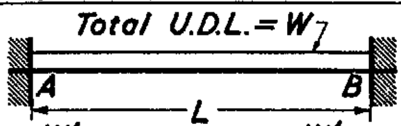
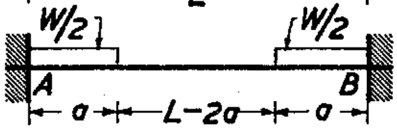
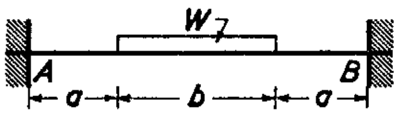
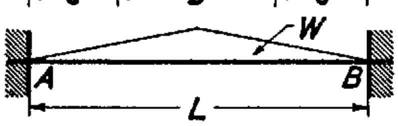
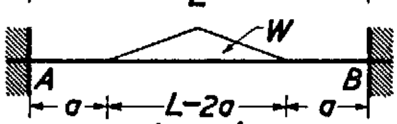
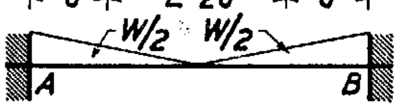
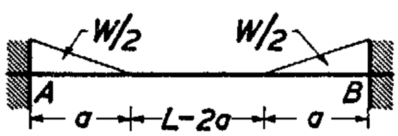
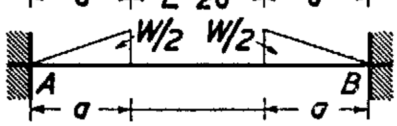
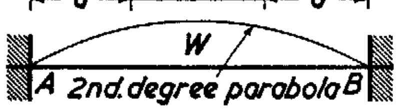
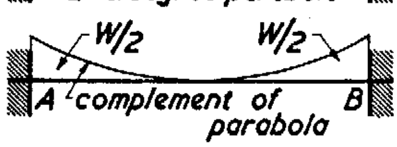
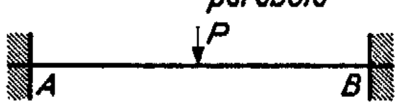
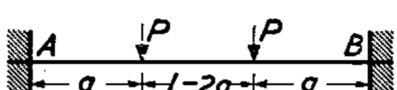
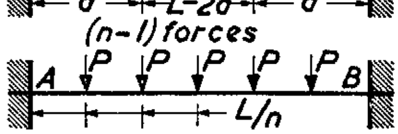
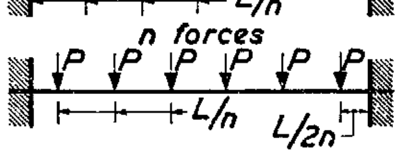
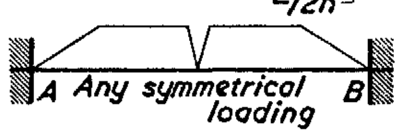
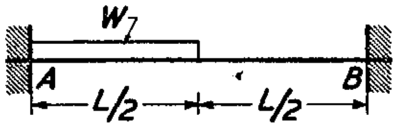
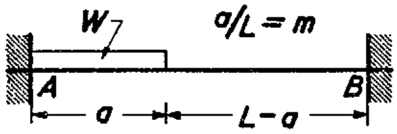
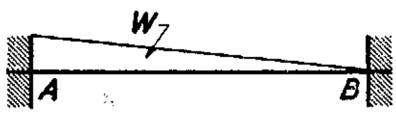
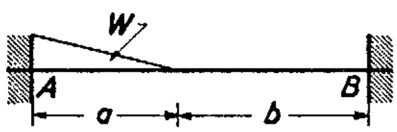
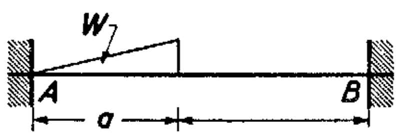
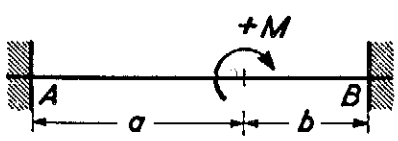
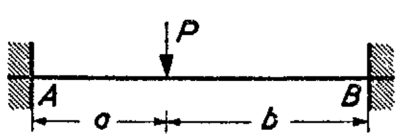
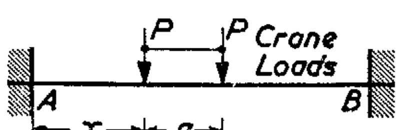
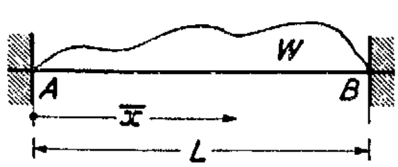
## FIXED-END MOMENTS

