DESIGN OF UNBRACED BEAMS

by

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1. INTRODUCTION

Although the AS4100-1990 [1] method of designing unbraced beams against lateral buckling introduces only one new concept to those of the previous working stress method, the presentation of the method is somewhat different, and designers may need assistance in understanding how the new method relates to lateral buckling behaviour, and the various effects that influence it.

The objectives of this paper are: -

- to illustrate the lateral buckling behaviour of unbraced beams,
- to explain the methodology when using AS4100 to design against lateral buckling,
- to provide explanations with illustrative and realistic examples that will assist in the classification of restraint and load height conditions, and
- to demonstrate the use of the AISC Design Capacity Tables [2] for the design of unbraced beams.

For simplicity, the scope of the paper is limited to equal flange I and channel section beams of constant cross-section which are restrained at both ends against lateral deflections and twist rotations. This includes most practical beams that are susceptible to lateral buckling. Subsequent papers will deal with cantilevers and other aspects of designing against lateral buckling.

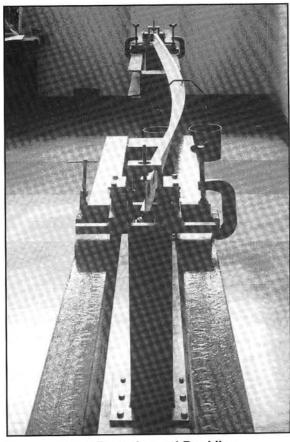


Fig. 1 Beam Lateral Buckling

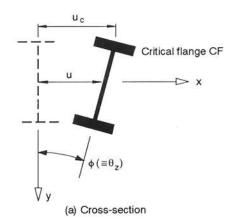
2. LATERAL BUCKLING BEHAVIOUR OF UNBRACED BEAMS

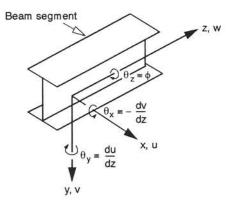
2.1 Elastic Buckling

A beam which is loaded in its major principal plane may suddenly buckle in a flexural-torsional mode out of the plane of loading by deflecting laterally u, rotating laterally du/dz ($\equiv \theta_y$), and twisting ϕ ($\equiv \theta_z$), as shown in Figs 1 and 2. This buckling action, which is often referred to as lateral buckling, can be thought of as an escape from the stiffer plane of the beam, which is permitted by the greater flexibility of the beam in bending in the minor principal plane and torsion about the longitudinal axis.

For a simply supported I or channel section beam in uniform bending, the moment resistance M_0 to elastic lateral buckling is

$$M_0 = \sqrt{\left\{ \left(\frac{\pi^2 E I_y}{L^2} \right) \left(G J + \frac{\pi^2 E I_w}{L^2} \right) \right\}}$$
 (1)





Prebuckling (in-plane) deformations v, w, $\theta_x = -dv/dz$ Buckling (out-of-plane) deformations u, $\theta_y = du/dz$, $\theta_z = \phi$

(b) Deformations

Fig. 2 Cross-Section at Buckling

In this equation, L is the distance between the lateral restraints (against u and ϕ) at the ends of the beam, El_y is the minor axis flexural rigidity, GJ is the uniform torsional rigidity, and El_w is the warping torsional rigidity. In uniform torsion, the shear stresses vary across the thickness and around the cross-section as indicated in Fig. 3a, while in warping torsion the shear stresses are constant across the flange thicknesses as indicated in Fig. 3b and parabolic across their widths. Further information on the torsional behaviour of steel members is given in [3].

Equation 1 shows that the elastic buckling resistance increases with the flexural and torsional rigidities and decreases with the unrestrained length L. Thus lateral buckling can often be prevented by increasing these rigidities (for example by using hollow section members), or by decreasing the distance L between lateral restraints by providing intermediate braces. Beams with closely spaced braces may reach their cross-section moment capacity $M_{\rm S}$ without buckling laterally.

2.2 Capacities of Real Beams

The elastic buckling resistance given by Equation 1 applies to beams which are initially perfectly straight and untwisted, and which remain elastic. Real beams always have some initial crookedness and twist, and have residual stresses induced by the method of manufacture. The initial crookedness and twist cause minor axis bending and torsion to take place before the elastic buckling moment is reached, and the residual stresses cause early yielding. Real beams therefore have reduced strengths as shown in Fig. 4.

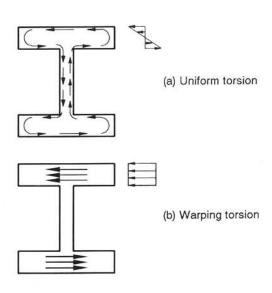


Fig. 3 Torsion Shear Stresses

The lower bound approximation used in AS4100 for these reduced uniform bending strengths is given by

$$M_{\rm bu} = \alpha_{\rm S} M_{\rm S} \tag{2}$$

in which

$$\alpha_{s} = 0.6 \left\{ \sqrt{\left(\left(M_{s} / M_{o} \right)^{2} + 3 \right) - M_{s} / M_{o} \right\}} \le 1$$
 (3)

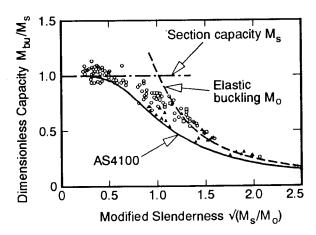


Fig. 4 Nominal Moment Capacities in Uniform Bending

2.3 Effect of the Bending Moment Distribution

Uniform bending rarely occurs in real beams, which usually have concentrated or distributed loads and varying bending moment diagrams. Uniform bending is the most severe case, and so AS4100 allows increased member capacities $M_{\rm b}$ to be estimated from

$$M_{\rm b} = \alpha_{\rm m} \alpha_{\rm s} M_{\rm s} \le M_{\rm s} \tag{4}$$

in which α_m is a moment modification factor. The value of α_m may be conservatively and most simply taken as 1.0. A more accurate and economical approximation may be obtained from the bending moment distribution by using

$$\alpha_{\rm m} = \frac{1.7 \ M_{\rm m}^{\star}}{\sqrt{\left[\left(M_{\rm o}^{\star}\right)^2 + \left(M_{\rm d}^{\star}\right)^2 + \left(M_{\rm d}^{\star}\right)^2\right]}} \le 2.5 \tag{5}$$

in which M_m^* is the maximum moment and M_2^* , M_3^* , and M_4^* are the moments at the quarter, mid, and three-quarter points, as shown in Fig. 5. Alternatively, α_m may be approximated even more accurately but less generally by using Table 5.6.1 of AS4100. The values for α_m are high for beams with small moments near midspan, and lower for beams with large moments near midspan, with the lowest value of $\alpha_m = 1.0$ being for uniform bending.

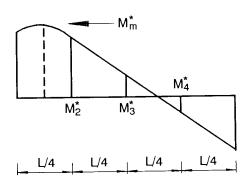


Fig. 5 Moment Distribution

2.4 Effective Length

The effects of cross-section distortion, load height, and rotational end restraint may vary the moment resistance to buckling M_0 . AS4100 allows for these effects approximately by using an effective length

$$L_{\mathsf{P}} = k_{\mathsf{L}} \, k_{\mathsf{L}} \, k_{\mathsf{L}} \, L \tag{6}$$

in place of the distance L between braces or lateral restraints. Descriptions of the factors $k_{\rm t}$, $k_{\rm l}$, and $k_{\rm r}$ are given in the following sub-sections.

2.4.1 Effect of cross-section distortion

Beams with deep thin webs may distort during lateral buckling as indicated in Fig. 6. This distortion increases the twist rotations φ, and lowers the effective torsional resistance, and thereby the resistance to lateral buckling. This effect is accentuated in beams which are only partially restrained against twist rotations at the supports or braces as shown in Fig. 6.

