressure tapings may be shaped and where they must be placed.
- How the tapings must be arranged in detail (diameter of drilled hole, smoothness of pipe wall, distance to possible flow disturbances etc.)

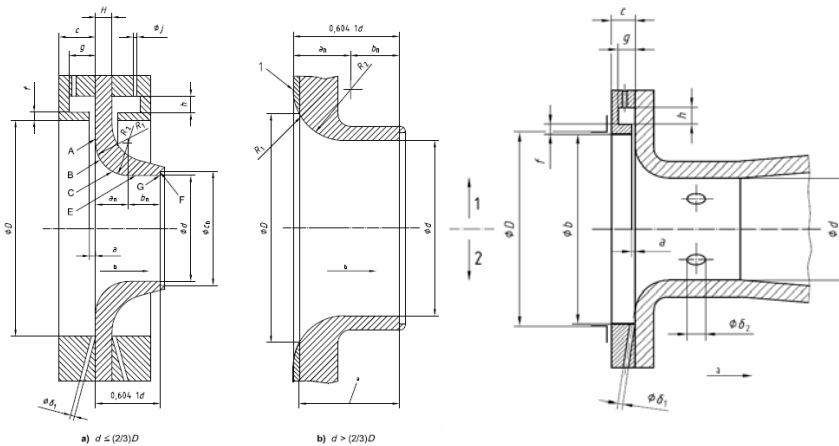
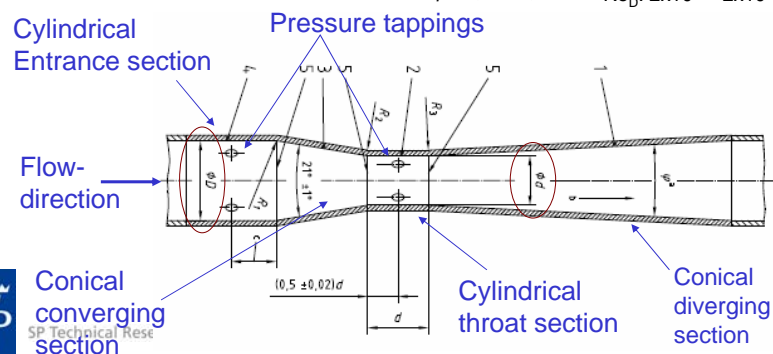


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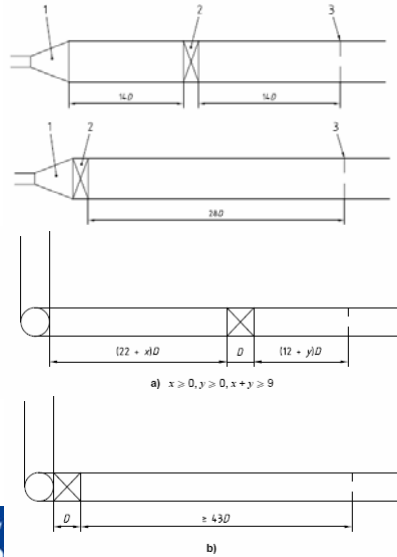
Orifice plates (part 2 of ISO 5167)



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$$\text{Re}_D: 1,5 \times 10^5 - 2 \times 10^6$$

$$\beta: 0,4 - 0,7 \quad \text{Re}_D: 2 \times 10^5 - 2 \times 10^6$$


The standard gives advice by installation examples



- 1: conical diameter increase
- 2: totally open valve
- 3: place for orifice
- 4: 90° elbow

As long as certain distances (in multiples of D) between the orifice (or other dp-devices) and upstream disturbances are kept, the uncertainties stated in the standard are valid.

No extra uncertainties need to be added.

A table for lowest distance to different installation disturbances are given in a table.



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Table for critical distances to upstream disturbances

Table 3 — Required straight lengths between orifice plates and fittings without flow conditioners

Values expressed as multiples of internal diameter, D

Diameter ratio β	Upstream (inlet) side of orifice plate													Downstream (outlet) side of the orifice plate
	Single 90° bend	Two 90° bends in any plane ($S > 30D$) ^a	Two 90° bends in the same plane: S-configuration ($30D > S > 10D$) ^a	Two 90° bends in the same plane: S-configuration ($10D > S$) ^a	Two 90° bends in perpendicular planes ($30D > S > 5D$) ^a	Two 90° bends in perpendicular planes ($5D > S$) ^{a, b}	Single 90° tee with or without an extension	Single 45° bend	Concentric reducer	Concentric expander	Full bore ball valve or gate valve fully open	Abrupt symmetrical reduction	Thermometer pocket or well ^c of diameter $\leq 0,03D$ ^d	
	2	3	4	5	6	7	8	9	10	11	12	13	14	
—	A ^e	B ^f	A ^e	B ^f	A ^e	B ^f	A ^e	B ^f	A ^e	B ^f	A ^e	B ^f	A ^e	B ^f
$\leq 0,20$	6	3	10	9	10	9	19	18	34	17	3	9	7	9
0,40	16	3	10	9	10	9	44	18	50	25	9	3	30	9
0,50	22	9	18	10	22	10	44	18	75	34	19	9	30	18
0,60	42	13	30	18	42	18	44	18	65 ^h	25	29	18	30	18
0,67	44	20	44	18	44	20	60	18	36	18	44	18	12	6
0,75	44	20	44	18	44	22	44	20	75	18	44	18	13	8

NOTE 1 The minimum straight lengths required are the lengths between various fittings located upstream or downstream of the orifice plate and the orifice plate itself. Straight lengths shall be measured from the downstream end of the curved portion of the reducer (or only bend or of the tee or the downstream end of the curved or conical portion of the reducer or the expander.

NOTE 2 Most of the bends on which the lengths in this table are based had a radius of curvature equal to $1,5D$.

