Practical methods for the analysis of differential strain effects

Doug Jenkins, Principal, Interactive Design Services, Sydney, Australia

Abstract: In the assessment of time and temperature related strain effects on the deflection of reinforced and prestressed concrete structures in a practical design context, creep and shrinkage effects are normally assumed to be uniform over the depth of the section, and differential temperature is usually treated as a purely short term effect. Both research and monitoring of actual structures however show that differential strain effects can have a significant effect on deflections, in some cases resulting in deflections exceeding specified contract limits. In this paper recent research on differential strain effects is reviewed, and a practical method for the assessment of differential creep, shrinkage and temperature effects is presented. Examples are presented of the application of the method to pre-tensioned bridge girders, showing that apparently anomalous behaviour can be explained by taking account of all differential strain effects.

Keywords: Differential strain, deflections, design methods.

1. Introduction

Both the Concrete Structures Code, AS 3600 (1), and the Bridge Design Code (Concrete), AS 5100.5 (2), have provisions for calculation of beam deflections by “refined calculation” which require the following factors to be considered:

- Cracking and tension stiffening.
- Shrinkage and creep properties of the concrete.
- Expected load history

These requirements are quite general, and are open to interpretation on the extent and detail of the analysis required. In practice, at least for standard structures such as concrete framed buildings and short to medium span bridges, the following simplifications are normally made (3),(4):

- Shrinkage strains and creep coefficients are treated as uniform across the section (or across each part of the section cast on the same day for composite sections).
- Differential temperature strains are treated as short term effects.

These simplifications have been found to introduce significant error in the calculated deflections, in some cases leading to deflections outside contract specified limits (5).

This paper presents a practical method of assessment of differential creep, shrinkage and temperature effects, and uses this approach to examine the behaviour of pre-tensioned Super-T bridge girders allowing for the effects of curing under hot, dry conditions and steam curing.

2. Current Procedures

Australian concrete design codes allow for “simplified” or “refined” methods of calculation of deflections. No guidance is provided on the application of refined methods, other than listing the factors to be considered, but in recent years many publications have focussed on this question, particularly in relation to flexural behaviour at just above the cracking moment, and the effects of tension stiffening and loss of tension stiffening (for example: 4, 6-8).

Other work has focussed on the effects of creep on long term deflections, particularly for long span segmentally erected prestressed concrete bridges, finding that major revisions were required to concrete creep models and to methods of analysis (9).

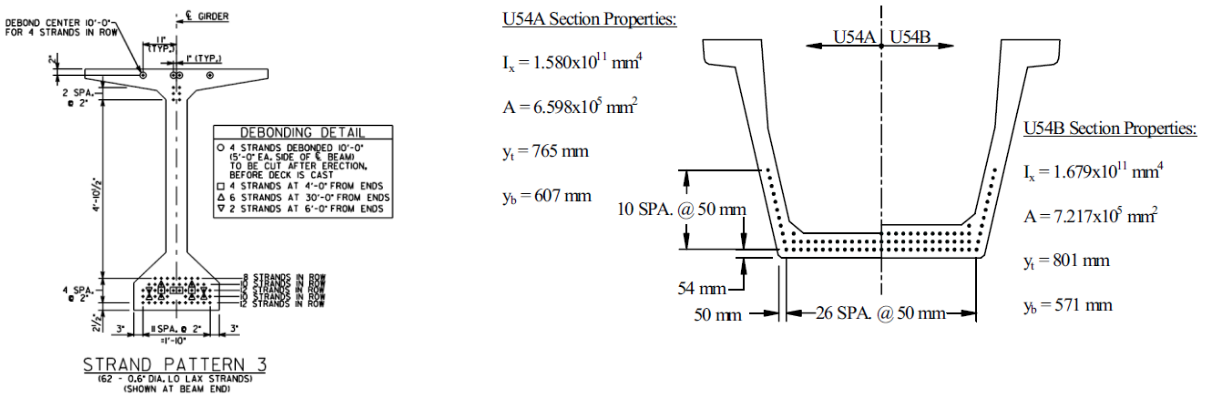
For smaller structures the effect of differential time and temperature related strains has received less attention, with uniform shrinkage and creep coefficients assumed to provide an acceptable approximation. Work reported by Connal (5) however suggests that differential strain effects may be significant, particularly for pre-tensioned bridge girders.