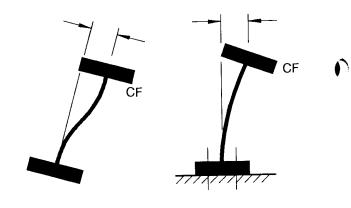


Fig. 6 Distortions of Thin Webs

The twist restraint effective length factor k_1 of Equation 6 depends on the dimensionless web flexibility $(d_1/L) (t_1/2t_w)^3$. This factor is greatest in deep, short span beams with thin webs.

2.4.2 Effect of load height

In some cases, beams may have gravity loads which act at the top flange, and which move with the flange during lateral buckling, as shown in Fig. 7. These loads induce additional torques about the beam axis, which increase the twist rotations ϕ and lower the resistance to lateral buckling. AS4100 allows for this effect approximately by using a load height factor k_{\parallel} to increase the effective length $L_{\rm e}$ given by Equation 6.

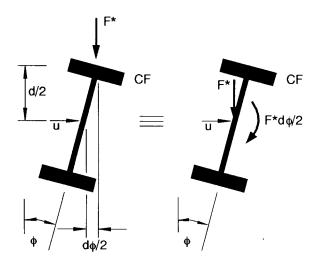


Fig. 7 Effect of Load Height

2.4.3 Effect of rotational end restraint

When a continuous beam buckles laterally, the buckling deflections u and twist rotations ϕ of the most critically loaded span may be reduced by lateral rotation end restraints provided by the flexural stiffnesses of the adjacent spans. These restraints are associated with the lateral end rotations du/dz ($\equiv \theta_y$ in Fig. 2). They are the beam equivalents of the end restraints which increase the flexural buckling loads of columns in rigid-jointed frames.

The restraining effects of adjacent spans are often less than might be expected, especially when those adjacent spans are subject to lateral buckling. When there are dependable end restraints, then AS4100 allows advantage to be taken of this by using a lateral rotational restraint factor $k_{\rm r}$ to reduce the effective length given by Equation 6.

2.5 Effect of Continuous Lateral Restraints

When a beam supports a plate type structure such as a floor slab, then this may provide continuous lateral restraints which will prevent lateral buckling. Clause 5.3.2.2 of AS4100 allows beams which are fully (F) or partially (P) restrained at both ends (see Section 4.1 below) to be treated as if they have full lateral restraint if they have continuous lateral restraints at the **critical flange** (CF). The critical flange for beams with both

ends restrained F or P is the compression flange.

When a beam has closely spaced restraints which prevent lateral deflection of the critical flange (CF), then these prevent lateral buckling and the beam can be considered to have full lateral restraint. Clauses 5.3.2.3 and 5.3.2.4 of AS4100 provide approximations for the maximum spacing of restraints which can be considered to prevent lateral buckling.

3. DESIGN METHODOLOGY

The AS4100 methodology for designing against lateral buckling is summarised in Fig. 8. First the in-plane analysis of the beam under the factored design loads is carried out to determine the distribution of the bending moment $M_{\rm x}^*$, and the maximum design moment $M_{\rm m}^*$.

The design section capacity $\phi M_{\rm S}$ is then evaluated using

$$M_{\rm S} = f_{\rm V} Z_{\rm e} \tag{7}$$

in which Z_e is the effective section modulus (Clause 5.2 of AS4100) and ϕ is the capacity factor, and used in

$$M_{\rm m}^{\star} \le \phi M_{\rm S}$$
 (8)

for the section capacity check.

The cross-sections at supports and braces are then classified as being fully restrained (F), partially restrained (P), laterally restrained (L), or unrestrained (U) against lateral deflection u and twist rotation ϕ (see Section 4.1).

The member can then be divided into segments (whose end sections are either F or P) and sub-segments (a sub-segment has one end section L, and the other F, P, or L). Lateral buckling must be considered for each segment and sub-segment. Only one of these will control the design, and often this will be easily identified because of its long length or high moment.

For each segment and sub-segment checked, the effective length $L_{\rm e}$ is determined from Equation 6, the reference moment $M_{\rm o}$ from Equation 1, and the slenderness reduction factor $\alpha_{\rm s}$ from Equation 3. The moment modification factor $\alpha_{\rm m}$ is then determined from Equation 5, or Table 5.6.1 of AS4100, or conservatively taken as 1.0, and the nominal moment capacity $M_{\rm b}$ is determined from Equation 4. This is then used in the member capacity check

$$M_{\rm m}^{\star} \le \phi M_{\rm b} \tag{9}$$

in which $M_{\rm m}^{\star}$ is the maximum moment in the segment or sub-segment under consideration. This process is greatly simplified in manual design by using the Design Capacity Tables [2] to evaluate $\phi \alpha_{\rm s} M_{\rm s}$.

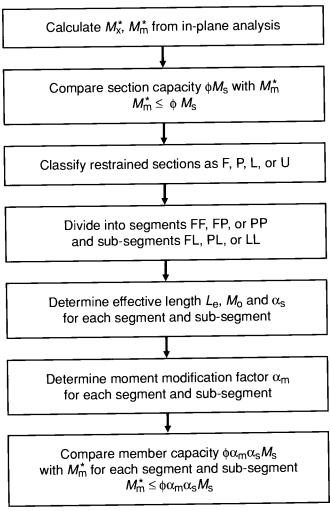


Fig. 8 Design Methodology

4. DETERMINATION OF EFFECTIVE LENGTH

4.1 Restraints at Cross-Sections

4.1.1 General

The conditions of restraint at a cross-section are classified in AS4100 as being either fully restrained (F), partially restrained (P), laterally restrained (L), or unrestrained (U). These classifications are unaffected by the conditions of lateral rotation restraint (against $du/dz \equiv \theta_y$ in Fig. 2), which are discussed in Section 4.3.4 below. The restraint conditions for each of these classifications are shown in Table 1, and examples are given in Figs A1-A13 in Appendix 1. In some cases, some engineering judgement may need to be exercised by the designer. Where the examples are of this type, then a conservative approach has been used in Figs A1-A13. The conditions of restraint are discussed in Sections 4.1.2 to 4.1.5 below.

4.1.2 Fully restrained (F) cross-sections

Fully restrained (F) cross-sections are **effectively** prevented from deflecting laterally ($u\approx 0$) and from twisting ($\phi\approx 0$). Some engineering judgement may need to be exercised by the designer, since the limiting theoretical case of rigid restraint ($u=0, \phi=0$) never occurs in practice. Examples of fully restrained cross-sections are given in Fig. 5.4.2.1 of AS4100, and in Figs A1-A3, A5-A11.

The degree of torsional restraint required depends on where the restraint against lateral deflection u acts. When this restraint acts **at the critical flange** (CF), which is the compression flange for beams with both ends fully (F) or partially (P) restrained (Clause 5.5.2 of AS4100), then less than full restraint against twisting is required to ensure that $\phi \approx 0$ (Fig. 5.4.2.1b of AS4100, and Connection Details 1, 5, 6, 11, 23, 32, 34 in Figs A1-A3, A7, A10, and A11).

When the restraint against lateral deflection u acts away from the critical flange (CF), then **effective** restraint against twisting is required to ensure that $\phi \approx 0$ (Fig 5.4.2.1c of AS4100, and Connection Details 3, 8, 12, 18, 20, 22, 25, and 27 in Figs A1-A3, A5-A9). Generally, effective torsional restraint is provided when all the elements between the brace support and the critical flange are stiff.

4.1.3 Partially restrained (P) cross-sections

Partially restrained (P) cross-sections are effectively prevented from deflecting laterally ($u \approx 0$), but only **partially** prevented from twisting ϕ . Again, some engineering judgement may need to be exercised by the designer.

