5

Footings

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5.1 GENERAL

The purpose of a footing is to transfer safely to the ground the dead load of the superstructure (weight), and all other external forces acting upon it. The latter ones consist not only of general live load (as in dwellings, warehouses, industrial buildings, etc.) or fill (as in silos, bunkers, tank supporting structures, and similar), but also of the ground effects of various lateral forces such as wind, blast, or earthquake. In case any of these forces are of dynamic character, an allowance for impact should be included in the estimate of their magnitude unless a more exact evaluation is required. Footings also include foundation elements that are designed to resist uplift and overturning, and comprise in general all structural elements that will provide a stable base for the superstructure and safe transfer of all applied forces down to the ground.

The ways and means by which the transfer of these forces into the subsoil can be obtained are manyfold. They depend primarily on the type and magnitude of the loading, on the stiffness and structural behaviour of the superstructure, on the bearing capacity of the soil in general and on that of a certain stratum in particular, and on the depth below ground surface where this soil stratum is encountered, and on the depth of the groundwater level below grade. The type of foundation is also influenced, though to a lesser degree, by the geographical location and climatic condition of the site; by the time of the year when the construction is to be executed; by the speed of construction that is desired; by the availability of material, equipment and manpower; by the prevailing economical conditions; and by many other circumstances. The importance of these apparently secondary considerations can grow with the magnitude of the job and may, in certain cases, influence the successful outcome of an entire project. It is, therefore, not enough to approach the selection of the type of foundation with a cool, calm engineering mind; very often, experienced judgment that can envision advantages and foresee difficulties deserves serious consideration.

The type and magnitude of the loading will usually be furnished by the engineer designing the superstructure. It is up to the foundation engineer to collect all information regarding the purpose of the superstructure, the materials that will be used in its construction, its sensitivity to settlements in general and to differential settlements in particular, and all other pertinent information that may influence the successful selection and execution of the foundation design.

The evaluation of the maximum bearing capacity of the soil is to be made on the basis of soil mechanical considerations. The engineer designing the foundation should select the soil stratum that is most suitable for the support of the

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superstructure; he must assume the appropriate safety factor to arrive at the allowable bearing pressure; and finally, decide on the most economical type of foundation to be used. For this reason it is essential for a foundation engineer to possess a good knowledge of the problems that are involved in the design and behaviour of the superstructure, a certain familiarity with the basic principles of soil mechanics, and a good understanding of the interaction between both.

A detailed treatment of above topics does not fall within the scope of this handbook; however, a short discussion of the basic considerations affecting the evaluation and distribution of the bearing pressures under footing bases is given below.

5.2 EVALUATION OF BEARING PRESSURES AT FOOTING BASES

5.2.1 General Principles

The distribution of the bearing pressures under a concentrically loaded, infinitely stiff footing, with frictionless base, resting on an ideal, cohesionless or cohesive subsoil,[1,2] is generally known, and shown in Fig. 5-1. Under ordinary conditions few soils will exhibit such a behavior; no footing could be considered to be infinitely stiff. The distribution of the bearing pressure under somewhat flexible footings and ordinary soil conditions will be similar to those shown in Fig. 5-2; or it may assume any intermediate distribution. The assumption of a uniform bearing pressure over the entire base area of a concentrically loaded footing, as shown in Fig. 5-3, seems to be justified, therefore, for reasons of simplicity, and is common design practice. This assumption not only represents an average condition, but is usually on the safe side because most of the common soil types will produce bearing pressure distributions similar to that shown in Fig. 5-2a. The foundation designer, however, shall keep in mind that the assumption of a uniform bearing pressure distribution was primarily made for reasons of simplicity and may, in special cases, require adjustment.

Any footing that is held in static equilibrium solely by bearing pressures acting against its base has to satisfy the following basic requirements regardless of whether it is an isolated or a combined footing:

1. The resultant of all bearing pressures, acting against

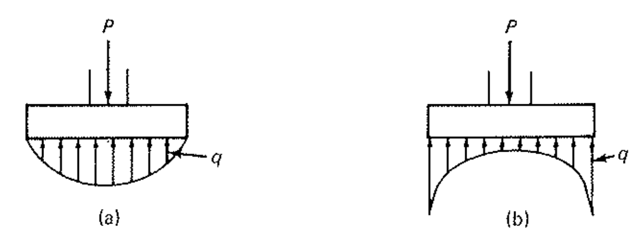


Fig. 5-1 Bearing pressure distribution for a stiff footing with frictionless base on ideal soil. (a) On cohesionless soil (sand); and (b) on cohesive soil (clay).

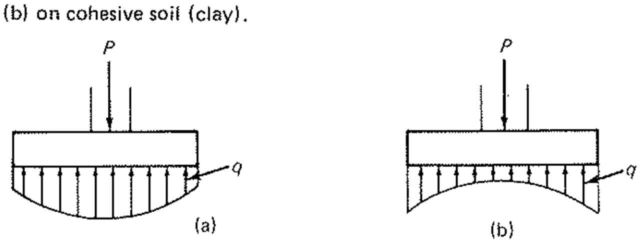


Fig. 5-2 Bearing pressure distribution for a flexible footing on ordinary soil. (a) On granular soil; and (b) on clayey soil.

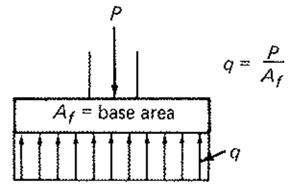


Fig. 5-3 Simplified bearing pressure distribution (commonly used).

the footing base (reaction), must be of equal intensity and and opposite direction as the resultant of all loads and/or vertical effects due to moments and lateral forces, acting on the footing element (action).

2. The location where the resultant vector of the reaction intersects the footing base must coincide with the location where the resultant vector of the action is applied. Action and reaction are as defined under (1) above.

3. The maximum intensity of the bearing pressures under the most severe combination of service loads must be smaller than, or equal to, the maximum bearing pressure allowed for this kind of loading and type of soil, as determined by principles of soil mechanics.

4. The resultant vector of the least favorable combination of vertical loads, horizontal shears, and bending moments that may occur under service load conditions, including wind or earthquake, must intersect the footing base within a maximum eccentricity that will provide safety against overturning.

The method most commonly used for the design of footings and related elements for ordinary building construction, is the one where static equilibrium is obtained by bearing pressures against the footing base only. This method is also the standard method that has been included in the "Building Code Requirements for Reinforced Concrete" ACI 318-71.

For zero eccentricities, the bearing pressures will be uniformly distributed over the entire base area of the footing as shown in Fig. 5-3 and will have the intensity of $q = P/A_F$.

If the footing shall restrain the column base, i.e., if a bending moment has to be resisted by the subsoil alongside with a concentric load, or if the column load is applied outside of the centroid of the base area of the footing, the bearing pressure distribution will vary depending on the magnitude of the eccentricity and its relationship to the kern distance c_k. The kern distance can generally be evaluated as shown in Fig. 5-4.

When the eccentricity is equal to, or smaller than, the kern distance c_k, the extreme (maximum or minimum) bearing pressures $q_{\min}^{\max}$ can be found by superposing the flexural bearing pressures over the axial bearing pressures, see Fig. 5-5a.

When the eccentricity becomes greater than the kern distance superposition cannot be applied anymore, because it would result in tensile stresses between soil and footing near the lifted edge of the base. Equilibrium can, however, be attained by resisting the load resultant by a bearing pressure resultant of equal magnitude and location. In this case the extreme bearing pressures at the edge of the base can be evaluated as shown in Fig. 5-5b. The maximum edge pressure $q_{\max}$ must, under all conditions, be smaller or equal than the maximum allowable soil pressure, q_a.

This condition applies until the excentricity, e, of the load, P, reaches the edge of the footing base. Any greater eccentricity will result in overturning. Such a condition, however, can only occur on rock or on very hard, stiff soils. For most practical cases, edge-yielding can make a footing

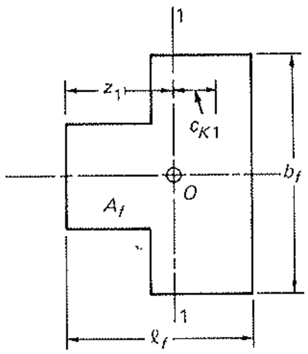


Fig. 5-4 Kern distance. (a) $c_{K1} = I_{F1}/A_{F1}z_1$; (b) I_{F1} = moment of inertia of footing base about neutral axis I-I; (c) z_1 = distance of extreme fiber at opposite side of desired kern distance; (d) for strips, $c_K = l_f/6$.

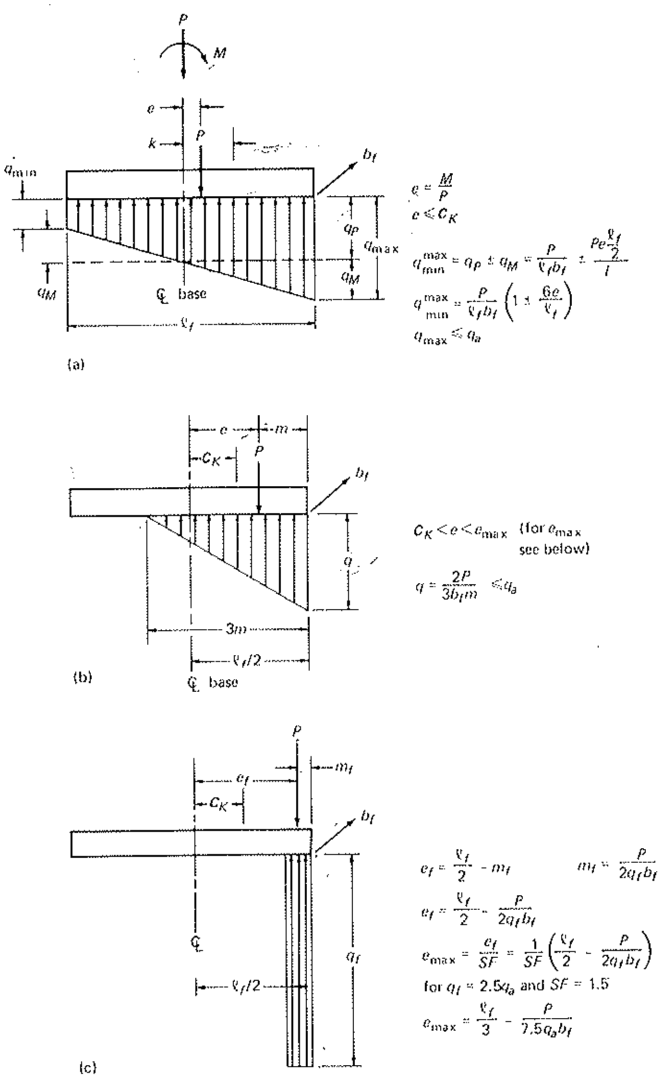


Fig. 5-5 Bearing pressure distribution under eccentric loading.

unusable and produce a condition that is equivalent to overturning. Edge-yielding will occur when the extreme bearing pressure at the pressed edge will cause failure in the bearing capacity of the subsoil. The eccentricity causing this condition will, therefore, limit the maximum useful eccentricity. Unless actual test results are available, the failure condition in the bearing capacity of the soil, q_f, can be assumed with about 2.5 times the allowable bearing capacity, q_a; the minimum safety factor against overturning is usually specified as 1.50; although somewhat greater safety factors are sometimes desirable. Introducing these requirements, we arrive in Fig. 5-5c at a maximum eccentricity, e_{max}, that can safely be utilized. How far the design engineer will take advantage of this condition will depend on his judgment of the soil and on the sensitivity of the superstructure to tolerate lateral tilting that may occur if a loading, causing such an eccentricity, is applied for a longer period.[5-3]

Moments occurring alongside with concentric loads, may be uniaxial or biaxial. If they occur in oblique directions it is most practical to have their influence divided into two perpendicular components, each of them parallel to the main axes of the footing mat, and superpose the resulting bearing pressures. Such conditions occur not only with isolated spread footings, but also with strip footings of limited length, as in the case of shear walls and similar.

Combined footings, (i.e., footings supporting more than one column load, such as exterior double-column footings), strip footings supporting spaced column loads, rafts, or mats can be designed as described above, as long as the entire foundation can be considered as infinitely stiff. In this case the resultant of all bearing pressures must be equal to the resultant of all loads, and its location must coincide with the eccentricity of the resultant. This approach is statically correct, but not necessarily close to the actual condition.

In certain cases it may be advisable to consider the footing as a beam on an elastic foundation and utilize the elasticity of the footing mat as well as that of the soil in the evaluation of the bearing pressures. The bearing pressures obtained by this method no longer follow a straight line distribution across the contact area. They show maximum accumulations immediately below, and in the vicinity of, concentrated loads and greatly reduced intensities between them, as shown in Fig. 5-6. Such a pressure distribution can reduce the maximum design moments of a foundation considerably and is therefore in many cases quite economical. This method is, in addition, intriguing in its setup and appealing especially to the mathematically inclined engineer.[5-4]

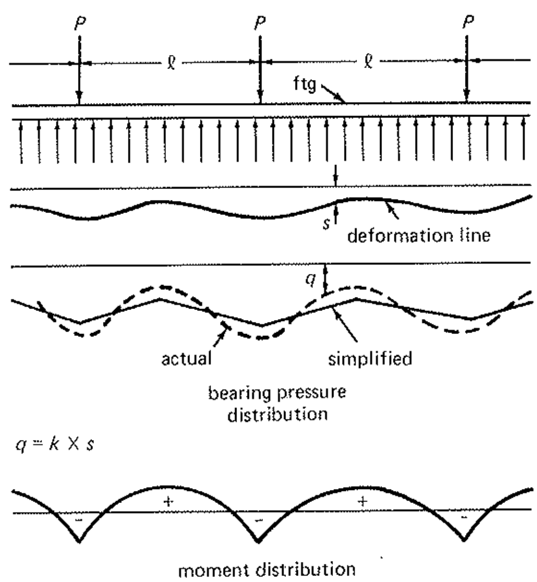


Fig. 5-6 General conditions for a beam on an elastic foundation.

In the analysis of a beam on elastic foundation, the soil is introduced as an elastic medium and the intensity of its reaction (bearing pressure) is assumed to be proportional to the deformation (settlement) of the footing, or soil, under the load.

Soils behave like elastic materials only to a limited degree and only small portions of the settlements can be recovered in case of unloading the superstructure. Settlements of the subsoil occur very seldom immediately or shortly after the load application. Most soils are rather insensitive to sudden load changes and react more to average loadings applied over an extended period. Variable configurations of the line of deformations during this period will cause a variable distribution of corresponding bearing pressures and internal stresses of the footing or foundation mat. In addition, the factor relating the magnitude of the bearing pressure to that of the deformation is not a constant but varies with the magnitude of the settlements.

Some of these characteristics have been overcome by refinements that have been introduced into this method, making it much more complex. If, however, foundations under combined footings are not too stiff and they follow the deformations of the subsoil, the beam on elastic foundation-method will furnish bearing pressure distributions that are closer related to the actual condition and more economical than the straight line distribution of the bearing pressures.

Due to the basic deficiencies of a foundation design executed with the help of elasticity relationships, as described in this chapter, it is advisable that results obtained by this method are not taken as exact solutions but more as a guide to arrive at a reasonable distribution of the bearing pressure; such foundations shall be designed with ample reserve capacity to sustain deviations from the theoretical findings. Under such considerations, the method can provide an excellent tool and a valuable aid in the design of sometimes rather difficult foundation problems.[5-5]

Figure 5-6 shows the basic relationships between loading, deformation and bearing pressure as well as the resulting moment distribution. Formulas giving the bearing pressures and bending moments for a strip designed as a beam on elastic foundation are given later in Fig. 5-22.

So far, bearing pressures against the footing base have been considered as the only means to resist external forces. Static equilibrium of a footing element subjected to lateral forces and/or moments in addition to vertical loads, can also be attained with the help of passive earth pressure, as shown in Fig. 5-7. In this case lateral (passive) pressure of the soil is developed while resisting either lateral movements or tilting tendencies of the footing element. This method is sometimes used in the design of footings supporting tall, light superstructures such as transmission towers, light poles, etc. One of the drawbacks of this method is that foundations are usually surrounded by backfill of questionable lateral resistance; but even where the footing is cast tightly against the original subsoil, passive earth pressure will only develop (unless the soil is very stiff or hard) after a certain movement or tilting of the superstructure has taken place.

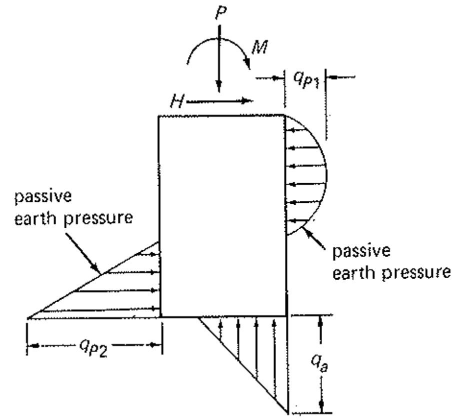


Fig. 5-7 General distribution of base and side pressures of a footing resisting overturning moments, where q_{p_1} and q_{p_2} are passive earth pressures due to loading and q_{p_1} and q_{p_2} are to be $\leq$ the maximum allowable passive earth pressure that can be utilized at each depth.

5.2.2 Strip Footings

-1. Concentrically loaded strip footings—The loadings of strip footings considered in the following section consist primarily of continuous line loads such as walls, or closely spaced, concentrated loads arranged in one direction. In many cases, the element causing the load, e.g., the wall itself, may provide sufficient stiffness to permit the assumption of a continuous line load, even if the intensity of the loading slightly varies (Fig. 5-8). Concentrated loads at close spacings sometimes require a stiffening member to transform the effects of the concentrated loads into a continuous loading. For small variations in the load intensity, the stiffening member may be the footing itself; or a foundation wall resting on top of the footings; or a beam formed by composite action of both. In either case the stiffening member has to be designed as an inverted continuous beam, upwardly loaded by a line load made up of the bearing pressure, q, and intermittently supported where the loads are applied, as shown in Fig. 5-9.

-2. Eccentrically loaded strip footings (or strip footings with concentric load and moment at base)—For stability considerations, a free standing wall is equivalent to one that is fully restrained at its base. Any shear and bending moment caused by lateral forces such as wind, fill, earth-pressure, simulated earthquake loadings, etc., as well as all moments and shears caused by eccentrically applied vertical loads acting on projecting piers or brackets, have to be transferred into the subsoil, see Fig. 5-10.

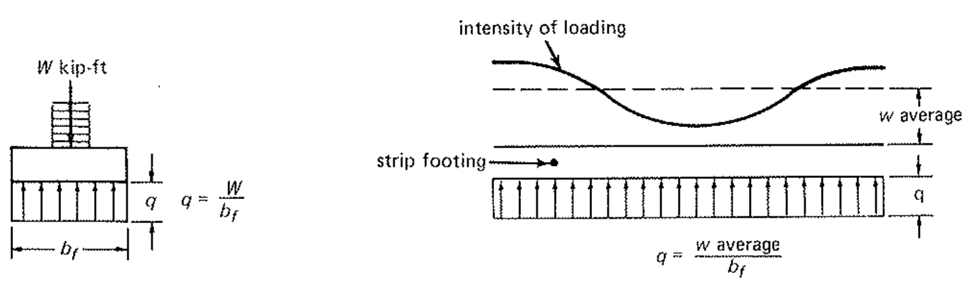


Fig. 5-8 Equalizing strip loadings of variable intensity.

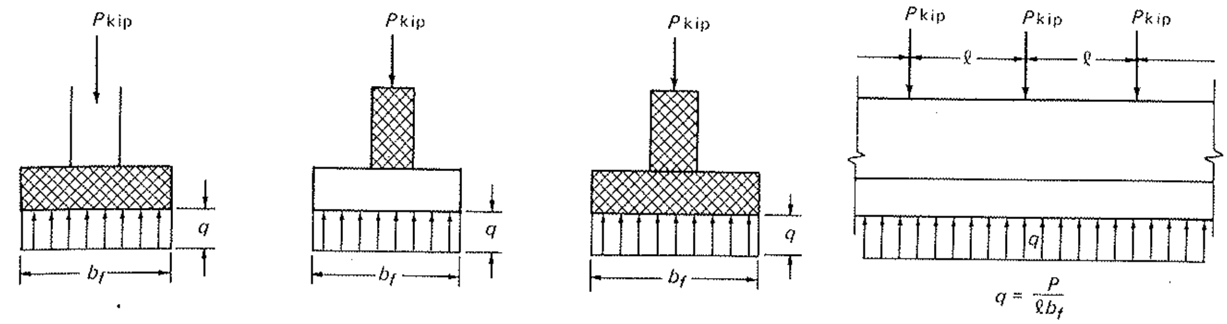


Fig. 5-9 Transforming concentrated loads into equal bearing pressures.

