TUBULAR STEEL STRUCTURES –

Theory and Design

SECOND EDITION

by

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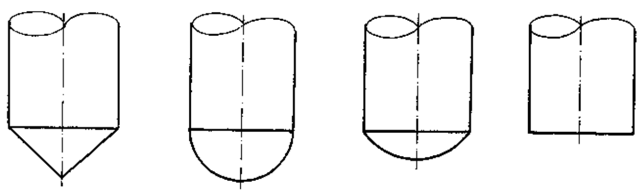

FIGURE 1.1 — Types of head-cylinder structures.

This is prevented at their junctions by the rigid attachment of the heads and cylinders, one to the other. Such a disturbance of the natural deformations, arising from internal pressure, produces shear and flexural stresses which obviously are most severe at or near the head-cylinder junctures. These stresses do not vary circumferentially because of the axial symmetry of the structures and decay ultimately, if not at once, both in the cylinders and in the heads as the axial distance from this point of stress disturbance increases. These juncture stresses exist in addition to the membrane stresses.

More particularly, the juncture stresses consist of axial shear stresses, axial flexural stresses, and circumferential membrane stresses. All vary in an axial direction only, as previously stated, due to the axial symmetry of the structure under consideration.

This same property of symmetry obviously excludes the existence of circumferential shear stresses, so that circumferentially only uniform membrane stresses and uniformly varying flexural stresses are obtained. These two types of stresses are additive, and the magnitudes of their algebraic sums, taken at any axial point on the structures, at both the inner and outer wall surfaces related to that point, represent the greatest and least combined circumferential stresses at the point.

It should be noted that when the greater stress is at the outer wall surface, then the lesser stress occurs at the inner wall surface, and, vice versa. Both stresses are principal stresses, due to the absence of any circumferential shear stresses to combine with them, and as axial distances from the head-cylinder juncture increase, the greater of the two ultimately decreases, both in the head and in the cylinder, down to the circumferential membrane stresses of those parts as a limit.

Likewise, the greater and lesser axial combined stresses are also derived as the algebraic sums of the axial membrane and axial flexural stresses, in the same manner as the greater and lesser circumferential stresses just described. They too, are principal stresses.

From the foregoing, it is seen that the important stresses, at any axial point on the middle surface in the head-cylinder structure, are the circumferential stresses on the inner and outer wall surfaces related to the point; the axial stresses on the inner and outer wall surfaces related to the point and the axial shear stress at the point.

A detailed stress analysis of the tubular structures at the junction will be given in Chapter 3.

CHAPTER 2

local and overall buckling of cylindrical shells

2.1 Introduction

The problem of determining the external pressure at which a thin-walled cylinder of large diameter will collapse confronts the designers of steel stacks, bins, tanks, pipelines, conveyor galleries, and similar structures.

In the design of the above structures the collapsing pressure and the evaluation of the effect of stiffeners upon the strength of tubular structure are frequently encountered. After determining the cylinder plate thickness in order to satisfy tensile stress requirements, the stability of the shell should be checked for compressive stresses against buckling. This analysis is more complex because the general and local buckling of thin-walled cylindrical shell under different loading conditions should be investigated.

In a linear shell theory, displacements are proportional to loads. The essence of shell buckling, however, is a disproportionate increase in displacement resulting from a small increase in load. It becomes obvious that a nonlinear shell theory is required. Thus, shell buckling is fundamentally a sub-topic of nonlinear shell theory.

The purpose of this Chapter is to discuss stress analysis of the local and overall buckling of thin-walled large diameter tubular steel structures. Although tubular structures are susceptible to buckling, most structural standards or codes do not give complete design information on the buckling analysis of such structures. Presumably, the column buckling formulas given in these standards may be applied to tubular structures, because the standards do not restrict the formulas to any particular shape. However, the tubular structures especially those having relatively large ration D/t, as experience shows, cannot be safely designed by formulas given in standards. Furthermore, no limitations based on local buckling of tubular structures under compression, bending, or combined loadings are given in these standards.

If the purpose of engineering analysis is to predict the behavior of structures under a variety of loading conditions, then experience tells us that the buckling is often the prime consideration in the design of tubular structures.