^a S is the separation between the two bends measured from the downstream end of the curved portion of the upstream bend to the upstream end of the curved portion of the downstream bend.

^b This is not a good upstream installation; a flow conditioner should be used where possible.

^c The installation of thermometer pockets or wells will not alter the required minimum upstream straight lengths for the other fittings.

^d A thermometer pocket or well of diameter between $0,03D$ and $0,13D$ may be installed provided that the values in Columns A and B are increased to 20 and 10 respectively. Such an installation is not, however, recommended.

^e Column A for each fitting gives lengths corresponding to "zero additional uncertainty" values (see 6.2.3).

^f Column B for each fitting gives lengths corresponding to "0,5 % additional uncertainty" values (see 6.2.4).

^g The straight length in Column A gives zero additional uncertainty; data are not available for shorter straight lengths which could be used to give the required straight lengths for Column B.

^h $95D$ is required for $Re_D > 2 \times 10^6$ if $S < 2D$.

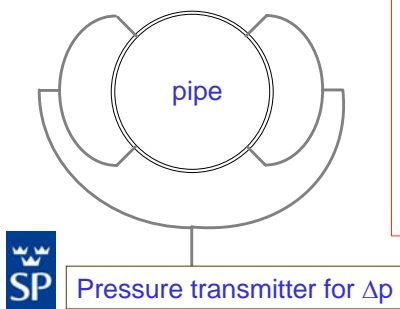


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Measurement of pressure and differential pressure

- In flow computers or intelligent transmitters the density for the known liquid or gas is calculated from equations of state if pressure, temperature and gas composition is known.
- The measurement of static pressure p and differential pressure Δp should be separated. A correct Δp is of outmost importance.

Pressure i triple –T form



To be observed in Δp -measurement

- Zero point adjustment
- Clogged pressure tappings/lines
- Pressure lines must be easily drained or ventilated
- No disturbances for flow close to the pressure tappings

Discharge coefficient C for orifice plates

Purpose: defines the relation of jet flow diameter to orifice diameter and distance to pressure tappings.

$$C = 0,5961 + 0,0261 \cdot \beta^2 - 0,216 \cdot \beta^8 + 0,000521 \cdot \left(\frac{10^6 \cdot \beta}{Re_D} \right)^{0,7} \\ + (0,0188 + 0,0063 \cdot A) \cdot \beta^{3,5} \cdot \left(\frac{10^6}{Re_D} \right)^{0,3} \\ + (0,043 + 0,080 \cdot e^{-10L_1} - 0,123 \cdot e^{-7L_1}) \cdot (1 - 0,11 \cdot A) \cdot \frac{\beta^4}{1 - \beta^4} \\ - 0,031 \cdot (M_2 - 0,8 \cdot M_2^{1,1}) \cdot \beta^{1,3}$$

Data on pipe
orifice plate
medium

Data on upstream
pressure tappings

Data on downstream
pressure tappings

$\beta = \frac{d}{D}$ relation between orifice and pipe diameter

A function of β and Re_D

$$A = \left(\frac{19000 \cdot \beta}{Re_D} \right)^{0,8} \\ M_2 = \frac{2 \cdot L_2}{1 - \beta}$$

M_2 function of β and L_2

Reader-Harris
Gallagher
Equation



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Standards are updated from time to time

ISO 5167 (1991) & ASME MFC-3M (2004)

ISO 5167 (1991)

Discharge coefficient

$$C = 0,5961 + 0,0261 \cdot \beta^2 - 0,216 \cdot \beta^8 + 0,000521 \cdot \left(\frac{10^6 \cdot \beta}{Re_D} \right)^{0,7} \\ + (0,0188 + 0,0063 \cdot A) \cdot \beta^{3,5} \cdot \left(\frac{10^6}{Re_D} \right)^{0,3} \\ + (0,043 + 0,080 \cdot e^{-10L_1} - 0,123 \cdot e^{-7L_1}) \cdot (1 - 0,11 \cdot A) \cdot \frac{\beta^4}{1 - \beta^4} \\ - 0,031 \cdot (M_2 - 0,8 \cdot M_2^{1,1}) \cdot \beta^{1,3}$$

$$C = 0,5959 + 0,0312 \beta^{2,1} - 0,1840 \beta^8 + 0,0029 \beta^{2,5} \cdot \left(\frac{10^6}{Re_D} \right)^{0,75} \\ + 0,0900 L_1 \cdot \beta^4 \cdot (1 - \beta^4)^{-1} - 0,0337 L_2 \cdot \beta^3$$

Stolz equation

Reader-Harris Gallagher equation

Expansibility factor

$$\varepsilon = 1 - (0,351 + 0,256 \cdot \beta^4 + 0,93 \cdot \beta^8) \cdot \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{1}{\kappa}} \right]$$

$$\varepsilon = 1 - (0,41 + 0,35 \cdot \beta^4) \cdot \frac{\Delta p}{\kappa \cdot p_1}$$



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Different flow result with new version from 2003

Difference in air flow calculation using different versions of ISO 5167

ISO 5167	Flow rate			Expansion coefficient			Discharge Coefficient		
	1991	2003	Diff [%]	1991	2003	Diff [%]	1991	2003	Diff [%]
Air 0 °C	887,69	893,51	0,651	0,9908	0,9909	0,008	0,60027	0,60416	0,643
	1261,25	1268,26	0,553	0,9827	0,9828	0,010	0,59940	0,60267	0,543
	1783,36	1791,27	0,442	0,9673	0,9673	-0,002	0,59872	0,60139	0,443
	2207,29	2215,05	0,350	0,9527	0,9524	-0,034	0,59838	0,60069	0,384
Air 20 °C	864,00	869,87	0,676	0,9909	0,991	0,008	0,60053	0,60457	0,667
	1210,12	1217,17	0,580	0,9827	0,9828	0,010	0,599616	0,60305	0,570
	1710,95	1719,00	0,468	0,9673	0,9672	-0,002	0,598888	0,60172	0,470
	2117,62	2125,62	0,377	0,9527	0,9524	-0,034	0,598526	0,60099	0,410
Air 50 °C	821,20	827,09	0,713	0,9909	0,991	0,008	0,600942	0,60521	0,704
	1150,09	1157,23	0,617	0,9827	0,9828	0,010	0,599936	0,6036	0,607
	1626,11	1634,37	0,505	0,9672	0,9672	-0,002	0,599135	0,60219	0,507
	2012,77	2021,11	0,413	0,9527	0,9523	-0,034	0,598736	0,60142	0,446