2.1 Status of analysis for creep and shrinkage

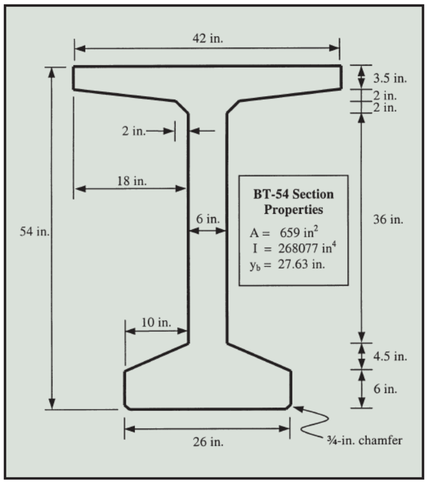
Yu and Bažant (9) suggest that a one-dimensional analysis of creep effects in large span prestressed box girder bridges is inadequate because it does not realistically capture the shear lag effects, and it does not account for the differences in drying creep properties and in shrinkage caused by the differences in the drying rates of slabs, resulting from different thicknesses and environmental exposure. They suggest that for smaller-span structures the simplified analysis is “likely harmless”, however examples in this paper will show that these effects may also be significant in small to medium span bridges under some conditions, especially for cross-sections where the top and bottom flanges have very different proportions, such as the Australian Super-T section.

2.2 Research on pre-tensioned bridge beam deflections

Research on the time-dependent behaviour of prestressed concrete bridge beams has been reported in a number of papers, for instance References (10) to (12). Typical cross sections from these papers are shown in Figures 1-3, compared with the Super-T section in Figure 4.



Figures 1,2. Typical beam sections in deflection studies.



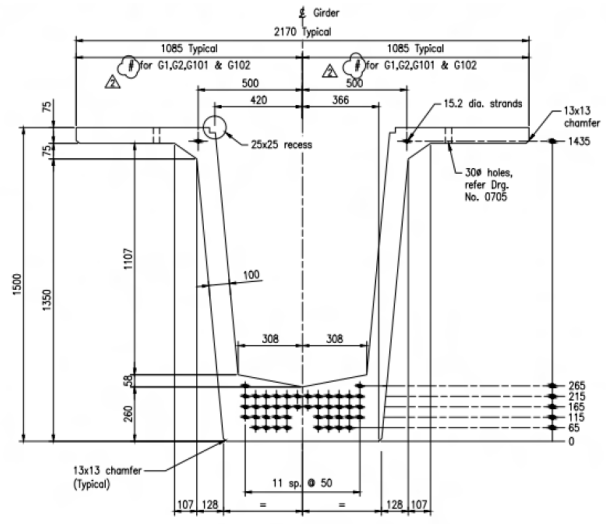


Figure 3. Typical beam sections in deflection studies. **Figure 4. Typical Super-T section**

It can be seen from these figures that the Super-T section has the thinnest top flange and webs, combined with the heaviest bottom flange. It is this combination that makes the Super-T section more sensitive to differential strain effects than other more symmetrical sections.

2.3 Problems with current procedures

The paper by Connal (5) details two projects where the hog deflection of Super-T girders at the time of installation was much less than the expected value, resulting in nett sag deflections after placing of in-situ concrete and superimposed dead loads. In both cases the beams had been stored in hot dry conditions prior to placement, and the deflections could not be explained using standard analysis procedures.

The opposite effect has also been reported, where Super-T girders have a much greater hog deflection than expected. This problem is often associated with steam curing.

