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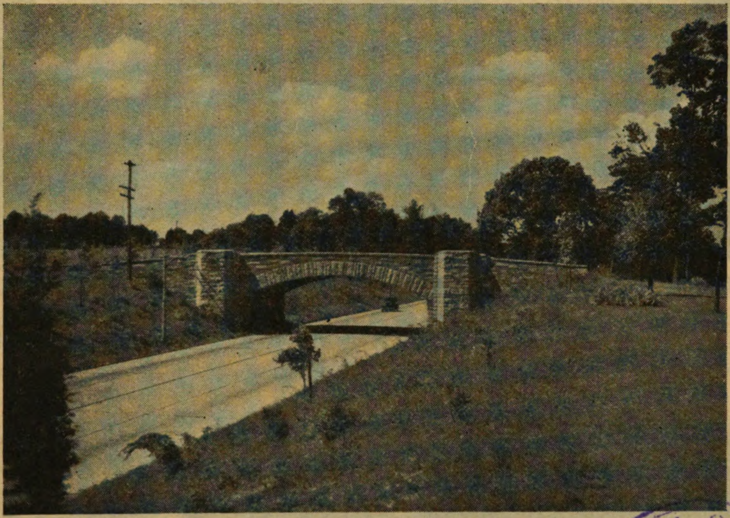

THE WELLINGTON UNDERPASS

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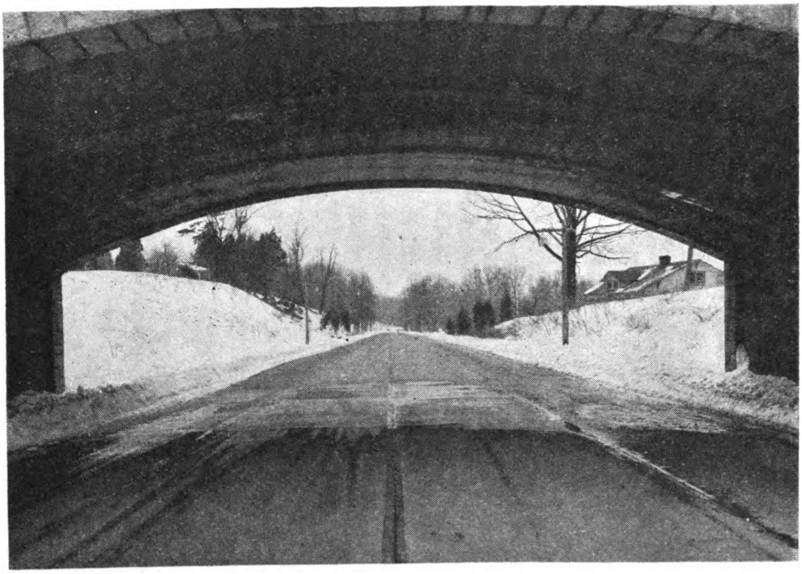
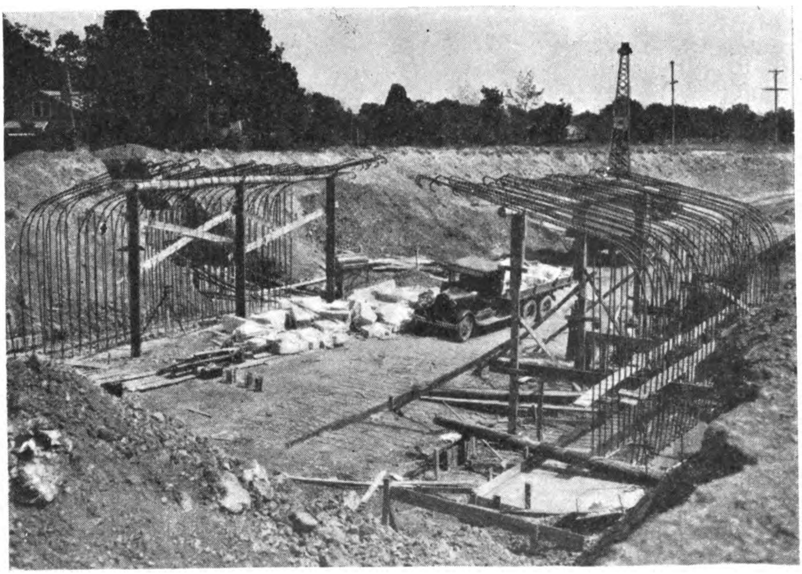


Underside of the Wellington underpass. The rigid-frame arch has architectural advantages over other types.



Constructing the Wellington underpass.

ANALYSIS OF A RIGID-FRAME CONCRETE ARCH BRIDGE

By C. D. GEISLER, *Highway Bridge Engineer, Division of Design, Bureau of Public Roads* [1]

The rigid-frame bridge is a type of curved beam or arch structure in which both horizontal and vertical reactions occur under vertical loading. Unless the frame is three-hinged, requiring a hinge at the crown, it is statically indeterminate.

For a true arch, analysis is made of an elastic rib sprung between rock or massive gravity abutments which are practically free from elastic deformation, and remain immovable under the reactions of the arch rib. In the rigid frame the abutments are relatively slender and flexible and approximately of the same general thickness as the rib spanning the abutments and therefore must be considered as integral parts of the elastic structure. Inasmuch as the interior face of the rigid frame is shaped to provide desired clearances and proportions of opening, the neutral axis of the entire rib usually will not conform to the curve of the equilibrium polygon or reaction line closely as in the case of the true arch. Consequently high bending moments will occur, requiring an adequate investigation of the bending and shearing stresses.

In accordance with the procedure of arch design, the dimensions of the entire rigid frame rib are first assumed, and the stresses computed on the basis of the elastic properties of the structure. The rib is then revised where required to reduce the stresses, and if necessary the frame is again analyzed.

