The Islamic University of Gaza
Department of Civil Engineering
Design of Special Reinforced Concrete Structures

Design of Circular Beams

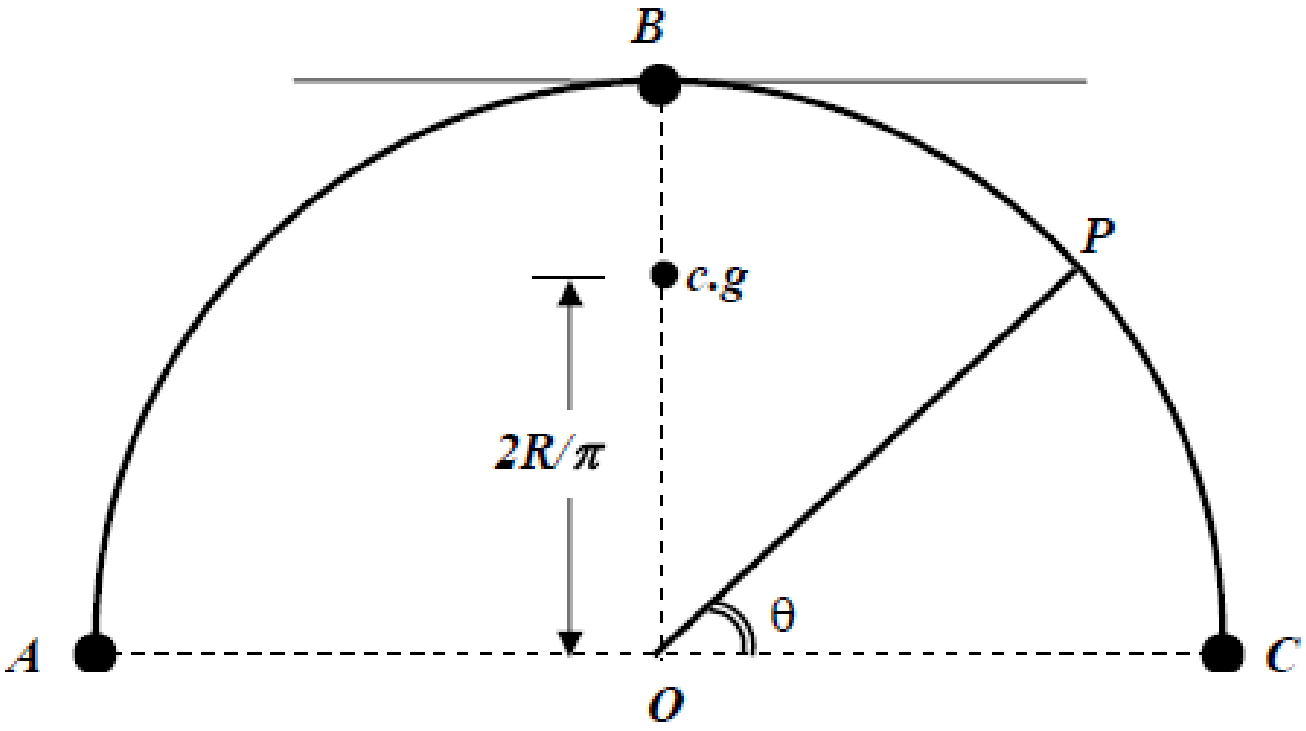
Circular Beams

- They are most frequently used in circular reservoirs, spherical dome, curved balconies … etc.
- Circular beams loaded and supported loads normal to their plans.
- The center of gravity of loads does not coincide with the centerline axis of the member.

Circular Beams

- Circular beam are subjected to torsional moments in addition to shear and bending moment
- The torsional moment causes overturning of the beams unless the end supported of the beams are properly restrained.

Semi-circular beam simply supported on three equally spaced supports



Semi-circular beam supported on three supports

Supports Reactions

Let

$R_A = R_C = V_1$

$R_B = V_2$

Taking the moment of the reaction about the line passing through B and parallel to AC

$$\sum M = 0$$

$$2V_1 R = wR\pi\left(R - \frac{2R}{\pi}\right)$$

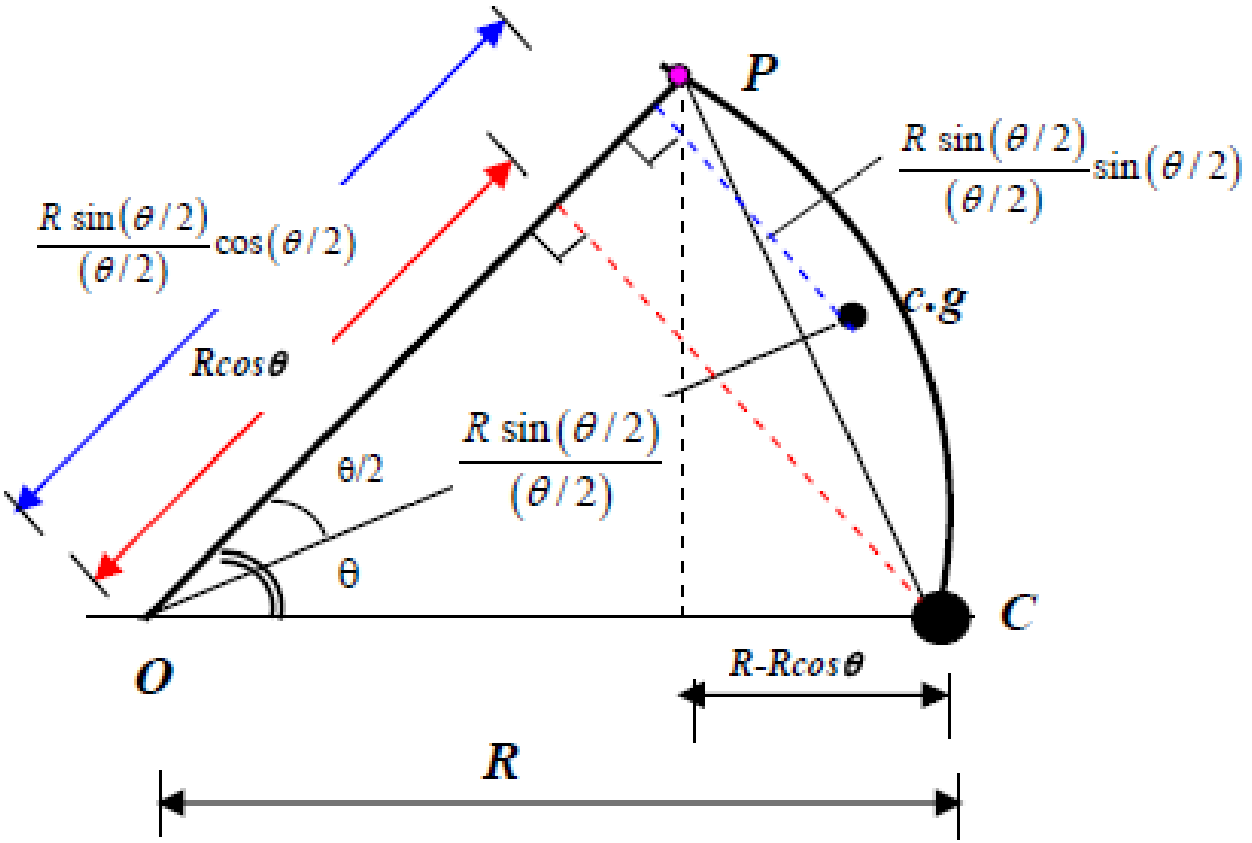
$$V_1 = \frac{WR}{2}(\pi - 2)$$

$$\sum F_y = 0$$

$$V_2 = wR\pi - 2\frac{WR}{2}(\pi - 2)$$

$$V_2 = 2Rw$$

S.F and B.M at Point P at angle θ from support C



Semi-circular beam supported on three supports

Taking the bending moment at point P at angle θ from the support say, C equal:

$$M_\theta = V_1 R \sin\theta - wR\theta\left(\frac{R\sin(\theta/2)}{(\theta/2)}\sin(\theta/2)\right)$$

$$M_\theta = \frac{wR}{2}(\pi-2)R\sin\theta - 2wR^2\sin^2(\theta/2)$$

$$M_\theta = wR^2\left[\frac{(\pi-2)}{2}\sin\theta - 2\sin^2(\theta/2)\right]$$

Semi-circular beam supported on three supports

Maximum negative moment occurs at point B

$$M_{\max.-ve} = M_B \ (at\ \theta = \frac{\pi}{2})$$

$$M_B = -0.429 wR^2$$

Maximum Positive moment

$$Max.M_{+ve} = where\ (\frac{dM_\theta}{d\theta} = 0)$$

$$\frac{dM_\theta}{d\theta} = wR^2\left[\frac{(\pi-2)}{2}\cos\theta - 2\sin(\theta/2)\cos(\theta/2)\right] = 0$$

$$\tan\theta = \frac{\pi-2}{2} \qquad \theta = 29^\circ\ 43'$$

$$M_{\max} = 0.1514 wR^2$$

Semi-circular beam supported on three supports

Torsional Moment at point P

$$T_\theta = V_1 \cdot \left(R - R\cos\theta\right) - wR\theta\left[R - \frac{R\sin\left(\theta/2\right)}{\left(\theta/2\right)}\cos\left(\theta/2\right)\right]$$

