$$H_F = \frac{\omega(2a+c)(a-c)}{12L} - \frac{x_{11}}{2L} + \frac{x_{12}}{2L}\left(\frac{k-1}{k+2}\right)$$

$$M_F = \frac{3k+1}{2(6k+1)}\left(\frac{\omega}{C}(2a+c)(a-c) - x_{11}\right) - \frac{x_{12}}{2}\left(\frac{1}{k+2} - \frac{3k}{6k+1}\right)$$

$$x_{11} = \frac{\omega(2a+c)(a-c)}{6} + x_{12}\cdot\frac{k-1}{k+2} - 24\cdot H_F = -\left(M_F + \frac{x_{12}}{2}\left(\frac{1}{k+2} - \frac{3k}{6k+1}\right)\right)\frac{2(6k+1)}{3k+1} + \frac{\omega}{C}(2a+c)(a-c) = x_{11}$$

$$x_{12}\cdot\frac{k-1}{k+2} - 24\,H_F = \frac{x_{12}}{2}\left(\frac{3k}{6k+1} - \frac{1}{k+2}\right)\cdot\frac{2(6k+1)}{3k+1} - M_F\cdot\frac{2(6k+1)}{3k+1}$$

$$x_{12}\left(\frac{k-1}{k+2}\right) - 24\,H_F = x_{12}\left(\frac{3k}{3k+1} - \frac{6k+1}{(k+2)(3k+1)}\right) - M_F\cdot\frac{2(6k+1)}{3k+1}$$

$$x_{12}\left(\frac{k-1}{k+2} - \frac{3k}{3k+1} + \frac{6k+1}{(k+2)(3k+1)}\right) = 24\,H_F - M_F\cdot\frac{2(6k+1)}{3k+1}$$

$$x_{12}\left(\frac{(k-1)(3k+1) - 3k(k+2) + 6k+1}{(k+2)(3k+1)}\right) = 24\,H_F - M_F\cdot\frac{2(6k+1)}{3k+1}$$

$$x_{12}\left(\frac{3k^2 - 2k - 1 - 3k^2 - 6k + 6k+1}{(k+2)(3k+1)}\right) = 24\,H_F - M_F\cdot\frac{2(6k+1)}{3k+1}$$

$$x_{12} = \left[M_F\cdot\frac{2(6k+1)}{3k+1} - 24\,H_F\right]\frac{(k+2)(3k+1)}{2k}$$

$$x_{12} = M_F\cdot\frac{(6k+1)(k+2)}{k} - 4H_F\cdot\frac{(k+2)(3k+1)}{k}$$

$$\boxed{x_{12} = \frac{M_F\cdot(6k+1)(k+2) - 4H_F(k+2)(3k+1)}{k}}$$

From Mathcad soln.

$$12(6x^2+7x+1) = 12(6x+1)(x+1) \qquad 15(9x^2+14x+3)$$
$$9(6x^2+7x+1) = 9(6x+1)(x+1) \qquad 6(6x^2+7x+1) = 6(6x+1)(x+1)$$
$$3(6x^2+7x+1) = 3(6x+1)(x+1)$$
$$5(9x^2+14x+3)$$

$$M_F = \frac{-W(a-c)}{120h^2(K+2)(6K+1)} \cdot \left(72K^2a^3 + 54K^2a^2c - 135K^2a^2h + 36K^2ac^2 - 90hK^2ac + 18K^2c^3 - 45hK^2c^2 \right.$$

$$\cdot \left(\begin{array}{l} 72k^2a^3 + 54K^2a^2c - 135hK^2a^2 + 36K^2ac^2 - 90hk^2ac + 18k^2c^3 - 45hk^2c^2 \\ +84Ka^3 + 63Ka^2c - 210hKa^2 + 42Kac^2 - 140hKac + 21Kc^3 - 70hKc^2 \\ +12a^3 + 9a^2c - 45ha^2 + 6ac^2 - 30hac + 3c^3 - 15hc^2 \end{array} \right)$$

