# Property Risk Consulting Guidelines 

## SPRINKLER SYSTEM HYDRAULICS

## INTRODUCTION

The two basic methods for designing sprinkler systems are the pipe schedule method and the hydraulic design method. In the pipe schedule method a specific number of sprinkler heads can be fed off each particular size pipe. A hydraulically designed system is one intended to fulfill the specified sprinkler density operating over a selected area of application in a fairly uniform manner.

The hydraulic design method allows the designer more freedom. The design criteria are selected based on the degree of hazard. When specifying the design, it is important that consideration be given to possible changes in occupancy so the protection can be maximized for the greatest intended usage. It is usually difficult to improve most hydraulically designed systems because pipe sizes are selected on a pressure loss basis in order to optimize the use of available water supply.

The restrictions on hydraulically designed sprinkler systems are found in NFPA 13. The minimum pipe size allowed is 1 in . ( 25 mm ). The design should be within the available water supply corrected to the base of the specific system riser. Correct the water supply for friction and elevation losses as well as anticipated hose demand

For many years only tree type pipe schedule systems where permitted by the NFPA 13. Today, the use of pipe schedule systems is restricted. The majority of sprinkler systems being installed today are hydraulically designed due to overall reduced system cost due to a reduction in:

- Design cost - computer generated design.
- Materials cost - the computer uses pipe on hand or what can be economically purchased. Smaller piping is cheaper.
- Installation cost - systems are less complex, require less manpower due to the use of smaller, lightweight pipe.

System design is expressed in terms of density, gpm/ft² $\left(\mathrm{L} / \mathrm{min} / \mathrm{m}^{2}\right)$, operating over the total design area of application. This means that the most remote head in the sprinkler system should be capable of delivering minimum flow and pressure if all of the sprinkler heads in the area of application are operating. The density and area of application design are based on NFPA 13 or other applicable standard, as interpreted in PRC.12.1.1.0. It is important that the system designer be aware of AXA XL Risk Consulting's requirements before committing to a particular design.

## SYSTEM CONFIGURATIONS

The three basic system configurations are the tree, the loop and the grid. The tree system is characterized by larger pipe sizes near the riser (see Figure 1). As the system extends toward the most remote area the piping gets smaller, just like branches on a tree. There is no looped piping in a tree system.

When tree systems are analyzed hydraulically, it is found that the general layout of the system has a great effect on the hydraulic demand. Systems that are laid out very symmetrically with short branch lines have relatively low demands when compared to those that are end corner fed and have long branch lines.

Looped systems are those that have interconnected pipes, which form looped paths for water flow (see Figure 2). These may vary from a single loop to more complex multiple loop systems. The advantage of the loop is that it gives the water several paths to get to the point of discharge. This means that any one path is carrying less water, resulting in less friction loss per path. In order for a loop to be effective, the alternate pipe paths must be large enough to carry a substantial part of the total flow.

Gridded systems are those that have adjacent branch lines looped throughout the system (see Figure 3). This provides a multitude of paths for the water to flow through, thereby taking the best hydraulic advantage of the available water supply.


Figure 1. Typical Tree System.


Figure 2. Typical Loop System.


Figure 3. Typical Grid System.
It is important that the pipes that connect the branch lines together are sized for the flow rates they are expected to carry. When converting a tree system to a gridded system, it is wise to replace the smaller diameter piping at the ends of the branches, as they would otherwise restrict the flow and limit the advantage of gridding the system.
The minimum theoretical flow demand (ideal flow) is obtained by multiplying the system design density by the design area of application. This is not a true measure of the flow demand because of frictional losses in the system. Higher pressures are available at flowing sprinklers closer to the riser. Higher pressures allow increased flow at each operating sprinkler resulting in a higher flow demand at the base of the riser.

Generally, a system with a reasonable amount of friction loss will have less than a $10 \%$ increase above ideal flow at the base of the riser. This increase is referred to as the flow overage and is expressed as a factor of approximately 1.1.

System overage is determined by dividing the required flow demand by the ideal flow demand. The larger the overage factor the less efficient the system piping. Poorly designed systems, such as endside feed tree systems, can have overage factors approaching 1.6. Using an overage factor of 1.1 for proposed systems is useful in approximating base of riser flow before the system has been designed.

## SELECTION OF AREA OF APPLICATION

If a fire starts in the open with uniform combustible loading, the fire growth should proceed in a circular fashion. Many older, hydraulically designed sprinkler systems were designed with remote design areas in the form of a square. Fires do not usually burn with precise symmetry. Fires that burn in a rectangular fashion have a higher water demand. NFPA 13 changed its requirements for the design area from a square to a rectangle with its longest side equivalent to 1.2 times the square root of the area of application.
AXA XL Risk Consulting uses a rectangle with its longest side equivalent to 1.4 times the square root of the area of application. The longest side of the rectangle is measured in the direction of the branch lines.


Figure 4. Typical Fire Spread.
Two very common fire scenarios are one where a fire starts near a wall, or a fire that starts in a storage area near an aisle resulting in a semicircular fire. These conceivable severe fires burn in a rectangle that is twice as wide as it is deep. Figure No. 4 shows an example of such a fire were the radius of the fire is $r$.
The dimensions of the area of application are $r \times 2 r$.
Area of application $=A=2 r^{2}$
Solving for $r$, the shortest side of the rectangle:

$$
r=\sqrt{\frac{A}{2}}
$$

Solving for $2 r$, the longest side of the rectangle:

$$
2 r=2 \sqrt{\frac{A}{2}}
$$

Multiplying the top and bottom of the right hand term by $\sqrt{2}$ results in the following equation:
$2 r=\sqrt{2} \sqrt{A}$
Since the $\sqrt{2}=1.414$ :
$2 r \approx 1.4 \sqrt{A}$

## SELECTION OF REFERENCE POINTS

Before hydraulic calculations are conducted it is important to select points of reference identifying specific junction points, or pipe nodes, on the piping system. These can be numbers, letters, or names assigned to these points. Reference points are normally used at a point where there is one or more of the following:

- A change in flow because of a flow split.
- A change in flow because of joining together of multiple flows.
- A change in pipe size.
- A need to identify a point for future reference.