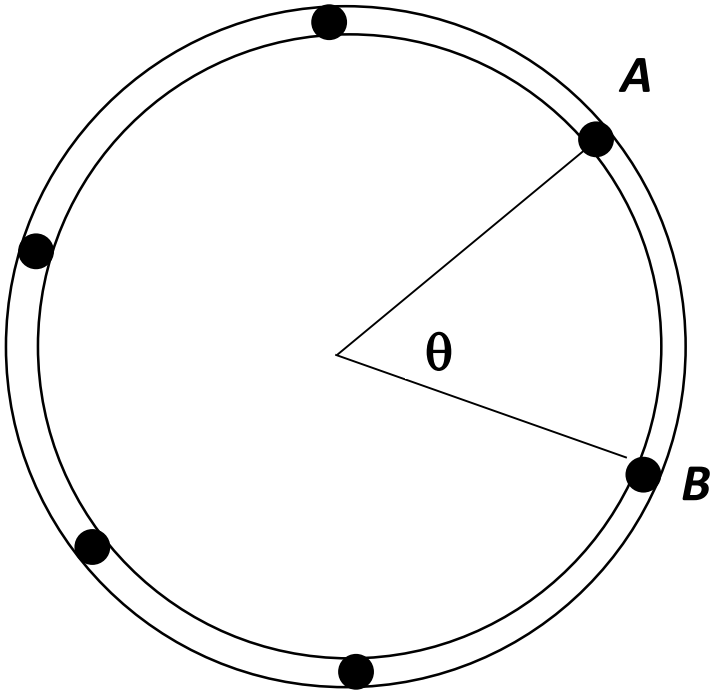
$$T_\theta = wR^2\left[\left(\frac{\pi-2}{2}\right) - \left(\frac{\pi-2}{2}\right)\cos\theta - \theta + \sin\theta\right]$$

The section of maximum torsion $\dfrac{dT_\theta}{d\theta} = 0$

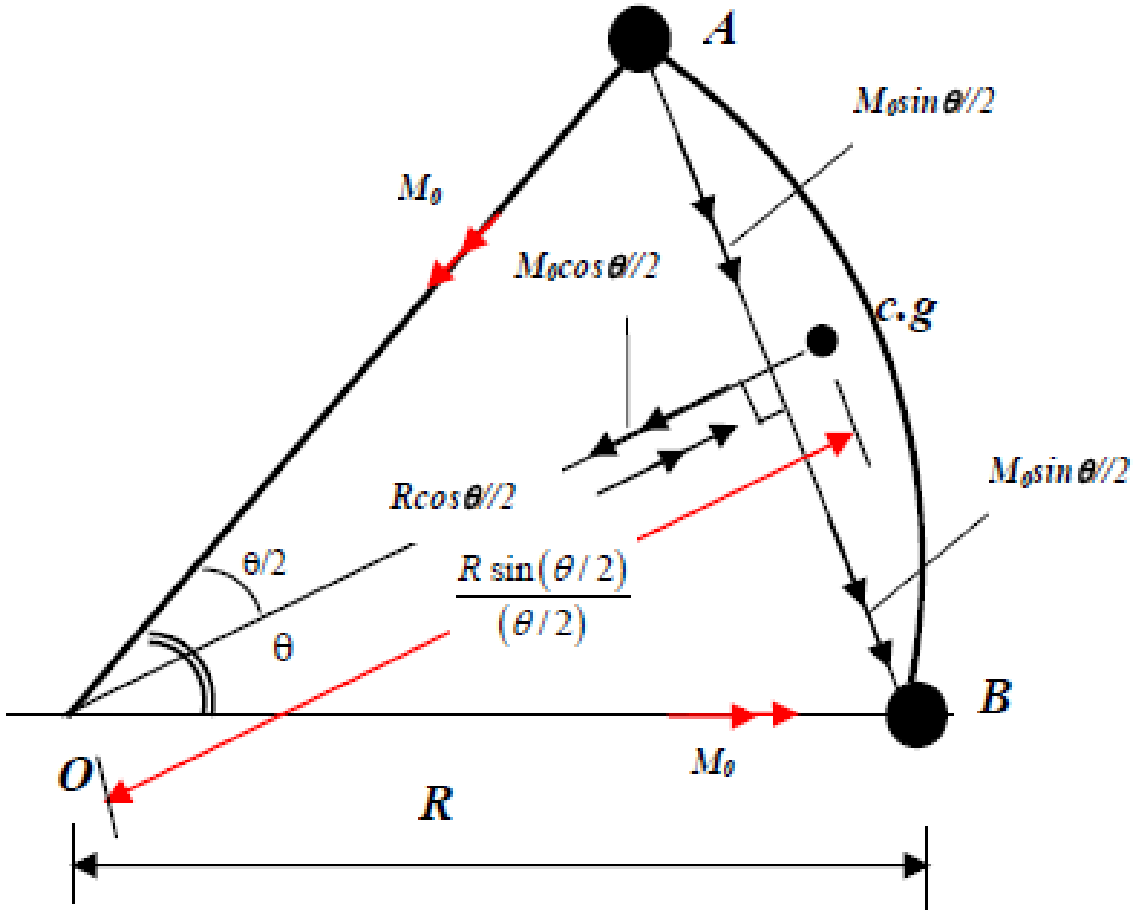
$$\frac{dT_\theta}{d\theta} = 0 \qquad at\ \ \theta = 59^\circ\ 26'$$

$$T_{max.} = 0.1042wR^2$$

Circular beams loaded uniformly and supported on symmetrically placed columns



Circular beams at symmetrically placed columns



Circular beams at symmetrically placed columns

- Consider portion AB of beam between two consecutive columns A and B be with angle θ
- The load on portion AB of beam $= wR\theta$
- The center of gravity of the load will lie at a distance $R \sin(\theta/2)/(\theta/2)$ from the center
- Let M_0 be the bending moment and V_0 be the shear at the supports

Circular beams at symmetrically placed columns

- From geometry $V_0 = wR\theta/2$
- The moment M_0 at the support can be resolved along the line AB and at right angle to it. The component along line AB is $M_0 \sin(\theta/2)$ and $M_0 \cos(\theta/2)$ at right angle of it.
- Taking the moment of all forces about line AB

Circular beams at symmetrically placed columns

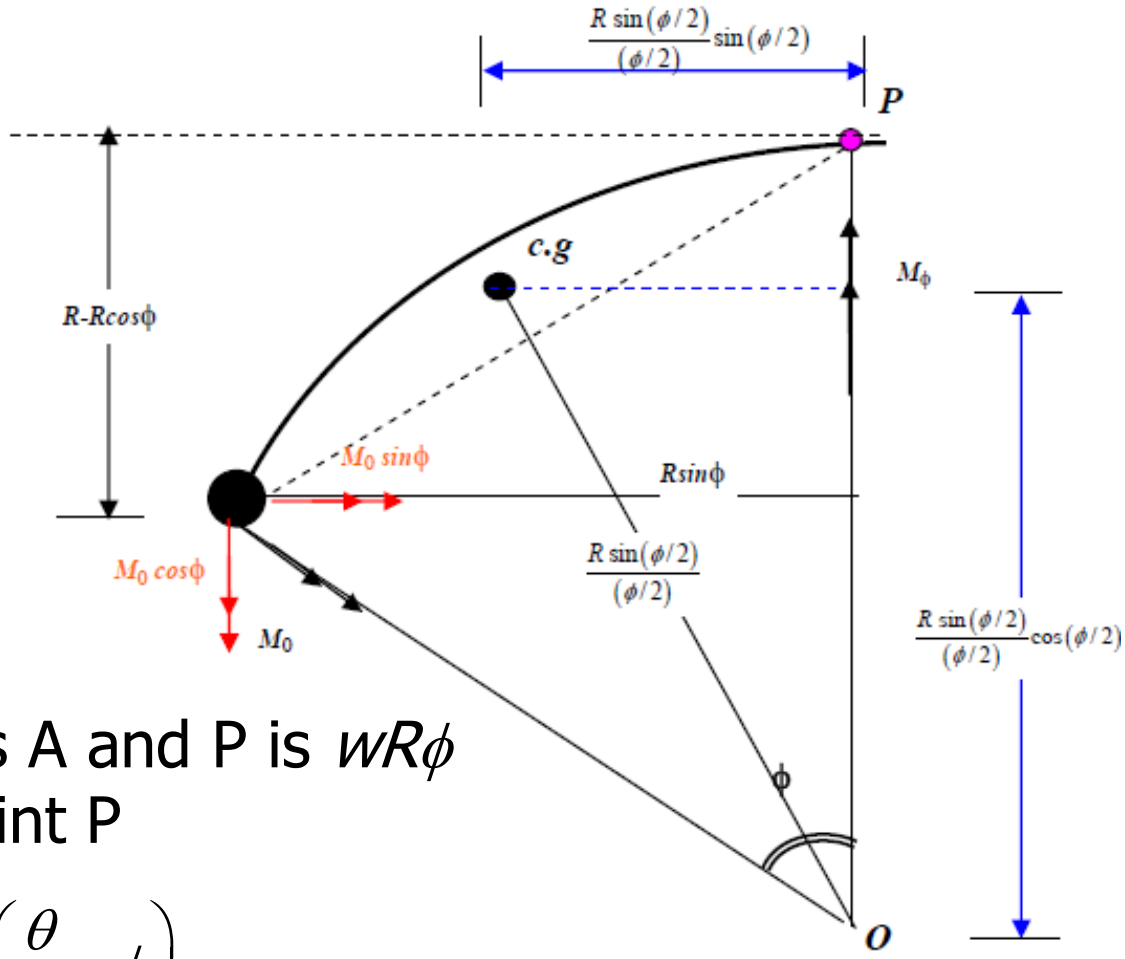
- Taking the moment of all forces about line AB