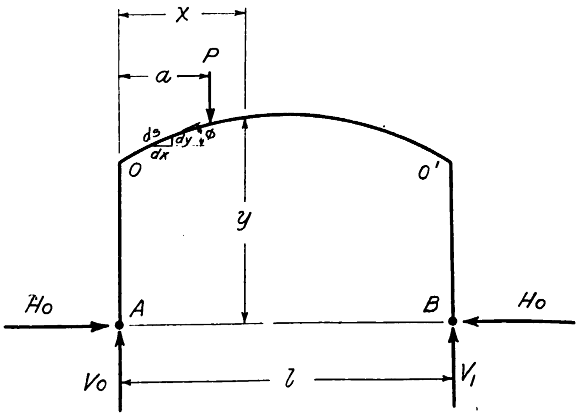
The abutments of a rigid-frame structure are usually set on light, spread footings of sufficient width to keep the foundation pressure within safe limits, but generally insufficient in size or mass to hold the lower ends of the abutments fixed against angular movement under flexure of the elastic frame. Since the exact degree of restraint furnished by the footings is questionable and frequently of negligible value, it is generally sufficiently accurate as well as on the side of safety to neglect any restraint afforded by the footings in the stress analysis. Thus a frame of single span may be assumed as "hinged" at the approximate center of the abutment bases. After the stresses have been determined, simply supplementary computations may be made to include the approximate restraining effect due to the footing reactions shifting to an eccentric position.

[1] E. J. Budge, associate highway bridge engineer, checked the computations and assisted in their arrangement.

GENERAL NOTES

FOR THE NOMENCLATURE AND THE DEVELOPEMENT OF THE FUNDAMENTAL FORMULAS USED IN THIS RIGID FRAME ANALYSIS, REFERENCE IS MADE TO THE ARTICLE " ANALYSIS OF CONCRETE ARCHES," BY W. P. LINTON AND C.D. GEISLER, REPRINTED FROM PUBLIC ROADS VOL. 8, NOS. 4 AND 5 JUNE AND JULY, 1927 AND OBTAINABLE FROM THE SUPERINTENDENT OF DOCUMENTS, GOVERNMENT PRINTING OFFICE, WASHINGTON, D.C. FOR 10 CENTS PER COPY. THE FORMULAS USED IN THE FOLLOWING ANALYSIS, TOGETHER WITH THEIR GENERAL APPLICATION AND THE METHOD OF DIVIDING UP THE AXIS OF THE FRAME, CONFORM WITH THE ABOVE TREATISE ON ARCHES EXCEPT AS NECESSARILY MODIFIED TO SUIT THE PARTICULAR SHAPE OF THE RIGID FRAME AND TO FULFILL THE ASSUMPTION THAT THE FRAME IS HINGED AT THE CENTER OF THE ABUTMENT FOOTINGS.

THE DERIVATION OF THE FINAL FORMULAS IS GIVEN AS FOLLOWS:



SYMMETRICAL TWO-HINGED RIGID FRAME

IN RIGID FRAME ANALYSIS, ASSUMPTION IS MADE THAT THE TOTAL SPAN LENGTH BETWEEN A AND B REMAINS CONSTANT. IN THE FOLLOWING TREATISE THE FOOTINGS OF THE FRAME ARE ALSO REGARDED AS FREE TO ROTATE OR "HINGED" AT THEIR CENTERS A AND B.

FOR M = BENDING MOMENT AT ANY POINT IN THE FRAME;

(1) ---------------------------------- $\sum_A^B My\Delta + etlE = 0;$

IN WHICH

$\Delta = \frac{ds}{I}$

e = COEFFICIENT OF EXPANSION DUE TO A CHANGE OF TEMPERATURE

t = THE NUMBER OF DEGREES FAHR. OF RISE OR FALL OF TEMPERATURE

E = YOUNG'S MODULUS OF ELASTICITY OF MATERIAL

I = MOMENT OF INERTIA OF A RADIAL SECTION OF THE FRAME

FOR A VERTICAL LOAD OF UNITY AT POINT a,

(2) ---------------------------- $M = V_0 x - H_0 y - \overset{x>a}{(x-a)}$

THEN $My\Delta = V_0 xy\Delta - H_0 y^2\Delta - \overset{x>a}{(x-a)} y\Delta$

AND (3) -------------- $\sum_A^B My\Delta = V_0 \sum_A^B xy\Delta - H_0 \sum_A^B y^2\Delta - \sum_a^B (x-a) y\Delta$

ALSO, FOR A VERTICAL LOAD OF UNITY AT POINT a,

(4) ------------------------- $V_0 = \frac{l-a}{l}$ AND (5) $V_1 = \frac{a}{l}$

SUBSTITUTING IN EQUATION (3) THE ABOVE VALUE FOR V_0, AND SIMPLIFYING, EQUATION (3) MAY TAKE THE FOLLOWING FORM,

(6) ---------------- $\sum_A^B My\Delta = \frac{a}{2} \sum_0^{0'} y\Delta - H_0 \sum_A^B y^2\Delta - \sum_0^a (a-x) y\Delta$

SOLVING EQUATIONS (1) AND (5) FOR H_0,

(7) -------------------- $H_0 = \dfrac{\frac{a}{2}\sum_0^{0'} y\Delta - \sum_0^a (a-x) y\Delta + etlE}{\sum_A^B y^2\Delta}$

EQUATION (2) MAY BE WRITTEN AS TWO EQUATIONS, ONE FOR $x<a$ AND ONE FOR $x>a$

FOR $X<a$; $M = V_0 X - H_0 y$

AND FOR $X>a$; $M = V_0 X - H_0 y - (X-a)$

THE LATTER EQUATION MAY BE SIMPLIFIED AS FOLLOWS:

$$M = a\left(1-\frac{X}{l}\right) - H_0 y$$

WE THEN HAVE

$$(8)\text{------}\begin{cases} M = V_0 X - H_0 y \text{..................(FOR } X<a) \\ M = a\left(1-\frac{X}{l}\right) - H_0 y \text{..........(FOR } X>a) \end{cases}$$

IF THE SPAN l IS DIVIDED INTO 10 EQUAL PARTS EACH EQUAL TO dx, WE MAY LET

$$X = Z.\frac{dx}{2}; \quad a = k.\frac{dx}{2}; \text{ AND } l = 20.\frac{dx}{2}$$

MAKING THESE SUBSTITUTIONS IN EQUATIONS (4), (7) AND (8), WE HAVE

$$(9)\text{---}V_0 = \frac{20-k}{20}$$

$$(10)\text{--}H_0 = \frac{k.\frac{1}{4}.\sum_{0}^{0'} y\Delta - \frac{1}{2}\sum_{0}^{a}(k-Z)y\Delta + 10etE}{\frac{1}{dx}\cdot\sum_{A}^{B} y^2\Delta}$$

$$(11)\text{----}\begin{cases} M = V_0 Z.\frac{dx}{2} - H_0 y \text{ .... FOR } Z<k \\ M = k\left(1-\frac{Z}{20}\right)\frac{dx}{2} - H_0 y \text{............. FOR } Z>k \end{cases}$$