All heads flowing water within the design area of application are assigned reference points, as are the points where the branch lines connect to the cross mains.
In gridded systems, reference points are needed at all junction points since water flowing through the system has the ability to divert flow in several directions at each junction point.

## TREE SYSTEM CALCULATION METHOD

The design density of a sprinkler system is expressed in terms of $\mathrm{gpm} / \mathrm{ft}^{2}\left(\mathrm{~L} / \mathrm{min} / \mathrm{m}^{2}\right)$ and anticipates that all sprinklers within the area of application will be operating.
The calculation process traditionally starts at the hydraulically most remote head since it is indicative of the worst condition in the system. Since there is friction loss between the end head and the next to last head, there is more pressure available at the next to last head and therefore results in a greater flow at the next to last head. In a branch line, this may result in a significant difference in flow between the most remote head and heads closer to the cross mains.

By using the equation $Q=K \sqrt{P}$ as derived in PRC.12.0.1, and established friction loss tables based on the Hazen/Williams equation, it is simply a matter of progressive calculation to determine the accumulated flow and pressure required at the base of the sprinkler riser.

In order to hydraulically calculate a sprinkler system certain data is required:

- The discharge coverage per head.
- The design density.
- The area of application.
- Sprinkler head $K$ factor.
- Pipe C factor.
- Whether or not velocity pressure is taken into account.
- The type of pipe used in the system.

In addition, piping details are needed which include the following:

- Pipe sizes.
- Lengths.
- Elevation changes.
- Connecting fittings.

If the sprinkler heads within the area of application are spaced such that each head covers a uniform number of square feet, then the minimum flow rate per head can be calculated by multiplying the density by the area covered by a single head. If the sprinkler heads are not spaced uniformly, the minimum flow from each end head on a branch line must be calculated by multiplying the density by the area covered by that particular head.

## Sample Tree Calculations

Consider the simple tree system in Figure 5 consisting of only 6 heads flowing in the $600 \mathrm{ft}^{2}\left(55.74 \mathrm{~m}^{2}\right)$ area of application. In this example, instead of calculating to the base of the riser, the calculations will be performed to the end of the 2 in . pipe. The spacing per head is $10 \mathrm{ft}(3.05 \mathrm{~m})$ between heads and 10 ft $(3.05 \mathrm{~m})$ between lines; each head will protect an area of $100 \mathrm{ft}^{2}\left(9.29 \mathrm{~m}^{2}\right)$.
If a density of $0.20 \mathrm{gpm} / \mathrm{ft}^{2}\left(8.15 \mathrm{~L} / \mathrm{min} / \mathrm{m}^{2}\right)$ were desired, the flow rate for the most remote sprinkler head would be $0.20-\mathrm{gpm} / \mathrm{ft}^{2} \times 100 \mathrm{ft}^{2}$ or 20 gpm . In metrics, the flow rate for the most remote sprinkler head would be $8.15 \mathrm{~L} / \mathrm{min} / \mathrm{m}^{2} \times 9.3 \mathrm{~m}^{2}$ or $75.70 \mathrm{~L} / \mathrm{min}$.

The pressure needed to get this particular flow through the most remote sprinkler head can be determined from the following equation:
$Q=K \sqrt{P}$
Where:

$$
\begin{aligned}
& Q=\text { flow } \\
& K=\text { coefficient of sprinkler orifice } \\
& P=\text { pressure }
\end{aligned}
$$

While sprinkler heads of various manufacturers have varying $K$ factors, they usually range between 5.5 and 5.75 ( 79.2 to 82.9 ) for a nominal in. ( 12.5 mm ) orifice head. For the purposes of this example a $K$ factor of 5.6 (80.7) will be used. A sprinkler head that has other than a nominal $1 / 2 \mathrm{in}$. ( 12.5 mm ) orifice will have a different $K$ factor which will significantly affect the calculations.


Figure 5. Sample Tree System.
Using the $K$ factor of 5.6 (80.7), the pressure required to obtain a flow of $20 \mathrm{gpm}(75.7 \mathrm{~L} / \mathrm{min})$ can be determined by rearranging the equation and solving for $P$.

$$
P=\frac{Q^{2}}{K^{2}}=\frac{20^{2}}{5.6^{2}}=12.76 \mathrm{psi}
$$

In metrics

$$
P=\frac{Q^{2}}{K^{2}}=\frac{75.7^{2}}{80.7^{2}}=0.88 \mathrm{bar}
$$

Once the end head flow and pressure are obtained, the calculations proceed along the branch line towards the riser. The calculations are performed in a direction opposite to the normal direction of flow. In this manner, the friction losses are pressure gains and losses because of elevation are also pressure gains. The demand at the base of the riser will include all of the pressure losses needed to get water to all the operating heads in the area of application, while maintaining a minimum density at the hydraulically most remote head within that area.

