Chapter 2

Application of Fluid Flow Equations to Gas Systems

2.1 Introduction

The aim of this chapter is to develop and present the fundamental equations for flow of gases through porous media, along with solutions of interest for various boundary conditions and reservoir geometries. These solutions are required in the design and interpretation of flow and pressure tests.

To simplify the solutions and application of the solutions, dimensionless terms are used. Assumptions and approximations necessary for defining the system and solving the differential equations are clearly stated. The principle of superposition is applied to solve problems involving interference between wells, variables flow rates, and wells located in noncircular reservoirs. The use of analytical and numerical solutions of the flow equations is also discussed. Formation damage or stimulation, turbulence, and wellbore storage or unloading are given due consideration. This chapter applies in general to laminar, single, and multiphase flow, but deviations due to inertial and turbulent effects are considered. For well testing purposes two-phase flow in the reservoir is treated analytically by the use of an equivalent single-phase mobility.

The equations of continuity, Darcy's law, and the gas equation of state are presented and combined to develop a differential equation for flow of gases through porous media. This equation, in generalized coordinate notation, can be expressed in rectangular, cylindrical, or spherical coordinates and is solved by suitable techniques. The next subsections describe steady-state, pseudo-steady-state, and unsteady-state flow equations including the gas radial diffusivity equation, basic gas flow equations, solutions, and one-, two-, and three-dimensional coordinate systems.

2.2 Steady-State Laminar Flow

Darcy's law for flow in a porous medium is

$$v = \frac{k}{\mu_g}\frac{dp}{dx} \quad \text{or} \quad q = vA = \frac{kA}{\mu_g}\frac{dp}{dx} \tag{2–1}$$

where

v = gas viscosity; q = volumetric flow rate; k = effective permeability; μ_g = gas viscosity; and $\frac{dp}{dx}$ = pressure gradient in the direction of flow

For radial flow, Eq. 2–1 becomes

$$q = \frac{k(2\pi rh)}{\mu_g}\frac{dp}{dx} \tag{2–2}$$

where r is radial distance and h is reservoir thickness,

Equation 2–2 is a differential equation and must be integrated for application. Before integration the flow equation must be combined with an equation of state and the continuity equation. The continuity equation is

$$\rho_1 q_1 = \rho_2 q_2 = \text{constant} \tag{2–3}$$

The equation of state for a real gas is

$$\rho = \frac{pM}{ZRT} \tag{2–4}$$

The flow rate of a gas is usually desired at some standard conditions of pressure and temperature, p_{sc} and T_{sc}. Using these conditions in Eq. 2–3 and combining Eqs. 2–3 and 2–4, we get

$$\rho q = \rho_{sc} q_{sc},$$

or

$$q\frac{pM}{zRT} = q_{sc}\frac{p_{sc}M}{z_{sc}RT_{sc}}$$

Solving for q_{sc} and expressing q_{sc} with Eq. 2–2 gives

$$q_{sc} = \frac{pT_{sc}}{p_{sc}zT}\frac{2\pi rhk}{\mu}\frac{dp}{dr}$$

The variables in this equation are p and r. Separating the variables and integrating:

$$\int_{p_w}^{\bar{p}} p dp = \frac{q_{sc} p_{sc} T \bar{\mu}_g \bar{z}}{T_{sc}\, 2\pi kh} \int_{r_w}^{r_c} \frac{dr}{r}$$

$$\frac{\bar{p}^2 - p_w^2}{2} = \frac{q_{sc} p_{sc} T \bar{\mu} \bar{z}}{T_{sc} 2\pi kh} \ln\left(\frac{r_e}{r_w}\right)$$

$$\text{or} \quad q_{sc} = \frac{\pi kh T_{sc}\left(\bar{p}^2 - p_w^2\right)}{p_{sc} T \bar{\mu}_g \bar{z} \ln\left(\frac{r_e}{r_w}\right)} \tag{2–5}$$

In this derivative it was assumed that μ_g and z were independent of pressure. They may be evaluated at reservoir temperature and average pressure in the drainage area such as

$$\bar{P} = \frac{P_e - P_w}{2}$$

In gasfield units, Eq. 2–5 becomes

$$q_{sc} = \frac{0.007027kh\left(\bar{P}^2 - P_w^2\right)}{\mu_g \bar{z} T \log\left(\frac{r_e}{r_w}\right)} \tag{2–6}$$

$$q_{sc} = \frac{0.000305kh\left(\bar{P}^2 - P_w^2\right)}{\mu_g \bar{z} T \ln\left(\frac{r_e}{r_w}\right)} \tag{2–7}$$

Where q_{sc} = mscf/d; k = permeability in mD; h = formation thickness in feet; p_e = reservoir pressure, psi, p_w = well bore pressure, psia, T = reservoir temperature, °R; r_e = drainage radius, ft; r_w = well bore radius, ft; $\bar{z}$ = average compressibility factor, dimensionless; and $\bar{\mu}_g$ = gas viscosity, cP.

This equation incorporates the following values for standard pressure and temperature:

$$p_{sc} = 14.7 \text{ psia},$$

$$T_{sc} = 60°\text{F} = 520°\text{R}$$

The gas flow rate is directly proportional to the pseudopressures. The pseudopressure is defined as

$$\psi(p) = 2\int_{p_{ref}}^{\bar{p}} \frac{p}{\mu z} dp \tag{2-8}$$

In Eq. 2–8, p_{ref} is a reference pressure. At the reference pressure, pseudopressure is assigned a datum value of zero. The Eqs. 2–6 and 2–7 in terms of pseudopressure become

$$q_{sc} = \frac{0.0007027kh[\psi(\bar{p}) - \psi(p_w)]}{T\ \ln\left(\frac{r_e}{r_w}\right)} \tag{2-9}$$

$$q_{sc} = \frac{0.000305kh[\psi(\bar{p}) - \psi(p_w)]}{T\log\left(\frac{r_e}{r_w}\right)} \tag{2-10}$$

p^2 and $\psi(p)$ have identical values up to 2500 psia. Above 2500 psia, p^2 and $\psi(p)$ exhibit different values. Thus, below 2500 psia, either p^2 or $\psi(p)$ can be used. Above 2500 psia, $\psi(p)$ should be used. Gas pseudopressure, $\psi(p)$, which is defined by Eq. 2–8, is considered, i.e.,

$$\psi(\bar{p}) - \psi(p_w) = 2\int_{p_{ref}}^{\bar{p}} \frac{pdp}{\mu_g z} - 2\int_{p_{ref}}^{p_w} \frac{pdp}{\mu_g z}$$

It is more difficult and generally engineers feel more comfortable dealing with pressure squared, p^2, rather than an integral transformation. Therefore, it is worthwhile, at this stage, to examine the ease with which these functions can be generated and used. We evaluate the integral in Eq. 2–8 numerically, using values for μ_g and z for the specific gas under consideration, evaluated at reservoir temperature. An example will illustrate this calculation.

Example 2–1 *Calculating Gas Pseudopressure*

Calculate the gas pseudopressure $\psi(p)$ for a reservoir containing 0.732 gravity gas at 250°F as a function of pressure in the range 400 to 4000 psia. Gas properties as functions of pressure are given in Table 2–1.

Solution For $p = 400$ psia:

$$\psi(400) = 2\int_{p_{ref}}^{p} \frac{p}{\mu_g z} dp$$

Table 2–1
Generation of Gas Pseudopressure as a Function of the Actual Pressure

Pressure, p (psia)	μ_g (cP)	Z –	$P/\mu_g z$ (psia/cP)	$\psi(P)$ (mm psia^2/cP)
400	0.014337	0.9733	28.665	11.47
800	0.014932	0.9503	56,378	45.48
1200	0.015723	0.9319	81,899	100.83
1600	0.016681	0.9189	104,383	175.33
2000	0.017784	0.9120	123,312	266.41
2400	0.019008	0.9113	138,552	371.18
2800	0.020329	0.9169	150,217	486.72
3200	0.021721	0.9282	158,719	610.28
3600	0.023151	0.9445	164,638	739.56
4000	0.024580	0.9647	168,689	872.92

$$= 2\frac{\left[\left(\frac{p}{\mu_g z}\right)_0 + \left(\frac{p}{\mu_g z}\right)_{400}\right]}{2}$$

$$= 2\left(\frac{0 + 28,665}{2}\right)(400 - 0)$$

$$= 11.466 \times 10^6 \text{ psia}^2/\text{cp}$$

For $p = 800$ psia:

$$\psi(800) = 11.466 \times 10^6 + 2\left(\frac{28,665 + 56,378}{2}\right)(800 - 400)$$

$$= 11.466 \times 10^6 + 34.017 \times 10^6$$

$$= 45.483 \times 10^6 \text{ psia}^2/\text{cp}$$

Proceeding in a similar way, we can construct Table 2–1. These results are plotted in Figure 2–1. This plot is used in the gas well test analysis, in which it is assumed that for high pressure, in excess of 2800 psia, the function is almost linear and can be described by

$$\psi(p) = [0.3218p - 416.85] \text{ mm psia}^2/\text{cp}$$

For low pressure, less than 2800 psia, the function is described by a polynomial equation of the form

$$\psi(p) = A + Bp + Cp^2 + Dp^3 + Ep^4 + Fp^5$$

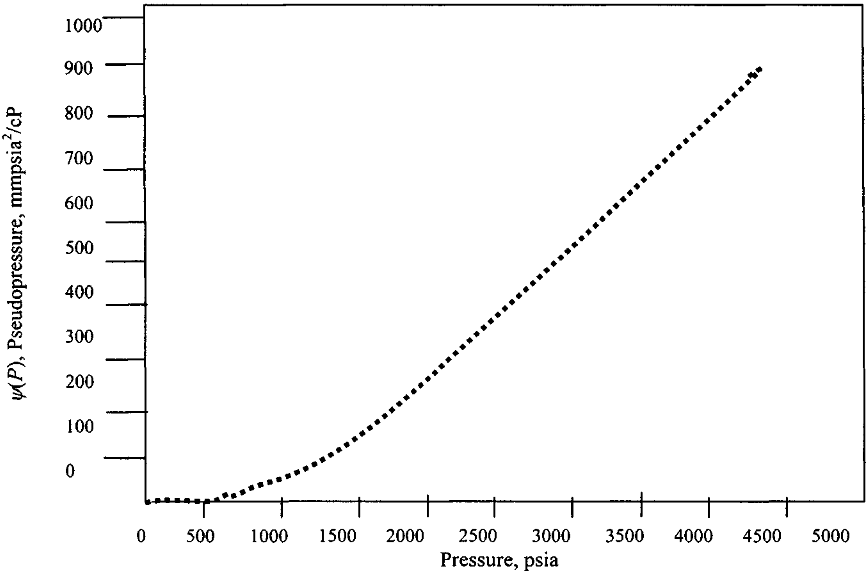

Figure 2–1. Gas pseudopressure $\psi(P)$ versus pressure, psia.

where A, B, C, D, E, and F are polynomial coefficients whose values are

$$A = 39{,}453;\ B = -222.976;\ C = 72.0827$$
$$D = 5.287041\text{E-}04;\ E = -1.993697\text{E-}06;\ \text{and}\ F = 1.92384\text{E-}10$$

These relationships and the plot can be used to convert from real to pseudopressure and vice versa.

Example 2–2 *Determining Wellbore Pressure Assuming Steady-State Flow Conditions*

Perform this calculation given the following data:

$k = 1.50$ mD, $h = 39$ ft, $q_{sc} = 3900$ mscfd, $p_e = 4625$ psia, $T = 712°$ R, $r_e = 550$ ft, $r_w = 0.333$, $\bar{\mu} = 0.02695$cp, $\gamma_g = 0.759$, $T_{sc} = 520°$R, $P_{sc} = 14.7$ psia.

Solution The solution is iterative since $\bar{z} = f(\bar{p})$, where $\bar{p} = (p_e + p_w)/2$, and p_w is the unknown. As a first estimate, assume $\bar{z} = 1.0$.

First trial using Eq. 2–6:

$$p_w^2 = p_e^2 - \frac{\bar{\mu} T \ln(r_e/r_w) q_{sc} \bar{z}}{0.0007027kh}$$

$$= 4625^2 - \frac{(.02695)(712)(550/.333)(3900) \times \bar{z}}{.0007027(1.5(30)}$$

$$= 2.139 \times 10^7 - 1.756 \times 10^7(1.0)$$

$$= 3.83 \times 10^6$$

or $p_w = 1957$ psia.

Second trial:

$$\bar{p} = \frac{4625 + 1957}{2} = 3291 \text{ psia}, \ \bar{z} \text{ at } 3291 \text{ psia} = 0.88$$

$$p_w^2 = 2.139 \times 10^7 - 1.756 \times 10^7(0.88)$$

$$= 5.937 \times 10^6$$

or $p_w = 2436$ psia.

Third trial:

$$\bar{p} = \frac{4625 + 2436}{2} = 3530 \text{ psia}, \ \bar{z} \text{ at } 3530 \text{ psia} = 0.890$$

$$= 2.139 \times 10^7 - 1.756 \times 10^7(0.89)$$

$$= 5.762 \times 10^6$$

or $p_w = 2400$ psia.

$$\bar{p} = \frac{4625 + 2400}{2} = 3512 \text{ psia and } \bar{z} \text{ at } 3512 \text{ psia} = 0.890$$

Since the value for $\bar{z}$ is the same as for second trial, the solution has converged and the required wellbore pressure is 2400 psia. The solution would have been more complicated if a constant value for μ had not been assumed. The above treatment of steady-state flow assumes no turbulence flow in the formation and no formation or skin damage around the wellbore.

2.3 Steady-State Turbulence Flow

The above treatment of steady-state flow assumes no turbulent flow in the formation and no skin damage around the wellbore. The pressure squared and pseudopressure representations of the steady-state equations including turbulence are

$$p_e^2 - p_w^2 = \frac{50.3 \times 10^6 \mu_g z T P_{sc} q_{sc}}{kh T_{sc}} \left[\ln \frac{r_e}{r_w} + s + D q_{sc} \right] \tag{2–11}$$

$$\psi(\bar{p}) - \psi(p_w) = \frac{1.422 \times 10^3 T q_{sc}}{kh} \left[\ln \frac{r_e}{r_w} - 0.5 + s + D q_{sc} \right] \tag{2–12}$$

where Dq_{sc} is interpreted as the rate-dependent skin factor, and

$$D = \frac{5.18 \times 10^{-5} \gamma_g}{\bar{\mu} h r_w k^{0.2}} \beta \tag{2–13}$$

Expression D is the non-Darcy flow coefficient in psia^2/cP/(mscf/d)^2 and is calculated from Eq. 2–13
where

$$\beta = \frac{2.33 \times 10^3}{k^{1.201}}, \text{ 1/ft} \tag{2–14a}$$

or

$$\beta = \frac{2.73 \times 10^{10}}{k^{1.1045}}, \text{ 1/ft} \tag{2–14b}$$

where k is the permeability near the wellbore region in mD. Values of the velocity coefficient β for various permeability and porosity can be obtained from Ref. 1 or calculated from Eq. 2–14a or 2–14b. The foregoing equations 2–11 and 2–12 have the forms

$$p_e^2 - p_w^2 = AA' q_{sc} + BB' q_{sc}^2 \tag{2–11a}$$

where

$$AA' = 50.3 \times 10^3 \frac{\mu_g z T P_{sc}}{kh T_{sc}} [\ln(r_e/r_w) - 0.75 + s] \tag{2–11b}$$

$$BB' = 50.3 \times 10^3 \frac{\mu_g z T P_{sc}}{kh T_{sc}} D \tag{2–11c}$$

$$\psi(\bar{p}) - \psi(p_w) = AA q_{sc} + Bb q_{sc}^2 \tag{2–12a}$$

where

$$AA = \frac{1.422 \times 10^3}{kh}[\ln(r_e/r_w) - .75 + s] \tag{2–12b}$$

$$BB = \frac{1.422 \times 10^3 T}{kh} D \tag{2–12c}$$

Example 2–3 *Calculating Influence of Turbulence in a Vertical Well Using Steady-State Flow Equation*

A vertical gas well is drilled in a 45-ft-thick sandstone reservoir with permeability of 12 mD. The initial reservoir pressure is 2150 psia and well spacing is 640 acres. The well could be operated with a minimum bottomhole pressure of 350 psia. The other data are $T = 590°\text{R}$, $\mu_g = 0.02$ cP, $z = 0.90$, $\gamma_g = 0.70$, $r_w = 0.29$ ft, $s' = 0$, perforated length $h_p = 45$ ft.

Use the p^2 equation to calculate the flow rate.