$$2M_0 \sin(\theta/2) = wR\theta\left[\frac{R\sin(\theta/2)}{\theta/2} - R\cos(\theta/2)\right]$$

$$M_0 = wR^2\left[\frac{\theta}{2\sin(\theta/2)}\cdot\frac{\sin(\theta/2)}{\theta/2} - \theta\frac{\cos(\theta/2)}{2\sin(\theta/2)}\right]$$

$$M_0 = wR^2\left[1 - \frac{\theta}{2}\cot(\theta/2)\right]$$

S.F and B.M. at point P, at angle ϕ from the support



- Load between points A and P is $wR\phi$
- Shearing force at point P

$$V_P = \frac{wR\theta}{2} - wR\phi = wR\left(\frac{\theta}{2} - \phi\right)$$

Shear force and bending moment at point P

The direction of vector representing bending moment at point P will act along line PO

$$M_\phi = V_0 R \sin\phi - M_0 \cos\phi - wR\phi \frac{R\sin(\phi/2)}{\phi/2}\sin(\phi/2)$$

$$M_\phi = \frac{wR^2\theta}{2}\sin\phi - wR^2\left[1-\frac{\theta}{2}\cot(\theta/2)\right]\cos\phi - 2wR^2\sin^2(\phi/2)$$

$$M_\phi = wR^2\left[\frac{\theta}{2}\sin\phi - \cos\phi + \frac{\theta}{2}\cot(\theta/2)\cos\phi - 2\sin^2(\phi/2)\right]$$

since $\cos\phi = 1 - 2\sin^2(\phi/2)$

$$M_\phi = wR^2\left[-1+\frac{\theta}{2}\sin\phi + \frac{\theta}{2}\cot(\theta/2)\cos\phi\right]$$

Shear force and bending moment at point P

The direction of vector representing torsion at point P will act at right angle to line PO

$$T_\phi = M_0 \sin\phi - \frac{wR\theta}{2}\left(R - R\cos\phi\right) + wR\phi\left(R - \frac{R\sin(\phi/2)}{\phi/2}\cos(\phi/2)\right)$$

since $R - R\cos\phi = R\left(1-\cos\phi\right) = 2R\sin^2(\phi/2)$

$$T_\phi = wR^2\left(1-\frac{\theta}{2}\cot(\theta/2)\right)\sin\phi - wR^2\theta\sin^2(\phi/2) + wR^2\phi\left(1-\frac{2}{\phi}\sin(\phi/2)\cos(\phi/2)\right)$$

$$T_\phi = wR^2\left(1-\frac{\theta}{2}\cot(\theta/2)\right)\sin\phi - wR^2\theta\sin^2(\phi/2) + wR^2\phi\left(1-\frac{2}{\phi}\frac{1}{2}\sin\phi\right)$$

$$T_\phi = wR^2\left[\sin\phi - \frac{\theta}{2}\cot(\theta/2)\sin\phi - \theta\sin^2(\phi/2) + \phi - \sin\phi\right]$$

$$T_\phi = wR^2\left[\phi - \frac{\theta}{2}\cot(\theta/2)\sin\phi - \theta\sin^2(\phi/2)\right]$$

since $2\sin^2(\phi/2) = 1-\cos\phi$

$$T_\phi = wR^2\left[\phi - \frac{\theta}{2} + \frac{\theta}{2}\cos\phi - \sin\phi - \frac{\theta}{2}\cot(\theta/2)\sin\phi\right]$$

Shear force and bending moment at point P

To obtain maximum value of torsional moment

$$\frac{dT_\phi}{d\phi} = 0$$

$$T_\phi = M_0 \sin\phi - \frac{wR\theta}{2}\left(R - R\cos\phi\right) + wR\phi\left(R - \frac{R\sin\left(\phi/2\right)}{\phi/2}\cos\left(\phi/2\right)\right)$$

$$\frac{dT_\phi}{d\phi} = wR^2\left[1 - \frac{\theta}{2}\sin\phi - \sin\phi - \frac{\theta}{2}\cot\left(\theta/2\right)\cos\phi\right] = 0$$

$$M_\phi = wR^2\left[1 - \frac{\theta}{2}\sin\phi - \sin\phi - \frac{\theta}{2}\cot\left(\theta/2\right)\cos\phi\right] = 0$$

i.e. point of maximum torsion will be point of zero moment

Circular beams loaded uniformly and supported on symmetrically *n* placed columns

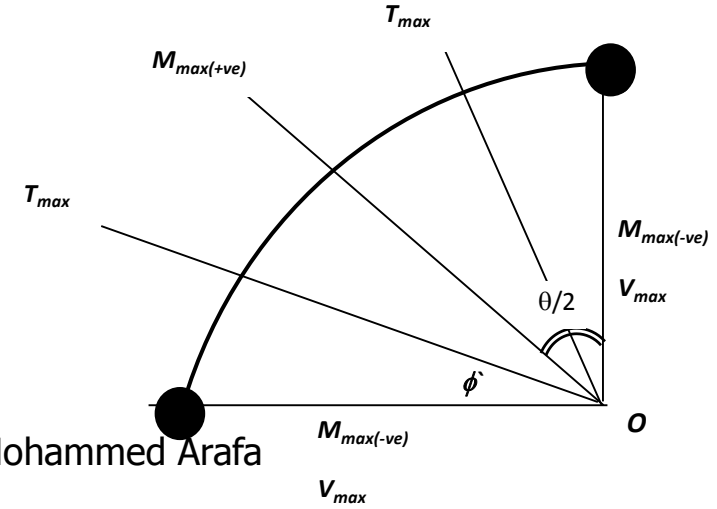
# of Supports	R	V	θ	K	K`	K``	Angle for Max Torsion
4	P/4	P/8	90	0.137	0.07	0.021	19 21
5	P/5	P/10	72	0.108	0.054	0.014	15 45
6	P/6	P/12	60	0.089	0.045	0.009	12 44
7	P/7	P/14	51.4 3	0.077	0.037	0.007	10 45
8	P/8	P/16	45	0.066	0.030	0.005	9 33
9	P/9	P/18	40	0.060	0.027	0.004	8 30
10	P/10	P/20	36	0.054	0.023	0.003	7 30
12	P/12	P/24	30	0.045	0.017	0.002	6 21

$$P = 2\pi rw$$

$$M_{\text{support}} = KwR^2\theta$$

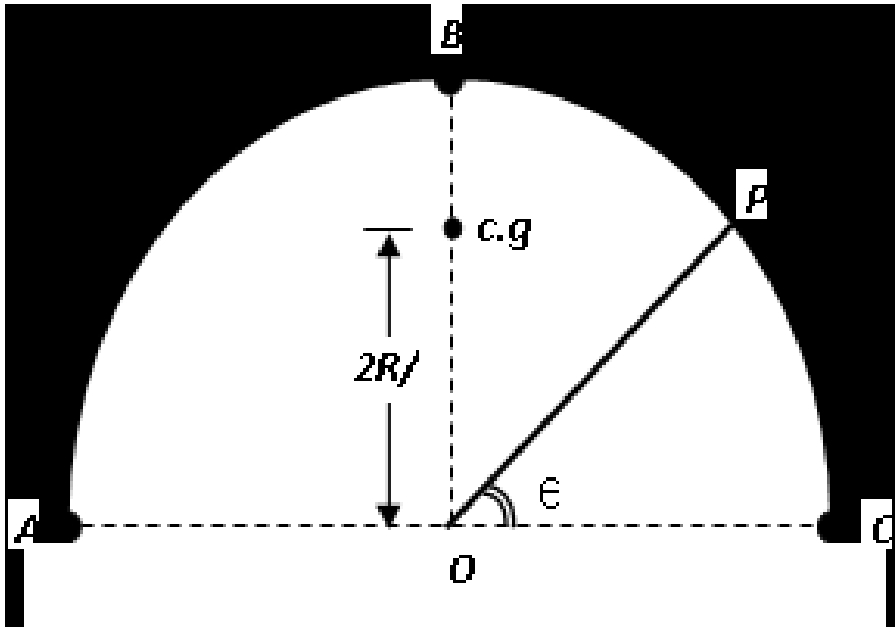
$$M_{\text{Midspan}} = K'wR^2\theta$$

$$T_{\max} = K''wR^2\theta$$



Example 1

Design a semi-circular beam supported on three-equally spaced columns. The centers of the columns are on a circular curve of diameter 8m. The beam is support a uniformly distributed factored load of 5.14 t/m in addition to its own weight.