Moving towards sprinkler head No. 2 the 20 gpm of water reaching head No. 1 must flow through 10 ft of 1 in . pipe. This results in a loss in pressure because of friction. To calculate the amount of pressure loss due to friction, the friction loss per ft must be determined. This may be done by using either the Hazen/Williams equation or tables generated from this equation. See PRC.12.0.1.

$$
F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}
$$

Where:
$F_{p}=$ friction loss in psi/ft
$Q=$ flow in gpm
$D=$ internal diameter in inches
C = roughness coefficient
In metrics

$$
F_{p}=\frac{6.06 \times 10^{5} Q^{1.85}}{C^{1.85} D^{4.87}}
$$

Where:

$$
\begin{aligned}
& F_{p}=\text { friction loss in bar/m } \\
& Q=\text { flow in } \mathrm{L} / \mathrm{min} \\
& D=\text { internal diameter in } \mathrm{mm} \\
& C=\text { roughness coefficient }
\end{aligned}
$$

In order to reduce confusion, the metric equivalents for each step of the calculations will be eliminated until these calculations are completed.
Substituting:

$$
\begin{aligned}
& C=120 \quad D=1.049 \quad Q=120 \\
& F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}=\frac{4.52(20)^{1.85}}{(120)^{1.85}(1.049)^{4.87}} \\
& \quad F_{p}=0.130 \mathrm{psi} / \mathrm{ft}
\end{aligned}
$$

The total equivalent length (actual pipe length) $L_{e q}=10 \mathrm{ft}$ :

$$
\begin{aligned}
& P_{f}=F_{p} L_{e q} \\
& P_{f}=10 \mathrm{ft} \times 0.130 \mathrm{psi} / \mathrm{ft} \text { or } 1.3 \mathrm{psi}
\end{aligned}
$$

The pressure loss in this segment of pipe is added to the pressure required to deliver minimum flow to the end sprinkler head. The pressure available at sprinkler head No. 2 is 12.76 psi plus 1.3 psi, or 14.06 psi. With this pressure available at head No. 2, it is possible to determine the expected flow based on the equation:

$$
Q=K \sqrt{P} \quad \text { or } Q=5.6 \sqrt{14.06}=21.0
$$

Note the increase in flow at head No. 2. The pipe feeding head No. 2 is carrying the combined flow of 20 gpm to head No. 1, plus 21 gpm to head No. 2 or 41.0 gpm .

This combined flow is coming through 5 ft of 1 in . pipe and out of the 1 in . "Tee." By taking each of these separately to simplify what is occurring, and by using the Hazen/Williams equation and substituting:

$$
\begin{aligned}
& C=120 \quad D=1.049 \quad Q=410 \\
& \qquad F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}=\frac{4.52(41)^{1.85}}{(120)^{1.85}(1.049)^{4.87}} \\
& \quad F_{p}=0.491 \text { psi/ft } \\
& \text { If } L_{e q}=5 \mathrm{ft} \text { of actual length: } \\
& \quad P_{f}=5 \mathrm{ft} \times 0.491 \text { psi/ft or } 2.45 \mathrm{psi}
\end{aligned}
$$

The combined pressure is 14.06 plus 2.45 , or 16.51 psi. The combined flow moves through the "Tee" at the top of the riser nipple. At this point, it is important to consider what is happening within the fitting. (See Figure 6.) Inside the "Tee" the flow coming up the riser nipple is splitting, allowing part of the flow to go to heads Nos. $1 \& 2$ and the remainder to go to head No. 3.


Figure 6. Typical Tee Showing Flow Split.
A total of 41.0 gpm is making a right hand $90^{\circ}$ turn toward head Nos. $1 \& 2$. The loss incurred in this side of the fitting is similar to flowing the same amount of water through an equivalent length of 1 in . pipe measuring 5 ft long.
Using the Hazen/Williams equation once again:
Substituting $C=120 \quad D=1.049 \quad Q=410$
$F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}=\frac{4.52(41)^{1.85}}{(120)^{1.85}(1.049)^{4.87}}$
$F_{p}=0.491 \mathrm{psi} / \mathrm{ft}$
Based on an equivalent length for the fitting, of $L_{e q}=5 \mathrm{ft}$ (see Table 3 in PRC.12.0.1):
$P_{f}=5 \mathrm{ft} \times 0.491 \mathrm{psi} / \mathrm{ft}$ or 2.45 psi
At reference point "AT" which is inside the tee is a pressure of 16.51 psi plus 2.45 psi or 18.97 psi based on the 41.0 gpm flow.

This takes the flow into the "Tee" at the point where the flow splits and goes to each branch line.
The flow and pressure at head No. 3 is again considered at the minimum required density of $0.20 \mathrm{gpm} / \mathrm{ft}^{2}$. Repeating the same procedure used at head No. 1, the required flow is 20 gpm and the required pressure is 12.76 psi .
This flow is coming through 5 ft of 1 in . pipe and out of the 1 in . "Tee." Taking each of these separately, using the Hazen/Williams equation and substituting:

$$
\begin{aligned}
& C=120 \quad D=1.049 \quad Q=20.0 \\
& F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}=\frac{4.52(20)^{1.85}}{(120)^{1.85}(1.049)^{4.87}} \\
& F_{p}=0.130 \mathrm{psi} / \mathrm{ft}
\end{aligned}
$$

If $L_{e q}=5 \mathrm{ft}$ :
$P_{f}=5 \mathrm{ft} \times 0.130 \mathrm{psi} / \mathrm{ft}$ or 0.65 psi

The combined pressure is 12.76 plus 0.65 or 13.41 psi. The flow moves through the "Tee" at the top of the riser nipple. Using the Hazen/Williams equation once again:

Substituting $C=120 \quad D=1.049 \quad Q=20.0$

$$
\begin{aligned}
& F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}=\frac{4.52(20)^{1.85}}{(120)^{1.85}(1.049)^{4.87}} \\
& \quad F_{p}=0.130 \mathrm{psi} / \mathrm{ft}
\end{aligned}
$$

Based on an equivalent length for the fitting of $L_{e q}=5 \mathrm{ft}$ :

$$
P_{f}=5 \mathrm{ft} \times 0.130 \mathrm{psi} / \mathrm{ft} \text { or } 0.65 \mathrm{psi}
$$

Reference point "AT" which is inside the tee is a pressure of 13.41 psi plus 0.65 psi or 14.06 psi based on the 20.0 gpm flow.