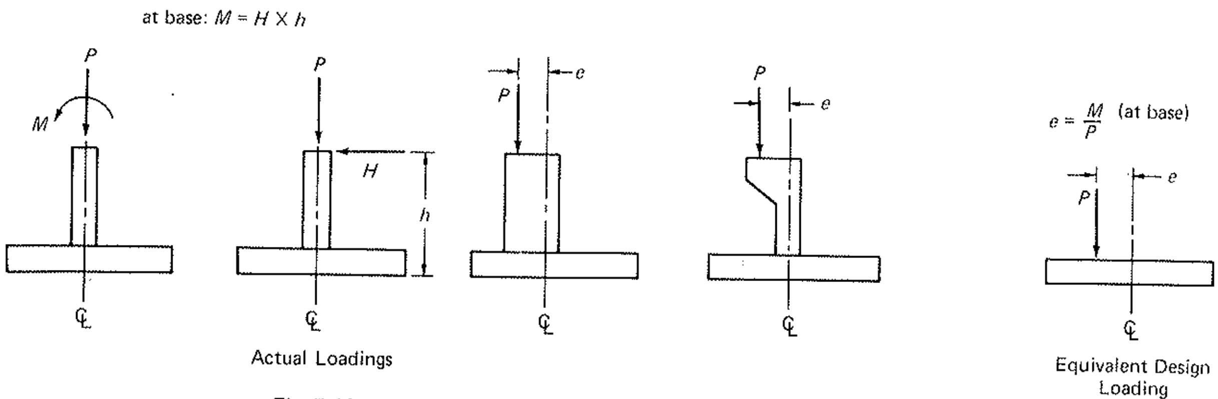


Fig. 5-10 Analogy between moments and eccentric loading conditions.

In cases where walls are not free standing but form a part of a frame work, such as in monolithic floor-wall constructions, the wall may transfer a certain amount of moment into the ground. How successful such a design assumption may be, deserves a brief discussion.

It can be proven by statics that only small, lopsided settlements are needed to reduce the restraining effect at the base of a frame work to such a degree that it may be rendered useless. Such a rotation would produce a condition at the base that is in effect similar to that of a hinge and would increase the moments in the upper frame by the amount lost at the base. Cautious designers assume and design the base of such frameworks as hinges and make the structure this way independent of the deformation of the subsoil.

Bending moments or eccentric loadings caused by wind or earthquake simulating forces may very well be taken by restraint at the base of the footing mat; such forces are of short duration and stable soils usually respond rather elastically to loads of short duration. It is however recommended that bending moments and eccentric forces caused by dead loads, fill, or average service live load conditions should not be transferred into the soil unless the soil is very stiff or well consolidated.

For intermediate conditions, partial restraint in an amount selected by judgement, may be utilized.

5.2.3 Isolated Spread Footings (square and oblong)

-1. Concentrically loaded isolated spread footings—Isolated spread footings are used to resist column loads that are concentric with the centroid of the footing base. Theoretically such footings may have any shape. Square and oblong bases are most common, but round and polygonal shapes are being used under certain conditions. Footings may be solid or without center portion, like a ring. We speak of a concentric loading if the resultant of all acting forces coincides with the centroid of the footing mat or bearing area, regardless of its shape. In this case the bearing pressure distribution to be used for the design is usually assumed to be of uniform intensity over the entire area, see Fig. 5-3.

-2. Eccentrically loaded isolated spread footings or isolated footings with concentric load and moment at base—In case of eccentric loadings caused by a moment at the column base, or by an eccentrically applied load, or by an unsymmetrical footing base, the bearing pressures will deviate from the uniform distribution and gradually vary across the footing base. The distribution of the bearing pressures will follow the rules as described above and shown in Fig. 5-5.[5-3] Fig. 5-11 gives the base area, moment of inertia; and kern distance for various shapes of footing bases[5-6]. For irregular shapes, it can be found as indicated in Fig. 5-4.

Round and polygonal footings, whether solid or ring-shaped, can only have a uniaxial eccentricity under the resultant of such a loading. Square and rectangular footings may have eccentricities in the direction of either one or both main axes, if the eccentricity falls in an oblique direction.

Figures 5-12 to 5-16 give the intensity of the extreme bearing pressure, and the location of the zero pressure, for various shapes of footings under eccentric loading.[5-6, 5-7]

Moments affecting the bearing pressures shall be determined about the elevation of the footing base. For evaluating the bearing pressures by means of Figs. 5-12 to 5-16, moments shall be converted first into eccentric loadings having the magnitude of the resultant loading, P, acting at the eccentricity $e = M_B/P$. In the evaluation of the resultant P it is important to investigate all conditions of loading that may occur simultaneously with the moment under consideration, i.e., the maximum as well as the minimum. In

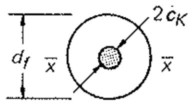
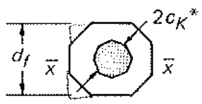
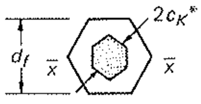
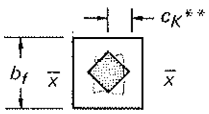
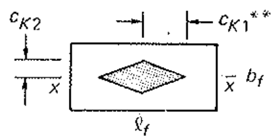
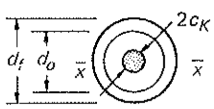
Shape of Base Area		Base Area A_F	Moment of Inertia I_x	Kern Distance c_k
CIRCLE		$0.785 d_f^2$	$0.049 d_f^4$	$0.125 d_f$
OCTAGON		$0.828 d_f^2$	$0.055 d_f^4$	$0.122 df$
HEXAGON		$0.866 d_f^2$	$0.060 d_f^4$	$0.120 df$
SQUARE		b_f^2	$0.083 b_f^4$	$0.167 bf$
RECTANGLE		$l_f b_f$	$0.083 l_f b_f^3$	$c_{k1} = 0.167 l_f$ $c_{k2} = 0.167 b_f$
RING		$0.785(d_f^2 - d_0^2)$	$0.049(d_f^4 - d_0^4)$	$\frac{d_f}{8}\left(1 + \frac{d_0^2}{d_f^2}\right)$

Fig. 5-11 Area properties of various cross sections. $^*c_K$ = radius of circle inscribed in polygonal kern area. $^{**}c_K$ = kern distance on main axis. Shaded area in diagrams indicates kern.

Values of C_1 and C_2 for Various Values of e/l_f

e/l_f	C_1	C_2
0.000	–	1.000
0.025	–	1.150
0.050	–	1.300
0.075	–	1.450
0.100	–	1.600
0.125	–	1.750
0.150	–	1.900
0.167	1.000	2.000
0.175	0.975	2.051
0.200	0.900	2.222
0.225	0.825	2.424
0.250	0.750	2.667
0.275	0.675	2.962
0.300	0.600	3.333
0.325	0.525	3.809
0.333	0.500	4.000
0.350	0.450	4.444
0.375	0.375	5.333
0.400	0.300	6.667
0.425	0.225	8.889
0.450	0.150	13.333
0.475	0.075	26.667
0.500	0.000	∞

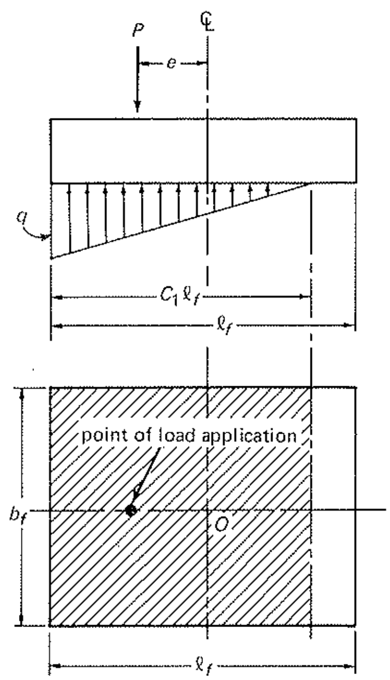


Fig. 5-12 Bearing pressure under square and oblong bases. Notation used includes: $A_F = l_f b_f$; $q_P = P/A_F$; $e = M/P$; $q = C_2 q_p$; and q_P = bearing pressure under concentric loading.

Values of C_1 and C_2 for Various Values of $e/b_f\sqrt{2}$

$e/b_f\sqrt{2}$	C_1	C_2
0.000	–	1.000
0.025	–	1.30
0.050	–	1.60
0.075	–	1.90
0.083	1.000	2.00
0.100	0.915	2.21
0.110	0.870	2.34
0.120	0.832	2.48
0.130	0.796	2.63
0.140	0.766	2.80
0.150	0.736	2.97
0.160	0.707	3.16
0.170	0.680	3.37
0.180	0.654	3.61
0.190	0.628	3.85
0.200	0.604	4.14
0.210	0.582	4.44
0.220	0.561	4.77
0.230	0.540	5.14
0.240	0.521	5.54
0.250	0.500	6.00
0.300	0.400	9.38
0.350	0.300	16.67
0.400	0.200	37.50
0.450	0.100	150.00
0.500	0.000	∞

Values of C_1 and C_2 for Various Values of e/d_f

e/d_f	C_1	C_2
0.000	–	1.00
0.025	–	1.20
0.050	–	1.40
0.075	–	1.60
0.100	–	1.80
0.125	1.000	2.00
0.150	0.910	2.23
0.175	0.830	2.48
0.200	0.755	2.76
0.225	0.685	3.11
0.250	0.615	3.55
0.275	0.550	4.15
0.294	0.500	4.69
0.300	0.485	4.96
0.325	0.420	6.00
0.350	0.360	7.48
0.375	0.295	9.93
0.400	0.235	13.87
0.425	0.175	21.08
0.450	0.120	38.25
0.475	0.060	96.10
0.500	0.000	∞

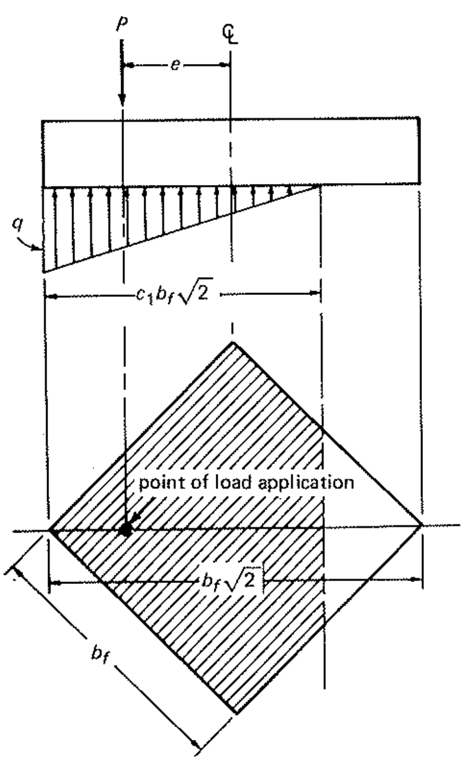


Fig. 5-13 Bearing pressure under square bases (about diagonal axis). Notation same as in Fig. 5-12 except for $A_F = b_f^2$.

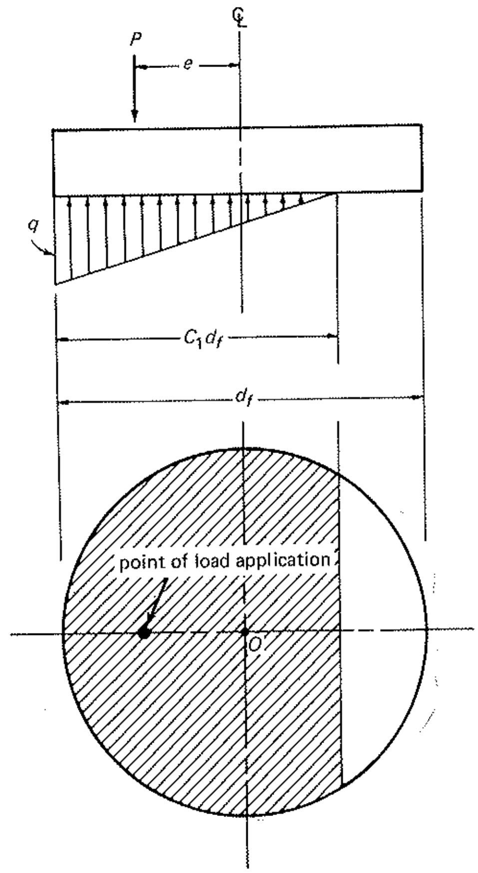


Fig. 5-14 Bearing pressure under circular and polygonal bases. Notation same as in Fig. 5-12 except for $A_F = d_f^2\pi/4$.

NOTE: For hexagonal bases use $d_f = 1.077\ d_{fo}$.

For octagonal bases use $d_f = 1.041\ d_{fo}$, where d_{fo} = diameter of inscribed circle.

Values of C_1 and C_2 for Various Values of e/d_f and d_o/d_f

	C_1 – Values d_o/d_f						
e/d_f	0.0	0.5	0.6	0.7	0.8	0.9	1.0
0.125	2.00	–	–	–	–	–	–
0.150	1.82	–	–	–	–	–	–
0.175	1.66	1.89	1.98	–	–	–	–
0.200	1.51	1.75	1.84	1.93	–	–	–
0.225	1.37	1.61	1.71	1.81	1.90	–	–
0.250	1.23	1.46	1.56	1.66	1.78	1.89	2.00
0.275	1.10	1.29	1.39	1.50	1.62	1.74	1.87
0.300	0.97	1.12	1.21	1.32	1.45	1.58	1.71
0.325	0.84	0.94	1.02	1.13	1.25	1.40	1.54
0.350	0.72	0.75	0.82	0.93	1.05	1.20	1.35
0.375	0.59	0.60	0.64	0.72	0.85	0.99	1.15
0.400	0.47	0.47	0.48	0.52	0.61	0.77	0.94
0.425	0.35	0.35	0.35	0.36	0.42	0.55	0.72
0.450	0.24	0.24	0.24	0.24	0.24	0.32	0.49
0.475	0.12	0.12	0.12	0.12	0.12	0.12	0.25
0.500	0.00	0.00	0.00	0.00	0.00	0.00	0.00

	C_2 – Values d_o/d_f						
e/d_f	0.0	0.5	0.6	0.7	0.8	0.9	1.0
0.000	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.025	1.20	1.16	1.15	1.13	1.12	1.11	1.10
0.050	1.40	1.32	1.29	1.27	1.24	1.22	1.20
0.075	1.60	1.48	1.44	1.40	1.37	1.33	1.30
0.100	1.80	1.64	1.59	1.54	1.49	1.44	1.40
0.125	2.00	1.80	1.73	1.67	1.61	1.55	1.50
0.150	2.23	1.96	1.88	1.81	1.73	1.66	1.60
0.175	2.48	2.12	2.04	1.94	1.85	1.77	1.70
0.200	2.76	2.29	2.20	2.07	1.98	1.88	1.80
0.225	3.11	2.51	2.39	2.23	2.10	1.99	1.90
0.250	3.55	2.80	2.61	2.42	2.26	2.10	2.00
0.275	4.15	3.14	2.89	2.67	2.42	2.26	2.17
0.300	4.96	3.58	3.24	2.92	2.64	2.42	2.26
0.325	6.00	4.34	3.80	3.30	2.92	2.64	2.42
0.350	7.48	5.40	4.65	3.86	3.33	2.95	2.64
0.375	9.93	7.26	5.97	4.81	3.93	3.33	2.89
0.400	13.87	10.05	8.80	6.53	4.93	3.96	3.27
0.425	21.08	15.55	13.32	10.43	7.16	4.50	3.77
0.450	38.25	30.80	25.80	19.85	14.60	7.13	4.71
0.475	96.10	72.20	62.20	50.20	34.60	19.80	6.72
0.500	∞	∞	∞	∞	∞	∞	∞

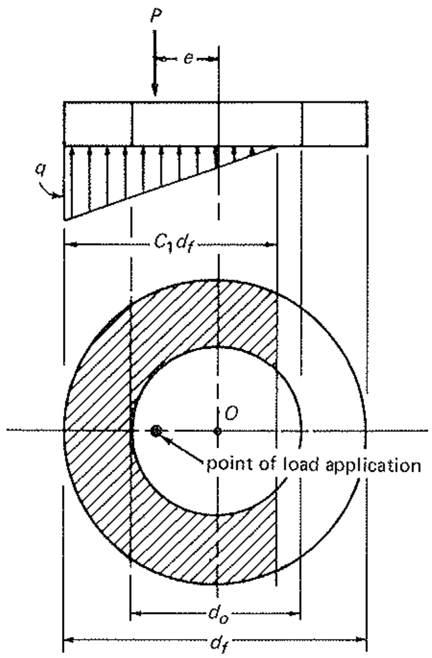


Fig. 5-15 Bearing pressure under ring-shaped bases. Notation same as in Fig. 5-12 except for $A_F = (d_f^2 - d_o^2)\,\pi/4$.

the calculation of the minimum loading it is advisable to reduce dead loads arbitrarily, similarly as required in the *ACI Building Code 318-71*, section 9.3, to ascertain that the bearing pressure and/or eccentricity resulting from this loading condition is a maximum. See also section 5.3.1.

5.2.4 Combined Footings

Independent, isolated footings for each column load, usually provide the most economical method to design a foundation. Support of two or more column loads on one combined footing should be attempted only when required by one of the following conditions:

a. Proximity of the building line or any other space limitation adjacent to a building column.
b. Overlapping of adjacent isolated column footings.
c. Insufficient bearing capacity of subsoil requiring large bearing areas.
d. Sensitivity of the superstructure to differential settlements.
e. Advantage in construction procedure, such as trench excavation or similar.

The first condition also includes all cases where elements of existing structures, underground facilities, excavations, embankments, etc., limit the extent of the foundation to be constructed; it is a common case for exterior building columns. If such a condition does not permit the design of a symmetrical column footing, the footing can be combined with the adjacent footing, located on the axis perpendicular to the limiting line, to balance its eccentricity. Depending on the difference in the intensity of the two combined column loads, as well as on the existence of any simultaneous moments or shears, the eccentricity of the resultant load and its magnitude will govern the shape and size of the required base area for the footing. Any shape of base area can be used for the footing as long as it will satisfy the basic requirements discussed in section 5.2.1. Rectangular, trapezoidal, and T-shaped footings are most commonly used. Figures 5-17 through 19 provide the necessary formulae to determine the required areas and geometrical

(*continued on p. 121*)

Values of C_3 for Various Values of e_l/l_f; e_b/b_f

e_l/l_f	e_b/b_f													
	0.50	0.40	0.30	0.20	0.18	0.16	0.14	0.12	0.10	0.08	0.06	0.04	0.02	0.00
0.50	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.40	∞	37.5	18.8	12.5	11.6	10.9	10.2	9.6	9.0	8.5	8.0	7.5	7.1	6.7
0.30	∞	18.8	9.4	6.2	5.8	5.4	5.1	4.8	4.5	4.2	4.0	3.8	3.5	3.3
0.20	∞	12.5	6.2	4.1	3.9	3.6	3.4	3.2	3.0	2.8	2.7	2.5	2.4	2.2
0.18	∞	11.6	5.8	3.9	3.6	3.4	3.2	3.0	2.8	2.6	2.5	2.3	2.2	2.1
0.16	∞	10.9	5.4	3.6	3.4	3.2	3.0	2.8	2.6	2.5	2.3	2.2	2.1	2.0
0.14	∞	10.2	5.1	3.4	3.2	3.0	2.8	2.6	2.5	2.3	2.2	2.1	2.0	1.8
0.12	∞	9.6	4.8	3.2	3.0	2.8	2.6	2.5	2.3	2.2	2.1	2.0	1.8	1.7
0.10	∞	9.0	4.5	3.0	2.8	2.6	2.5	2.3	2.2	2.1	2.0	1.8	1.7	1.6
0.08	∞	8.5	4.2	2.8	2.6	2.5	2.3	2.2	2.1	2.0	1.8	1.7	1.6	1.5
0.06	∞	8.0	4.0	2.7	2.5	2.3	2.2	2.1	2.0	1.8	1.7	1.6	1.5	1.4
0.04	∞	7.5	3.8	2.5	2.3	2.2	2.1	2.0	1.8	1.7	1.6	1.5	1.4	1.2
0.02	∞	7.1	3.5	2.4	2.2	2.1	2.0	1.8	1.7	1.6	1.5	1.4	1.2	1.1
0.00	∞	6.7	3.3	2.2	2.1	2.0	1.8	1.7	1.6	1.5	1.4	1.2	1.1	1.0

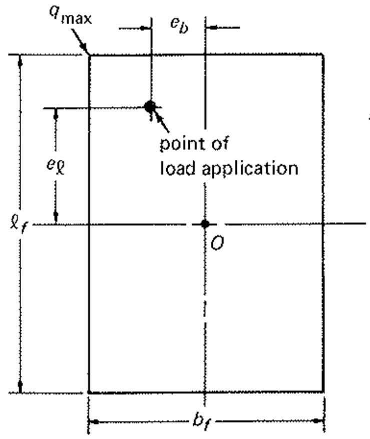
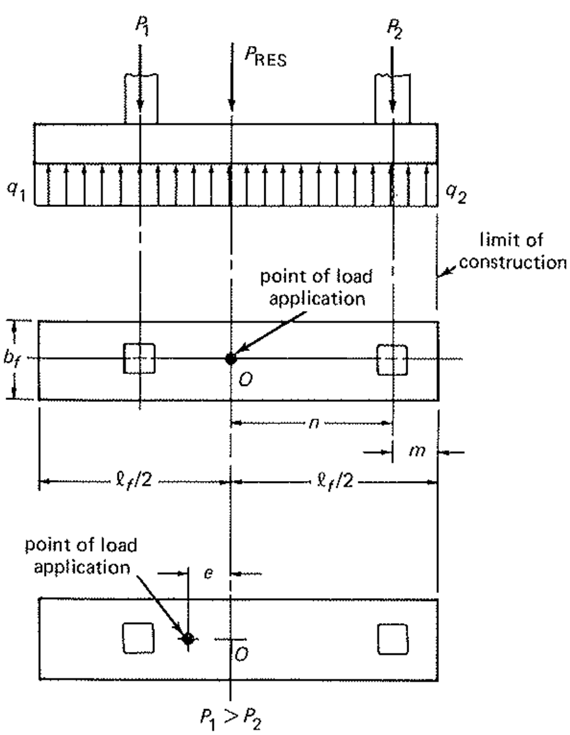


Fig. 5-16 Maximum corner pressure for biaxial eccentricities. Notation same as Fig. 5-12 except for $q_{max} = C_3 q_p$.