3. Sources of Deflections

Deflections in concrete members are associated with four broad categories of behaviour:

- Short term stress-strain and bond behaviour of the concrete and reinforcement.
- Time dependent behaviour of the concrete.
- Differential strain effects.
- Construction sequence and other load sequence effects

In addition to the elastic deformation of the concrete and steel the following short term stress-strain behaviour must be considered:

- Concrete flexural tensile strength.
- Concrete tension-stiffening effect.
- Loss of prestress due to elastic shortening
- Temperature effects due to steam curing

Time dependent behaviour includes:

- Concrete creep
- Concrete shrinkage
- Loss of tension stiffening
- Loss of flexural tensile strength
- Prestress steel relaxation

Significant differential strain effects are:

- Differential shrinkage and creep
- Differential temperature

Load sequence effects to be considered are:

- Storage environment and conditions
- Handling, transport and erection loads
- Variation in span at transfer and in storage
- Change in stiffness after overload.
- Timing of composite connections.
- Effect of varying axial load

Storage conditions may be significantly different even for items stored at the same location. Figure 5 for instance shows open and closed top Super-T girders in open and shaded positions, each of which will have significantly different rates of shrinkage and creep on the internal surfaces.

3.1 Effect of uniform shrinkage strain and creep coefficient

Uniform creep effects may be conveniently evaluated using the standard age-adjusted effective modulus method (AEMM) (3),(4). AS 3600 provides simplified methods for assessment of deflections due to uniform shrinkage, but a more rigorous approach is to apply a compressive prestress strain, equal to the concrete free shrinkage stress, to all the reinforcement (passive and prestressed). This allows shrinkage effects to be rigorously assessed within a closed form solution (10).

Figures 5. Differing storage conditions at one location.

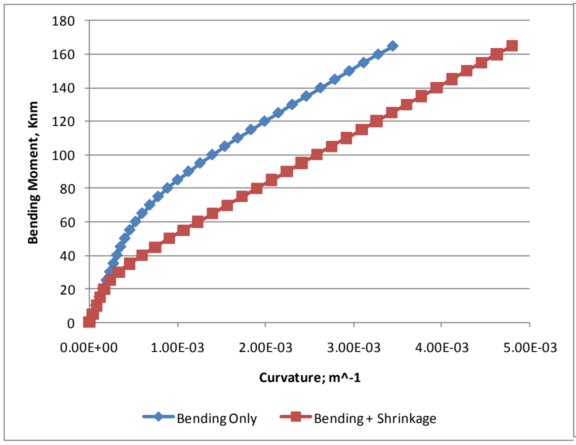


Figure 6. Effect of uniform shrinkage on reinforced section

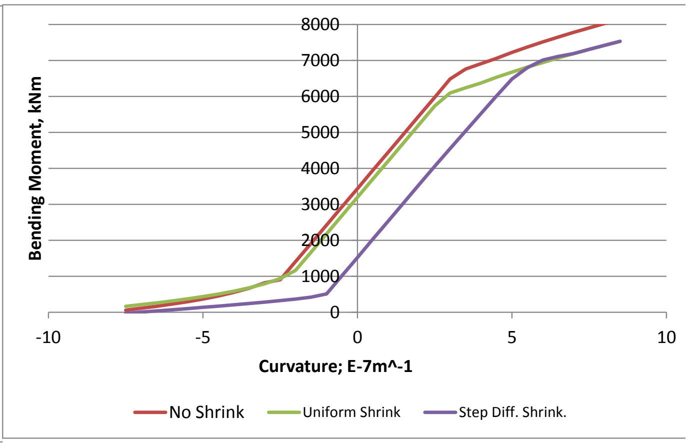


Figure 7. Effect of uniform and differential shrinkage on prestressed section

Figure 6 shows the effect of uniform shrinkage on the moment-curvature relationship for A typical reinforced concrete section. For a symmetrically reinforced section uniform shrinkage does not result in any curvature up to the cracking moment, but after cracking curvature is greatly increased. Figure 7 shows the effect of uniform and differential shrinkage on a typical prestressed Super-T bridge girder. It can be seen that for a prestressed section at bending moments up to about 6000 kNm the uniform shrinkage has very little effect.