This takes the flow into the "Tee" at the point where the flow splits and goes to each branch line. Only one pressure can exist at this point in the system. Since the two results are both based on minimum system requirements, the higher pressure must prevail in order to maintain the minimum required design density. This means with more pressure available at the split point within the "Tee" that more water will be discharged to head No. 3.
In order to determine the proper amount of water flowing to head No. 3, the flow at the "Tee" split must be balanced. Refer to Figure 6. There are three ways that this can be accomplished:

- Conventional Method
- Simple Proportion
- Additive K factors

The conventional method uses the $K$ factor determined for the flow to be balanced. The new flow is then calculated based on the square root of the required pressure (the higher pressure).
The proportion method does not use $K$ factors, but rather sets up a simple proportion of flows and the square root of the pressures.

The additive $K$ factor method determines the $K$ factor for each of the two branches entering the "Tee" and then calculates a total $K$ factor for the "Tee" based on the sum of the $K$ factors. The resulting $K$ factor is used with the required pressure to determine the total flow leaving the "Tee."
In the conventional method, in order to determine the $K$ factor for the flow being balanced:

$$
\begin{aligned}
& K_{\text {branch 2 }}=\frac{Q_{\text {branch 2 }}}{\sqrt{P_{\text {branch2 }}}} \\
& K_{\text {branch 2 }}=\frac{20}{\sqrt{14.06}}=5.33 \\
& Q_{\text {new }}=K_{\text {branch 2 }} \sqrt{P_{\text {pressure required }}} \\
& Q_{\text {new }}=5.333 \sqrt{18.97}=23.23
\end{aligned}
$$

Using the simple proportion:

$$
\frac{Q_{\text {new }}}{Q_{\text {branch2 }}}=\frac{\sqrt{P_{\text {branch } 2 \text { required }}}}{\sqrt{P_{\text {branch } 2 \text { calculated }}}}
$$

$$
\begin{aligned}
& Q_{\text {new }}=Q_{\text {branch } 2} \frac{\sqrt{P_{\text {branch } 2 \text { required }}}}{\sqrt{P_{\text {branch } 2 \text { calculated }}}} \\
& Q_{\text {new }}=20 \frac{\sqrt{18.97}}{\sqrt{14.06}}=23.23 \mathrm{gpm}
\end{aligned}
$$

In both these methods the total flow leaving the junction point in the "Tee" is the flow for the first branch line plus the corrected flow to the second branch line:

$$
Q_{\text {totala }}=41.0+23.23=64.23 \mathrm{gpm}
$$

In the additive $K$ factor method, the $K$ factor for each branch is determined. $K_{\text {branch } 2}$ was determined above and is 5.33 .

$$
\begin{aligned}
& K_{\text {branch 1 }}=\frac{Q_{\text {branch 1 }}}{\sqrt{P_{\text {branch 1 }}}} \\
& K_{\text {branch } 1}=\frac{41.0}{\sqrt{18.97}}=9.41 \\
& K_{\text {total }}=K_{\text {branch1 }}+K_{\text {branch2 }} \\
& K_{\text {total }}=9.41+5.33 \\
& Q_{\text {total }}=K_{\text {total }} \sqrt{P_{\text {required }}} \\
& Q_{\text {total }}=14.74 \sqrt{18.97}=64.23 \mathrm{gpm}
\end{aligned}
$$

All three balancing methods show identical results. It does not make any difference which method is used.
Once the required flow and pressure leaving the junction point are known, they are brought back through the riser nipple to reference point "A." The losses in the riser nipple consist of elevation loss and friction loss for the length of pipe, and the losses through the "Tee" at the bottom of the riser nipple.
The losses, which are due to elevation, are simple to calculate. For each foot of elevation the pressure changes by 0.433 psi. In this case, the flow is being taken down the 1 ft long riser nipple, which means that the calculated elevation loss of 0.433 psi will be added to the required pressure.
In addition, there is friction loss that is due to carrying 64.2 gpm through 1 ft of 1.25 in . pipe. Using the Hazen/Williams equation:

$$
\begin{aligned}
& C=120 \quad D=1.38 \quad Q=64.23 \\
& F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}=\frac{4.52(64.23)^{1.85}}{(120)^{1.85}(1.38)^{4.87}} \\
& F_{p}=0.296 \mathrm{psi} / \mathrm{ft} \\
& \text { If } L_{e q}=1 \mathrm{ft} \text { of actual length: } \\
& P_{f}=1 \mathrm{ft} \times 0.296 \text { psi/ft or } 0.296 \mathrm{psi}
\end{aligned}
$$

The "Tee" at the bottom of the riser nipple is subjected to the same $P_{f}=0.296$ psi/ft. Since the 1.25 in . "Tee" has an equivalent length of 5 ft :
If $L_{\text {eq }}=5 \mathrm{ft}$ :
$P_{f}=5 \mathrm{ft} \times 0.296 \mathrm{psi} / \mathrm{ft}$ or 1.78 psi