Solution To solve this problem, the Eq. 2–11a has the form

$$p_e^2 - p_w^2 = AA'q_{sc} + BB'q_{sc}^2$$

where

$$AA' = 50.3 \times 10^6 \frac{\mu_g z T P_{sc}}{kh T_{sc}}\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]$$

$$BB' = 50.3 \times 10^6 \frac{\mu_g z T P_{sc}}{kh T_{sc}} D$$

Substituting these parameters in the above equations, we have

$$AA' = 50.3 \times 10^6 \frac{(.02)(.9)(130)(590)(14.7)}{12(45)(520)}\left[\ln\left(\frac{2978}{.29}\right) - 0.75 + 0\right]$$
$$= 237.34$$

The value of BB' can be calculated using the preceding equation:

$$BB' = 50.3 \times 10^6 \frac{(0.02)(0.9)(590)(14.7)}{12(45)(520)} D$$
$$= 0.027965 \times 10^6 D$$

where

$$D = \frac{2.222 \times 10^{-15} \gamma_g kh\beta}{\mu_g r_w h_p^2}$$

and

$$\beta = 2.73 \times 10^{10}\, k^{-1.1045},\ 1/\text{ft}$$

$$= 2.73 \times 10^{10}(12)^{-1.1045} = 1.7547 \times 10^{9}\, 1/\text{ft}$$

$$\therefore D = \frac{2.222 \times 10^{-15}(0.7)(12)(45)}{(0.02)(0.29)(45)(45)}(1.7547 \times 10^{9})$$

$$= 1.255 \times 10^{-4},\ 1/\text{mscfd}$$

Substituting value of D into Eq. 2–11a, BB' is calculated as

$$BB' = 0.027965 \times 10^{3}(1.255 \times 10^{-4}) = 0.351\ 1/\text{mscfd}^2$$

Substituting values of AA' and BB' into Eq. 2–11a:

$$p_e^2 - p_w^2 = 237.34 q_{sc} + 0.351 q_{sc}^2$$

This quadratic equation is rearranged as

$$0.351 q_{sc}^2 + 237.34 q_{sc} - \left(p_e^2 - p_w^2\right) = 0$$

By solving the above quadratic equation the value of q_{sc} is calculated as

$$q_{sc} = \frac{-237.34 + \sqrt{(237.34)^2 + 4(0.351)(p_e^2 - p_w^2)}}{2(0.351)}$$

$$= \frac{-237.34 + \sqrt{56{,}330.271 + 1.404(p_e^2 - p_w^2)}}{0.7020}$$

Calculated values of q_{sc}, both with and without turbulence for various values of p_w, are summarized in Table 2–2. This table indicates a significant effect of turbulence on well productivity.

Table 2–2
Effect of Turbulence on Vertical Well Productivity

P_w (psia)	$p_e^2 - p_w^2$ (psia^2)	No turbulence, $D = 0$ q (mmscfd)	With turbulence q (mmscfd)
1800	138×10^4	5.816	1.673
1400	266×10^4	11.208	2.435
1000	362×10^4	15.252	2.891
500	437×10^4	18.412	3.207

2.4 Pseudo-Steady-State (Finite) Flow

The equations for pseudo-steady-state flow in terms of pressure squared and pseudopressure are:

In terms of pressure-squared treatment:

$$q_{sc} = \frac{0.0007027kh\left(\bar{p}_R^2 - p_w^2\right)}{T\bar{\mu}_g\bar{z}\ \ln(0.472r_e/r_w)} \tag{2–15}$$

The effects of skin damage and turbulence are included in Eq. 2–15 as follows:

$$q_{sc} = \frac{0.0007027kh\left(\bar{p}_R^2 - p_w^2\right)}{T\bar{\mu}_g\bar{z}[\ln(0.472r_e/r_w) + s + Dq_{sc}]} \tag{2–16}$$

It is frequently necessary to solve Eq. 2–16 for pressure or pressure drop for a known flow rate, q_{sc}.

$$p_R^2 - p_w^2 = \frac{1.422 \times 10^3 T\bar{\mu}_g\bar{z}q_{sc}}{kh}[\ln(0.472r_e/r_w) + s + Dq_{sc}] \tag{2–17}$$

Equation 2–17 may be written as follows:

$$\bar{p}_R^2 - p_w^2 = Aq_{sc} + Bq_{sc}^2 \tag{2–17a}$$

where

$$A = \frac{1.422 \times 10^3\bar{\mu}_g\bar{z}T}{kh}\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]$$

and

$$B = \frac{1.422 \times 10^3\bar{\mu}_g\bar{z}T}{kh}D$$

It is sometimes convenient to establish a relationship between the two parameters that indicate the degree of turbulence occurring in a gas reservoir. These parameters are the velocity coefficient β and the turbulence coefficient D. Equation 2–17a can be written for pseudo-steady-state flow as

$$\bar{p}_R^2 - p_w^2 = 1.422 \times 10^3\bar{\mu}_g\bar{z}T\left(\ln\frac{0.472r_e}{r_w} + s\right)q_{sc} + \frac{3.161 \times 10^{-12}\gamma_g\bar{z}T\beta}{r_w h^2}q_{sc}^2 \tag{2–17b}$$

This form of the equation includes the assumption that $r_e \gg r_w$. Equating the terms and multiplying q_{sc}^2 in Eqs. 2–17a and 2–17b yields

$$\frac{1.422 \times 10^3 \bar{\mu}_g \bar{z} T}{kh} D = \frac{3.161 \times 10^{-12} \gamma_g \bar{z} T}{r_w h^2} \beta$$

or

$$D = \frac{2.22 \times 10^{-15} \gamma_g k}{\bar{\mu}_g h r_w} \beta$$

Expressing β in terms of permeability from Eq. 2–14b, the preceding expression becomes

$$D = \frac{5.18 \times 10^{-5} \gamma_g}{\bar{\mu}_g h r_w k^{0.2}} \tag{2–17c}$$

In terms of pseudopressure treatment:

$$\psi(\bar{p}_R) - \psi(p_w) = A' q_{sc} + B' q_{sc}^2 \tag{2–17d}$$

where

$$A' = \frac{1.422 \times 10^3 T}{kh} \left[\ln\left(\frac{0.472 r_e}{r_w}\right) + s \right]$$

and

$$B' = \frac{1.422 \times 10^3 T}{kh} D$$

It is sometimes convenient to establish a relationship between the two parameters that indicate the degree of turbulence occurring in a gas reservoir. These parameters are the velocity coefficient β and the turbulence coefficient D. Equation 2–17d can be written for pseudo-steady-state flow as

$$\psi(\bar{p}_R) - \psi(p_w) = 1.422 \times 10^3 T \left(\ln \frac{0.472 r_e}{r_w} + s \right) q_{sc} + \frac{3.161 \times 10^{-12} \gamma_g T \beta}{r_w h^2} q_{sc}^2 \tag{2–17e}$$

This form of the equation includes the assumption that $r_e \gg r_w$. Equating the terms and multiplying q_{sc}^2 in Eqs. 2–17d and 2–17e yields

$$\frac{1.422 \times 10^3 \bar{\mu}_g \bar{z} T}{kh} D = \frac{3.161 \times 10^{-12} \gamma_g \bar{z} T}{r_w h^2} \beta$$

or

$$D = \frac{2.22 \times 10^{-15} \gamma_g k}{h r_w} \beta$$

Expressing β in terms of permeability from Eq. 2–14b, the preceding expression becomes

$$D = \frac{5.18 \times 10^{-5} \gamma_g}{h r_w k^{0.2}} \tag{2–17f}$$

2.5 Unsteady-State (Transient) Flow

A well flows in the unsteady-state or transient regime until the pressure disturbance reaches a reservoir boundary or until interference from other wells takes effect. Although the flow capacity of a well is desired for pseudo-steady-state or stabilized conditions, much useful information can be obtained from transient tests. This information includes permeability, skin factor, turbulence coefficient, and average reservoir pressure. The procedures are developed on transient testing and the relationship among flow rate, pressure, and time will be presented in this section for various conditions of well performance and reservoir types.

2.6 Gas Radial Diffusivity Equation

By combining an unsteady-state continuity equation with Darcy's law and the gas equation of state, one can derive the diffusivity equation. The equation is

$$\frac{\partial}{\partial x}\left(\frac{k_x \rho}{\mu}\frac{\partial p}{\partial x}\right) = \frac{\partial}{\partial t}(\phi \rho) \tag{2–18}$$

Equation 2–18 can be written in three-dimensional form:

$$\frac{\partial}{\partial x}\left(\frac{k_x \rho}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{k_y \rho}{\mu}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{k_z \rho}{\mu}\left(\frac{\partial p}{\partial z} + \rho\right)\right) = \frac{\partial}{\partial t}(\phi \rho) \tag{2–19}$$

Equation 2–19 represents a general form for the combination of the continuity equation and Darcy's law. The final differential equation, which will result from this equation, depends on the fluid and the equation of state of interest.

For the radial flow case we obtain in a similar manner

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r \rho k_r}{\mu}\frac{\partial p}{\partial r}\right) = \frac{\partial}{\partial t}(\phi \rho) \tag{2–20}$$

In the case of flow of a nonideal gas, the gas deviation factor z_g is introduced into the equation of state to give

$$\rho = \frac{M}{RT}\frac{\rho}{z_g} \tag{2–21}$$

If we assume laminar flow, neglect gravity, and assume constant rock properties, then Eq. 2–19 becomes, for isothermal conditions,

$$\frac{\partial}{\partial x}\left(\frac{p}{\mu z_g}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{p}{\mu z_g}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{p}{\mu z_g}\frac{\partial p}{\partial z}\right) = \frac{\phi}{k}\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) \tag{2–22}$$

For radial flow Eq. 2–22 can be expressed as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{p}{\mu z_g} r \frac{\partial p}{\partial r}\right) = \frac{\phi}{k}\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) \tag{2–23}$$

Equation 2–23 in gasfield units is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{p}{\mu z} r \frac{\partial p}{\partial r}\right) = \frac{\phi}{0.000264}\frac{\partial}{\partial t}\left(\frac{p}{z}\right) \tag{2–24}$$

Equation 2–24 can be modified to account for simultaneous flow of gas, oil, and water; the equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial z}\right) = \frac{\phi c_t}{0.000264\lambda_t}\frac{\partial p}{\partial t} \tag{2–25}$$

where

z = gas deviation factor
c_t = total system isothermal compressibility, psi^{-1}
λ_t = total mobility

$$c_t = c_g s_g + c_o s_o + c_w s_w c_f \tag{2–26}$$

$$\lambda_t = \frac{k_g}{\mu_g} + \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} \tag{2–27}$$

2.7 Basic Gas Flow Equations

Gas flow is characterized by Darcy's law and for a gas described by the equation of state:

$$\rho = \frac{M}{RT}\frac{p}{z} \tag{2–28}$$

Equation 2–19 becomes, for constant ϕ and k and negligible gravitational forces,

$$\frac{\partial}{\partial x}\left(\frac{\rho}{\mu z_g}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho}{\mu z_g}\frac{\partial p}{\partial y}\right)+\frac{\partial}{z}\left(\frac{\rho}{\mu z_g}\frac{\partial p}{\partial z}\right)$$

$$=\frac{\phi}{0.000264k}\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) \tag{2–29}$$

Equation 2–29 has a form similar to the following equation:

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}=\frac{\phi\mu c}{0.000264k}\frac{\partial p}{\partial t} \tag{2–30}$$

For radial flow, the corresponding equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)=\frac{\phi\mu c}{0.000264k}\frac{\partial p}{\partial t} \tag{2–31}$$

We define a pseudopressure,[1] $\Psi(p)$, as follows:

$$\psi(p)=2\int_{p_0}^{p}\frac{p}{\mu z_g}dp \tag{2–32}$$

where p_0 is a low base pressure, now:

$$\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right)=\frac{d\left(\frac{p}{z_g}\right)}{dp}\frac{\partial p}{\partial t}=\frac{c_g p}{z_g}\frac{\partial p}{\partial t}$$

because

$$c_g=\frac{1}{\rho}\frac{d\rho}{d\rho}=\frac{z_g}{p}\frac{d\left(\frac{p}{z_g}\right)}{dp}$$

and also

$$\frac{\partial\psi}{\partial t}=\frac{\partial\psi}{\partial p}\frac{\partial p}{\partial t}\frac{\partial p}{\partial x}$$

Similar expressions apply for $\frac{\partial\psi}{\partial y}$ and $\frac{\partial\psi}{\partial z}$. Thus, Eq. 2–29 becomes

$$\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial\psi}{\partial z}\right)=\frac{\phi\mu c_g}{0.000264k}\frac{\partial\psi}{\partial t} \tag{2–33}$$

For radial flow, the equivalent of Eq. 2–33 is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)=\frac{\phi\mu c_g}{0.000264k}\frac{\partial\psi}{\partial t} \tag{2–34}$$

2.8 One-Dimensional Coordinate Systems

Equation 2–29 may be expressed in terms of rectangular, cylindrical, or spherical coordinates:

$$\nabla^2 p=\frac{\phi\mu c}{k}\frac{\partial p}{\partial t} \tag{2–35}$$

where $\nabla^2 p$ is the Laplacian of p. The expression "one-dimensional" refers to a specified coordinate system. For example, one-dimensional flow in the x-direction in rectangular coordinates may be expressed in cylindrical coordinates.

Linear Flow

Flow lines are parallel, and the cross-sectional area of flow is constant and is represented by Eq. 2–36, which is in the rectangular coordinate system and is the one-dimensional form of Eq. 2–35:

$$\frac{\partial^2 p}{\partial x^2}=\frac{\phi\mu c}{k}\frac{\partial p}{\partial t} \tag{2–36}$$

Fractures often exist naturally in the reservoir, and the flow toward the fracture is linear.

Radial Cylindrical Flow

In petroleum engineering the reservoir is often considered to be circular and of constant thickness h, with a well opened over the entire thickness. The flow takes place in the radial direction only. The flow lines converge toward a central point in each point, and the cross-sectional area of flow decreases as the center is approached. Thus flow is directed toward a central line referred to as a line-sink (or line-source in the case of an injection well). In the petroleum literature it is often simply called radial flow in the cylindrical coordinate system and is given by one-dimensional form of Eq. 2–35:

$$\frac{\partial}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)=\frac{\phi\mu c}{k}\frac{\partial p}{\partial t} \tag{2–37}$$

Radial Spherical Flow

If the well is not opened to the entire production formation because of a thick reservoir (h is very large), then to measure vertical permeability, the one-dimensional form of Eq. 2–35, in the spherical coordinate system, is of interest. It is known as the radial–spherical flow equation and is given by

$$\frac{\partial}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\phi\mu c}{k}\frac{\partial p}{\partial t} \tag{2–38}$$

2.9 Radial Gas Flow Equations in Dimensionless Variables and Groups

Equation 2–35 and the relevant boundary conditions in dimensionless terms are:

$$\nabla^2(\Delta p_D) = \frac{\partial}{\partial t_D}(\Delta p_D) \tag{2–39}$$

where the subscript D means dimensionless, and the dimensionless terms are defined in the next section for various modes of flow.

Pressure Treatment

The pressure case will be considered along with the boundary and initial conditions. Assuming a well is producing at a constant rate q_g from an infinite reservoir, the equation governing flow is

$$\frac{\partial}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\phi\mu c}{k}\frac{\partial p}{\partial t} \tag{2–40}$$

with the following boundary and initial conditions:

Inner Boundary Condition:

Assuming at the wellbore, the flow rate is constant and from Darcy's law,

$$\left.\frac{q}{2\pi rh}\right|_{well} = \left.\frac{k}{\mu}\frac{\partial p}{\partial r}\right|_{well} \quad \text{for } t > 0 \tag{2–41}$$

That is,

$$\left.r\frac{\partial p}{\partial r}\right|_{well} = \frac{q\mu}{2\pi kh} \tag{2–42}$$

and in terms of standard conditions,

$$r\frac{\partial p}{\partial r}\bigg|_{well} = \frac{q_{sc}\mu}{2\pi kh}\frac{P_{sc}T\bar{z}}{\bar{p}T_{sc}} \tag{2–43}$$

Outer Boundary Condition:

At all times, the pressure at the outer boundary (radius = infinity) is the same as the initial pressure, p_i, that is,

$$p \to p_i \quad as\ r \to \infty$$

for all t.

Initial Condition

Initially, the pressure throughout the reservoir is constant, that is,

$$p = p_i \quad at\ t = 0$$

for all t.