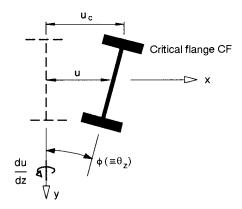
Examples of partially restrained cross-sections are given in Fig. 5.4.2.2 of AS4100 and in Figs A1-A8, A10, A11. Generally, these have only one flexible element (web, connection, or brace) between the brace support and the critical flange (CF) which allows some twist rotation.

4.1.4 Laterally restrained (L) cross-sections

Laterally restrained (L) cross-sections are effectively prevented from deflecting laterally at the critical flange CF ($u_{\rm c}\approx 0$), but are unrestrained against twisting. Despite this lack of restraint against twist rotation, laterally restrained intermediate cross-sections in beams which are fully or partially restrained at both ends act effectively as if fully restrained.

Examples of laterally restrained cross-sections are given in Fig. 5.4.2.4 of AS4100 and in Figs A6, A9-A11. These include examples for which there is more than one flexible element (web, connection, or brace) between the brace support and the critical flange (CF).

Table 1. Restraint at Cross-Sections



eF - effectively Fixed
Pr - Partially restrained
Ur - Unrestrained

R* - Restrained against lateral rotation

4.1.5 Unrestrained (U) cross-sections

If a cross-section does not satisfy the conditions for fully restrained (F), partially restrained (P), or laterally restrained (L), then it must be classified as unrestrained (U). Cross-sections which have no restraints are unrestrained.

Cross-sections which are not effectively prevented from deflecting laterally u are treated as unrestrained in AS4100, no matter how effective the restraint against twisting may be, as shown in Connection Details 36 and 38 in Figs A12 and A13.

Cross-sections which are effectively prevented from deflecting laterally at points away from their critical flanges (CF) and which do not have partial or effective restraints against twisting are also treated as unrestrained, as shown in Connection Details 19, 26, 28, 31, and 33 in Figs A6, A8-A11.

In some cases these definitions of unrestrained crosssections are conservative. More accurate account can be taken of the restraint conditions by using the method of design by buckling analysis of Clause 5.6.4 of AS4100, but only a few designers are likely to take advantage of this option.

4.2 Segments and Sub-Segment Lengths

When the cross-sections have been classified as fully (F), partially (P), or laterally (L) restrained or as unrestrained (U), the beam may be divided into segments and sub-segments. While each of these must have sufficient capacity against lateral buckling, only one will control design, and often this will be easily identified because of its long length or high moment.

Cross Section	Buckling Deformation							
Classific- ation	Deflection u	Flange Deflection u_c Rotation $\phi \equiv \theta_z$		Rotation du / dz				
F	eF	_	eF	Ur				
	_	eF	eF	Ur Ur				
	_	eF	Pr					
F,R*	A	As above for F						
Р	eF	_	Pr	Ur				
P, R*	eF	_	Pr	Pr				
L	-	eF	Ur	Ur				
	_	eF	Ur	Pr				
υ	eF	-	Ur	Ur, Pr				
	Ur	_	eF	Ur, Pr				
	Ur	_	Pr	Ur, Pr				
	Ur	_	Ur	Ur, Pr				

A segment (FF, FP, or PP) is a length between crosssections which are fully (F) or partially (P) restrained.

A segment may be divided into sub-segments. Each sub-segment (FL, PL, or LL) has one end laterally (L) restrained, and the other end fully (F), partially (P) or laterally (L) restrained.

Although their definitions are slightly different, there are no differences in the lateral buckling design of segments and sub-segments, provided the beams of which they form parts are fully (F) or partially (P) restrained at **both** ends.

4.3 Effective Lengths

4.3.1 General

The effective length $L_{\rm e}$ of a segment or sub-segment of a beam is calculated from its length L by using Eq. 6 and the twist restraint factor $k_{\rm t}$, the load height factor $k_{\rm l}$, and the lateral rotation restraint factor $k_{\rm r}$ discussed below.

4.3.2 Twist restraint factor k_1

The twist restraint factor k_t given in Table 5.6.3(1) of AS4100 depends on the geometry of the cross-section, the length L, and the restraint conditions at the segment or sub-segment ends.

The twist restraint factor k_t is unity when both ends are fully (F) or laterally (L) restrained.

The twist restraint factor $k_{\rm t}$ is increased when one or both ends are partially (P) restrained. The increase depends on the ratio of $d_1(t_{\rm f}/2t_{\rm w})^3$ to the length L. Values of $d_1(t_{\rm f}/2t_{\rm w})^3$ for the BHP sections and RHS and SHS are given in Tables 2 and 3. The increases are usually quite small for a large range of practical beams, and only become substantial for short length beams with deep, thin webs.

4.3.3 Load height factor k_1

The load height factors k_l of Table 5.6.3(2) of AS4100 are shown in Fig. 9.

The factor k_{\parallel} is equal to 1.0 when the load acts at the shear centre of the beam.

The factor $k_{\rm I}$ is also equal to 1.0 if the load acts at a segment or sub-segment end which is fully (F), partially (P) or laterally (L) restrained, regardless of the load height. This is because the restraint at this restrained cross-section effectively prevents the twist rotation which leads to the additional torques shown in Fig. 7.

The load height factor k_{\parallel} is greater than 1.0 for beams which are fully (F) or partially (P) restrained at both ends when the load is a gravity load acting above the shear centre and at a position within a segment or sub-segment length. When this is the case, then $k_{\parallel} = 1.4$ for top flange loading.

Load Position	End Restraints	Shear Centre	Top Flange		
_Within	FF, FP, FL PP, PL, LL	1.0	1.4		
Segment	FU, PU	1.0	2.0		
At	FF, FP, FL PP, PL, LL	1.0	1.0		
Segment End	FU, PU	1.0	2.0		

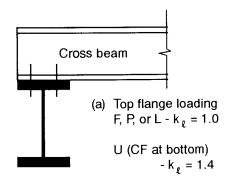
 $imes \equiv$ full or partial restraint

Examples of top flange loading include crane runway girders, and beams which are loaded by cross beams or purlins connected to the top flange, as shown in Fig. 10a.

Fig.9 Load Height Factors k1

A cross beam often braces the top flange against lateral deflection so that $k_{\parallel}=1.0$. However, there is one special case of a beam whose top flange is braced against lateral deflection at the load point which requires the use of $k_{\parallel}=1.4$. This is the case for which the top flange is not the critical flange (CF), so that the cross-section at the load point must be classified as unrestrained (U). An additional torque is induced by top flange loading because the cross-section twists during buckling by rotating about the top flange, as shown in Fig. 11.

The load height factors $k_{\rm l}$ of AS4100 for gravity loads may also be used for uplift loading, provided that the wording "top flange" is replaced by "bottom flange". Thus the load height factor for uplift loading is only greater than 1.0 when the load acts at the actual bottom flange.



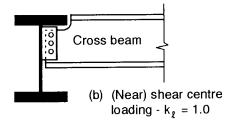


Fig. 10 Loading by Cross Beams

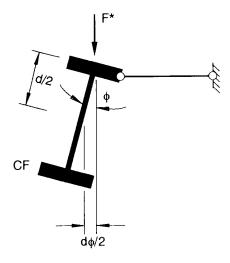


Fig. 11 Load Height Effect at Braced Load Point

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4.3.4 Lateral rotation restraint factor k_r

The lateral rotation restraint factors $k_{\rm r}$ of Table 5.6.3(3) of AS4100 are equal to 1.0 if neither segment end (F or P) is restrained against lateral rotation du/dz ($\equiv \theta_{\rm y}$), 0.85 if one end is restrained, and 0.70 if both ends are restrained.

Generally, $k_{\rm f}=1.0$ should be used because lateral rotation restraints are often not dependable, as for example when there are not adequate moment connections to the restraining element, or when the restraining element is of low stiffness, as is the case for the torsional stiffnesses of I-section columns. Thus none of the connection details of Figs A1-A11 are assumed to restrain lateral rotation.