Considering the stability analysis of tubular structures it is necessary to clarify the situation regarding such terminology as buckling and collapse.

Since the terms "buckling" and "collapse" are often used interchangeably, we define a buckle as a localized failure in the form of a wrinkle or indentation caused by overstress or instability of the pipe wall on the compressive side of a pipe subjected to bending.

Collapse, on the other hand, is defined as a general failure usually in the form of a flattening of the pipe cross-section over a considerable length as the result of the action of external pressure on the pipe.

2.2 Overall and Local Buckling

In the design of tubular structures, after determining the shell plate thickness in order to satisfy tensile stress requirements, the stability of the shell should be checked for compressive stresses against buckling.

A thin-walled cylindrical shell subjected to compression in the direction of its longitudinal axis may fail either by the instability of the shell as a whole, involving bending of the axis, or by the local instability of the wall of the shell which may not at all involve lateral distortion of the axis. The former type of failure is that investigated by Euler, when the strength depends on the ratio of length to the radius of gyration of the shell. The latter type of failure has been called by various authors: secondary flexure, crinkling, wrinkling, or local buckling, and is often the governing consideration in the design of thin-walled cylinders.

The stability against local buckling depends on the ratio of thickness to the radius of the shell wall (t/R). Wrinkling is local in nature and depends upon the combined compressive stresses at the point under consideration. Failure of this type is due to the formation of characteristic wrinkles or bulges, circular or lobed in shape, Figure 2.1.

In studying thin-walled tubular structures, two considerations are of importance. First, local buckling should be prevented at stresses below yield strength; secondly, a more severe restriction is that the tendency to buckle locally should not reduce the general buckling load of a whole structure.

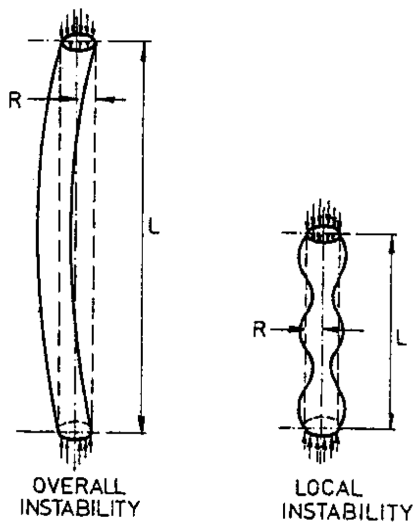

FIGURE 2.1 — Overall and local buckling.

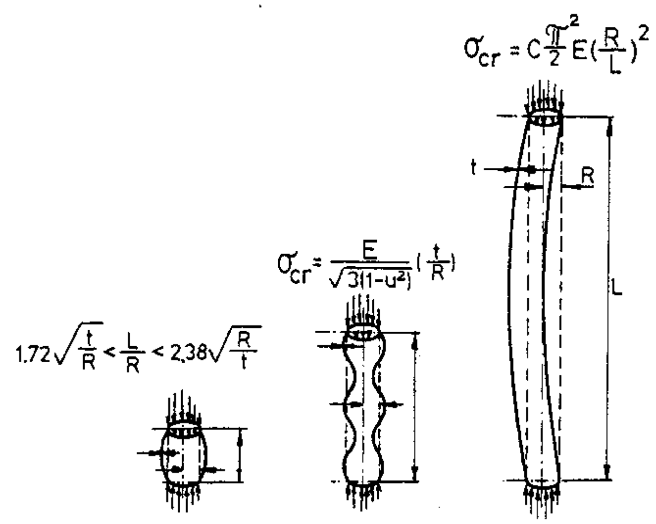

FIGURE 2.2 — Cylindrical shell subjected to axial compression.

2.3 A Paradox in the Buckling Analysis of Cylindrical Shell Under Axial Compression

To investigate the phenomena of the local and overall buckling of tubular structures, the classical approach was to investigate the fundamental case, namely — the buckling of cylindrical shell under axial compression. Solutions to this problem were obtained more than sixty years ago, first by Lorenz [2.1] in 1908, then by Timoshenko [2.2] in 1910 and by others. Lorenz used Euler's method and substantially simplified the problem, assuming that upon the loss of stability all generatrix of the cylinder were bent in equal manner, and that the axially-symmetric shape at the loss of stability had taken place, Figure 2.2.