The difference is predominantly caused by the discharge coefficient. The change in C decreases with increasing flow

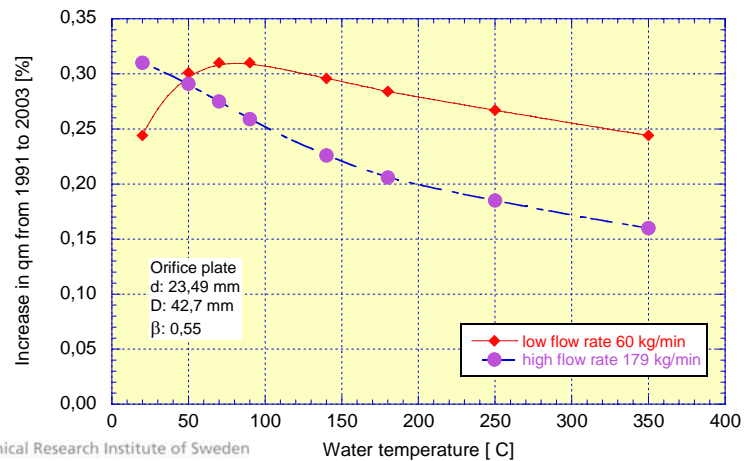


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Changes in flow rate calculation according to ISO 5167 between 1991 and 2003

With the same conditions the latest version of ISO 5167 translates the differential pressure Δp to 0,2 - 0,3 % higher flow rate q_m

Difference in mass flow calculation q_m



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Change in discharge coefficient between versions

Steam at ~12 bar and ~188 °C

Δp [Pa]	q_m [kg/s]	C 1991	C 2003	Diff [%]
2573	0,540	0,59890	0,60072	0,30%
5030	0,753	0,59835	0,59973	0,23%
9943	1,058	0,59792	0,59885	0,16%
15031	1,300	0,59771	0,59838	0,11%

The change in discharge coefficient decreases with increasing flow rate.



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How are a dp-devices often used?

- The primary flow device is bought with a belonging discharge coefficient C – (one value).
 - calculated to a standard for a specified application (medium, nominal flow rate, pressure, temperature $\Rightarrow Re_D$)
 - or
 - calibrated in a similar reference condition $\Rightarrow Re_D$

- Actual flow $q_m(\text{act})$ is calculated in relation to nominal flow rate $q_m(\text{nom})$ using the square root dependency from Δp

$$q_m(\text{act}) = \sqrt{\frac{\Delta p(\text{act})}{\Delta p(\text{nom})}} \cdot q_m(\text{nom})$$

- For a different application a new relation $q_n \Leftrightarrow \Delta p$ must be applied
- What must be known to calculate flow rate correctly?



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How correct does the simplified model work?

$$q_m(\text{act}) = \sqrt{\frac{\Delta p(\text{act})}{\Delta p(\text{nom})}} \cdot q_m(\text{nom})$$

Example for orifice plate
In water at 90 °C
 $\Delta p = 0,1 \text{ bar (10000 Pa)}$
 $\beta = 0,7361$
 $Re = \sim 540\,000$

Change in indicated Δp	Change in Calculated flow q_n	Error in simplified calculation
+ 1%	0,5 %	0,002 %
+10 %	-4,86 %	0,023 %
-10%	-5,11 %	-0,026 %

Conclusion: The formula works well if **only** Δp changes

Question: What happens if temperature, viscosity or media pressure varies?



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Not registered process parameters changes lead to measurement errors

Example: Hot water flow at 90 °C

Increase in temperature $\Delta T=3\text{ °C}$ (influences directly and indirectly)
 viscosity -3,17 %, density -0,21 %, Re +3,15 %, C -0,02 %
 FLOW RATE Δq_m -0,12 %

Increase in viscosity $\Delta \mu_i=3\text{ %}$ (influences indirectly)
 Re -2,89 %, C + 0,02 %,
 FLOW RATE Δq_m +0,02 %

Increase in density $\Delta \rho=0,3\text{ %}$ (influences directly)
 Re +1,49 %, C - 0,001 %,
 FLOW RATE Δq_m +0,15 %



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Errors do not add statistically!

What more information is needed?

q_m calculated from Δp
 if everything else is known

$$q_m = \frac{C}{\sqrt{1-\beta^4}} \cdot \varepsilon \cdot \frac{\pi}{4} \cdot d^2 \cdot \sqrt{2\Delta p \cdot \rho_1}$$

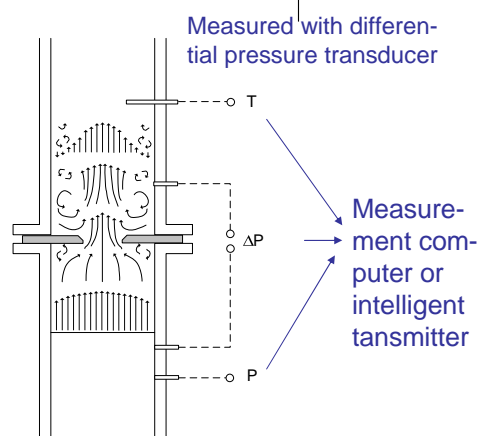
Calculation parameters
 for mass flow q_m :