$f_c^{'} = 350\,kg/cm^2$ and $f_y = 4200\,kg/cm^2$

$L = r\left(\pi/2\right) = 6.28\,m$

$h_{\min} = L/18.5 = 628/18.5 = 33.94cm$

use Beam 40×70 cm

o. w. of the beam $=0.4(0.7)(2.5)(1.2)$

$= 0.84$ t/m'

Total load= 5.14+0.84=5.98 t/m'

Example 1

$$M_{\max(-ve)} = 0.429wR^2 = 0.429(5.98)(4)^2 = 41.05t.m$$

$$M_{\max(+ve)} = 0.1514wR^2 = 0.1514(5.98)(4)^2 = 14.48t.m$$

$$T_{\max} = 0.1042wR^2 = 0.1042(5.98)(4)^2 = 9.97t.m$$

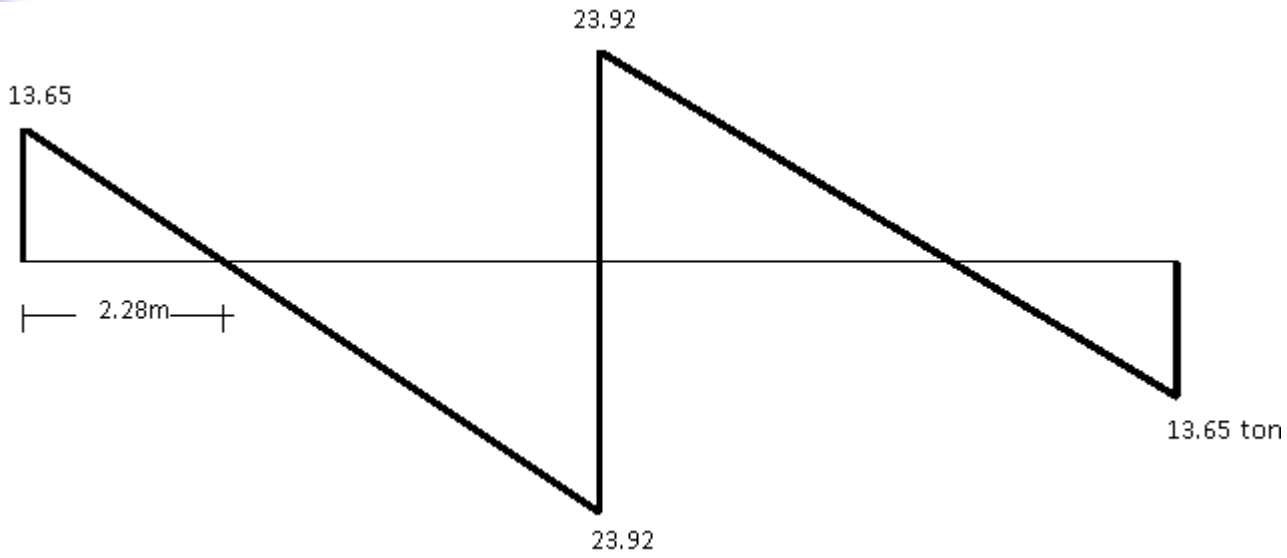
Reactions

$$R_A = \frac{wR}{2}(\pi - 2) = \frac{5.98(4)}{2}(\pi - 2) = 13.65ton$$

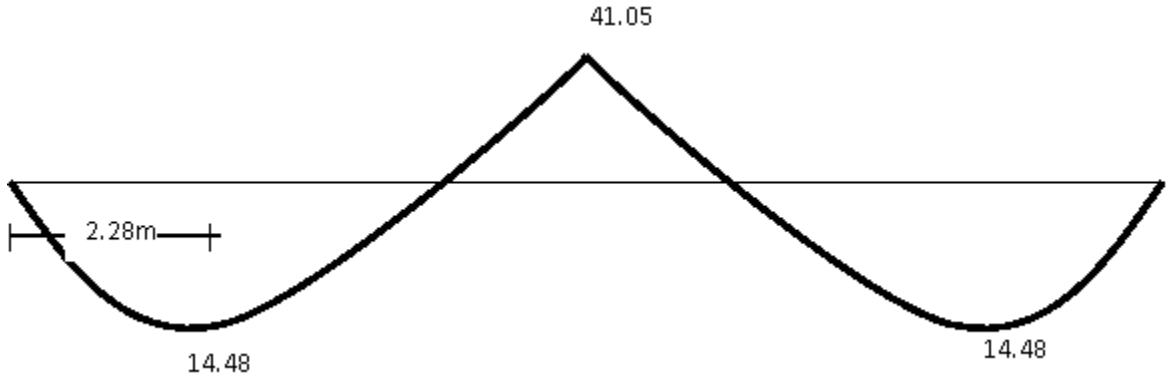
$$R_B = 2wR = 2(5.98)(4) = 47.84ton$$

Shear at point P: $$V_\theta = \frac{wR}{2}(\pi - 2) - wR\theta$$

Example 1



Shear Force Diagram



Bending Moment Diagram

Example 1

Design for Reinforcement

d=70-4.0-0.8-1.25=63.95

$$\rho_{-ve} = \frac{0.85 \cdot 350}{4200}\left[1-\sqrt{1-\frac{2.61 \cdot 10^5 (41.05)}{40 \cdot 63.95 \cdot 350}}\right] = 0.00697 > \rho_{\min}$$

$$A_{s(-ve)} = (0.00697)(40)(63.95) = 17.85 cm^2$$

$$\rho_{-ve} = \frac{0.85 \cdot 350}{4200}\left[1-\sqrt{1-\frac{2.61 \cdot 10^5 (14.48)}{40 \cdot 63.95 \cdot 350}}\right] = 0.00238 < \rho_{\min}$$

$$A_{s(+ve)} = (0.0033)(40)(63.95) = 8.44 cm^2$$

Example 1

Design for Shear

$$V_{max} = 23.92\,ton \qquad (T = 0.0 \text{ at middle support})$$

$$\phi V_c = 0.53(0.75)\sqrt{350}(40)(63.95) = 19.02\,ton$$

$$V_S = \frac{23.92 - 19.02}{0.75} = 6.53\ ton$$

$$6.53 \cdot 1000 = \frac{A_V}{S} 4200(63.95)$$

$$\frac{A_V}{S} = 0.0243\ cm^2/cm$$

Example 1

Design for Torsion

$$if \quad T_u \le \frac{0.265\phi\sqrt{f_c'}A_{cp}^2}{p_{cp}} \Rightarrow \qquad \text{neglect torsion}$$

$A_{cp} = bh$, $p_{cp} = 2(b+h)$, $A_{0h} = x_0 y_0$, $p_h = 2(x_0 + y_0)$ and $A_0 = 0.85 x_0 y_0$

The following equation has to be satisfied to check for ductlity

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 2.12\sqrt{f_c'}\right)$$

Reinforcement

$$\frac{A_T}{S} = \frac{T_u}{2\phi f_{ys} A_0}$$

$$A_l = \left(\frac{A_T}{S}\right) p_h \quad \text{and} \quad A_{l,\min} = \frac{1.3\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_T}{S}\right) p_h$$

Example 1

Design for Torsion

At section of maximum torsion is located at θ=59.43

T=9.97 t.m

$$V_u = \frac{wR}{2}(\pi - 2) - wR\theta = 13.65 - 5.98(4)(\frac{59.43}{360}2\pi) = -11.6 ton$$

$$M = 0.0$$

$$\phi V_c = 0.53(0.75)\sqrt{350}(40)(63.95) = 19.02 ton > V_u \quad (\text{No shear reinf. required})$$

$$\frac{0.265\phi\sqrt{f_c'}A_{cp}^2}{p_{cp}} = \frac{0.265(0.75)\sqrt{f_c'}(40\cdot 70)^2}{2(40+70)10^5} = 1.32 t.m < T_u$$

i.e torsion must be considered

Example 1

Design for Torsion: Ductility Check

The following equation has to be satisfied

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)} \le \phi\left(\frac{V_c}{b_w d}+2.12\sqrt{f_c'}\right)$$

$$x_0 = 40-4-4-0.8 = 31.2$$

$$y_0 = 70-4-4-0.8 = 61.2$$

$$p_h = 2\left(x_0+y_0\right) = 2\left(31.2+61.2\right) = 184.8\,cm$$

$$A_{oh} = 31.2\cdot 61.2 = 1909.44\ cm^2$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)} = \sqrt{\left(\frac{11.16\cdot 10^3}{40\cdot 63.95}\right)^2+\left(\frac{\left(9.97\cdot 10^5\right)\left(184.8\right)}{1.7\left(1909.44\right)^2}\right)} = 30.04\ kg/cm^2$$

$$\phi\left(\frac{V_c}{b_w d}+2.12\sqrt{f_c'}\right) = 0.75\left(\frac{19.02}{40\cdot 63.95}+2.12\sqrt{350}\right) = 49.6\ kg/cm^2 > 30.04$$

i.e section dimension are adequate for preventing brittle failure due to combined shear stresses.