FOR TEMPERATURE

$$(12)\text{-----}M_t = H_t y$$

IF WE LET $\frac{y\Delta}{dx}$ REPRESENT THE INTENSITY OF AN "ELASTIC" LOAD, AND ASSUME THE dx LENGTH OF THIS LOAD TO BE CONCENTRATED AT THE CENTER OF THE dx DIVISION, THE ELASTIC SHEAR AT THE IMMEDIATE LEFT OF ANY POINT X WILL BE THE SUMMATION OF ALL ELASTIC LOADS ON THE LEFT OF POINT X, OR

$$\sum_{0}^{X-1} \frac{y\Delta}{dx} \cdot dx$$

THE ELASTIC CANTILEVER MOMENT AT POINT X WILL BE THE SUMMATION OF THE ELASTIC SHEARS ON THE LEFT OF POINT X, OR

$$\sum_{0}^{X} \sum_{0}^{X-1} \frac{y\Delta}{dx} \cdot dx \cdot dx$$

AT POINT a THE ELASTIC MOMENT WILL BE

$$\sum_{0}^{a} \sum_{0}^{X-1} \frac{y\Delta}{dx} \cdot dx \cdot dx$$

THIS ELASTIC MOMENT AT POINT a MAY ALSO BE WRITTEN IN THE FORM:

$$\sum_{0}^{a} (k-z)\left(\frac{dx}{2}\right) \frac{y\Delta}{dx} \cdot dx$$

WE THEN MAY EQUATE;

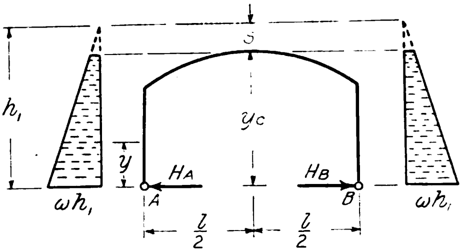
$$\sum_{0}^{a} (k-z)\left(\frac{dx}{2}\right) \frac{y\Delta}{dx} \cdot dx = \sum_{0}^{a} \sum_{0}^{X-1} \frac{y\Delta}{dx} \cdot dx \cdot dx$$

$$\text{OR} \quad \frac{1}{2} \sum_{0}^{a} (k-z) y\Delta = \sum_{0}^{a} \sum_{0}^{X-1} y\Delta$$

MAKING THIS SUBSTITUTION IN FORMULA (10), THE LATTER BECOMES;

$$(13) \text{-------------------} H_0 = \frac{k \cdot \frac{1}{4} \cdot \sum_{0}^{0'} y\Delta - \sum_{0}^{a} \sum_{0}^{X-1} y\Delta + 10\, etE}{\frac{1}{dx} \cdot \sum_{A}^{B} y^2\Delta}$$

HORIZONTAL EARTH PRESSURE



ω = UNIT WEIGHT OF AN EQUIVALENT FLUID

y_c = RISE OF FRAME AT CROWN

S = SURCHARGE

THE ROADWAY GRADE OVER THE STRUCTURE IS SYMMETRICAL SO THE FRAME WILL BE ANALYZED FOR BALANCED EARTH PRESSURES AT EACH END, ASSUMING THAT THE BACK-FILL IS PLACED SYMMETRICALLY BEHIND THE ABUTMENTS.

INASMUCH AS THE BACK-FILL WILL TEND TO PUSH THE ABUTMENTS INWARD, H_A AND H_B WILL ACT OUTWARD AND WILL BE GIVEN A − SIGN.

FOR BALANCED PRESSURES, $H_A = H_B$, AND AT ANY POINT $V = 0$.

IF M = MOMENT AT ANY POINT,

(14) $$M = -H_A y - \frac{1}{2}\omega h_1 y^2 + \frac{1}{6}\omega y^3$$

THEN $My\Delta = -H_A y^2\Delta - \frac{1}{2}\cdot\omega h_1 y^3\Delta + \frac{1}{6}\omega y^4\Delta$ AND

(15) $$\sum_A^B My\Delta = -H_A\sum_A^B y^2\Delta - \frac{1}{2}\cdot\omega h_1\sum_A^B y^3\Delta + \frac{1}{6}\omega\sum_A^B y^4\Delta$$

BUT, WE HAVE FROM EQUATION (1),

$$\sum_A^B My\Delta = 0$$

SOLVING FOR H_A,

(16) $$H_A = -\frac{\omega}{2}\cdot\frac{\frac{1}{2}\sum_A^B (h_1 - \frac{1}{3}y)y^3\Delta}{\frac{1}{2}\sum_A^B y^2\Delta}$$

AT ANY POINT, y;

(17) $H = H_A + \omega y(h_1 - \frac{1}{2}y)$ AND (18) $M = y[-H_A - \frac{1}{2}\cdot\omega y(h_1 - \frac{1}{3}y)]$

M = (MAX.) WHEN $y = h_1 - (h_1^2 + \frac{2H_A}{\omega})^{\frac{1}{2}}$

ANALYSIS OF THE WELLINGTON RIGID FRAME

ALTHOUGH THE WELLINGTON RIGID FRAME IS FACED WITH STONE, THE ANALYSIS OF ONLY THE INTERIOR STRUCTURAL FRAME IS CONSIDERED HERE.

THE SPAN AND VERTICAL CLEARANCES ARE SHOWN IN THE ACCOMPANYING SKETCH. THE FRAME IS DESIGNED FOR A WEIGHT OF EARTH FILL OF 110 LBS. PER CUBIC FOOT AND FOR FUTURE 8-INCH PAVEMENT AT 100 POUNDS PER SQUARE FOOT. THE LIVE LOAD IS TO BE EITHER A UNIFORM LOAD OF 125 POUNDS PER SQUARE FOOT OR THE A.A.S.H.O. H-15-TON TRUCK CONCENTRATED LOADING. THE LATERAL EARTH PRESSURE AGAINST THE ABUTMENTS IS ASSUMED AS THAT OF AN EQUIVALENT FLUID WEIGHING 35 POUNDS PER CUBIC FOOT. THE FRAME WILL ALSO BE ANALYZED FOR A TEMPERATURE VARIATION OF + 25 DEGREES AND - 35 DEGREES FAHRENHEIT.