At this stage, the variables which affect the solution of Eq. 2–40 are p, p_I, r, r_w, q_{sc}, μ_g, k, h, ϕ, c, and t. Let

$$\Delta p = p_i - p$$

$$r_D = \frac{r}{r_w}\text{(dimensionless)}$$

$$\Delta p'_D = \frac{p_i - p}{p_i}$$

Then Eq. 2–43 becomes

$$r_D\frac{\partial}{\partial r_D}(\Delta p'_D)\bigg|_{r_D=1} = \frac{-q_{sc}\mu_g P_{sc}T\bar{z}}{p_i 2\pi kh\bar{p}}T_{sc} \tag{2–44}$$

Let the dimensionless flow rate be

$$q_D = \frac{q_{sc}\mu P_{sc}T\bar{z}}{p_i 2\pi kh\bar{p}T_{sc}}$$

Equation 2–44 becomes

$$r_D\frac{\partial}{\partial r_D}\left[\frac{(\Delta p'_D)}{q_D}\right]\bigg|_{r_D=1} = -1 \tag{2–45}$$

Let the dimensionless pressure drop be

$$\Delta p_D = \frac{(\Delta p'_D)}{q_D} = \frac{p_i - p}{p_i q_D}$$

Then Eq. 2–45 becomes

$$r_D \frac{\partial}{\partial r_D}(\Delta p_D)\bigg|_{r_D=1} = -1$$

Equation 2–37 becomes

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left[r_D \frac{\partial}{\partial r_D}(\Delta p_D)\right] = \frac{\phi \mu c r_w^2}{k}\frac{\partial}{\partial t}(\Delta p_D) \tag{2–46}$$

Let dimensionless time be

$$t_D = \frac{kt}{\phi \mu c r_w^2}$$

Equation 2–37, the radial cylindrical flow equation, may now be expressed in dimensionless terms by

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left[r_D \frac{\partial}{\partial r_D}(\Delta p_D)\right] = \frac{\partial}{\partial t_D}(\Delta p_D) \tag{2–47}$$

with the boundary and initial conditions as follows:

1. $r_D \frac{\partial}{\partial r_D}(\Delta p_D)\bigg|_{r_D=1} = -1 \quad \text{for } t_D > 0$
2. $\Delta p_D \to 0 \quad \text{as } r_D \to \infty \quad \text{for all } t_D$
3. $\Delta p_D = 0 \quad \text{at } t_D = 0 \quad \text{for all } r_D$

The solution of Eq. 2–47, which is the dimensionless form of Eq. 2–40, now involves only Δp_D, t_D, and r_D. The dimensionless terms in terms of pressure treatment case are defined in gasfield units as follows:

$$t_D = \frac{0.0002637kt}{\phi \bar{\mu}_g \bar{c} r_w^2} \tag{2–48}$$

$$\Delta p_D = \frac{p_i - p}{p_i q_D}, \tag{2–49}$$

and

$$q_D = \frac{7.085 \times 10^5 q_{sc} \bar{\mu}_g T \bar{z}}{\bar{p} k h p_i} \tag{2–50}$$

where k = formation permeability, mD; t = time, hours; ϕ = porosity, fraction; $\bar{\mu}_g$ = average gas viscosity, cP; T = reservoir temperature, °R; $\bar{z}$ = gas compressibility factor at average pressure; ΔP_D = dimensionless average reservoir pressure, psia; p_i = initial reservoir pressure, psia; h = reservoir thickness, ft; q_{sc} = gas flow rate, mmscfd; T_{sc} = base temperature, °R; P_{sc} = base pressure, psia; and $\bar{c}$ = gas compressibility, psi^{-1}.

Pressure Squared Treatment

Dimensionless variables in terms of pressure squared treatment are defined in gasfield units as follows:

$$t_D = \frac{0.0002637kt}{\phi\bar{\mu}_g\bar{c}r_w^2} \tag{2–51}$$

$$p_D = \frac{p_i^2 - p^2}{p_i^2 q_D} \tag{2–52}$$

and

$$q_D = \frac{1.417 \times 10^6\, \bar{z}Tq_{sc}\bar{\mu}_g}{khp_i^2} \tag{2–53}$$

Pseudopressure Treatment

Dimensionless variables in terms of pseudopressure treatment are defined in gasfield units as follows:

$$t_D = \frac{0.0002637kt}{\phi\bar{\mu}_g\bar{c}r_w^2} \tag{2–54}$$

$$\Delta p_D = \frac{\psi_i - \psi_{wf}}{\psi_i q_D} \tag{2–55}$$

and

$$q_D = \frac{1.417 \times 10^6 Tq_{sc}}{kh\psi_i} \tag{2–56}$$

Example 2–4 *Calculating Dimensionless Quantities Using p, p^2, and $\psi(p)$ Treatment*

A gas reservoir was produced at a constant rate q_{sc} of 6.5 mmscfd for a time, t, of 36 hours. The sandface pressure, p_{wf}, at that time was 1750 psia. General data are as follows:

$\bar{p} = 1925$ psia, $p_i = 2100$ psia, $z_I = 0.842$, $z_i = 0.849$, $z_{1750} = 0.855$, $c_i = 0.000525$ psi^{-1}, $c_{1750} = 0.000571$ psi^{-1}, $\bar{c} = 0.000548$ psi^{-1}, $k = 18.85$ mD, $T = 595°$R, $r_w = 0.39$ ft, $\mu_i = 0.01495$ cp, $\bar{\mu} = 0.01430$ cp, $\mu_{1,750} = 0.01365$ cp, $h = 40$ ft, and $\phi = 0.138$ fraction.

Calculate the dimensionless quantities t_D, P_D, and q_D using the p, p^2, and ψ treatments.

Solution Pressure treatment, p, from Eq. 2–48:

$$t_D = \frac{0.0002637kt}{\phi\bar{\mu}\bar{c}r_w^2}$$

$$\therefore t_D = \frac{0.0002637(18.85)(36)}{(0.138)(0.01430)(0.000548)(0.39)^2} = 1{,}087{,}925$$

From Eq. 2–50:

$$q_D = \frac{7.085 \times 10^5 q_{sc}\bar{\mu}T\bar{z}}{\bar{p}khp_i}$$

$$\therefore q_D = \frac{7.085 \times 10^5(6.5)(0.0143)(595)(0.849)}{(1925)(18.85)(40)(2100)} = 0.010914$$

From Eq. 2–49:

$$\therefore \Delta p_D = \frac{p_i - p}{p_i q_D} = \frac{2100 - 1750}{2100(0.010914)} = \frac{350}{22.92} = 15.27$$

Pressure-squared treatment, p^2, from Eq. 2–51:

$$t_D = \frac{0.0002637kt}{\phi\bar{\mu}\bar{c}r_w^2}$$

$$\therefore t_D = \frac{0.0002637(18.85)(36)}{(0.138)(0.01430)(0.000548)(0.39)^2} = 1{,}087{,}925$$

From Eq. 2–53:

$$q_D = \frac{1.417 \times 10^6 \bar{z}Tq_{sc}\bar{\mu}}{khp_i^2}$$

$$= \frac{1.417 \times 10^6(0.849)(595)(6.5)(0.0143)}{(18.85)(40)(2100)^2} = 0.020010$$

From Eq. 2–52:

$$\Delta p_D = \frac{p_i^2 - p^2}{p_i^2 q_D}$$

$$= \frac{2100^2 - 1.750^2}{2100^2(0.020010)} = 15.27$$

Pseudopressure treatment, ψ, from Eq. 2–54:

$$t_D = \frac{0.0002637kt}{\phi\bar{\mu}\bar{c}r_w^2}$$

$$\therefore t_D = \frac{0.0002637(18.85)(36)}{(0.138)(0.01430)(0.000548)(0.39)^2} = 1{,}087{,}925$$

From Eq. 2–56:

$$q_D = \frac{1.417 \times 10^6 T q_{sc}}{kh\psi_i}$$

$$p_I = 2100 \text{ psia} \leftrightarrow \psi_i = 335 \text{ mmpsia}^2/\text{cp}$$

$$\therefore q_D = \frac{1.417 \times 10^6(595)(6.5)}{(18.85)(40)(335 \times 10^6)} = 0.021696$$

From Eq. 2–55:

$$\Delta p_D = \frac{\psi_i - \psi_{wf}}{\psi_i q_D}$$

$$p = 1{,}750 \text{ psia} \leftrightarrow \psi(p) = 223 \text{ mmpsia}^2/\text{cp}$$

$$\therefore \Delta p_D = \frac{(335 - 223)10^6}{335 \times 10^6(0.021696)} = 15.41$$

Calculating Gas-Pseudopressure $\psi(p)$ Function

Accuracy of gas well test analysis can be improved in some cases if the pseudopressure $\psi(p)$ is used instead of approximations written in terms of pressure or pressure squared. In this section, we discuss the calculations of pseudopressure. Detailed discussion, including systematic development of working equations and application to drawdown, buildup, and deliverability tests, is provided in Ref. 2. The applications of real gas pseudopressure $\psi(p)$ to flow in gas wells under practical conditions are as follows:

1. When turbulence is not present, the drawdown test provides accurate results. When turbulence is significant, the drawdown test can be misleading.
2. The buildup test can be interpreted accurately even with extreme turbulence.
3. The use of a p^2 well-test plot is usually equivalent to the $\Delta(p)$ method, when well pressures are below 2000 psi.
4. Flow capacity can be determined accurately from $(p)^2$ or p well-test plots if point values, rather than average values, are used for slopes and gas properties.

Calculation of Pseudopressure

Gas pseudopressure, $\psi(p)$, is defined by the integral

$$\psi(p) = 2\int_{p_{BASE=0}}^{P} \frac{p}{\mu z} dp \tag{2–57}$$

An example will illustrate this calculation.

Example 2–5 *Calculating Gas Pseudopressure*

Given data are gas gravity = 0.7, T = 200°F. Gas properties as functions of pressure are given in Table 2–3.

Solution Use the trapezoidal rule for numerical integration.

For p = 150 psia,

$$\psi(150) = 2\int_{p_{base}}^{p} \frac{p}{\mu z} dp = 2\frac{\left[\left(\frac{p}{\mu z}\right)_0 + \left(\frac{p}{\mu z}\right)_{150}\right]}{2}(150 - 0)$$

$$= 2\frac{[0 + 12{,}290]}{2}(150) = 1.844 \times 10^6 \text{ psia}^2/\text{cp}$$

Table 2–3
Gas Properties as Functions of Pressure

Pressure P (psia)	Gas viscosity (cP)	Compressibility factor z	$p/\mu z$ (psia/cP)
150	0.01238	0.9856	12,290
300	0.01254	0.9717	24,620
450	0.01274	0.9582	36,860

For $p = 300$ psia,

$$\psi(300) = 1.844 \times 10^6 + 2\frac{\left[\left(\frac{p}{\mu z}\right)_{150} + \left(\frac{p}{\mu z}\right)_{300}\right]}{2}(300 - 150)$$

$$= 1.844 \times 10^6 + 2\frac{(12{,}290 + 24{,}620)}{2}(300 - 150)$$

$$= 7.381 \times 10^6 \text{ psia}^2/\text{cp}$$

2.10 Analytical Solutions of Gas Flow Equations

Radial flow geometry is of greatest interest in gas well testing. This radial flow equation was developed in terms of dimensionless variables in previous sections. It is Eq. 2–47 and is repeated below.

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left[r_D\frac{\partial}{\partial r_D}(\Delta p_D)\right] = \frac{\partial}{\partial t_D}(\Delta p_D) \tag{2–58}$$

Equation 2–58 can be solved for pressure as a function of flow rate and time. Solutions to Eq. 2–47 depend on the reservoir type, the boundary and initial conditions. Direct analytical solutions will be presented in this section.

Constant Production Rate, Radial Cylindrical Flow, Infinite-Acting Reservoir (Transient)

The Eq. 2–58 is reduced to an ordinary differential equation by applying the Boltzmann transformation $X = r_D^2/(4t_D)$. This is then solved by separating the variables and integrating with the above three conditions. The equation form of the solution is

$$\Delta p_D = -0.5E_i\left(-\frac{r_D^2}{4t_D}\right) \tag{2–59}$$

or

$$\Delta p_D = -0.5E_i\left(-\frac{\phi\mu c r^2}{0.0002637kt}\right) \tag{2–60}$$

Values of Δp_D versus t_D can be found in Ref. 5 for various reservoir sizes, that is, for various values of r_D. E_i is the exponential integral and is defined by

$$E_i(-x) = \int_x^\infty \frac{e^{-u}du}{u} = \ln(1.781) - \frac{x}{1!} + \frac{x^2}{2 \times 2!} - \frac{x^3}{3 \times 3!} + \frac{x^4}{4 \times 4!} \cdots + \frac{(-x)^n}{n \times n!} \quad (2\text{–}61)$$

For values of x less than 0.02, Eq. 2–62 can approximate the exponential integral with an error of less than 0.6:

$$E_i(-x) = \ln(1.781x) \quad \text{for } x < 0.02 \quad (2\text{–}62)$$

For computing pressures at the borehole such as drawdown pressures or buildup pressures Eq. 2–61 may be used. However, if practical units are used and logarithms to the base 10 are used, constants for Eq. 2–62 must be evaluated. Darcy units apply to Eq. 2–62. Table 2–4 lists Darcy units and practical units.

For $x \geq 10.9$ the exponential integral is closely approximated by zero. To evaluate the E_i function, we can use Table 2–5 for $0.02 < x$ 10.9.

Thus Eq. 2–59 becomes

$$p_D = 0.5 \ln\left(\frac{4t_D}{1.781r_D^2}\right) \quad \text{for } \frac{4t_D}{r_D^2} > 100 \quad (2\text{–}59a)$$

$$p_D = 0.5\left[\ln\left(\frac{t_D}{r_D^2}\right) + 0.80907\right] \quad \text{for } \frac{t_D}{r_D^2} > 25 \quad (2\text{–}63)$$

Table 2–4
Darcy and Practical Units for Parameters in the Exponential Solution of the Diffusivity Equation

Parameter or variables	Darcy units	Practical units
C	vol/vol/atm	vol/vol/psi
ϕ	Porosity	Porosity
h	cm	ft
K	Darcy	Millidarcies
μ	Centipoise	Centipoise

Table 2–5
Values of the Exponential Integral, $-E_i(-x)$ (after Lee, © SPE, *Well Testing*, 1982)[5]

$-E_i(-x)$, 0.000 < 0.209, interval – 0.001										
X	0	1	2	3	4	5	6	7	8	9
0.00	∞	6.332	5.639	5.235	4.948	4.726	4.545	4.392	4.259	4.142
0.01	4.038	3.944	3.858	3.779	3.705	3.637	3.574	3.514	3.458	3.405
0.02	3.355	3.307	3.261	3.218	3.176	3.137	3.098	3.062	3.026	2.992
0.03	2.959	2.927	2.897	2.867	2.838	2.810	2.783	2.756	2.731	2.706
0.04	2.681	2.658	2.634	2.612	2.590	2.568	2.547	2.527	2.507	2.487
0.05	2.468	2.449	2.431	2.413	2.395	2.378	2.360	2.344	2.327	2.311
0.06	2.295	2.280	2.265	2.249	2.235	2.220	2.206	2.192	2.178	2.164
0.07	2.251	2.138	2.125	2.112	2.099	2.087	2.074	2.062	2.050	2.039
0.08	2.027	2.016	2.004	1.993	1.982	1.971	1.960	1.950	1.939	1.929
0.09	1.919	1.909	1.899	1.889	1.879	1.870	1.860	1.851	1.841	1.832
0.10	1.823	1.814	1.805	1.796	1.788	1.770	1.770	1.762	1.754	1.745
0.11	1.737	1.729	1.721	1.713	1.705	1.697	1.690	1.682	1.675	1.667
0.12	1.660	1.652	1.645	1.638	1.631	1.623	1.616	1.609	1.603	1.696
0.13	1.589	1.582	1.576	1.569	1.562	1.556	1.549	1.543	1.537	1.530
0.14	1.524	1.518	1.512	1.506	1.500	1.494	1.488	1.482	1.476	1.470
0.15	1.465	1.459	1.453	1.448	1.442	1.436	1.431	1.425	1.420	1.415
0.16	1.409	1.404	1.399	1.393	1.388	1.383	1.378	1.373	1.368	1.363
0.17	1.358	1.353	1.348	1.343	1.338	1.333	1.329	1.324	1.319	1.315
0.18	1.310	1.305	1.301	1.296	1.292	1.287	1.283	1.278	1.274	1.269
0.19	1.265	1.261	1.256	1.252	1.248	1.244	1.239	1.235	1.231	1.227
0.20	1.223	1.219	1.215	1.211	1.207	1.203	1.199	1.195	1.191	1.187
$-E_i(-x)$, 0.00 < x < 2.09, interval = 0.01										
0.0	∞	4.0380	3.3548	2.9592	2.6813	2.4680	2.2954	2.1509	2.0270	1.9188
0.1	1.8230	1.7372	1.6596	1.5890	1.5242	1.4645	1.4092	1.3578	1.3099	1.2649
0.2	1.2227	1.1830	1.1454	1.1099	1.0763	1.0443	1.0139	0.9850	0.9574	0.9310
0.3	0.9057	0.8816	0.8584	0.8362	0.8148	0.7943	0.7745	0.7555	0.7372	0.7195
0.4	0.7024	0.6860	0.6701	0.6547	0.6398	0.6354	0.6114	0.5979	0.5848	0.5721
0.5	0.5598	0.5479	0.5363	0.5350	0.5141	0.5034	0.4931	0.4830	0.4732	0.5721
0.6	0.4544	0.4454	0.4366	0.4281	0.4197	0.4116	0.4036	0.3959	0.3884	0.3810
0.7	0.3738	0.3668	0.3600	0.3533	0.3468	0.3404	0.3342	0.3281	0.3221	0.3163
0.8	0.3107	0.3051	0.2997	0.2944	0.2892	0.2841	0.2791	0.2742	0.2695	0.2648
0.9	0.2602	0.2558	0.2514	0.2471	0.2429	0.2388	0.2348	0.2308	0.2270	0.2232
1.0	0.2194	0.2158	0.2122	0.2087	0.2053	0.2019	0.1986	0.1954	0.1922	0.1891
1.1	0.1861	0.1831	0.1801	0.1772	0.1744	0.1716	0.1689	0.1662	0.1636	0.1610
1.2	0.1585	0.1560	0.1536	0.1512	0.1488	0.1465	0.1442	0.1420	0.1398	0.1377
1.3	0.1355	0.1335	0.1314	0.1294	0.1274	0.1255	0.1236	0.1217	0.1199	0.1181
1.4	0.1163	0.1146	0.1129	0.1112	0.1095	0.1079	0.1063	0.1047	0.1032	0.1016