He arrived at the classical expressions of buckling where the critical stress is equal

$$\sigma_{cr} = \frac{Et}{R\sqrt{3(1-\mu^2)}} = 0.6E(\frac{t}{R}) \qquad (2.1)$$

and the "classical buckling load" is

$$P_{c\ell} = \frac{Et}{\sqrt{3(1-\mu^2)}}(\frac{t^2}{R}) = 0.6E(\frac{t^2}{R}) \qquad (2.2)$$

where

- R = is the radius of the shell, that is, the inside radius plus one-half of the wall thickness
- t = thickness of the shell wall
- E = modulus of elasticity of material
- μ = 0.3 Poisson's ratio

Southwell [2.3], Dean [2.4] and Prescott [2.5] give for a lobed form of buckling the following formula

$$\sigma_{cr} = \frac{E}{\sqrt{3(1-\mu^2)}}(\frac{t}{R})\frac{n^2-1}{n^2+1} \qquad (2.3)$$

in which

- n = the number of lobes in the wrinkle.

In these derivations, it is assumed that the elastic limit of the material is not exceeded. In general, although different methods of approach were used, the same results were obtained, namely, for a uniform circular bulge or wrinkle, as shown in Figure 2.1. In any case, where the number of lobes is greater than 3, Eq. (2.3) gives substantially the same critical stress as Eq. (2.1).

It may be noted that Eqs. (2.1) and (2.2) for local buckling do not involve the length of the shell. That is, the critical local buckling stress is independent of the length of the shell. Nevertheless, in the case of long slender shells, the total load-carrying capacity is affected by the ratio of the length to the radius of gyration. For, if there is a tendency to buckle, the stress will no longer be uniform over a section, and failure will occur when the maximum stress on the section becomes equal to the critical buckling stress. When we pass from the local loss of stability to the overall, the ratio L/R lies within the following limits [2.6], Figure 2.3.

$$1.72\sqrt{t/R} < L/R < 2.38\sqrt{R/t} \qquad (2.4)$$

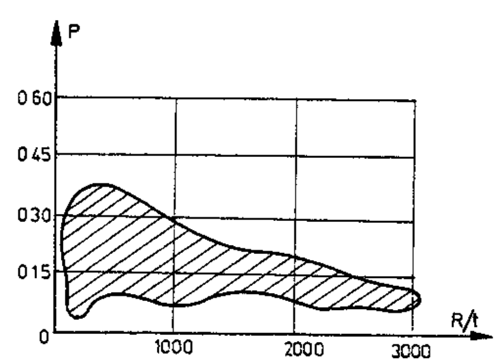

FIGURE 2.3 — Slenderness Ratios (L/R) for local buckling.

For a long slender cylindrical shell, failure does not occur by wrinkling, but by the type of buckling investigated by Euler. The unit stress at which buckling is likely to occur in this case is given by the formula

$$\sigma_{cr} = C\frac{\pi^2 E}{(L/r)^2} \qquad (2.5)$$

where

- σ_{cr} = unit stress at failure
- r = radius of gyration of the cross-section of the cylinder
- C = a constant, depending on the end conditions

For a very thin shell $r^2 = R^2/2$, and Eq. (1.5) becomes

$$\sigma_{cr} = C\frac{\pi^2}{2}E(\frac{R}{L})^2 \qquad (2.6)$$

When the experiments were made to check the validity of Eq. (2.1), certain behavioral patterns were observed which were completely at variance with the theory.

There is a serious disagreement between the results of classical and experimental stress for the buckling of isotropic cylindrical shells under axial compression.

Similar discrepancies can be observed for other loading conditions. These experiments indicated critical stress levels in the order of ⅓ of those given by the classical linear theory.

This paradox puzzled the investigators for more than 30 years!