ONE-HALF OF THE COMPLETE FRAME INCLUDING ABUTMENTS AND FOOTINGS, IS PLOTTED ON DETAIL PAPER TO A SCALE OF 1 INCH TO 4 FEET. THROUGH THE CENTER OF THE ABUTMENT FOOTING, WHICH IS THE LOCATION OF THE ASSUMED HINGE OF THE "TWO-HINGED" FRAME, A VERTICAL LINE IS DRAWN EXTENDING UPWARD TO APPROXIMATELY THE TOP OF THE ABUTMENT OR VERTICAL LEG OF THE FRAME. THIS VERTICAL LINE IS THE AXIS OF THE VERTICAL LEG. NEXT, DRAW THE AXIS OF THE CURVED RIB, WHICH IS A CURVE LYING HALF-WAY BETWEEN THE EXTRADOS AND INTRADOS, INTERSECTING THE AXIS OF THE VERTICAL LEG. DRAW A HORIZONTAL LINE FROM THE "HINGE POINT" AT THE BASE OF THE FOOTING TO THE CENTER OF THE SPAN, AND DIVIDE THIS HORIZONTAL LINE INTO 5 EQUAL PARTS. THE LENGTH OF EACH PART WILL BE EQUAL TO dx. AT THE CENTER OF EACH dx ERECT A PERPENDICULAR TO INTERSECT THE AXIS OF THE CURVED RIB, AND MARK THESE INTERSECTIONS 1, 2, 3, 4, AND 5. THE POINT OF INTERSECTION OF THE AXIS OF THE CURVED RIB AND VERTICAL AXIS OF THE ABUTMENT LEG WILL BE MARKED O, AND THE POINT OF ASSUMED HINGE AT THE LEFT ABUTMENT FOOTING MARKED A. THE CORRESPONDING HINGE POINT AT THE RIGHT ABUTMENT FOOTING WILL BE INDICATED AS B IN THE FORMULAS. THE VERTICAL AXIS O-A OF THE ABUTMENT IS THEN ARBITRARILY DIVIDED INTO TWO EQUAL PARTS AND THE CENTER OF EACH OF THESE PARTS, BEGINNING AT THE TOP OF THE ABUTMENT LEG, WILL BE MARKED O_1 AND O_2.

THE DIMENSIONS AND ORDINATES OF THE CURVED RIB AND VERTICAL LEG OF THE FRAME AT THE ABOVE POINTS ARE THEN TABULATED, AND THE PROCEDURE OF ANALYSIS FROM HERE ON FOLLOWS IN A GENERAL WAY SIMILAR TO THAT GIVEN IN THE TREATISE ON THE "ANALYSIS OF CONCRETE ARCHES."

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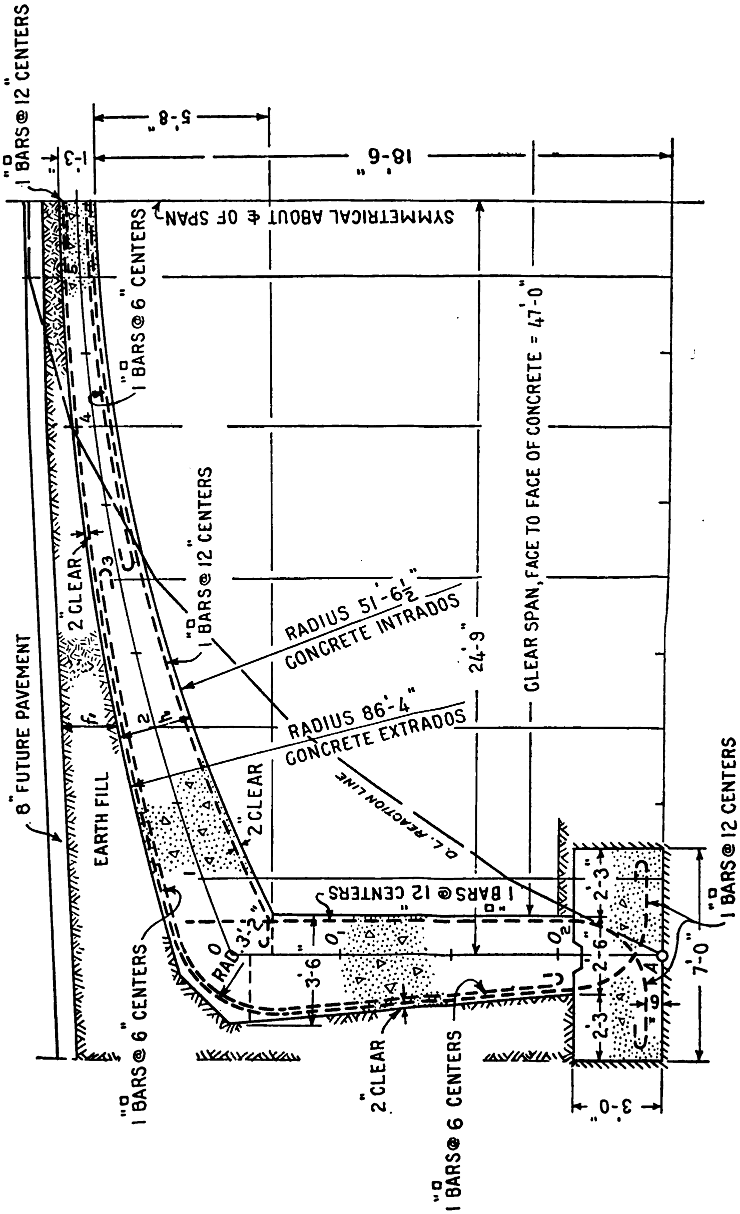