3.2 Effect of differential shrinkage

Figure 7 shows that for applied moments between about 1000 kNm and 6500 kNm the differential shrinkage results in greater compressive strain in the top flange, relative to the bottom flange which causes a substantial additional sagging curvature.

3.3 Effect of differential temperature and steam curing

During steam curing the internal concrete temperature may be much greater than the steam temperature, due to the heat of hydration of the cement. Mak, Vessalas et al. (11) report temperatures reaching 90 °C at the core of the bottom flange of Super-T girders steam cured at 55 °C (Figure 8). Temperatures at 100 mm inside the surface of the bottom flange reached 80 °C. The temperatures in the webs and top flange were not reported, but clearly the maximum temperature reached in members only 100 mm or 75 mm thick would be significantly less than for the bottom flange.

These high temperatures during curing give rise to locked in thermal strains which will result in greater effective shrinkage strains in the hotter zones, and hence an increased hogging curvature. Estimated curvatures of a steam cured Super-T girder, with and without thermal strains, are shown in Figure 9.

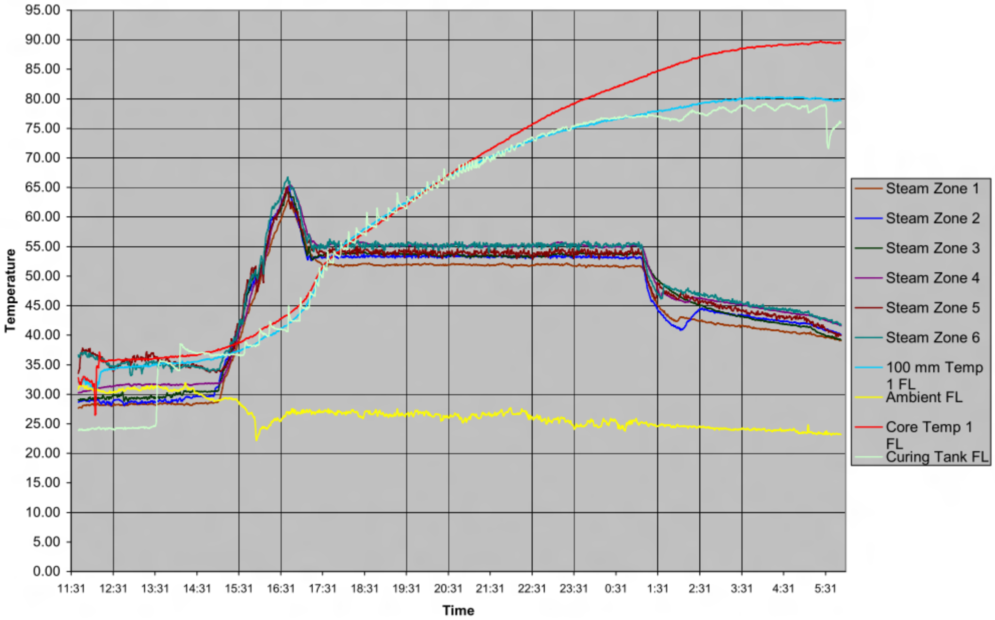


Figure 8 Typical temperature profile recorded during girder manufacture (11)