Example 1

Design for Torsion

$$\frac{A_T}{S}=\frac{T_u}{2\phi f_{ys}A_0}=\frac{9.97\cdot 10^5}{2(0.75)4200(0.85\cdot 1909.44)}=0.097\ cm^2/cm$$

$$\left(\frac{A_T}{S}\right)_{\min}=\frac{3.5b_w}{2f_{ys}}=\frac{3.5(40)}{2(4200)}=0.0167 cm^2/cm$$

$$\left(\frac{A_V}{S}\right)+\left(\frac{A_T}{S}\right)=0+0.097=0.086 cm^2/cm$$

Use ϕ12mm @11.5cm (closed stirrups) with $\frac{A}{S}=\frac{1.13}{11.5}=0.0983 cm^2/cm$

Example 1

Design for Torsion

$$A_l = \left(\frac{A_T}{S}\right) p_h = 0.086(184.8) = 15.98 cm^2$$

$$A_{l,\min} = \frac{1.3\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_T}{S}\right) p_h = \frac{1.3\sqrt{350}(40 \cdot 70)}{4200} - 15.98 = 0.234 cm^2$$

$$A_l = \frac{15.98}{3} = 5.33 cm^2$$

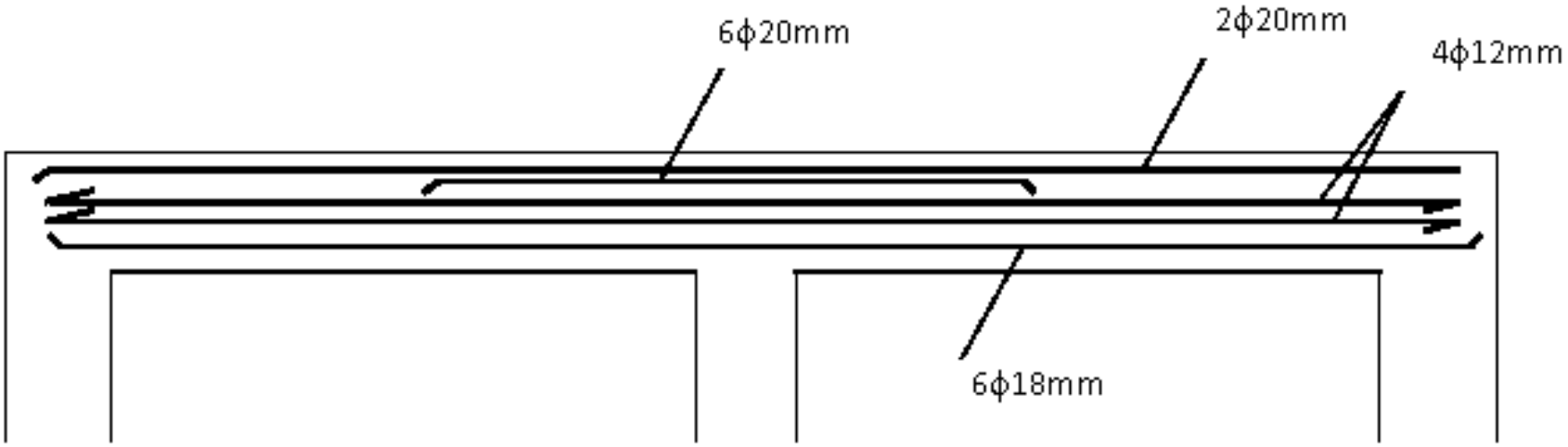
Use $2\phi 20$ top and $4\phi 14$ skin reinforcement

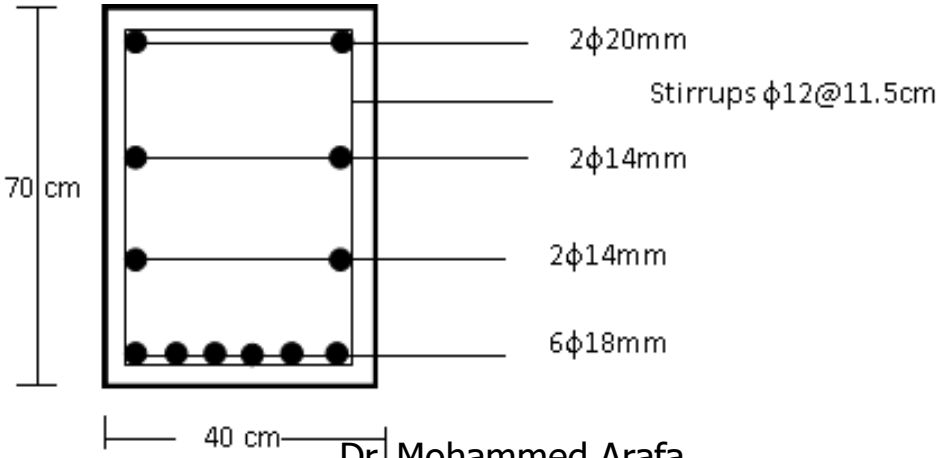
$$\left(A_{s,+ve}\right)_{total} = 8.44 + 5.33 = 13.77 cm^2 \quad \text{use } 6\phi 18\text{mm}$$

$$\left(A_{s,-ve}\right)_{total} = 17.83 + 5.33 = 23.16 cm^2 \quad \text{use } 8\phi 20\text{mm}$$

Example 1

Design for Torsion





Internal Forces in Circular beams Using polar coordinate

$$\frac{dV}{ds} = -w \qquad ds = r\,d\varphi$$

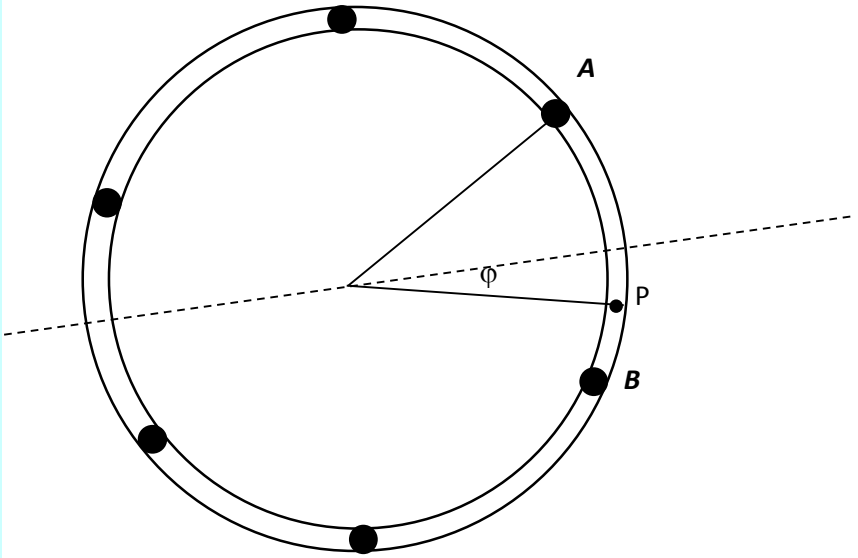
$$\frac{dV}{d\varphi} = -wr$$

$$\frac{dM}{ds} + \frac{dT}{dr} = V \qquad \text{as } r = \text{constant}$$

$$\frac{dT}{dr} = \frac{T}{r}$$

$$\frac{dM}{ds} + \frac{T}{r} = V$$

$$\frac{dM}{d\varphi} + T = Vr$$



Internal Forces in Circular beams Using polar coordinate

The component of the moment M along *ds* in the radial direction must be equal to the difference of the torsional moments along the same element

$$dT = M d\varphi = \frac{M ds}{r}$$

$$\frac{dT}{d\varphi} = M$$

This means that the torsional moment is maximum at point of zero bending moment

Internal Forces in Circular beams Using polar coordinate

Differentiating the equation

$$\frac{dM}{d\varphi} + T = Vr$$

$$\frac{d^2M}{d\varphi^2} + \frac{dT}{d\varphi} = \frac{d(Vr)}{d\varphi} = r\frac{dV}{d\varphi} = -r^2 w$$

$$\frac{d^2M}{d\varphi^2} + M = -wr^2$$

The solution of this deferential equation gives

$$M = A\sin\varphi + B\cos\varphi - wr^2$$

Internal Forces in Circular beams Using polar coordinate

$$M = A\sin\varphi + B\cos\varphi - wr^2$$

- The constants can be determined from conditions at supports
- The internal forces are in this manner statically indeterminate. In many cases, they can be determined from the conditions of equilibrium alone because the statically indeterminate values are either known or equal to zero.

