12 Serviceability Limit State

The New Zealand code approach for the design of reinforced concrete structures identifies two limit states:

(i) Serviceability limit state
(ii) Ultimate limit state.

The serviceability limit state is concerned with behaviour under operating conditions expected to occur a number of times during the life of the structure. Under the serviceability limit state loading combinations, the structure shall satisfy the following criteria:

(i) the deflections shall not impair the performance of the structure,
(ii) its dynamic characteristics shall not impair the performance of the structure,
(iii) where necessary it shall satisfy the fatigue criteria.

The load factors for the serviceability limit state are *less* than for the strength limit state. Unless noted otherwise, loading upon the structure is calculated as:

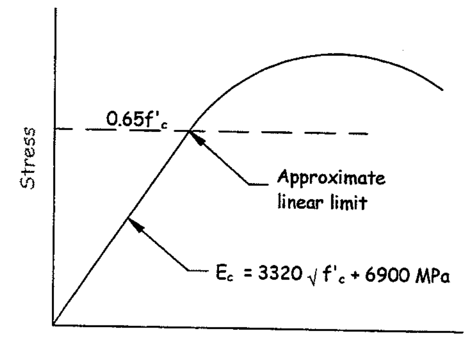
$$1.0\,G + 1.0\,Q_s$$

Similarly, for serviceability conditions the strength reduction factor should be taken as:

$$\phi = 1.0$$

12.1 Material Properties

Under serviceability conditions the structure should remain elastic. Concrete compression stresses should remain in the linear elastic range, and reinforcement should not yield. As inelastic behaviour does not develop, redistribution of bending moments is not permitted for serviceability conditions.



Concrete Stress-Strain characteristics in compression

12.2 Short Term Loads

For short term loadings:

(a) Concrete stress-strain curve is essentially linear to $0.65f'_c$
(b) Elastic modulus of concrete varies with:
  (i) aggregate type
  (ii) cement, water content, and the use of admixtures
  (iii) concrete strength
  (iv) moisture content when tested

In the New Zealand code (NZS 3101-95) the elastic modulus of concrete is given by (see 3.8.1.2):

$$E_c = 3320\sqrt{f'_c} + 6900MPa$$

but in practice much variation occurs i.e. $E_c \pm \underline{25\%}$. For instance, when considering f'_c, there are several factors to consider:

(i) As f'_c is the 28 day minimum specified strength, should an allowance be made for strength increase with age?
(ii) What is the mean concrete strength? Is it great than the specified (5% lower characteristic) strength?
(iii) What is the variation in f'_c between test specimens and the structure?

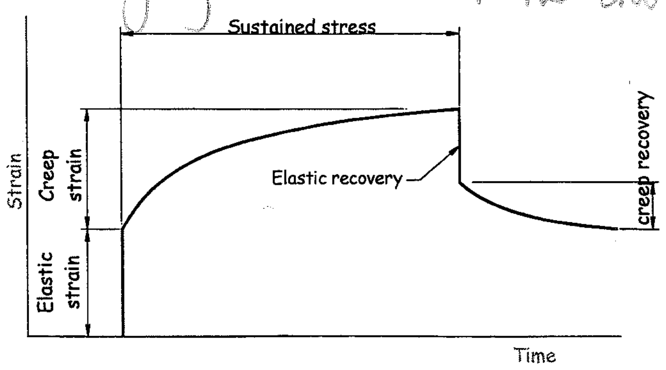
In practice, different f'_c values are used by engineers. In this course, use the minimum specified strength, f'_c.

12.3 Creep Under Sustained Loads

Under sustained stresses concrete creeps. Typically the creep strain is 1.5 to 2.5 times the elastic strain.

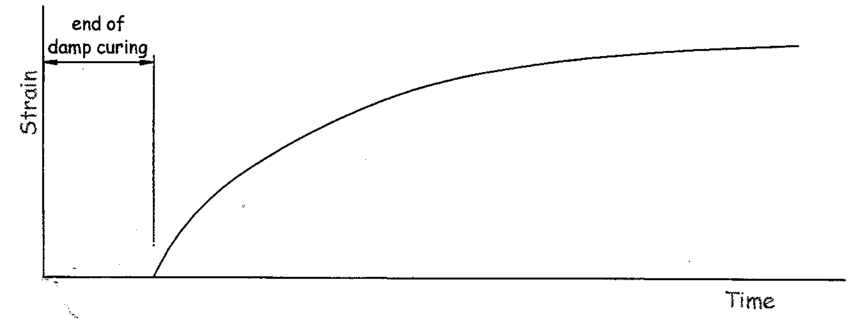
The magnitude of creep strain is influenced by:

(i) Age at loading
(ii) Thickness of member
(iii) Concrete constituents
(iv) Drying conditions in the environment



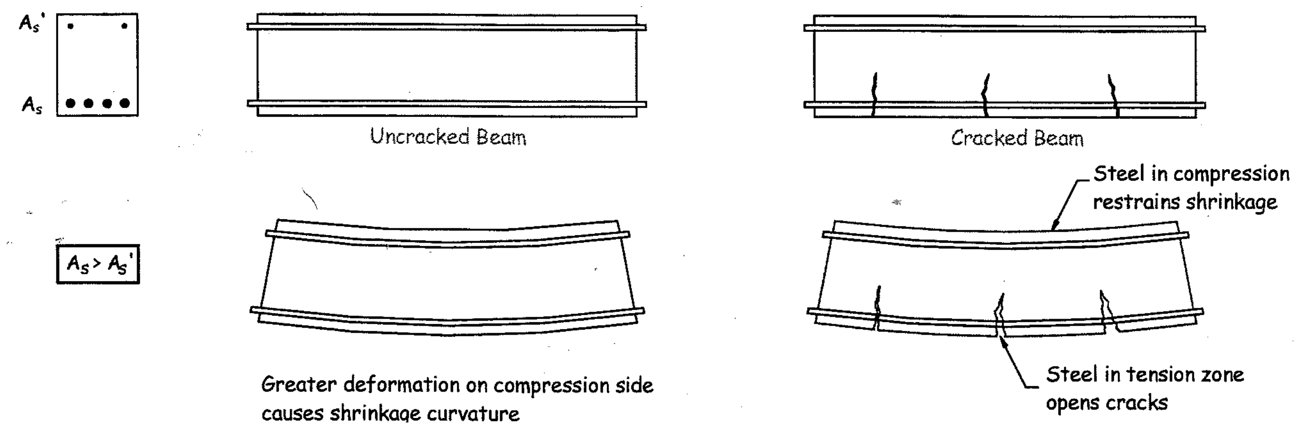
12.4 Shrinkage

As concrete dries (cures), it shrinks. Typical shrinkage strain values are 0.0002 - 0.0004.



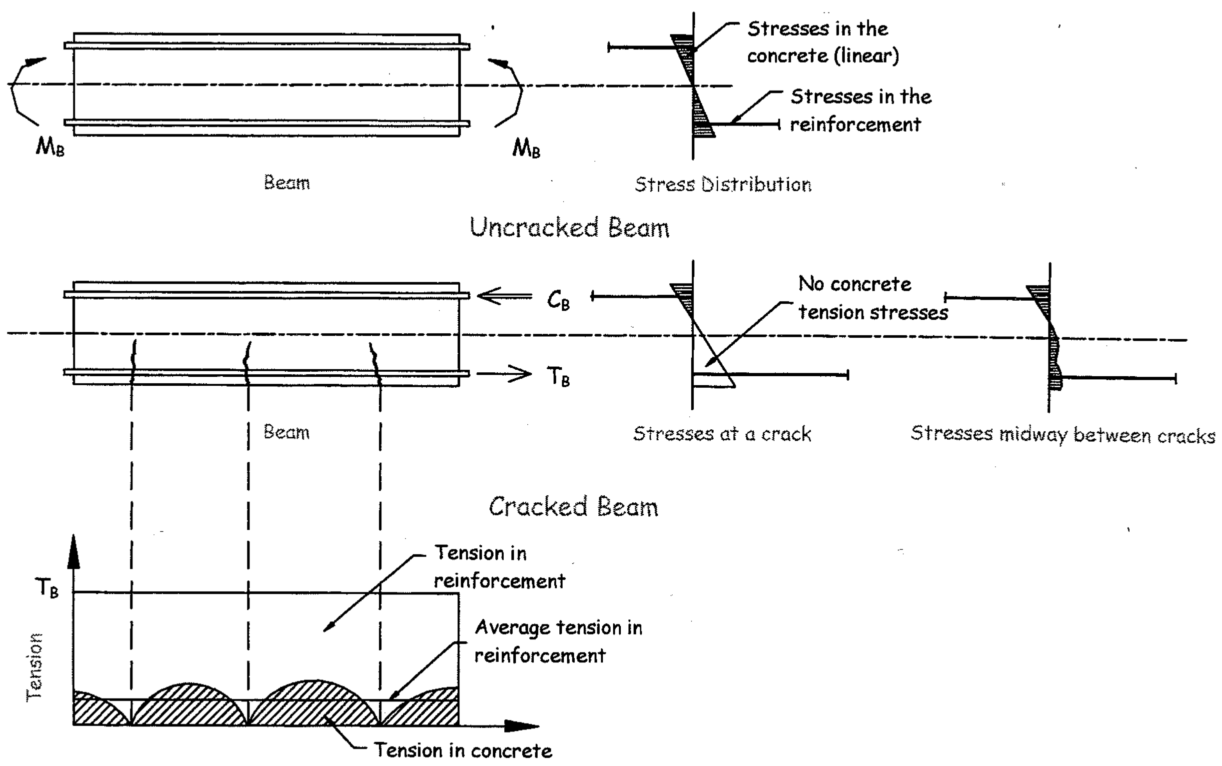
Shrinkage depends upon:

