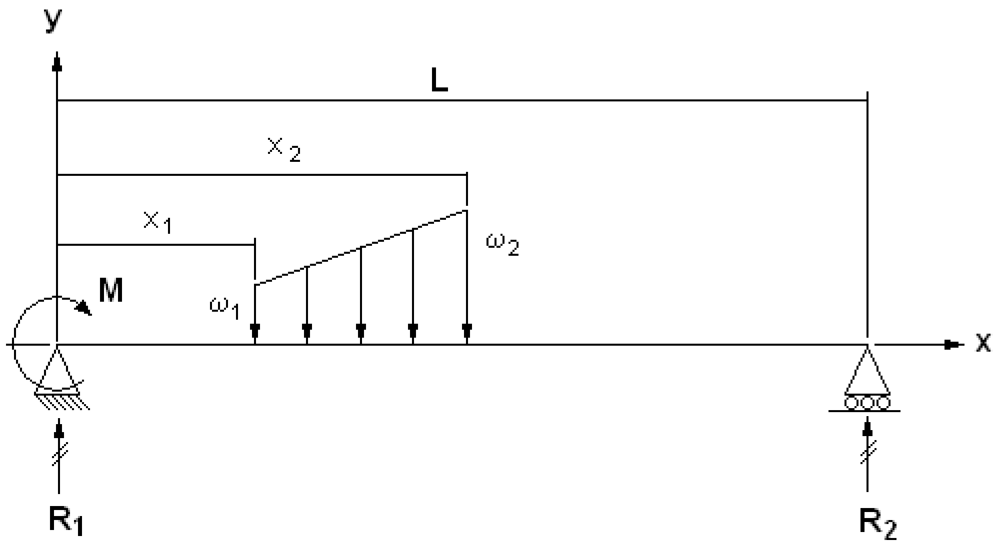
Beam Analysis



$L := 10 \cdot m$ ... beam length

$I := 0.0500 \cdot m^4$ ... beam inertia

$E := 210 \cdot GPa$ ... material modulus

1st distributed loading data ... $\omega_{11} := 10 \cdot \frac{kN}{m}$ ... distributed loading at x_{11}, where ... $x_{11} := 2 \cdot m$

Note, additional distributions can be added, e.g for 2nd ...

$\omega_{12} := 40 \cdot \frac{kN}{m}$ ... distributed loading at x_{12}, where ... $x_{12} := 5 \cdot m$

ω_{21} ... at ... x_{21}

ω_{22} ... at ... x_{22}

and ... $\Delta\omega_2$

$\Delta\omega_1 := \frac{(\omega_{12} - \omega_{11})}{(x_{12} - x_{11})}$ $\Delta\omega_1 = 10 \frac{kN}{m^2}$ ... change in 1st distributed loading

End restraint conditions ... at x = 0 ... $Restaint_1 := 1$ ... note, $Restraint_n$ = 0 for free condition
$Restraint_n$ = 1 for fixed condition

at x = L ... $Restaint_2 := 1$

Shear and Moment for 1st Distribution

Shear functions ...

$$V_{\omega 1}(x') = \int_{x_{11}}^{x'} \left[\omega_{11} + \Delta\omega_1\cdot\left(x - x_{11}\right)\right] dx = \omega_{11}\cdot\left(x' - x_{11}\right) + \frac{1}{2}\cdot\Delta\omega_1\cdot\left(x' - x_{11}\right)^2 \quad \text{... when} \quad x_{11} < x < x_{12}$$

$$V_{\omega 1} = \int_{x_{11}}^{x_{12}} \left[\omega_{11} + \Delta\omega_1\cdot\left(x - x_{11}\right)\right] dx = \omega_{11}\cdot\left(x_{12} - x_{11}\right) + \frac{1}{2}\cdot\Delta\omega_1\cdot\left(x_{12} - x_{11}\right)^2 \quad \text{... when} \quad x \ge x_{12}$$

Combining ...

$$\begin{aligned} V_{\omega 1}(x) := & \left[\omega_{11}\cdot\left(x - x_{11}\right) + \frac{1}{2}\cdot\Delta\omega_1\cdot\left(x - x_{11}\right)^2\right]\cdot\left(x_{11} < x < x_{12}\right) \ldots \\ & + \left[\omega_{11}\cdot\left(x_{12} - x_{11}\right) + \frac{1}{2}\cdot\Delta\omega_1\cdot\left(x_{12} - x_{11}\right)^2\right]\cdot\left(x \ge x_{12}\right) \end{aligned}$$