Circular beams loaded uniformly and supported on symmetrically *n* placed columns

$$2\varphi_0 = \frac{2\pi}{n} \qquad \varphi_0 = \frac{\pi}{n}$$

$$R = \frac{2\pi rw}{n} \qquad V = \pm\frac{\pi rw}{n}$$

- The bending moment and shearing force V at any section at an angle φ from the centerline between two successive supports are given by:

$$M = r^2 w\left(\frac{\pi\cos\varphi}{n\sin\varphi_0} - 1\right)$$

$$T = -r^2 w\left(\frac{\pi\sin\varphi}{n\sin\varphi_0} - \varphi\right)$$

$$V = -rw\,\varphi$$

SEMICIRCULAR BEAM FIXED AT END SUPPORTS

- If a semicircular beam supports a concrete slab, as shown the ratio of the length to the width of the slab is 2r/r = 2, and the slab is considered a one-way slab.
- The beam will be subjected to a distributed load, which causes torsional moments in addition to the bending moments and shearing forces.
- The load on the curved beam will be proportional to its distance from the support AB

SEMICIRCULAR BEAM FIXED AT END SUPPORTS

Load on the curved beam per unit length

$$w' = \frac{wr\sin\theta}{2}$$

Shear Force

$$V_A = V_B = \left(\frac{\pi}{8}\right) wr^2 = 0.39wr^2$$

Bending Moment at support

$$M_A = M_B = -\frac{wr^3}{3}$$

Torsional Moment at support

$$T_A = -0.11wr^3$$

SEMICIRCULAR BEAM FIXED AT END SUPPORTS

Bending Moment at section N

$$M_N = wr^3\left[\frac{\pi}{8}\sin\theta - \frac{1+\cos^2\theta}{6}\right] + T_A \sin\theta$$

$$M_N = wr^3\left[\frac{\pi}{8}\sin\theta - \frac{1+\cos^2\theta}{6} - 0.11\sin\theta\right]$$

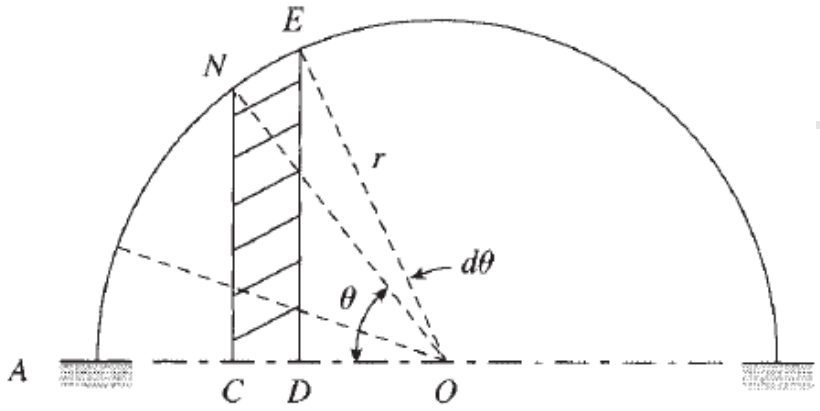
$$M_{Midspan} = 0.116wr^3 \qquad \text{at } \theta = \frac{\pi}{2}$$

Torsional Moment at section N

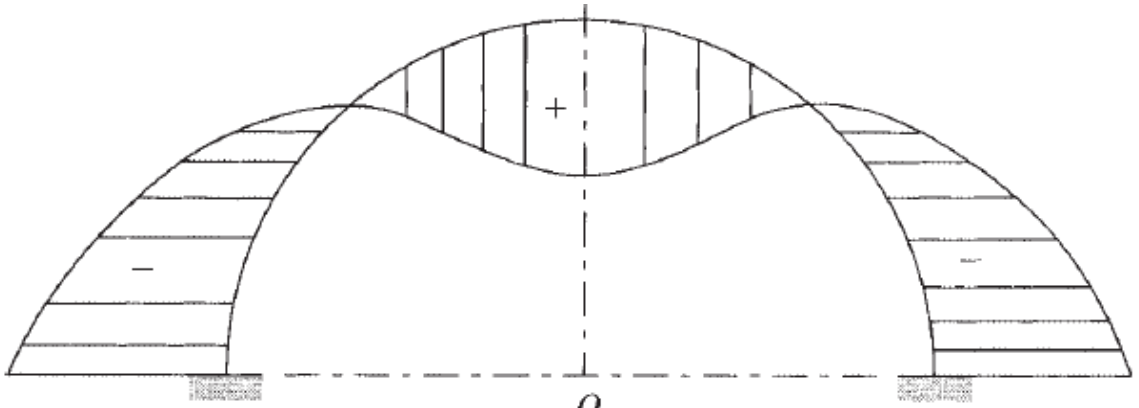
$$T_N = wr^3\left[\frac{\pi}{8}(\cos\theta - 1) + \frac{\theta}{4} + \frac{1}{24}\sin 2\theta\right] + T_A \cos\theta$$

$$T_N = wr^3\left[\frac{\pi}{8}(\cos\theta - 1) + \frac{\theta}{4} + \frac{1}{24}\sin 2\theta - 0.11\cos\theta\right]$$

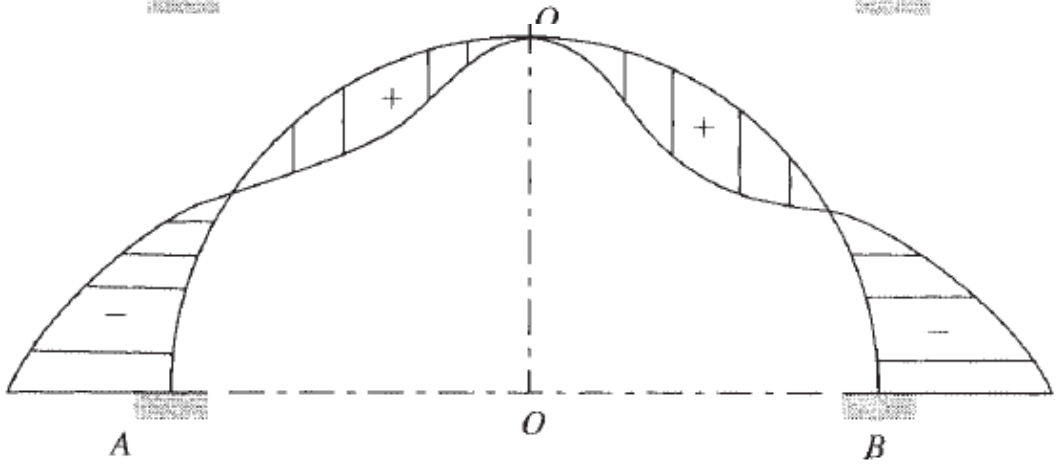
$$T_{Midspan} = 0 \qquad \text{at } \theta = \frac{\pi}{2}$$



$$NE = r\,d\theta$$
$$OC = r\cos\theta$$
$$NC = r\sin\theta$$
$$CD = d(OC) = d(r\cos\theta)/d\theta$$
$$= r\sin\theta\,d\theta$$