(i) member thickness
(ii) concrete constituents
(iii) drying characteristics of the environment.



12.5 Behaviour of Beams - Short Term Loading

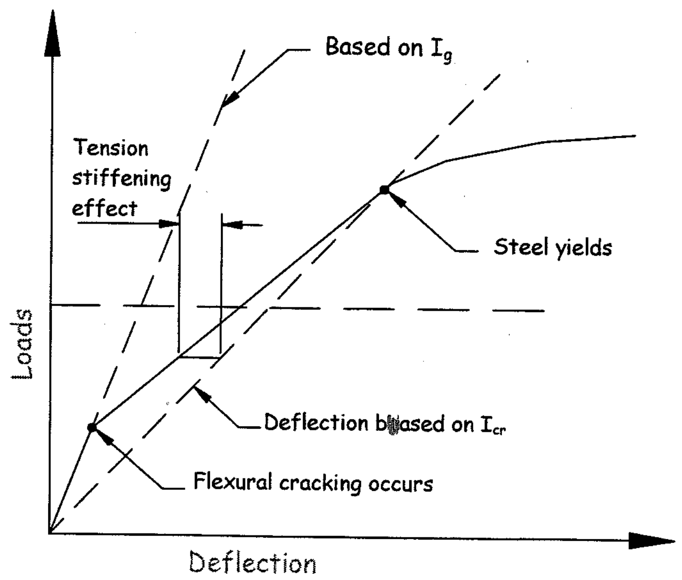
For short term loading a distinction is made between uncracked and cracked sections. If the section is uncracked, the gross section stiffness, I_g, should be used. More accurately, the uncracked transformed section should be employed, however this is generally not necessary.



Tension Stiffening

Tension carried by the concrete between the cracks reduces the average tensile strain in the reinforcement, effectively making the section stiffer.

When the section is cracked, the concrete continues to carry tension in the regions between the cracks. This reduces the average tensile strain in the reinforcement, resulting in a stiffer section. Cracked section response is typically calculated on the basis of the transformed cracked section for calculating stresses. This neglects tension stiffening effects. Where these are important, for example in calculating deflections, a nominal allowance is made. Note that the flexural stiffness of a cracked beam is typically about half that of the uncracked equivalent. Where tension stiffening is important, as for example in calculating deflections, nominal allowance is made for this effect.



12.6 Cracked Transformed Sections

When determining member stresses, it is necessary to ensure member force equilibrium:

$$C_c + C_s = T \text{ or } f_c A_c + f_s' A_s' = f_s A_s$$

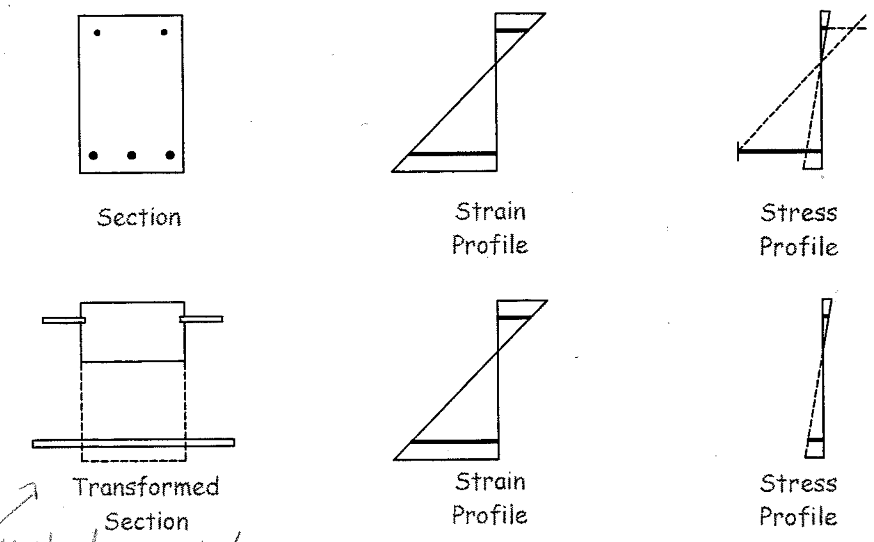
Working from a linear strain assumption for both the concrete and the reinforcement, and recognising that strains will remain in the elastic regime, the stress in each component can be related to the strain using:-