SKETCH OF FRAME

TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
PT.	h	ds	$h \cdot ds$	$150 h \cdot ds$	f_1	$110 f_1 \cdot dx$	PAVE.	D.L.	h^3	$I_c = h^3/12$	$h/2$	$(h/2 - d')$	$(h/2 - d')^2$	I_s	$I = I_c + I_s$	$\Delta = ds/I$	y	$y\Delta$	$y^2\Delta$
0_2	2.53	7.00	17.70	2655	—	—	—	—	16.19	1.35	1.27	1.07	1.15	0.36	1.71	4.09	3.50	14.31	50.08
0_1	3.18	7.00	22.25	3340	3.40	—	—	—	32.16	2.68	1.59	1.39	1.93	0.60	3.28	2.14	10.50	22.47	235.94
1	3.26	5.30	17.27	2590	2.80	1525	495	4610	34.65	2.89	1.63	1.43	2.04	0.64	3.53	1.50	14.97	22.45	336.08
2	2.45	5.15	12.62	1890	1.85	1010	495	3395	14.70	1.22	1.23	1.03	1.06	0.33	1.55	3.32	16.63	55.21	918.14
3	1.88	5.05	9.50	1425	1.20	655	495	2575	6.65	0.55	0.94	0.74	0.55	0.17	0.72	7.02	17.82	125.10	2229.28
4	1.50	5.00	7.50	1125	0.75	410	495	2030	3.38	0.28	0.75	0.55	0.30	0.09	0.37	13.52	18.65	252.15	4702.60
5	1.28	4.97	6.36	953	0.55	300	495	1748	2.10	0.17	0.64	0.44	0.19	0.06	0.23	21.60	19.03	411.05	7822.28
$\frac{1}{2}\sum_A^B =$																			16294.40

SPAN = l = 49.50′ RISE = 19.125′ $dx = \frac{1}{10} l = 4.95'$ $\frac{1}{2} dx = 2.475'$

REINFORCEMENT – 1″□ BARS @ 6″ CENTERS AND 12″ CENTERS AS SHOWN

$A_S = \frac{3 \times 1}{144} = 0.0208^{\square'}$ $I_S = 15 A_S \left(\frac{h}{2} - d'\right)^2 = 0.312 \times \text{COL. } 14$

TABLE 2

21	22	23	24	25	26	27	28
PT.	Z OR k	$y\Delta$	$\sum_0^{x-1} y\Delta$	$\sum_0^{a} \sum_0^{x-1} y\Delta$	$k \cdot \frac{1}{4} \sum_0^{0'} y\Delta$	COL. 26 – COL. 25	H_0
1	1	22.45	0	0	432.98	432.98	.0657
2	3	55.21	22.45	22.45	1298.94	1276.49	.1939
3	5	125.10	77.66	100.11	2164.90	2064.79	.3136
4	7	252.15	202.76	302.87	3030.86	2727.99	.4144
5	9	411.05	454.91	757.78	3896.82	3139.04	.4768
$\frac{1}{2} \sum_0^{0'} y\Delta =$		865.96	757.78	1183.21	10824.50	9641.29	1.4644
$\frac{1}{4} \sum_0^{0'} y\Delta =$		432.98					

FOR CHECK:– Σ COL. 27 = Σ COL. 26 – Σ COL. 25; Σ COL. 28 = Σ COL. 27 ÷ C_h

$$H_0 = \frac{k \cdot \frac{1}{4} \sum_0^{0'} y\Delta - \sum_0^{a} \sum_0^{x-1} y\Delta}{C_h} = \frac{\text{COL. } 27}{C_h}$$

$$C_h = \frac{1}{dx} \sum_A^B y^2 \Delta = \frac{2}{dx} \cdot \Sigma \text{ COL. } 20$$

$$C_h = \frac{2}{4.95} \times 16294.40 = 6583.6$$

$$H_t = \frac{10\, e t E_c}{C_h} = \frac{10 \times .000006 \times 144 \times 2{,}000{,}000}{C_h} = \frac{17{,}280}{C_h} \cdot t$$

FOR $t = {+25^\circ F \atop -35^\circ F}$ $H_t = {+65 \text{ LBS.} \atop -92 \text{ LBS.}}$

TABLE 3

29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
PT.	k	V_0	H_0	Pt. 3, $y = 17.82$, $Z = 5$	$Z\frac{dx}{2} = 12.375$, $(1-\frac{Z}{20})\frac{dx}{2} = 1.8562$		Pt. $5\frac{1}{2}$, $y = 19.125$, $Z = 10$	$Z\frac{dx}{2} = 24.75$, $(1-\frac{Z}{20})\frac{dx}{2} = 1.2375$		D.L.	DEAD LOAD			
				m_x	$H_0 y$	M_3	m_x	$H_0 y$	$M_{5\frac{1}{2}}$	LBS.	V_0 LBS.	H_0 LBS.	M_3 FT. LBS.	$M_{5\frac{1}{2}}$ FT. LBS.
1	1	0.950	.0657	1.856	1.170	+0.686	1.238	1.257	-0.019	4610	4379	302.9	+3162.0	- 87.6
2	3	0.850	.1939	5.568	3.455	+2.113	3.712	3.708	+0.004	3395	2886	658.3	+ 7173.6	+ 13.6
3	5	0.750	.3136	9.281	5.588	+3.693	6.187	5.998	+0.189	2575	1931	807.5	+ 9509.5	+ 486.7
4	7	0.650	.4144	8.044	7.385	+0.659	8.662	7.925	+0.737	2030	1320	841.2	+ 1337.8	+ 1496.1
5	9	0.550	.4768	6.806	8.497	- 1.691	11.137	9.119	+2.018	1748	961	833.4	- 2955.9	+ 3527.5
5'	11	0.450	.4768	5.569	8.497	-2.928	11.137	9.119	+2.018	1748	787	833.4	- 5118.1	+ 3527.5
4'	13	0.350	.4144	4.331	7.385	-3.054	8.662	7.925	+0.737	2030	710	841.2	- 6199.6	+ 1496.1
3'	15	0.250	.3136	3.094	5.588	-2.494	6.187	5.998	+0.189	2575	644	807.5	- 6422.1	+ 486.7
2'	17	0.150	.1939	1.856	3.455	-1.599	3.712	3.708	+0.004	3395	509	658.3	- 5428.6	+ 13.6
1'	19	0.050	.0657	0.619	1.170	-0.551.	1.238	1.257	- 0.019	4610	231	302.9	- 2540.1	- 87.6
		5.000	2.9288			+7.151			+5.896	28,716	14,358	6,886.6	- 7,481.5	+ 10,872.6
						-12.317			-0.038					

$$V_0 = \frac{20-k}{20}$$

For D.L. ~ $M_0 = -H_0 y_0 = -6886.6 \times 14 = -96{,}412$ FT. LBS.