1) For uniform bearing pressure

$$l_f = 2(m+n), \quad P_{RES} = P_1 + P_2$$

$$b_f = \frac{P_{RES}}{q\, l_f}, \qquad q_1 = q_2 = q$$

2) For eccentric loading e on either side of O; q_1 = bearing pressure at loaded edge

for $e \leqslant \dfrac{l_f}{6}$

$$\frac{q_1}{q_2} = \frac{P_{RES}}{b_f\, l_f}\left(1 \pm \frac{6e}{l_f}\right)$$

for $e > \dfrac{l_f}{6} < e_{max}$

$$q_1 = \frac{2P_{RES}}{3\left(\dfrac{l_f}{2} - e\right) b_f}$$

$q = 0$ at a distance $3\left(\dfrac{l_f}{2} - e\right)$ from loaded edge (for e_{max} see Fig. 5-5c).

Fig. 5-17 Strap footing.

For uniform bearing pressure (q) (general relationship)

$$\frac{b_f}{b_{f1}} = \frac{3(n+m) - l_f}{2l_f - 3(n+m)}$$

$$(b_f + b_{f1}) = \frac{2P_{\text{RES}}}{q l_f}$$

$$C_1 = \frac{l_f(2b_f + b_{f1})}{3(b_f + b_{f1})}; \quad C_2 = \frac{l_f(b_f + 2b_{f1})}{3(b_f + b_{f1})}$$

for uniform bearing pressure of known magnitude (q).*

for $P_1 > P_2$

$$b_f = \frac{6P_{\text{RES}}}{l_f^2 q}\left(n_1 + m - \frac{l_f}{3}\right)$$

$$b_{f1} = \frac{2P_{\text{RES}}}{q l_f} - b_f$$

for $P_1 < P_2$

$$b_f = \frac{6P_{\text{RES}}}{l_f^2 q}\left[\frac{2}{3} l_f - n_2 - m\right]$$

$$b_{f1} = \frac{2P_{\text{RES}}}{q l_f} - b_f$$

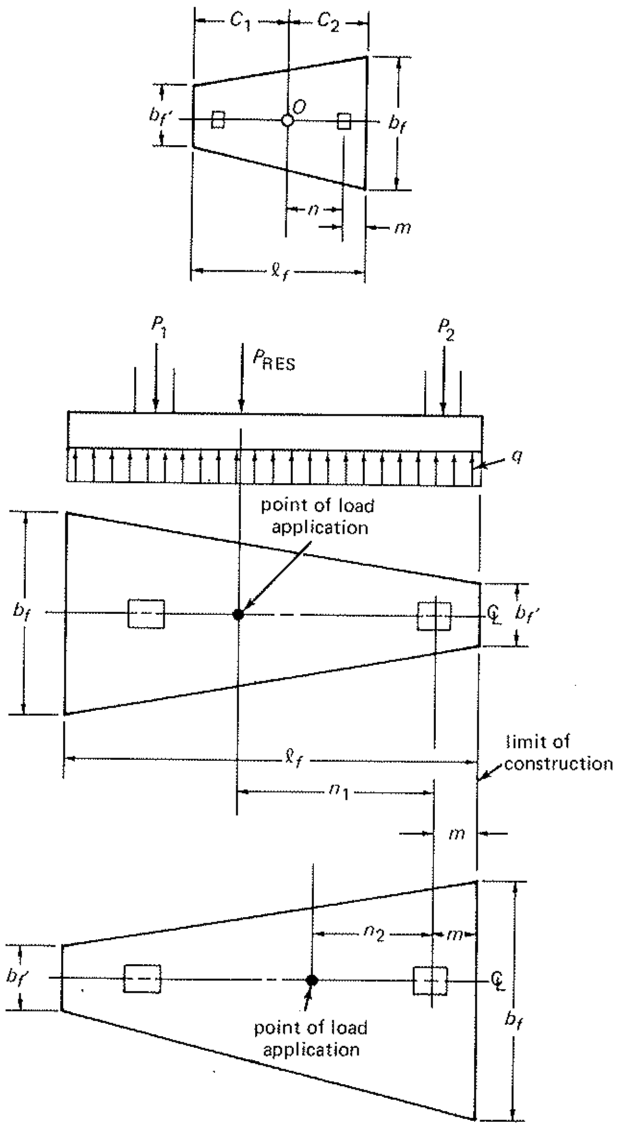


Fig. 5-18 Trapezoidal footing. *For non-uniform bearing pressure under eccentric loadings see: A. Zweig, "Eccentrically loaded trapezoidal or round footings," *Proceedings ASCE*, ST-1, 161-168, Feb. 1966.

$$P_1 < P_2$$

for uniform bearing pressure $q_1 = q_2 = q$

$$b_1 = \frac{P_{\text{RES}}}{q}\left[\frac{2(n+m) - l_2}{l_1(l_1 + l_2)}\right]$$

$$b_2 = \frac{P_{\text{RES}}}{l_2 q} - \frac{l_1 b_1}{l_2}$$

area $A_F = l_1 b_1 + l_2 b_2$

$$n + m = \frac{l_1^2 b_1 + 2 l_1 b_1 l_2 + l_2^2 b_2}{2(l_1 b_1 + l_2 b_2)}$$

$$c_{K1} = \frac{I}{(n+m) A_F}, \quad c_{K2} = \frac{I}{[l - (n+m)] A_F}$$

for eccentric load application:

$$e \leqslant c_k$$

$$q_1 = \frac{P_{\text{RES}}}{A_F} \mp \frac{P_{\text{RES}}\, e[l - (n+m)]}{I}$$

$$q_2 = \frac{P_{\text{RES}}}{A_F} \pm \frac{P_{\text{RES}}\, e(n+m)}{I}$$

The upper sign applies if e is towards wide side. The lower sign applies if e is towards narrow side.

$$e > c_K$$

if e is towards wide side

$$q_2 = \frac{2P_{\text{RES}}}{3b[(n+m) - e]}$$

if e is towards narrow side

$$q_1 = \frac{2P_{\text{RES}}}{3b_1[l - (n+m+e)]}$$

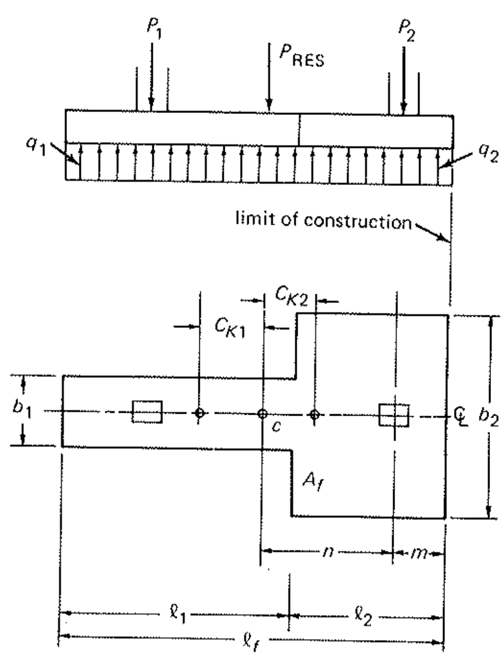


Fig. 5-19 T-shaped footings.

relations to design such footings for uniform bearing pressure; or to evaluate the extreme bearing pressures for a given footing shape and size in case of eccentric load applications. It is recommended to maintain bearing pressures over the entire base area of a combined footing, i.e., to design it in such a way that the eccentricity does not exceed the kern distance. Under extreme conditions, however, combined footings may also be designed for partial bearing with exclusion of tensile stresses at the footing base, as indicated.

If the nearest column, which can be utilized for the design of a combined footing, is too far away to permit a combined footing to be built economically, counterweights, or deadmen, can be provided to balance the eccentric loading of a footing as shown in Fig. 5-20. In such a case it is advantageous to design the footing so as to establish concentric loading for each element; otherwise, edge pressure conditions have to be investigated carefully. In the evaluation of the available weight on a deadman, it is recommended to rely primarily on the actual developed weight, and disregard load spreading or internal friction as far as possible. The safety factor to be used in the design of such foundations shall be at least the same as used for overturning.

Referring to condition (b), if overlapping of isolated single footings occurs, combination of the two footings into one will permit greater lateral spreading at maintaining a symmetrical base area, see Fig. 5-21.

In (c) and (d) above, it can easily be seen that a combination of two or more column loads by means of strips, rafts, or mats will not only spread the load over a bigger area, but will also give the foundation a monolithic quality which will

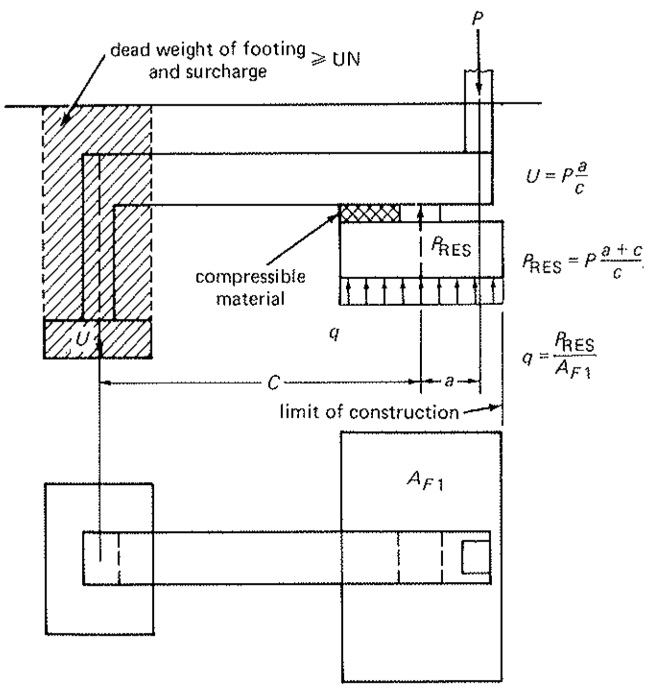


Fig. 5-20 Footing with deadman.

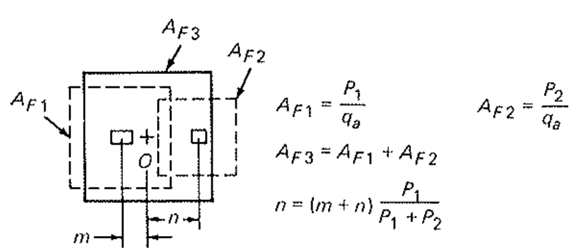


Fig. 5-21 Combined footing areas.

help to bridge over soft spots in the subsoil. This will reduce the risk of differential settlements. Such a design can be structurally desirable and may also be economical on soft or irregular subsoils. If the foundation is stiff enough (either by means of its own thickness or by means of well placed, monolithic basement walls), then the foundation may be considered as one unit. The shape of its base and the resulting bearing pressures can be evaluated as a combined footing. The stiffness of a strip, raft, or mat foundation alone is often not great enough to produce sufficient rigidity; in this case, the footing is best treated as a beam on elastic foundation. Figure 5-22 gives the basic equations required to design and evaluate the bearing pressures and moments using the simpified method.[5-3, 5-5]

As far as the last condition goes, the advantage that may be gained is primarily economic.

5.3 BASIC DESIGN PROCEDURE–REINFORCED CONCRETE FOOTINGS WITH EQUALLY DISTRIBUTED BEARING PRESSURES

5.3.1 Footing Size

It is important to keep the determination of the footing size completely separate from the design of the footing strength. The determination of the footing size, or of its width, as in the case of a strip footing or foundation raft, depends on the following criteria:

a. The evaluation of the service loads. (The term service is used here in order to differentiate the actual quantities from the factored loads to be used in the strength design.)
b. The evaluation of the bearing pressures at the footing base under the service loads, and the maximum allowable soil pressure.

In the evaluation of the service loads, the entire dead load (weight) of the superstructure, including the weight of the footing with surcharge and all floor live loads, should be used in their actual intensity. Floor live loads of multistory buildings, having an intensity of 100 lb/ft^2 or less may, according to the requirements of many local building codes, be reduced on the assumption that the full live load will hardly ever occur simultaneously on all floors. Reductions for live loads exceeding 100 lb/ft^2 are seldom permitted by building codes because such loads usually apply to warehouses or storage facilities.

Floors of industrial buildings are often designed for heavy loads in anticipation of machines that may have to be moved across the floor or will have to be set up at certain unforeseen locations. It is up to the judgment of the designing engineer and his personal knowledge of the manufacturing processes, to suggest a justified live load reduction to be used for the design of columns and footings and have it approved by the respective authority.

Crane loads, hoist loads, equipment loads; and the like have to be included in full, even if they are only of short duration.

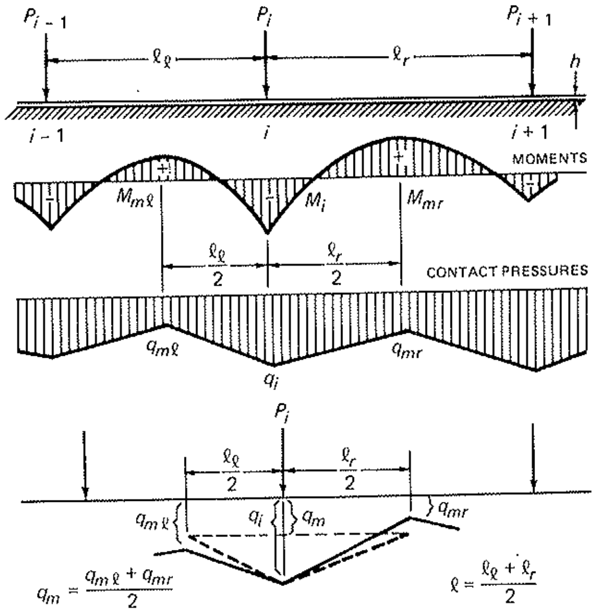
Impact caused by the occasional passing of a crane need not be considered in the design load of a footing; however, impact caused by continuously operating hammers or reciprocating machines shall be considered in the loading. Footings resting on loose, granular subsoil may require special provisions to prevent the transfer of impact or vibrations to the subsoil.

Loadings caused directly or indirectly by fill, lateral earth pressure, or water pressure have to be considered in full.

Lateral loads due to wind and their related effects, such as vertical forces and moments caused by them, have to be determined and considered in the design.

In certain geographical areas, earthquake-simulating lateral

(continued on p. 123)



$$l = \frac{l_l + l_r}{2}, \quad \frac{1.75}{\lambda} < l < \frac{3.50}{\lambda}$$

minimum total length: $\frac{5.25}{\lambda}$ or 3 bays

$$\lambda = 2.9\sqrt[4]{\frac{K_{si}F}{f'_c h^3}}$$

Form Factors, F

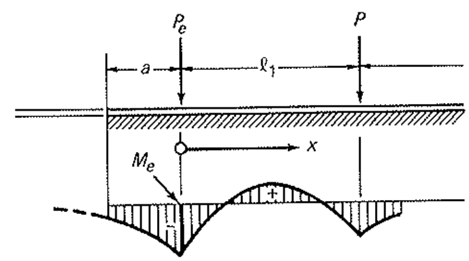
Sand	b	5	10	15	LARGE
	F	0.36	0.30	0.275	0.25
Clay	l/b	1	2	3	STRIP
	F	1	0.83	0.77	0.67

	Sand			*Clay*		
N	LOOSE 4–10	MEDIUM 10–30	DENSE 30–50	STIFF 8–15	VERY STIFF 15–30	HARD >30
K_{si}	20–60	60–300	300–1000	50–100	100–200	>200

at column (i): $$M_i = -\frac{P_i}{4\lambda}(0.24\lambda l + 0.16) \leqslant -\frac{P_i l}{12}$$

for about equal spans

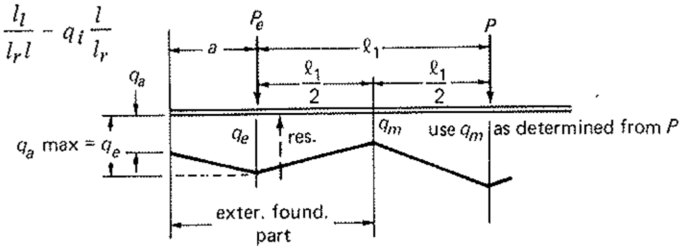
$$q_1 = \frac{5P_i}{l} + \frac{48M_i}{l^2}$$



at midspan left: $$q_{ml} = 2P_i\frac{l_r}{l_l l} - q_i\frac{l}{l_l}$$

for equal spans $q_m = \frac{2P_i}{l} - q_i$

at midspan right: $$q_{mr} = 2P_i\frac{l_l}{l_r l} - q_i\frac{l}{l_r}$$



at midspan left: $$M_{ml} = M_{ol} + \frac{M_l + M_i}{2}$$

$$M_{ol} = \frac{l_l^2}{48}(q_l + 4q_{ml} + q_i)$$

at midspan right: $$M_{mr} = M_{or} + \frac{M_r + M_i}{2}$$

$$M_{or} = \frac{l_r^2}{48}(q_r + 4q_{mr} + q_i)$$

End Condition: (The index e in P_e, M_e, and q_e refers to the end column.)

at end column: 1) $M_e = -\frac{P_e}{4\lambda}(0.13\lambda l_1 + 1.06\lambda a - 0.50)$

$$q_e = \frac{4P_e + 6M_e/a - q_m l_1}{a + l_1} \qquad q_a = -\frac{3M_e}{a^2} - \frac{q_e}{2}$$

2) $$M_e = -\left(\frac{4P_e - q_m l_1}{4a + l_1}\right)\frac{a^2}{2}$$

$$q_e = q_a = \frac{4P_e - q_m l_1}{4a + l_1}$$

The smaller M_e governs.

Fig. 5-22 Simplified design of combined footings.[5-5]

forces have to be applied at all mass centers of the structure and have to be transmitted through the foundation into the subsoil. These forces can act in any direction, but do not have to be considered simultaneously with the wind forces. Whichever force will have the greater effect on the element under consideration will govern.

Building codes usually permit an increase in the allowable stresses by 33 percent where wind or seismic forces are included. The load combination resulting in the largest required base area shall be used.

In general, footings shall be designed to be at least as strong as the design loads of the columns which they support; all possible combinations of forces that can act simultaneously on the footing under consideration have to be considered. For the design of footings for equal bearing pressures under average service load conditions, see Section 5.4.2.

The "Building Code Requirements for Reinforced Concrete," ACI 318-71, stipulates the following combinations of load effects as generally applicable:

1. $(D + L)$ The resulting maximum bearing pressure must be smaller than or equal to the maximum allowable soil pressure, $q \leqslant q_a$.

2. $(D + L + W)$, or $(D + L + E)$ The resulting maximum caused by either loading (W or E), must be smaller than $1.33q_a$. This increase of 33% above the allowable bearing pressure is usually accepted but ought to be checked with the local building code requirements.

In case of uplift or overturning the most critical combinations shall be investigated. Live loads shall only be considered with uplift forces where they contribute to the overturning; wherever dead load counteracts the overturning it shall be introduced with only 0.9 of its actual value to be on the safe side. The required safety factor against overturning shall not be less than 1.50, but shall be checked with the local building code requirements.

The evaluation of the intensity and distribution of the bearing pressures shall be done along the lines discussed in Section 5.2.