$$f_c = E_c \varepsilon_c \text{ and}$$

Consequently the stress profile is not linear, as shown below. By transforming the reinforcement to concrete by the modular ratio:

$$n = \frac{E_s}{E_c}$$

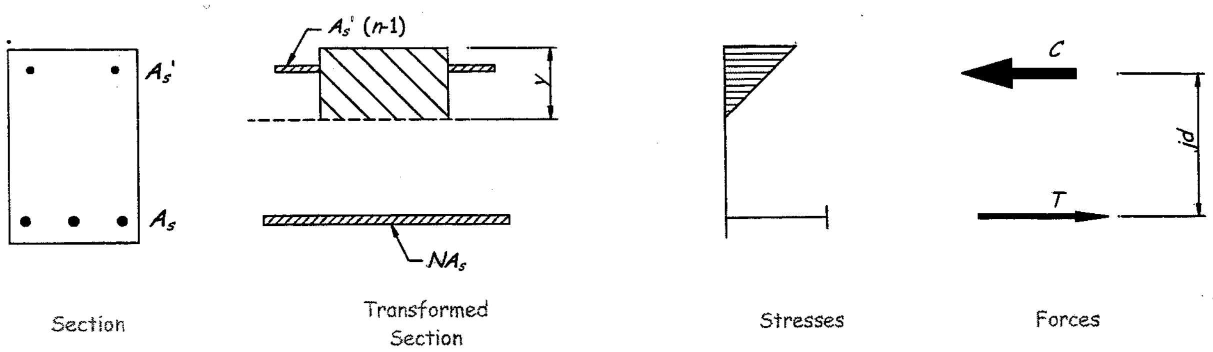
an equivalent concrete section is obtained, having both a linear strain profile and a linear stress profile. Using similar triangles, it is then relatively simple to develop equations that have the neutral axis depth as the only unknown. Note that this implies that at the serviceability limit state the neutral axis depth is constant, regardless of the stresses acting upon the section.



equivalent concrete

12.6.1 Transformed Section Analysis Procedure

When concrete is subjected to flexural tension it is assumed to crack, and support no tension forces. Consequently it is disregarded in the calculation.



The method used is:

(i) Find the neutral axis position by:

(a) 1st moment of areas (which is ok if there is no axial load)

or (b) trial and error in balancing forces

(ii) Find I of the transformed section

(iii) To find stresses in the section, two different methods may be used:

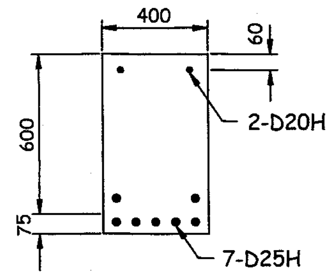
(a) use $f = \frac{My}{I}$ allowing for modular ratio n

prone to errors

(b) find jd from the stress distribution, then find $T = \frac{M}{jd}$ etc

Example

If f'_c = 25 MPa and the serviceability bending moment is 330 kNm, find the stresses in the concrete and the tension reinforcement.



$E_c = 3320\sqrt{25} + 6900 =$ 23500 MPa

E_s = 200000 MPa

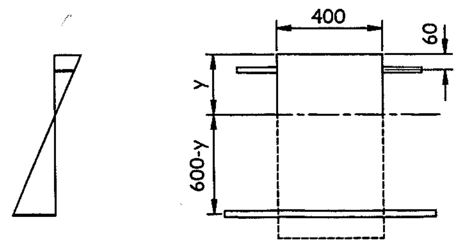
$n = \frac{E_s}{E_c} = \frac{200000}{23500} =$ 8.51

A_s' = 2-D20H = 2 x 314 = 628 mm²

$(n\text{-}1)A_s'$ = 4719 mm²

A_s = 7-D25H = 7 x 491 = 3436 mm²

nA_s = 29241 mm²



For force equilibrium the sum of the first moments of transformed area about the neutral axis = 0.