$$= \frac{-W(a-c)}{120h^2(K+2)(6K+1)} \cdot \left(\begin{array}{l} 12a^2(6K+1)(K+1) + 9a^2c(6K+1)(K+1) + 6ac^2(6K+1)(K+1) + 3c^3(6K+1)(K+1) \\ -15ha^2(9K^2+14K+3) - 10hac(9K^2+14K+3) - 5hc^2(9K^2+14K+3) \end{array} \right)$$

$$= \frac{-W(a-c)}{120h^2(K-2)(6K+1)} \left[3\left(\tfrac{4}{3}a^3 + \tfrac{3}{2}a^2c + 2ac^2 + c^3 \right)(6K+1)(K+1) - 5h\left(15h^3a^2 + 10hac + 5hc^2 \right)(9K^2+14K+3) \right]$$

$$\boxed{M_F = \frac{-W(a-c)}{120h^2(K+2)(6K+1)} \left[3\underbrace{(4a^3 + 3a^2c + 2ac^2 + c^3)(6K+1)(K+1)}_{A} - 5h\underbrace{(3a^2 + 2ac + c^2)(9K^2+14K+3)}_{B} \right]}$$

$$-9K^2 - 14K - 3 + 9K^2 + 12K + 3$$
$$-2k$$

$$9K^2 + 14K + 3 - 9K^2 - 12K - 3$$

$$6K^2 + 5K + 1 - 6K^2 - 7K - 1$$
$$-2k$$

$$3k^2 - 2k - 1 + 6K^2 + 13k + 2$$
$$9K^2 + 11K + 1$$

$$6K^2 + 7K + 1 - 6K^2 - 5K - 1$$
$$2k - 3A$$

$$H_E = \frac{-w(a-c)}{40h^3(k+2)} \left(\begin{array}{l} 8ka^3 + 2kc^3 + 4kac^2 + 6ka^2c - 15kha - 5khc - 10ach \\ + 4a^3 + c^3 + 2ac^2 + 3a^2c - 15ha^2 - 5hc^2 - 10ach \end{array} \right)$$

$$H_F = \frac{-w(a-c)}{40h^3(k+2)} \left(\underbrace{(2k+1)(4a^3 + 3a^2c + 2ac^2 + c^3)}_{A} - \underbrace{5h(k+1)(3a^2 + 2ac + c^2)}_{B} \right)$$

$$X_{12} = \frac{-w(a-c)}{120h^2(k+2)(6k+1)} \left[\frac{3A(6k+1)(k+1) - 5hB(9k^2 + 14k+3)}{k} \right] (6k+1)(k+2)$$

$$+ \frac{w(a-c)}{40h^2(k+2)} \left[\frac{(2k+1)A - 5h(k+1)B}{k} \right] (k+2)(3k+1)$$

$$= \frac{-w(a-c)}{120h^2 K} \left[3A(6k+1)(k+1) - 5hB(9k^2 + 14k+3) - 3(2k+1)(3k+1)A + 15h(k+1)(3k+1)B \right]$$

$$= \frac{-w(a-c)}{120h^2 K} \left[3A(6k^2 + 7k+1 - 6k^2 - 5k-1) - 5hB(9k^2 + 14k+3 - 9k^2 - 12k-3) \right]$$

$$= \frac{-w(a-c)}{120h^2 K} \left[2k(3A - 5hB) \right]$$

$$\boxed{X_{12} = \frac{-w(a-c)}{60h^2} \left(3A - 5hB \right)}$$

$$X_{11} = \frac{w}{6}(2a+c)(a-c) + X_{12} \cdot \frac{k-1}{k+2} - 2Lh\varepsilon \quad (\text{pg 1 line 2})$$

$$= \frac{w}{6}(2a+c)(a-c) - \frac{w(a-c)}{60h^2}(3A-5hB)\frac{k-1}{k+2} + \frac{37h\,w(a-c)}{\cancel{40h^3}(k+2)}\left[(2k+1)A - 5h(k+1)B\right]$$

$$= \frac{w(a-c)}{60h^2}\left[10h^2(2a+c) - (3A-5hB)\frac{k-1}{k+2} + \frac{3}{k+2}\left[(2k+1)A - 5h(k+1)B\right]\right]$$