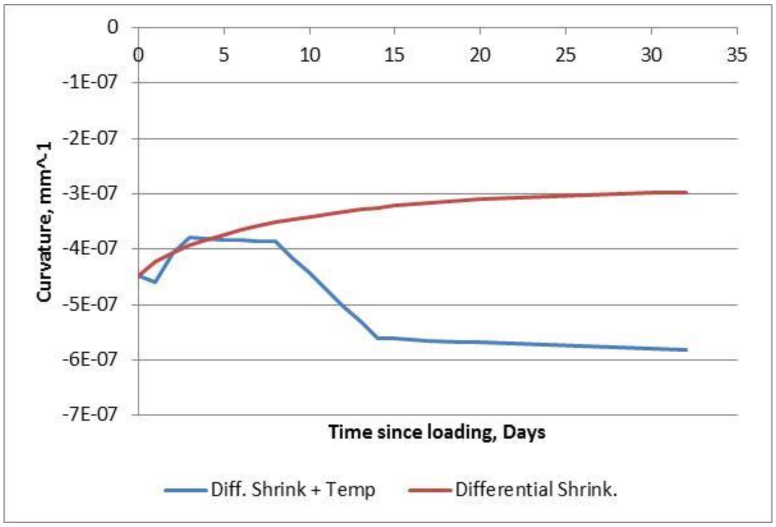


Figure 9 Effect of differential temperature due to steam curing

4. Procedure for assessment of differential strain effects

The procedure used above for the assessment of uniform shrinkage effects is not applicable to differential shrinkage because the concrete stress distribution is no longer linear. The procedure adopted to assess the effect of differential time and temperature related strains is as follows:

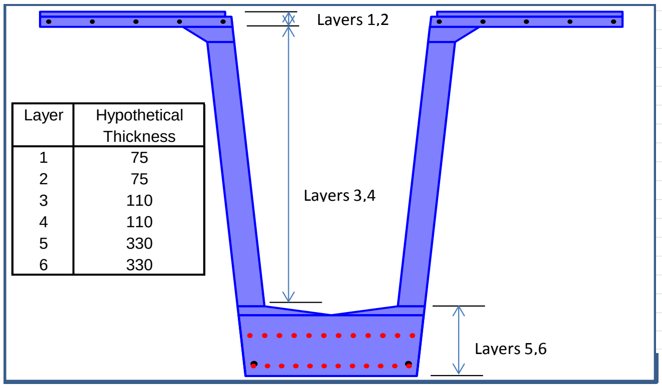
1. Divide the section into trapezoidal layers.
2. Estimate the "hypothetical thickness" for each layer.
3. Use AS 3600 provisions to estimate shrinkage strain and creep coefficient for each layer.
4. Calculate the shrinkage, creep and temperature related strain at the top and bottom of each layer.
5. Calculate temperature strain for each reinforcement layer (both passive and prestressed), and relaxation strain in prestressed layers.
6. Assume an initial total strain at the top face of the section, and a Neutral Axis depth.

7. Calculate stresses at the top and bottom of each layer, taking account of the maximum tensile stress in the concrete, and crack depths from previous stages (if any).
8. Calculate the resultant force and moment.
9. Adjust the Neutral Axis depth and top face strain to satisfy force and moment equilibrium

The stress at any level is given by:

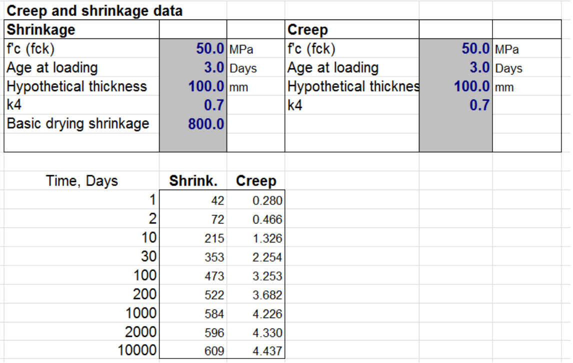
$$\sigma = (\varepsilon_T - \varepsilon_{ttr})E \qquad (1)$$

Where ε_T = total strain calculated from the assumed top face strain and Neutral Axis depth, assuming that plane sections remain plane, and ε_{ttr} = total time and temperature related strain, including shrinkage, creep and temperature change from the temperature during initial curing.



