

**NorthWoods Software**

**Program Name:** Composite-W\_PL

**Project Name:** -

**Project Number:** -

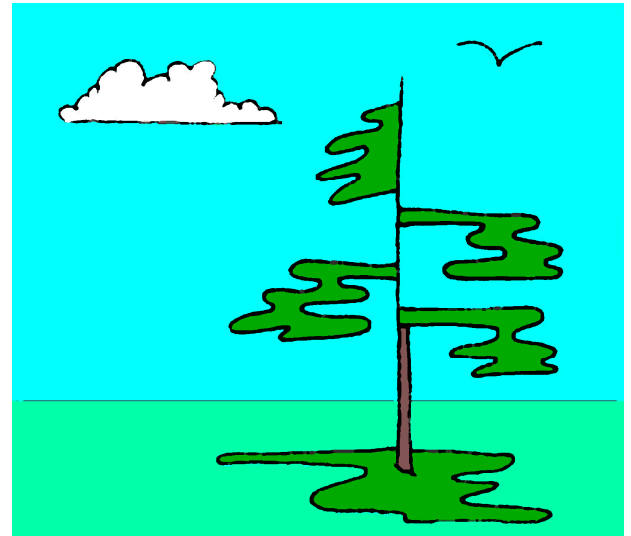
**Project Description:** -

**Project Designer:** Dik

**Last Revised (yy-mm-dd):** 21-04-11

**Reference:** NBCC

Created using SMath Studio, a MathCAD workalike from <https://en.smath.info/view/SMathStudio>. The User is responsible to verify data using an alternative method



**Menu:**

- ..... Input Data
- ..... Important Output
- ..... Logical Constructs
- Blue Units
- ..... Sum / For
- Red Important Note

**Defined Units:**

$K := kip$	Force
$K\_ft := K ft$ $kN\_m := kN m$ $K\_in := K in$ $kN\_mm := kN mm$ $lb\_in := lbf in$	Moment
$pcf := \frac{lbf}{ft^3}$ $kNpcm := \frac{kN}{m^3}$ $kgpcm := \frac{kg}{m^3}$	Density
$Klf := \frac{K}{ft}$ $plf := \frac{lbf}{ft}$ $kNpm := \frac{kN}{m}$ $Kpi := \frac{K}{in}$ $kNpmm := \frac{kN}{mm}$	Force/Unit Length
$psf := \frac{lbf}{ft^2}$ $Ksf := \frac{K}{ft^2}$ $Ksi := \frac{K}{in^2}$ $kNpsm := \frac{kN}{m^2}$ $psi := \frac{lbf}{in^2}$	Pressure
$pci := \frac{lbf}{in^3}$	Subgrade Modulus
$psfpf := \frac{psf}{ft}$ $kPapm := \frac{kPa}{m}$	Pressure per Depth
$pmcf := \frac{lb}{ft^3}$ $lb := lbf$	Force
$mph := \frac{mi}{hr}$ $kph := \frac{km}{hr}$	Velocity
$ispf := \frac{in^2}{ft}$ $mmspm := \frac{mm^2}{m}$	Area per Unit Length

**User Defined Functions:**

```

Check (arg) := if arg = 1
                Check := "...OK"
            else
                Check := "...NG"
```

```

Check (2 = 3) = "...NG"
Check (2 ≠ 3) = "...OK"
```

```

Check (2 ≤ 3) = "...OK"
Check (3 ≥ 2) = "...OK"
```