| BUILT-IN BEAMS OF CONSTANT CROSS-SECTION | |
|---|---|
| *Symmetrical loadings* | *Values of* $+M_A$ *and* $-M_B$ |
|  | $-\frac{WL}{12}$ |
|  | $-\frac{Wa}{12L}(3L-2a)$ |
|  | $-\frac{W}{24L}(3L^2-b^2)$ |
|  | $-\frac{5WL}{48}$ |
|  | $-\frac{W}{48L}(5L^2+4aL-4a^2)$ |
|  | $-\frac{WL}{16}$ |
|  | $-\frac{Wa}{12L}(2L-a)$ |
|  | $-\frac{Wa}{12L}(4L-3a)$ |
|  | $-\frac{WL}{10}$ |
|  | $-\frac{WL}{20}$ |
|  | $-\frac{PL}{8}$ |
|  | $-\frac{Pa}{L}(L-a)$ |
|  | $-\frac{PL}{12n}(n^2-1)$ |
|  | $-\frac{PL}{24n}(2n^2+1)$ |
|  | $-A_S/L$ Where $A_S$ is the area of the 'free' bending moment diagram |

## FIXED-END MOMENTS

### BUILT-IN BEAMS OF CONSTANT CROSS-SECTION

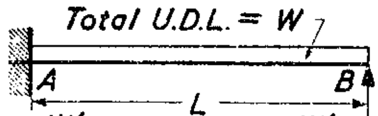
| Asymmetrical loadings | Values of $M_A$ and $M_B$ |
|---|---|
|  | $M_A = -\frac{11}{96}WL$<br>$M_B = +\frac{5}{96}WL$ |
|  | $M_A = -\frac{WL}{12}\cdot m(3m^2-8m+6)$<br>$M_B = +\frac{WL}{12}\cdot m^2(4-3m)$ |
|  | $M_A = -\frac{W}{12L^2b}\left[e^3(4L-3e)-c^3(4L-3c)\right]$<br>$M_B = +\frac{W}{12L^2b}\left[d^3(4L-3d)-a^3(4L-3a)\right]$ |
|  | $M_A = -\frac{WL}{10}$<br>$M_B = +\frac{WL}{15}$ |
|  | $M_A = -\frac{Wa}{30L^2}(3a^2+10bL)$<br>$M_B = +\frac{Wa^2}{30L^2}(5L-3a)$ |
|  | $M_A = -\frac{Wa}{15L^2}(10L^2-15aL+6a^2)$<br>$M_B = +\frac{Wa^2}{10L^2}(5L-4a)$ |
|  | $M_A = +M\frac{b}{L^2}(3a-L)$<br>$M_B = +M\frac{a}{L^2}(3b-L)$ |
|  | $M_A = -\frac{Pab^2}{L^2}$<br>$M_B = +\frac{Pa^2b}{L^2}$ |
|  | When $x = \frac{4L-3a-\sqrt{4L^2-9a^2}}{6}$<br>$M_A\ max = -\frac{P}{54L^2}\left[8L^3+(4L^2-9a^2)^{3/2}\right]$<br>corresponding $M_B = +\frac{P}{54L^2}\left[4L^3+(2L^2+9a^2)\sqrt{4L^2-9a^2}\right]$ |
|  | $M_A = -\left[\frac{4A_S}{L}-\frac{6A_S\bar{x}}{L^2}\right]$<br>$M_B = +\left[\frac{6A_S\bar{x}}{L^2}-\frac{2A_S}{L}\right]$ |

Where $A_S$ is the area of the 'free' B.M. Diagram and $\bar{x}$ is the distance from A to its centroid

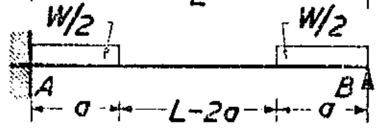
## FIXED-END MOMENTS

### PROPPED CANTILEVERS OF CONSTANT CROSS-SECTION

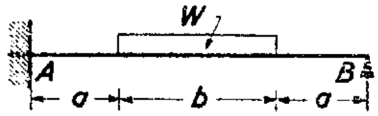
**Symmetrical loadings** — **Values of fixing moment $M_A$**