The maximum allowable soil pressure shall be determined by principles of soil mechanics. Unless the engineer designing the foundation has determined the maximum allowable soil pressure himself, it is important for him to understand the basis on which it was determined and the safety factor which was used in its evaluation. It is also important for him to find out whether the weight of the overlying soil surcharge was included in the evaluation of the allowable bearing pressure, and what the minimum depth below ground surface is at which this bearing pressure can be developed. He should also know the influence that raising or lowering of a footing may have on the magnitude of the allowable bearing pressure, and the elevation and possible fluctuations of the groundwater level. All these factors can be of important influence on the design of the foundation.

5.3.2 Footing Strength

-1. General principles—The strength design, also called Ultimate Strength Design (USD), of practically all types of footings can proceed along the lines described below.

The size of the footing or foundation, and the resulting bearing pressures, are determined from the service loads (actual loads). In order to perform the strength design of a footing, the various types of loads have to be multiplied by the respective load factors; and the bearing pressures have to be reevaluated for these factored loads. It has to be kept in mind, however, that these newly evaluated bearing pressures are of purely mathematical nature and have no soil-mechanical significance. They are calculated reactions to an imaginary factored loading condition resisted by the strength capacity of the foundation element.

The strength design requires that the minimum capacity of every structural element be sufficient to resist the factored loadings in their most severe combination. For this purpose it is advisable to assemble all different types of loadings and their related effects, such as shears and bending moments, independently for dead load, live load, wind, and earthquake (if so required), so that the sum of each type of loading can be multiplied by the respective load factor. ACI 318-71 requires in Section 9.3 that the following load factors be used:

1.4, for dead loads, fills and liquid loads
1.7, for live loads, wind loads and earth pressures
1.87, for earthquake loads

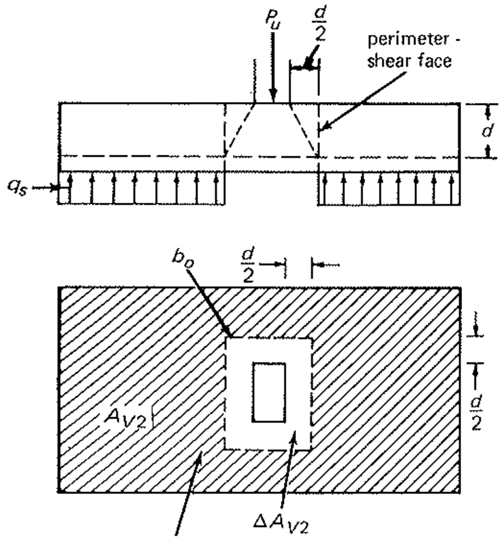
After the most severe strength combinations have been determined for axial loads, bending moments, and shears according to Section 9.3 of the *ACI Building Code*, the footing has to be designed strong enough to resist these factored load combinations and their resulting bearing pressures.

It has to be kept in mind, however, that factored loadings do not always furnish the same eccentricities as service loads. This is true where lateral loads, or other loads causing eccentricities or overturning, are of a different origin from the loads or load combinations causing the concentric loading. Consequently, the various loadings will have to be multiplied by different load factors, and will thus result in bearing pressure distributions that are, in principle, different from those obtained for the unfactored service load conditions. Since this is not the intent of the design, it is advisable to determine the resulting bearing pressure distribution from the service loads and then multiply it by an appropriate load factor for the strength design. In such a case, either the maximum load factor of 1.7 may be used, or an approximate load factor between 1.7 and 1.4 may be selected.

A footing, depending on its type, can be compared with a heavily loaded bracket, or with a column capital, or with a beam or slab, all in inverted position.

Because of the heavy (inverted) loading due to the bearing pressure, the thickness (depth) of a reinforced concrete footing is, with the exception of long-span rafts and mats, usually governed by shear. Due to the different working conditions under which footings often have to be constructed, it is common practice to design them without shear reinforcement; however, the *ACI Building Code* permits the use of shear reinforcement, if so required, except for mats or slabs less than 10 in. thick. Since the column, pier, or pedestal is usually much narrower than the size of the footing or the width of the footing strip, oneway shear action (also called beam shear), *and* two-way shear action (also called slab or perimeter shear), have to be investigated. With the exception of oblong footings or long rafts, two-way shear action will usually govern the design. It is therefore advisable to investigate it first.

-2. Investigation for two-way shear action (slab or perimeter shear)—The investigation for two-way shear action is the same whether shear reinforcement is provided or not; only the permissible value of the nominal shear stress varies depending on whether shear reinforcement is used or not. The nominal shear stress is to be determined along a line concentric with the loaded area, usually provided by the pier or pedestal, and is located at a distance $d/2$ from the face of the loaded area. The line forms the perimeter of the base of a truncated cone or pyramid, through which bearing



$$A_{V2} = A_F - \Delta A_{V2}$$
$$V_{2u} = A_{V2} q_s = (A_F - \Delta A_{V2}) q_s$$
$$v_{2u} = 1000 V_{2u} / \phi b_o d$$

Fig. 5-23 Two-way (perimeter) shear action.

stresses are considered to be transferred straight into the supporting subsoil. Considering the case of a concentrically loaded, isolated spread footing with uniform bearing pressure, the force V_{2u}, as shown in Fig. 5-23, is pressing the footing portion located outside of ΔA_{v2} upward. This force is resisted by the two-way shear of average intensity, v_{2u}, acting along the perimeter area $b_0 d$.

The nominal shear stress for two-way shear action is therefore

$$v_{2u} = \frac{V_{2u}}{\phi b_0 d} 1000$$

where ϕ is the capacity reduction factor for shear ($\phi = 0.85$) from Section 9.2.1.3 of ACI 318-71.

The maximum permissible two-way shear stress under ultimate loading is $v_{2u} = 4\sqrt{f_c'}$ and is applicable to all concrete footings without shear reinforcement. Here, as well as in the following sections and examples, all concrete is assumed to be normal weight concrete. Where sand-lightweight, or all-lightweight concrete is used (which is rather unusual in connection with footings), the permissible shear values have to be reduced in accordance with the requirements of Chapter 11 of ACI 318-71. If shear reinforcement is provided in accordance with Section 11.11.1 of ACI 318-71, the maximum permissible shear stress is

$$v_{2u} = 6\sqrt{f_c'}$$

If shearhead reinforcement is provided in accordance with Section 11.11.2 of ACI 318-71

$$v_{2u} = 7\sqrt{f_c'}$$

However, for practical reasons, footings ought to be designed whenever possible without the use of shear reinforcement.

In the following, reference is frequently made to tables contained in other chapters of this handbook, or in the "Ultimate Strength Design Handbook," ACI SP-17. Numbers of tables contained in other handbooks have been omitted herein and the tables are referred to by their general name only, with ACI SP-17 added. Tables that are similar to those referred to in the USD SP-17, can also be found in other design aids and textbooks, and can be used in a similar manner.

Tables to determine the footing depth as required by perimeter shear have been provided in ACI SP-17 to simplify the calculation. These tables are prepared for square footings, concentrically loaded by square, round, or polygonal pedestals or piers. If the loading pressure, q_c, at the base of the column, pier, or pedestal, and the bearing pressure at the footing base, q_s, due to the factored load are known; then the effective depth of the reinforced concrete footing can be determined from the ratio d/t, which can be taken from the tables.

Similar tables are also provided in the USD SP-17 for rectangular footings loaded by square or rectangular columns, piers, or pedestals. These tables can also be used for round and polygonal columns, piers, and pedestals, if their loading areas are transformed into squares of equal cross sectional area.

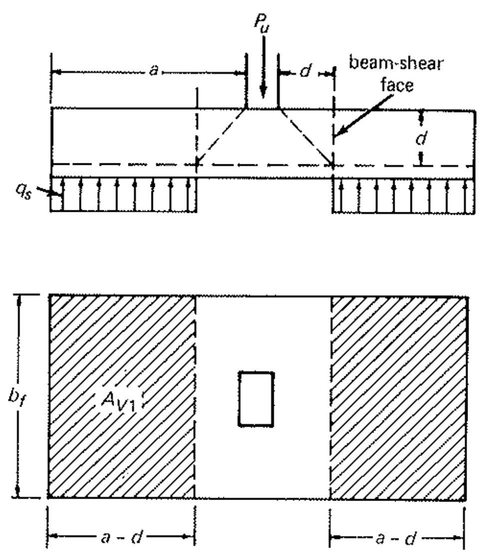
-3. Investigation for one-way shear action—The nominal shear stress due to one-way shear action shall be calculated as in an ordinary reinforced concrete beam, along a plane perpendicular to the footing, located at a distance d from the face of the column, pier, or pedestal. This plane shall extend across the entire footing and the shear stresses shall be assumed to be uniformly distributed over this plane, as shown in Fig. 5-24. The governing shear force, V_{1u}, for one-way shear action consists, therefore, of the sum of all bearing pressures, q_s, acting outside of the critical section.

The average nominal shear stress acting along the critical section is then

$$v_{1u} = \frac{1000 V_{1u}}{\phi(12 b_f) d}$$

where ϕ is the capacity reduction factor for shear ($\phi = 0.85$) from Section 9.2.1.3 of ACI 318-71. If no shear reinforcement is provided, which is the usual case, and the concrete is of normal weight taking all of the shear stresses, then the maximum permissible stress is

$$v_c = 2\sqrt{f_c'}$$



$$A_{V1} = b_f\left(\frac{a-d}{12}\right)$$
$$V_{1u} = A_{V1} q_s = \frac{(a-d)}{12} b_f q_s$$
$$v_{1u} = 1000 V_{1u} / \phi 12 b_f d$$

Fig. 5-24 One-way (beam) shear action.

In the exceptional cases, where shear reinforcement is provided in footings, it needs to be designed only for the excessive shear stress $(v_{1u} - v_c)$ in accordance with Section 11.6 of the *ACI Code*. In no case shall $(v_{1u} - v_c)$ exceed $8\sqrt{f'_c}$.

-4. Anchorage development of column dowels—After the minimum footing thickness that will satisfy both shear requirements has been determined, it should be checked to see whether it provides sufficient depth for the development of the column dowels. Anchorage development in compression is the most common condition for column bars; however where bending moments or uplift forces have to be transmitted to the footing, the bars must also satisfy the anchorage requirements for tension bars. Right-angle, or 90°, hooks at the bottom end of column dowels (common practice), are of no help in the development of anchorage for compression bars, see Fig. 5-25. Chapter 1 of this handbook contains Tables 1-10a to g with minimum required anchorage lengths for bar development in compression and in tension, with and without hooks. Dowels of smaller diameter than the column bars can be used to reduce the required anchorage length, and, therefore, thickness of the footing. Dowels of bigger diameter than the column bars may be used also if desirable; the *Code*, however, does not permit the diameter of the dowel to exceed that of the column bar by more than 0.15 in. ACI 318-71 provides in Section 15.6 that under certain conditions not all longitudinal column reinforcement needs to be extended into the footings, but only enough to cover the excess beyond the permissible bearing stress of the supporting or the supported member, whichever is smaller. In this connection, column steel, which has to be counted on at or above the contact area, has to be extended or developed in the column above and in the footing below. This also applies to areas where high edge pressures are caused by eccentric loadings or moments.

-5. Investigation for flexure—The flexural strength of a footing can be determined in a similar manner to that of an ordinary beam or cantilever member. The condition is shown in Fig. 5-26.

The reference lines about which the bending moments are to be determined shall extend all the way across the footing and shall be located as follows:

a. At the face of the supported element, for footings supporting columns, piers or pedestals.

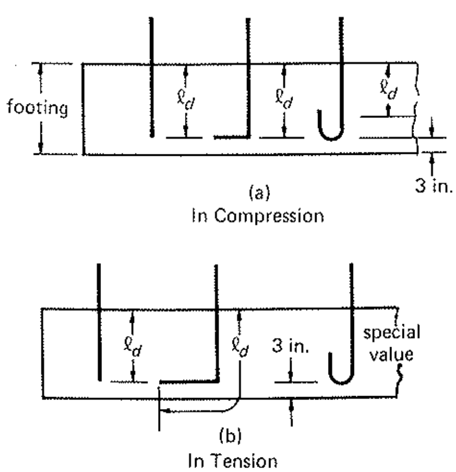
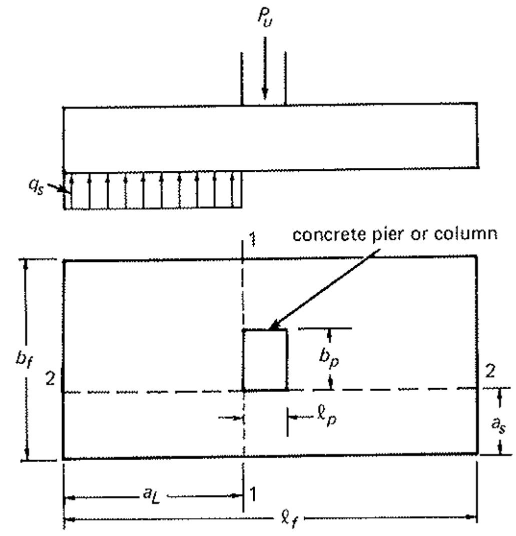


Fig. 5-25 Development (anchorage) of column bars or dowels. (a) In compression; and (b) in tension.



about 1 - 1 axis $M_u = \left(\frac{a_L}{12} b_f q_s\right) \frac{a_L}{2 \times 12} \frac{1}{\phi}$ (kft)

about 2 - 2 axis $M_u = \left(\frac{a_s}{12} \ell_f q_s\right) \frac{a_s}{2 \times 12} \frac{1}{\phi}$ (kft)

Fig. 5-26 Flexure.

b. Halfway between the center and the edge of the supported masonry, for footings supporting masonry construction.

c. Halfway between the face of the column and the edge of the base plate, for footings supporting steel base plates and steel columns.

The flexural moment, M_u, to be used in the calculation is therefore

$$M_u = \left(\frac{a}{12} b_f q_s\right)\left(\frac{a}{2 \times 12}\right)\frac{1}{\phi} \quad \text{in kip-ft}$$

For rectangular footings or for square footings supporting rectangular columns, pedestals, or piers, the larger moment, i.e., the moment in the direction of the longer projection, may influence the selection of the footing depth; to determine the reinforcement, however, the moments have to be calculated for each direction. The capacity reduction factor, ϕ, in the above equation is 0.9 for flexure according to Section 9.2.1.1 of ACI 318-71. If Tables 1-2 in Chapter 1 of this handbook, or if tables or graphs of USD SP-17 are used, the ϕ-factor can be disregarded because it has generally been included in the handbook.

The flexural strength shall also be investigated at all changes in the cross section of the footing element. Such checks are important in the design of stepped or sloping footing mats.

The Flexural Tables 1-2 provided in Chapter 1 of this handbook, for the design of ordinary beam and slab problems, can be used just as well for the flexural design of a footing. In this case, the F-factor can be determined from the selected size of the footing as

$$F = (12 b_f) d^2 / 12000, \quad \begin{array}{l} b_f \text{ in ft} \\ d \text{ in in.} \end{array}$$

or can be read from the F-table. With the help of $K_u = M_u / F$, the corresponding a_u or ρ can be taken from the tables and the cross sectional area of the reinforcement be determined from

$$A_s = M_u / a_u d \quad \text{or} \quad A_s = \rho 12 b_f d$$

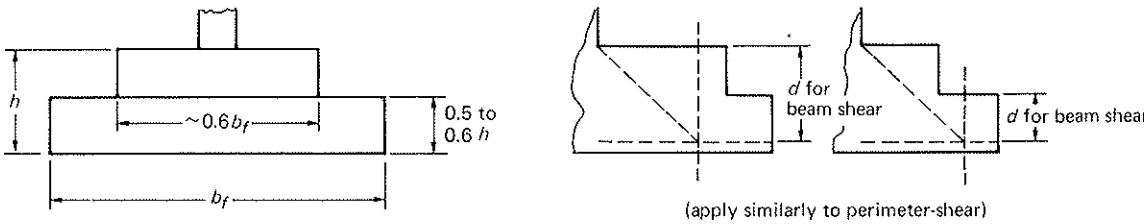


Fig. 5-27 Stepped footing.

If the required percentage exceeds the maximum permissible percentage $\rho_{max} = 0.75\rho_b$ or if the size and spacing of the reinforcing bars appears to be impractical, the footing thickness has to be increased to satisfy the requirements.

If the Flexural Graphs of USD SP-17 are used, the moments, M_u, must be determined either for a 1 ft wide strip if the slab graphs are used; or for a 10 in. wide strip if the beam graphs are used; and the obtained reinforcement must be multiplied by b_f for slabs or $1.2b_f$ for beams to find the total amount required in each direction. Attention must be drawn to the selection of the bar diameter to be used for the reinforcement, because it must, in addition to providing the required cross sectional area, also satisfy the anchorage requirements (see 6. below).

USD SP-17 contains also a special set of Footing Tables which are rather practical to use. These tables cover a wide range of bearing pressures due to factored loadings from $q_s = 3.33$ kip/ft^2 to $q_s = 26.67$ kip/ft^2 and can be used for both structural plain and reinforced concrete footings. Since the base area (size) of the footing must be known before the strength design is attempted, its greatest projection beyond the critical line is also known. The effective footing depth, d, can be selected so that the permissible perimeter shear and beam shear values are satisfied for the given projection. After these checks have been made, the required amount of reinforcement can be read from the table.

–6. *Development of footing reinforcement*–Most of the footings or portions thereof consist of short, heavily loaded cantilever sections. In the design of such elements it is important to see that the reinforcing bars are sufficiently anchored on either side of the critical section, as described above, to develop the full tension required. Since the projection of the footing beyond the critical section is a given length, and to satisfy the anchorage requirements, the diameters of the reinforcement selected should be small enough to provide sufficient embedment length on either side of the critical section. Most footing tables prepared for Ultimate Strength Design, including those in USD SP-17, contain the maximum diameters of bars that may be used to satisfy whatever projection the footing has.

5.4 SPECIAL CONDITIONS

5.4.1 Stepped Footings

In the design of isolated spread footings, the calculated thickness is required at the various critical locations near the center of the footing, but not at the edge of it. It has been common practice to give footings a tapered or stepped cross section in order to save the concrete in areas where it is not structurally needed. This method is structurally sound but getting out of practice for the following reasons:

a. The extra cost of the formwork is often greater than the saving in the amount of concrete used; and
b. the monolithic action between the upper (cap) and lower (mat) portion of the footing, which is structurally essential, is in practice difficult to obtain if cap and mat are not cast simultaneously, see Fig. 5-27. Unless special provisions are made, this method of construction will develop a "cold" joint and a cleavage plane will separate the two pieces.

Steps, however, are still in use in cases where the mat is getting excessively thick (usually more than 3 ft). If footings are not cast monolithically, key ways or shear-friction reinforcement have to be provided to transfer the horizontal shear and obtain monolithic action.

The size and thickness of the caps have to be designed in such a way that at each step (change in cross section) all shear stresses and flexural requirements are satisfied.

5.4.2 Footings Designed for Equal Bearing Pressures

Since settlements are practically independent of short time fluctuations in the loading, foundations for apartment houses, office buildings, institutional buildings, and the like are often designed for equal bearing pressures under average service load conditions, with the intent to obtain equal settlements over the entire building area. Full dead load plus one-half of the live load are often considered to represent an "average service load"; however, other ratios may be substituted depending on the judgement of the designing engineer. Such an approach will, at its best, reduce the amount of differential settlements to some degree because mutual influence, dishing, and, especially, variations in footing sizes will influence the settlement of each footing in a different way, regardless of the equal bearing pressure.

If such a design is desired, proceed as follows:

a. Determine the live load to dead load ratio for each column footing.
b. Determine the average (reduced service load for all column footings usually assumed with full dead load plus one-half live load).
c. Select the column with the greatest ratio of live load to dead load (from item a) and design its footing for the maximum allowable soil pressure under full load.
d. Determine the bearing pressure q_{av} for the same footing under the average service load (as determined under item b).
e. Design the size of all other footings for the average service load (as determined under b) and the average bearing pressure (as determined under d).
f. Ascertain for all footings, that the bearing pressure under the maximum load does not exceed the maximum allowable bearing pressure.
g. Make strength design of all footings at least for the factored maximum load and the bearing pressure caused by it or preferably for the q_s determined from the maximum allowable bearing pressure factored according to the applicable dead load to live load ratio.

The final result will be a somewhat overdesigned foundation, having equal bearing pressures under average service load conditions.

5.4.3 Footings of Structural Plain Concrete

Structural plain concrete footings on soil are permitted by the code; such footings are designed without reinforcement. The critical sections at which shear and flexure are to be determined are the same as for reinforced concrete footings, so are also the permissible shear stresses assigned to the unreinforced concrete. For all ordinary cases, flexure will govern the design.

The flexural stresses are calculated as for a homogeneous monolithic section. In this case the maximum tensile stress becomes

$$f_t = M_u z_t / I_c$$

The maximum tensile stress, f_t, in a structural plain concrete footing must not exceed $5\phi\sqrt{f_c'}$. Since a capacity reduction factor of $\phi = 0.65$ is prescribed by the *ACI Building Code* in Section 9.2.1.5, the maximum permissible tensile stress for structural plain concrete assumes the value of

$$f_t = 3.25\sqrt{f_c'}.$$

Footing Tables in USD SP-17 contain also values to select the minimum thickness of footings made of structural plain concrete for given projections and bearing pressures under the factored loading.