Bending moment functions ...

$$M_{\omega 1}(x') = \int_{x_{11}}^{x'} \left[\omega_{11} + \Delta\omega_1\cdot\left(x - x_{11}\right)\right]\cdot(x' - x)\, dx = \frac{1}{6}\cdot\left(x' - x_{11}\right)^2\cdot\left[3\cdot\omega_{11} + \Delta\omega_1\cdot\left(x' - x_{11}\right)\right] \text{... when} \quad x_{11} < x < x_{12}$$

$$M_{\omega 1}(x') = \int_{x_{11}}^{x_{12}} \left[\omega_{11} + \Delta\omega_1\cdot\left(x - x_{11}\right)\right]\cdot(x' - x)\, dx = \frac{1}{6}\cdot\left(x_{12} - x_{11}\right)\cdot\left[\begin{array}{l} 3\cdot\Delta\omega_1\cdot x'\cdot\left(x_{12} - x_{11}\right) \ldots \\ + -3\cdot\omega_{11}\cdot\left(x_{12} + x_{11} - 2\cdot x'\right) \ldots \\ + -\Delta\omega_1\cdot\left(2\cdot x_{12} + x_{11}\right)\cdot\left(x_{12} - x_{11}\right) \end{array}\right] \quad x \ge x_{12}$$

Combining ...

$$\begin{aligned} M_{\omega 1}(x) := & \left[\frac{1}{6}\cdot\left(x - x_{11}\right)^2\cdot\left[3\cdot\omega_{11} + \Delta\omega_1\cdot\left(x - x_{11}\right)\right]\right]\cdot\left(x_{11} < x < x_{12}\right) \ldots \\ & + \left[\frac{1}{6}\cdot\left(x_{12} - x_{11}\right)\cdot\left[\begin{array}{l} 3\cdot\Delta\omega_1\cdot x\cdot\left(x_{12} - x_{11}\right) \ldots \\ + -3\cdot\omega_{11}\cdot\left(x_{12} + x_{11} - 2\cdot x\right) \ldots \\ + -\Delta\omega_1\cdot\left(2\cdot x_{12} + x_{11}\right)\cdot\left(x_{12} - x_{11}\right) \end{array}\right]\right]\cdot\left(x \ge x_{12}\right) \end{aligned}$$

Note, for 2nd create ... $V_{\omega 2}(x)$ ... and ... $M_{\omega 2}(x)$ ... by copying the 1st, modifying for 2nd and adding to shear and moment equations

$V(x, R_1) := R_1 - V_{\omega 1}(x)$ ... shear as a function of x and R_1 at x = 0

$M(x, R_1, M_1) := M_1 + R_1 \cdot x - M_{\omega 1}(x)$ ... moment as a function of x, R_1 and M_1 at x = 0