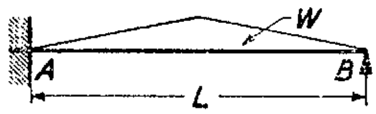

$-\frac{WL}{8}$



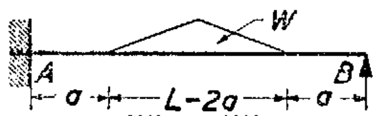

$-\frac{Wa}{8L}(3L-2a)$



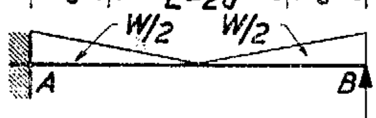

$-\frac{W}{16L}(3L^2-b^2)$



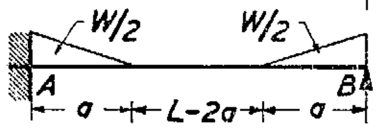

$-\frac{5WL}{32}$



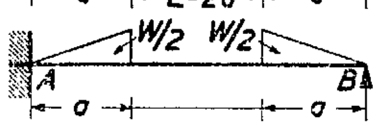

$-\frac{W}{32L}(5L^2+4aL-4a^2)$



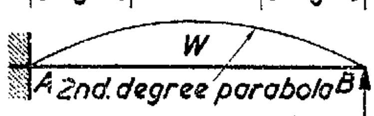

$-\frac{3WL}{32}$



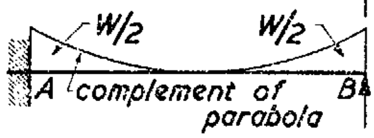

$-\frac{Wa}{8L}(2L-a)$



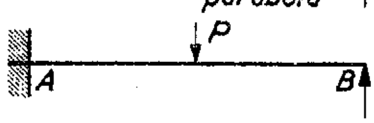

$-\frac{Wa}{8L}(4L-3a)$



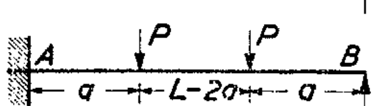

$-\frac{3WL}{20}$



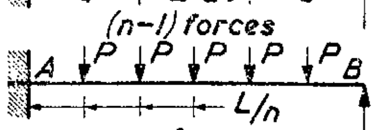

$+\frac{3WL}{40}$



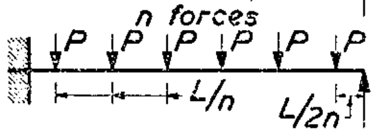

$-\frac{3PL}{16}$




$-\frac{3Pa}{2L}(L-a)$




$-\frac{PL}{8n}(n^2-1)$




$-\frac{PL}{16n}(2n^2+1)$

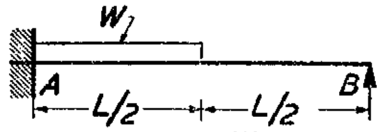

$-3A_S/2L$
Where $A_S$ is the area of the 'free' bending moment diagram

For cantilevers of opposite hand, the fixing moments $M_B$ are of opposite sign.

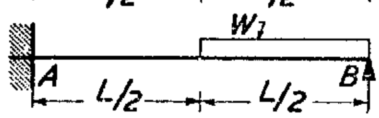
## FIXED-END MOMENTS

### PROPPED CANTILEVERS OF CONSTANT CROSS-SECTION

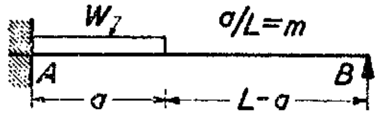
**Asymmetrical loadings — Values of fixing moment $M_A$**