Alternatively, recognising that stress is linearly related to the distance from the neutral axis:

$$f_c \propto \frac{y}{2},\ f_s' \propto (y-60),\ f_s \propto (600-y)$$

Force equilibrium may be expressed as:

$$C_c + C_s = T$$

$$\Rightarrow A_c f_c + (n-1)A_s' f_s' = nA_s f_s$$

$$\Rightarrow 400y\frac{y}{2} + 4719\,(y-60) = 29241\,(600-y)$$

$200y^2 + 4719y - 283140 + 29241y - 17544600 = 0$

$200y^2 + 33960y - 1782774 = 0$

The solution to this expression can then be obtain by rearranging to form a quadratic equation, or by finding a solution by trial and error:

Try y = 200 mm	8,000,000 + 660,660 = 8,660,660 (LHS) 11,696,400 (RHS) → Therefore increase y
Try y = 220 mm	9,680,000 + 755,040 = 10,435,040 (LHS) 11,111,580 (RHS) → Therefore increase y
Try y = 225 mm	10,125,000 + 778,635 = 10,903,635 (LHS) 10,965,375 (RHS) → Therefore OK

12.6.2 Cracked Section Stiffness

Once the location of the neutral axis has been determined, the cracked section stiffness can be found using the parallel axis theorem. Note that generally the reinforcement will have little moment of inertia about its local axis, so that only rotation of the reinforcement about a displaced axis needs to be considered.

$$\text{hence, } I_{cr} = \frac{400 \text{ x } 225^3}{12} + 400 \text{ x } 225 \text{ x} \left(\frac{225}{2}\right)^2 + 4719\,(225-60)^2 + 29241\,(600-225)^2$$

$$= 5.76 \times 10^9 \text{ mm}^4$$

Note that I_g of the untransformed section (ignoring reinforcement) is:

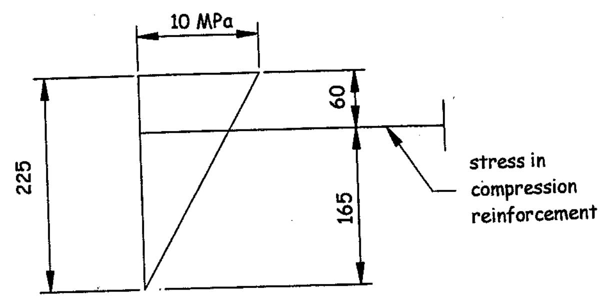
$$\frac{bh^3}{12} = \frac{400 \text{ x } (600+75)^3}{12} = 10.25 \times 10^9 \text{ mm}^2$$

$$\text{Therefore,} \quad I_{cr} = \frac{5.76 \text{ x } 10^9}{10.25 \text{ x } 10^9} = 0.56\, I_g$$

Note also that the above calculation has assumed a cracked section, but has not been dependent upon the applied loading.

12.6.3 Determining Stresses

To find *jd*, it is necessary to find the centroid of the compression forces in the concrete and reinforcement. As the section is behaves linearly (constant neutral axis depth and constant *jd*), the forces for any stress in the top fibre can be calculated and these values can be used to find *jd*. This may be done by assuming a compression stress at the extreme compression fibre, or by conducting the calculation treating the extreme compression stress as an unknown.



Assume f = 10 MPa

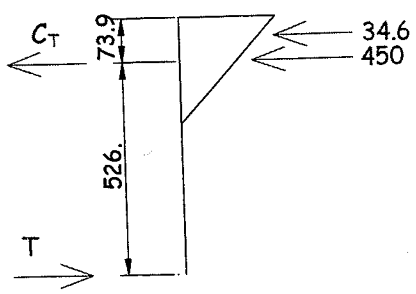
$$C_c = 400 \text{ x } \underbrace{225}_{\text{depth}} \text{ x } \underbrace{10/2}_{\text{assumed stress}} = 450 \text{ kN}$$

x = 225/3 = 75.0 mm

Stress in 2-D20H = 165/225 x 10 x 7.51 = 55.1 MPa (n-1)

should be (31.2kN)