Even in cases with adequate moment connections and seemingly adequate restraint stiffnesses, the actual stiffness of the restraining element may be greatly

Table 2. Values of $d_1(t_f/2t_w)^3$ – (mm) (Note: $a = d_1(t_f/2t_w)^3$ in this table)

Welded Bear	ns	Universal Be	ams	Welded Colu	Welded Columns Universal Colum		
Section	а	Section	а	Section	а	Section	а
1200 WB 455	2188	760 UB 244	383	500 WC 440	50.0	310 UC 283	153
423	1595	220	387	414	97.7	240	152
392	1120	197	388	383	71.2	198	151
342	1120	173	310	340	118	158	140
317	750	148	224	290	154	137	135
278	534	690 UB 140	290	267	110	118	134
249	534	125	214	228	56.3	96.8	130
1000 WB 322	960	610 UB 125	320	400 WC 361	43.8	250 UC 89.5	126
296	643	113	264	328	128	72.9	127
258	458	101	195	303	93.0	200 UC 59.5	80.5
215	234	530 UB 92.4	224	270	91.8	52.2	86.3
900 WB 282	2039	82.0	163	212	85.4	46.2	77.4
257	1366	460 UB 82.1	226	181	43.8	150 UC 37.2	49.7
218	972	74.6	216	144	43.8	30.0	50.2
175	498	67.1	178	350 WC 280	100	23.4	24.1
800 WB 192	2085	410 UB 59.7	210	258	73.1	100 UC 14.8	28.5
168	1484	53.7	140	230	72.1		
146	760	360 UB 56.7	179	197	94.3		
122	389	50.7	163			-	
700 WB 173	1811	44.7	117				
150	1289	310 UB 46.2	193				
130	660	40.4	165				
115	338	250 UB 37.3	144				
		31.4	82.0				
		200 UB 29.8	83.1				
		25.4	57.2				
		180 UB 22.2	92.0				
		18.1	81.4				
		150 UB 18.0	67.5				
		14.0	46.6				

Universal Bearing Piles		Parallel F Chann	-	Taper Flange Channels		Taper Flange Beams	
Section	а	Section	а	Section	а	Section	а
310 UBP 78.8	34.6	380 PFC	231	125 TFC	32.3	125 TFB	66.3
250 UBP 84.9	28.3	300 PFC	268	100 TFC	35.9	100 TFB	37.1
62.6	28.3	250 PFC	181	75 TFC	17.0		
200 UBP 146	47.4	230 PFC	162			-	
122	22.5	200 PFC	176				
		180 PFC	122				
		150 PFC	65.0				

Table 3. Values of $d_1(t_f/2t_w)^3$ - (mm) (Note: $a = d_1(t_f/2t_w)^3$ in this table)

Rectangular Hollow Sections Square Hollow Sections							
Section Section		Section Section	а				
	a						
250 x 150 x 9.0 RHS	29.0	250 x 250 x 9.0 SHS	29.0				
6.0 RHS	29.8	6.0 SHS	29.8				
5.0 RHS	30.0	200 x 200 x 9.0 SHS	22.8				
200 x 100 x 9.0 RHS	22.8	6.0 SHS	23.5				
6.0 RHS	23.5	5.0 SHS	23.8				
5.0 RHS	23.8	150 x 150 x 9.0 SHS	16.5				
4.0 RHS	24.0	6.0 SHS	17.3				
150 x 100 x 6.0 RHS	17.3	5.0 SHS	17.5				
5.0 RHS	17.5	125 x 125 x 9.0 SHS	13.4				
4.0 RHS	17.8	6.0 SHS	14.1				
150 x 50 x 5.0 RHS	17.5	5.0 SHS	14.4				
4.0 RHS	17.8	4.0 SHS	14.6				
3.0 RHS	18.0	100 x 100 x 9.0 SHS	10.3				
125 x 75 x 6.0 RHS	14.1	6.0 SHS	11.0				
5.0 RHS	14.4	5.0 SHS	11.3				
4.0 RHS	14.6	4.0 SHS	11.5				
3.8 RHS	14.7	3.8 SHS	11.6				
3.3 RHS	14.8	3.3 SHS	11.7				
3.0 RHS	14.9	3.0 SHS	11.8				
100 x 50 x 6.0 RHS	11.0	89 x 89 x 6.0 SHS	9.63				
5.0 RHS	11.3	5.0 SHS	9.88				
4.0 RHS	11.5	3.5 SHS	10.3				
3.5 RHS	11.6	75 x 75 x 6.0 SHS	7.88				
3.3 RHS	11.7	5.0 SHS	8.13				
3.0 RHS	11.8	4.0 SHS	8.38				
75 x 50 x 6.0 RHS	7.88	3.5 SHS	8.50				
5.0 RHS	8.13	3.3 SHS 3.0 SHS	8.55				
4.0 RHS	8.38	0.0 0.10	8.63				
3.0 RHS	8.63	65 x 65 x 6.0 SHS	6.63				
65 x 35 x 3.0 RHS	7.38	5.0 SHS	6.88				
50 x 25 x 3.0 RHS	5.50	4.0 SHS	7.13				
50 x 20 x 3.0 RHS	5.50	3.0 SHS	7.38				
		50 x 50 x 5.0 SHS	5.00				
		4.0 SHS	5.25				
		3.0 SHS	5.50				
		40 x 40 x 4.0 SHS	4.00				
		3.0 SHS	4.25				
		35 x 35 x 3.0 SHS	3.63				
		30 x 30 x 3.0 SHS	3.00				
		25 x 25 x 3.0 SHS	2.38				

reduced by its own loading. For example, for the braced beam shown in Fig. 12, the left hand segment cannot receive any lateral rotation restraint from the right hand segment because the right segment buckles simultaneously but in the opposite direction.

When the lateral rotation restraints are dependable, as in the case where the restraining element has full lateral support so that it will not buckle laterally, then a reduced value for $k_{\rm f}$ of 0.85 or 0.70 may be used.

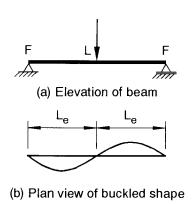


Fig. 12 Zero Lateral Rotation Restraint

5 EXAMPLES

5.1 General

Worked examples of the determination of the effective length are given in Problems 5.11 and 5.12 of [8].

Five additional examples are shown in Fig. 13. In these, the circled numbers indicate the Connection Detail number of Figs A1-A3, A6, A7, while in each case the beam is a 610UB125 of Grade 250 steel, and the load acts at the top flange, as indicated by the circled T. The design values of the loads and moments which are shown include allowances for self weight.

The solution of Example 1 is shown in Table 4 and discussed in Section 5.2 below. Table 4 has blank spaces for writing in the values of the various quantities used in Examples 2–5.

5.2 Example 1

The maximum moment in the beam of Example 1 is $M_{\rm m}^{\star}$ = 600 kNm, and this is less than the design section capacity $\phi M_{\rm S}$ = 828 kNm, as shown in Table 4.