Pressure difference Δp
 Density of medium ρ_1
 Orifice diameter d
 Diameter relation $\beta=d/D$
 Expansibility factor ε
 Discharge Coefficient C

Except β all other factors
 change with pressure
 and/or temperature

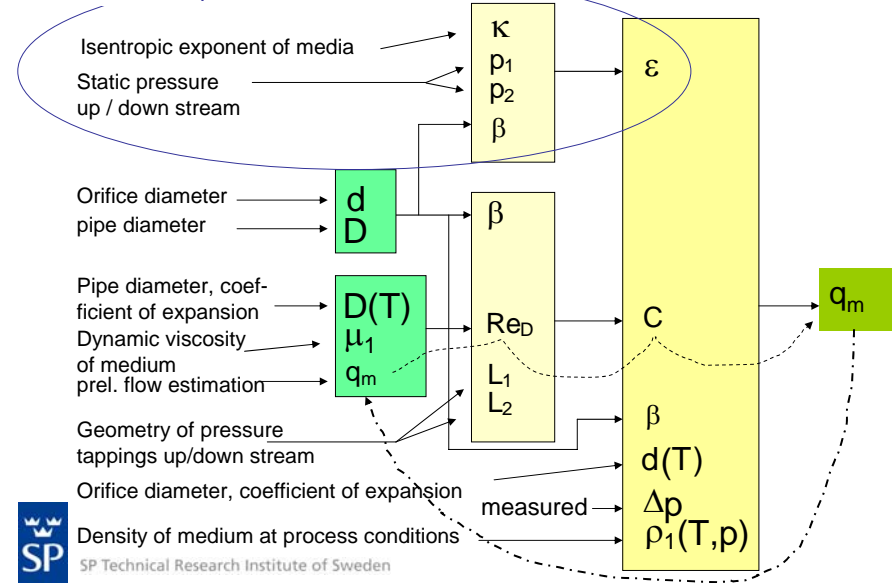


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How are the variables and parameters interrelated?

Not relevant for liquids



Necessary knowledge about process conditions

How much does viscosity change with temperature?

How much does density change with temperature?

How much does density change with pressure (steam)?

How much does temperature change with pressure (steam)?

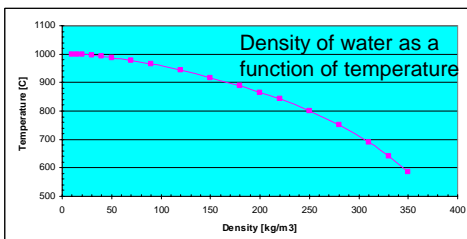
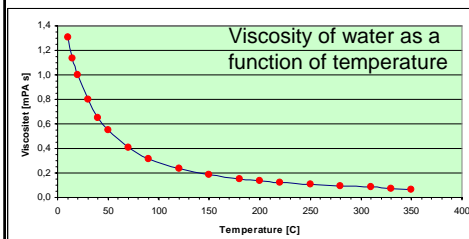
Important for calculating

Reynolds number Re_D

Discharge coefficient C

(and Expansion coefficient ϵ)

Viscosity and density of water
as a function of temperature

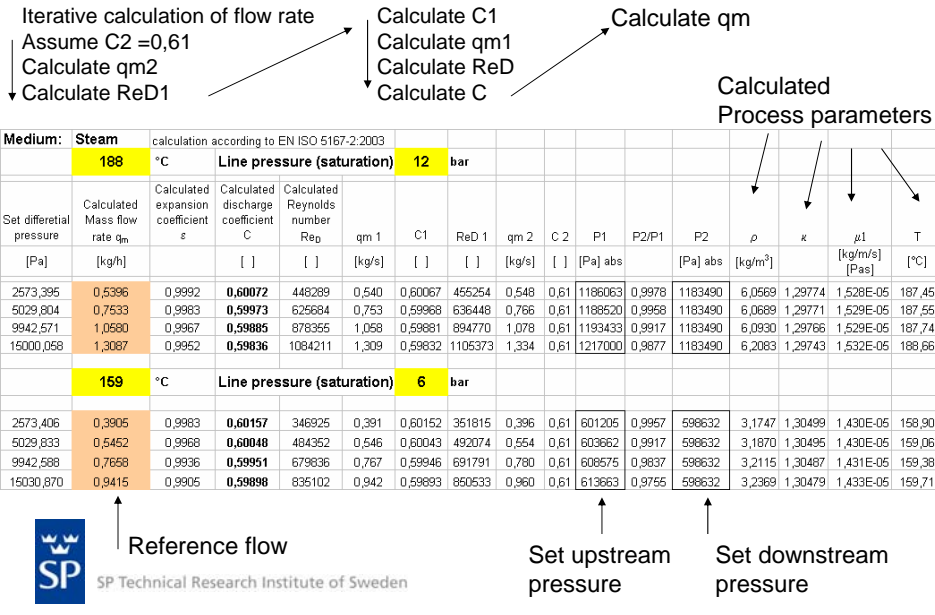


For hot water these relations are reasonably well known - IAPWS



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Reference calculation for testing - simulating saturated steam



How is flow measurement affected?

Unrecognized process parameter change: **Temperature change** + 1,5 °C at 90 °C
+ 3 °C at 200 °C

Temperature influences flow rate qm

directly

$$q_m = \frac{C}{\sqrt{1-\beta^4}} \cdot \varepsilon \cdot \frac{\pi}{4} d^2 \cdot \sqrt{2\Delta p \cdot \rho_1}$$

indirectly

$$C = f(Re_D, \beta, L)$$

$$Re = \frac{4 \cdot q_m}{\pi \cdot \mu_1 \cdot D}$$

Change in Density and Viscosity
90 + 1,5 °C ⇒ -0,10 % ⇒ -1,72 %
200 + 3 °C ⇒ -0,38 % ⇒ -1,52 %

Δp = is assumed constant

Orifice/pipe
d: 23,491 mm
D: 42,7 mm
β: 0,5501

Flow changes at high low flow
90 + 1,5 °C ⇒ -0,05 to -0,06 %
200 + 3 °C ⇒ -0,18 to -0,19 %

Effect on flow rate due to an unrecognized temperature increase.