The following design examples were prepared with and without the help of tables and graphs. In the first part of the design examples, the procedure was presented in easy-to-follow steps. A considerable amount of explanatory text was also added to assist in the understanding of the basic requirements. It cannot be stressed enough that a full understanding of these basic approaches makes them applicable to footings of any type or character. This may make the procedure appear rather lengthy, it is, however, evident that a great part of the extra steps will become unnecessary in the solution of an actual design problem; however, they were included here for illustrative purposes.

EXAMPLE 5-1: Without using footing tables and graphs, design a *concentrically loaded, square, spread footing* for the following conditions:

column load

$$P_D = 350.0 \text{ kips}, \quad P_L = 275.0 \text{ kips}$$

pier

$$b_p = 18 \text{ in.}, \quad l_p = 24 \text{ in.}$$

$$q_a = 4.50 \text{ k/ft}^2, \quad d_f = 5 \text{ ft}$$

$$w_L = 100 \text{ lb/ft}^2$$

$$f_c' = 3000 \text{ psi, normal weight concrete}, \quad f_y = 40{,}000 \text{ psi}$$

Design footing by strength design method (USD).

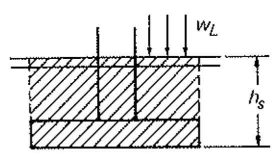


Fig. 5-28 Evaluation of surcharge.

Step 1: Determine footing size from service loads. The approximate weight of the surcharge is $\Delta q = \gamma_s h_s + w_L$ where γ_s is the assumed average unit weight for all material above the footing base. Since it consists of concrete and soil it can be estimated close enough between 150 and 100 lbs/ft^3. In this example it is assumed to be 130 lbs/ft^3, see Fig. 5-28.

$$\Delta q = \gamma_s h_s + w_L = 0.13 \times 5 + 0.1 = 0.75 \text{ kip/ft}^2$$

Determine effective soil pressure (allowable soil pressure that can be utilized)

$$q_e = q_a - \Delta q = 4.50 - 0.75 = 3.75 \text{ kip/ft}^2$$

to find the minimum required base area of footing

$$A_{F\min} = \frac{P_D + P_L}{q_e} = \frac{350 + 275}{3.75} = 167 \text{ ft}^2$$

selected footing size

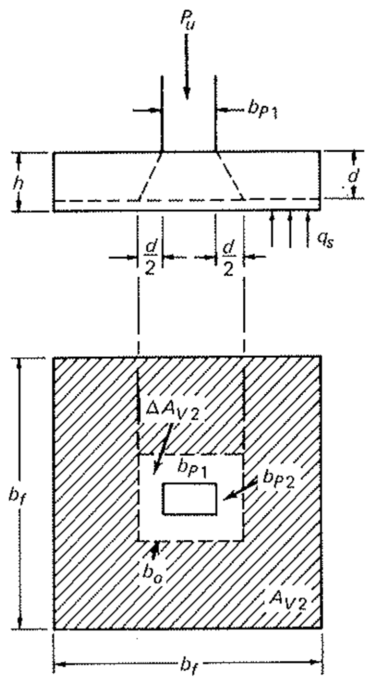
$$\text{use 13-ft square}, \quad A_F = 169 \text{ ft}^2$$

Step 2: Determine bearing pressure to be used for strength design (USD).

$$q_s = \frac{P_{Du} + P_{Lu}}{A_F} = \frac{350 \times 1.4 + 275 \times 1.7}{169} = 5.70 \text{ kip/ft}^2$$

Step 3: Determine thickness of footing mat. In the case of a square spread footing, concentrically loaded by a square column, the mat is always governed by two-way (slab or perimeter) shear action.[8] It is, therefore, advisable to investigate such a footing first for this condition. Only if the footing is oblong, or if the column or pier has a rectangular cross section, does the footing thickness have to be checked also for one-way (beam) shear action (see also Step 3 of Ex. 5-2), and may be governed by it. The tentatively selected mat thickness must also be checked for dowel embedment length and flexural requirements.

a. When investigating for two-way shear action, the most practical design approach is to assume a footing thickness and check it for its required strength; if incorrectly assumed, it can easily be adjusted to the correct thickness. The location of the base of the truncated shear cone or pyramid is concentric with that of the pier and located at a distance $d/2$ outside of it, as shown in Fig. 5-29.



assume: $d = 26''$

ΔA_{v2} = Base Area of truncated shear pyramid (ACI 318-71, Sec. 11.10.1 & 2)

$$\Delta A_{v2} = \left(b_{P1} + 2\frac{d}{2}\right) \times \left(b_{P2} + 2\frac{d}{2}\right)$$

$$= (24 + 26)(18 + 26)$$

$$= 2200 \text{ sq in}$$

$$= 15.3 \text{ sq ft}$$

$$A_{v2} = A_F - \Delta A_{v2} = 169.0 - 15.3 = 153.7 \text{ sq ft}$$

$$V_{2u} = A_{v2} q_s = 153.7 \times 5.7 = 875.0 \text{ k}$$

$$v_{2u} = \frac{V_{2u}}{\phi b_o d} = \frac{875.0 \times 1000}{0.85\,[2\,(50 + 44)]\,26} = 212 \text{ psi}$$

Fig. 5-29 Two-way shear action.

The permissible two-way shear stress (ACI 318-71, Section 11.10.3) is

$$v_{2u} = 4\sqrt{f_c'} = 220 \text{ psi}$$

NOTE: It is advisable to design a footing thick enough to satisfy the permissible shear stresses for unreinforced concrete. When, under extreme conditions, the footing cannot be made thick enough, the shear capacity of the footing has to be strengthened by reinforcement the same way as in a column-slab intersection. If shear reinforcement (usually bent up bars) is used, the permissible average shear stress may be increased by 50%; if a steel shearhead reinforcement is provided, the permissible average shear stress may be increased by 75%. In each case the concrete section can only be stressed to the permissible stress value of $4\sqrt{f_c'}$ and the remainder has to be carried by the reinforcement.

b. Investigation for one-way shear action begins by investigating the footing mat in the direction in which the distance between footing edge and critical section is largest. The critical section for

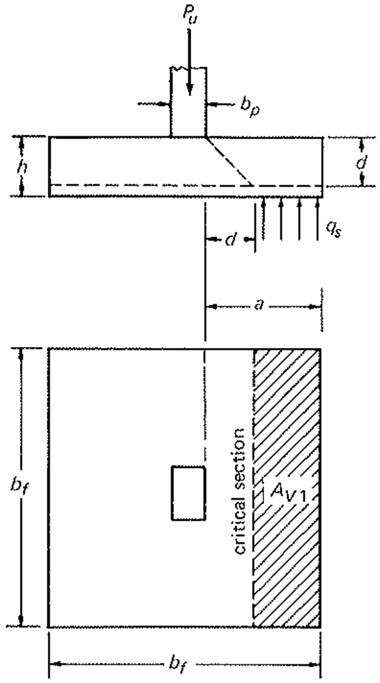


Fig. 5-30 One-way shear investigation.

one-way shear runs parallel to each pier face, and at the distance d away from it, across the entire footing mat, see Fig. 5-30.

$$a = \frac{b_f - b_p}{2} = \frac{13.0 - 1.5}{2} = 5.75 \text{ ft} = 69 \text{ in}$$

$$A_{v1} = \frac{(a-d)}{12} b_f = \frac{69 - 26}{12} \; 13.0 = 46.6 \text{ ft}^2$$

$$V_{1u} = A_{v1} q_s = 46.6 \times 5.7 = 268 \text{ kips}$$

$$v_{1u} = \frac{V_{1u}}{\phi b_f d} = \frac{268.0 \times 1000}{0.85(13 \times 12)26} = 78 \text{ psi}$$

The permissible shear stress for one-way action is

$$v_{1u} = 2\sqrt{f_c'} = 110 \text{ psi}$$

NOTE: In essence, the note provided at the end of Step 3a applies here also. (ACI 318-71, Section 11.10.3)

Step 4: Investigation for flexure. If the projection of the footing beyond the critical section varies, the flexural investigation has to be made for each direction. In each case the critical section extends across the entire footing.

For concrete piers or columns, the critical section is located at the face of the pier (ACI 318-71, section 15.4.2). (See also Fig. 5-26, $l_f = b_f$.)

a. Flexural computation in the direction of the longer footing dimension

$$a_L = \frac{l_f - l_p}{2} = \frac{13.0 - 1.5}{2} = 5.75 \text{ ft} = 69 \text{ in.}$$

$$M_u = b_f a_L q_s \frac{a_L}{2} = 13.0 \times 5.75 \times 5.7 \frac{5.75}{2} = 1220 \text{ kip-ft}$$

$$F = \frac{b_f d^2}{12{,}000} = \frac{(13 \times 12)\, 26^2}{12{,}000} = 8.75, \quad K_u = M_u/F = \frac{1220}{8.75} = 139$$

$$\rho = 0.004. \; A_s = \rho b_f d = 0.004\,(13 \times 12)\,26 = 16.2 \text{ in.}^2$$

Minimum reinforcement required (ACI 318-71, section 7.13)

$$\rho_{min} = 0.0018$$

$$A_{s\,min} = \rho_{min} b_f h = 0.0018\,(13 \times 12)\,31 = 8.8 \text{ in.}^2$$

For evaluation of h see below.

$$A_s = 16.2 \text{ in.}^2 \text{ governs.}$$

b. Flexural computation in the direction of the shorter footing dimension

$$a_s = \frac{b_f - b_p}{2} = \frac{13.0 - 2.0}{2} = 5.5 \text{ ft} = 66 \text{ in.}$$

$$M_u = l_f a_s q_s \frac{a_s}{2} = 13.0 \times 5.5 \times 5.7 \frac{5.5}{2} = 1120 \text{ kip-ft}$$

$$F = 8.75 \text{ as above}$$

$$K_u = \frac{M_u}{F} = \frac{1120}{8.75} = 128, \quad \rho = 0.0037$$

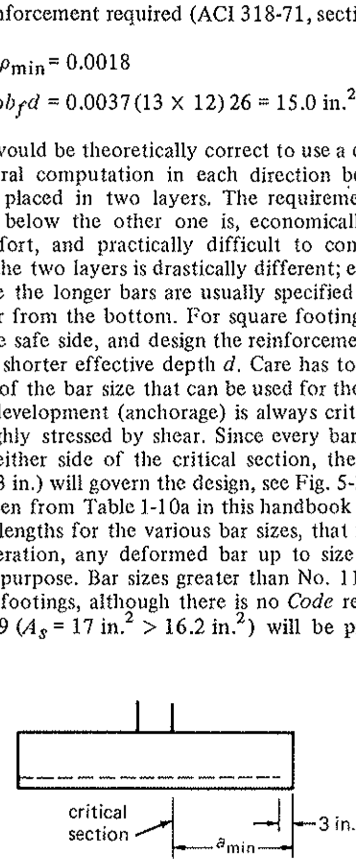
Minimum reinforcement required (ACI 318-71, section 7.13)

$$\rho_{min} = 0.0018$$

$$A_s = \rho b_f d = 0.0037\,(13 \times 12)\,26 = 15.0 \text{ in.}^2 \text{ governs.}$$

NOTE: It would be theoretically correct to use a different d-value for the flexural computation in each direction because the reinforcement is placed in two layers. The requirement of placing a certain layer below the other one is, economically, only seldom worth the effort, and practically difficult to control, unless the character of the two layers is drastically different; e.g., in an oblong footing where the longer bars are usually specified to be placed in the first layer from the bottom. For square footings it is advisable to stay on the safe side, and design the reinforcement in both directions for the shorter effective depth d. Care has to be exercised in the selection of the bar size that can be used for the reinforcement, because bar development (anchorage) is always critical in members which are highly stressed by shear. Since every bar has to be fully anchored at either side of the critical section, the shorter length, which is (a – 3 in.) will govern the design, see Fig. 5-31.

It can be seen from Table 1-10a in this handbook showing the bar development lengths for the various bar sizes, that for the example under consideration, any deformed bar up to size No. 11 can be used for this purpose. Bar sizes greater than No. 11 are commonly not used for footings, although there is no *Code* restriction in this respect. 17-#9 (A_s = 17 in.^{2} > 16.2 in.^{2}) will be provided in each direction.



a_{min} - 3 in. = 65 - 3 = 62 in.

Fig. 5-31 Bar development (anchorage).

The total thickness of the footing can be found, under consideration of the above, as shown in Fig. 5-32.

$$h = d + 1.5 d_b + c_c = 26 + 1.5 \times 1.13 + 3 = 30.7 \text{ in.}$$
$$\cong 31.0 \text{ in.}$$

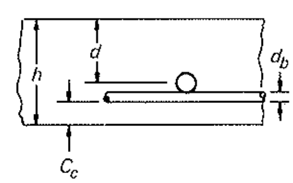


Fig. 5-32 Effective depth of spread footings.

EXAMPLE 5-2: Below, ex. 5-1 is solved with the help of footing tables and graphs contained in the "Ultimate Strength Design Handbook SP-17." *Step 1* and *Step 2* have to be performed as before.

Step 3:

a. Use Footing Graphs of USD SP-17.

$$\frac{P_u}{A_c} = \frac{P_{Du} + P_{Lu}}{l_p \times b_p} = \frac{350.0 \times 1.4 + 275.0 \times 1.7}{24 \times 18} = 2.21 \text{ kip/in.}^2$$

enter graph with $P_u/A_p = 2.21$ and read at intersection with $q_s = 5.7$, a ratio of $d/b_p = 1.25$.

The graph was prepared for square piers, but it also can be used for slightly rectangular piers if the rectangle is transformed into a square of equal area. The size of the equivalent square is then $b_p = \sqrt{24 \times 18} = 20.7$, $d = 1.25 \times 20.7 = 26$ in. (Since the intersection with the q_s-curves cannot be read with accuracy it is recommended to verify the selected value by calculation.)

b. From Footing Tables of USD SP-17, select the table which is closest to the required q_s, or interpolate if necessary. Footing Table for $q_s = 6.67$ will be on the safe side. By entering this table with the selected depth $d = 26$ in. (interpolate between 24 and 28), we find the appropriate a_b-value equal to 78.25 in. This value represents the maximum projection that can be used for one-way shear action. Since the maximum projection, a, in the example is 69 in. (which is smaller than 78.25), the selected depth of 26 in. is satisfactory.

Step 4: The footing must also be deep enough to develop the column dowels, or the column dowels must be selected so that their necessary development length is satisfied by the depth of the footing. In case the dowels have to be anchored for compression only, the l_d must be smaller than or equal to 26 in., which is satisfied by #11 bars. In case of tension, the anchorage can be increased through hooks or bends.

Step 5: The same footing table can be used to determine the reinforcement. Enter the table with the value of $d = 26$ and $a = 69$ in. Double interpolation furnishes a value of 1.22 in.^{2}/ft for the reinforcement in each direction. The total required reinforcement is therefore

$$A_s = A_s/\text{ft (from Table)} \times b_f = 1.22 \times 13 = 15.9 \text{ in.}^2,$$

against 16.2 in.^{2} found by calculation in Step 4a) of Ex. 5-1. The column at the right hand side of the Table indicates the maximum bar size that can be used corresponding to each projection a.

EXAMPLE 5-3: Without the help of footing tables and graphs, design an oblong, concentrically loaded, spread footing for the following conditions: all data identical with those of ex. 5-1 except that the width of the footing is restricted to 8 ft, and the long side of the pier is, for architectural reasons, perpendicular to the long-side of the footing.

Step 1: Determine footing size from service loads. Evaluation of the minimum base area remains unchanged

$$A_F = 167 \text{ ft}^2$$

Selected footing size

$$b \times l_f = 8 \times 21 \text{ ft}, \quad A_F = 168 \text{ ft}^2$$

Step 2: Determine bearing pressure to be used in strength design. Unchanged from example 8.5.1

$$q_s = 5.7 \text{ kip/ft}^2$$

Step 3: Determine the thickness of the footing. In the case of an oblong, concentrically loaded, spread footing, the footing thickness is often governed by one-way (beam) shear action, depending on the length to width ratio of the footing. It is therefore necessary to investigate the footing for this condition first, and check the tentatively selected thickness afterwards for two-way (slab) shear action (see also Step 3 of Example 5-1). The tentatively selected footing thickness must also be checked for dowel anchorage and flexural requirements.

a. When investigating for one-way shear action, consider that the critical section for one-way shear extends across the entire footing parallel to each pier face at the distance d away from it. Similarly to Step 3 of Example 5-1, we assume the footing depth, in this case to be 36 in., see Fig. 5-33.

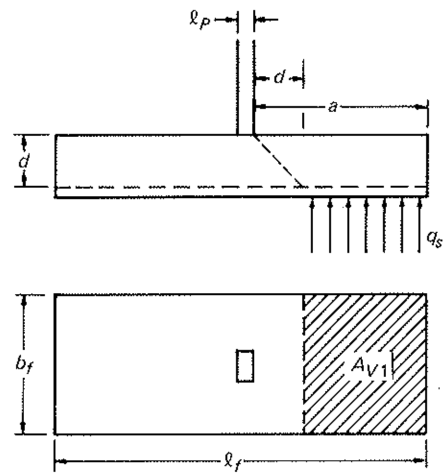


Fig. 5-33 One-way shear in oblong footings.

$$a = \frac{l_f - l_p}{2} = \frac{21.0 - 1.0}{2} = 10.0 \text{ ft} = 120 \text{ in.}$$

$$A_{V1} = \frac{(a - d)}{12} b_f = \frac{120 - 36}{12} 8.0 = 56.0 \text{ ft}^2$$

$$V_{1u} = A_{V1}\, q_s = 56.0 \times 5.7 = 312.0 \text{ kip}$$

$$v_{1u} = \frac{V_{1u}}{\phi b_f d} = \frac{312 \times 1000}{0.85 (8 \times 12) \times 36} = 108 \text{ psi}$$

The permissible shear stress for one-way action is

$$2\sqrt{f_c'} = 2\sqrt{3000} = 110 \text{ psi}$$

b. Investigation for two-way shear.

$$\Delta A_{V2} = (l_p + 2d/2)(b_p + 2d/2) = (18 + 36)(24 + 36) = 3240 \text{ in.}^2 = 22.4 \text{ ft}^2$$

$$V_{2u} = A_{V2}\, q_s = (l_f \times b - \Delta A_{V2}) q_s = (8.0 \times 21.0 - 22.4)\, 5.7 = 830.0 \text{ kip}$$

$$v_{2u} = V_{2u}/\phi b_0 d = 830 \times 1000/0.85[2(54 + 60)]\, 36 = 120 \text{ psi}$$

which is well below the permissible value of $4\sqrt{f_c'} = 220$ psi. The correctness of this result is questionable, if we look at the plan of the footing in Fig. 5-34 which is illustrating the condition. An even distribution of the two-way shear along the perimeter is hardly probable if we consider the narrow width of the influence area parallel to the short sides of the pier.

A more reasonable result is obtained if we divide the influence area in four portions, and investigate the greatest shear stress caused by each part separately.

$$A'_{V2} = \frac{l_c - (l_p + d)}{2} = \frac{21.0 \times 12 - (18 + 36)}{2 \times 12} \times 8 - 2 \times \frac{18^2}{2 \times 144} = 63.75 \text{ ft}^2$$

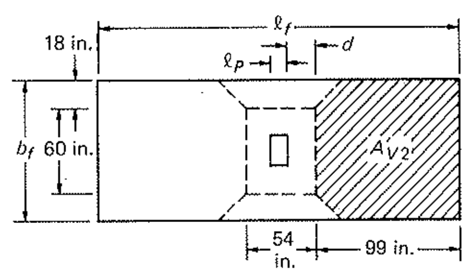


Fig. 5-34 Two-way shear in oblong footings.

$V'_{2u} = A'_{V2}\, q_s = 63.75 \times 5.7 = 365$ kip

$$v'_{2u} = \frac{V'_{2u}}{\phi(b_p + d)d} = \frac{365 \times 1000}{0.85(24 + 36)36} = 200 \text{ psi}$$

which is considerably greater and more realistic than the first value of v_{2u} found above, but still within the permissible limit of 220 psi.

Step 4: The depth of 36 in. will permit the use of any bar size for dowels, regarding their development in compression.