The modern phase of the investigation of the buckling of thin-walled cylindrical shells subjected to axial compression began in 1940, with two papers published by Von Karman [2.7], [2.8] and his collaborators. They showed that the significant difference between the buckling stresses predicted by the theory and those observed in experiment could be attributed to the fundamentally nonlinear nature of the buckling process.

They obtained a lower value which is three times smaller than that given in the classical theory, or

$$\sigma_{cr} = 0.195E(\frac{t}{R}) \qquad (2.7)$$

Von Karman and Tsien also suggested that imperfections which were inevitable in the fabrication, such as initial irregularities in shape in the test cylinders, might cause a round-off of the sharp peak between the linear and nonlinear branches of the load-displacement curve, and, thus, result in a lower maximum point.

TABLE 2.1 — Critical Buckling Stress σ_{cr}.

Author	Year	Buckling Stress	References
Michielson	1948	$\sigma_{cr} = 0.194E(\frac{t}{R})$	[2.9]
Kirste	1954	$\sigma_{cr} = 0.187E(\frac{t}{R})$	[2.10]
Kempner	1954	$\sigma_{cr} = 0.182E(\frac{t}{R})$	[2.11]
Pogorelov	1967	$\sigma_{cr} = 0.160E(\frac{t}{R})$	[2.12]

Several investigators extended the Von Karman-Tsien analysis and found the lowest values of the buckling stress as shown in Table 2.1 [2.9, 2.10, 2.11, 2.12].

2.4 Imperfections of the Shell Shape and Edge Effect

Theoretical analyses [2.12; 2.13; 2.14; 2.16] of the effect of imperfections on the buckling behavior of cylinders have clearly demonstrated that relatively small imperfection amplitudes can drastically reduce the critical load of the shell.

Despite the substantial theory available, few experimental data [2.17; 2.18; 2.19] exist describing the effects of specific imperfections in shape in reducing the static buckling load. Consequently, it was of particular interest to determine the buckling load reduction caused by an initial axissymmetric imperfection in shape defined by a simple trigonometric function. This problem was investigated by Koiter.

Koiter [2.20; 2.21] developed a rigorous theory of the maximum load and showed that thin-walled circular cylindrical shells are very sensitive to small deviations in the initial, unstressed state from the exact circular cylindrical shape.

The results of the Koiter analysis indicate that an initial imperfection amplitude equal to the shell thickness is sufficient to reduce the buckling load to only 20 percent of the corresponding value for the perfect cylinder.

Investigations by Stein [2.22], Ohira [2.23] and Hoff [2.24] showed lower buckling load due to the edge effect. In the classical analysis the influence of the edge restraint on the prebuckling deformation was neglected. However, in reality, the diameter of the restrained cylinder tends to increase under axial compression due to Poisson's effect. This increase is prevented at the ends of the cylinder by its boundary restraints.

Hence, the generators of the cylinder are distorted prior to buckling and axial forces in the cylinder at the ends are eccentric relative to portions of the shell wall near their mid-length. When this eccentricity is considered, the theoretical prebuckling equilibrium becomes nonlinear.

2.5 Practical Application of Experimental Data

A disagreement between the theory and experiments considering the buckling of thin cylindrical shells under axial compression has lasted a long time, causing much disappointment, and inducing a number of new theories. And yet, when the final clarification arrived, the answer was simple and clear: one should have made better experiments and more extensive calculations!

Small imperfections of the test specimen, nonuniformity in loading, and small uncertainties in the control of boundary conditions have had large effect. To compare theory with experiment, a very careful analysis and experimental control would have to be made.

When we turn to experimental data in considering the magnitude of critical stress, this data is somewhat contra-

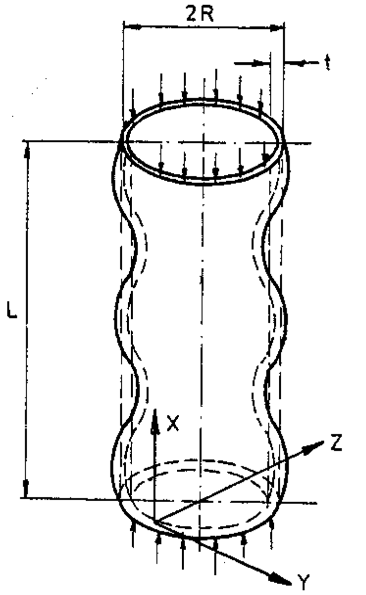

FIGURE 2.4 — Region of experimental data for critical compression stress.

dictory, since it strongly depends on initial imperfections in the form of the shell and in conditions of loading.