The total at the base of the riser nipple at reference point " $A$ " is
$18.97+0.433+0.296+1.78=21.47 \mathrm{psi}$
Determining the $K$ factor for this demand will be helpful, since this is a significant point in the system and the $K$ factor will be used later.

$$
\begin{aligned}
& K_{p o \mathrm{int} A}=\frac{Q_{\mathrm{atA}}}{\sqrt{P_{\mathrm{at} A}}} \\
& K_{p o \mathrm{int} A}=\frac{64.23}{\sqrt{21.47}}=13.86
\end{aligned}
$$

The pressure loss from reference point " $A$ " to reference point " $B$ " depends on flow through 10 ft of 1.25 in. pipe.

$$
\begin{aligned}
& C=120 \quad D=1.38 \quad Q=64.23 \\
& F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}=\frac{4.52(64.23)^{1.85}}{(120)^{1.85}(1.38)^{4.87}} \\
& F_{p}=0.296 \mathrm{psi} / \mathrm{ft} \\
& \text { If } L_{e q}=10 \mathrm{ft} \text { of actual length: } \\
& P_{f}=10 \mathrm{ft} \times 0.296 \mathrm{psi} / \mathrm{ft} \text { or } 2.96 \mathrm{psi} \\
& P_{\text {pointB }}=21.47+2.96=24.44 \mathrm{psi}
\end{aligned}
$$

This is the first time reference " $B$ " occurs in the calculation.
The next step is to determine the flow and pressure at sprinkler head No. 4. Again, minimum flow and pressure are established at this point. Examining the system, note that sprinkler heads No. 4 through 6 are laid out exactly the same as heads 1 through 3.
The procedure for calculating heads 4 through 6 is identical and will result in exactly the same results as heads 1 through 3. The demand at reference point " B " is again 64.23 gpm at 21.47 psi . This is the second time reference " $B$ " is reached in the calculation process. There are two calculated pressures at "B." However, only one can exist. The most demanding pressure is the greater of the two or 24.44 psi. Since the flow and pressure demands are the same, it stands to reason that:

$$
\begin{aligned}
& K_{\text {point } B}=K_{\text {point } A} \\
& Q_{\text {new }}=K_{\text {point } B} \sqrt{P_{\text {requiredat } B}} \\
& Q_{\text {new }}=13.861 \sqrt{24.44}=68.52 \mathrm{gpm}
\end{aligned}
$$

The total flow leaving reference point " B " is $64.23+68.52=132.7 \mathrm{gpm}$
Taking this flow through the remaining 20 ft of 2 in . pipe:

$$
\begin{aligned}
& C=120 \quad D=1.38 \quad Q=132.73 \\
& \qquad \begin{aligned}
C & =\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}=\frac{4.52(132.73)^{1.85}}{(120)^{1.85}(2.067)^{4.87}} \\
F_{p} & =0.1585
\end{aligned} \\
& \text { If } L_{e q}=20 \mathrm{ft}: \\
& P_{f}=20 \mathrm{ft} \times 0.1585 \mathrm{psi} / \mathrm{ft} \text { or } 3.17 \mathrm{psi} \\
& P_{f}=24.44+3.17=27.61 \mathrm{psi}
\end{aligned}
$$

A computer run of the above calculations is shown in Table 1. They show a demand of 132.7 gpm (502.38 L/min) at 27.61 psi (1.90 bar).

Neither the manual calculations nor the computer calculations took the effects of system velocity pressure into account. To do so results in negligible changes in the calculations, reduces conservatism, and complicates the hand calculation process.

At any particular point in a piping system, the total pressure is the combination of both the normal pressure and the velocity pressure. (See PRC.12.0.1.)
$P_{n}=P_{f}-P_{v}$
Where:

$$
P_{v}=\frac{0.001123 Q^{2}}{D^{4}}
$$

TABLE 1
Calculation Without Velocity Pressure

| Hd <br> No | Added gpm | Total gpm | Pipe ID | Fit | Length | Elev | Fric psi/ft | Loss Tot | Elev psi | Req psi | Ref Pt |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  | 12.76 |  |
| 1 | 20.0 | 20.0 | 1.05 |  | 10.00 | 0.00 | 0.130 | 1.30 | 0.00 | 14.06 |  |
| 2 | 21.0 | 41.0 | 1.05 |  | 5.00 | 0.00 | 0.491 | 2.45 | 0.00 | 16.51 |  |
| 0 | 0.0 | 41.0 | 1.05 | TE | 5.00 | 0.00 | 0.491 | 2.45 | 0.00 | 18.97 | AT |
|  |  |  |  |  |  |  |  |  |  | 12.76 |  |
| 3 | 20.0 | 20.0 | 1.05 |  | 5.00 | 0.00 | 0.130 | 0.65 | 0.00 | 13.41 |  |
| 0 | 0.0 | 20.0 | 1.05 | TE | 5.00 | 0.00 | 0.130 | 0.65 | 0.00 | 14.06 | AT |
|  | 23.2 | 64.2 |  |  | Adjustment |  |  |  |  | 18.97 | AT |
| 0 | 0.0 | 64.2 | 1.38 |  | 1.00 | 1.00 | 0.296 | 0.30 | 0.43 | 19.70 |  |
| 0 | 0.0 | 64.2 | 1.38 | TE | 6.00 | 0.00 | 0.296 | 1.78 | 0.00 | 21.47 | A |
| 0 | 0.0 | 64.2 | 1.38 |  | 10.00 | 0.00 | 0.296 | 2.96 | 0.00 | 24.44 | B |
|  | 64.2 | 64.2 |  |  | From: | A |  |  |  | 21.47 | B |
|  | 68.5 | 132.7 |  |  | Adjustment |  |  |  |  | 24.44 | B |
| 0 | 0.0 | 132.7 | 2.07 |  | 20.00 | 0.00 | 0.159 | 3.17 | 0.00 | 27.61 |  |