**Input Data**

**Material Property Factors:**

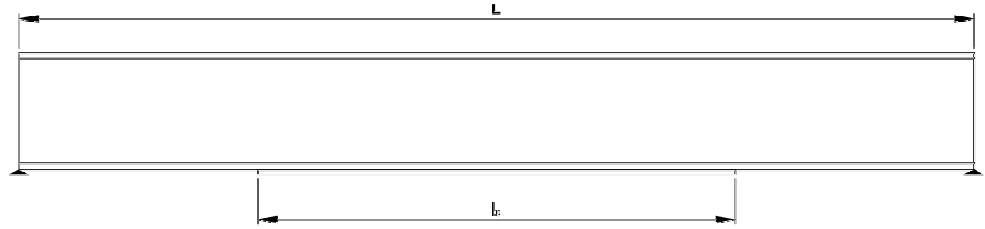
$\phi_s := 0.90$

$\phi_w := 0.67$

**Load Factors:**

$\alpha_L := 1.50$

$\alpha_D := 1.25$



**Steel Properties:**

**W Properties**  $stl_{NDX} := 1$

NDX	des	fy	Fu
1	"G40.21-350W"	50 Ksi	65 Ksi
2	"G40.21-300W"	44 Ksi	65 Ksi
3	"A36"	36 Ksi	58 Ksi

$desI_1 := stl_{NDX} 2$        $f_{y1} := stl_{NDX} 3$   
 $F_{u1} := stl_{NDX} 4$        $E_s := 29000 \text{ Ksi}$

$\nu := 0.3$        $G_s := \frac{E_s}{2 \cdot (1 + \nu)}$        $G_s := 11154 \text{ Ksi}$        $\gamma_s := 489 \text{ pcf}$

**PL Properties**  $stl_{NDX} := 1$

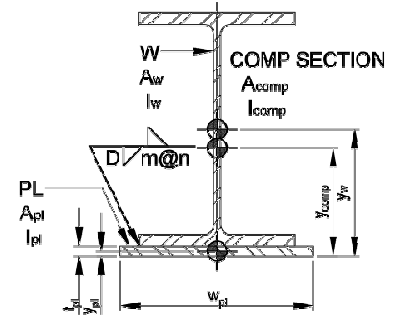
**Weld**  $desI_2 := stl_{NDX} 2$        $F_{u2} := stl_{NDX} 4$        $f_{y2} := stl_{NDX} 3$

**Beam Span:**  $L := 19.33 \text{ ft}$        $L = 5.89 \text{ m}$       **Span Length**

**Electrodes**  $we_{NDX} := 2$

NDX	DesI	DesM	UTS
1	"E60xx"	"E410xx"	60 ksi
2	"E70xx"	"E480xx"	70 ksi
3	"E80xx"	"E550xx"	80 ksi
4	"E90xx"	"E620xx"	90 ksi

$desI_w := we_{NDX} 2$        $X_u := we_{NDX} 4$   
 $desM_w := we_{NDX} 3$



Size  $ws_{NDX} := 4$

	NDX	desI	desM	D
$ws :=$	1	""	"3mm"	0.11811 in
	2	"1/8"	""	0.125 in
	3	""	"4mm"	0.15748 in
	4	"3/16"	""	0.1875 in
	5	""	"5mm"	0.19685 in
	6	""	"6mm"	0.23622 in
	7	"1/4"	""	0.25 in
	8	""	"7mm"	0.275591 in
	9	"5/16"	""	0.3125 in
	10	""	"8mm"	0.314961 in
	11	""	"9mm"	0.354331 in
	12	"3/8"	""	0.375 in

$$desI_D := ws_{NDX}^2 \quad desM_D := ws_{NDX}^3$$

$$D := ws_{NDX}^4$$

$$m := 3 \text{ in}$$

$$m = 76.2 \text{ mm}$$

Weld Length

$$n := 12 \text{ in}$$

$$n = 304.8 \text{ mm}$$

Weld Spacing

### W Section

$$desI_w := "W16x26"$$

$$desM_w := "W410x39"$$

$$d_w := 15.69 \text{ in}$$

$$d_w = 398.5 \text{ mm}$$

W Beam Depth

$$b_w := 5.50 \text{ in}$$

$$b_w = 139.7 \text{ mm}$$

W Beam Width

$$t_w := 0.345 \text{ in}$$

$$t_w = 8.8 \text{ mm}$$

W Beam Depth

$$w_w := 0.25 \text{ in}$$

$$w_w = 6.35 \text{ mm}$$

W Beam Width

$$A_w := 2 \cdot b_w \cdot t_w + (d_w - 2 \cdot t_w) \cdot w_w$$

$$A_w = 7.545 \text{ in}^2$$

$$A_w = 4868 \text{ mm}^2$$

W Beam Area

$$Ix_w := \frac{1}{12} \cdot \left( b_w \cdot d_w^3 - (b_w - w_w) \cdot (d_w - 2 \cdot t_w)^3 \right)$$