In Figure 2.4, a region of experimental values is shown of the buckling coefficient p in equation

$$\sigma_{cr} = pE\left(\frac{t}{R}\right) \qquad (2.8)$$

based on experiments performed by different investigators [2.25].

A significant part of the experiments leads to the values of p lying above 0.18. However, certain values lie below this magnitude and in separate cases turn out to be equal to 0.06 — 0.15. Figure 2.4 indicates an evident tendency of p to drop during the growth of ratio R/t. It is necessary to recall that with an increase of R/t, the probability of the appearance of initial imperfections should increase. This, undoubtedly, should lead to a lowering of the average magnitude of real critical stresses.

2.6 Allowable Design Stresses

In view of discrepancy between theoretical and experimental results, it seems advisable to rely largely on test results for developing adequate design provisions to safeguard against local buckling.

Donnell [2.26] developed the following formula for the ultimate buckling stress of circular cylinders in compression

$$\sigma_c = E\,\frac{0.6\frac{t}{R} - 10^{-7}\frac{R}{t}}{1 + 0.004\frac{E}{F_y}} \qquad (2.9)$$

where F_y is the yield stress of the material.

The formula is designed to give the average strengths to be expected, and if it is desired to know the minimum strength likely to be encountered under any circumstances, some factor must be used with it.

A systematic evaluation of test evidence obtained by a number of investigators was analyzed by Plantema [2.29]. The permissible compressive stress is given by the following formula:

$$\sigma_{cr} = \frac{662}{D/t} + 0.399\,F_y \qquad (2.10)$$

The ratio D/t is valid for

$$\frac{3{,}300}{F_y} < \frac{D}{t} < \frac{13{,}000}{F_y} \qquad (2.11)$$

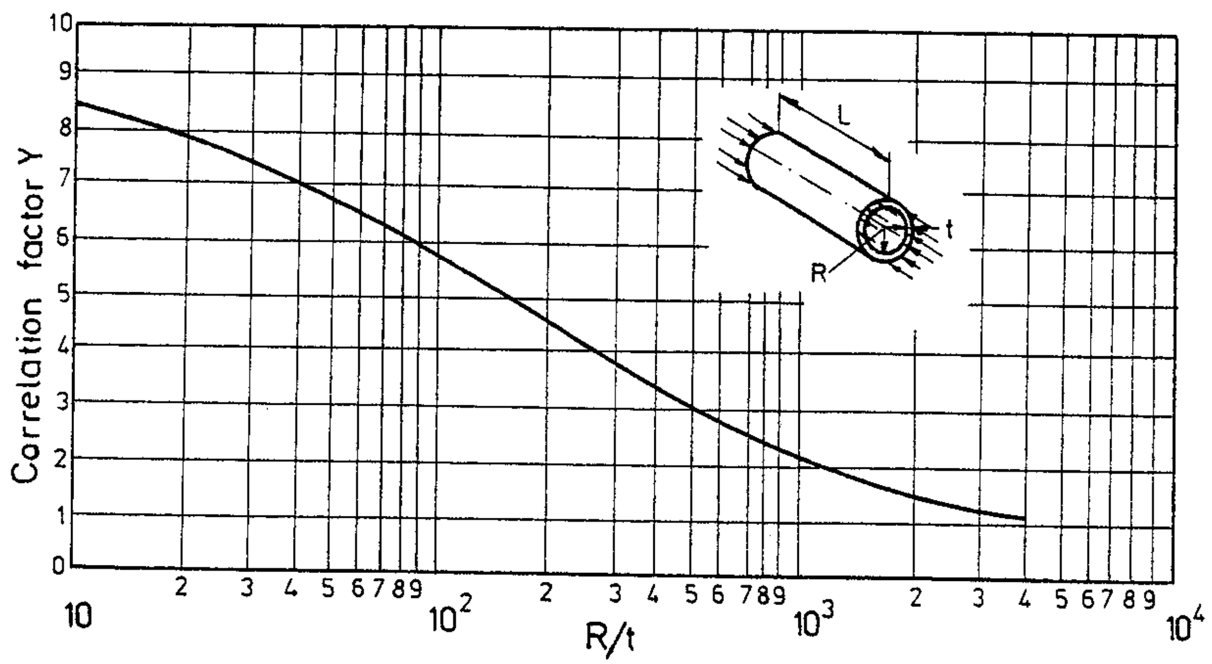

FIGURE 2.5 — Correlation factors for unstiffened circular cylinders subjected to axial compression.

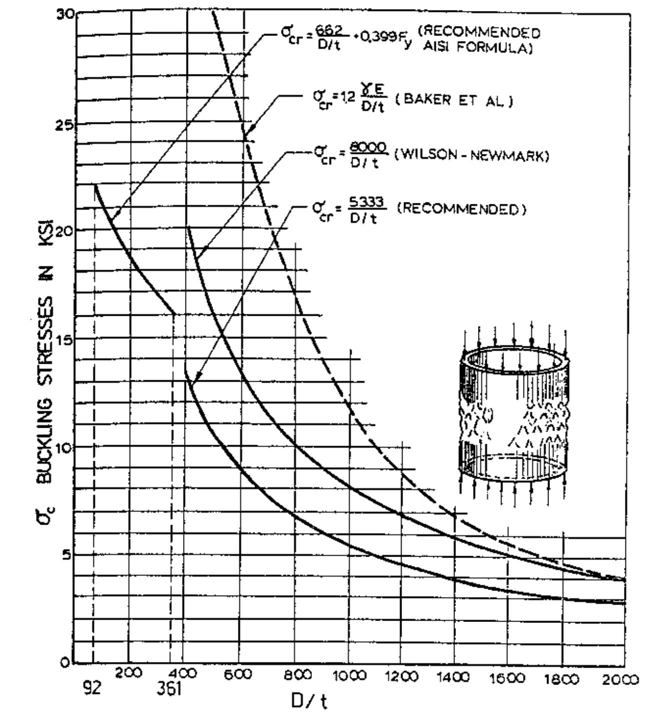

FIGURE 2.6 — The recommended allowable buckling stresses.

At yield point for mild steel F_y = 36 ksi, the limits are

$$92 < \frac{D}{t} < 361 \qquad (2.12)$$

Formula (2.10) is recommended by the American Iron and Steel Institute [2.28].

Baker et al [2.29] proposed the following formula for the determination of the local buckling stress for cylindrical shells of moderate length.

$$\sigma_{cr} = 0.6\gamma E\left(\frac{t}{R}\right) \qquad (2.13)$$

where the values of the correlation factor γ in the function of the ratio (R/t), are shown in Figure 2.5. The correlation γ is introduced to account for the difference between theoretical and experimental results.

Wilson and Newmark [2.30] carried tests using tubular steel compression members having a large D/t ratio. In the elastic range, the magnitude of the critical buckling stress is expressed as follows

$$\sigma_{cr} = \frac{8{,}000}{D/t}, \text{ ksi} \qquad (2.14)$$

Assuming the factor of safety = 1.5, the allowable critical stress for local buckling is

$$\sigma_{cr} = \frac{5{,}333}{D/t}, \text{ ksi} \qquad (2.15)$$

The recommended allowable stresses for local buckling in the function of D/t are shown in Figure 2.6 [2.31].

Long cylindrical shells must be checked for overall buckling as an Euler column, by the formula (2.6).

Buckling analysis of the cylindrical shell indicates that there has been little agreement between theoretical and experimental results for critical loads of shell structure, since apparently, infinitesimal deviations in boundary conditions and in the shape of the shell yield drastic reduction in critical loads. It is believed that accurate formulation of a problem in terms of nonlinear theory and exact solution of the equations would result in a close agreement between theoretical and experimental results. At present, for actual practice, however, this procedure is prohibitively difficult. Nonlinear theory serves to broaden our knowledge of shell buckling analysis and to clarify the meaning and limitations of linear stability theory, but at present, it is not a design tool for direct determination of the buckling load.