In metrics

$$
P_{v}=\frac{2.25 Q^{2}}{D^{4}}
$$

Using velocity pressure in the calculations involves a "trial and error" process. Generally, discharge from a nozzle at the end of a branch line converts all of the available energy (pressure) into flow. In order to determine the pressure loss in a segment of pipe from Head 1 to Head 2, the friction loss between Head 1 to Head 2 must be calculated. If there is a pipe size change at Head 2, there will be a change in velocity, resulting in a change in the velocity pressure. This adjustment affects the pressure feeding Head 1, resulting in a lesser flow, which means the original friction loss calculation was in error. An adjustment to the flow is made until the resulting total pressure at each nozzle results in a balanced condition. This can easily be done by computer but is very tedious by hand calculation.
Table 2 shows the same sprinkler system calculated taking velocity pressure into account. They show a demand of $128.63 \mathrm{gpm}(486.75 \mathrm{~L} / \mathrm{min})$ at $26.99 \mathrm{psi}(1.86 \mathrm{bar})$. The results do not differ much from the calculations where velocity pressure is ignored.

## HARDY CROSS CALCULATION METHOD

Most designs and calculations submitted for review are computer assisted. Many are gridded systems made up of numerous interconnected loops. Interconnected piping results in what is called a network. Solution of pressure drops through the system can be determined, but they are much more complex than tree systems where the path and direction of water flow are always known. Looped piping calculations are solved by using either the Hardy Cross Method or some other regression method. While the other methods are solved using simultaneous equations, the Hardy Cross method is concerned with one loop at a time using a reiterative process. Due to its simplicity the Hardy Cross method will be the primary method described in this section.
The Hardy Cross method is a reiterative process that calculates by a "trial and error" method. This makes hand calculating difficult or nearly impossible on large systems.

TABLE 2
Calculations With Velocity Pressure

| $\begin{aligned} & \mathrm{Hd} \\ & \text { No } \end{aligned}$ | Added gpm | Total gpm | Pipe <br> ID | Fit | Length | Elev | $\begin{aligned} & \text { Fric } \\ & \text { psi/ft } \end{aligned}$ | $\begin{gathered} \text { Loss } \\ \text { Tot } \end{gathered}$ | Elev psi | Req psi | $\begin{aligned} & \text { Ref } \\ & \mathrm{Pt} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  | 12.76 |  |
| 1 | 20.0 | 20.0 | 1.05 |  | 10.00 | 0.00 | 0.130 | 1.30 | 0.00 | 14.06 |  |
| 2 | 19.9 | 39.9 | 1.05 |  | 5.00 | 0.00 | 0.466 | 2.33 | 0.00 | 16.39 |  |
|  | Vel | Press |  |  | Adjustment |  |  |  |  | 1.474 |  |
| 0 | 0.0 | 39.9 | 1.05 | TE | 5.00 | 0.00 | 0.491 | 2.33 | 0.00 | 18.72 | AT |
|  |  |  |  |  |  |  |  |  |  | 12.76 |  |
| 3 | 20.0 | 20.0 | 1.05 |  | 5.00 | 0.00 | 0.130 | 0.65 | 0.00 | 13.41 |  |
| 0 | 0.0 | 20.0 | 1.05 | TE | 5.00 | 0.00 | 0.130 | 0.65 | 0.00 | 14.06 | AT |
|  | 23.08 | 62.9 |  |  | Adjustment |  |  |  |  | 18.72 | AT |
| 0 | 0.0 | 62.9 | 1.38 |  | 1.00 | 1.00 | 0.285 | 0.29 | 0.43 | 19.44 |  |
| 0 | 0.0 | 62.9 | 1.38 | TE | 6.00 | 0.00 | 0.285 | 1.71 | 0.00 | 21.15 | A |
| 0 | 0.0 | 62.9 | 1.38 |  | 10.00 | 0.00 | 0.285 | 2.85 | 0.00 | 24.00 | B |
|  | 62.9 | 62.9 |  |  | From: | A |  |  |  | 21.15 | B |
|  | 65.62 | 132.7 |  |  | Adjustment |  |  |  |  | 24.00 | B |
|  | Vel | Press |  |  | Adjustment |  |  |  |  | 1.017 |  |
| 0 | 0.0 | 128.6 | 2.07 |  | 20.00 | 0.00 | 0.150 | 2.99 | 0.00 | 26.99 |  |

The following conditions of flow must be satisfied in the piping network:

- The algebraic sum of the pressures around the loop must be zero.
- The algebraic sum of the flows into any node must be zero.

Complex piping with multiple loops requires that the flows within adjacent loops to balance so that the common pipe between them is carrying the same quantity of water for each loop.

The procedure suggested by Hardy Cross requires that the flow in each pipe be assumed.
Successive corrections to the assumed flow are computed for each loop in order to satisfy the above conditions.

$$
\begin{equation*}
Q=Q_{\text {est }}+q \tag{1}
\end{equation*}
$$

Where:
$Q=$ Corrected flow
$Q_{\text {est }}=$ Estimated flow
$q=$ Correction factor

The correction factor usually gets smaller each time resulting in many trials before the results are within acceptable limits, which is usually about $0.5 \mathrm{gpm}(1.89 \mathrm{~L} / \mathrm{min})$ maximum per loop or 0.5 psi ( 0.03 bar ) imbalance at a node.