Table 2–5 (Continued)

1.5	0.1002	0.0987	0.0972	0.0958	0.0944	0.0930	0.0917	0.0904	0.0890	0.0878
1.6	0.0865	0.0852	0.0840	0.0828	0.0816	0.0805	0.0793	0.0782	0.0771	0.0760
1.7	0.0749	0.0738	0.0728	0.0718	0.0708	0.0698	0.0679	0.0669	0.0669	0.0660
1.8	0.0651	0.0642	0.0633	0.0624	0.0616	0.0607	0.0599	0.0591	0.0583	0/0575
1.9	0.0567	0.0559	0.0552	0.0545	0.0537	0.0530	0.0523	0.0516	0.0509	0.0503
2.0	0.0496	0.0490	0.0483	0.0477	0.0471	0.0465	0.0459	0.0453	0.0448	0.0432

$2.0 < x < 10.9$, interval = 0.1

2	4.89	4.26	3.72	3.25	2.84	2.49	2.19	1.92	1.69	1.48
3	1.30	1.15	1.01	8.94	7.89	6.87	6.16	5.45	4.82	4.27
4	3.78	3.35	2.97	2.64	2.34	2.07	1.84	1.64	1.45	1.29
5	1.15	1.02	9.08	8.09	7.19	6.41	5.71	5.09	4.53	4.04
6	3.60	3.21	2.86	2.55	2.28	2.03	1.82	1.62	1.45	1.29
7	1.15	1.03	9.22	8.24	7.36	6.58	5.89	5.26	4.71	4.21
8	3.77	3.37	3.02	2.70	2.42	2.16	1.94	1.73	1.55	1.39
9	1.24	1.11	9.99	8.95	8.02	7.18	6.44	5.77	5.17	4.64
10	4.15	3.73	3.34	3.00	2.68	2.41	2.16	1.94	1.74	1.56×60^{-6}

Δp_D varies with the boundary conditions, but for the case of constant productivity rate from an infinite-acting reservoir, Δp_D is given by

$$\Delta p_D = -0.5E_i\left(-\frac{1}{4t_D}\right) \tag{2–64}$$

When $r = r_w$, $r_D = 1$. In terms of the logarithmic approximation, from Eq. 2–63

$$\Delta p_D = 0.5\ (\ln t_D + 0.809) \quad \text{for } t_D > 25 \tag{2–65}$$

It is evident that p_D for an infinite-acting reservoir is identical to the $r_D = 1$ curve for p_D, is expressed in dimensionless terms, and is the value at the well without inertial-turbulent and skin effects.[1] The effects of skin inertial-turbulent flow are treated earlier.

Example 2–6 *Calculating Flowing Pressure at the Well due to Laminar Flow in an Infinite-Acting Reservoir Using p, p^2, and Pseudopressure Treatments.*

Using the following data, calculate the pressure at the well after a flowing time of 24 hours using p, p^2, and ψ treatment. Given data are $h = 40$ ft, $k = 20$ mD, $p_i = 2000$ psia, $r_w = 0.399$ ft, $T = 580°$R, $q_{sc} = 7.0$ mmscfd, $\phi = 0.16$, $\bar{z} = 0.850$, $\bar{\mu} = 0.0152$ cP, $\bar{c} = 0.00061$ psi^{-1}.

Solution Pressure treatment:

From Eq. 2–54:

$$t_D = \frac{0.0002637kt}{\phi\bar{\mu}\bar{c}r_w^2}$$

$$= \frac{0.0002637(20)(24)}{0.16(0.0152)(0.00061)(0.399)^2} = 535{,}935$$

From Eq. 2–65, since $t_D > 25$:

$$\therefore \Delta p_D = 0.5(\ln t_D + 0.809)$$

$$= 0.5(\ln(535{,}935) + 0.809)) = 7.00$$

The value of Δp_D can also be obtained from Ref. 5, $r_D = 1.0$ curve.

First trial:

Assume

$$\bar{p} = p_i = 2000 \text{ psia}$$

From Eq. 2–50:

$$q_D = \frac{7.085 \times 10^5 \bar{z}Tq_{sc}\bar{\mu}}{\bar{p}khp_i}$$

$$= \frac{7.085 \times 10^5(0.85)(580)(7.0)(0.0152)}{(2000)(20)(40)(2000)} = 0.01161$$

Using Eq. 2–49:

$$\Delta p_D = \frac{p_i - p}{p_i q_D}$$

$$p = p_i - p_i \Delta p_D q_D = 2000 - 2000(0.01161)(7.00)$$

$$= 2000 - 163 = 1837 \text{ psia}$$

Second trial:

Assume

$$\bar{p} = \frac{p_i + p}{2} = \frac{2000 + 1837}{2} = 1919 \text{ psia}$$

From Eq. 2–50:

$$q_D = \frac{7.085 \times 10^5(0.85)(580)(7.0)(0.0152)}{1919(20)(40)(2000)} = 0.01210$$

or

$$p = 2000 - 2000(7.0)(0.01210) = 1831 \text{ psia}$$

Third trial:
Assume

$$\bar{p} = \frac{p_i + p}{2} = \frac{2000 + 1831}{2} = 1916 \text{ psia}$$

$$q_D = \frac{7.085 \times 10^5(0.85)(0.0152)(580)(7.0)}{1916(20)(40)(2000)} = 0.01212$$

or

$$p = 2000 - 2000(7.0)(0.01212) = 1830 \text{ psia}$$

Pressure-squared treatment:
Assuming $\bar{\mu}$, $\bar{z}$, and $\bar{c}$ are constants, therefore, using Eqs. 2–65 and 2–53:
From Eq. 2–53:

$$q_D = \frac{1.417 \times 10^6 \bar{z} T q_{sc} \bar{\mu}}{khp_i^2}$$

$$= \frac{1.417 \times 10^6(0.85)(580)(7.00)(0.0152)}{(20)(40)(2000)^2} = 0.02323$$

From Eq. 2–52:

$$\Delta p_D = \frac{p_i^2 - p^2}{p_i^2 q_D}$$

or

$$p = \sqrt{p_i^2 - p_i^2 \Delta p_D q_D} = [2000^2 - 2000^2(7.00)(0.02323)]^{0.5}$$

$$= 1830 \text{ psia}$$

(the same as the results from the pressure treatment).

Pseudopressure treatment:

The values of z_i, μ, and c_i are calculated at p_i; therefore

$$\psi_i = 329.6 \text{ mmpsia}^2/\text{cP}, \quad z_i = 0.84, \quad \mu_i = 0.0156 \text{ cP},$$
$$c_i = 0.00058 \text{ psi}^{-1}$$

From Eq. 2–54:

$$t_D = \frac{0.0002637kt}{\phi \mu_i c_i r_w^2}$$

$$= \frac{0.0002637(20)(24)}{(0.16)(0.0156)(0.00058)(0.399)^2} = 549{,}203$$

Since $t_D > 25$ and using Eq. 2–65:

$$\Delta p_D = 0.5(\ln t_D + 0.809)$$

$$= 0.5[\ln(549{,}203) + 0.809] = 7.013$$

From Eq. 2–56:

$$\Delta p_D = \frac{1.417 \times 10^6 \, T q_{sc}}{kh\psi_i}$$

$$= \frac{1.417 \times 10^6(580)(7.0)}{(20)(40)(329.6 \times 10^6)} = 0.02182$$

From Eq. 2–55:

$$\Delta p_D = \frac{\psi_i - \psi_{wf}}{\psi_i q_D}$$

Therefore:

$$\psi_{wf} = \psi_i - \psi_i q_D \Delta p_D$$

$$= 329.6 \times 10^6 - 329.6 \times 10^6(0.02182)(7.013)$$

$$= 279.16 \text{ mmpsia}^2/\text{cP} = 1818 \text{ psia}$$

The values of p_{wf} calculated by the p, p^2, and ψ treatments are 1830, 1830, and 1818 psi respectively.

Example 2–7 *Calculating Flowing Pressure away from the Well due to Laminar Flow in an Infinite-Acting Reservoir Using p, p^2, and Pseudopressure Treatments*

A gas well is situated in an infinite-acting reservoir. Calculate the flowing pressure, due to laminar flow, at a radius of 100 feet from the well, after 24 hours of production. Reservoir and well data are as follows:

$p_i = 2000$ psia, $\psi_i = 329.6$ mmpsia^2/cP, $z_i = 0.835$, $\mu_i = 0.0159$ cP, $c_i = 0.00055$ psia^{-1}, $r = 50$ ft, $r_w = 0.33$ ft, $\phi = 0.15$, $k = 20$ mD, $t = 24$ hours, $q_{sc} = 7.50$ mmscfd, $T = 580°$R, $h = 40$ ft.

Solution From Eq. 2–54:

$$t_D = \frac{0.0002637kt}{\phi\mu_i c_i r_w^2}$$

$$= \frac{0.0002637(20)(24)}{(0.15)(0.0159)(0.00055)(0.33)^2} = 886{,}079$$

$$r_D = \frac{r}{r_w} = \frac{50}{.33} = 152$$

$$\therefore \frac{t_D}{r_D^2} = \frac{886{,}079}{152^2} = 38.35$$

Since $\frac{t_D}{r_D^2} > 25$ and using Eq. 2–63:

$$\Delta p_D = 0.5\left[\ln\left(\frac{t_D}{r_D^2}\right) + 0.80907\right]$$

$$= 0.5[\ln(38.35) + 0.80907] = 2.228$$

From Eq. 2–56:

$$q_D = \frac{1.417 \times 10^6 T q_{sc}}{kh\psi_i}$$

$$= \frac{1.417 \times 10^6(580)(7.50)}{(20)(40)(329.6 \times 10^6)} = 0.02338$$

From Eq. 2–55:

$$\Delta p_D = \frac{\psi_i - \psi_{wf}}{\psi_i - q_D}$$

$$\therefore \psi_{wf} = \psi_i - \psi_i \Delta p_D q_D$$

$$= 329.6 \times 10^6 - 329.6 \times 10^6(2.228)(0.02338)$$

$$= 327.88 \text{ mmpsia}^2/\text{cP}$$

Using the ψ-p curve, $p_{wf} = 1942$ psia.

Radial–Cylindrical Flow, Finite Reservoir, Constant Production Rate, with No Flow at Outer Boundary (Pseudo-Steady-State)

Equation 2–58 can be written as follows:

$$\frac{\partial^2}{\partial r_D^2}(\Delta p_D) + \frac{1}{r_D}\frac{\partial}{\partial r_D}(\Delta p_D) = \frac{\partial}{\partial t_D}(\Delta p_D) \tag{2–66}$$

Using Laplace transform[15] and Bessel functions, Δp_D, which is the solution at the well, is obtained as follows.

For values of $t_D < 0.25\, r_{eD}^2$:

$$\Delta p_D = 0.5\ \ln(t_D + 0.80907) \tag{2–67}$$

For $\frac{t_D}{r_{eD}^2} > 0.25$: the equation of the form solution is

$$\Delta p_D = \frac{2t_D}{r_{eD}^2} + \ln(0.472\, r_{eD}) \tag{2–68}$$

where

$$r_{eD} = \frac{r_e}{r_w}$$

Values of Δp_D versus t_D can be found in Ref. 5 for various reservoir sizes. At early times the solution is represented by Eq. 2–61 and for large times and where $r_w \ll r_e$, the solution at the well is given by Eq. 2–68. The transition from infinite to finite behavior occurs at

$$t_D \approx 0.25\, r_{eD}^2 \tag{2–68a}$$

Example 2–8 *Calculating Flowing Sandface Pressure in Finite-Acting (Closed) Reservoir*

A gas well in a finite-acting (closed) reservoir ($r_e = 1850$ ft) was produced at a constant rate of 7.5 mmscfd. Assuming gas composition, reservoir, and well data pertinent to the test are the same as in Example 2–1, calculate the flowing sandface pressure, p_{wf}, after 80 days of production.

Solution Since the gas is the same as that of Example 2–1, the $\psi - p$ curve already constructed for Figure 2–1 is applicable to this problem.

$$t = 80 \times 24 = 1920 \text{ hours}$$

From Eq. 2–54:

$$t_D = \frac{0.0002637kt}{\phi\mu_i c_i r_w^2}$$

$$= \frac{0.0002637(20)(1{,}920)}{(0.15)(0.0159)(0.00055)(0.33)^2} = 70{,}886{,}315$$

From Eq. 2–56:

$$q_D = \frac{1.417 \times 10^6 T q_{sc}}{kh\psi_i}$$

$$= \frac{1.417 \times 10^6(580)(7.5)}{(20)(40)(329.6 \times 10^6)} = 0.02338$$

$$r_{eD} = \frac{r_e}{r_w} = \frac{1850}{.33} = 5606$$

$$r_{eD}^2 = 5606^2 = 31{,}427{,}236$$

$$\therefore t_D/r_{eD}^2 = \frac{70{,}886{,}315}{31{,}427{,}236} = 2.256$$

Since $t_D/r_{eD}^2 > 0.25$, Δp_D is given by Eq. 2–68:

$$\Delta p_D = \frac{2t_D}{r_{eD}^2} + \ln(0.472\, r_{eD})$$

$$= 2(2.256) + \ln(0.472 \times 5606)$$

$$= 12.392$$

$$\therefore \psi_{wf} = \psi_i - \psi_i \Delta p_D q_D$$

$$= 329.6 \times 10^6 - 329.6 \times 10^6(12.392)(0.02338)$$

$$= 234.11 \text{ mmpsia}^2/\text{cP}$$

$$p_{wf} = 1790 \text{ psia} \quad \text{(Figure 2–1)}$$

The transition from infinite to finite behavior occurs at

$$t = \frac{0.25 r_{eD}^2 \phi\mu_i c_i r_w^2}{0.0002637k}$$

$$= \frac{0.25(31{,}427{,}236)(0.15)(0.0159)(0.00055)(0.33)^2}{0.0002637(20)} = 212.8 \text{ hours}$$

Radial–Cylindrical Flow, Finite Circular Reservoir, Constant Production Rate with Constant Pressure at Outer Boundary (Steady-State Conditions)

The conditions for this situation are:

1. Flow rate at the well is constant
2. The pressure at the boundary is constant at all times, $p_e = p_i$ for all t
3. Initially the pressure throughout the reservoir is uniform

By the use of the Laplace transform, Bessel functions,[3] and the above boundary conditions, the solution of the Eq. 2–66 is found to be (Carslaw and Jaeger, 1959, p. 334)[20]

$$\Delta p_D = \ln r_{eD} \quad \text{for } t_D > 1.0\, r_{eD}^2 \quad \text{(approximately)} \tag{2–69}$$

This equation may also be derived directly by integration of Darcy's law for a radial flow. Equation 2–69 represents the steady-state condition. Values of Δp_D versus t_D can be found in Ref. 5 for various reservoir sizes, which are for various values of r_D.

Example 2–9 *Calculation of Flowing Bottom-Hole Pressure Assuming Steady-State Conditions*

Rework Example 2–9, assuming a steady-state condition is achieved after long producing time. Calculate the flowing bottom hole pressure, p_{wf}, after 1920 hours of production.

Solution From Example 2–8, we have $t_D = 70{,}886{,}315$, $q_D = 0.02338$, $r_{eD} = 5606$, $(r_{eD})^2 = 31{,}427{,}236$. Since $t_D > 1.0 r_{eD}^2$, Δp_D is given by Eq. 2–69,

$$\Delta p_D = \ln(r_{eD}) = \ln(5606) = 8.632$$

From Eq. 2–55:

$$\psi_{wf} = \psi_i - \psi_i \Delta p_D q_D$$

$$= 329.6 \times 10^6 - 329.6 \times 10^6 (8.632)(0.02338)$$

$$= 263.8 \text{ mmpsia}^2/\text{cP}$$

From Figure 2–1, $p_{wf} = 1970$ psia.

Radial–Cylindrical Flow, Infinite and Finite Circular Reservoir, Constant Production Rate, Solution at the Well

The Δp_D functions may also be expressed in steady-state form by introducing the idea of an effective drainage radius. This concept, along with the concepts of radius of investigation and time to stabilization, is discussed in detail hereafter. Possible expressions for the effective drainage radius for various systems are as follows.

Infinite reservoir:

$$\ln\left(\frac{r_d}{r_w}\right) = \frac{1}{2}(\ln t_D + 0.809) \quad \text{for } t_D > 25. \tag{2–70}$$

Closed outer boundary:

$$r_d = 0.472 r_e \quad \text{for } t_D > 0.25 r_{eD}^2 \tag{2–70a}$$

Constant-pressure outer boundary:

$$r_d = r_e \quad \text{for } r_d = r_e$$

In terms of pressure treatment:

$$\Delta p_D = \frac{\bar{p}_R - p_{wf}}{p_i q_D} = \ln\left(\frac{r_d}{r_w}\right) \tag{2–71}$$

In terms of pressure-squared:

$$\frac{\bar{p}_R^2 - p_{wf}^2}{p_i^2 q_D} = \ln\left(\frac{r_d}{r_w}\right) \tag{2–72}$$

In terms of pseudopressure:

$$\frac{\bar{\psi}_R - \psi_{wf}}{\psi_i q_D} = \ln\left(\frac{r_d}{r_w}\right) \tag{2–73}$$

Radial–Cylindrical Flow, Constant Well Pressure, Infinite and Finite Circular Reservoir

When the well is producing at a constant pressure, the flow rate is not constant but declines continuously. The cumulative production is given by Katz et al. (1959, p. 414)[21] and may be written as

$$G_p = 2\pi\phi c r_w^2 h \frac{T_{sc}}{T}\frac{p_i}{P_{sc}}(p_i - p_{wf}) Q_{pD} \tag{2–74}$$

where

Gp = cumulative gas produced, and
Q_{pD} = dimensionless total production number which has been tabulated for certain boundary conditions, and can be found in Ref. 5.