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CHAPTER 3

edge effect at tubular structures

3.1 Physical Concept

Stresses and deformations in thin-walled shells, determined by applying the membrane theory are correct only in zones located at certain distances from the changes of such geometrical parameters as shape, dimensions and stiffness and also from the places of sharp changes in acting forces.

For sections having changes in geometry, apart from forces, stresses and deformations which are determined in applying the membrane theory, additional forces, stresses and deformation which also originate are called — the edge effect.

Due to the elastic resistance of the adjoining parts, the edge effect does not spread too far, but rather acts upon relatively narrow zones. The edge effect is spread by relatively fast diminishing waves, the general character of which is shown in Figure 3.1, where at the axis x are ordinates of the wave curve and along the axis y - are plotted lengths of the generatrix of the shell.

The physical causes of origination of the edge effect are:

a. An absence of free deformation of the shell, under membrane stresses in a circumferential direction.

b. Sudden changes or eccentricity of the generatrix, which lead under axially-symmetrical loading to the origin of local forces, distributed along the circumference of shell as projections of meridional forces on a plane, normal to the axis, or on moments due to its eccentricity.

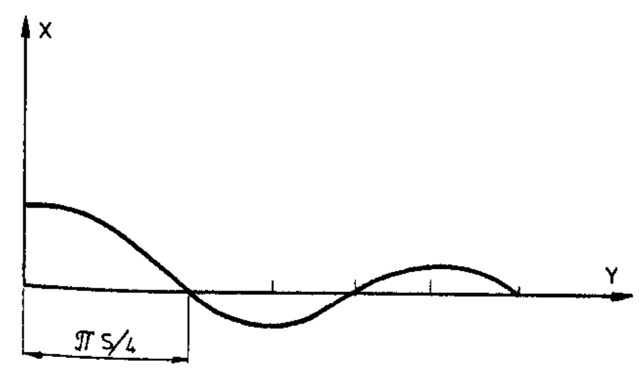

FIGURE 3.1 — Diagram of diminshing of edge effect. Axis Ox — location of origin of edge effect.

An example, shown in Figure 3.2 illustrates the original causes of edge effect.

At the elevation a-a under free deformation and loading P_r an increase of radii may be expressed as follows:

For a cylinder

$$\Delta r = \frac{r}{E}(\sigma_2 - \nu\sigma_1) = \frac{r^2 p}{E t_{cyl}}\left(1 - \frac{\nu}{2}\right) \qquad (3.1)$$

For a cone

$$\Delta R = \frac{R}{E}(\sigma_2 - \nu\sigma_1) = \frac{R^2 p}{E t_{cone}}\left(1 - \frac{\nu}{2}\right) \qquad (3.2)$$

where

σ_1 = longitudinal stress

σ_2 = circumferential stress

t_{cyl}, t_{con} = thicknesses of the walls of a cylinder and cone, respectively

ν = Poisson's ratio

E = modulus of elasticity of shell materials

In general cases

$$\Delta r \neq \Delta R \sin\beta \qquad (3.3)$$

and at the free elastic deformations results in a relative displacement at section a-a. However, the interconnection between the cylindrical and conical shells prevents free deformation at these shells at a level a-a which results in the origin of local bending at this level. Apart from this, the edge effect originates due to a break of the generatrix and the existence of the local circumferential forces in the plan a-a, which are projections of the meridional forces of conical shell. The presence of these forces cause different deformations and stresses to those of the parts of a shell located relatively at a distance from section a-a. This results in the local bending of shells at their interconnections. Therefore, in general, as shown in Figure 3.2, the edge effect is due to both causes.

In particular, at the ratio of a shell thickness

$$\frac{t_{cyl}}{t_{cone}} = \sin\beta \qquad (3.4)$$

and under internal or external uniform pressure, we have

$$\Delta r = \Delta R \sin\alpha \qquad (3.5)$$