Layer	Hypothetical Thickness
1	75
2	75
3	110
4	110
5	330
6	330

Figure 10 Hypothetical thickness for each layer



Creep and shrinkage data

Shrinkage			**Creep**		
f'c (fck)	50.0	MPa	f'c (fck)	50.0	MPa
Age at loading	3.0	Days	Age at loading	3.0	Days
Hypothetical thickness	100.0	mm	Hypothetical thicknes	100.0	mm
k4	0.7		k4	0.7	
Basic drying shrinkage	800.0				

Time, Days	Shrink.	Creep
1	42	0.280
2	72	0.466
10	215	1.326
30	353	2.254
100	473	3.253
200	522	3.682
1000	584	4.226
2000	596	4.330
10000	609	4.437

Figure 11 Shrinkage and creep coefficients for each layer

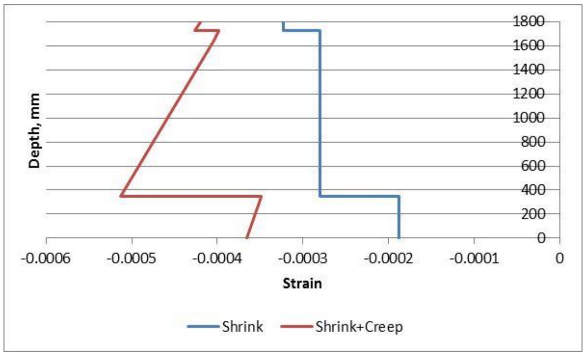


Figure 12 Creep and shrinkage strains

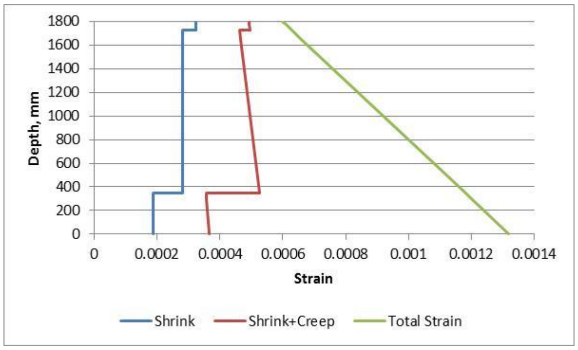


Figure 13 Total strain found by iteration

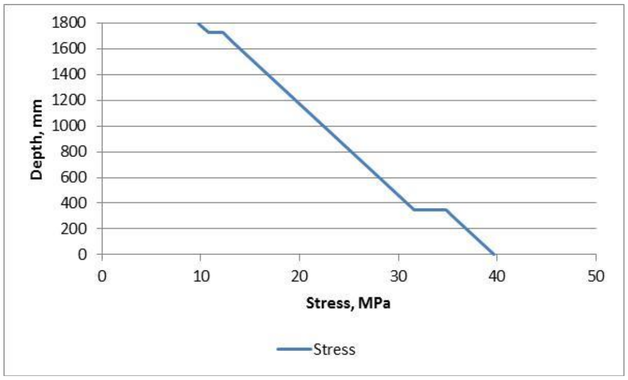
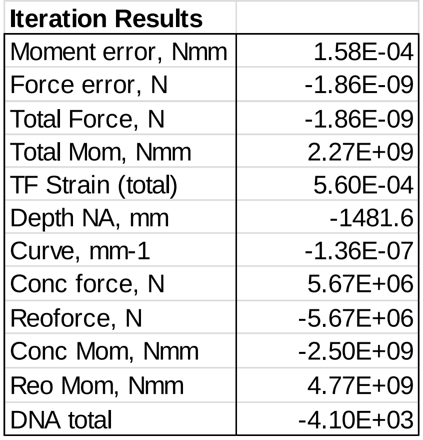


Figure 14 Stresses for each layer



Iteration Results	
Moment error, Nmm	1.58E-04
Force error, N	-1.86E-09
Total Force, N	-1.86E-09
Total Mom, Nmm	2.27E+09
TF Strain (total)	5.60E-04
Depth NA, mm	-1481.6
Curve, mm-1	-1.36E-07
Conc force, N	5.67E+06
Reoforce, N	-5.67E+06
Conc Mom, Nmm	-2.50E+09
Reo Mom, Nmm	4.77E+09
DNA total	-4.10E+03

Figure 15 Iterate to find force and moment equilibrium

In sections where the maximum tensile stress exceeds the flexural tensile strength of the concrete, or where the section is cracked from a previous load condition, tension stiffening effects may be included using an empirical method such as that given in Eurocode 2 (12). Alternatively a strain-softening stress-strain curve may be adopted, such as that proposed by Ng, Lam et al. (13, 14).