$$Ix_w = 294 \text{ in}^4$$

$$Ix_w = 1.22 \cdot 10^8 \text{ mm}^4$$

W Beam Ix

$$y_w := \frac{d_w}{2}$$

$$y_w = 7.85 \text{ in}$$

$$y_w = 199.3 \text{ mm}$$

W Centroid Distance

$$Sx_w := \frac{1}{6 \cdot d_w} \cdot \left( b_w \cdot d_w^3 - (b_w - w_w) \cdot (d_w - 2 \cdot t_w)^3 \right)$$

$$Sx_w = 37.4 \text{ in}^3$$

$$Sx_w = 6.14 \cdot 10^5 \text{ mm}^3$$

W Beam Sx

### PL Section

$$desI_{pl} := "PL .25x6"$$

$$desM_{pl} := "PL 6x150"$$

$$w_{pl} := 6.0 \text{ in}$$

$$w_{pl} = 152.4 \text{ mm}$$

PL Width

$$t_{pl} := 0.25 \text{ in}$$

$$t_{pl} = 6.4 \text{ mm}$$

PL Thickness

$$\begin{array}{lll}
 A_{pl} := w_{pl} \cdot t_{pl} & A_{pl} = 1.500 \text{ in}^2 & A_{pl} = 967.7 \text{ mm}^2 \\
 Ix_{pl} := \frac{w_{pl} \cdot t_{pl}^3}{12} & Ix_{pl} = 0.008 \text{ in}^4 & Ix_{pl} = 3251.8 \text{ mm}^4 \\
 y_{pl} := \frac{t_{pl}}{2} & y_{pl} = 0.125 \text{ in} & y_{pl} = 3.2 \text{ mm}
 \end{array}$$

PL Area  
PL Ix  
PL Centroid Dist

### Composite Section Centroid

$$\begin{array}{lll}
 A_{comp} := A_w + A_{pl} & A_{comp} = 9.05 \text{ in}^2 & A_{comp} = 5835 \text{ mm}^2 \\
 y_b := \frac{A_w \cdot (y_w + t_{pl}) + A_{pl} \cdot y_{pl}}{A_{comp}} & y_b = 6.77 \text{ in} & y_b = 172.0 \text{ mm}
 \end{array}$$

### Moment of Inertia (x-Axis)

$$\begin{array}{lll}
 Ix_{comp} := Ix_w + \left( A_w \cdot (y_w + t_{pl} - y_b)^2 \right) + Ix_{pl} + \left( A_{pl} \cdot (y_{pl} - y_b)^2 \right) & & \\
 Ix_{comp} = 373 \text{ in}^4 & Ix_{comp} = 1.55 \cdot 10^8 \text{ mm}^4 & 
 \end{array}$$

Design shear stress is determined based on the uniform distributed load that produces the maximum resisting moment for the fully braced span under consideration.

### Section Modulus (x-Axis)

$$\begin{array}{lll}
 y_t := d_w + t_{pl} - y_b & y_t = 9.17 \text{ in} & y_t = 232.8 \text{ mm} \\
 Sx_t := \frac{Ix_{comp}}{y_t} & Sx_t = 40.7 \text{ in}^3 & Sx_t = 6.67 \cdot 10^5 \text{ mm}^3 \\
 Sx_b := \frac{Ix_{comp}}{y_b} & Sx_b = 55.1 \text{ in}^3 & Sx_b = 9.03 \cdot 10^5 \text{ mm}^3
 \end{array}$$

### Moment Resistance

$$\begin{array}{lll}
 M_{rt} := \phi_s \cdot Sx_t \cdot f_{y1} & M_{rt} = 152.7 \text{ K\_ft} & M_{rt} = 207.0 \text{ kN\_m} \\
 M_{rb} := \phi_s \cdot Sx_b \cdot f_{y2} & M_{rb} = 206.6 \text{ K\_ft} & M_{rb} = 280.2 \text{ kN\_m} \\
 M_r := \min \left( \left[ M_{rt} \ M_{rb} \right] \right) & M_r = 152.7 \text{ K\_ft} & M_r = 207 \text{ kN\_m}
 \end{array}$$