$y''_b(x, R_1, M_1) := \dfrac{M(x, R_1, M_1)}{E \cdot I}$ ... 2nd derivative

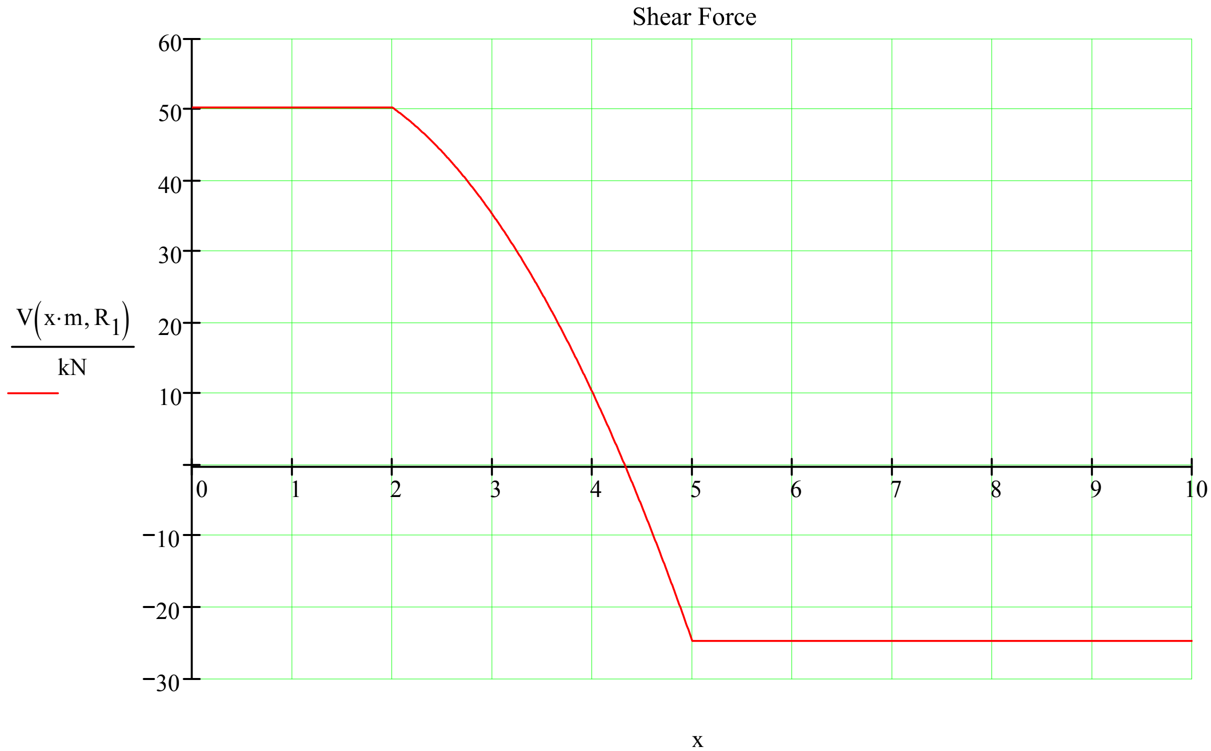
$y'_b(x', R_1, M_1, \theta_1) := \theta_1 + \int_0^{x'} y''_b(x, R_1, M_1)\, dx$ ... 1st derivative (gradient)

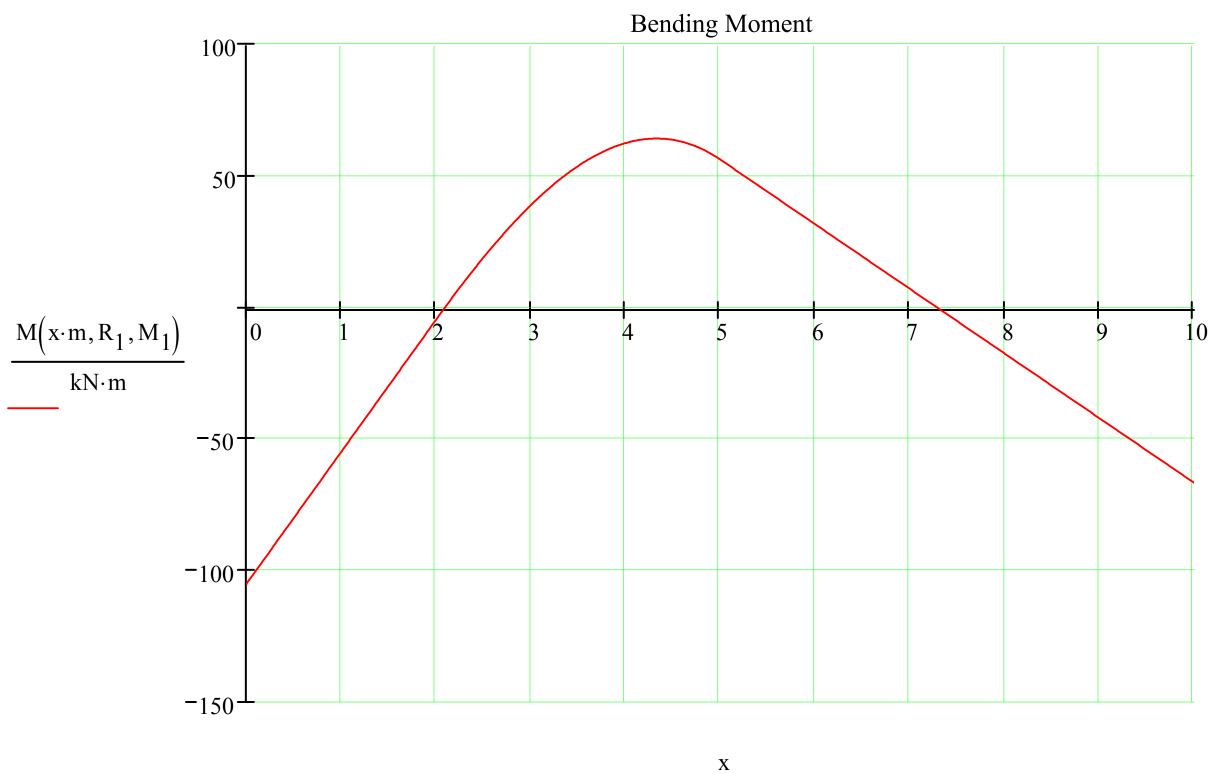
$y_b(x', R_1, M_1, \theta_1) := \int_0^{x'} y'_b(x, R_1, M_1, \theta_1)\, dx$ ... displacement