This procedure is illustrated in Figures 10 to 15. Having calculated the time and temperature related curvature at sections along the beam the deflection may be found by numerical integration, or with the use of a simple beam analysis program.

5. Example calculations

5.1 Differential shrinkage under hot, dry conditions

Deflections of a 32 metre span Super-T girder under self-weight were calculated at transfer and after 30 days assuming uniform shrinkage (Figure 16) and differential shrinkage (Figure 17). Creep coefficients and shrinkage strains were calculated in accordance with AS 3600, assuming a k_4 factor of 0.7; appropriate for arid conditions. Differential temperature strains were not included in this analysis. It can be seen that the predicted hog deflection under self-weight has reduced from 45 mm to 15 mm when differential shrinkage and creep is considered, which would be likely to result in sag deflections in the finished structure.

5.2 Differential strain effects due to steam curing

The same beam was analysed under the combined effect of differential shrinkage and creep and differential temperature, assuming a maximum temperature of 90 °C in the bottom flange and 55 °C in the webs and top flange. The temperature was assumed to return to 30 °C over a period of 10 days. It can be seen in Figure 18 that the inclusion of differential temperature effects has increased the hog deflection to over 50 mm. This would be substantially increased if differential shrinkage and creep effects were reduced.

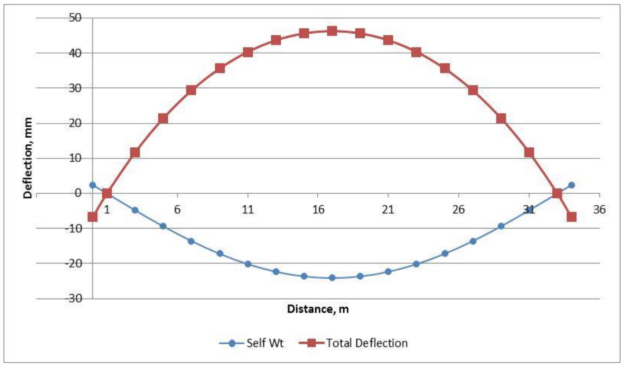


Figure 16 Self-weight + uniform shrinkage

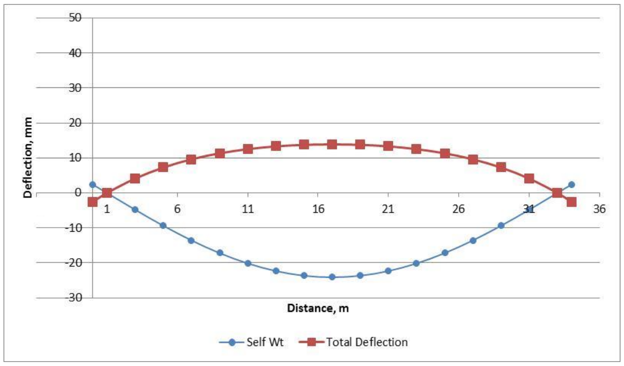


Figure 17 Self-weight + differential shrinkage

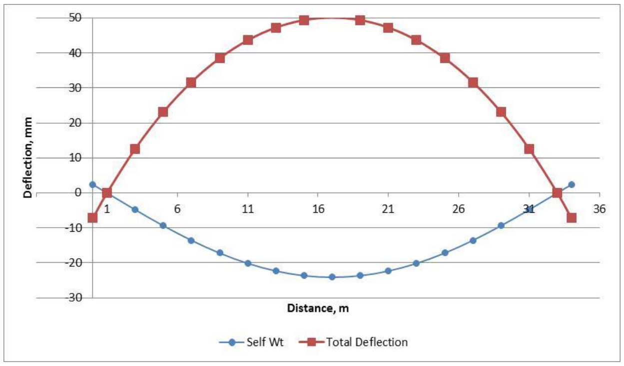


Figure 18 Differential shrinkage + differential temperature

6. Conclusions

It has been shown that differential strain effects during curing have a significant effect on the deflections of pretensioned concrete bridge girders, of the same order of magnitude as immediate deflections due to self-weight and prestress. These effects will be greatest in members that have top and bottom flanges with significantly different effective thickness, such as the Super-T girders examined in this paper. Deflections due to differential temperature under steam cured conditions were found to act in the opposite direction to those resulting from differential shrinkage and creep, but the relative magnitude of these effects will depend on many factors, including environmental conditions, cement content, and temperatures generated during steam curing.

A straightforward method of evaluating differential strain effects was presented, which could be implemented in a practical design context. Evaluation of appropriate strains due to shrinkage and creep is straightforward, using data in applicable design codes. Differential strain effects due to temperature during curing are less well documented however, especially for locked in stresses due to steam curing.

