at 1 atm. The damage left by the mild explosion at 23 atm., shown at center, was much less than that from the brisant detonation at 1 atm., shown at right.

The apparatus, nevertheless, was damaged by the mild detonation at 23 atm. The control experiment with liquid N_2-O_2 was made at the same pressure, 23 atm., to assess how much damage was unique to NO. The inside appearance of the two pipes after shocking N_2-O_2 and NO, both at 23 atm., is shown in Figure 4. Both pipes in this picture have been sawed down their mid-planes through the instrument ports. The inside of the pipe that contained NO was severely oxidized and bulged, while the control shows no evidence of oxidation or explosion in the liquid.

The bulges caused by the low order explosion in NO at 23 atm. (Figure 4, right) are interesting because they occur at, or a short distance past, the instrument ports in the pipe. The sensing devices seem to have momentarily increased the reaction rate, causing strong pressure surges at these locations. The pressure surges, roughly estimated from the strength properties of 304 stainless steel, were about 2000 atm. A velocity of roughly 2000 meters per second was indicated in the second half of the low order propagation in NO at 23 atm. Very inefficient conversion of NO to N_2 and O_2 was shown by the copious quantities of NO_2 fumes that remained after this explosion.

In each experiment with NO, the pressure in the pipe, after condensation of NO, was significantly higher than the vapor pressure of NO; considerable gassing at 1 atm. occurred in each experiment even while N_2 was still in the coolant jacket. A simple explanation is that the NO contained more than enough N_2 impurity to saturate the liquid and solid phases of NO.

The explosion behavior of all three experiments with NO can now be explained by gas-bubble initiation theory as follows: When NO was boiling at 1 atm., NO vapor bubbles were there to form adiabatic hot spots for the fast chemical initiation reactions required by the brisant mode of propagation. When the pressure was 3 atm. over nonevaporating liquid-solid NO, bubbles of N_2 impurity, rather than NO vapor, made enough hot spots to propagate the brisant mode. When the pressure on liquid NO at its normal boiling point was elevated to 23 atm., however, both the vapor pressure of NO and the Henry's law pressure of N_2 in NO were exceeded, and no hot-spot–forming bubbles were in the liquid NO. The chemical rate was too slow for the brisant mode and a lower order explosion took its place.

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COMMUNICATION

PERFORATED-PIPE DISTRIBUTORS

A method for calculating the pressure drop in perforated-pipe distributors by separating the pipe into discrete sections and using the summation method is presented. For short distributors or distributors with a small number of holes, this method gives an applicable equation which is more reliable than prior equations.

THE pressure drop in pipe distributors is calculated by separating the pipe into discrete sections and using the summation technique rather than the integral approach to arrive at the solution. For long distributors with a large number of holes, the final solution presented here does not contradict prior correlations such as those presented by Lapple (1951) and Acrivos *et al.* (1959); however, for short distributors and/or distributors with a small number of holes, this development gives rise to an extremely applicable equation and one more reliable than the prior equations.

Although the theory is not new, it is presented to clarify the development.

To describe appropriately the pressure forces existing in the pipe distributor, friction terms, flow inefficiencies, and momentum recovery contributions must be considered. If the distributor is divided into n equal sections, the length of each section is represented by L/n where L is the distributor length. If there is no maldistribution and if the orifice holes are of the same size, A_o, the volumetric flow rate from each side port will be A_oV_o.

For section 1 in the flow distributor, a mass balance can be written for the fluid stream (see Figure 1).

$$\text{Mass in} = \text{mass out}$$

$$\rho_L V_1 A_1 = \rho_L V_2 A_1 + \rho_L V_o A_o \quad (1)$$

Since equal distribution is assumed, the velocity in section 2 is:

$$V_2 = (1 - 1/n)\, V_1 \quad (2)$$

For the case of n holes, the general form of the velocity in section i can be written as:

$$V_i = \left(1 - \frac{i-1}{n}\right) V_1 \quad (3)$$

The equation representing the frictional pressure loss in each section can be written using the Fanning friction factor in the form

$$\left(\frac{P_i - P_{i+1}}{\rho}\right)_i = \frac{2f_iL_iV_i^2}{Dg_c} \quad (4)$$

where i = section number.