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How is flow measurement affected?

Unrecognized process
parameter change:

Viscosity change: + 1 % or + 2 %
at 90 and 200 °C

Viscosity influences flow rate
indirectly

$$q_m = \frac{C}{\sqrt{1-\beta^4}} \cdot \varepsilon \cdot \frac{\pi}{4} \cdot d^2 \cdot \sqrt{2\Delta p \cdot \rho_1}$$

$$C = f(Re_D; \beta; L)$$

$$Re = \frac{4 \cdot q_m}{\pi \cdot \mu_1 \cdot D}$$

Δp = is assumed constant

Flow changes at 90 °C high low flow
 $\Delta\mu_1=1\% \Rightarrow 0,003$ to $0,005 \%$
 $\Delta\mu_1=2\% \Rightarrow 0,006$ to $0,010 \%$

Flow changes at 200 °C
 $\Delta\mu_1=1\% \Rightarrow 0,004$ to $0,007 \%$
 $\Delta\mu_1=2\% \Rightarrow 0,008$ to $0,014 \%$

Orifice/pipe
d: 23,491 mm
D: 42,7 mm
 β : 0,5501



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Both flow rate and temperature have marginal influence

How is flow measurement affected?

Unrealized error in pipe diameter **D** +0,5 mm or +1 mm at 90 °C
at 200 °C

Pipe diameter influences flow rate
indirectly

$$q_m = \frac{C}{\sqrt{1-\beta^4}} \cdot \varepsilon \cdot \frac{\pi}{4} \cdot d^2 \cdot \sqrt{2\Delta p \cdot \rho_1}$$

$$C = f(Re_D; \beta; L)$$

$$\beta = \frac{d}{D} \quad Re = \frac{4 \cdot q_m}{\pi \cdot \mu_1 \cdot D}$$

Δp = is assumed constant

$\Delta D = 0,5 \text{ mm} \Rightarrow 1,17 \%$ of D
 $\Delta D = 1 \text{ mm} \Rightarrow 2,34 \%$
 $\Delta D = 0,5 \text{ mm} \Rightarrow \Delta\beta -1,16 \%$
 $\Delta D = 1 \text{ mm} \Rightarrow \Delta\beta -2,29 \%$

Flow changes at 90 °C high low flow
 $\Delta D=0,5 \text{ mm} \Rightarrow -0,25$ to $-0,26 \%$
 $\Delta D = 1 \text{ mm} \Rightarrow -0,49$ to $-0,51 \%$

Flow changes at 200 °C
 $\Delta D=0,5 \text{ mm} \Rightarrow -0,249$ to $-0,254 \%$
 $\Delta D = 1 \text{ mm} \Rightarrow -0,483$ to $-0,496 \%$

Orifice/pipe
d: 23,491 mm
D: 42,7 mm
 β : 0,5501

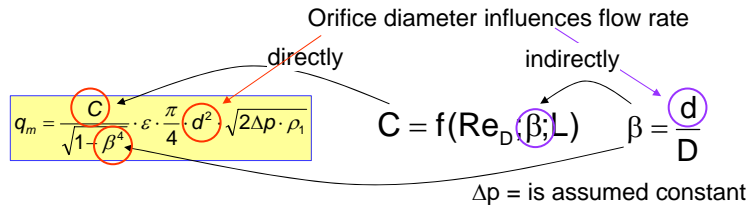


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The flow rate and temperature influences marginal

How is flow measurement affected?

Unrealized error in orifice diameter Δd +0,05 mm or +0,1 mm at 90 °C
at 200 °C



$\Delta d = 0,05 \text{ mm} \Rightarrow 0,21 \% \text{ of } d$
 $\Delta d = 0,1 \text{ mm} \Rightarrow 0,43 \% \text{ of } d$
 $\Delta d = 0,05 \text{ mm} \Rightarrow \Delta \beta -0,21 \%$
 $\Delta d = 0,1 \text{ mm} \Rightarrow \Delta \beta -0,43 \%$

Flow changes at 90 °C high low flow
 $+0,05 \text{ mm} \Rightarrow -0,473 \text{ to } -0,474 \%$
 $+0,1 \text{ mm} \Rightarrow -0,948 \text{ to } -0,950 \%$

Flow changes at 200 °C
 $+0,5 \text{ mm} \Rightarrow -0,472 \text{ to } -0,473 \%$
 $+1 \text{ mm} \Rightarrow -0,946 \text{ to } -0,947 \%$

Orifice/pipe
 $d: 23,491 \text{ mm}$
 $D: 42,7 \text{ mm}$
 $\beta: 0,5501$



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Uncertainty in discharge coefficient C (ISO 5167)

Venturi tubes:	a) casted	0,7 %
U(C) depends on	b) machined	1,0 %
production method	c) welded	1,5 %
Venturi nozzles.	$U(C) = (1,2 + 1,5 \cdot \beta^4) \%$	$\beta = 0,736: 1,64 \%$
ISA nozzles	$U(C) = 0,8 \%$ if $\beta \leq 0,6$	0,8 %
	$U(C) = (2 \cdot \beta - 0,4) \%$ if $\beta > 0,6$	$\beta = 0,736: 1,072 \%$