The beam is restrained only at its ends A and C and is unrestrained (U) at the load point B, so that the beam has only one segment (AC). The left hand end has the bearing pad connection with a full depth end plate shown as Connection Detail 3 of Fig. A1, which is classified as fully restrained (F). The right hand end has the

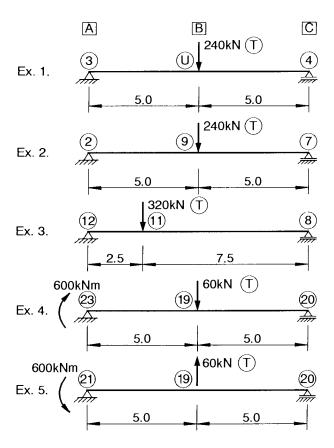


Fig. 13 Examples 1-5

Table 3.3-5

DESIGN MOMENT CAPACITIES FOR

MEMBERS WITHOUT FULL LATERAL RESTRAINT

SUBJECT TO BENDING about x-axis

UNIVERSAL BEAMS – GRADE 250

Designation	Design Moment Capacities ϕM_b (kNm) for Effective Length in metres						
	2	2 14 14.45 16					
690UB140	989	243	1	208			
125	863	194	\	164			
610UB125			187	166			
113	701	158		136			
101	612	125		107			

Fig. 14 Design Capacity Table

partial depth end plate connection shown as Connection Detail 4 of Fig. A1, which is classified as partially restrained (P).

The value of $d_1(t_f/2t_w)^3 = 320$ mm for the 610UB125 given in Table 2 allows the value of $k_t = 1.032$ to be

Table 4. Table for Solution of Examples

Beam	1		2	3	4	5
<i>M</i> _m (kNm)	600					
Section f_y (MPa) Z_e (mm ³) ϕM_s (kNm)	610UB129 250 3680 x 10 828					
Segment or Sub-segment	AC	-				
End restraints	FP	_				
L (m) kt kι kr Le (m) (Mo) (kNm) (αs)	10.0 1.032 1.4 1.0 14.45 (241) (0.225)					
α_{m}	1.35	_				
$\phi \alpha_{\rm m} \alpha_{\rm s} M_{\rm s} ({\rm kNm})$	251	_		2		
<i>M</i> [⋆] _m (kNm)	600	-				

calculated. The load acts at the top flange at the point B which is within the segment length, so that $k_{\rm I}=1.4$. There are no restraints against lateral rotation at the ends A and C (see also Connection Details 3 and 4 of Fig. A1), so that $k_{\rm F}=1.0$. Substituting these values into Eq. 6 leads to $L_{\rm e}=14.45$ m.

The values of $M_{\rm O}$ and $\alpha_{\rm S}$ can be obtained by substitution into Eqs 1 and 3. However, these calculations can be avoided by using Table 3.3-5 of the AISC Design Capacity Tables [2], part of which is reproduced in Fig. 14. Interpolating in this table for $L_{\rm e}=14.45$ m leads to $\phi\alpha_{\rm S}M_{\rm S}=187$ kNm approximately, so that $\phi\alpha_{\rm m}\alpha_{\rm S}M_{\rm S}=1.35$ x 187=252 kNm approximately, which is less than the design moment $M_{\rm m}^{\star}=600$ kNm. Thus the 610UB125 is inadequate for this example.

Note that further design checks must be made such as for shear, combined shear and bending, serviceability, etc., but these are outside the scope of this paper.

6. CONCLUSIONS

This paper explains the methodology when using AS4100 to design unbraced beams against lateral buckling. The beams considered are equal flange I and channel section beams of constant cross-section which are supported and restrained against lateral deflections and twist rotations at both ends.

The lateral buckling behaviour of beams is discussed, including elastic buckling, the strengths of real beams, and the effects of the bending moment distribution, cross-section distortion, load height, rotational end restraints, and continuous lateral restraints.

The determination of the effective length is summarised. Practical examples are given for the classification of cross-section restraint, the division of a beam into segments and sub-segments is discussed, and the determination of the effective length factors is explained.

A series of examples are presented, and the solution of one of these is given in tabular form. The use of the AISC Design Capacity Tables is demonstrated for this example.

7. REFERENCES

- [1] Standards Australia, *AS4100-1990 Steel Structures*, Standards Australia, Sydney, 1990.
- [2] AISC, *Design Capacity Tables for Structural Steel*, Australian Institute of Steel Construction, Sydney, 1991.
- [3] Trahair, NS and Bradford, MA, *The Behaviour and Design of Steel Structures*, Revised Second Edition, Chapman and Hall, London, 1991.
- [4] Trahair, NS, *Flexural-Torsional Buckling of Structures*, E and FN Spon, London, 1993.
- [5] BHP, *Hot-Rolled and Structural Products*, 1991 Edition, BHP Steel, Melbourne, 1991.
- [6] AISC, *Design Capacity Tables for Structural Steel Hollow Sections*, Australian Institute of Steel Construction, Sydney, 1992.
- [7] Nethercot, DA and Lawson, RM, *Lateral Stability of Steel Beams and Columns Common Cases of Restraint*, Steel Construction Institute, Ascot, 1992.
- [8] Bradford, MA, Bridge, RQ, and Trahair, NS, *Worked Examples for Steel Structures*, 2nd edition, Australian Institute of Steel Construction, Sydney, 1992.