Step 5: Investigation for flexure. The oblong shape of the footing requires the design to be made independently for both directions. It is good practice, and reasonable, to assume that the reinforcement in the long direction will be placed at the bottom in order to utilize a greater depth. It is safer, however, and recommended, to design both layers for the shorter effective depth to stay independent of the field inspection.

a. In the long direction of the mat

$$a_L = \frac{21.0 - 1.0}{2} = 10.0$$

$$M_u = a_L\, b_f q_s \frac{a_L}{2} = 10.0 \times 8.0 \times 5.7 \times \frac{10}{2} = 2280 \text{ kip-ft}$$

$$F = b_f d^2/12{,}000 = (8 \times 12) \times 36^2/12{,}000 = 10.4,$$

$$K_u = 2280/10.4 = 218$$

$a_u = 2.85$, from Flexure Table 1-2 in Chapter 1 of this handbook

$$A_s = \frac{M_u}{a_u d} = \frac{2280}{2.85 \times 36} = 22.2 \text{ in.}^2$$

The minimum available anchorage length in this direction is $a_L - 3 = 10 \times 12 - 3 = 117$ in. It can be seen from Tables 1-10 in Chapter 1 of this handbook that this length permits the use of any bar size up to #11. Select

$$A_s = 15 - \#11, \quad A_s = 23.4 \text{ in.}^2$$

b. In the short direction of the mat

$$a_s = \frac{8.0 - 1.5}{2} = 3.25 \text{ ft}$$

$$M_u = a_s l_f q_s \frac{a_s}{2} = 3.25 \times 21 \times 5.7 \times \frac{3.25}{2} = 630 \text{ kip-ft}$$

$$F = \frac{l_f d^2}{12{,}000} = \frac{(21 \times 12) \times 36^2}{12{,}000} = 27.2, \quad K_u = \frac{540}{27.2} = 20$$

$$a_u = 2.96$$

only minimum reinforcement will be required.

$$A_s = \frac{1.33 M_u}{a_u d} = \frac{1.33 \times 540}{2.96 \times 36} = 6.8 \text{ in.}^2$$

or $\quad A_{s\min} = \rho_{\min} l_f h = 0.0018(21 \times 12)41 = 18.7 \text{ in.}^2$

For evaluation of h see below. The smaller value can be used.

The minimum available anchorage length in this direction is $a_s - 3 = (3 \times 12) - 3 = 33$ in. According to Tables 1-10 in Chapter 1 of this handbook, the maximum permissible bar size which can be used in this respect is #9. Because of the large footing length a greater number of bars is desirable and we select #7 bars.

In an oblong footing, section 15.4.4 of ACI 318-71 requires that a portion of the reinforcement in the short direction A_{s1}, be distributed over a width b_f and the balance of the reinforcement be spread evenly over the rest of the footing length, eq. (15-1).

If A_{sT} = total required reinforcement in the short direction, then

$$A_{s1} = \frac{A_{sT} \times 2}{(S + 1)} = \frac{6.8 \times 2}{(2.65 + 1)} = 3.7 \text{ in.}^2$$

$$S = l_f/b_f = 21/8 = 2.65$$

To avoid unequal spacings of reinforcement, the total reinforcement A_{sT} may be increased to A_{s2} in order to be spread evenly over the entire length of the footing.

$$A_{s2} = \frac{2A_{sT}S}{(S + 1)} = \frac{2 \times 6.8 \times 2.65}{(2.65 + 1)} = 9.8 \text{ in.}^2 \text{ still less than } 18.7 \text{ in.}^2$$

Reinforcement used is 31 #7 bars, $A_s = 18.6$ in.^{2} and the total thickness of the footing is

$$h = d + 3 + d_{bL} + \frac{1}{2} d_{bs} = 36 + 3 + 1.41 + \frac{0.88}{2} = 40.85 \cong 41 \text{ in.}$$

where d_{bL} and d_{bs} are the bar diameters in the long and short direction.

EXAMPLE 5-4: Stepped footing. If the thickness of the footing investigated in ex. 5-3 is considered to be uneconomical or otherwise excessive, it may be designed as a stepped footing and required to be cast monolithically, see Fig. 5-27.

It is common practice to make the cap (the upper portion) about $0.6\, b_f$ long, however this length should be checked as described below. The cap thickness is usually assumed with a fraction of the total footing thickness depending on the number of steps to be used. In this case the shear investigations of ex. 5-3, for one-way and for two-way shear, are applicable only as long as the critical sections fall within the extent of the cap; otherwise, the depth occurring at the critical section has to be used. The shear-cone or pyramid must not intersect the step, or the size of the cap has to be adjusted as needed. The entire shear investigation has to be repeated along the perimeter of the cap.

In the flexural investigation, care has to be exercised so that only that portion of the cross section that is in compression is utilized for determining the F-value, $bd^2/12{,}000$, and consequently the amount of reinforcement. In addition to the maximum moment occurring at the critical section, the amount at the edge of each step has to be investigated, and the amount of reinforcement at these locations checked for the reduced available depth.

EXAMPLE 5-5: Structural plain concrete footing

column dead load:	40 kips
column live load:	60 kips
total column load: (service)	100 kips

allowable soil pressure

$$P_a = 4.0 \text{ kip/ft}^2$$

$$f'_c = 3000 \text{ psi}, \quad \text{pier size } 12 \times 12 \text{ in.}$$

footing size

$$A_F = \frac{100}{4} = 25 \text{ ft}^2, \quad \text{or} \quad 5 \times 5 \text{ ft}$$

Start with the flexural investigation because it governs the design of a square, structural plain concrete footing supporting a square column or pier. See Fig. 5-35.

$$P_{uD} = 40.0 \times 1.4 = 56.0 \text{ kips}$$

$$P_{uL} = 60.0 \times 1.7 = 102.0 \text{ kips}$$

$$P_u = 158.0 \text{ kips}$$

$$q_s = \frac{P_u}{A_F} = \frac{158}{25} = 6.35 \text{ kips-ft}^2$$

$$M_u = b_f a q_s \frac{a}{2} = 5 \times \frac{24}{12} \times 6.35 \times \frac{24}{2 \times 12} = 63.5 \text{ kip-ft}$$

$$S_F = \frac{12 b_f \times h^2}{6} = 10h^2, \quad \text{for } b_f = 5.0$$

The permissible flexural strength is, according to sections 15.7.2 and 9.2.1.5 of ACI 318-71

$$f_t = 5.0 \times \phi \times \sqrt{f'_c} = 5 \times 0.65 \times \sqrt{3000} = 179 \text{ psi}$$

$$S_F(\text{required}) = \frac{M_u \times 12{,}000}{f_t} = \frac{63.5 \times 12{,}000}{179} = 4230$$

$$4230 = 10h^2, \quad h^2 = 423, \quad h = 20.5 \text{ in., say 21 in.}$$

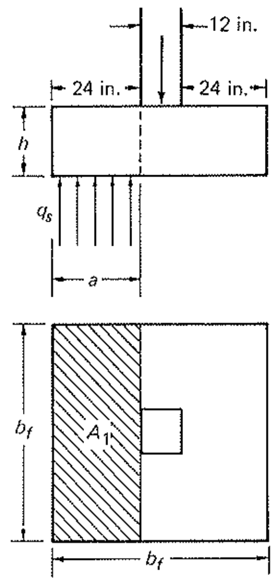


Fig. 5-35 Flexural design of structural plain concrete footings.

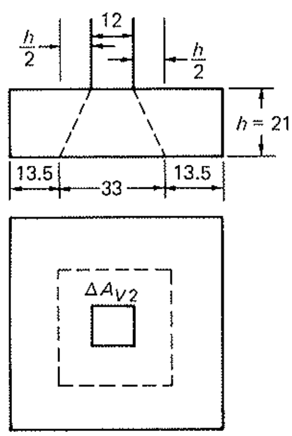


Fig. 5-36 Two-way shear in structural plain concrete footings.

Check footing thickness for two-way shear (for illustrative purposes only). See Fig. 5-36.

$$\Delta A_{V2} = \left(\frac{33}{12}\right)^2 = 7.55 \text{ ft}^2$$

$$\Delta V_{2u} = \Delta A_{V2} \times q_s = 7.55 \times 6.35 = 48 \text{ kips}$$

$$P_u - \Delta P_u = 158 - 48 = 110 \text{ kip}$$

$$v_{2u} = \frac{(P_u - \Delta P_u)\,1000}{b_o d} = \frac{110 \times 1000}{4 \times 33 \times 21} = 40 \text{ psi}$$

permissible

$$v_{2u} = 4\phi\sqrt{f_c'} = 4 \times 0.85 \times \sqrt{3000} = 187 \text{ psi}$$

5.5 REINFORCED CONCRETE FOOTINGS WITH CONCENTRATED REACTIONS (PILE CAPS)

5.5.1 General Principles

Where soil conditions do not favor the design or construction of shallow foundations (spread footings), but a firm soil stratum can be found at greater depth, piles can be used to transfer the loads from the superstructure down to the soil stratum, where the required resistance is available. The piles may develop this resistance by end bearing (bearing piles) on the firm stratum; or by skin friction (friction piles) developed by driving the piles into the firm stratum. Foundation piers or caissons can also be used for similar purposes but do not form a part of this discussion.

Similar to the action of a spread footing, a footing on piles (commonly called pile cap) has to distribute the column load to the piles in each group, which in turn will transmit it to the subsoil. The main difference between the two types of footings lies in the application of the base reactions which, in the case of a footing on piles, consists of a number of concentrated loads. If we divide the sum of all pile reactions in a group, just for reasons of comparison, by the base area of the pile cap, we obtain an equivalent bearing pressure caused by the bearing capacities of the individual piles. Such an average bearing pressure would be quite high because of the large bearing capacities of the individual piles. These large pile capacities were brought about by great progress made in the theoretical understanding of the soil resistance; by improvement in the quality of the materials used; and by the higher power and reliability of modern driving procedures and equipment.

The allowable bearing capacity that can be expected from a pile is usually based on the information gained from exploratory soil borings, and evaluated with the help of soil mechanical principles; it should be confirmed, however, by performance tests made on the site to ascertain the actual conditions. Depending on the availability of rock, hardpan, or other firm soil stratum and on their distance below grade, the engineer will decide whether bearing piles can be used economically. Otherwise, he has to resort to friction piles of some sort to utilize the available soil condition.

Lack of a firm soil stratum at reasonable depth can sometimes be treated also with the help of floating (boatlike) foundations which do not form a part of this discussion. The structural design of a pile cap is, in principle, not affected by the type of pile to be used, because it is primarily dependent on the magnitude of the pile reaction; however, a few explanations are necessary for a better understanding in the evaluation of the basic design approach.

5.5.2 Number of Piles Required

In the case of a spread footing, the size of the footing is determined from the total load on the footing and the allowable bearing pressure; hence, the size of the footing is rather made to order. In the case of a pile group, however, the number of piles is determined from the total group and the allowable load bearing capacity of each individual pile. Since the addition of a pile will raise the capacity of the whole group by a considerable amount, some of the pile groups may have, in order to be on the safe side, a capacity that exceeds that of the column load by a substantial amount. Furthermore, it is common practice to use, for reasons of stability, a minimum of three piles in a free standing pile group; a minimum of two piles if a foundation beam or similar provides lateral support; and a single pile only if lateral support can be provided in two directions. These minimum requirements have to be satisfied even if the capacity provided by the pile group far exceeds the amount of the load to be supported. It is good practice to design the pile caps in any case for the full allowable capacity of the group. This is done whether required by the column load or not, and in spite of the waste that may be connected with it, to permit full utilization of the pile capacity under any circumstances.

In the case of bearing piles, every pile in a group may be considered to act as an independent pier down to the bearing stratum and to share equally in the carrying of the load. In the case of friction piles, the number of piles in a group affects their carrying capacity, especially that of the interior piles. Although this deficiency is usually averaged over the entire pile group, as far as the capacity of the group is concerned, the variations in the capacity of each individual pile requires, sometimes, consideration in the design of the pile cap.

In either case, whether we are dealing with bearing piles or friction piles, there is always a chance that some piles in the group may develop a smaller (or greater) resistance than others; a pile cap ought to be stiff enough to equalize this condition. It is therefore advisable not to keep the effective depth of a pile cap down to the minimum required, but to increase it somewhat wherever possible.

The design of a pile cap follows in general the same rules and regulations as that of a spread footing, except that the base reactions (pile reactions) are applied as concentrated loads in the center of each pile. Attention is drawn to section 15.5.5 of ACI 318-71 which states that "in computing the external shear on any section through a footing supported on piles, the entire reaction from any pile whose center is located $d_p/2$ (d_p is the pile diameter at the upper end) or more outside the section shall be assumed as producing shear on the section. The reaction from any pile whose center is located $d_p/2$ or more inside the section shall be assumed as producing no shear on the section. For intermediate positions of the pile center, the portion of the pile reaction to be assumed as producing shear on the section shall be based on straight line interpolation between full value at $d_p/2$ outside the section and zero value at $d_p/2$ inside the section."

For evaluation of pile reactions under various loading conditions see the following section.

The considerable intensity of the concentrated pile reactions requires that more than usual attention be given to the design for shear in the concrete cap and the development (anchorage) of the reinforcement in the section. Due to the importance of a crack free entity of a pile cap in the distribution of the column load to the supporting pile group, the use of plain concrete is not permitted for pile caps.

5.5.3 Evaluation of Pile Reactions

-1. Concentric loading conditions—After the allowable pile reaction, R_{pa} (often incorrectly called "allowable pile capacity"), has been determined or evaluated by principles of soil mechanics,* the minimum number of piles for each column load can be determined as follows:

The effective pile reaction, R_{pe} (kips), consists of the allowable pile reaction, R_{pa} (tons), less the weight of the pile cap per pile, W_p. Any eventual surcharge shall be added to the weight of the pile cap.

$$R_{pe} \text{ (in kips)} = 2R_{pa} - W_p$$

The number of piles, n_p, required to support the unfactored total column load P is then $n_p = P/R_{pe}$, where n_p is to be rounded off to the next whole number.

*Verification of the validity of this "allowable pile reaction" is usually established by one or more pile loading tests performed at the site under actual driving conditions and at the beginning of construction. A safe assumption of the allowable pile reaction, however, has to be made, at a much earlier date to enable the engineer to design the foundation ahead of the actual construction.

Unless special conditions require a spreading of the piles, they are assembled in tight patterns to arrive at the most economical design for the pile caps. An often recommended spacing, c_p, is about three times the butt diameter of the pile, usually not less than 2½ ft. The most common spacing for piles of an average pile reaction ranging from 30 to 70 tons is 3 ft.

-2. Eccentric loading condition or concentric loading with moment at base—To transform eccentric loading conditions into concentric loadings with moment at base proceed as follows:

a. find pile reaction R_p for concentric loading condition
b. find pile reaction R_{pM} for moment at base
c. superpose 1 and 2

$$R_p + R_{pM} \leqslant 2R_{pa}$$

Where wind or earthquake are included, the R_{pa} can be increased by 33% if so allowed by the local building code. The extreme pile reaction due to a moment M is

$$R_{pM} = \frac{M}{I_{pG}/z_{pG}}$$

To calculate the moment of inertia of a pile group I_{pG}, first find the centroid of the pile group and moment of inertia of all units in the group about the centroidal axis.

$$I_{pG} = \sum_{1}^{n} y^2$$

where y is the distance of each pile in the group from the centroidal axis.

Where a pile group consists of m equal, parallel rows of piles, the moment of inertia of the entire group is

$$I_{pG} = mI_p/\text{Row} = m\,\frac{n_{pr}(n_{pr}^2 - 1)}{12}\,c_p^2$$

and the section modulus for the extreme piles in the group is

$$S_{pG} = m\,\frac{n_{pr}(n_{pr} + 1)}{6}\,c_p$$

However, if the parallel rows are not of the same configuration, sum up the moments of inertia for the various rows and find the section modulus of the extreme pile by dividing the moment of inertia of the entire group by the distance of the extreme pile from the centroid, as

$$S_{pG} = I_{pG}/z_{pG}$$

EXAMPLE 5-6: As discussed in section 5.3.2 for ordinary spread footings, the number of piles or their arrangement in the pile group depends only on the unfactored loading conditions, as shown in Fig. 5-37, and the strength design of the pile cap has to be done by converting all loads and reactions to the factored conditions.

column load:

$$\begin{array}{r} D = 400 \text{ kip} \\ L = 520 \\ \hline \text{total} = 920 \text{ kip} \end{array} \qquad R_{pa} = 50 \text{ tons}$$

$$R_{pe} = 2R_{pa} - W_p = 2 \times 50 - 6.5 = 93.5 \text{ kip}$$

$$n_p = \frac{920}{93.5} = 9.8 \cong 10 \text{ piles}$$

$$W_p = A_p(\Delta q) = 3^2(50 + 75 + 150 + 450)$$

$$= 6525 \text{ lb} \cong 6.5 \text{ kip}$$

where $\Delta q = [w_L + \text{slab} + \text{fill} + \text{cap}]$. See Fig. 5-37.

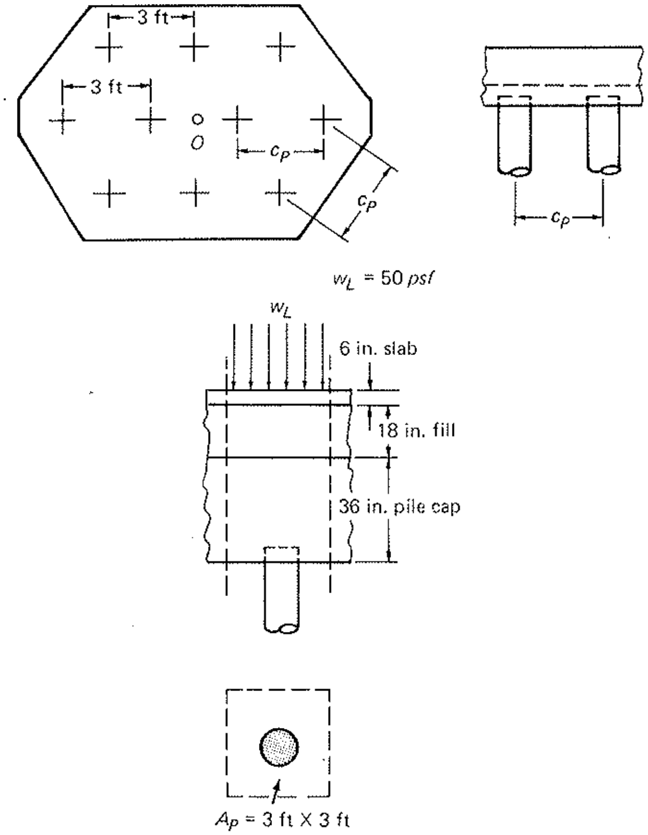


Fig. 5-37 Pile cap. Conventional pile arrangement in ten-pile cap.

EXAMPLE 5-7: Investigate ex. 5-6 for an additional wind moment of 450 kip-ft in the long direction of the pile group.

The moment of inertia of the entire pile group can be considered as the sum of the moments of inertia of each row of piles or

$$I_{pG} = \sum \frac{n_{pr}(n_{pr}^2 - 1)}{12} c_p^2 = \left[2 \times \frac{3(3^2 - 1)}{12} + 1 \times \frac{4(4^2 - 1)}{12} \right] 3^2 = 81 \text{ ft}^3$$

The section modulus of the extreme pile in longitudinal direction is then

$$S_{pG} = I_{pG}/1.5\, c_p = 81/1.5 \times 3 = 18 \text{ ft}^2$$

and the reaction on this pile due to the wind moment is

$$R_{pM} = \frac{M}{S_{PG}} = \frac{450}{18} = 25 \text{ kip}$$

Summing up, we obtain a total maximum pile reaction under wind of

$$R_p + R_{pM} = 98.5 + 25.0 = 123.5 \text{ kip}$$

Since the maximum allowable pile reaction under wind is

$$R_{pa(w)} = 1.33 \times R_{pa} = 1.33 \times 100 = 133 > 123.5 \text{ kip}$$

no increase in the number of piles is required due to wind.

EXAMPLE 5-8: Strength design of pile cap. The column load and allowable pile capacity is the same as in ex. 5-6.

$$f_c' = 3000 \text{ psi}, \quad \text{and} \quad f_y = 60{,}000 \text{ psi}$$

Pier size is 22 X 22 in., the butt diameter of the piles is 14 in. Determine the thickness and reinforcement of the pile cap.

The strength design of the pile cap is based on the R_{pu} which is determined from the factored loading, similar to the q_s for the spread footings, and has also here no other significance.

from dead load

$$\frac{400 \times 1.4}{10} = 56.0 \text{ kip}$$

from live load

$$\frac{520 \times 1.7}{10} = 88.4$$

The factored pile reaction is then 56.0 + 88.4 = 144.4 kips; it is, however, recommended to design the pile cap for the maximum factored pile reaction based on the average load factor.

average load factor

$$\frac{400 \times 1.4 + 520 \times 1.7}{920} \cong 1.6$$

maximum factored pile reaction due to column load is

$$93.5 \times 1.6 \cong 150 \text{ kip}$$

Fig. 5-38 shows the layout for a ten-pile cap and the various approaches that need to be followed in the evaluation of its strength design.