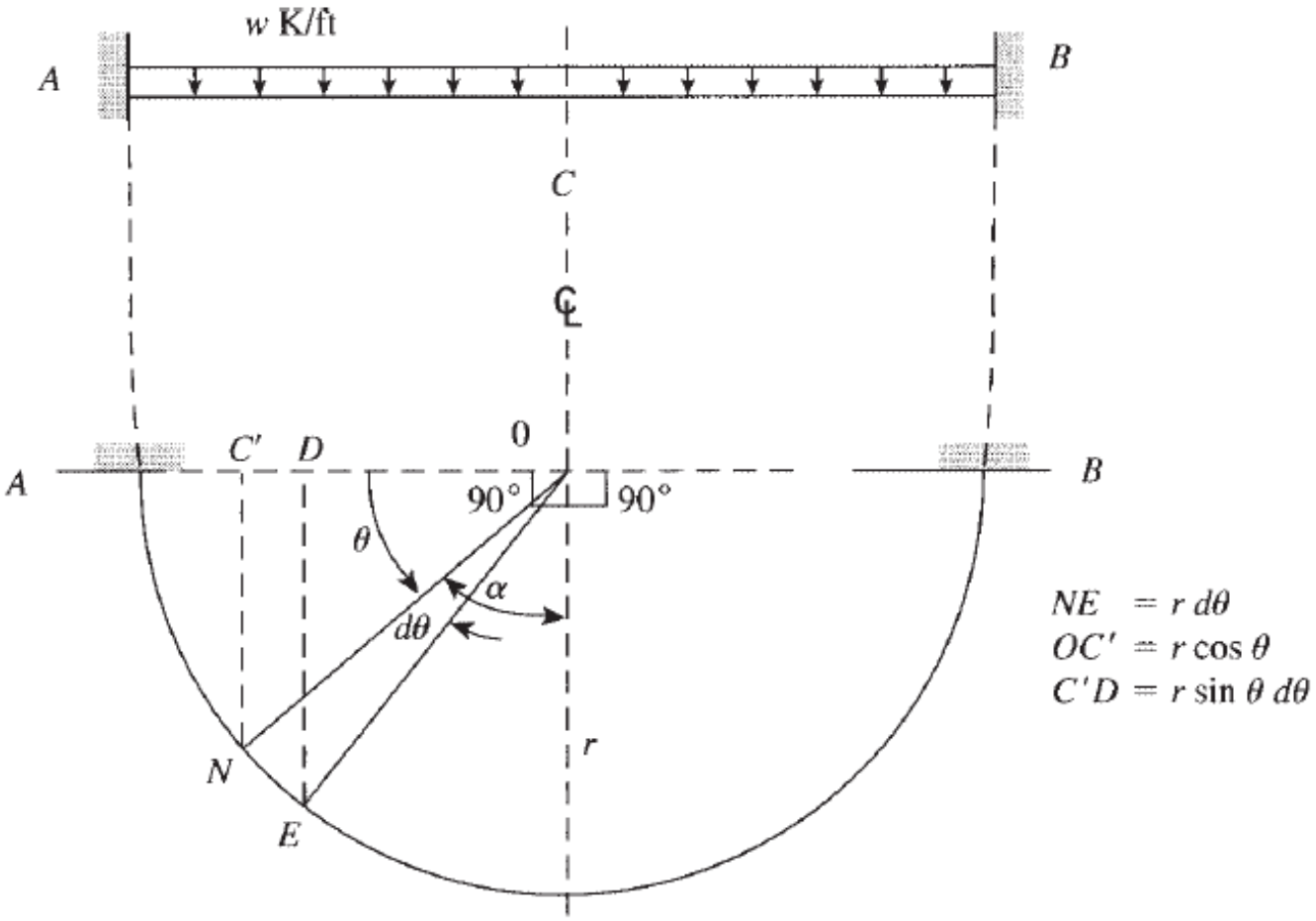


BMD

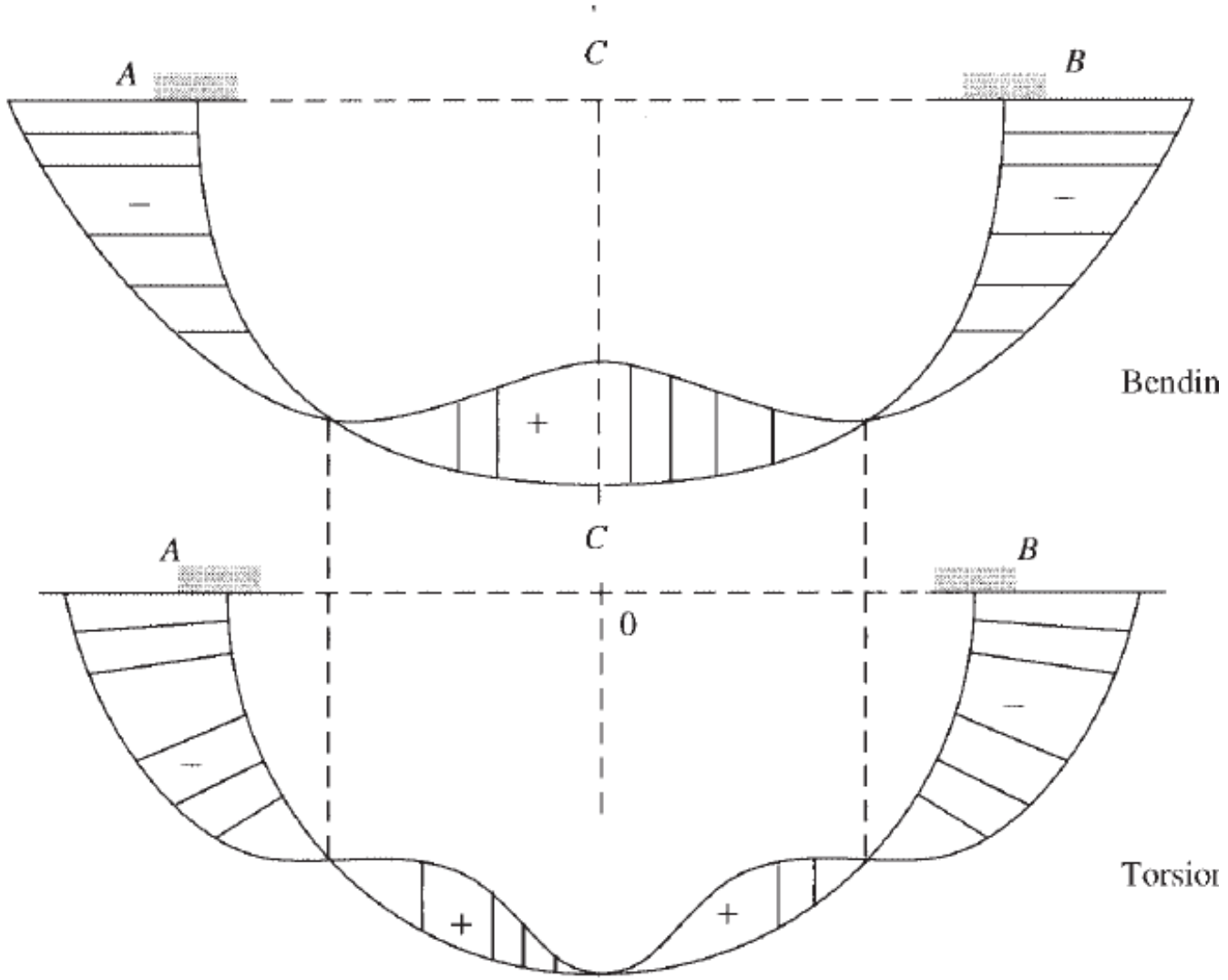


TMD

Fixed-end Semicircular Beam Under Uniform Loading



Fixed-end Semicircular Beam Under Uniform Loading



Fixed End Semicircular Beam with Uniform Loading

Load on the curved beam per unit length w

Shear Force

$$V_A = V_B = 1.57wr$$

Bending Moment at support

$$M_A = M_B = -wr^2$$

Torsional Moment at support

$$T_A = -0.3wr^2$$

Fixed End Semicircular Beam with Uniform Loading

Bending Moment at section N

$$M_N = wr^2\left[\frac{\pi}{2}\sin\theta - 1\right] + T_A \sin\theta$$

$$M_N = wr^2\left[\frac{4}{\pi}\sin\theta - 1\right]$$

$$M_{Midspan} = 0.273wr^2 \qquad \theta = \frac{\pi}{2}$$

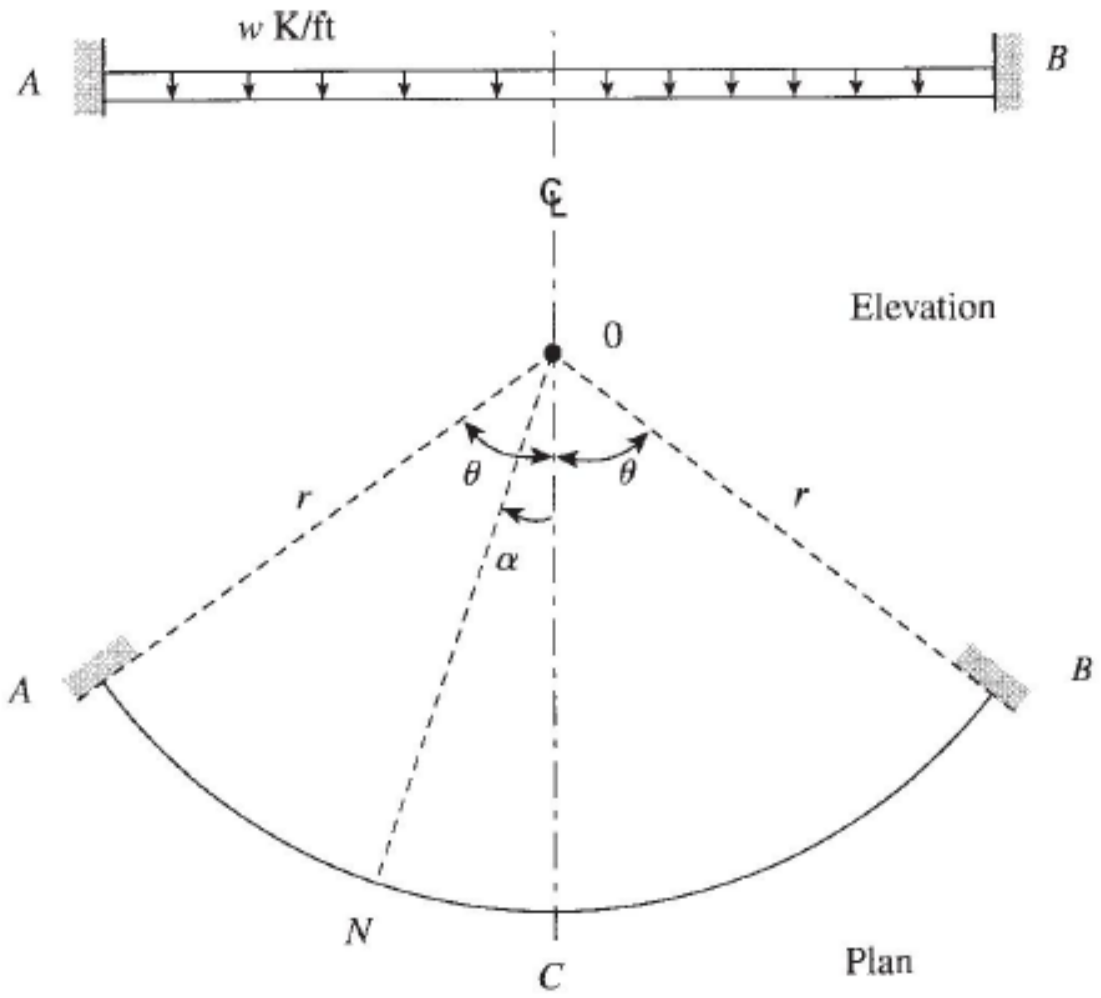
Torsional Moment at section N

$$T_N = wr^2\left[\frac{\pi}{2}\cos\theta - \frac{\pi}{2} + \theta\right] + T_A \cos\theta$$

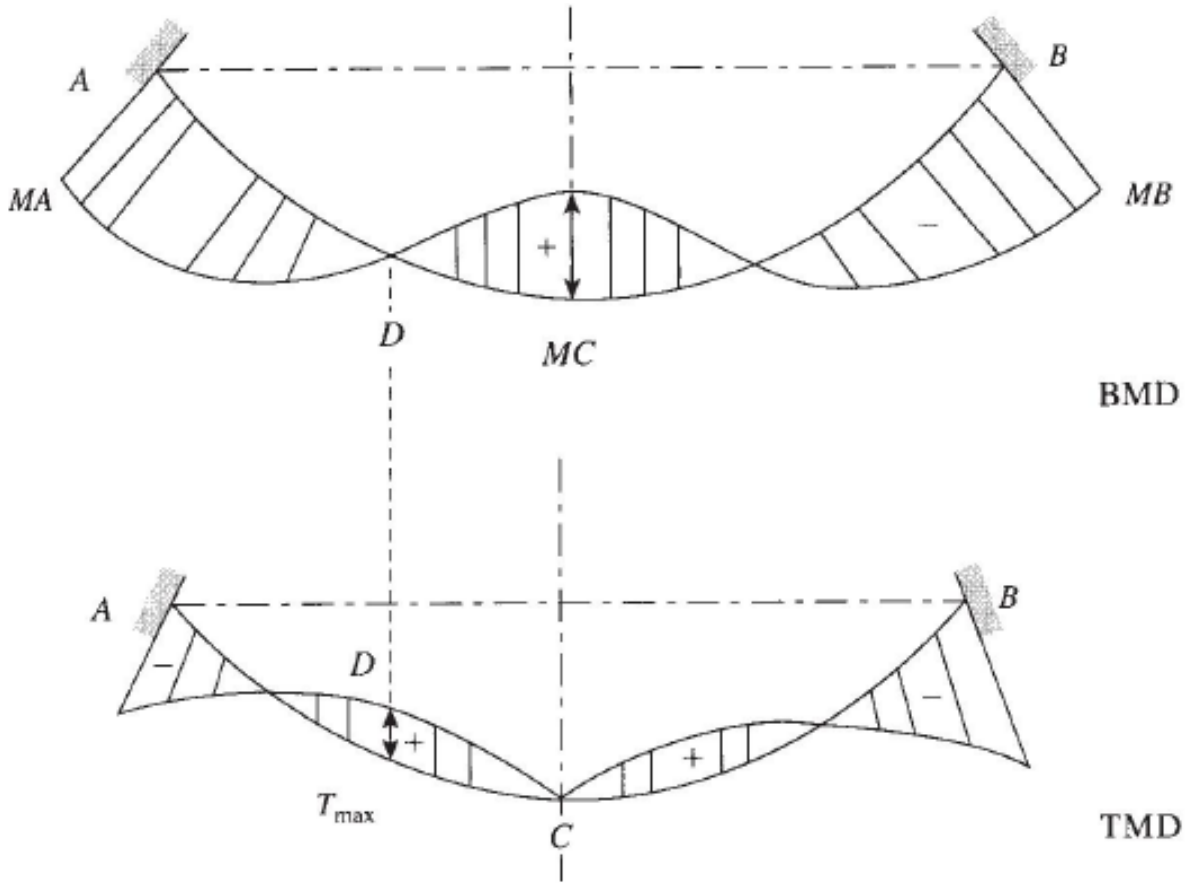
$$T_N = wr^2\left[\theta - \frac{\pi}{2} + \frac{4}{\pi}\cos\theta\right]$$

$$T_{Midspan} = 0 \qquad \theta = \frac{\pi}{2}$$

Fixed End Circular Beam under Uniform Loading



Fixed End Circular Beam under Uniform Loading



Fixed End Semicircular Beam with Uniform Loading

Load on the curved beam per unit length w

Bending Moment at Centerline

$$M_C = \frac{wr^2}{K_4}\left[\lambda\left(K_1 + K_2 - K_3\right) + \left(K_1 - K_2\right)\right]$$

Torsional Moment at Centerline

$$T_C = 0$$

Bending Moment at Section N

$$M_N = M_C \cos\alpha - wr^2\left(1 - \cos\alpha\right)$$

Torsional Moment at Section N

$$T_N = M_C \sin\alpha - wr^2\left(\alpha - \sin\alpha\right)$$

Fixed End Semicircular Beam with Uniform Loading

Bending Moment at Support A

$$M_A = M_C \cos\theta - wr^2\left(1-\cos\theta\right)$$

Torsional Moment at Support A

$$T_A = M_C \sin\theta - wr^2\left(\theta-\sin\theta\right)$$

where

$\lambda = EI/GJ$
$K_1 = 2(2\sin\theta - \theta)$
$K_2 = 2\sin\theta\cos\theta = \sin 2\theta$
$K_3 = 4\theta\cos\theta$
$K_4 = 2\theta(\lambda+1) - (\lambda-1)\sin 2\theta$