$$= \frac{w(a-c)}{60h^2}\left[10h^2(2a+c) + \frac{1}{k+2}\left[-3A(k-1)+5hB(k-1) + 3(2k+1)A - 15h(k+1)B\right]\right]$$

$$= \frac{w(a-c)}{60h^2}\left[10h^2(2a+c) + \frac{1}{k+2}\left[3A(2k+1-k+1) + 5hB(k-1-3k-3)\right]\right]$$

$$= \frac{w(a-c)}{60h^2}\left[10h^2(2a+c) + \frac{1}{k+2}\left[3A(k+2) - 5hB(2k+4)\right]\right]$$

$$\boxed{X_{11} = \frac{w(a-c)}{60h^2}\left[10h^2(2a+c) + 3A - 10hB\right]}$$

from mathcad:

$$M_a = \frac{-w(a-c)}{120h^2(k+2)(6k+1)}\left(\begin{array}{l}72k^2a^3 + 54k^2a^2c + 36k^2ac^2 + 18k^2c^3 - 225hk^2a^2c - 150hk^2ac^2 - 75hk^2c^2 \\ +84ka^3 + 63ka^2c + 42kac^2 + 21kc^3 - 390hka^2 - 260hkac - 130hkc^2 \\ +12a^3 + 9a^2c + 6ac^2 + 3c^3 - 45ha^2 - 30hac - 15hc^2 \\ +240k^2ah^2 + 120k^2ch^2 \\ +520kah^2 + 260kch^2 \\ +80ah^2 + 40ch^2\end{array}\right)$$

$$= \frac{w(a-c)}{120h^2(k+2)(6k+1)}\left(\begin{array}{l}\overbrace{(12a^3+9a^2c+6ac^2+3c^3)}^{3A}(6k+1)(k+1) - 15ha^2(15k^2+26k+3) - 10hac(15k^2+26k+3) \\ -5hc^2(15k^2+26k+3) + 40ah^2(6k^2+13k+2) + 20ch^2(6k^2+13k+2)\end{array}\right)$$

$$= \frac{w(a-c)}{120h^2(k+2)(6k+1)}\left(3A(6k+1)(k+1) - 54\underbrace{(3a^2+2ac+c^2)}_{B}(15k^2+26k+3) + 20h^2(2a+c)(6k+1)(k+2)\right)$$

$$M_a = \frac{w(a-c)}{120h^2}\left[3A\frac{k+1}{k+2} - 54B\frac{15k^2+26k+3}{(k+2)(6k+1)} + 20h^2(2a+c)\right]$$

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$$X_{12} = \frac{-w(a-c)}{60h^2}\left(12a^3 + 9a^2c + 6ac^2 + 3c^3 - 15a^2h - 10ach - 5c^2h\right]$$

$$= \frac{-w}{60h^2}\left(12a^4 + 9a^3c - 12a^3c + 6a^2c^2 - 9a^2c^2 + 3ac^3 - 6ac^3 - 3c^4 - 15a^3h - 10a^2ch + 15a^2ch - 5ach^2 \right.$$
$$\left. + 10ach^2 + 5c^3h\right)$$

$$= \frac{-w}{60h^2}\left(12a^4 - 3a^3c - 3a^2c^2 - 3ac^3 - 3c^4 - 15a^3h + 5a^2ch + 5ac^2h + 5c^3h\right)$$

$$= \frac{-w}{60h^2(a-c)}\left(12a^5 - 3a^4c - 12a^4c - 3a^3c^2 + 3a^3c^2 - 3a^2c^3 + 3a^2c^3 - 3ac^4 + 3ac^4 + 3c^5 - 15a^4h + 5a^3ch + 15a^4h \right.$$
$$+ 5a^2c^2h - 5a^2c^2h$$
$$+ 5ac^3h - 5ac^3h$$
$$\left. - 5c^4h\right)$$

$$= \frac{+w}{60h^2(a-c)}\left(-12a^5 + 15a^4c + 3c^5 + 15a^4h + 20a^3ch + 5c^4h\right)$$

$$= \frac{w}{60h^2(a-c)}\left[15ha^4 - 15hc^4 + 15ca^4 - 15c^5 - 12a^5 + 12c^5 + 20hc^4 - 20hca^3\right]$$

$$= \frac{w}{60h^2(a-c)}\left[15h(a^4-c^4) + 15c(a^4-c^4) - 12(a^5-c^5) - 20ch(a^3-c^3)\right]$$

$$\boxed{X_{12} = \frac{w}{60h^2(a-c)}\left[15(h+c)(a^4-c^4) - 12(a^5-c^5) - 20ch(a^3-c^3)\right]}$$