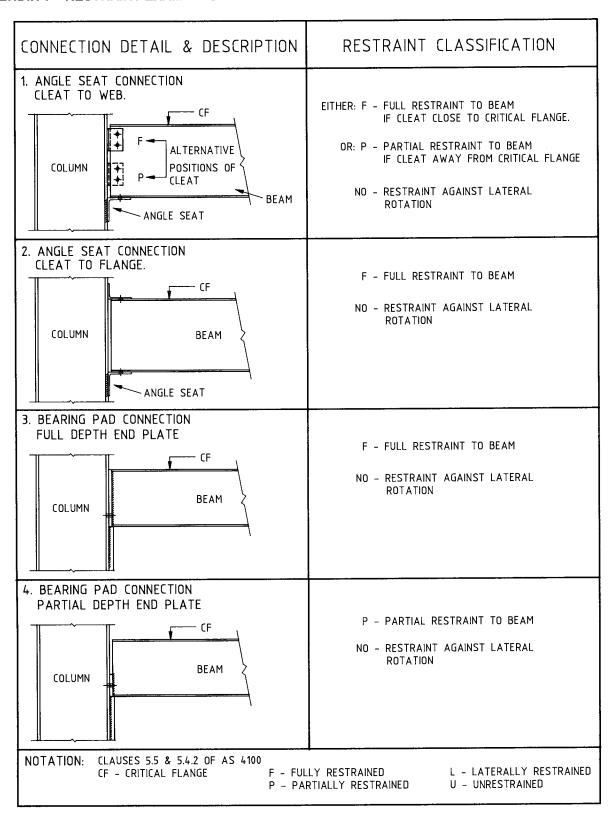


Fig. A1 Restraint at Beam - Column Connections

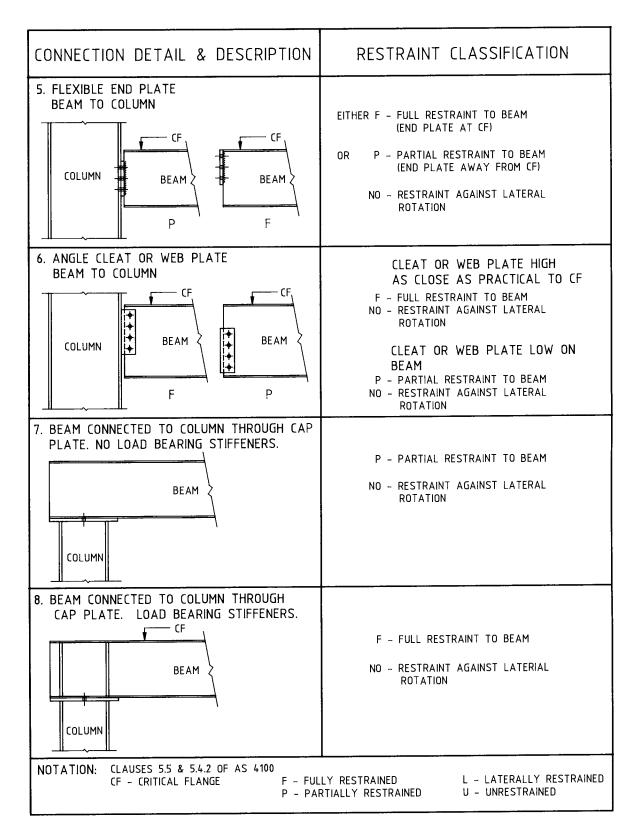


Fig. A2 Restraint at Beam – Column Connections

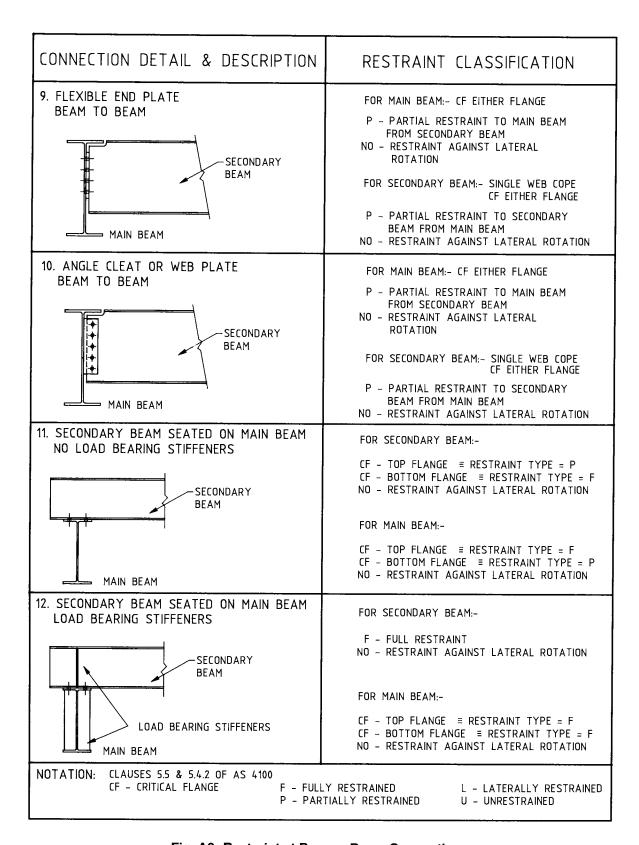


Fig. A3 Restraint at Beam – Beam Connections

CONNECTION DETAIL & DESCRIPTION	RESTRAINT CLASSIFICATION
13. FLEXIBLE END PLATE DOUBLE WEB COPE - SMALL COPES	FOR MAIN BEAM:- CF EITHER FLANGE P - PARTIAL RESTRAINT TO MAIN BEAM FROM SECONOARY BEAM
SECONOARY BEAM	NO - RESTRAINT AGAINST LATERAL ROTATION FOR SECONOARY BEAM:- CF EITHER FLANGE P - PARTIAL RESTRAINT TO SECONDARY BEAM FROM MAIN BEAM NO - RESTRAINT AGAINST LATERAL ROTATION
14. FLEXIBLE END PLATE DOUBLE WEB COPE - LARGE COPE AT BOTTOM	FOR MAIN BEAM:- AS CASE 13 ABOVE
SECONO ARY BEAM	FOR SECONDARY BEAM:- CF - TOP FLANGE = RESTRAINT TYPE = P CF BOTTOM FLANGE = RESTRAINT TYPE = U NO - RESTRAINT AGAINST LATERAL ROTATION
15. ANGLE CLEAT OR WEB PLATE DOUBLE WEB COPE - SMALL COPES	FOR MAIN BEAM:- CF EITHER FLANGE P - PARTIAL RESTRAINT TO MAIN BEAM FROM SECONDARY BEAM
SECONOARY BEAM MAIN BEAM	NO - RESTRAINT AGAINST LATERAL ROTATION FOR SECONOARY BEAM:- CF EITHER FLANGE P - PARTIAL RESTRAINT TO SECONOARY BEAM FROM MAIN BEAM NO - RESTRAINT AGAINST LATERAL ROTATION
16. ANGLE CLEAT OR WEB PLATE DOUBLE WEB COPE - LARGE COPE AT BOTTOM	FOR MAIN BEAM:- AS CASE 13 ABOVE
MAIN BEAM	FOR SECONOARY BEAM:- CF - TOP FLANGE = RESTRAINT TYPE = P CF - BOTTOM FLANGE = RESTRAINT TYPE = U NO - RESTRAINT AGAINST LATERAL ROTATION
	LLY RESTRAINEO L - LATERALLY RESTRAINED RTIALLY RESTRAINEO U - UNRESTRAINED