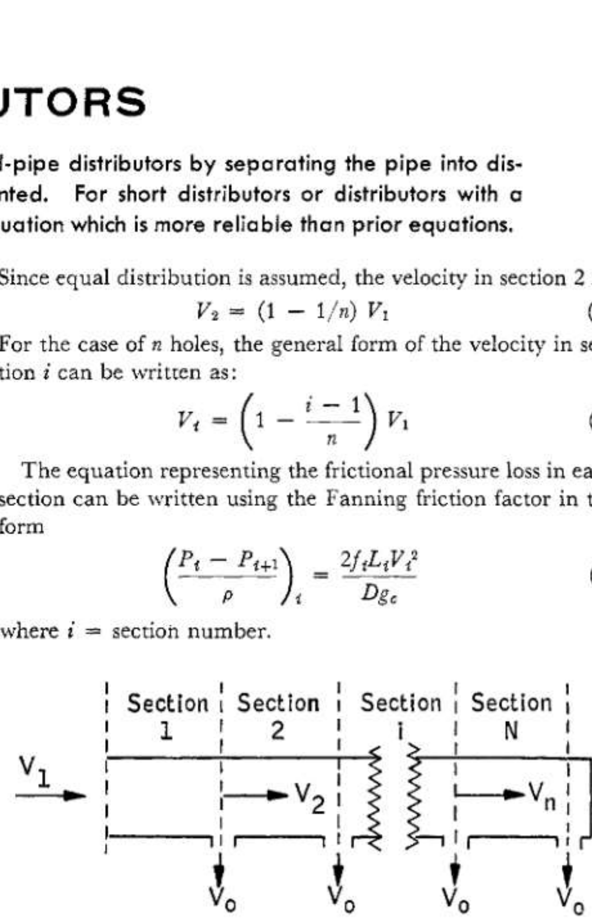

Figure 1. Expanded schematic of pipe distributor

The total pressure drop over the distributor due to friction can then be found by summing the pressure drops over each section.

$$\left(\frac{\Delta P}{\rho}\right)_{\text{friction}} = \frac{2f_1(L/n)\ V_1^2}{Dg_c} + \frac{2f_2(L/n)\ V_2^2}{Dg_c} + \ldots \frac{2f_n(L/n)\ V_n^2}{Dg_c} \quad (5)$$

where $\Delta P = (P_1 - P_n)$.

For a given distributor diameter, and fluid density and viscosity, the friction factors are solely functions of velocity in the respective sections using smooth tube correlations. Therefore, f may be written as a function of $V_1, V_2, \ldots V_n$ in Equation 5. Although this seems to complicate calculations, a simple computer program can easily handle the computations. To facilitate hand calculations, an approximate average friction factor may be defined. One method is to calculate an average fluid velocity in the distributor and calculate f_{av}. Therefore, Equation 5 reduces to:

$$\left(\frac{\Delta P}{\rho}\right)_{\text{friction}} = \frac{2f_{av}(L/n)\ V_1^2}{Dg_c} + \frac{2f_{av}(L/n)V_2^2}{Dg_c} + \ldots \frac{2f_{av}(L/n)V_n^2}{Dg_c} \quad (6)$$

Simplifying Equation 6:

$$\left(\frac{\Delta P}{\rho}\right)_{\text{friction}} = \frac{2f_{av}(L/n)}{Dg_c}\left[\sum_{i=1}^{n} V_i^2\right] \quad (7)$$

Combining Equations 3 and 6 and simplifying, the result is:

$$\left(\frac{\Delta P}{\rho}\right)_{\text{friction}} = \frac{2f_{av}LV_1^2}{Dg_c}\sum_{i=1}^{n}\left[\frac{n-(i-1)}{n}\right]^2\Big/n \quad (8)$$

Equation 8 has the unique advantage over other equations reported in the literature for calculating frictional losses in pipe distributors that it is a function of the number of holes, n. In other equations previously presented, either an integral approach was used which reduces to a continuous slot distributor, or the expression for the pressure drop through the orifices was substituted in the equation for distributor pressure drop to obtain a solution.