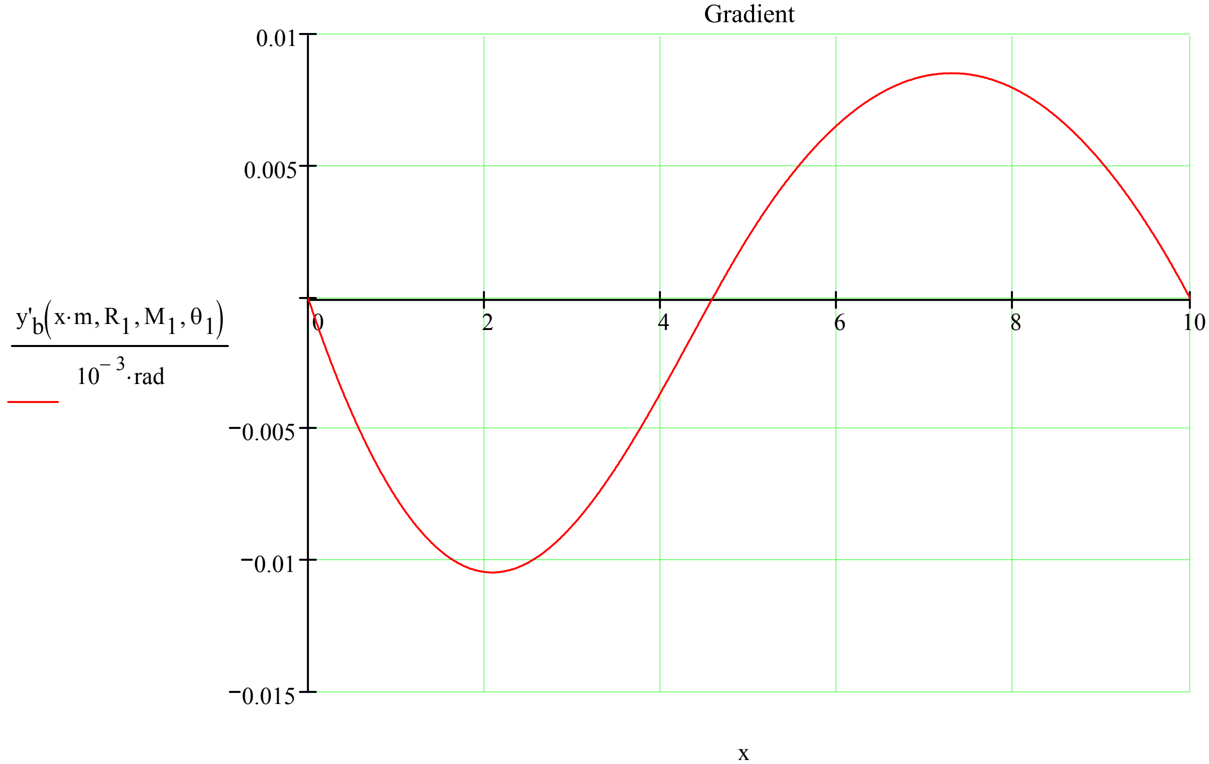
Given $\left| \begin{array}{l} M_1 \ \text{ if } (\text{Restaint}_1 = 0) \\ \theta_1 \ \text{ otherwise} \end{array} \right. = \left| \begin{array}{l} 0 \cdot N \cdot m \ \text{ if } (\text{Restaint}_1 = 0) \\ 0 \ \text{ otherwise} \end{array} \right.$ ... defining moment or gradient at x = 0 ... Equ 1