For $t_D < 0.01$:

$$Q_{pD} = \left(\frac{t_D}{\pi}\right)^{0.5} \tag{2–75}$$

For $t_D \geq 200$ or

$$t_D \propto Q_{pD} = \frac{-4.29881 + 2.02566 t_D}{\ln t_D}: \tag{2–76}$$

$$t_D = \frac{0.0002637kt}{\phi \bar{\mu} \bar{c} r_w^2}$$

In terms of pressure-squared treatment:

$$G_p = \frac{0.111 \phi h r_w^2 c \left(p_i^2 - p_{wf}^2\right)}{\bar{z} T} Q_{pD} \tag{2–77}$$

where

G_p = cumulative gas produced, mscf, and
$r_D = r/r_w$

Values of Q_{pD} as a function of dimensionless time t_D and dimensionless radius can be found in tabular form in Ref. 5.

Linear Flow, Constant Production Rate, Infinite Reservoir

When flow is in the vicinity of a fracture (of length x_f), the flow will be linear and the pressure at any distance x from the sandface ($x \neq 0$) is given by Katz *et al.* (1959, p. 411)[21] as

$$\Delta p_D = \frac{2}{\sqrt{\pi}} \left(\frac{t_D}{x_D^2}\right)^{0.5} \exp\left(-\frac{x_D^2}{4t_D}\right) - \text{erfc}\left[0.5\left(\frac{x_D^2}{t_D}\right)^{0.5}\right] \tag{2–78}$$

where

$$t_D = \frac{0.0002637kt}{\phi \bar{\mu} \bar{c} x_f^2} \tag{2–79}$$

x_f is half fracture length, ft

$$x_D = \frac{x}{x_f}$$

In terms of pressure treatment:

$$q_D = \frac{4.467 \times 10^6 \bar{z} T q_{sc} \bar{\mu}}{\bar{p} k h p_i} \tag{2–80}$$

In terms of pressure-squared treatment:

$$q_D = \frac{8.933 \times 10^6 \bar{z} T q_{sc} \bar{\mu}}{k h p_i^2} \tag{2–81}$$

In terms of pseudopressure treatment:

$$q_D = \frac{8.933 \times 10^6 T q_{sc}}{k h \psi_i} \tag{2–82}$$

and erf is the error function defined as

$$\text{erf}\, x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{2–83}$$

$$\text{erf}\, x = \frac{2}{\sqrt{\pi}} \left(x - \frac{x^2}{1!3} + \frac{x^5}{2!5} - \frac{x^7}{3!7} + - \cdots \right) \tag{2–83a}$$

$\text{erf}(\infty) = 1$, the complementary error function, and is defined by

$$\text{erfc}\, x = 1 - \text{erf}\, x = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt \tag{2–83b}$$

The values of error and complementary functions are given in Table 2–6.

Radial–Spherical Flow, Constant Production Rate, Infinite Reservoir

The dimensionless Δp_D, at any radius r, is given by (Carslaw and Jaeger, 1959, p. 261)[20]

$$\Delta p_D = \frac{1}{2} \text{erfc} \left(\frac{r_D^2}{4 t_D} \right)^{0.5} \tag{2–84}$$

Table 2–6
Complementary Error Function (after Katz *et al.*, 1959, © McGraw-Hill)[21]

X	erf x	erfc $x = 1 - \text{erf } x$
0.0	0.0000	1.0000
0.1	0.1114	0.8887
0.2	0.2227	0.7773
0.3	0.3256	0.6745
0.4	0.4284	0.5716
0.5	0.5162	0.4839
0.6	0.6039	0.3961
0.7	0.6730	0.3268
0.8	0.7421	0.2579
0.9	0.7924	0.2076
1.0	0.8427	0.1573
1.1	0.8765	0.1235
1.2	0.9103	0.0897
1.3	0.9313	0.0687
1.4	0.9523	0.0477
1.5	0.9643	0.0356
1.6	0.9763	0.0237
1.7	0.9827	0.0173
1.8	0.9891	0.0109
1.9	0.9922	0.0078
2.0	0.9953	0.0047
2.1	0.9967	0.0033
2.2	0.9981	0.0019
2.3	0.9987	0.0013
2.4	0.9993	0.0007
2.5	0.9996	0.0005
2.6	0.9998	0.0002
2.7	0.9999	0.0001
2.8	0.9999	0.0001
2.9	1.0000	0.0000
3.0	1.0000	0.0000
3.1	1.0000	0.0000
3.2	1.0000	0.0
3.3	1.0000	0.0
3.4	1.0000	0.0
3.5	1.0000	0.0
3.6	1.0000	0.0
3.7	1.0000	0.0
3.8	1.0000	0.0
3.9	1.0000	0.0
4.0	1.0000	0.0

where

$$t_D = \frac{0.0002637kt}{\phi\bar{\mu}\bar{c}r_w^2} \tag{2–85}$$

In terms of pressure treatment:

$$q_D = \frac{7.110 \times 10^5 \bar{z}T q_{sc}\bar{\mu}}{\bar{p}krp_i} \tag{2–86}$$

In terms of pressure-squared treatment:

$$q_D = \frac{1.422 \times 10^6 \bar{z}T q_{sc}\bar{\mu}}{krp_i^2} \tag{2–87}$$

In terms of pseudopressure treatment:

$$q_D = \frac{1.422 \times 10^6 T q_{sc}}{kr\psi_i} \tag{2–88}$$

In thick formations, radial–spherical flow may exist in the vicinity of the well when only a limited portion of the formation is opened to flow.

2.11 Application of Superposition Techniques

Superposition may be considered to be a problem-solving technique in which the pressure behavior at any point at any time is the sum of the histories of each of the effects that may be considered to affect the solution at that point. Particular applications of superposition, which are important in the analysis of pressure test data, are discussed in the following section.

Investigating for Rate Change Effects

The following example will illustrate the principle of superposition as applied to the pressure drawdown due to two different flow rates. The method may be extended to any number of changing flow rates. Thus the total pressure drop for the well would be

$$(\Delta\psi)_{total} = |\psi_i \Delta p_{D1} q_{D1}|_{q1} + |\psi_i \Delta p_{D2} q_{D2}|_{q2-q1} + |\psi_i \Delta p_{D3} q_{D3}|_{q3-q2} + \cdots. \tag{2–89}$$

$$\psi_{mf} = \psi_i - (\Delta\psi)_{total} \tag{2–90}$$

The variable-rate production history is illustrated in Figure 2–2.

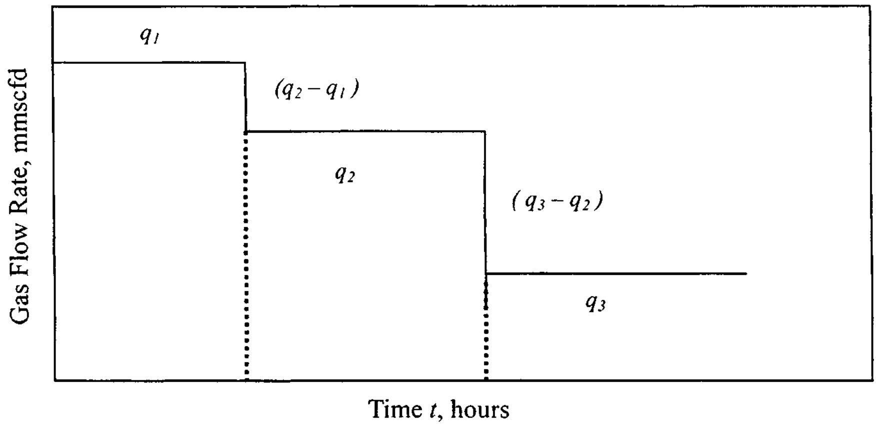

Figure 2–2. Variable-rate production of a gas well.

Example 2–10 *Calculating Flowing Sandface Pressure Accounting for Rate Change Effects*

A well situated in an infinite-acting reservoir was produced at constant rate of 5 mmscfd for 55 hours, at which time the flow rate was changed to 15 mmscfd. The stabilized shut-in pressure, $\bar{p}_R$, prior to the test was 2100 psia. General data pertinent to the test are as follows: $k = 25$ mD, $T = 600°$R, $r_w = 0.35$ ft, $h = 35$ ft, $\phi = 0.16$, $c_i = 0.00053$ psi^{-1}, $\mu_i = 0.0147$ cP, $\psi_i = 320$ mmpsia^2/cP, $t_1 = 45$ hours, $t_2 = 70$ hours, $q_1 = 5$ mmscfd, $q_2 = 15$ mmscfd.

Using the principle of superposition, calculate the flowing sandface pressure, p_{wf}, after 40 hours of production at the increased flow rate.

Solution Total production time $= t_1 + t_2 = 45 + 70 = 115$ hours.

From Eq. 2–54:

$$t_D = \frac{0.0002637kt}{\phi\mu_i c_i r_w^2}$$

$$t_{D1} = \frac{0.0002637(25)(115)}{(0.16)(0.0147)(0.00053)(0.35)^2} = 4{,}964{,}765$$

$$t_{D1} = \frac{0.0002637(25)(115-45)}{(0.16)(0.0147)(0.00053)(0.35)^2} = 3{,}022{,}031$$

From Eq. 2–56:

$$q_D = \frac{1427 \times 10^3 T q_{sc}}{kh\psi_i}$$

$$q_{D1} = \frac{1427 \times 10^3 (600)(5)}{(25)(35)(320 \times 10^6)} = 0.01518$$

$$q_{D2} = \frac{1427 \times 10^6 (600)(10)}{(25)(35)(320 \times 10^6)} = 0.03036$$

Since the reservoir is infinite-acting, Eq. 2–65 applies, so that

$$\Delta p_D = 0.5\,[\ln t_D + 0.809]$$

$$\Delta p_{D1} = 0.5\,[\ln(4{,}964{,}765) + 0.809] = 8.1134$$

$$\Delta p_{D2} = 0.5\,[\ln(3{,}022{,}031) + 0.809] = 7.86522$$

$$(\Delta\psi)_{total} = \psi_i \Delta p_{D1} q_{D1} + \psi_i \Delta p_{D2} q_{D2}$$

$$= 320 \times 10^6 (8.1134)(0.01518) + 320 \times 10^6 (7.86522)(0.03036)$$

$$= 115.82 \text{ mmpsia}^2/\text{cP}$$

$$\psi_{wf} = \psi_i - (\Delta\psi)_{total}$$

$$= 320 \times 10^6 - 115.82 \times 10^6 = 204.18 \text{ mmpsia}^2/\text{cP}$$

from Figure 2–1; $\therefore\ p_{wf} = 1604$ psia.

Estimating for Effects of More Than One Well

In some cases more than one well is producing from a common reservoir. As an example, consider three wells A, B, and C that start to produce at the same time, from an infinite-acting reservoir, the pressure at a point C in the producing wells (see Figure 2–3). Thus the pressure at a point C in the reservoir is obtained by superposing (adding) the solution at point C due to well A to that at point C due to well B. Each of these solutions is independent of the other and, to obtain it, the pressure behavior at any point r in the reservoir is required: that is, the general solution of the partial differential equation and

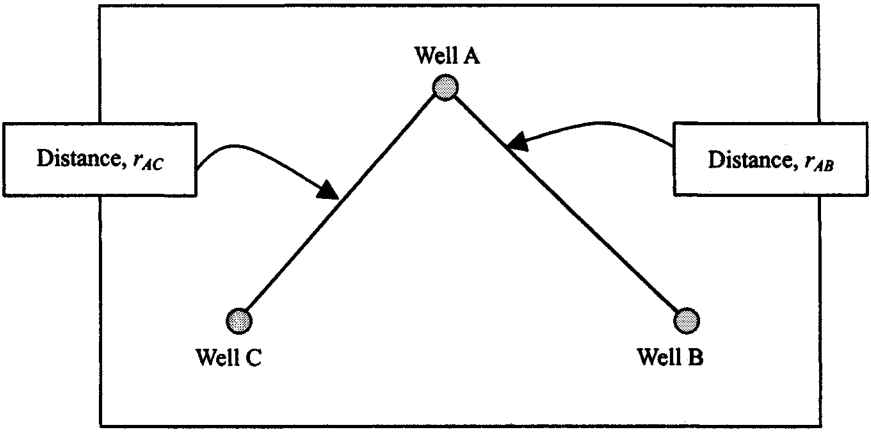

Figure 2–3. Three wells in an infinite reservoir.

not just the solution at the well. Thus

$$\Delta p|_{PointC} = p_i q_{AD}\left[-0.5E_i\left(\frac{r_{AD}^2}{4t_D}\right)\right] + p_i q_{BD}\left[-0.5E_i\left(\frac{r_{BD}^2}{4t_D}\right)\right] \quad (2\text{–}91)$$

where

r_A = distance from C to well A.
$r_{AD} = r_A/r_w$
r_B = distance from C to well B
$r_{BD} = r_B/r_w$

This is the basis of "interference" type tests used to determine reservoir characteristics. In such a test, point C is really an observation well and the interference of other producing wells is measured at C. Figure 2–3 illustrates this concept.

Example 2–11 *Accounting for the Effects of More Than One Well*

Consider the three wells in Figure 2–4. Well B is put on production at rate of 3.0 mmscfd after well A has produced for 2 months at a rate of 5.2 mmscfd. After well A has produced 3 months, what is the pressure at well C, where a well C is to be drilled? Rock and fluid properties are as follows:

p_i = 3700 psia, ψ_i = 772.56 mmpsia^2/cP, c_i = 0.00023 psi^{-1}, μ_i = 0.0235 cP, ϕ = 0.1007 fraction, r_w = 0.4271 ft, T = 710°R, h = 41 ft, k = 8.5 mD.

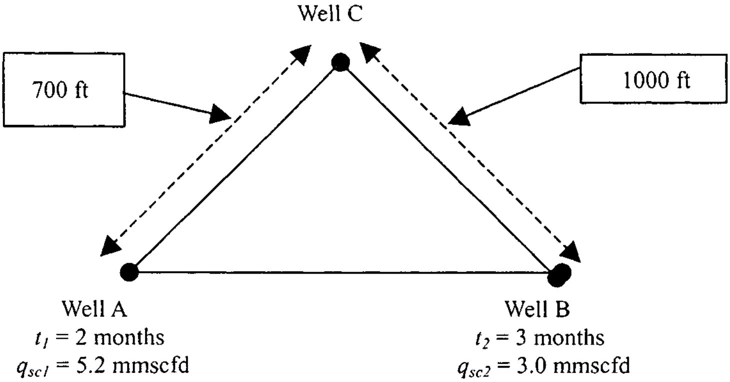

Figure 2–4. Illustration of three wells in infinite system.

Solution From Eq. 2–51:

$$t_D = \frac{0.0002637kt}{\phi \mu_i c_i r_w^2}$$

$$t_{DA} = \frac{0.0002637 \times 8.5 \times 2 \times 30.5 \times 24}{0.1007(0.0235)(0.00023)(0.4271)^2} = 33{,}051{,}092.58$$

$$t_{DB} = \frac{0.0002637 \times 8.5 \times 3 \times 30.5 \times 24}{0.1007(0.0235)(0.00023)(0.4271)^2} = 49{,}576{,}638.87$$

From Eq. 2–56:

$$q_D = \frac{1427 \times 10^3 T q_{sc}}{kh\psi_i}$$

$$q_{DA} = \frac{1427 \times 10^3(710)(5.2)}{(8.5)(41)(772.56 \times 10^6)} = 0.019568$$

$$q_{DB} = \frac{1427 \times 10^3(710)(3.0)}{(8.5)(41)(772.56 \times 10^6)} = 0.011289$$

r_A = distance from well C to well A = 700 ft

$$r_{AD} = \frac{r_A}{r_w} = \frac{700}{0.4271} = 1638.96$$

r_B = distance from well C to well B = 1000 ft

$$r_{BD} = \frac{1000}{0.4271} = 2{,}341.37$$

Using Eq. 2–91:

$$\Delta p|_{well\,C} = p_i(q_{AD})\left[0.5E_i\left(\frac{r_{DA}^2}{4t_{DA}}\right)\right] + p_i(q_{DB})\left[0.5E_i\left(\frac{r_{DB}^2}{4t_{DB}}\right)\right]$$

$$= 3700(0.019568)\left[0.5E_i\left[\frac{(1{,}638.96)^2}{4(33{,}051{,}092.58)}\right]\right]$$

$$+\,3{,}700(0.011289)\left[0.5E_i\left(\frac{(2{,}341.37)^2}{4(49{,}576{,}638.87}\right)\right]$$

$$= 72.4016[0.5E_i(0.020318)] + 41.7693[0.5E_i(0.027644)]$$

From Table 2–5, $E_i(0.020318) = 3.355$ and $E_i(0.027644) = 3.062$

$$\therefore\ \Delta p|_{well\,C} = 72.4016[0.5(3.355)] + 41.7693[0.5(3.062)]$$

$$= 185.40 \text{ psia}$$

Pressure at well C = 3700 − 185.50 = 3515 psia.