Orifice plates	$U(C) = (0,7 - \beta) \%$	for $\beta 0,1 \text{ to } 0,2$	0,6 – 0,5 %
	$U(C) = 0,5 \%$	for $\beta 0,2 \text{ to } 0,6$	0,5 %
	$U(C) = (1,667\beta - 0,5) \%$	for $\beta 0,6 \text{ to } 0,75$	0,5 – 0,75 %
If $D < 71,12 \text{ mm}$	$U(C) = +0,9 \cdot (0,75 - \beta) \cdot \left(2,8 - \frac{D}{25,4}\right) \%$		
		$D = 70 \text{ mm } \beta = 0,3$	+0,018 %
		$D = 60 \text{ mm } \beta = 0,4$	+1,38 %
If moreover $\beta > 0,5$ och $\text{Re}_D < 10\,000$			+0,5 %

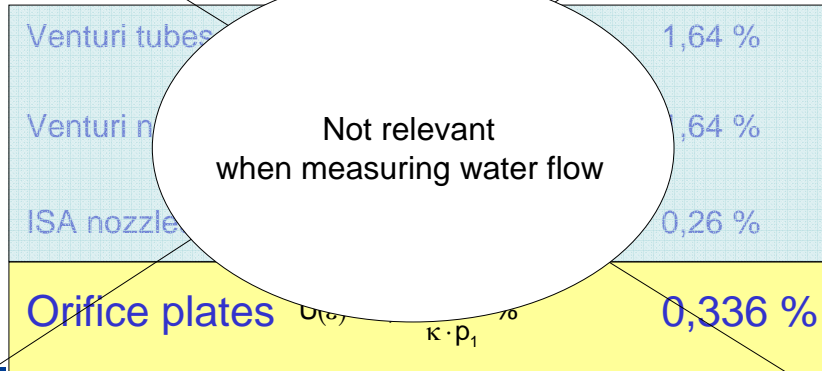
Uncertainty in expansibility factor ε

Uncertainty depends on geometry and pressure conditions.

With $\beta = 0,7361$ ($d=73,61$ mm / $D=100$ mm)

$\Delta P = 15000$ Pa (0,15 bar); $P_1 = 115400$ Pa (1,15 bar absolute)

$\kappa = 1,399$ (for air at 50 °C)



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Problems with direct temperature and density measurement

- Fluid temperature and density should be measured 5 – 15 D downstream from orifice plate in order not disturb flow.
- Reynolds no Re_D needing this information apply however for upstream conditions.
- For very accurate measurements (especially non ideal gases) the temperature drop over the orifice must be calculated:

$$\Delta T = \mu_{JT} \cdot \Delta \varpi$$

$$\mu_{JT} = \frac{R_u \cdot T^2}{p \cdot c_{m,p}} \cdot \left. \frac{\partial Z}{\partial T} \right|_p$$

Joule Thomson coefficient

R_u : universal gas constant

T : absolute Temperature

p : static pressure in medium

$c_{m,p}$: molar heatcapitet at const. p

Z : compressibility factor

$$\Delta \varpi = \frac{\sqrt{1 - \beta^4 \cdot (1 - C^2)} - C\beta^2}{\sqrt{1 - \beta^4 \cdot (1 - C^2)} + C\beta^2} \cdot \Delta p$$

Pressure drop over orifice

β : diameter ratio

C : discharge coefficient

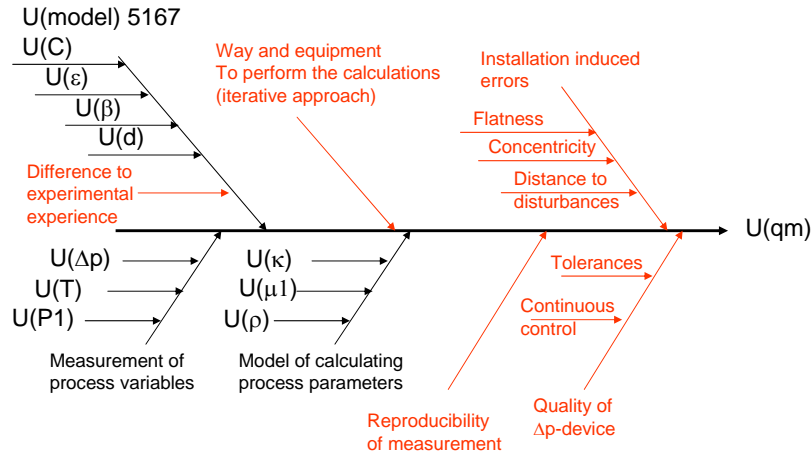
Δp : pressure difference

$$\Delta \varpi \cong 1 - \beta^{1,9} \Delta p$$



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What uncertainties then must we expect in measurement?



Analysis of the measurement uncertainty according to GUM for an idealized situation using GUM-Workbench



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Model equation in GUM Workbench

```
GUM Workbench Pro - Uncertainty air 50 C orifice.smu
File Edit View Option Tools Help
Model Observation Correlation
Title Model Equation Quantity Data
Equation:
qm=C/(1-β^4)^0.5*ε*π/4*d^2*(2*Δp*ρo1)^0.5;
C=0.5961+0.0261*β^2-0.216*β^8+0.000521*(1000000*β/ReD)^0.7
+(0.0188+0.0063*A)*β^3.5*(1000000/ReD)^0.3+(0.043+0.08*exp(-10*L1)
-0.123*exp(-7*L1))*(1-0.11*A)*β^4/(1-β^4)
-0.031*(M2-0.8*(M2^1.1))*β^1.3+Cfit;
β=d/D;
L1=1/D;
L2=12/D;
M2=(2*L2)/(1-β);
A=(19000*β/ReD)^0.8;
ε=1-(0.351+0.256*β^4+0.93*β^8)*(1-(p2/p1)^(1/κ))+εfit;
ReD=4*qm1/(π*ρmu1*D);
D=D0*(1+α1*(T-20));
d=d0*(1+α1*(T-20));
```