Fig. A4 Restraint at Beam – Beam Connections

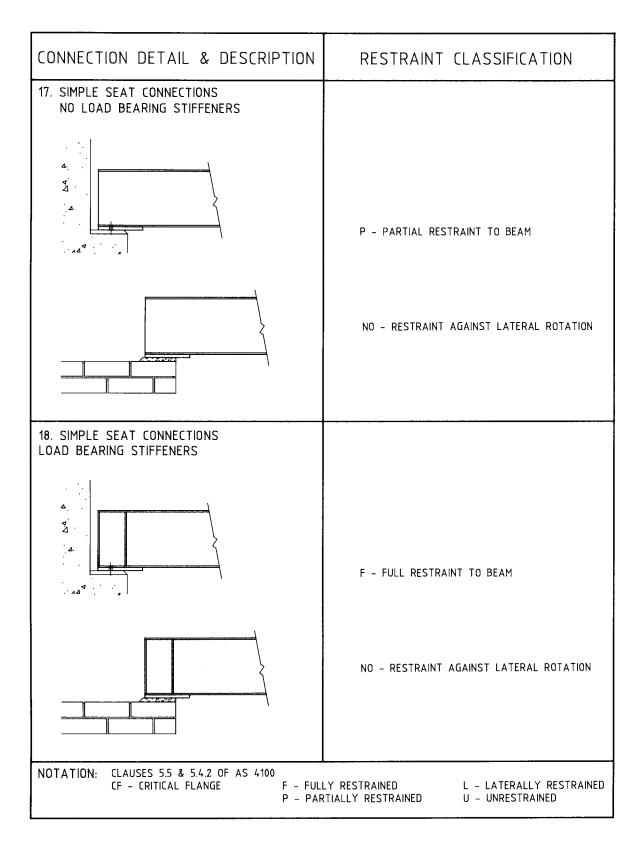


Fig. A5 Restraint at Beam Seats

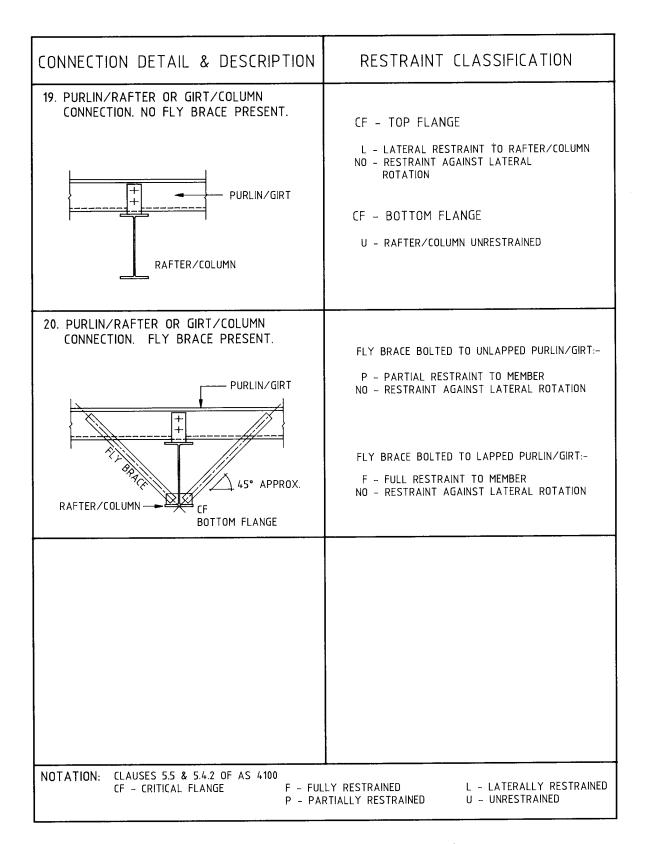


Fig. A6 Restraint at Purlin and Girt Connections

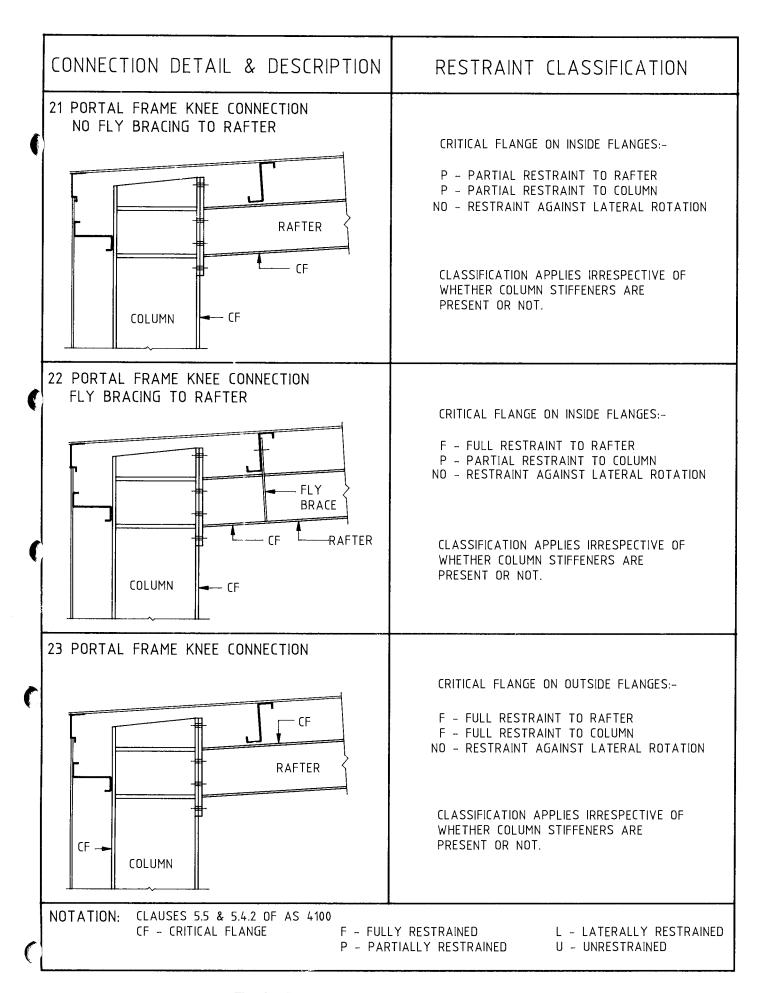


Fig. A7 Restraint at Knee Joints in Portal Frames

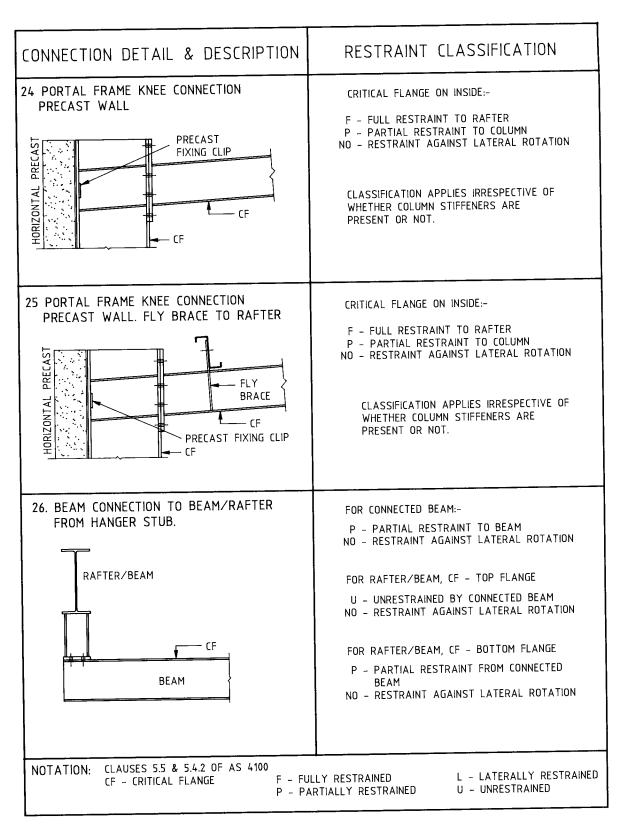


Fig. A8 Restraint at Knee Joints in Portal Frames and at Hanger

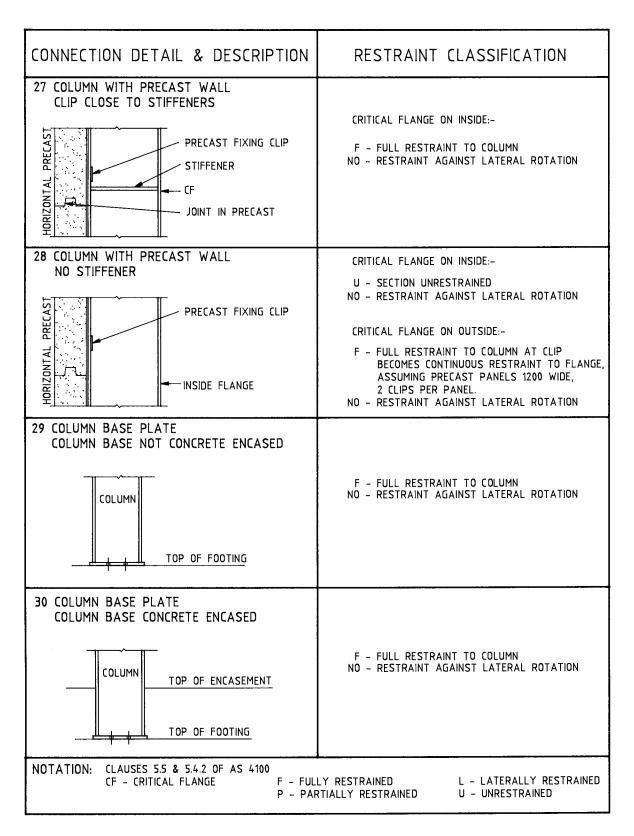


Fig. A9 Restraint at Columns

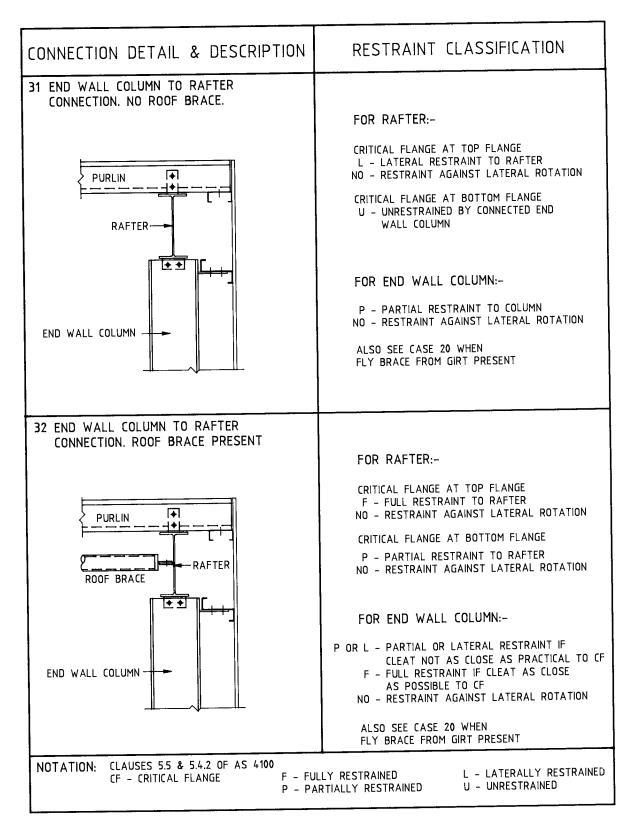


Fig. A10 Restraint at End Wall Columns

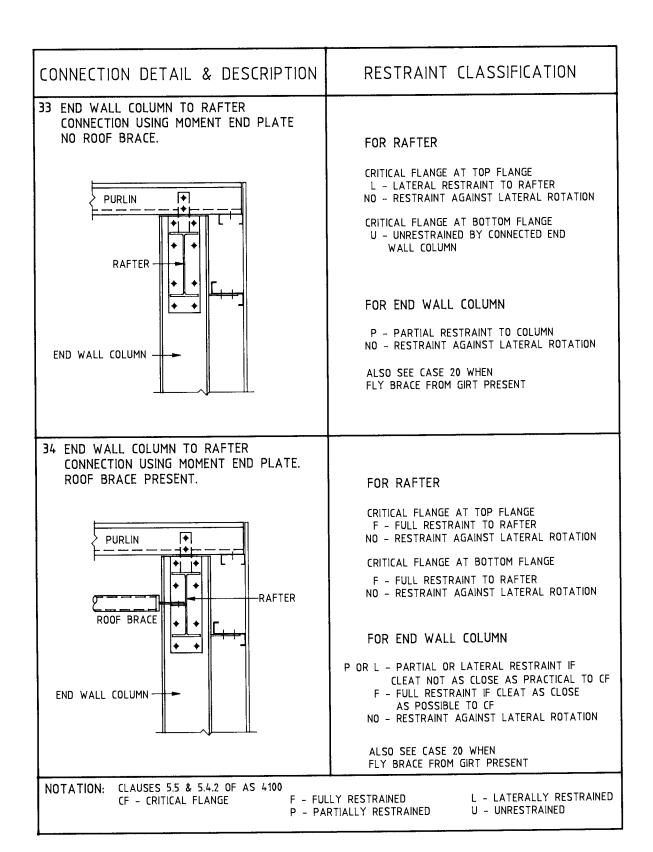


Fig. A11 Restraint at End Wall Columns

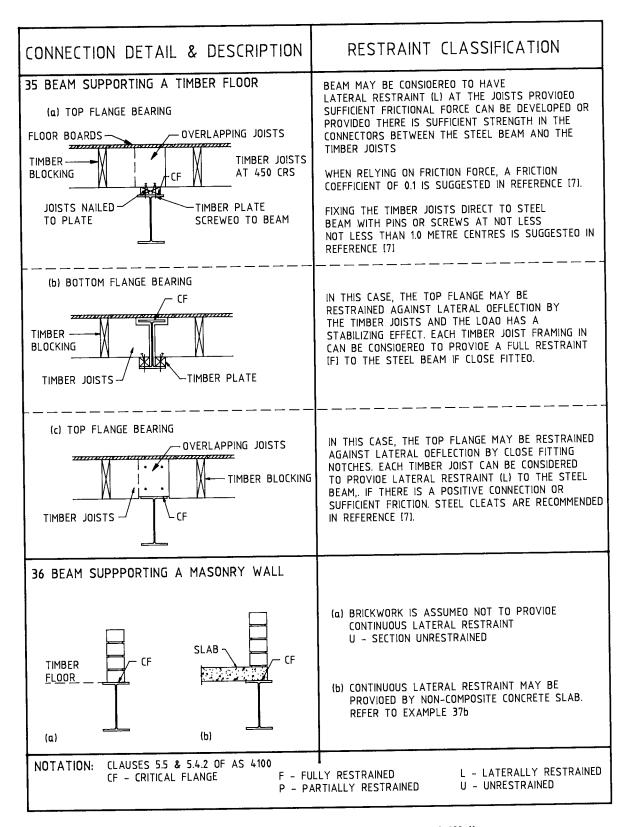


Fig. A12 Restraint at Timber Floors and Brick Walls

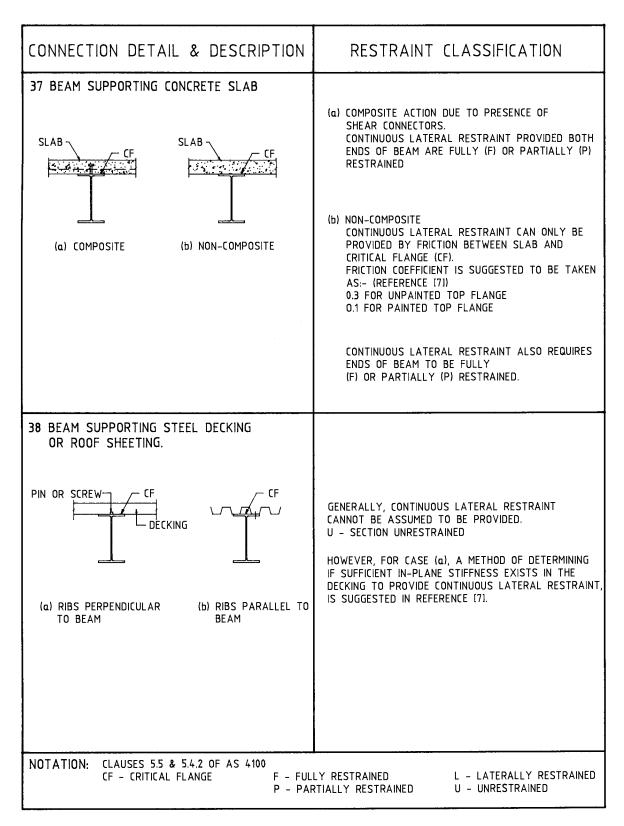


Fig. A13 Restraint at Concrete Slabs and Steel Decking

APPENDIX 2 – SOLUTIONS OF EXAMPLES