The Hardy Cross method is based on the assumption that the unbalanced losses in a loop of any pipe network is the differential of the Hazen-Williams equation.

$$
\begin{equation*}
F_{p}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}} \tag{2E}
\end{equation*}
$$

Where:
$F_{p}=$ friction loss per ft
$Q=$ flow in gpm
$D=$ internal diameter in inches
$C=$ roughness coefficient
In metrics

$$
\begin{equation*}
F_{p}=\frac{6.06 \times 10^{-5} Q^{1.85}}{C^{1.85} D^{4.87}} \tag{2S}
\end{equation*}
$$

Where:
$F_{p}=$ friction loss per m
$Q=$ flow in L/min
$D=$ internal diameter in mm
$C=$ roughness coefficient
Since:

$$
\begin{equation*}
P_{f}=F_{p} L \tag{3}
\end{equation*}
$$

Where

$$
\begin{aligned}
& P_{f}=\text { Pressure loss due to friction } \\
& L=\text { equivalent length }
\end{aligned}
$$

Then substituting Equation 2E into Equation 3:

$$
\begin{equation*}
P_{f}=\frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}} L \tag{4E}
\end{equation*}
$$

In metrics

$$
\begin{equation*}
P_{f}=\frac{6.06 \times 10^{-5} Q^{1.85}}{C^{1.85} D^{4.87}} L \tag{4S}
\end{equation*}
$$

Substituting all constant terms on the right side of the equation except for $Q^{1.85}$ with a single constant $K$ where:

$$
\begin{equation*}
K=\frac{4.52 L}{C^{1.85} D^{4.87}} \tag{5E}
\end{equation*}
$$

In metrics

$$
\begin{equation*}
K=\frac{6.06 \times 10^{-5} L}{C^{1.85} D^{4.87}} \tag{5S}
\end{equation*}
$$

Substituting for $K$ and rearranging the equation to solve for $Q$ results in the following:

$$
\begin{equation*}
Q=K\left(P_{f}\right)^{\frac{1}{1.85}} \tag{6}
\end{equation*}
$$

Taking the differential of equation (6)

$$
\begin{equation*}
d q=\frac{\frac{d P_{f}}{1.85 P_{f}}}{Q} \tag{7}
\end{equation*}
$$

Then the sum of all the partial flow corrections is:

$$
\begin{equation*}
q=-\frac{\sum P_{f}}{\sum \frac{1.85 P_{f}}{Q}} \tag{8}
\end{equation*}
$$

Rearranging equation (6) and substituting for $P_{f}$ in equation (8)

$$
\begin{equation*}
q=-\frac{\sum K Q^{1.85}}{\sum 1.85 K Q^{0.85}} \tag{9}
\end{equation*}
$$

It is important to be able to calculate the $K$ factor for each piece of pipe. For an illustrative example the following explanation will describe the rudimentary steps of performing the Hardy Cross procedure to the simple system shown in Figure 7. This system consists of two loops with only 2 heads flowing. All piping is 1 in . ( 25 mm ); head and line spacing is on $10 \mathrm{ft}(3.05 \mathrm{~m})$ centers.
Consider the first loop. Moving in a counter clockwise direction starting at reference point $A$, the following reference points are encountered; $A, 1,2, A A, B B, B$, and $A$. The sum of the flows around this loop must be zero. Flows that move clockwise are considered negative.


Figure 7. Sample Grid System.
Assume a flow and direction for each segment of pipe. Assuming that the first estimate is incorrect, a flow correction can be applied to each of the flows around the loop on successive trials.

$$
\begin{gather*}
K_{A-1}\left(Q_{A-1}+q\right)^{1.85}+K_{1-2}\left(Q_{1-2}+q\right)^{1.85}+K_{2-A A}\left(Q_{2-A A}+q\right)^{1.85}+K_{A A-B B}\left(Q_{A A-B B}+q\right)^{1.85}+ \\
K_{B B-B}\left(Q_{B B-B}+q\right)^{1.85}+K_{B-A}\left(Q_{B-A}+q\right)^{1.85}=0 \tag{10}
\end{gather*}
$$

The flow correction $q$ can be determined from equation (8).
Although this equation is not exact, it does give a very close approximation.
The first column in Table 3 indicates the segment of pipe. The second column is an equivalent length that includes the actual length and the tees at each end of the branch line. The 1 in . ( 25 mm ) tees have an equivalent length of $5 \mathrm{ft}(1.52 \mathrm{~m})$. The third column is the pipe diameter, in this case all 1 in .
( 25 mm ) pipe. The fourth column is the calculated $K$ factor per equation (5). The fifth column is an estimated flow and direction. The sixth column $K Q^{1.85}$ calculated for each segment. $\sum K Q^{1.85}$ is shown for each loop in this column. The seventh column $K Q^{0.85} \cdot \sum K Q^{0.85}$ is shown for each loop in this column. Column eight is the calculated flow correction using the $\sum$ from columns six and seven. Column nine is the correction to the calculated flow, which is due to the same pipe being effected by two adjoining loops. Column ten is the new calculated estimated flow determined by applying column eight and nine to column five.
The next trial basically replaces all of the numbers in column five with those in column ten, and the process is repeated. This continues until the maximum per loop and imbalance at a node are reduced to acceptable limits or until the degree of accuracy desired is reached.
Table 4 shows a compilation of the first 6 trials. Table 5 shows a computer run of the same system out to 20 iterations.
Although this calculation method can be done by hand, calculation of loop and gridded systems are best left to computers for solution.