At each side port, two additional phenomena contribute to the pressure drop calculation. There is a fluid momentum effect at each port. The fluid flowing out each port ideally decelerates to zero in the main direction of flow in the distributor, makes a right-angle turn, and flows out the side port. At each side port, a momentum balance can be made over a control volume as in Figure 2. Because of a loss of mass of fluid changing direction from X to Y, the conservation of linear momentum dictates that momentum in the X direction must be conserved. Hence the velocity decrease must be accompanied by a pressure increase. For section i,

$$\left(\frac{P_{i+1} - P_i}{\rho}\right)_{\substack{\text{momentum}\\\text{recovery}}} = \left(\frac{V_i^2 - V_{i+1}^2}{g_c}\right) \quad (9)$$

Since the fluid at each port does not exactly make a 90° change in direction and does not decelerate exactly to zero, Equation 9 must be modified to take this inefficiency into account. By incorporating into Equation 9 a constant k, where $1 > k > 0$, the momentum recovery and fluid flow inefficiencies may be written in the form

$$\left(\frac{P_{i+1} - P_i}{\rho}\right) = \frac{k}{g_c}\ (V_i^2 - V_{i+1}^2)$$

For the entire distributor,

$$\left(\frac{P_n - P_1}{\rho}\right) = \frac{k}{g_c}\ [(V_1^2 - V_2^2) + (V_2^2 - V_3^2) + \ldots (V_{n-1}^2 - V_n^2)] \quad (10)$$

Although k is assumed to be a constant for the entire distributor, it is recognized that practical situations exist—e.g., V_o not constant—which lead to varying velocity and pressure profiles along the pipe length, hence varying values for k. Simplifying and combining Equations 3 and 10, the total pressure recovery for n holes is:

$$-\left(\frac{\Delta P}{\rho}\right)_{\substack{\text{momentum}\\\text{recovery}}} = \frac{k}{g_c}\ [1 - 1/n^2]V_1^2 \quad (11)$$

Adding the pressure drop due to friction and that due to momentum recovery and fluid inefficiencies, the total change in pressure over the perforated pipe distributor is:

$$\left(\frac{\Delta P}{\rho}\right)_{\text{total}} = \frac{2f_{av}LV_1^2}{Dg_c}\left\{\sum_{i=1}^{n}\left[\frac{n-(i-1)}{n}\right]^2\Big/n\right\} - \frac{k}{g_c}\ [1 - 1/n^2]V_1^2 \quad (12)$$

Equation 12 can be simplified for practical applications. The summation in the first term can be carried out explicitly and is given for all values of n by the quantity $(n + 1)(2n + 1)/6n^2$. This quantity obviously has the limit 1/3 for large values of n. If the summation term is taken and plotted with respect to the number of holes, an asymptote of 0.33 exists for large values of n (Figure 3). When the number of holes reaches 20,

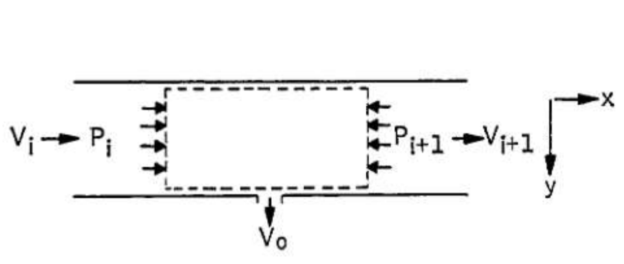

Figure 2. Pressure and velocity vectors over a control volume at a port

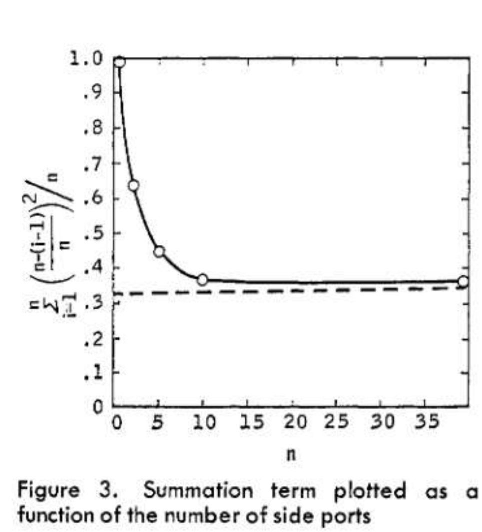

Figure 3. Summation term plotted as a function of the number of side ports

the value of the summation is approximately 0.35; however, as n decreases below 10, the value sharply increases. Furthermore, when n is sufficiently large, $[1 - 1/n^2]$ in the second term in Equation 12 goes to 1. For sufficiently large numbers of side ports, Equation 12 reduces to

$$\left(\frac{\Delta P}{\rho}\right)_{\text{total}} = \frac{2f_{av}LV_1^2}{Dg_c}(0.33) - \frac{k\,V_1^2}{g_c} \qquad (13)$$

Lapple (1951) has derived the pressure drop through a perforated pipe distributor assuming ideal momentum recovery—i.e., $k = 1$—and uniform flow of fluid along the entire length of the pipe distributor—i.e., the ports are very close to one another, simulating a continuous slot distributor. Lapple's pressure drop equation for a horizontal distributor is given as:

$$\frac{\Delta P}{\rho} = \frac{V_1^2}{2g_c}\left[\frac{1}{3}\left(\frac{4fL}{D}\right) - 2\right] \qquad (14)$$

This is equivalent to Equation 13 for $k = 1$ and n very large. For small values of n—i.e., less than 10—Equation 12 would be more appropriate to use than Equation 14.