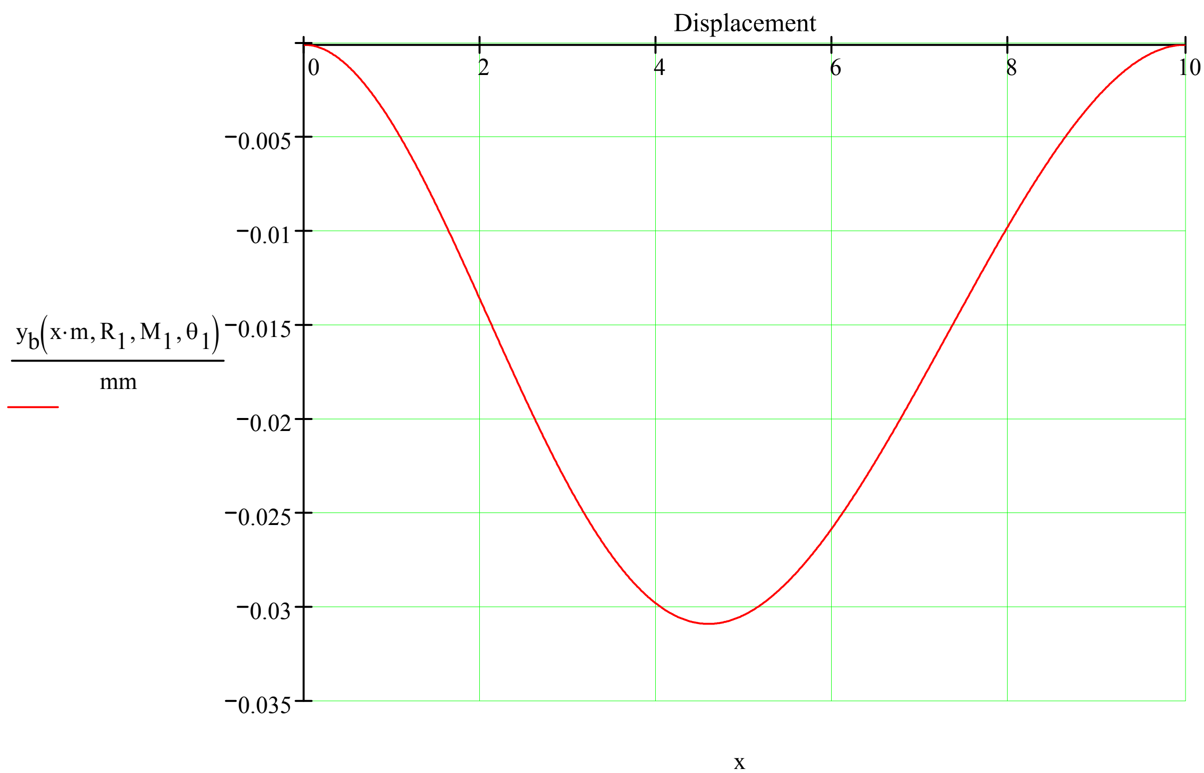
$\left| \begin{array}{l} M(L, R_1, M_1) \ \text{ if } (\text{Restaint}_2 = 0) \\ y'_b(L, R_1, M_1, \theta_1) \ \text{ otherwise} \end{array} \right. = \left| \begin{array}{l} 0 \cdot N \cdot m \ \text{ if } (\text{Restaint}_2 = 0) \\ 0 \ \text{ otherwise} \end{array} \right.$ ... defining moment or gradient at x = L ... Equ 2

$y_b(L, R_1, M_1, \theta_1) = 0 \cdot m$ ... defining displacement at x = L ... Equ 3


Shear Force
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
−10
−20
−30
$\frac{V(x\cdot m, R_1)}{kN}$
x


Bending Moment
100
50
0
1
2
3
4
5
6
7
8
9
10
−50
−100
−150
$\frac{M(x\cdot m, R_1, M_1)}{kN\cdot m}$
x


Gradient
0.01
0.005
−0.005
−0.01
−0.015
0
2
4
6
8
10
y'b(x·m, R1, M1, θ1) / 10^−3·rad
x


Displacement
0
2
4
6
8
10
−0.005
−0.01
−0.015
−0.02
−0.025
−0.03
−0.035
yb(x·m, R1, M1, θ1) / mm
x

Results

Shear forces ... at x = 0 ... $R_1 = 50352\,N$

at x = L ... $R_2 := -V(L, R_1)$ $R_2 = 24648\,N$

check ... $R_1 + R_2 - V_{\omega 1}(x_{12}) = 0\,N$

Bending moments ... at x = 0 ... $M_1 = -105135\,N\cdot m$

at x = L ... $M_2 := M(L, R_1, M_1)$ $M_2 = -66615\,N\cdot m$

Max. BM occurs at ... $x_{BM\_max} = 4.327\,m$ ... with a value of ... $M_{max} = 64663\,N\cdot m$

Max. deflection occurs at ... $x_{y\_max} = 4.588\,m$ ... with a value of ... $y_{max} = -0.0309\,mm$