TABLE 3
First Hardy Cross Trial

| Leg | Len | Diam | K Factor | Q est | KQ ${ }^{(1.85)}$ | KQ ${ }^{(.85)}$ | q | Corr | Q new |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A-1 | 10 | 1.049 | 0.008125 | 25.00 | 3.13342 | 0.12534 | 1.98533 | 0.00000 | 26.98533 |
| 1-2 | 10 | 1.049 | 0.008125 | 0.00 | 0.00000 | 0.00000 | 1.98533 | 0.00000 | 1.98533 |
| 2-AA | 10 | 1.049 | 0.008125 | -25.00 | -3.13342 | 0.12534 | 1.98533 | 0.00000 | -23.01467 |
| AA-BB | 10 | 1.049 | 0.008125 | -25.00 | -3.13342 | 0.12534 | 1.98533 | 0.00000 | -23.01467 |
| BB-B | 30 | 1.049 | 0.024375 | -12.50 | -2.60756 | 0.20860 | 1.98533 | 4.69553 | -5.81914 |
| B-A | 10 | 1.049 | 0.008125 | 25.00 | 3.13342 | 0.12534 | 1.98533 | 0.00000 | 26.98533 |
|  |  |  |  |  | -2.60756 | 0.70995 |  |  |  |
| B-BB | 30 | 1.049 | 0.024375 | 12.50 | 2.60756 | 0.20860 | -4.69553 |  | 5.81914 |
| BB-CC | 10 | 1.049 | 0.008125 | -12.50 | -0.86919 | 0.06953 | -4.69553 | 1.98533 | -17.19553 |
| CC-C | 30 | 1.049 | 0.024375 | -12.50 | -2.60756 | 0.20860 | -4.69553 | 0.00000 | -17.19553 |
| C-B | 10 | 1.049 | 0.008125 | 37.50 | 6.63419 | 0.17691 | -4.69553 | 0.00000 | 32.80447 |
|  |  |  |  |  | 5.76500 | 0.66366 |  | 0.00000 |  |

TABLE 4
First Six Hardy Cross Trials

| Leg | Estimated | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | Trial 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A-1 | 25.00 | 26.98533 | 25.91548 | 26.00285 | 25.96165 | 25.96513 | 25.96349 |
| 1-2 | 0.00 | 1.98533 | 0.91548 | 1.00285 | 0.96165 | 0.96513 | 0.96349 |
| 2-AA | -25.00 | -23.01467 | -24.08452 | -23.99715 | -24.03835 | -24.03487 | -24.03651 |
| AA-BB | -25.00 | -23.01467 | -24.08452 | -23.99715 | -24.03835 | -24.03487 | -24.03651 |
| BB-B | -12.50 | -5.81914 | -7.28144 | -6.99028 | -7.04870 | -7.03709 | -7.03941 |
| B-A | 25.00 | 26.98533 | 25.91548 | 26.00285 | 25.96165 | 25.96513 | 25.96349 |
| B-BB | 12.50 | 5.81914 | 7.28144 | 6.99028 | 7.04870 | 7.03709 | 7.03941 |
| BB-CC | -12.50 | -17.19553 | -16.80307 | -17.00688 | -16.98965 | -16.99778 | -16.99709 |
| CC-C | -12.50 | -17.19553 | -16.80307 | -17.00688 | -16.98965 | -16.99778 | -16.99709 |
| C-B | 37.50 | 32.80447 | 33.19693 | 32.99312 | 33.01035 | 33.00222 | 33.00291 |

TABLE 5
Computerized Grid Calculation

| $\begin{aligned} & \text { Ref } \\ & \mathrm{Pt} \end{aligned}$ | $\begin{aligned} & \mathrm{Hd} \\ & \mathrm{No} \end{aligned}$ | Added gpm | Total gpm | Pipe Diam | Fit | Pipe Leng | $\begin{aligned} & \text { Loss } \\ & \text { psi/ft } \end{aligned}$ | Total psi | $\begin{gathered} \text { Req } \\ \text { psi } \end{gathered}$ | Ref psi |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | 0 | 0.0 | 50.0 | 1.049 | 0 | . 08 | 0.00 | 0.00 | 27.73 | RS |
| AA | 0 | 0.0 | 24.0 | 1.049 | 0 | 10.00 | 0.18 | 1.83 | 23.87 | BB |
| BB | 0 | 0.0 | 17.0 | 1.049 | 0 | 10.00 | 0.10 | 0.96 | 24.84 | CC |
| A | 0 | 0.0 | 26.0 | 1.049 | 0 | 10.00 | 0.21 | 2.11 | 24.44 | B |
| B | 0 | 0.0 | 33.0 | 1.049 | 0 | 10.00 | 0.33 | 3.29 | 27.73 | C |
| A | 0 | 0.0 | 26.0 | 1.049 | $5=T E$ | 5.00 | 0.21 | 2.11 | 20.22 |  |
|  | 1 | 25.0 | 1.0 | 1.049 | 0 | 10.00 | 0.00 | 0.000 | 20.22 |  |
|  | 2 | 25.0 | 24.0 | 1.049 | $5=T E$ | 5.00 | 0.18 | 1.83 | 22.05 | AA |
| A | 0 | 0.0 | 7.0 | 1.049 | $10=2 \mathrm{TE}$ | 10.00 | 0.02 | 0.57 | 23.87 | BB |
| B | 0 | 0.0 | 17.0 | 1.049 | $10=2 \mathrm{TE}$ | 10.00 | 0.10 | 2.89 | 24.84 | CC |