The value of k to be used in the momentum recovery term in Equation 12 still needs additional investigation. Acrivos *et al.* (1959) present some values of k for air systems, ranging from approximately 0.6 to 0.9 and apparently not correlating with fluid maldistribution. On the same basis, data based on the work of Soucek and Zelnick (1945) in long 6-inch square channels with square side ports were compared with the air system. The value of k for water varied from approximately 0.7 to 0.4 with increasing maldistribution. It has been suggested that k may be better correlated with fluid velocity or pressure drop through the side ports; however, additional data are needed to substantiate this.

Nomenclature

A_1 = distributor cross-sectional area, sq. ft.
A_o = port cross-sectional area, sq. ft.
D = distributor diameter, ft.
f = Fanning friction factor
g_c = gravitational constant, 32.2 (ft. lb.) (lb. force sec.^{2})
i = section number
k = momentum recovery correction factor
L = distributor length, ft.
n = number of side ports or orifices
P_1 = pressure at distributor inlet, p.s.i.
P_n = pressure at closed end of distributor, p.s.i.
ΔP = total pressure drop over distributor, p.s.i.
V = velocity, ft./sec.
V_o = velocity through side ports, ft./sec.
V_1 = inlet velocity to distributor, ft./sec.
X, Y = direction vectors
ρ_L = liquid density, lb./cu. ft.

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CORRESPONDENCE

OPTIMAL ADIABATIC BED REACTOR WITH COLD SHOT COOLING

SIR: The optimal design of adiabatic bed reactors with cold shot cooling has been treated in detail by Lee and Aris (1963). In a recent communication Malengé and Villermaux (1967) have shown that the optimizing algorithm proposed by Lee and Aris does not lead to the optimal design conditions; in fact, by a direct search method on the set of six decision variables appearing in the expression for the profit of a three-bed reactor they could substantially improve the profit as obtained by Lee and Aris. However, here it is shown that even the solution of Malengé and Villermaux does not yield the true optimum and that neither of the previous solutions, although giving a profit close to the maximum profit, leads to the optimal design conditions.

Although the optimizing algorithm used by Lee and Aris fails, their mathematical formulation of the problem is correct and it suits perfectly a discrete maximum principle approach. In this note we use the notation of Lee and Aris, although this notation is more appropriate to a dynamic programming formulation than to the maximum principle formulation used by us. A stage consists of the catalyst bed and the preceding bypass mixing chamber or the preceding heater (for the Nth stage).

The state of the process stream at each stage can be described by the set of state variables: entrance conversion g, exit conversion g', exit temperature t', cumulative relative mass flow rate λ/λ_N, cumulative profit per unit of mass flow through that stage P. The decisions to be made at each stage are the entrance temperature, t, and the holding time, θ.

The following set of equations results, corresponding to Equations 15, 14, 16, and 18 of Lee and Aris, respectively:

$$\theta_n = \frac{W_n}{G(\lambda_n/\lambda_N)/(\lambda_1/\lambda_N)} = \int_{g_n}^{g'_n} \frac{dg}{R_n(g)} \qquad (1)$$

$$t'_n = t_n + (g'_n - g_n) \qquad (2)$$

$$\lambda_n/\lambda_N = \lambda_{n+1}/\lambda_N(t'_{n+1}/t_n) \qquad (3)$$

$$g_n = g'_{n+1}(t_n/t'_{n+1}) \qquad (4)$$

$$P_N = (g'_N - g_N) - \delta\theta_N - \mu t_N \qquad (5')$$

$$P_n = P_{n+1}\frac{\lambda_{n+1}/\lambda_N}{\lambda_n/\lambda_N} + (g'_n - g_n) - \delta_n \quad n \neq N \qquad (5)$$

Equations 1 to 5 are implicit forms of the "performance equations" of Fan and Wang (1964). Since the total mass