C'_s = 628 x 55.1 = 34163.2 N = 34.6 kN



Location of the centroid of C:

$$\bar{x} = \frac{(34.6 \text{ x } 60 + 450 \text{ x } 75)}{(34.6+450)} = 73.9 \text{ mm}$$

therefore, jd = 600 - 73.9 = 526.1 mm

hence $\quad T = \frac{M}{jd} = \frac{330}{0.5261} = 627 \text{ kN}$

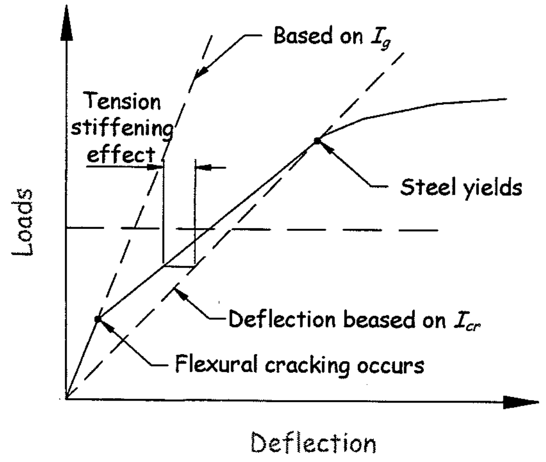
and $\quad f_s = \frac{T}{A_s} = \frac{627 \text{ x } 1000}{3436} = 182.5 \text{ MPa}$

$\therefore$ actual $f_c = \frac{627}{1} \times \frac{10}{34.6+450} = 12.9$ MPa $\quad$ actual $f_s' = 627 \times \frac{55.1}{34.6+450} = 71.3$ MPa

12.7 Deflection Calculations

To calculate deflections an effective moment of inertia, I_e, is used for the beam, or part of the beam, where:

$$I_g \geq I_e \geq I_{cr}$$



The value of I_e varies with the stress level in the beam. It is greater than I_{cr} to allow for tension stiffening effects, as discussed earlier. In the New Zealand code the effective moment of inertia is given by (see 3.3.2.3):

$$I_e = \left[\frac{M_{cr}}{M_a}\right]^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

actual moment

where: I_{cr} = moment of inertia of cracked transformed section,

I_g = moment of inertia of gross section,

M_{cr} = flexural cracking moment (Note: f_r is not a reliable strength),

$= f_r \frac{I_g}{y_t}$

y_t = distance from the neutral axis to the tension fibre,

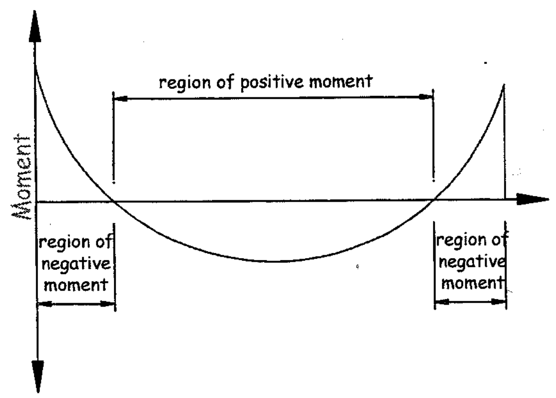
f_r = modulus of rupture = $0.6\sqrt{f'_c}$ MPa (Cl 3.8.1.3)

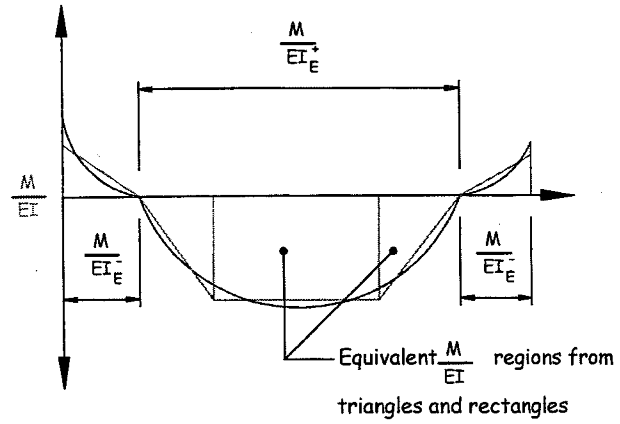
M_a = the maximum bending moment in the beam (or part of the beam).

For a simply supported beam, only one I_{cr} and one M_a value is used. The I_{cr} value is calculated for the section at which M_a acts. For continuous beams the member is divided into the positive and negative flexural zones. Cracking moments, maximum moments, and effective moments of inertia are calculated for each zone, so that several values of I_e may be determined. Where the magnitude of these values are close an average value may be used.

12.7.1 Using Moment-Area Theorem to Find Deflections

The deflection of any beam (including members subjected to both positive and negative moments) can be found using the moment-area theorem by subdividing the member into the various regions of positive and negative bending, and then substituting the appropriate cracked section stiffness for these regions.