### Maximum UDL

$$\begin{array}{lll}
 Q_{max} := \frac{8 \cdot M_r}{L} & Q_{max} = 3.3 \text{ Kl f} & Q_{max} = 47.7 \text{ kNpm}
 \end{array}$$

### Maximum Reaction (Shear)

$$\begin{array}{lll}
 V_{max} := Q_{max} \cdot \frac{L}{2} & V_{max} = 31.6 \text{ K} & V_{max} = 140.5 \text{ kN}
 \end{array}$$

### Shear Stress at W and WT Junction

$$\begin{array}{lll}
 A' := A_{pl} & & \\
 y_{bar} := y_b - \frac{t_{pl}}{2} & & \\
 \tau := \frac{V_{max}}{Ix_{comp} \cdot \min \left( \left[ w_{pl} \ b_w \right] \right)} \cdot A' \cdot (y_{bar}) & \tau = 0.15 \text{ Ksi} & \tau = 1.06 \text{ MPa}
 \end{array}$$

**Weld Design:****Weld Material Strength**

$$v_{rw} := \phi_w \cdot 0.67 \cdot X_u \cdot \frac{1}{\sqrt{2}} \quad v_{rw} = 22.22 \text{ ksi} \quad v_{rw} = 153.20 \text{ MPa}$$

**Weld Material Strength / Unit Length**

$$V_{rw} := v_{rw} \cdot D \quad V_{rw} = 4.17 \text{ Kpi} \quad V_{rw} = 0.73 \text{ kNpmm}$$

**Base Metal Strength**

$$v_{rb} := \phi_s \cdot \min \left( \left[ \begin{array}{c} f_{y1} \\ f_{y2} \end{array} \right] \right) v_{rb} = 45.00 \text{ Ksi} \quad v_{rb} = 310.26 \text{ MPa}$$

**Base Metal Strength / Unit Length**

$$V_{rb} := v_{rb} \cdot D \quad V_{rb} = 8.44 \text{ Kpi} \quad V_{rb} = 1.48 \text{ kNpmm}$$

**Minimum Weld Capacity**

$$V_{rmin} := \min \left( \left[ \begin{array}{c} V_{rw} \\ V_{rb} \end{array} \right] \right) \quad V_{rmin} = 4.17 \text{ Kpi} \quad V_{rmin} = 0.73 \text{ kNpmm}$$

**Connection Shear per Unit Length**

$$\tau' := \left( \tau \cdot n \cdot \min \left( \left[ \begin{array}{c} w_{pl} \\ b_w \end{array} \right] \right) \right) \\ \tau' = 10.1 \text{ K} \quad \tau' = 45.1 \text{ kN}$$

**Weld Provided**

$$\tau_r := 2 \cdot V_{rmin} \cdot m \quad \tau_r = 25.0 \text{ K} \quad \tau_r = 111.2 \text{ kN}$$

**Connection Force at End:****Length of Reinforcement**

$$M_{rw} := \phi_s \cdot f_{y1} \cdot Sx_w \quad M_{rw} = 140.4 \text{ K\_ft} \quad M_{rw} = 190.4 \text{ kN\_m}$$

$$M_{rcomp} := \phi_s \cdot f_{y1} \cdot Sx_b \quad M_{rcomp} = 206.6 \text{ K\_ft} \quad M_{rcomp} = 280.2 \text{ kN\_m}$$

$$\Delta_M := M_{rcomp} - M_{rw} \quad \Delta_M = 66.2 \text{ K\_ft} \quad \Delta_M = 89.8 \text{ kN\_m}$$

$$l_r := \sqrt{\frac{8 \cdot \Delta_M}{Q_{max}}} \quad l_r = 12.73 \text{ ft} \quad l_r = 3.88 \text{ m}$$