Determining Pressure Change Effects

Superposition is also used in applying the constant pressure-rate case. In cases where two pressure changes have occurred, the constant-pressure solution will be applied to each individual pressure change. This means that in this particular case we have to use Eq. 2–92 two times. The following generalized form of Eq. 2–92 will be used in applying the principle of superposition to pressure changes in the constant-pressure case:

$$G_p = \frac{0.111\phi h r_w^2 c}{T} \sum_{j=1}^{j=m} \left(\frac{\Delta p_j^2}{\bar{z}}\right) Q_{pD} \tag{2–92}$$

$$\Delta p_j^2 = p_{old}^2 - p_{new}^2$$

and

$\bar{z}$ is calculated at $\left(\dfrac{p_{old} + p_{new}}{2}\right)$

For illustration, let us assume that a well has experienced the pressure history shown in Figure 2–5.

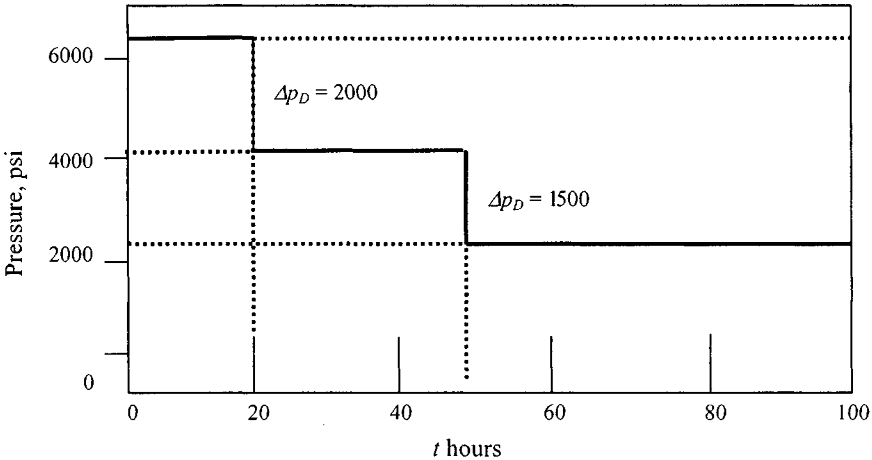

Figure 2–5. Variable pressure history of a gas well.

Simulating Boundary Effects

The principle of superposition concept can be applied to infinite-acting solutions to reservoirs that are limited in one or more direction, i.e., pressure behavior in bounded fault. Figure 2–6 shows a well, A, located at a distance $L/2$ from a no-flow barrier and producing at a constant rate. This system can be treated by replacing the barrier by an imaging well A′ identical to the real well but situated at a distance L from it. Thus the pressure history of the well will be that of an infinite-acting well at A, plus the effect at point A′ of an infinite-acting well at A′, that is,

$$\Delta p_D|_{well} = p_i q_D \left[-0.5 E_i \left(-\frac{\phi \mu c r_w^2}{0.00105 k t} \right) \right]$$

← caused by A →

$$+\left[-0.5 E_i \left(-\frac{\phi \mu c L^2}{0.00105 k t} \right) \right] \qquad (2\text{–}93)$$

→ effect of A′ at A →

Equation 2–67 may approximate the first E_i term because the agreement is usually less than 0.01 for all practical times. However the second E_i term is not true because of the presence of L^2 (usually a large number) in the argument.

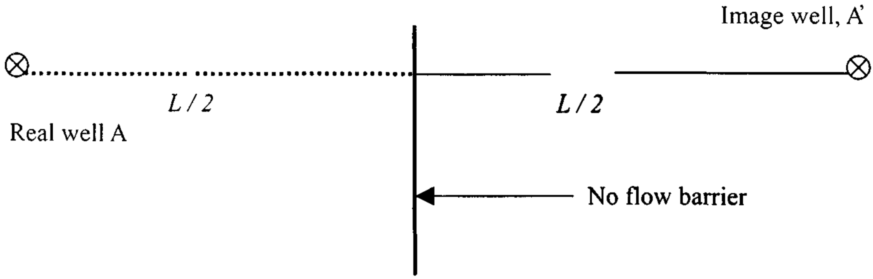

Figure 2–6. Well near no-flow boundary illustrating use of imaging.

Therefore:

$$\Delta p_D = p_i q_D\left[0.5(\ln t_D + 0.809) - 0.5E_i\left(-\frac{\phi\mu c L^2}{0.00105kt}\right)\right] \tag{2–94}$$

The following example will illustrate the principle of superposition applied to the simulation of no-flow barriers within a reservoir.

Example 2–12 *Simulating No-Flow Boundaries within a Reservoir*

In an infinite-acting gas reservoir, a well is situated 150 ft from a barrier and produced at a constant rate of 5 mmscfd for 36 hours. The stabilized shut-in reservoir pressure, p_R, prior to the test was 2100 psia. Calculate the flowing bottom hole pressure. Other data are as follows:

$k = 25\,\text{mD}, T = 580°\text{R}, h = 41\,\text{ft}, r_w = 0.35\,\text{ft}, \phi = 0.16, \mu_i = 0.0157\,\text{cP}$, $c_i = 0.00059\ \text{psi}^{-1}$, $p_i = 2{,}100$ psia, $\psi_i = 320$ mmpsia²/cP.

Solution From Eq. 2–51:

$$t_D = \frac{0.0002637kt}{\phi\mu_i c_i r_w^2}$$

$$= \frac{0.0002637(25)(36)}{(0.16)(0.0157)(0.00059)(0.35)^2} = 1{,}307{,}209$$

From Eq. 2–56:

$$q_D = \frac{1417 \times 10^3 T q_{sc}}{kh\psi_i}$$

$$= \frac{1417 \times 10^3(580)(5)}{(25)(41)(320 \times 10^6)} = 0.01253$$

Equation 2–55 may be written in terms of pseudopressure as

$$\psi_{wf} = \psi_i - \psi_i \Delta p_D q_D$$

where

$$\Delta p_D = 0.5(\ln t_D + 0.809) - 0.5E_i\left(-\frac{\phi\mu_i c_i L^2}{0.00105kt}\right)$$

$$= 0.5(\ln 1{,}307{,}209 + 0.809)$$

$$-0.5\ E_i\left(-\frac{(0.16)(0.0157)(0.00059)(150)^2}{(0.00105)(25)(36)}\right)$$

$$= 7.446 - 0.5E_i(-0.353) = 7.447 - 0.5(2.75) = 6.07$$

Therefore

$$\psi_{wf} = 320 \times 10^6 - 320 \times 10^6(6.07)(0.01253)$$

$$= 320 \times 10^6 - 24.34 \times 10^6 = 295.66 \text{ mmpsia}^2\text{cP}$$

from $\therefore p_{wf} = 1865$ psia.

Use of Horner's Approximation

In 1951, Horner[11] introduced an approximation that could be used in many cases to avoid the use of the tedious superposition principle as applied to model production history of a variable-rate well instead of using the sequence of E_i functions, i.e., one E_i function for each rate change. With the help of this approximation, we are able to use one equation with one single producing rate and one single producing time.

Thus, mathematically,

$$t_p = \frac{24G_p}{q_{last}} \tag{2–95}$$

where

G_p = cumulative production, mmscf, and
q_{last} = constant-rate just before shut-in, mmscfd.

Accounting for Different Reservoir Geometry

Ramey[22] has presented models of pseudo-steady-state flow in more general reservoir shapes. For practical applications, the concept of the shape factor, C_A, which depends on the shape of the area and the well position, is quite useful. Defining a dimensionless time based on drainage area, A, as

$$t_D = \frac{0.0002637kt}{\phi\mu c A} \tag{2–51}$$

$$t_{DA} = t_D \frac{r_w^2}{A} \tag{2–96}$$

$$p_i - p_{wf} = p_i q_D \frac{1}{2}\left[\ln\left(\frac{2.2458At_{DA}}{r_w^2}\right) + 4\pi t_{DA} - F\right] \tag{2–97}$$

where dimensionless pressure Δp_D is

$$\Delta p_D = \frac{1}{2}\left[\ln\left(\frac{2.2458At_{DA}}{r_w^2}\right) + 4\pi t_{DA} - F\right] \tag{2–98}$$

and F is the Matthews, Brons, and Hazebroek[23] dimensionless pressure function that has been evaluated for various reservoir shapes and well locations. For small values of t_{DA}, that is, the transient region of flow, the well is infinite-acting and

$$F = 4\pi t_{DA} \tag{2–99}$$

and

$$\Delta p_D = 0.5\ln\left(\frac{2.2458At_{DA}}{r_w^2}\right) \tag{2–100}$$

For large values of t_{DA}, when all the boundaries have been felt, that is, at pseudo-steady state,

$$F = \ln(C_A t_{DA}) \tag{2–101}$$

and

$$\Delta p_D = 0.5\ln\left(\frac{2.2458A}{r_w^2 C_A}\right) + 2\pi t_{DA} \tag{2–102}$$

The late transient between transient and pseudo-steady-state varies with each situation. During this period, the pressure drop function may be obtained from

$$\Delta p_D = 0.5\left[\ln\left(\frac{2.2458At_{DA}}{r_w^2}\right) + 4\pi t_{DA} - F\right] \tag{2–103}$$

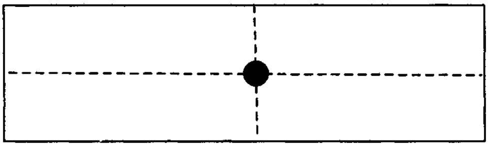

Figure 2–7. Gas well is situated in the center of a rectangle.

Dimensionless pressure function i is obtained from Table B–1[23] or graphically[23] from Figures B–1 through B–7. Shape factors C_A for various drainage shapes and well locations can be found from Table B–1.[13]

Example 2–13 *Accounting for Different Reservoir Geometry*

A gas well is situated in the center of a rectangle, as shown in Figure 2–7, having closed no-flow boundaries and an area A of 8×10^6 sq ft, was produced at a constant rate of 5 mmscfd. The stabilized shut-in reservoir pressure, $\bar{p}_R$, prior to the test was 2100 psia. Use gas composition given in Example 2–1. Other data are as follows: $k = 25$ mD, $T = 580°$R, $h = 41$ ft, $r_w = 0.35$ ft, $\phi = 0.16$, $\mu_i = 0.0157$ cP, $c_i = 0.0059$ psi^{-1}, $\bar{p}_R = 2100$ psia, $\bar{\psi}_R = 320$ mmpsia^2/cP.

Calculate flowing pressure, p_{wf}, after 40 and 2000 hours of production.

Solution Since the gas is the same as that of Example 2–1, the $\psi - p$ curve already constructed (Figure 2–1) is applicable to the problem.

$t = 40$ hours:
From Eq. 2–51:

$$t_{DA} = \frac{0.0002637kt}{\phi\mu_i c_i A}$$

$$= \frac{0.0002637(25)(40)}{(0.16)(0.0157)(0.00059)\ (8 \times 10^6)} = 0.02224$$

From Eq. 2–56:

$$q_D = \frac{1417 \times 10^3 T q_{sc}}{kh\psi_i}$$

$$= \frac{1417 \times 10^3(580)(5)}{(25)((41)(320 \times 10^6)} = 0.01253$$

Calculate F from Table B–1:[23] $F = 0.2806$.

From Eq. 2–103:

$$\Delta p_D = 0.5\left[\ln\frac{2.2458At_{DA}}{r_w^2} + 4\pi t_{DA} - F\right]$$

$$= 0.5\left[\ln\frac{2.2458(8\times10^6)(0.02224)}{(0.35)^2} + 4(22/7)(0.01253) - 0.2806\right]$$

$$= 7.29$$

Also,

$$\Delta p_D = \frac{\psi_i - \psi_{mf}}{\psi_i q_D}$$

After rearranging:

$$\psi_{wf} = \psi_i - \psi_i \Delta p_D q_D$$

$$= 320\times10^6 - 320\times10^6(7.29)(0.01253) = 290.77\ \text{mmpsia}^2/\text{cP}$$

From the $\psi - p$ curve (Figure 2–1), $P_{wf} = 1845$ psia.

$t = 2000$ hours:

From Eq. 2–51:

$$t_{DA} = \frac{0.0002637kt}{\phi\mu_i c_i A}$$

$$= \frac{0.0002637(25)(2000)}{(0.16)(0.0157)(0.00059)(8\times10^6)} = 1.1120$$

From Eq. 2–56:

$$q_D = \frac{1417\times10^3 T q_{sc}}{kh\psi_i}$$

$$= \frac{1417\times10^3(580)(5)}{(25)(41)(320\times10^6)} = 0.01253$$

Calculate F from Table B–1:[23] $F = 3.2000$

From Eq. 2–103:

$$\Delta p_D = 0.5\left[\ln\frac{2.2458 A t_{DA}}{r_w^2} + 4\pi t_{DA} - F\right]$$

$$= 0.5\left[\ln\frac{2.2458(8\times 10^6)(1.1120)}{(0.35)^2} + 4(22/7)(0.01253) - 3.200\right]$$

$$= 14.84$$

Also,

$$\Delta p_D = \frac{\psi_i - \psi_{mf}}{\psi_i q_D}$$

After rearranging the preceding equation:

$$\psi_{wf} = \psi_i - \psi_i \Delta p_D q_D$$

$$= 320\times 10^6 - 320\times 10^6(14.84)(0.01253) = 260.50 \text{ mmpsia}^2/\text{cP}$$

From the $\psi - p$ curve (Figure 2–1), $P_{wf} = 1746$ psia.

Alternatively, from Table B–2,[13] t_{DA} required for stabilization equals 0.15 and $C_A = 21.8369$. Because t_{DA} at 2000 hours $= 1.1120 > 0.15$, Eq. 2–102 can be used to evaluate Δp_D.

From Eq. 2–102:

$$\Delta p_D = 0.5\ln\left(\frac{2.2458A}{r_w^2 C_A}\right) + 2\pi t_{DA}$$

$$= 0.5\ln\left(\frac{2.2458(8\times 10^6)}{(0.35)^2(21.8369)}\right) + 2(22/7)(1.1120) = 14.84$$

Therefore,

$$\psi_{wf} = \psi_i - \psi_i \Delta p_D q_D$$

$$= 320\times 10^6 - 320\times 10^6(14.84)(0.01253) = 260.50 \text{ mmpsia}^2/\text{cP}$$

From the $\psi - p$ curve (Figure 2–1), $P_{wf} = 1746$ psia.

2.12 Choice of Equation for Gas Flow Testing and Analysis

This section will discuss correlation of the gas flow solutions in terms of the pressure; pressure squared, and real-gas pseudopressure approaches. An analysis of these approaches has been conducted by Aziz, Mattar, Ko, and Brar.[7] They consider the analytical solution at the well for an infinite reservoir given by Eq. 2–104:

$$\Delta p_D = -0.5E_i\left(-\frac{1}{4t_D}\right) \tag{2–104}$$

Calculate the sandface pressure from this equation, using different approaches.

Pressure Case

For pressure >3000 psi the simpler form is in terms of pressure, p. The differential equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) = \frac{\phi\mu c}{0.0002637k}\frac{\partial p}{\partial t} \tag{2–105}$$

The diffusivity equation in dimensionless variables becomes

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left[r_D\frac{\partial}{\partial r_D}(\Delta p_D)\right] = \frac{\partial}{\partial t_D}(\Delta p_D) \tag{2–106}$$

The dimensionless time, t_d, in Eq. 2–106 is defined by

$$t_D = \frac{0.0002637kt}{\phi r_w^2}\left(\frac{1}{\mu c}\right) \tag{2–107}$$

The definition of Δp_D, however, is different for this approach. For the pressure case,

$$\Delta p_D = \frac{p_i - p}{\frac{70.85\times 10^4 Tq_{sc}}{kh}\left(\frac{\mu z}{p}\right)} \tag{2–108}$$

Both quantities $(\frac{1}{\mu c})$ and $(\frac{\mu c}{p})$ in Eqs. 2–107 and 2–108 are evaluated at $(p_i + p)/2$.

Pressure-Squared Case

For pressure <2000 psi a simple form in terms of p^2 is more generally applicable.

$$\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r}\frac{\partial p^2}{\partial r} = \frac{\phi \mu c}{0.0002637k}\frac{\partial p^2}{\partial t} \tag{2–109}$$

The diffusivity equation in dimensionless variables becomes

$$\frac{\partial^2 \Delta p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial \Delta p_D}{\partial r_D} = \frac{\partial}{\partial t_D}(\Delta p_D) \tag{2–110}$$

The definition of Δp_D, however, is different for this approach. For the pressure-squared case,

$$\Delta p_D = \frac{p_i^2 - p^2}{\frac{1{,}417\times 10^3 T q_{sc}}{kh}(\mu z)} \tag{2–111}$$

The quantities $(\frac{1}{\mu c})$ and (μz) in Eqs. 2–107 and 2–108 are evaluated at p_i.