Table 5. Solutions of Examples

Beam	1		2		3		4		5	
M _m [⋆] (kNm)	600		600		600	600			600	
Section fy (MPa) Ze (mm³) ϕM_s (kNm)	610UB129 250 3680 x 10 828		610UB125 250 3680 x 10 ³ 828		610UB125 250 3680 x 10 ³ 828		610UB125 250 3680 x 10 ³ 828		610UB125 250 3680 x 10 ³ 828	
Segment or Sub-segment	AC	-	AB	вс	AB	ВС	AB	вс	AC	_
End restraints	FP	_	FP	PP	FF	FF	FL	LF	PF	_
L (m) k_t k_l k_r L_e (m) (M_o) (kNm) (α_s)	10.0 1.032 1.4 1.0 14.45 (241) (0.225)		5.0 1.064 1.0 1.0 5.32 (1000) (0.624)	5.0 1.128 1.0 1.0 5.64 (908) (0.596)	2.5 1.0 1.0 1.0 2.5 (3881) (0.907)	7.5 1.0 1.0 0.85 6.38 (745) (0.536)	5.0 1.0 1.0 1.0 5.0 (1109) (0.655)	5.0 1.0 1.0 1.0 5.0 (1109) (0.655)	10.0 1.032 1.0 1.0 10.32 (370) (0.326)	- - - - -
α_{m}	1.35	-	1.75	1.75	1.75	1.75	1.131	1.75	1.403	-
φα _m α _s M _s (kNm)	251	-	(904)	(864)	(1314)	776	613	(949)	379	_
M [⋆] _m (kNm)	600	-	600	600	600	600	600	450	600	_