**Stress at Bottom of Reinforcement**

$$\sigma_b := \frac{M_{rcomp} \cdot Y_b}{Ix_{comp}} \quad \sigma_b = 45.00 \text{ Ksi} \quad \sigma_b = 310.26 \text{ MPa}$$

**Stress at Top of Reinforcement**

$$\sigma_t := \frac{M_{rcomp} \cdot (Y_b - t_{pl})}{Ix_{comp}} \quad \sigma_t = 43.3391 \text{ Ksi} \quad \sigma_t = 298.8123 \text{ MPa}$$

**Force Centroid**

$$e_c := \frac{\sigma_t \cdot t_{pl} \cdot \frac{t_{pl}}{2} + (\sigma_b - \sigma_t) \cdot \frac{t_{pl}}{2} \cdot \frac{2 \cdot t_{pl}}{3}}{\sigma_t \cdot t_{pl} + (\sigma_b - \sigma_t) \cdot \frac{t_{pl}}{2}} \quad e_c = 0.126 \text{ in} \quad e_c = 3.19 \text{ mm}$$

$$F_e := w_{pl} \cdot t_{pl} \cdot \frac{(\sigma_t + \sigma_b)}{2} \quad F_e = 66.25 \text{ K} \quad F_e = 294.71 \text{ kN}$$

**Connection Force**

$$F_c := F_e \cdot e_c \cdot \frac{2}{l_r} \quad F_c = 0.11 \text{ K} \quad F_c = 0.49 \text{ kN}$$

**Summary:**

Material Property Factor (Welds)	$\phi_w = 0.67$	
Material Property Factor (Steel)	$\phi_s = 0.9$	
Dead Load Factor	$\alpha_D = 1.25$	
Live Load Factor	$\alpha_L = 1.5$	
<b>Steel</b>		
Designation (Beam)	$desI_1 = \text{"G40.21-350W"}$	
Yield Strength (Beam)	$f_{y1} = 50 \text{ Ksi}$	$f_{y1} = 345 \text{ MPa}$
Ultimate Strength (Beam)	$F_{u1} = 65 \text{ Ksi}$	$F_{u1} = 448 \text{ MPa}$
Young's Modulus	$E_s = 29000 \text{ Ksi}$	$E_s = 1.9995 \cdot 10^5 \text{ MPa}$
Shear Modulus	$G_s = 11154 \text{ Ksi}$	$G_s = 76904 \text{ MPa}$
Density	$\gamma_s = 489 \text{ pcf}$	$\gamma_s = 76.8 \text{ kNpcm}$
<b>Welds</b>		
Electrode Designation (Imperial)	$desI_w = \text{"W16x26"}$	
Electrode Designation (Metric)	$desM_w = \text{"W410x39"}$	
Ultimate Strength (Welding Electrode)	$X_u = 70 \text{ Ksi}$	$X_u = 483 \text{ MPa}$
Size Designation	$desI_D = \text{"3/16"}$	$desM_D = \text{""}$
Fillet Weld Size	$D = 0.1875 \text{ in}$	$D = 4.7625 \text{ mm}$
<b>W Section</b>		
Designation	$desI_w = \text{"W16x26"}$	$desM_w = \text{"W410x39"}$
Depth	$d_w = 15.7 \text{ in}$	$d_w = 398.5 \text{ mm}$
Area	$A_w = 7.5 \text{ in}^2$	$A_w = 4867.7 \text{ mm}^2$
Moment of Inertia (x-axis)	$Ix_w = 293.8 \text{ in}^4$	$Ix_w = 1.2 \cdot 10^8 \text{ mm}^4$
Centroid Distance	$y_w = 7.8 \text{ in}$	$y_w = 199.3 \text{ mm}$
<b>PL Section</b>		
Designation (Imperial)	$desI_{pl} = \text{"PL .25x6"}$	
Designation (Metric)	$desM_{pl} = \text{"PL 6x150"}$	
Width	$w_{pl} = 6 \text{ in}$	$w_{pl} = 152.4 \text{ mm}$
Thickness	$t_{pl} = 0.250 \text{ in}$	$t_{pl} = 6.4 \text{ mm}$
Area	$A_{pl} = 1.5 \text{ in}^2$	$A_{pl} = 967.7 \text{ mm}^2$
Moment of Inertia (x-axis)	$Ix_{pl} = 0 \text{ in}^4$	$Ix_{pl} = 3251.8 \text{ mm}^4$
Centroid Distance	$y_{pl} = 0.1 \text{ in}$	$y_{pl} = 3.2 \text{ mm}$
<b>Composite Section</b>		
Area	$A_{comp} = 9.05 \text{ in}^2$	$A_{comp} = 5835 \text{ mm}^2$
Moment of Inertia (x-axis)	$Ix_{comp} = 373 \text{ in}^4$	$Ix_{comp} = 1.55 \cdot 10^8 \text{ mm}^4$
Centroid Distance (top)	$y_t = 9.17 \text{ in}$	$y_t = 232.8 \text{ mm}$
Centroid Distance (bottom)	$y_b = 6.77 \text{ in}$	$y_b = 172.0 \text{ mm}$