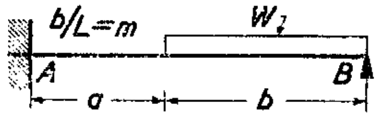

$$-\frac{9}{64}WL$$



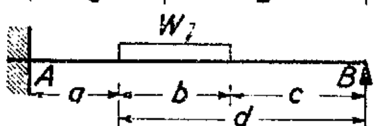

$$-\frac{7}{64}WL$$



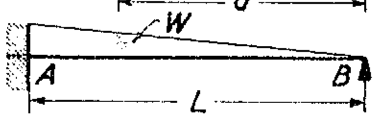

$$-\frac{Wa}{8}(2-m)^2$$



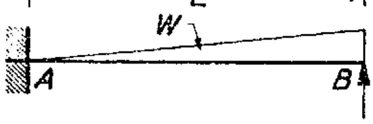

$$-\frac{Wb}{8}(2-m^2)$$



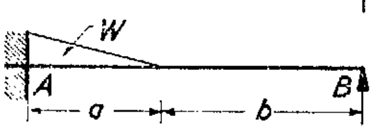

$$-\frac{W}{8L^2b}(d^2-c^2)(2L^2-c^2-d^2)$$



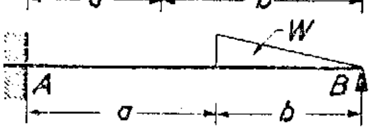

$$-\frac{2}{15}WL$$



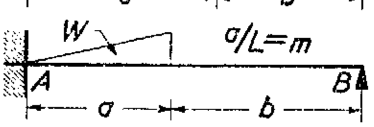

$$-\frac{7}{60}WL$$



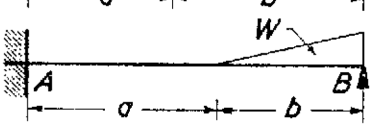

$$-\frac{Wa}{60L^2}(3a^2-15aL+20L^2)$$



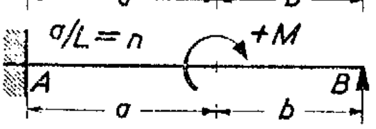

$$-\frac{Wb}{15L^2}(5L^2-3b^2)$$



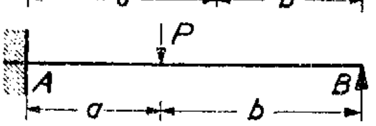

$$-Wa\left(\frac{m^2}{5}-\frac{3m}{4}+\frac{2}{3}\right)$$



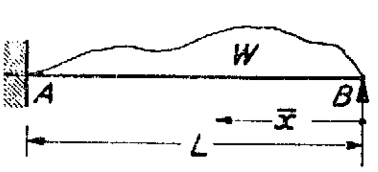

$$-\frac{Wb}{60L^2}(10L^2-3b^2)$$




$$+\frac{M}{2}(2-6n+3n^2)$$



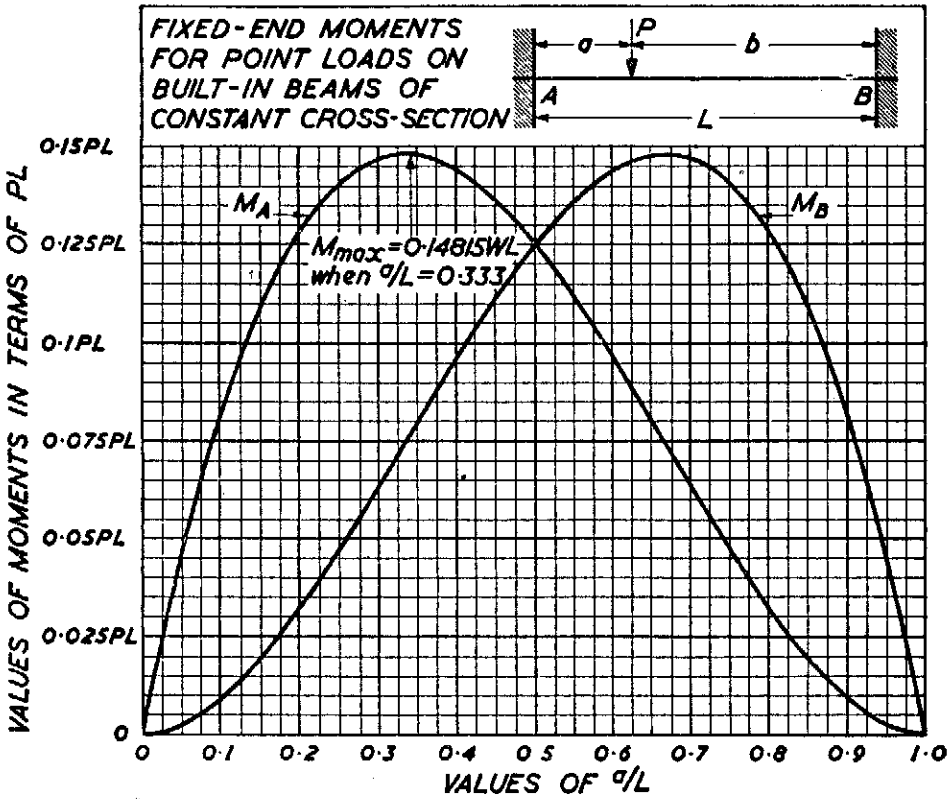