Pseudopressure Case

For both low and high pressures the equation in terms of pseudopressure is best fitted to this role, is denoted by $\psi(p)$, and is defined by the integral[10]

$$\psi(p) = 2\int_{p_{base}}^{p} \frac{p}{\mu z}\, dp \tag{2–112}$$

The differential equation in terms of this approach is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = \frac{\phi \mu c_g}{0.0002637k}\frac{\partial \psi}{\partial t} \tag{2–113}$$

The diffusivity equation in dimensionless variables becomes

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial \Delta \psi_D}{\partial r_D}\right) = \frac{\partial \Delta \psi_D}{\partial t_D} \tag{2–114}$$

The definition of $\Delta \psi_D$ is

$$\Delta \psi_D = \frac{\psi_i - \psi}{\frac{1{,}417\times 10^3 T q_{sc}}{kh}} \tag{2–115}$$

The properties are evaluated at initial conditions.

2.13 Skin, IT Flow, and Wellbore Storage Effects

In the derivation of the equations it was assumed that the porous medium was homogeneous and isotropic and that flow was single-phase and obeyed Darcy's law. It was also supposed that opening and shut-in of the well was done at the sandface. In actual fact these idealizations are not realistic, and derivations from the ideal model are too frequent and important to be ignored. Ways of accounting for skin effects; IT flow, and wellbore storage will be treated in the following sections.

Accounting for Effects of Formation Damage

The permeability of the formation immediately around the well can be damaged by the well drilling process or improved by fracturing or acidizing the well on completion. To account for this altered permeability a skin factor was defined by Van Everdingen[8] as

$$(\Delta p_D)_{skin} = s, \text{ a constant} \qquad (2\text{–}116)$$

so that

$$\Delta p_D|_{well} \text{ (including skin)} = p_D + s \qquad (2\text{–}117)$$

This essentially states that there will be an added pressure difference due to the skin effect given by Eq. 2–117. A position value of s indicates a damaged well, and a negative value, an improved well. Hawkins[9] proposed that the skin be treated as a region of radius r_{skin} with permeability k_{skin}, with the skin factor given by

$$s = \left(\frac{k}{k_{skin}} - 1\right) \ln \frac{r_{skin}}{r_w} \qquad (2\text{–}118)$$

Equation 2–118 is valid for both positive skin ($k_{skin} < k$) and negative skin ($k_{skin} > k$) but there is no unique set of values of k_{skin} and r_{skin} for a particular s.

An alternative treatment of the skin effect is that of an "effective wellbore radius" (Matthews and Russell, 1967, p. 21),[15] defined as that radius which makes the pressure drop in an ideal reservoir equal to that in an actual reservoir with skin. Thus:

$$r_w \text{(effective)} = r_w e^{-s} \qquad (2\text{–}119)$$

For positive skin, r_w (effective) $< r_w$, that is, the fluid must travel through additional formation to cause the observed pressure drop, Δp. For negative skin, r_w (effective) $> r_w$. This is a useful concept in hydraulically fractured wells.

Accounting for Effects of Turbulence

For gas flow, however, inertial and/or turbulent (IT) flow effects, not accounted for by Darcy's law, are frequently of significance and should not be ignored. IT flow is most pronounced near the well and results in an additional pressure drop similar to the skin effect, except that it is not a constant but varies directly with flow rate.[24] Smith[25] confirmed with actual test results and with numerical solutions that IT flow could be treated as an additional, rate-dependent skin effect.

$$(\Delta p_D)_{IT} = Dq_{sc} \tag{2–120}$$

Where D = IT flow factor for the system, the pressure at the well is given by

$$\Delta p_D|_{well} = p_D + s + Dq_{sc} \tag{2–121}$$

or

$$s' = (\Delta p_D)_{skin} + (\Delta p_D)_{IT} = s + Dq_{sc} \tag{2–122}$$

The following example will show how pressure drop is attributed to laminar flow, skin, and IT flow effects. It assumes negligible effects of viscosity on turbulence.

Example 2–14 *Calculating Pressure Drop due to Laminar Skin and IT Flow Effects*

In an infinite-acting gas reservoir, a well was produced at a constant rate, q_{sc1}, of 8 mmscfd for a period of 35 hours. The flowing bottom hole pressure, p_{wf1}, at that time was 1550 psia. The same well was produced at a constant rate, q_{sc2}, of 11 mmscfd for a time of 25 hours. The flowing bottom hole pressure, p_{wf2}, at that time was 1300 psia. The stabilized shut-in pressure, $\bar{p}_R$, prior to each of the two flowing periods, was 2100 psia. Other data pertinent to the test are given below:

$k = 25$ mD, $r_w = 0.35$ ft, $h = 35$ ft, $T = 600°$R,
$\phi = 0.16$, $\mu_i = 0.0147$ cP, $c_i = .00053$ psi^{-1}, $\psi_i = 320.00$ mmpsia^2/cP
$t_1 = 35$ hours, $q_{sc1} = 8$ mmscfd, $p_{wf1} = 1550$ psia
$t_2 = 25$ hours, $q_{sc2} = 11$ mmscfd, $p_{wf2} = 1300$ psia

Calculate the skin and IT flow effects, s and D, respectively. Also calculate, for the second flow rate, using the same gas composition given in Example 2–2:

(a) the pressure drop due to the laminar flow effect
(b) the pressure drop due to skin effects

(c) the pressure drop due to IT flow effects
(d) total pressure drop

Solution From Eq. 2–54:

$$t_D = \frac{0.0002637kt}{\phi\mu_i c_i r_w^2}$$

Therefore

$$t_{D1} = \frac{0.0002637(25)(35)}{(0.16)(0.0147)(0.00053)(0.35)^2} = 1{,}511{,}015$$

and

$$t_{D2} = \frac{0.0002637(25)(25)}{(0.16)(0.0147)(0.00053)(.35)^2} = 1{,}077{,}296$$

From Eq. 2–56:

$$q_d = \frac{1417 \times 10^3 T q_{sc}}{kh\psi_i}$$

Therefore

$$q_{D1} = \frac{1417 \times 10^3(600)(8)}{(25)(35)(320 \times 10^6)} = 0.02429$$

$$q_{D2} = \frac{1417 \times 10^3(600)(11)}{(25)(35)(320 \times 10^6)} = 0.03340$$

Since the reservoir is infinite-acting, Eq. 2–65 applies, so that

$$p_t = p_D = 0.5\,[\ln t_D + 0.809]$$

Therefore,

$$p_{t1} = p_{D1} = 0.5\,[\ln(1{,}511{,}015) + 0.809] = 7.519$$

$$p_{t2} = p_{D2} = 0.5\,[\ln(1{,}079{,}296) + 0.809] = 7.351$$

From Eq. 2–55:

$$\Delta p_D = \frac{\psi_i - \psi_{wf}}{\psi_i q_D}$$

From the $\psi - p$ curve, $P_{wf1} = 1550$ psia $\leftrightarrow$ $\psi_{wf1} = 207 \times 10^6$ psia^2/cP

$p_{wf2} = 1300$ psia $\leftrightarrow$ $\psi_{wf2} = 145 \times 10^6$ psia^2/cP

Therefore,

$$\Delta p_{D1} = \frac{320 \times 10^6 - 207 \times 10^6}{320 \times 10^6(0.02429)} = 14.54$$

$$\Delta p_{D2} = \frac{320 \times 10^6 - 145 \times 10^6}{320 \times 10^6(0.03340)} = 16.37$$

From Eq. 2–121:

$$\Delta p_D = p_D \quad \text{or} \quad p_t = s + Dq_{sc}$$

Substituting the calculated values of Δp_D, p_D, or p_t and q_{sc} in the above equation gives

$$14.54 = 7.519 + s + 8D$$
$$16.37 = 7.351 + s + 11D$$

Solving these equations simultaneously gives

$$D = \frac{(16.37 - 14.54) - (7.351 - 7.519)}{(11 - 8)} = 0.666$$

$$s = 14.54 - 7.519(8)(0.666) = 1.69$$

For the second production rate, q_{sc2} is as follows:

(a) Pressure drop due to laminar flow effects is given by

$$p_{t2} = \frac{\psi_i - \psi}{\psi_i q_{D2}}$$

Therefore

$$\begin{aligned}\psi &= \psi_i - \psi_i p_{t2} q_{D2}\\ &= 320 \times 10^6 - 320 \times 10^6(7.351)(0.3340)\\ &= 241.43 \text{ mmpsia}^2/\text{cP}\\ &= 1720 \text{ psia (from } \psi - p \text{ curve)}\end{aligned}$$

and $\Delta p_{laminar\,flow} = p_i - p = 2100 - 1720 = 380$ psia.

(b) Pressure drop due to skin effects is given by

$$s = \frac{\psi_i - \psi}{\psi_i q_{D2}}$$

$$\begin{aligned}\therefore \psi &= \psi_i - \psi_i s q_{D2} = 320 \times 10^6 - 320 \times 10^6 \times 1.69 \times 0.03340\\ &= 302 \text{ mmpsia/cP} \leftrightarrow p = 1910 \text{ psia}\end{aligned}$$

$$\Delta p_{skin} = p_i - p = 2100 - 1910 = 190 \text{ psia}$$

(c) Pressure drop due to IT flow effects is given by

$$Dq_{sc2} = \frac{\psi_i - \psi}{\psi_i q_{D2}}$$

$$\therefore \psi = \psi_i - \psi_i Dq_{sc2}\, q_{D2}$$

$$= 320 \times 10^6 - 320 \times 10^6 \times 0.666 \times 11 \times 0.03440$$

$$= 239.35\ \text{mmpsia}^2/\text{cP} \leftrightarrow p = 1690\ \text{psia}$$

$$\therefore \Delta p_{IT\,flow} = p_i - p = 2100 - 1690 = 410\ \text{psia}$$

(d) Total pressure drop $= \Delta p_{laminar\,flow} + \Delta p_{skin} + \Delta p_{IT\,flow} = 380 + 190 + 410 = 980$ psia.

Wellbore Storage Effects

Wellbore storage effects are associated with a continuously varying flow rate in the formation. One solution[8] is to assume that the rate of unloading of, or storage in, the wellbore per unit pressure difference is constant. This constant is known as the wellbore storage constant, C_S, and is given by

$$C_S = V_{WS} \times C_{WS} \quad (2\text{–}123)$$

where

V_{WS} = Volume of the wellbore tubing (and annulus, if there is no packer) ft^3
$V_{WS} = \pi r_w^2 L$, ft^3
L = well depth, ft
C_{WS} = compressibility of the wellbore fluid evaluated at the mean wellbore pressure and temperature, psi^{-1}

The wellbore storage constant may be expressed in a dimensionless term as

$$C_{SD} = \frac{0.159 C_S}{\phi h C r_w^2} \quad (2\text{–}124)$$

The rate of flow of fluid from the formation may then be obtained from

$$q = q_{sc}\left[1.0 - C_{SD}\frac{\partial}{\partial t_D}(\Delta p_D)\Big|_{wellbore}\right] \quad (2\text{–}125)$$

The time for which wellbore storage effects are significant is given by

$$t_{WSS} = 60C_{SD} \tag{2–126}$$

The time at which wellbore storage effects become negligible is given by

$$t_{WS} = \frac{36,177\mu C_S}{kh}, \text{ hours} \tag{2–127}$$

Example 2–15 *Finding the End of Wellbore Storage Effects*

The following characteristics are given: well depth = 5500 ft, $r_w = 0.39$ ft., $C_{WS} = 0.000595$ psi^{-1}, $h = 5$ ft, $k = 25$ mD, $\mu = 0.0175$ cP. Assume there is no bottomhole packer. Calculate the time required for wellbore storage effects to become negligible.

Solution From Eq. 2–123:

$$V_{WS} = \pi r_w^2 L = 22/7(0.39)^2(5500) = 2629 \text{ ft}^3$$

From Eq. 2–123: $C_S = C_{WS}V_{WS} = 0.000595 \times 1629 = 1.565$ ft^3/psi^{-1}
From Eq. 2–127:

$$t_{WS} = \frac{36,177(0.0175)(1.565)}{25(45)} = 0.88 \text{ hours}$$

After a time of 0.88 hours, wellbore storage effects become negligible and the analytical solutions for transient flow apply.

Radius of Investigation

The radius of investigation has several uses in pressure transient test analysis and design:

1. Provides a guide for well test design
2. Estimates the time required to test the desired depth in the formation
3. Provides a means of estimating the length of time required to achieve "stabilized" flow (i.e., the time required for a pressure transient to reach the boundaries of a tested reservoir)

An infinite reservoir may be considered to be a limited reservoir with a closed outer boundary at r, provided r is allowed to increase with t_D. This changing value of r is defined as the radius of investigation, r_{inv}, that is,

$$t_D = 0.25r_{eD}^2$$

or (2–128)

$$r_{eD}^2 = 4t_D$$

$$\left(\frac{r_{inv}}{r_w}\right)^2 = 4t_D \tag{2–128a}$$

$$r_{inv} = \left(\frac{0.00105kt}{\phi\mu c_t}\right)^{0.5}, \text{ ft}, \quad \text{for } r_{inv} \leq r_e \tag{2–128b}$$

If the value of r_{inv} obtained from Eq. 2–128a is greater than r_e, then the radius of investigation is taken to be r_e.

Time of Stabilization

If a well is centered in a cylindrical drainage area of radius r_e, then setting $r_{inv} = r_e$, the time required for stabilization, t_S, is defined as follows:

$$t_D = 0.25r_{eD}^2$$
$$= \frac{1}{4}r_{eD}^2$$

or

$$t_S = \frac{1}{4} \cdot \frac{\phi\mu C r_e^2}{0.0002637k}$$
$$= \frac{948\phi\mu C r_e^2}{k}, \text{ hours} \tag{2–129}$$

Example 2–16 *Estimating Radius of Investigation*

We want to conduct a flow test on an exploratory gas well for a long enough time to ensure that the well will drain a radius of more than 1500 ft. Well and fluid data are as follows: $\phi = 0.18$ fraction, $k = 9.0$ mD, $r_i = 1500$ ft, $\mu_i = 0.0156$ cP, $C_{ti} = 2.2 \times 10^{-4}$ psi^{-1}. What length of flow test appears advisable? What flow rate do you suggest?

Solution From Eq. 2–128a, the time required is

$$r_{inv} = \left(\frac{0.00105kt}{\phi\mu c_t}\right)^{0.5}, \text{ ft}, \quad \text{for } r_{inv} \leq r_e$$

In principle, any flow rate would sufficient required to achieve a particular radius of investigation is dependent of flow rate.

2.14 Numerical Solutions of Partial Differential Equations

Numerical methods must be used for cases where the partial differential equation and its boundary conditions cannot be linearized, where the reservoir shape is irregular, or when the reservoir is heterogeneous. In some complex situations, analytical solutions may be so difficult to apply that numerical methods are preferred. In this section a brief discussion of the numerical approach is presented including difference equations.

Three-Dimensional Models

Gas flow equations are different from those for liquid flow in that the equations of state that are used are quite different in functional form from those for liquids. The ideal gas law gives the equation of state for an ideal gas:

$$pV = \frac{m}{M}RT \quad \text{and} \quad \frac{m}{V} = \frac{M}{RT}P = \rho$$

where ρ is the density.

In the case of flow of a nonideal gas, the gas deviation factor z_g is introduced into the equation of state to give

$$\rho = \frac{M}{RP}\frac{\rho}{z_g} \tag{2–130}$$

If we assume laminar flow, neglect gravity effects, and assume constant rock properties, Eq. 2–130 becomes

$$\frac{\partial}{\partial x}\left(\frac{p}{\mu z_g}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{p}{\mu z_g}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{p}{\mu z_g \frac{\partial p}{\partial z}}\right) = \frac{\phi}{k}\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) \tag{2–131}$$

In field units Eq. 2–131 can be written as

$$\frac{\partial}{\partial x}\left(\frac{p}{\mu z_g}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{p}{\mu z_g}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{p}{\mu z_g \frac{\partial p}{\partial z}}\right) = \frac{\phi}{0.000264k}\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) \tag{2–132}$$

In terms of pseudopressure, $\psi(p)$, the equation can be written as follows:

$$\psi(p) = 2\int_{p_0}^{p} \frac{p}{\mu z_g}\,dp \tag{2–133}$$

where p_0 is a low base pressure. Now,

$$\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) = \frac{d\left(\frac{p}{z_g}\right)}{dp}\frac{\partial p}{\partial t} = \frac{c_g p}{z_g}\frac{\partial p}{\partial t},$$

because

$$c_g = \frac{1}{\rho}\frac{d\rho}{dp} = \frac{z_g}{p}\frac{d\left(\frac{p}{z_g}\right)}{dp}$$

Also note that

$$\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial p}\frac{\partial p}{\partial t} = \frac{2p}{\mu z_g}\frac{\partial p}{\partial t}$$

and

$$\frac{\partial \psi}{\partial x} = \frac{2p}{\mu z_g}\frac{\partial p}{\partial x}$$

Similar expressions apply for $\frac{\partial \psi}{\partial y}$ and $\frac{\partial \psi}{\partial z}$. Thus Eq. 2–131 becomes

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \psi}{\partial z}\right) = \frac{\phi \mu c_g}{0.000264k}\frac{\partial \psi}{\partial t} \quad (2\text{–}134)$$

Equations 2–131 and 2–134 are in three-dimensional form for single-phase flows and can be used for the study of completely heterogeneous reservoirs.