It is recommended that differential strain effects be considered for all Super-T bridge girders and other pretensioned beams with a similar section. Further research is desirable in this area, in particular to investigate differential temperatures occurring during and after steam curing, the magnitude of resulting stresses, and interaction with early age creep effects.

7. References

1. Standards Australia, “Concrete Structures (AS 3600, 2009)”, SAI Global, 2009, Sydney.
2. Standards Australia, “Bridge Design - Concrete (AS 5100.5, 2004)”, Standards Australia, 2004, Sydney.
3. Warner, R.F., Rangan, B.V. et al., “Concrete Structures”, Longman, 1999, Melbourne
4. Gilbert, R.I., Ranzi, G., “Time-Dependent Behaviour of Concrete Structures”, Spon Press, 2011, Abingdon, UK
5. Connal, J., “Deflections of Precast Pretensioned Beams”, Proceedings, Austroads Bridge Conference 2006, Perth, Australia.
6. Kilpatrick, A., Gilbert, R.I., “Predictions of short-term deflections in reinforced concrete one-way continuous members”, Proceedings of the 24th Biennial Conference of the Concrete institute of Australia, 2009, Concrete institute of Australia, Sydney.
7. Wu, H.Q., Gilbert, R.I., “Effect of shrinkage on the time-dependent deflection of reinforced concrete slabs”, Proceedings of the 24th Biennial Conference of the Concrete institute of Australia, 2009, Concrete institute of Australia, Sydney.
8. Gilbert, R.I., Ranzi, G., “Design of Reinforced Concrete for Deflection Control”, Concrete in Australia, vol. 36, no. 2, 2010, pp. 20 – 26
9. Yu, Q., Bažant, Z.P., “Improved Algorithm for Efficient and Realistic Creep Analysis of Large Creep-Sensitive Concrete Structures”, ACI Structural Journal, 109-S58, 2012, pp 665-676.
10. Jenkins, D., “Predicting the Deflection of Concrete Structures in Practice”, Proceedings of the 24th Biennial Conference of the Concrete institute of Australia, 2009, Concrete institute of Australia, Sydney.
11. Mak, J., Vessalas, K. et al., “Evaluation of Factors Pertaining to Delayed Ettringite Formation in Steam Cured Precast Concrete Members”, The New Zealand Concrete Industry Conference, 2012, NZ Concrete Industry Conference, Auckland, NZ.
12. Eurocode 2, “Design of concrete structures Part 1-1: General rules for buildings”, British Standard BS EN 1992-1-1:2004, European Committee for Standardisation (CEN).
13. Ng, P.L. et al., “Tension Stiffening in concrete beams. Part1 FE Analysis”, Structures and Buildings, Institution of Civil Australia, Vol 163 Issue SB1, Feb 2010, pp 19-28.
14. Lam, J.Y.K. et al., “Tension Stiffening in concrete beams. Part 2 member Analysis”, Structures and Buildings, Institution of Civil Australia, Vol 163 Issue SB1, Feb 2010, pp 29-39.