2θ = total central angle of the ends of the beam, angle AOB

Values of λ and J

- The value of G/E for concrete may be assumed to be equal to 0.43.

The Value of J

For circcular section $J = \frac{1}{2}\pi r^4$

For square section of side x $J = 0.141x^4$

For rectangular section of short side x and long side y

$J = K'xy^3$

Values of λ and J

For rectangular section of short side x and long side y

$$J = K' \times y^3$$

The values of K' are calculated as follows:

$$K' = \frac{1}{16}\left[\frac{16}{3} - 3.36\frac{x}{y}\left(1 - \frac{x^4}{12y^4}\right)\right]$$

whereas

$$\lambda = \frac{EI}{GJ} = \left(\frac{1}{0.43}\right)\left(\frac{xy^3}{12}\right)\left(\frac{1}{K'yx^3}\right) = \frac{1}{5.16\,K'}\left(\frac{y}{x}\right)^2$$

Values of K' and λ are both shown in Table 21.2.

Table 21.2 Values of K' and λ for Different Values of y/x

y/x	0.5	1.0	1.1	1.2	1.25	1.3	1.4	1.5	1.6
K'	0.473	0.141	0.154	0.166	0.172	0.177	0.187	0.196	0.204
λ	0.102	1.37	1.52	1.68	1.76	1.85	2.03	2.22	2.43
y/x	1.7	1.75	2.0	2.5	3.0	4.0	5.0	6.0	10
K'	0.211	0.214	0.229	0.249	0.263	0.281	0.291	0.300	0.312
λ	2.65	2.77	3.39	4.86	6.63	11.03	16.5	23.3	62.1

Problems

P1- Design a circular beam that supported on six equally spaced columns, and its centerline lies on a circle 7m in diameter. The beam carries a uniform dead load of 12 t/m` and live load of 6kg/m'. Use $f_c^{'} = 300\,kg/cm^2$ and $f_y = 4200\,kg/cm^2$

P2. Find the values of factored shear, moment and torsion at critical sections for the fixed end semicircular beam, if the diameter of the circle is 10 m. The beam is apart of floor slab that caries a uniform dead load (including its own) weight of 800 kg/m^2 and live load of 200kg/m^2.

P3. Find the values of factored shear, moment and torsion at critical sections for the 60^o V-shape beam, with a=8m. The beam carries a uniform dead load (including its own) weight of 10 t/m` and live load of 4t/m`. Assume the ratio of long to short side of the rectangular section is 2.