Radial One-Dimensional Model

For radial flow, the equivalent of Eq. 2–131 is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{p}{\mu z_g} r \frac{\partial p}{\partial r}\right) = \frac{\phi}{0.000264k}\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) \quad (2\text{–}135)$$

In terms of pseudopressure, $\Psi(p)$ is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = \frac{\phi}{0.000264k}\frac{\partial \psi}{\partial t} \quad (2\text{–}136)$$

For single-well problems, the use of the cylindrical coordinates provides greater accuracy than other coordinate systems. For the study of multiwell systems it is usually necessary to use rectangular coordinates with closely spaced grid points near the well.

Radial Two-Dimensional Coning Model

Where vertical flow is important, a two-dimensional radial model must be considered. The equation to be solved in this case is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{p}{\mu z_g} r \frac{\partial p}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{p}{\mu z_g}\frac{\partial p}{\partial z}\right) = \frac{\phi}{0.000264k}\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) \qquad (2\text{–}137)$$

In terms of pseudopressure, $\Psi(p)$ is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \psi}{\partial z}\right) = \frac{\phi \mu c_g}{0.000264k}\frac{\partial}{\partial t}\left(\frac{p}{z_g}\right) \qquad (2\text{–}138)$$

Models of this type can be used to study the effects of anisotropy on the transient pressure analysis of buildup and drawdown tests.

Areal Two-Dimensional Models

Multiwell problems can be solved through the solution of Eq. 2–139:

$$\frac{\partial}{\partial x}\left(\frac{p}{z_g}\frac{hk_x}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{p}{z_g}\frac{hk_y}{\mu}\frac{\partial p}{\partial y}\right) = \frac{\partial}{\partial t}\left(\frac{\phi hp}{z_g}\right) + q(x, y, t) \qquad (2\text{–}139)$$

The injection or production from different wells is accounted for by the q term. The reservoir shape may be completely arbitrary and there may be different types of boundary conditions such as no-flow or constant pressure. This model can also be used for interference test analysis.

Studies of this type for Darcy's flow have been reported in the literature, for example, by Carter.[12]

Multiphase (Gas-Condensate Flow) Model

In this section we outline a detailed derivation of an equation describing radial, and a multiphase mixture of gas, condensate, and water. We assume that a porous medium contains gas condensate and water, and that each phase has saturation-dependent effective permeability (k_g, k_o, and k_w); time-dependent saturation (S_g, S_o, and S_w); and pressure-dependent viscosity (μ_g, μ_o, and μ_w). When gravitational forces and capillary pressures are negligible, the differential equation describing this type of flow is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = \frac{\phi_t c_t}{0.000264\lambda_t}\frac{\partial \psi}{\partial t} \qquad (2\text{–}140)$$

where

$$c_t = S_g c_g + S_o c_o + S_w c_w + c_f \tag{2–141}$$

c_t is the effective total compressibility and is the sum of the fractional compressibilities. The fractional compressibility of a fluid is its compressibility multiplied by the fraction of the pore space that it occupies (that is, its saturation). The effective total mobility, $(k/\mu)_t$, is given in terms of the in situ permeability to each of the phases by

$$\lambda_t = \left(\frac{k}{\mu}\right)_t = \frac{k_g}{\mu_g} + \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} \tag{2–142}$$

The in situ permeability to each phase is the product of the permeability of the formation and the relative permeability to that phase. This latter factor depends on the prevailing saturation conditions. The effective total production rate is simply the sum of the individual fluid flow rates.

$$q_t = q_g + q_o + q_w \tag{2–143}$$

Substituting these effective total properties and the total porosity, ϕ_t, for their single-phase equivalents in Eq. 2–108 makes it possible to use the solutions of this equation for multiphase (gas-condensate flow) problems.

Compositional (Multicomponent) Model

In a reservoir system there are generally several species of chemical compounds. These components vary in composition in different phases, and each phase flows at a different rate. Therefore a mass balance must be made on every flowing fraction instead of each phase. Figure 2–8 shows compositional mass balance on element. Detailed discussion and numerical equations can be found in Refs. 16 and 17.

Compositional Mass Balance on Element

There are N species of chemical compounds flowing into the reservoir element in three phases. With the element there are changes due to either or all of the following:

1. Pressure change
2. Production
3. Injection

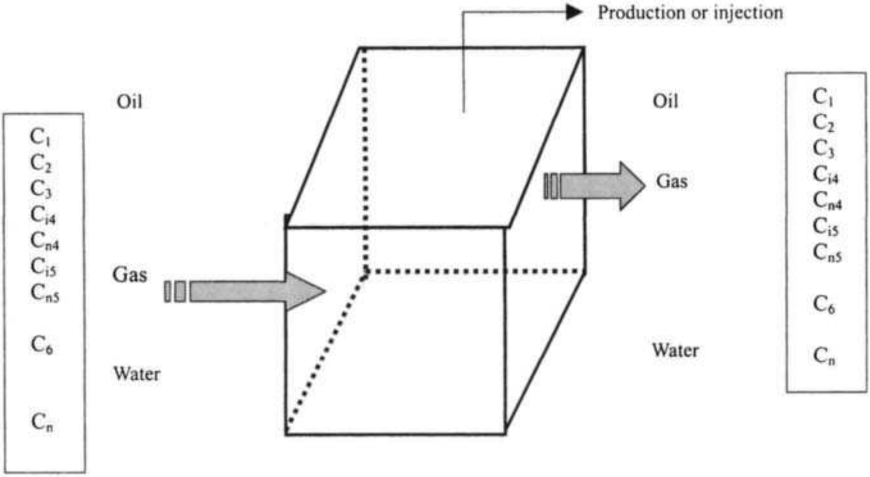

Figure 2–8. Composition mass balance on element (after Roebuck *et al.* © SPE, AIME 1969).[16]

Then we can write

$$\frac{\partial}{\partial x}\left(\frac{k_o\rho_o}{\mu_o}C_{Moj}\frac{\partial p_o}{\partial x}+\frac{k_g\rho_g}{\mu_g}C_{Mgj}\frac{\partial p_g}{\partial x}+\frac{k_w\rho_w}{\mu_w}C_{Mwj}\frac{\partial p_w}{\partial x}\right)$$
$$=\frac{\partial}{\partial t}\left(\phi S_o\rho_o C_{Moj}+\phi S_g\rho_g C_{Mgj}+\phi S_w\rho_w C_{Mwj}\right) \quad (2\text{–}144)$$

Consider the conservation of mass applied to one compound. Let

C_{Moj} = mass fraction of jth component in oil
C_{Mgj} = mass fraction of jth component in gas
C_{Mwj} = mass fraction of jth component in water

Equation 2–117 describes the flow of a single component, e.g., CH_4 in a linear system without any sources or sinks. Equation 2–117 also shows that each term on the left represents the mass flux of the jth component in each phase, which is simply derived by the following:

$$\text{Total mass flux} = \text{Density} \times \text{Volumetric rate}$$
$$=\rho_o q_o=\frac{k_o\rho_o}{\mu_o}\frac{\partial p_o}{\partial x} \quad (2\text{–}145)$$

$$\text{Component mass flux} = C_{Moj}\frac{k_o\rho_o}{\mu_o}\frac{\partial p_o}{\partial x} \quad (2\text{–}146)$$

Table 2–7

Unknown	Number
C_{mij}	$3N$
p_i	3
S_i	3
ρ_I	3
μ_I	3
k_I	3
	$3N + 15$

Note: $C_{mij} = 1, 2, 3$ $j = 1, \cdots, N$; total $= 3N$

Similarly, the accumulation term embodies the changes in each phase of the specific component:

$$\text{Mass rate of change} = \frac{\text{Mass at time } (t + \Delta t) - \text{Mass at time } t}{\Delta t}$$

A general equation for the N species under observation will be of the form

$$\frac{\partial}{\partial x}\left(\sum_{i=1}^{3} \frac{k_i \rho_i}{\mu_i} C_{Mij} \frac{\partial p_i}{\partial x}\right) = \frac{\partial}{\partial t}\left(\sum_{i=1}^{3} \phi S_i \rho_i C_{Mij}\right), \quad j = 1, \ldots, N \tag{2–147}$$

where

i = represents the phases and
j = the number of components.

We must determine the number of independent variables in the system. These data are listed in Table 2–7 for an N-component system.

In order to solve the system we must have $3N+15$ independent relationships. These relationships come from several sources:

1. Differential equations
2. Phase equilibrium
3. PVT data
4. Relative permeability data
5. Conservation principles
6. Capillary data

Relationship Development

Develop the necessary relationships as follows:

1. Write one partial differential equation for each component in the system, thus providing N relationships.
2. Since the pore space is always fluid-filled, the fluid phase saturations must always sum to unity:

$$S_o + S_g + S_w = 1 \tag{2–148}$$

This is one relationship.

3. The mass fraction of each component in each fluid phase must sum to unity, since mass conservation of each component is required. Thus:

$$\begin{aligned} \sum_{j=1}^{N} C_{Moj} &= 1 \\ \sum_{j=1}^{N} C_{Mgj} &= 1 \\ \sum_{j=1}^{N} C_{Mwj} &= 1 \end{aligned} \tag{2–149}$$

This provides three relationships.

4. The following can be obtained from the PVT data.

$$\begin{aligned} \mu_o &= f\,(p_o, C_{Moj}) \\ \mu_g &= f\,(p_g, C_{Mgj}) \\ \mu_w &= f\,(p_w, C_{Mwj}) \end{aligned} \tag{2–150}$$

$$\begin{aligned} \rho_o &= f\,(p_o, C_{Moj}) \\ \rho_g &= f\,(p_g, C_{Mgj}) \\ \rho_w &= f\,(p_w, C_{Mwj}) \end{aligned} \tag{2–151}$$

Note: These provide six more relationships. Viscosity and density are computed experimentally or from well-known correlations, which relate these parameters to compositions and pressures.

5. For mobility calculations, we need relative permeability data:

$$\begin{aligned} k_o &= f\,(S_g, S_o, S_w) \\ k_g &= f\,(S_g, S_o, S_w) \\ k_w &= f\,(S_g, S_o, S_w) \end{aligned} \tag{2–152}$$

This provides three more relationships.

6. For distribution of a component between its liquid and gaseous states, the equilibrium constant can be derived from thermodynamic principles. For example,

$$\frac{C_{Mgj}}{C_{Moj}} = K_{jgo} \qquad \frac{C_{Mgj}}{C_{Mwj}} = K_{jgw} \tag{2–153}$$

These equilibrium constants are a function of several variables:

$$K_{jgo} = f(p, T, C_{ij}) \qquad K_{jgw} = f(p, T, C_{ij}) \tag{2–154}$$

from which

$$\frac{K_{jo}}{K_{jw}} = \frac{K_{jgw}}{K_{jgo}} = K_{gow} \tag{2–155}$$

Equations 2–154 and 2–155 provide an independent relationship when written for each component in the system.

7. Capillary pressure provides the remaining relationship:

$$p_g - p_o = p_{cgo} = f(S_g, S_o, S_w) \qquad p_o - p_w = p_{cow} = f(S_g, S_o, S_w) \tag{2–156}$$

These relationships are summarized in Table 2–8.

Therefore, according to Table 2–8, we have $3N + 15$ independent unknown and $3N + 15$ independent relationships that can be used to solve the system.

Assumptions

Several simplifying assumptions are usually made to make the problem more amenable to solution:

Table 2–8

Relationship	Unknown	Equations
Differential equation	N	2–147
Phase equilibrium	$2N$	2–153
PVT data	6	2–150 and 2–151
Relative permeability	3	2–152
$\sum$ Mass fraction	3	2–149
$\sum$ Saturation	1	2–148
Capillary pressure	2	2–156

1. Capillary pressure between oil and gas is generally neglected.
2. Several components are grouped together, e.g., a system containing the following nine components will be grouped as shown below:

C_1 Component 1

C_2, C_3, C_{i4}, C_{in}, C_{i5}, C_{n5}, C_6 Component 2

C_{7+} Component 3

3. The mass fraction of components present in the water is so small that the C_{Mwj} terms are also zero. This means that oil and gas are the only phases in which mass transfer occurs. The equation for the water present is still needed.

Sources and Sinks

Sources and sinks can be included in Eq. 2–139 by the addition of a term representing the source or sink:

$$\frac{\partial}{\partial x}\left(\sum_{i=1}^{3}\frac{K_i\rho_i}{\mu_i}C_{Mij}\frac{\partial p_i}{\partial x}\right)-\sum_{i=1}^{3}q_i\alpha_{ij}\delta(x)=\frac{\partial}{\partial t}\left(\sum_{i=1}^{3}\phi_i S_i\rho_i C_{Mij}\right) \qquad (2\text{–}157)$$

where

q_i = Mass injection rate of phase in suitable units
α_{ij} = Mass fraction of *j*th component in *i*th phase
$\delta(x)$ = Delta function

The delta function $\delta(x)$ is defined as follows:

Production or injection in all at $x : \delta(x) = 1$
No production or injection in all at $x : \delta(x) = 0$

The locations of these wells are shown in Figure 2–9.

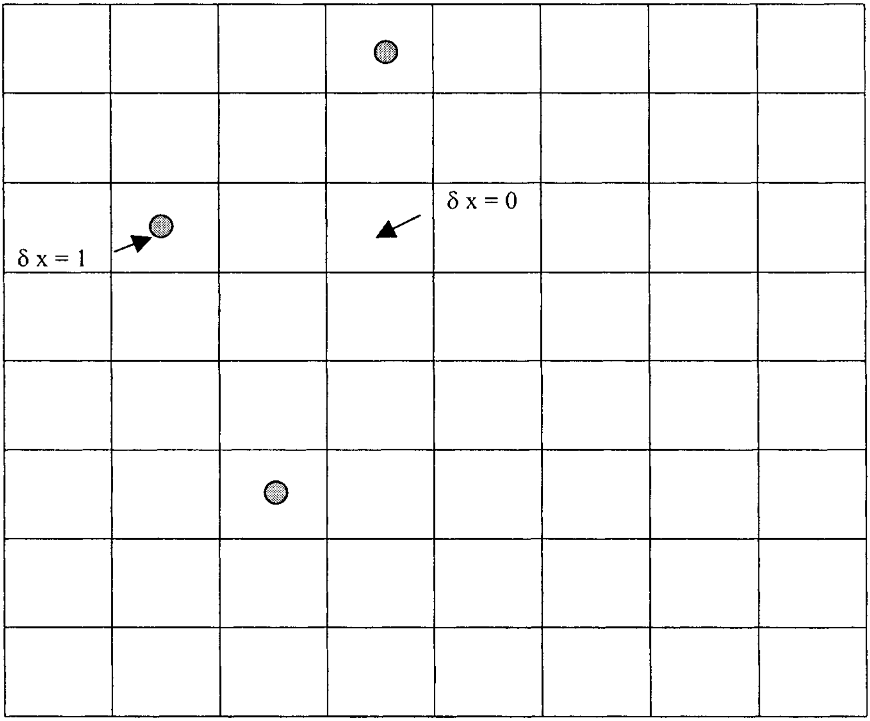

Figure 2–9. Well locations.

Procedure Outline for Solution of Flow Equations

The solution of the compositional model is an iterative one. The process indicated in Figure 2–10 is essentially the solution outline.

2.15 Summary

Chapter 2 provides the basic flow theory for gas well testing and analysis techniques. General equations are used for transient pressure behavior with dimensionless pressure solutions desired. Some important dimensionless pressure functions are presented in this chapter and references to others are provided. The dimensionless pressure approach provides a way to calculate pressure response and to devise techniques for analyzing transient tests in a variety of systems. Sections covering turbulence, wellbore storage effects, wellbore damage, and improvement are included, since the effects have a significant influence on transient well response.

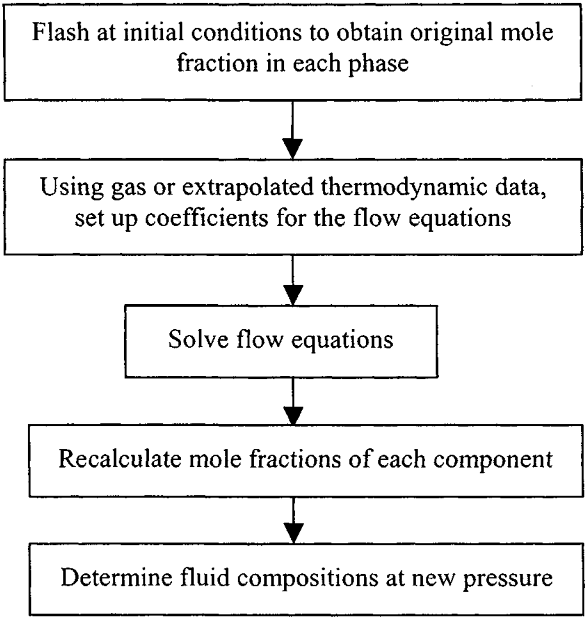

Figure 2–10. Solution Outline.

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