Test situation for air at 50 °C
Orifice plate with β=0,7361



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Variable declaration

Quantity	Unit	Definition
ε		Expansion factor
π		constant
d	m	orifice diameter
Δp	Pa	pressure drop
ρ	kg/m ³	medium density
ReD		Reynolds number
A		Area factor function of beta and Reynolds number
$L1$	m	Realtion upstream tapping length to to pipediameter
$M2$	m	relation down stream tapping length to 1-beta
C_{fit}		An estimator of the uncertainty of the actual discharge coefficient
D	m	Pipe inner diameter as fuction of temperature
$l1$	m	Distance upstream tapping to orificie face
$L2$	m	Realtion downstream tapping length to to pipediameter
$l2$	m	Distance downstream tapping to orificie face
$p2$	Pa	Down stream pressure = standard atmosphere
$p1$	Pa	Upstream pressure
κ		Isentropic coefficient
c_{fit}		An estimator for the uncertainty in the model formula for the expansion factor
$qm1$	kg/s	estimated mass flow rate
μ		dynamic viscosity
$D0$	m	Pipe inner diameter at 20 C
α	/C	Linear expansion coefficient for pipe
T	C	Actual medium temperature
$d0$	m	Orifice diameter at 20 C
$\alpha1$	/C	Linear expansion coefficient for orifice



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Estimation of all uncertainty components

GUM Workbench Pro - Uncertainty air 50 C orifice.smu

File Edit View Option Tools Help

Model Observation

Title	Model Equation	Quantity Data
qm		
C		
β		
ε		
π		
d		
Δp		
ρ		
ReD		
A		
$L1$		
$M2$		
C_{fit}		

Orifice diameter at 20 C

Type: Type B

Distribution: Rectangular

Value: 0.07361 m

Halfwidth of Limits: 0.0001 m

Description:

Orifice diameter $d0$ at 20 Celsius is 73,61 mm - nominal value.
Let us assume this value has an uncertainty of +/- 0,1 mm due to handling difficulties and we assume further a rectangular distribution



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The total uncertainty budget

calculated mass flow						
Uncertainty Budget:						
Quantity	Value	Standard Uncertainty	Distribution	Sensitivity Coefficient	Uncertainty Contribution	Index
C	0.60507	$2.22 \cdot 10^{-3}$				
β	0.73610	$2.20 \cdot 10^{-3}$				
π	3.1415926535898					
d	0.0736475 m	$57.8 \cdot 10^{-6}$ m				
Δp	2753.40 Pa	6.36 Pa	rectangular	$43 \cdot 10^{-6}$	$270 \cdot 10^{-6}$ kg/s	4.2 %
ρ_{H_2O}	1.10980 kg/m ³	$6.35 \cdot 10^{-3}$ kg/m ³	rectangular	0.11	$680 \cdot 10^{-6}$ kg/s	25.7 %
ReD	149.79 10 ³	2000				
A	0.15002	$1.57 \cdot 10^{-3}$				
M2	$7.57 \cdot 10^{-3}$ m	$4.37 \cdot 10^{-3}$ m				
Crit	0.0	$2.20 \cdot 10^{-3}$	normal	0.39	$860 \cdot 10^{-6}$ kg/s	41.5 %
D	0.100051 m	$289 \cdot 10^{-6}$ m				
l1	0.0 m	$289 \cdot 10^{-6}$ m	rectangular	0.098	$28 \cdot 10^{-6}$ kg/s	0.0 %
L2	$999 \cdot 10^{-6}$ m	$577 \cdot 10^{-6}$ m				
l2	$100.0 \cdot 10^{-6}$ m	$57.7 \cdot 10^{-6}$ m	rectangular	-0.29	$-17 \cdot 10^{-6}$ kg/s	0.0 %
p2	101.300 10 ³ Pa	234 Pa	rectangular	$840 \cdot 10^{-3}$	$200 \cdot 10^{-6}$ kg/s	1.1 %
p1	103.873 10 ³ Pa	240 Pa	rectangular	$-820 \cdot 10^{-3}$	$-200 \cdot 10^{-6}$ kg/s	1.1 %
κ	1.39900	$8.08 \cdot 10^{-3}$	rectangular	$1.5 \cdot 10^{-3}$	$12 \cdot 10^{-6}$ kg/s	0.0 %
sft	0.0	$328 \cdot 10^{-6}$	normal	0.24	$79 \cdot 10^{-6}$ kg/s	0.3 %
qm1	0.23200 kg/s	$1.33 \cdot 10^{-3}$ kg/s	rectangular	$-8.7 \cdot 10^{-3}$	$-12 \cdot 10^{-6}$ kg/s	0.0 %
ρ_{mu1}	$19.710 \cdot 10^{-6}$	$231 \cdot 10^{-9}$	rectangular	100	$24 \cdot 10^{-6}$ kg/s	0.0 %
D0	0.100000 m	$289 \cdot 10^{-6}$ m	rectangular	-1.6	$-460 \cdot 10^{-6}$ kg/s	12.0 %
α_{H_2O}	$17.000 \cdot 10^{-6}$ /C	$981 \cdot 10^{-9}$ /C	rectangular	-4.8	$-4.7 \cdot 10^{-6}$ kg/s	0.0 %
T	50.000 C	0.289 C	rectangular	$8.1 \cdot 10^{-6}$	$2.3 \cdot 10^{-6}$ kg/s	0.0 %
d0	0.0736100 m	$57.7 \cdot 10^{-6}$ m	rectangular	8.7	$500 \cdot 10^{-6}$ kg/s	13.9 %
α_{H_2O}	$17.000 \cdot 10^{-6}$ /C	$981 \cdot 10^{-9}$ /C	rectangular	19	$19 \cdot 10^{-6}$ kg/s	0.0 %
qm	0.23753 kg/s	$1.34 \cdot 10^{-3}$ kg/s				
Result:						
Value:	Expanded Uncertainty:	Coverage Factor:	Coverage:			
0.2376 kg/s	± 1.1 % (relative)	2.00	95% (normal)			