Step 1: For two-way shear section (a), as indicated in the lower left quadrant of Fig. 5-38, let us assume that the necessary depth has been evaluated with 30 in. and is checked herewith: the critical

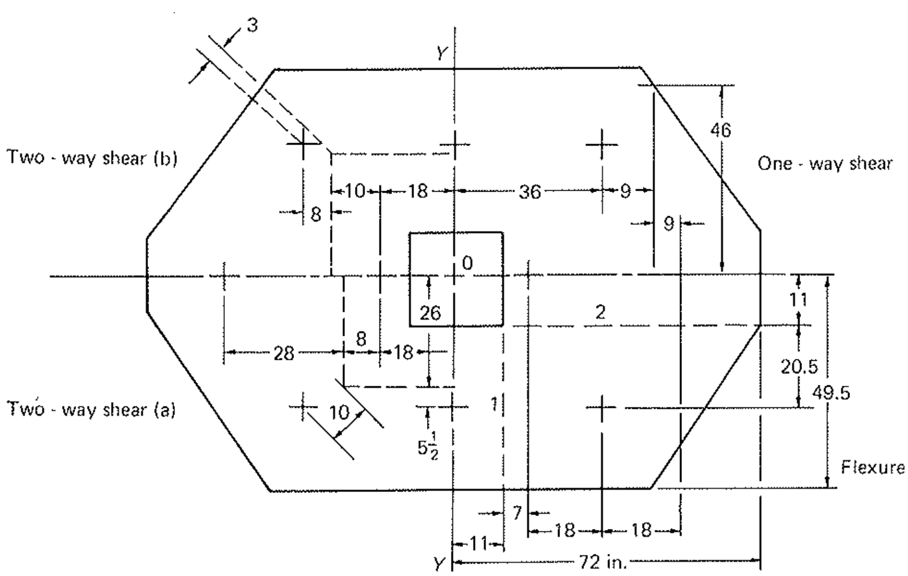


Fig. 5-38 Stress evaluation in pile caps.

base size of the truncated pyramid is 22 + 2 × 30/2 = 52 in. The critical shear force V_{2u} is then

$$V_{2u} = 6 \times 150.0 + 2 \times 16 = 932 \text{ kip}$$

where the contribution of the outer piles located on the $y - y$ axis is

$$150 \times \frac{1.5}{14} = 16 \text{ kip}, \quad d_p = 14 \text{ in.}, \quad \frac{d_p}{2} - 5.5 = 1.5 \text{ in.}$$

$$v_{2u} = \frac{V_{2u}}{b_0 d\phi} = \frac{932 \times 1000}{(4 \times 52) \times 30 \times 0.85} = 170 \text{ psi}$$

which is smaller than $4\sqrt{f'_c} = 220$ psi.

Two-way shear action (b), is indicated in the upper left quadrant of Fig. 5-38. It can be realized, by inspection of Fig. 5-38, that the actual shear distribution is unequal and will be much greater in the long direction. If we take an approach similar to ex. 5-3(b) we require a much greater cap thickness, as evaluated in the approach (a) described above. In this respect we divide the shear action again into two portions separated by a 45° line placed at the corner of the truncated pyramid base. Let us assume again that the necessary thickness of 34 in. has been evaluated before and is checked below.

The critical base size of the truncated pyramid is here 22 + 2 × 34/2 = 56 in. and the shear force for the most stressed quadrant becomes

$$V'_{2u} = 1 \times 150 + 2 \times 107 = 364.0 \text{ kip}$$

where the contribution of the outer piles is

$$150 \times 10/14 = 107.0 \text{ kip}, \quad d_p/2 + 3 = 10 \text{ in.}$$

$$v_{2u} = \frac{V'_{2u}}{b'_0 d\phi} = \frac{364 \times 1000}{56 \times 34 \times 0.85} = 225 > 220 \text{ psi}$$

but acceptable. The greater depth of 34 in. is, therefore, selected.

Step 2: One-way shear action, as indicated in the upper right quadrant of Fig. 5-38. The critical line is 22/2 + 34 = 45 in. away from the $y - y$ axis. The critical shear force is then $V_{1u} = 150.0$ kip,

$$v_{1u} = \frac{V_{1u}}{b_f d\phi} = \frac{150 \times 1000}{92 \times 34 \times 0.85} = 56 \text{ psi}$$

which is smaller than $2\sqrt{f'_c} = 110$ psi.

Step 3: The critical sections for flexure, as indicated in the lower right quadrant of Fig. 5-38, are at the face of the pier; the moments and reinforcements are determined for these sections.

critical section 1:

$$M_u = 150.0\left(\frac{7 + 2 \times 25 + 43}{12}\right) = 1250 \text{ kip-ft}$$

$$F = \frac{99 \times 34^2}{12{,}000} = 9.6, \quad K_u = 1250/9.6 = 130, \quad a_u = 4.37$$

$$A_s = \frac{M_u}{a_u d} = \frac{1250}{4.37 \times 34} = 8.4 \text{ in.}^2, \quad \rho_{min} = \frac{200}{60{,}000} = 0.0033$$

$A_{s\,min} = 0.0033 \times 99 \times 34 = 11.3$ in.^{2}, or $1.33 \times 8.4 = 11.2$ in.^{2} which governs

critical section 2:

$$M_u = 3 \times 150 \times 20.5/12 = 770 \text{ kip-ft}$$

$$F = \frac{144 \times 34^2}{12{,}000} = 13.7, \quad K_u = \frac{770}{13.7} = 56, \quad a_u = 4.45$$

$$A_s = \frac{M_u}{a_u d} = \frac{770}{4.45 \times 34} = 5.1 \text{ in.}^2$$

$A_{s\,min} = \rho_{min} bd = 0.0033 \times 144 \times 34 = 16.1$ in.^{2}, or $1.33 \times 5.1 = 6.8$ in.^{2} which governs

The selection and distribution of the bars is done as described in Example 5-1, Step 4b. The maximum bar size that may be used has to be selected in such a way that the development (anchor) length of the bar is smaller or equal than the shortest available embedment length of the bar at either side of the critical section.

5.6 RETAINING WALLS

5.6.1 General

A retaining wall is a structure designed for the purpose of providing one-sided lateral confinement of soil or fill.

All retaining walls, with the exception of true cantilever walls anchored to rock, are in principle gravity walls, i.e., their action depends primarily on their developed weight. In common practice, however, only those retaining walls are called gravity walls where the dead weight required to make the resultant vector intersect the base within safe allowable limits is made up solely by the dead weight of the concrete. Such walls are usually designed unreinforced, Fig. 5-39a. The commonly called cantilever walls are in principle gravity walls where reinforcement is used to reduce and modify the cross section of the concrete in such a way that portions of the soil or fill are utilized for developing the necessary rightening moment, Fig. 5-39b.

In every retaining wall design, regardless of the type used, three resultant forces, namely, the lateral confinement pressure, Q, the total developed weight, P, and the soil reaction or bearing resistance, R, have to be brought into equilibrium Fig. 5-40a; in addition all internal stresses in the structure and all external soil reactions have to be within the permissible limits.

Retaining walls of the commonly called cantilever type can be subdivided into two main groups:

1. Continuous walls of constant cross section, where every foot of wall length is providing its own equilibrium, Fig. 5-39b.
2. Sectional walls, where crosswalls introduced at certain

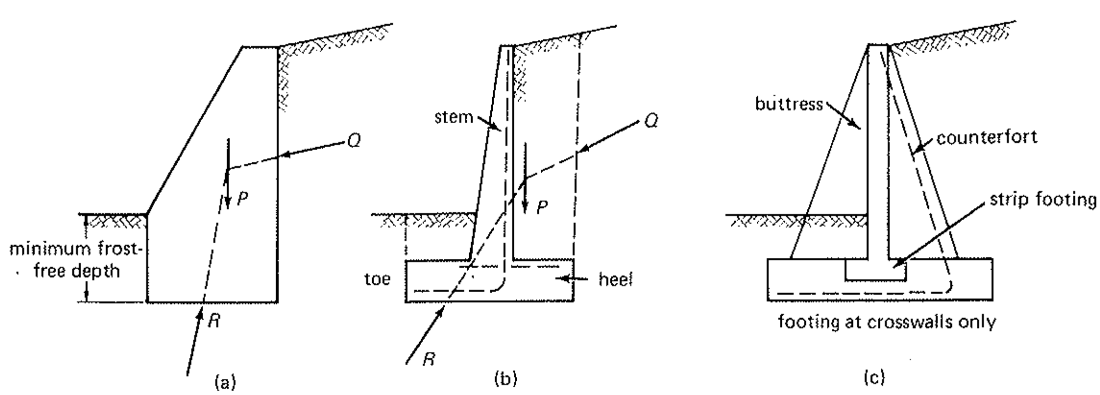


Fig. 5-39 Types of retaining walls.

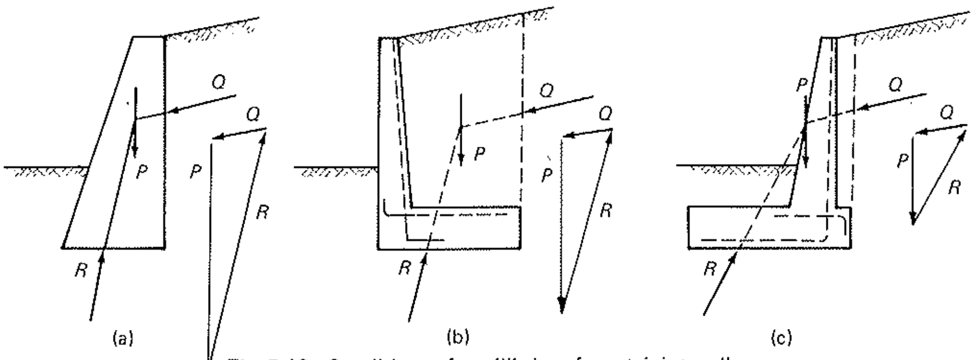


Fig. 5-40 Conditions of equilibrium for retaining walls.

spacings, provide all stability requirements and the walls between them act only as intermediate elements, Fig. 5-39c.

Where the crosswalls are visible in front they are called buttresses, where they are behind the wall and inside the soil, they are called counterforts. Some retaining walls are designed to have both.

The portion of a continuous cantilever wall or crosswall which is pressed downward into the soil is called the toe, and the portion which is lifted upward is called the heel. The vertical portion is called the stem, Fig. 5-39b.

The footing of a continuous retaining wall or crosswall must be large enough–

1. to resist the resultant vector due to confinement pressures, dead weight of the concrete and developed weight of the soil by means of safe bearing pressures; and
2. to keep the wall safely from overturning.

5.6.2 Confinement Pressure

The magnitude and distribution of the lateral pressures exerted by the confined soil or backfill depends on the kind of material, its moisture content, existence and depth of the groundwater, slope of backfill, and eventual surcharge due to live loads, storage, building loads, etc., applied close enough to be of influence. These pressures and their distribution have to be evaluated by principles of soil mechanics and do not form a part of this discussion. Most textbooks on Soil Mechanics contain detailed information on this subject.[5-1,5-2]

The resultant of these pressures is located at the centroid of the pressure wedge. The angle of inclination between the resultant and a line perpendicular to the back of the wall indicates the wall friction and is usually expressed as a fraction of the angle of internal friction, ϕ, of the fill material; it is often assumed with $\phi/2$. It is, however, important to keep in mind that this angle is also influenced by the slope, material and compaction of the backfill, by the surface texture of the concrete (at the back of the wall), by the existence of ground water behind the wall, or merely, by moisture in the soil which can act like a lubricant and reduce the friction angle to a minimal value. It is common practice with many designers to disregard the wall friction and to apply the resultant perpendicular to the back of the wall, in order to be on the safe side.

Spaced weep holes or continuous backdrains are often provided to alleviate the heavy pressure condition that can be caused by groundwater accumulating behind the back of the wall. Since weep holes and other drainage provisions may be clogged, it is recommended to investigate a retaining wall for a condition of full, or at least increased, water pressure. Such loading, however, represents an emergency condition and it is up to the engineer's judgement to reduce the safety factor for such a design as he sees fit.

5.6.3 Bearing Pressure

The pressure distribution under the footing of a retaining wall follows the same rules and is determined by the same methods as the pressure distribution under an eccentrically loaded footing. It is of course desirable, and to be attempted wherever possible, to keep the intersection of the resultant of all active forces (confinement pressures and developed weight) within the kern of the footing base. (In the case of a strip footing under a continuous retaining wall, the kern distance is $1/6$ of the footing size in the direction of the loading.) However, in many cases this is not economically feasible and greater eccentricities have to be accepted with a pressure distribution extending only over a part of the footing. The maximum edge pressure is then determined as discussed in sections 5.2.2 and 5.2.3 and shown in Fig. 5-5.

5.6.4 Overturning

Overturning can also be treated similarly as for regular column or wall footings. Where the base of the retaining wall is resting on rock or very hard soil, overturning may be calculated about the pressed edge and the safety factor can be expressed by the ratio $SF = M_Q/M_R$, where M_Q is the overturning moment caused by the confinement pressures acting about the pressed edge, and M_R is the resisting moment consisting of the dead weight of the retaining structure plus the developed weight of the fill material and any other frictional or passive resistances in the soil that may be mobilized during the overturning. The safety factor may also be expressed according to the "Suggested Design Procedures for Combined Footings and Mats"[3] as the ratio of the distance of the pressed edge from the base centroid to the eccentricity, e. In this ratio, $e = M_B/P$, where M_B is the sum of all moments about the base centroid and P is the sum of all forces acting perpendicular to the base. The safety factor against overturning should customarily be not less than 1.5.

Where the retaining wall rests on soil, the investigation regarding overturning may proceed along similar lines, except that the critical line about which the overturning and resisting moments are to be calculated is not at the pressed edge but somewhat inside, at the distance e_f from the centroid which is the center of gravity of the pressure block evaluated for an ultimate soil bearing condition. The evaluation of this condition is similar to the one described in section 5.2.1 and shown in Fig. 5-5c. Where the ultimate soil

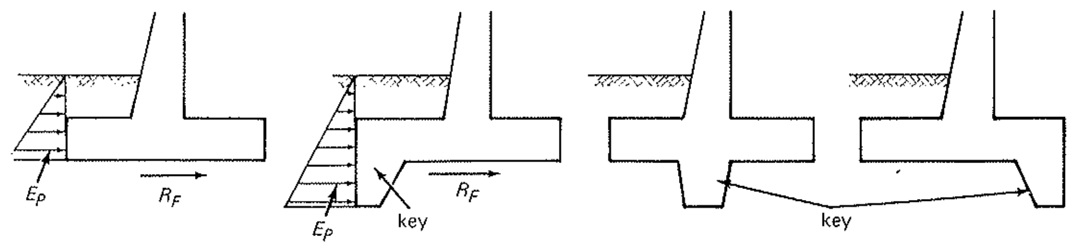


Fig. 5-41 Lateral resistance at base for retaining walls.

pressure is not known, it may be assumed to be 2.5 times the allowable soil pressure.[5-3]

5.6.5 Sliding Resistance

Lateral resistance against the horizontal component of the confinement pressure, commonly called resistance against sliding, has to be supplied by static friction at the footing base and by passive earth pressure against the embedded front portion of the retaining wall. Where this resistance is insufficient, the passive pressure can be increased by extending a key or lug into the soil below the footing base. Since the preference for the location of the lug can be argued about, it is probably located best where it is most practical with regard to construction and placement of reinforcement, Fig. 5-41.

5.6.6 Type Selection

The quality of the available subsoil and its allowable bearing pressure, as well as the height of the wall itself and the available construction space in front of the slope, can influence the selection of an economical type of wall to a great degree.

Where the construction space is ample, the allowable soil pressure is reasonable, and the height of the structure is not excessive, gravity walls of structural plain concrete can be used with advantage. Their weight is usually large enough to develop all the necessary base friction; however, the provision of keys or lugs projecting into the firm subsoil are rather common.

Where the available bearing pressure is large and construction space is available, the designer can utilize a long heel and develop much back fill weight to deflect the horizontal pressure resultant sharply down to the ground Fig. 5-40b. However, where the available bearing pressure is small, too much of it would have to be used to carry the developed weight of the backfill. In this case, the designer will have to be satisfied with a smaller total dead weight which will decrease the inclination of the resultant. Consequently he will have to increase the toe to keep the resultant sufficiently within the base to satisfy the allowable bearing pressure, and to keep the wall from overturning Fig. 5-40c. For ordinary conditions it may be considered good practice to keep the sizes of heel and toe about the same. The base of the toe has to be placed at frost free level.

Butresses and/or counterforts are used presently only for retaining walls of greater height (more than 20 to 25 ft, unless buttresses are architecturally desired) because of the great expense usually involved in the formwork. This type of design may see a kind of revival in combination with the use of precast wall panels.

For the enclosure of underground storage or extra large basements areas, freestanding retaining walls are sometimes used as basement walls.

In certain cases retaining walls are designed to act as such only temporarily until a full tie-in with the rest of the structure is achieved. In other cases such walls are designed only for the final loading condition, but not for the loading during construction, and will therefore require temporary shoring to maintain their stability. Some retaining walls may during construction change from a fully independent free standing structure to a top and bottom supported basement wall and in such a case every condition of loading ought to be considered for their stability as well as for their strength.

The use of precast sections may also here see a considerable field of application in the future. Such sections may, in the form of sheetings or in the form of entire wall panels, be driven or inserted into the ground before the main excavation has taken place. In such a case the same unit may serve successively as protective sheeting, retaining wall and final basement enclosure.[5-9]

Free standing retaining walls may also be constructed entirely of precast units. Such designs may simulate either the action of a sectional cantilever wall as shown in Fig. 5-39c, or that of a gravity wall in which case they are called cribbings.

5.6.7 Cribbings[5-10]

Cribbings consist in general of two types of units, namely, face and anchor units, also called stretchers and headers.

The structural design of cribbings is usually based on an empirical evaluation. The open faced units provide excellent drainage; in other cases drainage has to be provided to prevent groundwater from backing up and exerting pressure and unsightly leakage.

The satisfactory behavior of such precast cribbings depends to a considerable degree on the quality of the compacted backfill, which shall be installed in close coordination with the placement of the sections.

The face units are straight precast members of various cross sections, often with protruding lugs at their ends to connect them to the adjacent face and/or anchor members. Face members can be designed to open, closed, and flush-type manner, depending on the architectural requirements. They are usually set with a batter of about 1:6 at the front and can also be placed so as to form a curved face, up to 20°, without special units.

The anchor units usually come in two types of design, fishtail units or continuous back units. A fishtail unit is a T-shaped, precast section, placed perpendicular to the face of the wall with the purpose of tying two adjacent face units together and anchor them back to the fill material. Continuous back units are used together with cross wall units to form box like openings that are filled with compacted soil. There exist also combined units, where face and anchor units are cast together, simplifying erection where transportation permits.

5.6.8 Pile Supported Retaining Walls

Retaining walls may also be supported on piles which is often the case along waterfronts, or where the subsoil does not have a sufficient bearing capacity or lateral stability.

In such designs the vertical component of the pressure resultant is to be taken by the piles similarly as for a strip footing and the horizontal component is to be taken either by batter piles, tie backs, or similar devices.

Where the subsoil is also capable of providing uplift anchorage for the piles, the necessary developed weight may be reduced accordingly.

5.6.9 Design Procedure

The static design and the stability investigation of a retaining wall shall be based on service load conditions; the size of the footing and its location with regard to the wall itself is, therefore, entirely governed by the actual fill and/or liquid pressures under consideration of the allowable soil bearing pressures and allowable values for soil friction and passive resistance. The structural design of the stem and footing sections and their reinforcement, however, shall be based on the ultimate strength design method. For this purpose, the actual loads and active confinement pressures have to be multiplied by the appropriate load factors, and the resulting bearing pressures and other resistances have to be evaluated from these factored loading conditions. The procedure is not always easy to be executed because of variations in eccentricities that may be caused by it (see ex. 5-9, section 7). It is important to keep in mind, just as in the case of the ordinary footing, that these reactions are only caused by assumed factored loading conditions, but have no relationship to ultimate soil bearing values or similar.

After the ultimate loadings and corresponding reactions have been determined, every part of the retaining wall (stem, toe, or heel) has to be designed independently as a fully restrained cantilever section protruding from a mass center and carrying all applied loads.

The design to be performed for the service load conditions consists of a trial-and-error approach. The result is not too sensitive to slightly incorrect assumptions and can usually be adjusted in a second trial, if so desired. Due to the multitude of possible combinations it is rather difficult to arrive at reliable recommendations in this respect, however, if the conditions are not too much out of the ordinary, the footing size perpendicular to the wall directions can be assumed to be about $^2/_5$ to $^2/_3$ of the total wall height, the footing thickness about $^1/_8$ to $^1/_{12}$ of the footing size, and the base of the stem about $^1/_{10}$ to $^1/_{12}$ of its height. The following example has not been cut down to the minimum size, i.e., it could be adjusted and recalculated for further savings; such a procedure would be mostly repetitive and more confusing than helping. The top of the stem should not be made less than 8 in. to be able to place into it two layers of reinforcement, if so required. It is customary to place a second layer of reinforcement inside the exposed face for shrinkage and cracking control. It is practical to coordinate the reinforcements of stem and toe in such a way that the reinforcing bars of the stem can be bent right into the toe. For greater wall heights it is economical to run only a part of the vertical wall reinforcement to the top and stop the remainder at lower elevations.