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$$x_{11} = \frac{\omega(a-c)}{60h^2}\left[10h^2(2a+c) + 3(4a^3+3a^2c+2ac^2+c^3) - 10h(3a^2+2ac+c^2)\right]$$

$$a-c = h-b$$
$$c = h-d$$
$$a-c = d-b$$

$$x_{11} = \frac{\omega(d-b)}{60h^2}\left[10h^2(2h-2b+h-d) + 12(h-b)^3 + 9(h-b)^2(h-d) + 6(h-b)(h-d)^2 + 3(h-d)^3 - 30h(h-b)^2 \right.$$
$$\left. - 30h(h-b)(h-d) - 10h(h-d)^2\right]$$

$$= \frac{\omega(d-b)}{60h^2}\left[30h^3 - 20h^2b - 10h^2d + 12h^3 - 36h^2b + 36hb^2 - 12b^3 + \frac{(9h^2-18hb+9b^2)(h-d)}{} + \frac{(h-b)(6h^2-12hd+6d^2)}{} + 3h^3 - 9h^2d + 9hd^2 \right.$$
$$\left. - 3d^3 \right]$$

$$= \frac{\omega(d-b)}{60h^2}\left[30h^3 - 20h^2b - 10h^2d + 12h^3 - 36h^2b + 36hb^2 - 12b^3 + \frac{9h^3 - 18h^2b + 9hb^2 - 9h^2d + 18hbd - 9b^2d}{} + \frac{6h^3 - 12h^2d + 6hd^2 - 6h^2b + 12hbd - 6b^2d}{} + 3h^3 - 9h^2d + 9hd^2 - 3d^3\right]$$

$$= \frac{\omega(d-b)}{60h^2}\left[h^3(30+12+9+6+3) + h^2b(-20-36-18-6) + h^2d(-10-9-12-9) + hb^2(36+9-30) + hd^2(6+9-10) \right.$$
$$\left. + hbd(18+12-20) + b^3(-9) + bd^2(-6) + b^2d(-12) + d^3(-3)\right]$$

$$= \frac{\omega(d-b)}{60h^2}\left[15h^3b + 54hd^3 + 104hbd - 96b^2d - 12b^3 - 3d^3\right]$$

$$= \frac{\omega}{60h^2}\left[15h^3b + 54hd^3 + 104hbd - 96b^2d - 12b^3 - 3d^3 - 15h^3b^2 - 54hb^2d^2 - 10hb^2d + 96b^3d + 12b^4 + 3bd^3\right]$$

$$= \frac{\omega}{60h^2}\left[12b^4 - 36b^3d - 36b^2d^2 - 3d^4 - 15hb^3 + 54hb^2d + 54hbd^2 + 54d^3\right]$$

$$= \frac{\omega}{60h^2}\left[-12b^5 + 36b^4d + 12b^4d + 3b^2d^2 - 3b^2d^3 - 3b^2d^3 - 36b^3d^4 - 3d^5 + 15hb^4 - 54hb^3d - 54hb^2d^2 + 54b^2d^3 + 54b^3d^2 - 54b^3d^3 + 54d^4\right]$$

$$= \frac{\omega}{60h^2(d-b)}\left[-12b^5 + 15b^4d - 3d^5 + 15hb^4 - 20hb^3d + 15b^4(d+h) - 12b^5\right]$$

$$\boxed{x_{11} = \frac{\omega}{60h^2(d-b)}\left[5hd^4 - 3d^5 - 20hb^3d + 15b^4(d+h) - 12b^5\right]}$$

$$\boxed{x_{11} = \frac{\omega}{60h^2(d-b)}\left[5hd^4 - 3d^5 - 20hb^3d + 15b^4(d+h) - 12b^5\right]}$$

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