The bending moment diagram can be redrawn as an M/EI_e diagram, where the appropriate I_e is used for each region. The area of these M/EI_e regions, and the distance to the centroids of these various regions can then be found using geometric properties (if known). Alternatively, a sufficiently accurate solution may be obtained by considering the area as a series of regions having straight edges and being comprised of rectangles and triangles, so that areas and centroids may be readily determined.

Using the effective moment of inertia, the short term loading can be determined. Note also that as the accuracy of predicted deflections is not high, it is sufficient to work to ±10% and making simplifying assumptions.

12.8 Long term deflections

Shrinkage and creep due to sustained loads cause additional deflections over and above those that occur when loads are first placed on the structure. These additional deflections are called "long-term deflections". Such deflections are influenced by temperature, humidity, curing conditions, age at time of loading, quantity of compression reinforcement, magnitude of sustained loading, and other factors. To predict long term deflections accurately is complex and difficult. Codes use a simplified method.

The additional long term deflection, Δl, is given by:

$$\Delta l = k_{cp}\Delta_s$$

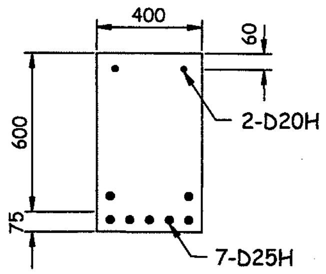
where Δs is the short term deflection due to the long term loads, and

$$k_{cp} = \left[2 - 1.2\left(\frac{A_s'}{A_s}\right)\right] > 0.6 \qquad (cl\ 3.3.2.3b(ii))$$

where A'_S is the area of longitudinal reinforcement in the flexural compression zone and A_S is the area of longitudinal reinforcement in the flexural tension zone.

Example

Calculate the deflection of the simply supported beam with the section properties used in the previous example. The span is 10m, the self weight plus imposed dead load is 15 kN/m and the live load is 11.4 kN/m. One third of the live load is considered to act on a long term basis.



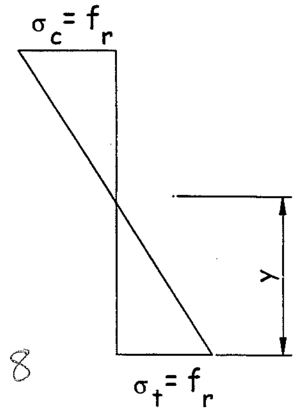
$w_G + w_Q$ = 15.0 + 11.4 = 26.4 kN/m

M_{max} = $wL^2/8$ = 26.4x10³ x 10.0²/8 = 330kNm

I_{cr} = 5.76 x 10⁹ mm⁴ (see section 12.6.2)
I_g = 10.25 x 10⁹ mm⁴ (see section 12.6.2)

(Note that generally the longitudinal reinforcement is neglected.
However, it may be included. In this case including the steel increases I_g by close to 19%).

M_a = 330kNm (from above)



$$M_{cr} = \left(\frac{I}{y}\right)f_r = \frac{10.25x10^9}{337.5}\ x\ 0.6\sqrt{25} = \ 91.1 \times 10^6$$

675/2 = 337.5

therefore:

$$\frac{M_{cr}}{M_a} = \frac{91.1}{330} = 0.276, \quad \left(\frac{M_{cr}}{M_a}\right)^3 = 0.021, \quad \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] = 0.978$$

hence: I_e = 0.021 x 10.25 x 10⁹ + 0.978 x 5.76 x 10⁹ = 5.85×10⁹ mm⁴

(Note that in this case the actual value of I_g has little effect on I_e)

E_C = 23500MPa
and the short term deflection due to short term loads is given by:

$$\Delta = \frac{5}{384}\frac{WL^4}{EI} = \frac{5}{384} \text{x} \frac{26.4 \text{ x } 10000^4}{23500 \text{ x } 5.85 \text{ x } 10^9} = 25.0\text{mm}$$

The short term deflection due to long term loads may be estimated by interpolating:

long term load

$$\Delta_s = 25.0 \text{ x} \left[\frac{15+11.4/3}{15+11.4}\right] = 25.0 \text{ x } 0.71 = 17.75\text{mm}$$

short term load

The long term multiplier for assessing the additional deflection due to creep and shrinkage is given by:

$$k_{cp} = \left[2 - 1.2\left(\frac{628}{3167}\right)\right] = 1.76 > 0.6 \textit{ as required}$$

hence the maximum deflection is:

$$\Delta = \Delta + k_{cp}\Delta_s = 25.0 + 1.76 \text{ x } 17.75 = 56.2\text{mm}$$

Note that the actual value is likely to be:
$\pm 30\%$ *of this value*

$$\text{The } \frac{l}{\Delta} = \frac{10000}{56.2} = 178$$

this is a little high (see Table), but in most situations an initial camber could be provided, and the deflections be limited subject to the secondary non-structural elements not being too rigid, and not constructed until a considerable portion of this deflection had developed.