EXAMPLE 5-9: Design a continuous retaining wall (cantilever wall) for the following conditions:

a. The difference between upper and lower level is 12 ft.
b. The upper level shall sustain a surcharge of 200 lb/ft^2 (medium heavy parking).
c. Soil of satisfactory bearing capacity is encountered at a depth of 4 ft below lower level (must be equal or greater than frost-free depth).
b. Provide weep holes to relieve water pressure; maximum expected ground water level 3 ft above lower level.
e. Use concrete with minimum $f_c' = 3000$ psi, and reinforcing steel having a minimum yield point of 60,000 psi.
f. Soil mechanical considerations provided the following values: The bearing soil is a medium dense, silty sand with an allowable bearing pressure of 3000 lb/ft^2. The weight of the moderately dry soil can be assumed with 100 lbs/ft^3 and its angle of internal friction $\phi = 32°$.

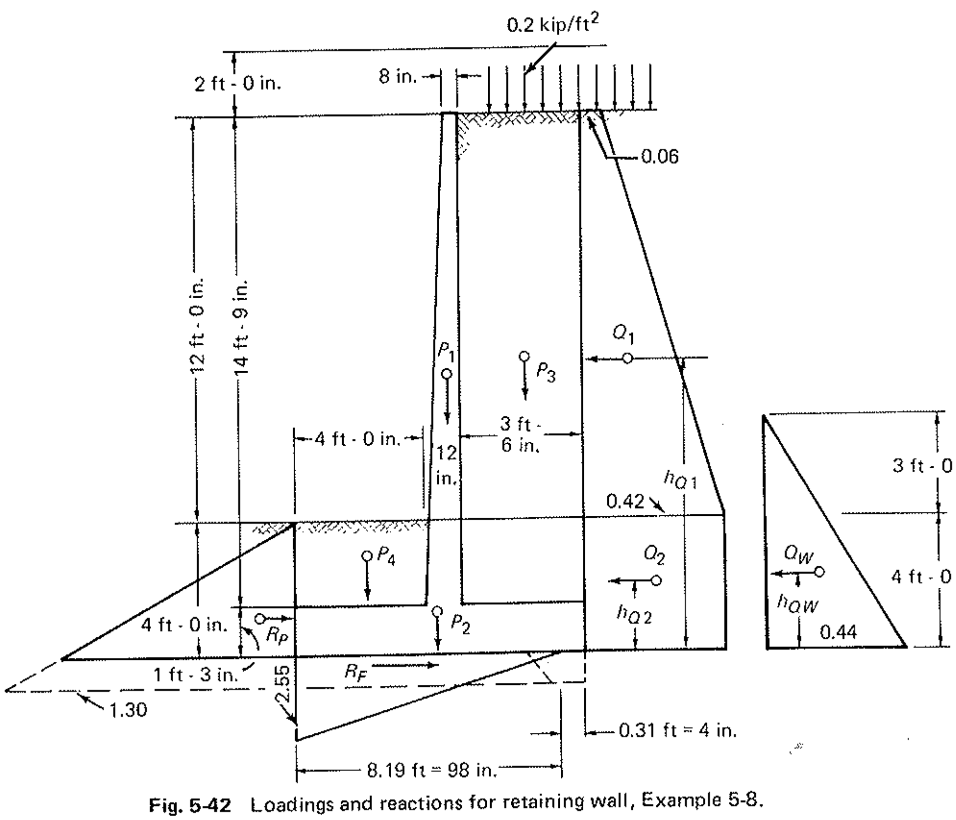


Fig. 5-42 Loadings and reactions for retaining wall, Example 5-8.

-1. Assumption of size and evaluation of loadings—Based on the requirements and other informations given above, the designer makes an assumption for the shape, sizes, and other relationships, and arrives at an arbitrary cross section as given in Fig. 5-42. Using these assumptions he arrives at the following values:

$$P_1 = \frac{(8+12)}{12 \times 2} \times 14.75 \times 0.15 \qquad = \quad 1.85 \text{ kip}$$

$$P_2 = 8.50 \times 15/12 \times 0.15 \qquad = \quad 1.60 \text{ kip}$$

$$P_3 = 3.50\,(14.75 \times 0.1 + 0.20) \qquad = \quad 5.90 \text{ kip}$$

$$P_4 = 2.75 \times 4.00 \times 0.1 \qquad = \quad \underline{1.10 \text{ kip}}$$

$$\text{total dead load } P = P_1 + P_2 + P_3 + P_4 = 10.45 \text{ kip/foot of wall}$$

eccentricity of total dead load P from centroid of base area, o;

$$e_P = \frac{P_1 \times 0.05 - P_3 \times 2.50 + P_4 \times 2.25}{P} = -1.15 \text{ ft}$$

the active earth pressure

$$p_a = \gamma h \tan^2 (45° - \phi/2)$$

Introducing above values the pressure increment can be evaluated with $p_a = 0.03$ kip/ft^2 per foot of depth. Transforming the given surcharge into an equivalent height of additional soil we obtain, for the active earth pressure at the upper level, $p_{a1} = 2 \times 0.03 = 0.06$ kip/ft^2, and at the lower level $p_{a2} = 14 \times 0.03 = 0.42$ kip/ft^2.

Below this elevation the active earth pressure remains constant.

$$Q_1 = \frac{0.06 + 0.42}{2} \times 12 = 2.88 \text{ kips}$$

$$Q_2 = 0.42 \times 4.0 \qquad = 1.68 \text{ kips}$$

total active earth pressure is

$$Q = Q_1 + Q_2 = 4.56 \text{ kip/lin-ft of wall.}$$

The elevations at which they are applied are for Q_1

$$h_{Q1} = 4.0 + \frac{12}{3}\left(\frac{2 \times 0.06 + 0.42}{0.06 + 0.42}\right) = 4.0 + 4.5 = 8.5 \text{ ft}$$

for Q_2

$$h_{Q2} = 2.0 \text{ ft}$$

and for the resultant active earth pressure, Q

$$h_Q = \frac{2.88 \times 8.5 + 1.68 \times 2.0}{4.56} = 6.1 \text{ ft}$$

-2. Calculation of bearing pressures—The resultant eccentricity at the base of the retaining wall can be found from

$$e = \frac{M_Q \pm M_P}{P} = \frac{Q \times h_Q \pm P \times e_P}{P}$$

The (±) sign depends on the location of e_P with regard to the base centroid, o.

$$e = \frac{4.56 \times 6.1 - 10.45 \times 1.15}{10.45} = \frac{15.9}{10.45} = 1.52 \text{ ft}$$

which is slightly outside the kern distance $c_k = \dfrac{8.5}{6} = 1.43$ ft.

The maximum toe pressure can be calculated according to Fig. 5-5 as

$$q = \frac{2P}{3mb} = \frac{2 \times 10.45}{3(4.25 - 1.52)\,1} = 2.55 \text{ kip/ft}^2 < 3.0 \text{ kip/ft}^2$$

-3. Overturning—Overturning according to Fig. 5-5c can be computed as follows:

$$e_f = \frac{l_f}{2} - \frac{P}{2q_f b_f} = \frac{8.5}{2} - \frac{10.45}{2(2.5 \times 3)\,1} = 3.55 \text{ ft}$$

$$SF = \frac{e_f}{e} = \frac{3.55}{1.52} = 2.30 > 1.5$$

Overturning under full water pressure is computed using: the total water pressure above base elevation

$$Q_W = (7.0 \times 0.0624)\,\frac{7.0}{2} = 1.55 \text{ kip}$$

and the additional moment due to the waterpressure about the base

$$M_W = 1.55 \times \frac{7.0}{3} = 3.6 \text{ kip-ft}$$

The eccentricity under this extreme condition would be

$$e_W = \frac{M_Q \pm M_P + M_W}{P_{\text{RED}}} = \frac{28.0 - 12.1 + 3.6}{9.57} = 2.0 \text{ ft}$$

The P in this equation was reduced to account for the buoyancy of the submerged portions.

$$SF = \frac{e_f}{e_W} = \frac{3.55}{2.0} = 1.77 > 1.5$$

-4. Sliding—A friction coefficient of 0.45 was assumed from soil mechanical considerations.

The total horizontal force is $Q = 4.56$ kip, then the frictional resistance that can be developed at the base is

$$R_F = 0.45P = 0.45 \times 10.45 = 4.75 \text{ kip}$$

The passive earth pressure that can be developed at the front of the retaining wall is

$$p_{p1} = \gamma \tan^2 (45° + \phi/2)h = 0.1 \tan^2\left(45° + \frac{32°}{2}\right) 4.0 = 1.3 \text{ kip/ft}^2$$

$$R_P = p_{p1}\, h/2 = 1.3 \times \frac{4.0}{2} = 2.6 \text{ kip}$$

The total resistance is therefore

$$R = R_F + R_P = 4.75 + 2.60 = 7.35 \text{ kip}$$

and the safety factor

$$SF = \frac{7.35}{4.56} = 1.6 > 1.5$$

This safety factor appears to be satisfactory. However, under full water pressure the horizontal force will be larger and the frictional resistance smaller due to the lubricating effect of the water. Provision of a key at the base is therefore recommended.

-5. Design of concrete thicknesses and reinforcement—In order to proceed with the strength design of the elements the actual service loads have to be multiplied by the appropriate load factors, and the respective strengths of the elements be evaluated under consideration of the appropriate ϕ-factors.

-6. Stem design—Moment about base of stem

$$M_u = 1.7\left[Q_1 (h_{Q1} - 1.25) + p_{a2}\,\frac{(2.75)^2}{2}\right]$$

$$= 1.7\left[2.88(8.50 - 1.25) + 0.42 \times \frac{(2.75)^2}{2}\right]$$

$$= 38.2 \text{ kip-ft}$$

for 2 in. concrete protection,

$$d_{\text{eff}} = 12 - 2.5 = 9.5 \text{ in.}$$

$$F = \frac{bd^2}{12{,}000} = \frac{12 \times 9.5^2}{12{,}000} = 0.09, \quad K_u = \frac{M_u}{F} = \frac{38.2}{0.09} = 425$$

a_u from Table 1-2 of Chapter 1, $a_u = 4.03$

$$A_s = M_u/a_u d = 38.2/4.03 \times 9.5 = 1.01 \text{ in.}^2 \ \#7 \ @\ 7$$

Check for full water pressure:

$$M_u = 38.2 + 1.4\left[\frac{(7-1.25)^2}{2}\ 0.0624 \times \frac{(7-1.25)}{3}\right]$$

$$= 38.2 + 1.4\,[1.04 \times 1.9] = 40.95 \text{ kip-ft}$$

$$K_u = \frac{40.95}{0.09} = 455,\quad a_u = 4.0,\quad A_s = \frac{40.95}{4.0 \times 9.5} = 1.07$$

$$A_s \text{ provided} = \#7 \,@\, 7 = 1.03 \text{ in.}^2;\ \frac{1.07}{1.03} = 1.04, \text{ O.K.}$$

-7. *Toe design*—It is advisable to check the shear condition first. In connection with bearing pressures, it is sometimes difficult to apply the corresponding load factors properly if the bearing pressures were caused by loadings with different load factors. Such a procedure may sometimes cause relocation of the resulting eccentricities and lead to pressure distributions that are in principle different from those obtained under service load conditions. Since this is not considered to be the intent of the design it is recommended to select in such cases either the highest load factor, 1.7, in order to be on the safe side, or to select by judgment an approximate load factor between 1.4 and 1.7, to apply to the bearing pressures obtained from the investigation of the service load conditions.

In this example a load factor of 1.7 was used.

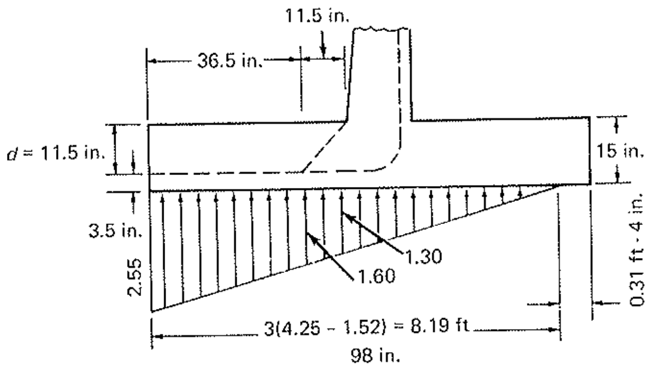


Fig. 5-43 Toe Design

$$d_{\text{eff}} = 15 - 3.5 = 11.5 \text{ in.}$$

$$V_{1u} = 1.7\left(\frac{2.55 + 1.60}{2}\right)\frac{36.5}{12} = 10.8 \text{ kips}$$

$$v_{1u} = \frac{1000\,V_{1u}}{\phi B d} = \frac{1000 \times 10.8}{0.85 \times 12 \times 11.5} = 92 \text{ psi}$$

$$v_{c\,\text{all}} = 2\sqrt{f'_c} = 110 \text{ psi}$$

$$92 < 110$$

For flexure, the bearing pressure outside the critical line (face of stem) is

$$1.7\left(\frac{2.55 + 1.30}{2}\right)4 = 13.0 \text{ kip}$$

The centroid of the trapezoidal pressure distribution is 2.25 ft away from the face of the stem. Hence

$$M_u = 13.0 \times 2.25 = 29.5 \text{ kip-ft},\quad F = \frac{12 \times 11.5^2}{12{,}000} = 0.132$$

$$K_u = \frac{29.5}{0.132} = 220,\quad a_u = 4.27,\quad A_s = \frac{29.5}{4.27 \times 11.5} = 0.62 \text{ in.}^2$$

Since it is practical to bend the stem reinforcement right into the toe, the provided stem reinforcement, $A_s = 1.03$ in.^{2}, is compared with the required one and found to be ample.

-8. *Heel design*—

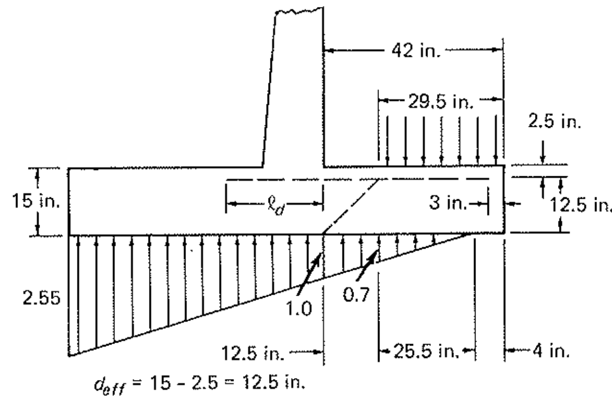


Fig. 5-44 Heel Design.

The shear force is

$$V_{1u} = 1.7\left(\frac{P_3 \times 29.5}{42} - \frac{0.7}{2} \times \frac{25.5}{12}\right) = 6.0 \text{ kip}$$

$$v_{1u} = \frac{1000\,V_{1u}}{\phi b d} = \frac{1000 \times 6.0}{0.85 \times 12 \times 12.5} = 47 \text{ psi} < 110 \text{ psi}$$

The flexural moment to be resisted is

$$M_u = 1.4 \times P_3 \times 1.75 - 1.7\left[\frac{1.0}{2}\left(\frac{38}{12}\right)^2 \frac{1}{3}\right] = 14.5 - 2.8$$

$$= 11.7 \text{ kip-ft}$$

$$F = \frac{12 \times 12.5^2}{12{,}000} = 0.155,\quad K_u = \frac{11.7}{0.155} = 76 \text{ less than min.}$$

$$A_s = \frac{1.33\,M_u}{a_u d}, \text{ or } \rho_{\min}\,bd$$

$$A_s = \frac{1.33 \times 11.7}{4.42 \times 12.5} = 0.28 \text{ in.}^2, \text{ or } 0.0033 \times 12 \times 12.5 = 0.49 \text{ in.}^2$$

$$A_s = 0.28 \text{ in.}^2 \text{ governs}$$

A bar size has to be selected for which the development length is smaller than the available embedment 42 - 3 = 39 in. Use #5 bars @ 12 in. o.c. See Tables 1-10 of Chapter 1. The bars have to be extended for the full development length beyond the face of the stem.

NOTATION

A_F = base area of footing, ft^2

A_P = cross sectional area of pier, in.^{2}

A_{ip} = average influence area per pile, ft^2

A_{V1} = influence area of bearing pressure for one-way shear, ft^2

A_{V2} = influence area of bearing pressure for two-way shear, ft^2

ΔA_{V2} = base area of truncated cone or pyramid for two-way shear, ft^2

a = footing projection in general, ft

a_L = projection of footing beyond critical face in long direction, in.

a_s = projection of footing beyond critical face in short direction, in.

a_u = factor used in determining $A_s = M_u/a_u d$

b = width of combined footing, ft

b_f = side of square footing, ft

b_f = short side of oblong footing, ft

b_o = base perimeter of truncated cone or pyramid for two-way shear, in.
b_p = dimension of pier parallel to footing side b_f, in.
c_c = concrete protection, in.
c_k = kern distance, ft
c_P = pile spacing, ft
D = dead loads or their related internal moments and forces
d = effective depth of section, in.
d_b = bar diameter, in.
d_f = diameter of round or polygonal footing, ft
d_o = diameter of opening in circular or polygonal footings, ft
d_P = pile diameter, in.
E = earthquake loads or their related moments and forces
e = eccentricity of load resultant from footing centroid, ft
e_f = eccentricity, as under e, causing failure pressure q_f at edge, ft
e_{max} = maximum permissible eccentricity to prevent overturning, ft
F = $bd^2/12{,}000$
γ_s = average unit weight of footing + surcharge, kip/ft^3
H = horizontal force, kip
h = lever arm of horizontal force, ft
h_f = thickness of footing, in.
h_Q = lever arm of force Q, ft
h_s = depth of footing base below floor, ft
I_c = moment of inertia of concrete cross section, in.^{4}
I_F = moment of inertia of footing base area, ft^4
I_{PG} = moment of inertia of pile group, ft^2
K_u = factor used in determining $F = M_u/K_u$
K_{si} = coefficient of vertical subgrade reaction for a 1 ft^2 area, tons/ft^2/ft
L = live loads and their related internal moments and forces
l = long dimension of combined footing, ft
l = distance between column centers, ft
l_d = development length (anchorage) of bar, in.
l_f = long side of rectangular footing, ft
l_p = dimension of pier parallel to footing side l_f, in.
M, M_u = flexural moment, kip-ft
M_B = moment about base, kip-ft
m = distance of eccentric load from pressed edge of footing, ft
m = general footing dimension, ft
m = number of pile rows of equal configuration
N = blow count of standard penetration test
n = general footing dimension, ft
n_P = number of piles in group
n_{Pr} = number of piles in row
o = centroid of footing or pile group
P, P_u = total column load at base, kip
P_D, P_{Du} = column dead load, kip
P_L, P_{Lu} = column live load, kip
P_{RES} = resultant load, kip
p_a = active earth pressure, kip/ft^2
p_p = passive earth pressure, kip/ft^2
Q = lateral force against retaining wall, kip
q = bearing pressure, kip/ft^2
Δq = weight of footing + surcharge, kip/ft^2
q_a = allowable bearing pressure, kip/ft^2
q_c = bearing pressure at base of column or pier, kip/in.^{2}
q_e = effective bearing pressure ($q - \Delta q$), kip/ft^2
q_f = bearing pressure at failure, kip/ft^2
q_m = bearing pressure due to moment, kip/ft^2
q_P = bearing pressure under concentric loading, kip/ft^2
q_s = bearing pressure at footing base due to factored loading, kip/ft^2
R = lateral resistance, kip
R_F = frictional resistance at base, kip
R_M, R_{Mu} = pile reaction due to moment, kip
R_P, R_{Pu} = pile reaction due to concentric loading, kip
R_{P_a} = allowable pile reaction, tons
R_{P_e} = effective pile reaction ($2R_{P_a} - W_P$), kip
ρ = percentage of reinforcement
S = ratio of long side to short side of footing
s = soil deformation
S_{PG} = section modulus of pile group, ft
S_{PR} = section modulus of pile row, ft
SF = safety factor
U = uplift force, kip
V_{1u} = shear force for one-way shear, kip
V_{2u} = shear force for two-way shear, kip
v_{1u} = average shear stress for one-way shear, psi
v_{2u} = average shear stress for two-way shear, psi
W = wind loads or their related moments and forces
W_P = weight of pile footing + surcharge, per pile, kip
w = line load, kip/ft
w_L = live load on floor, kip/ft^2
z = distance of extreme fiber from centroid, ft
z_{PG} = distance of extreme pile from centroid of pile group, ft
z_t = distance of extreme tensile fiber from centroid, in.
z_x = distance of reference line from centroid, ft

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