$$m_x = \begin{cases} k\left(1-\frac{Z}{20}\right)\frac{dx}{2} \text{ .... FOR } k < Z \\ V_0 Z \frac{dx}{2} \text{ .... FOR } k > Z \end{cases}$$

$$M_x = m_x - H_0 y$$

$$M_t = -H_t \cdot y$$

$$M_t(0) = \begin{cases} +25^\circ = -65 \times 14.0 = -910 \text{ FT. LBS.} \\ -35^\circ = \ \ 92 \times 14.0 = +1{,}290 \text{ FT. LBS.} \end{cases}$$

$$M_t(3) = \begin{cases} +25^\circ = -65 \times 17.82 = -1{,}160 \text{ FT. LBS.} \\ -35^\circ = \ \ 92 \times 17.82 = +1{,}640 \text{ FT. LBS.} \end{cases}$$

$$M_t(5\tfrac{1}{2}) = \begin{cases} +25^\circ = -65 \times 19.125 = -1{,}240 \text{ FT. LBS.} \\ -35^\circ = \ \ 92 \times 19.125 = +1{,}760 \text{ FT. LBS.} \end{cases}$$

MAXIMUM LIVE LOAD MOMENTS WITH CORRESPONDING THRUSTS AND SHEARS

CASE 1 UNIFORM LIVE LOAD OF 125 LBS. PER SQ. FT.
LIVE LOAD PER LOAD POINT = 125 × 4.95 = 618 LBS.

POINT (0) :-

H = 618 × 2.929 = 1,810 LBS.
$-M$ = 618 × 2.929 × 14 = −25,340 FT. LBS.
V = 618 × 5 = 3,090 LBS.

POINT (3) :-

$-M$ = 618 × 12.317 = −7,613 FT. LBS.
H = 618 × 1.941 = 1,200 LBS.
V = 618 × 1.80 = 1,110 LBS.

$+M$ = 618 × 7.151 = +4,419 FT. LBS.
H = 618 × 0.988 = 611 LBS.
V = 618 × (3.2 − 2.5) = 432 LBS.

POINT ($5\frac{1}{2}$) :-

$+M$ = 618 × 5.896 = +3,644 FT. LBS.
H = 618 × 2.797 = 1,728 LBS.
V = 618 × 0 = 0 LBS.

$-M$ = 618 × 0.038 = −23 FT. LBS.
H = 618 × 0.1314 = 81 LBS.
V = 618 × 0 = 0 LBS.

CASE 2

CONCENTRATED LIVE LOAD. A. A. S. H. O. 15-TON TRUCK WITH 30 %
IMPACT (LATERAL DISTRIBUTION OF EACH WHEEL = 4.5 FEET)

POINT (0) :-

H = (12000 × .488 + 3000 × .254) × 1.3 ÷ 4.5 =
= 3467 × .488 + 867 × .254 = 1,910 LBS.

M = −1910 × 14 = −26,740 FT. LBS.
V = (3467 × .50 + 867 × .20) = 1,910 LBS.
V (MAX.) = (3467 × 1.0 + 867 × .7) = 4,080 LBS.

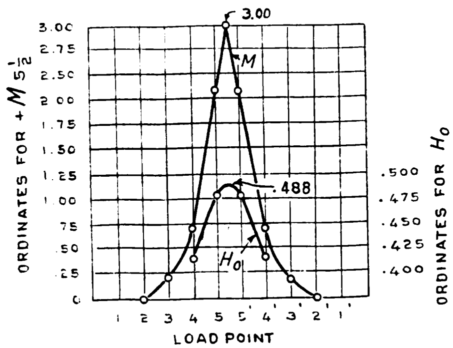
LIVE LOAD MOMENTS, THRUSTS, AND SHEARS

CASE 2 CONTINUED

POINT (3):-

+M	= 3467	× 3.693	+ 867 × 0	=	+12,800	FT. LBS
H	= 3467	× .314	+ 867 × 0	=	1,090	LBS.
V	= 3467	× 0.75	+ 867 × 0	=	2,600	LBS.
−M	= 3467	× 3.054	+ 867 × .551	=	−11,070	FT. LBS.
H	= 3467	× .414	+ 867 × .0657	=	1,490	LBS.
V	= 3467	× 0.35	+ 867 × .050	=	1,260	LBS.

POINT ($5\frac{1}{2}$):-



+M	= 3467	× 3.00	+ 867 × .12	=	+10,500	FT. LBS.
H	= 3467	× 0.488	+ 867 × .254	=	1,910	LBS.
V	= 3467	× 0.50	+ 867 × .20	=	1,910	LBS.
−M	= 3467	× .019	+ 867 × 0	=	−66	FT. LBS.
H	= 3467	× .0657	+ 867 × 0	=	230	LBS.
V	= 3467	× .050	+ 867 × 0	=	170	LBS.

HORIZONTAL EARTH PRESSURE

$$H_A = H_B = -\frac{\omega}{2} \times \frac{\frac{1}{2}\sum_A^B (h_1 - \frac{1}{3}y) y^3 \Delta}{\frac{1}{2}\sum_A^B y^2 \Delta} = -\frac{\omega}{2} \times \frac{\Sigma \text{ COL.48}}{\Sigma \text{ COL. 20}}$$

AT ANY POINT
$$\begin{cases} H = H_A + \omega y (h_1 - \frac{y}{2}) \\ M = y[-H_A - \frac{\omega y}{2}(h_1 - \frac{y}{3})] \\ V = 0 \end{cases}$$

$$M \text{ MAX. WHEN } y = h_1 - (h_1^2 + \frac{2H_A}{\omega})^{\frac{1}{2}}$$

TABLE 4

44	45	46	47	48
PT.	$\frac{1}{3}y$	$h_1 - \frac{1}{3}y$	$y^3\Delta$	$(h_1 - \frac{1}{3}y)y^3\Delta$
0_2	1.17	20.83	175.	3,645.
0_1	3.50	18.50	2,477.	45,825.
1	4.99	17.01	5,030.	85,560.
2	5.54	16.46	15,269.	251,328
3	5.94	16.06	39,726.	638,000.
4	6.22	15.78	87,703.	1,383,953.
5	6.34	15.66	148,858.	2,331,116.
			$\frac{1}{2}\sum_A^B =$	4,739,427.