Control Phenomenon	Element	Explanatory Comment	Deflection limit	Loading Regime	Action being Considered
Sensory					
Seen	Roof Systems	Rafters, purlins	Span/300	G & $\psi_l Q$	Sag
		Cladding	Span/85	Q_k = 1.0 kN	Indentation
	Ceilings	Flat finish	Span/500	G	Ripple
		Textured finish	Span/250	G	Ripple
		Suspended systems	Span/360	G	Ripple
	Ceiling supports		Span/360	G	Sag
	Glazing systems		Span/400	G	Bowing
	Columns		Height/500	G; W_s	Side sway
	Floors		Span/300	G & $\psi_l Q$	Ripple
			Span/180	G & $\psi_l Q$	Sag
	Beams	Line of sight Along invert	Span/500	G & $\psi_l Q$	Sag
		Line of sight Across soffit	Span/250	G & $\psi_l Q$	Sag
Felt	Walls		Height/150	W_s	Face Loading
			Height/200	Q_k = 0.7 kN	Impact
	Floors		< 1 mm	Q_k = 1.0 kN	Transient
					Vibrations
		Acceleration limit	0.01 g	W_s	Sway
Functionality	Flat roof		Span/400	G & $\psi_l Q$	Drainage
			Span/500	G & S_s	Ponding
	Walls		Height/300	W_s ψ	In-plane
	Floors	Normal Occupancy	Span/400	G & $\psi_s Q$	Sag
		Specialist floors	Span/60	G & $\psi_s Q$	Sag
	Lintels		Span/240	G & $\psi_s Q$	Jamming
			or <25 mm	G & $\psi_s Q$	
Protection of Non-Structural elements	Roofs	Brittle claddings	Span/150	W_s	Cracking
	Ceilings	Stopped systems	Span/200	G & $\psi_s Q$; W_s	Cracking
	Walls	Brittle claddings	Span/150	W_s	Cracking
	Glazing Elements	Facades, and curtain wall fixed glazing	Span/250	W_s; E_s	In-plane
			2 x glass clearance	W_s; E_s	In-plane
	Masonry walls		Height/600	W_s; E_s	In-plane
			Height/400	W_s; E_s	Face loading
	Plaster/gypsum walls		Height/400	W_s	In-plane
			Height/200	W_s; E_s	Face loading
			Height/200	Q_k = 0.7 kN	Impact
			Height/160	Q_k = 0.7 kN	Impact
	Moveable partitions Walls		Height/300	W_s; E_s	Face loading
	Portal Frames		Spacing/200	W_s; E_s	In-plane
	Floors	Supporting masonry walls	Span/500	G & $\psi_s Q$	Wall cracks
		Supporting framed walls	Span/500	G & $\psi_s Q$	Joint cracks

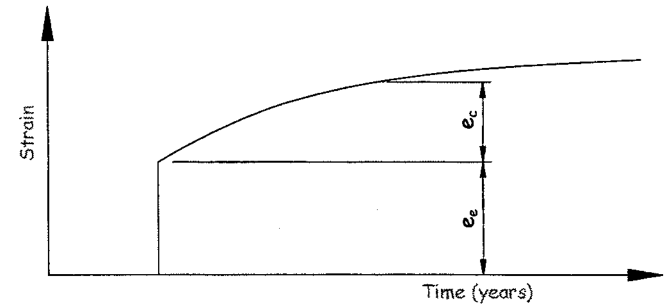
12.8.1 Effective Modulus Approach

The code approach for accounting for creep and shrinkage is to determine an effective moment of inertia. However, flexural stiffness is a function of both I and E:

$e_c = \phi e_s$ where ϕ is creep factor

$e_e = \dfrac{f}{E_c}$ where f is stress and E_c is elastic modulus

$e_t = e_e + e_c = (1+\phi)$



Therefore, $E_{eff} = \dfrac{E_c}{1+\phi}$

Consequently, an effective modulus for concrete allowing for creep can be developed. However, only approximate solutions can be obtained this way as generally stresses are not constant.