Section Modulus (top)	$Sx_t = 40.7 \text{ in}^3$	$Sx_t = 6.67 \cdot 10^5 \text{ mm}^3$
Section Modulus (bottom)	$Sx_b = 55.1 \text{ in}^3$	$Sx_b = 9.03 \cdot 10^5 \text{ mm}^3$
<b>Composite Connection Design</b>		
Beam Span	$L = 19.33 \text{ ft}$	$L = 5.892 \text{ m}$
Resisting Moment (top) based on fy1	$M_{rt} = 152.7 \text{ K\_ft}$	$M_{rt} = 207 \text{ kN\_m}$
Maximum Resisting (bottom) based on fy2	$M_{rb} = 206.6 \text{ K\_ft}$	$M_{rb} = 280.2 \text{ kN\_m}$
Minimum Resisting Moment	$M_r = 152.7 \text{ K\_ft}$	$M_r = 207 \text{ kN\_m}$
Maximum Uniformly Distributed Load	$Q_{max} = 3.3 \text{ Klf}$	$Q_{max} = 47.7 \text{ kNpm}$
Maximum Design Shear	$V_{max} = 31.6 \text{ K}$	$V_{max} = 140.5 \text{ kN}$
Shear Stress at Interface	$\tau = 0.153 \text{ Ksi}$	$\tau = 1.058 \text{ MPa}$
<b>Shear Resistance per Unit Length</b>		
Weld Length	$m = 3 \text{ in}$	$m = 76.2 \text{ mm}$
Weld Spacing	$n = 12 \text{ in}$	$n = 304.8 \text{ mm}$
Total Shear per Unit Length	$\tau' = 10.1 \text{ K}$	$\tau' = 45.1 \text{ kN}$
Minimum Weld Capacity	$V_{rmin} = 4.17 \text{ Kpi}$	$V_{rmin} = 0.73 \text{ kNpmm}$
Shear Resistance per Unit Length	$\tau_r = 25 \text{ K}$	$\tau_r = 111.2 \text{ kN}$
<b>Check <math>(\tau_r \geq \tau')</math> = "...OK"</b>		
<b>Connection at End</b>		
Moment Resistance of Unreinforced Section	$M_{rw} = 140.4 \text{ K\_ft}$	$M_{rw} = 190.4 \text{ kN\_m}$
Moment Resistance of Composite Section	$M_{rcomp} = 206.6 \text{ K\_ft}$	$M_{rcomp} = 280.2 \text{ kN\_m}$
Moment Difference	$\Delta_M = 66.2 \text{ K\_ft}$	$\Delta_M = 89.8 \text{ kN\_m}$
Equivalent Span	$l_r = 12.73 \text{ ft}$	$l_r = 3.880 \text{ m}$
Force Eccentricity in Reinforcing	$e_c = 0.126 \text{ in}$	$e_c = 3.2 \text{ mm}$
Total Force in Reinforcing	$F_e = 66.3 \text{ K}$	$F_e = 294.7 \text{ kN}$
Connection Force at End of Reinforcing	$F_c = 0.11 \text{ K}$	$F_c = 0.49 \text{ kN}$