Uncertainty contributions at idealized conditions

Process media: **Air at 50 C!**

D=100 mm, d=73,61 mm $\beta=0.7361$

$\rho=1.1098$ kg/m³, $\mu_1: 1.971 \cdot 10^{-5}$ Pa s, $\kappa=1.399$

P1=1,038 bar, $\Delta p=27.53$ mbar

Important uncertainty contributions:

- o Discharge coefficient U(C): 0,73 % - at best 0,5 % (41,5 %)
- o Density U(ρ): 1 % (25,7 %)
- o Diameter U(d): 0,1 mm of 73,61 mm (0,14 %) (13,9 %)
- o Diameter U(D): 0,5 mm of 100 mm (0,5 %) (12 %)
- o Differential pressure U(Δp): 0,4 % of reading (4,2 %)
- o Pressure U(P1,P2): 0,4 % of reading (1,1 %)

of calculated flow rate $q_m=0.2376 \pm 0.0027$ kg/s (1,1 %)

With iteration process $q_m=0.2376$ kg/s



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Uncertainty contributions at idealized conditions

Process media: **Water at 50 C!**

D=100 mm, d=73,61 mm $\beta=0,7361$

$\rho=988,1 \text{ kg/m}^3$, $\mu_1: 5,485 \cdot 10^{-5} \text{ Pa s}$, $P_1=1,038 \text{ bar}$, $\Delta p=150 \text{ mbar}$

Important uncertainty contributions and their relative importance:

	contribution to total uncertainty
➤ Discharge coefficient $U(C)$: 0,73 % - at best 0,5 %	(57,1 %)
➤ Diameter $U(d)$: 0,1 mm of 73,61 mm (0,14 %)	(19,1 %)
➤ Diameter $U(D)$: 0,5 mm of 100 mm (0,5 %)	(16,5 %)
➤ Differential pressure $U(\Delta p)$: 0,4 % of reading	(5,8 %)
➤ Temperature $U(T)$: 3 °C	(1,1 %)
➤ Density $U(\rho)$: 1 kg/m ³ (0,1 %)	(0,4 %)
➤ All others contributions below 0,1 %	

Calculated flow rate $q_m = 16,61 \pm 0,159 \text{ kg/s}$ ($k=2$)

$\pm 0,96 \%$



Installation induced systematic effects add linearly to uncertainty
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About measurement quality

Standard ISO 5167 specifies a measurement uncertainty for C och ε if all demands are fulfilled concerning:

- production (according to tolerances)
- installation (with respect to disturbances)
- usage (within the limits of the standard)

The standard expects to examine the orifice plates on a regular basis, to verify pipe measures (roundness, dimension, roughness), to have good knowledge of the process conditions of the medium (i.e. pressure fluctuation, single-phase, pressure > liquid evaporation pressure, temperature > condensation point of gases, $\tau=p_2/p_1 \geq 0,75$).

At low gas flow and large temperature differences between medium and surrounding – insulation is necessary.

Any violation of ideal flow conditions must be explored. Swirl and non-symmetric flow profiles must be avoided. Flow conditioners are highly recommended.



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Total measurement uncertainty

If all process parameters were known without any error:

The best measurement uncertainty (orifice plates) according to standard amounts to:

0.7 % on a 95 % confidence level.

The uncertainty in C, the dominating contribution can possibly be decreased by testing.

But other contributions from installation can easily generate larger uncertainty contributions.



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Orifice plates -- their + and -

- Simple and cheap production (especially for big diameters)
- Cheap in installation
- No moving parts - robust
- Works as "primary element" – no calibration !?
- Good characterized behavior
- Simple function control
- Suits most fluids and gases – even extreme conditions
- Available in many dimensions and a large flow range
- Less dynamic flow range for fixed diameter
- Large pressure loss
- Non linear output, complex calculation - needs urgent calculation help
- Process parameters must be known for flow calculation
- Problems with different phases or flow variations
- Can be sensitive for wear and disposition
- Very sensitive for disturbance and installation caused effects
- To achieve the measurement uncertainties indicated by ISO 5167 many conditions must be fulfilled



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Limitations for Orifice Plates – med ISO-5167

Pipe diameter D:	50 – 1200 mm
Reynolds no Re_D :	>3150 turbulent flow is demanded No upper limit – but there is no proof
Medium:	single phase liquid, steam or gas
Flow:	steady, no fast changes or pulsations
Pressure tappings:	corner-, flange eller D & D/2 tappings no support for other tappings by the standards
Pressure measurement:	Static pressure absolute units
Definition:	p_1 upstream, p_2 downstream
Pressure difference:	$\Delta p = p_1 - p_2$ only for specified tappings
Geometry:	$\beta = d/D$, d orifice diameter, D pipe diameter $d > 12,5$ mm; $0,1 \leq \beta \leq 0,75$



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To measure flow traceable with Δp devices we need

1. Transfer model between differential pressure Δp measurement and the momentary flow rate q_m . ✓
2. Precise definition of the selected primary device with the related discharge coefficient. Use the same standard ISO 5167. ✓
3. Design and production tolerances of all geometric measures that determine the $\Delta p \Leftrightarrow q_m$ relation. ✓
4. Clear advice for installation to keep disturbing effects below a significant error level. ✓
5. Additional contributions to uncertainty due to violation of point 4.
- quantified tolerances for distance to certain perturbations. ✓
6. Consider process parameter changes in calculation. ✓
7. **A routine to ensure that original conditions will be preserved in future!**
8. **A proven method to determine the discharge coefficient C at extreme temperatures and pressure.**



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