$$-\frac{Pb}{2L^2}(L^2-b^2)$$

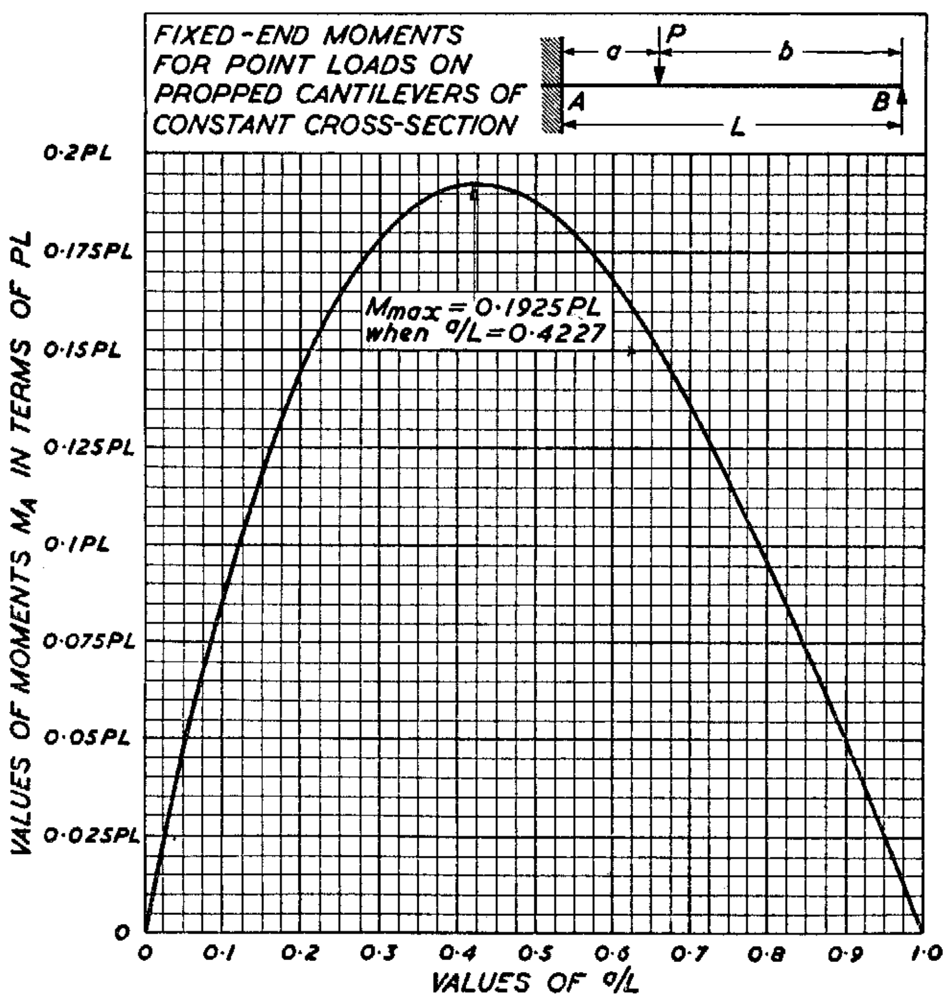



$$-\frac{3A_s\bar{x}}{L^2}$$

*Where $A_S$ is the area of the 'free' B.M. diagram, considering AB as a beam, and $\bar{x}$ is the distance from B to its centroid*

For cantilevers of opposite hand, the fixing moments $M_B$ are of opposite sign.


FIXED-END MOMENTS
FOR POINT LOADS ON
BUILT-IN BEAMS OF
CONSTANT CROSS-SECTION
a
P
b
A
B
L
$M_A$
$M_B$
$M_{max}$=0·14815WL
when a/L=0·333
0·15PL
0·125PL
0·1PL
0·075PL
0·05PL
0·025PL
0
VALUES OF MOMENTS IN TERMS OF PL
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
VALUES OF a/L


FIXED-END MOMENTS
FOR POINT LOADS ON
PROPPED CANTILEVERS OF
CONSTANT CROSS-SECTION
a
P
b
A
B
L
$M_{max}$ = 0·1925 PL
when a/L=0·4227
0·2PL
0·175PL
0·15PL
0·125PL
0·1PL
0·075PL
0·05PL
0·025PL
0
VALUES OF MOMENTS $M_A$ IN TERMS OF PL
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
VALUES OF a/L