ω = 35 LBS. PER CU. FT.

h_1 = 22.0 FT.

FOR M MAX.; –

$$y = 22.0 - (22^2 - \frac{2 \times 5{,}090}{35})^{\frac{1}{2}}$$

$$= 22.0 - 13.9 = 8.1 \text{ FT.}$$

$$H_A = -\frac{35}{2} \times \frac{4{,}739{,}427.}{16{,}294.40} = -5{,}090$$

AT POINT 0;

$$H = -5{,}090 + 35 \times 14 (22 - \frac{14}{2}) = +2{,}260 \text{ LBS.}$$

$$M = 14[5{,}090 - \frac{35 \times 14}{2} \times (22 - \frac{14}{3})] = +11{,}800 \text{ FT. LBS.}$$

AT POINT 3;

$$H = -5{,}090 + 35 \times 17.82 (22 - \frac{17.82}{2}) = +3{,}080 \text{ LBS.}$$

$$M = 17.82[5{,}090 - \frac{35 \times 17.82}{2} \times 16.06] = +1{,}460 \text{ FT. LBS.}$$

AT POINT $5\frac{1}{2}$,

$$H = -5{,}090 + 35 \times 19.125 \times (22 - \frac{19.125}{2}) = +3{,}240 \text{ LBS.}$$

$$M = 19.125 [5{,}090 - \frac{35 \times 19.125}{3} \times (22 - \frac{19.125}{3})] = -2{,}680 \text{ FT. L}$$

$$M \text{MAX.} = 8.1 [5{,}090 - \frac{35 \times 8.1}{2} \times (22 - \frac{8.1}{3})] = +19{,}070 \text{ FT. LBS.}$$

TABLE 5

49	50	51	52	53	54	55	56
		-MOMENT			+MOMENT		
		M	H	V	M	H	V
PT. 0. COS ϕ = .772 SIN ϕ = .641	D.L.	-96,412	+6887	14,358	-96,412	+6,887	14,358
	L.L.	-26,740	+1910	1910	0	0	0
	HOR.EARTH	—	—	—	+11,800	+2,260	0
	T.	-910	+65	0	+1,290	-92	0
	TOTAL	-124,062	+8,862	16,268	—	—	—
		$H \cdot \cos\phi$ = 6,840 $V \cdot \sin\phi$ = 10,420 N = 17,260			$H \cdot \cos\phi$ = $V \cdot \sin\phi$ = N		
PT. 3 COS ϕ = .980 SIN ϕ = .204	D.L.	-7,482	+6,887	5,065	-7,482	+6,887	5,065
	L.L.	-11,070	+1,490	1,260	+12,800	+1,090	2,600
	HOR.EARTH	—	—	—	+1,460	+3,080	0
	T	-1,160	+ 65	0	+1,640	-92	0
	TOTAL	-19,712	+8,442	6,325	+8,418	+10,965	7,665
		$H \cdot \cos\phi$ = 8,270 $V \sin\phi$ = 1,290 N = 9,560			$H \cdot \cos\phi$ = $V \cdot \sin\phi$ =		
PT. 5½ COS ϕ = 1.0 SIN ϕ = 0	D.L.	+10,873	+6887	0	+10,873	+6,887	0
	L.L.	- 66	+ 230	170	+10,500	+1910	1910
	HOR.EARTH	-2,680	+3,240	0	—	—	—
	T.	-1,240	+ 65	0	+1,760	-92	0
	TOTAL	—	—	—	+23,133	+8,705	1910
					$H \cdot \cos\phi$ = 8,705 $V \cdot \sin\phi$ = 0 8,705		

57 — MAXIMUM UNIT STRESSES

PT. 0. CONSIDER STEEL IN TENSION FACE ONLY
FROM HOOL AND JOHNSON'S HANDBOOK, DIAGRAM, PAGE 405

$$e' = \frac{M}{N} + \left(\frac{h}{2} - d'\right) = \frac{124,062}{17,260} + 1.55 = 8.75$$

$$\frac{e'}{d} = \frac{8.75}{3.4} = 2.57 \qquad p = \frac{2.0}{3.4 \times 1 \times 144} = .0041$$

$$K = \frac{17,260 \times 8.75 \times 12}{12 \times 3.4 \times 3.4 \times 144} = 90.8$$

f_s = 16,300#/SQ. IN. f_c = 580#/SQ. IN.

PT. 3 CONSIDER STEEL IN BOTH TOP AND BOTTOM
FROM HOOL AND JOHNSON'S HANDBOOK, DIAGRAMS, PAGES 399-402

$$x_0 = \frac{M}{N} = \frac{19,712}{9,560} = 2.05 \qquad \frac{x_0}{t} = \frac{2.05}{1.88} = 1.09 \qquad k = .33$$

$$\frac{d'}{t} = \frac{0.2}{1.88} = 0.106 \qquad p_0 = \frac{2.0}{1.88 \times 1 \times 144} = .0074 \qquad L = .118$$

$$f_c = \frac{19,712 \times 12}{12 \times 22.5 \times 22.5 \times .118} = 330\#/\text{SQ. IN.}$$

$$f_s = 15 \times 330 \left(\frac{1.68}{1.88 \times .33} - 1\right) = 8,450\#/\text{SQ. IN.}$$

PT. 5½ CONSIDER STEEL IN BOTH TOP AND BOTTOM
FROM HOOL AND JOHNSON'S HANDBOOK, DIAGRAMS, PAGES 399-402

$$x_0 = \frac{M}{N} = \frac{23,133}{8,705} = 2.66 \qquad \frac{x_0}{t} = \frac{2.66}{1.25} = 2.13 \qquad k = .38$$

$$\frac{d'}{t} = \frac{.02}{1.25} = 0.16 \qquad p_0 \frac{4.0}{1.25 \times 144} = .0222 \qquad L = .179$$

$$f_c = \frac{23,133 \times 12}{12 \times 15 \times 15 \times .179} = 575\#/\text{SQ. IN.}$$

$$f_s = 15 \times 575 \times \left(\frac{1.05}{1.25 \times 38} - 1\right) = 10,400\#/\text{SQ. IN.}$$

$$N = H \cos\phi + V \sin\phi$$

FOUNDATION SOIL PRESSURES

VERTICAL LOADS :-

DEAD LOAD:

VERTICAL REACTION AT POINT 0		= 14360 LBS
WEIGHT OF CONCRETE IN ABUTMENT LEG = 2655 + 3340		= 5995 LBS
ADDITIONAL WEIGHT OF CONCRETE IN FOOTING = 2.25 × 3.0 × 2 × 150		= 2025 LBS
EARTH ABOVE ABUTMENT AND FOOTING = $\left[(2.25+1.25)\times\frac{11}{2}+3.5\times4.5\right]\times110$		= 3850 LBS.
PAVEMENT ABOVE " " " = 3.5 × 100		= 350 LBS
TOTAL D.L VERTICAL REACTION		= 26.580 LBS
LIVE LOAD (UNIFORM LOAD) = 125 × (24.75 + 3.50)		= 3530 LBS
TOTAL VERTICAL LOAD		= 30,110 LBS

AVERAGE VERTICAL SOIL PRESSURE $= \frac{30.110}{7\times2000} = 2.2$ TONS PER SQ. FT.

HORIZONTAL THRUSTS

(HORIZONTAL PRESSURE OF BACKFILL NEGLECTED)

H_0	FOR DEAD LOAD	= 6,887 LBS
H_0	FOR UNIFORM LIVE LOAD	= 1810 LBS
$+H$	FOR TEMPERATURE	= 65 LBS.
	TOTAL HORIZONTAL THRUST	= 8762 LBS.

HORIZONTAL SOIL PRESSURE AT REAR FACE OF FOOTING :-

$= \frac{8762}{3\times2000} = 1.45$ TONS PER SQ FT., NEGLECTING RESISTANCE TO SLIDING

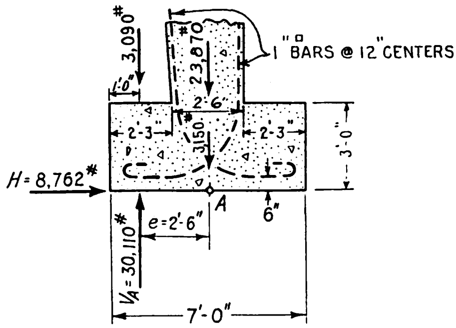
THE ABOVE SOIL PRESSURES ARE CONSIDERED SATISFACTORY FOR THE SHALE ROCK FOUNDATION UNDER THIS STRUCTURE.

A FOUNDATION SOIL OF YIELDING CHARACTER WOULD PROBABLY HAVE REQUIRED THE PLACING OF TIES BETWEEN THE FOOTINGS, AS A SLIGHT OUTWARD MOVEMENT OF THE ABUTMENTS IN A RIGID FRAME OF THIS TYPE WILL PRODUCE A MARKED INCREASE IN THE STRESSES AT THE CROWN. IN THE ABOVE STRUCTURE, AN INCREASE IN SPAN LENGTH OF $\frac{1}{2}$ AN INCH WOULD INTRODUCE STRESSES THROUGHOUT THE FRAME EQUIVALENT TO A DECREASE IN TEMPERATURE OF :

$$\frac{\frac{1}{2}\times\frac{1}{12}}{0.000006\times49.5} = 140 \text{ DEGREES FAHRENHEIT}$$

MAXIMUM STRESSES IN FOOTING

FOR DETERMINATION OF MAXIMUM STRESSES AT THE ABUTMENT FOOTING, ASSUMPTION IS MADE THAT THE REACTION LINE WILL PASS 1'-0" INSIDE THE REAR EDGE OF THE FOOTING. THIS IS A REASONABLE ASSUMPTION, AND WILL REPRESENT THE CONDITION OF THE FOOTING WHEN IT TENDS TO ROTATE ABOUT THE ASSUMED POINT OF HINGE. (THE FOOTING WILL TEND TO ROTATE OUTWARD ABOUT POINT "A" WHEN THRUST H IS POSITIVE)



MOMENT ACROSS BASE OF ABUTMENT STEM AT TOP OF FOOTING:-

$M = (30{,}110 - 3{,}090) \times 2.5 - 8{,}762 \times 3.0 = 41{,}260$ FT. LBS.

$N = 23{,}870$ LBS.

$p_o = \dfrac{2.0}{2.5 \times 144} = .0056$ $\qquad x_o = \dfrac{41{,}260}{23{,}870} = 1.73$ $\qquad \dfrac{x_o}{t} = \dfrac{1.73}{2.5} = 0.69$

$\dfrac{d'}{t} = \dfrac{0.2}{2.5} = 0.08$ $\qquad k = .37$ $\qquad L = .106$

$f_c = \dfrac{41{,}260 \times 12}{12 \times 30 \times 30 \times .106} = 433$ LBS. PER SQ. IN

$f_s = 15 \times 433 \times \left(\dfrac{2.3}{2.5 \times .37} - 1\right) = 9{,}700$ LBS PER SQ IN.

UNIT SHEAR; $v = \dfrac{8762}{27.5 \times 12 \times .875} = 30$ LBS. PER SQ. IN.

MOMENT IN VERTICAL SECTION OF FOOTING AT EDGE OF ABUTMENT STEM:-

$M = 27{,}020 \times 1.25 - 8{,}762 \times 1.5 = 20{,}640$ FT. LBS $\qquad N = 8762$ LBS. $\qquad p = \dfrac{1.0}{30 \times 12} = .0028$

$e' = \dfrac{20{,}640}{8762} + 1.0 = 3.36$ $\qquad \dfrac{e'}{d} = \dfrac{3.36}{2.50} = 1.35$ $\qquad K = \dfrac{8762 \times 3.36 \times 12}{12 \times 2.5 \times 2.5 \times 144} = 32.7$

$f_s = 5{,}000$ LBS. PER SQ IN. $\qquad f_c = 200$ LBS. PER SQ. IN

UNIT SHEAR:- $v = \dfrac{27{,}018}{30 \times 12 \times .875} = 86$ LBS. PER SQ. IN.

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