*†***15-4** The trailer hitch from Figure 1-1 (p. 11) has loads applied as shown in Figure P15-1. The tongue weight of 100 kg acts downward and the pull force of 4 905 N acts horizontally. Using the dimensions of the ball bracket in Figure 1-5 (p. 14), draw a free-body diagram of the ball bracket and find the tensile and shear loads applied to the two bolts that attach the bracket to the channel in Figure 1-1. Size and specify the bolts and their preload for a safety factor of at least 1.7.



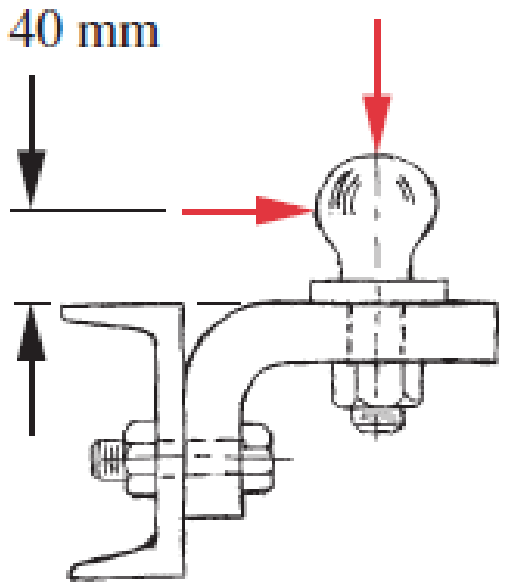



FIGURE P15-1

Problems 15-4 to 15-6

## PROBLEM 15-4

**Statement:** The trailer hitch from Figure 1-1 (p. 12) has loads applied as shown in Figure P3-2. The tongue weight of 100 kg acts downward and the pull force of 4905 N acts horizontally. Using the dimensions of the ball bracket in Figure 1-5 (p. 15), draw a free-body diagram of the ball bracket and find the tensile and shear loads applied to the two bolts that attach the bracket to the channel in Figure 1-1. Size and specify the bolts and their preload for a safety factor of at least 1.7.

**Given:**

Hitch dimensions:

$a := 40 \cdot mm$

$b := 31 \cdot mm$

$c := 70 \cdot mm$

$d := 20 \cdot mm$

$t := 19 \cdot mm$

| | |
|---|---|
| Tongue weight | $M_{tongue} := 100 \cdot kg$ |
| Pull force | $F_{pull} := 4.905 \cdot kN$ |
| Number of bolts | $N_{bolts} := 2$ |
| Young's modulus | $E := 206.8 \cdot GPa$ |
| Design safety factor | $N_d := 1.7$ |
| Bolt modulus | $E_{bolt} := E$ |
| Member modulus | $E_{memb} := E$ |

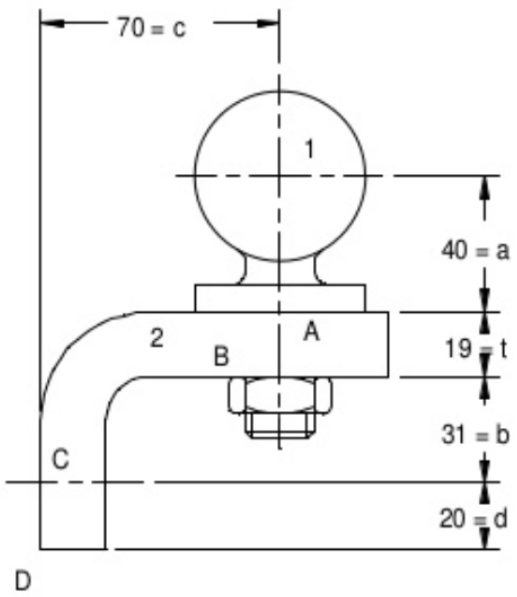
**Assumptions:** The shear load will be taken by friction between the hitch and the support.

**Design Choices:**

Use M12 x 1.75 , class 8.8 bolts.

Material properties for class 8.8:

| | | | |
|---|---|---|---|
| Proof strength | $S_p := 600 \cdot MPa$ | Bolt diameter | $d_b := 12 \cdot mm$ |
| Yield strength | $S_y := 660 \cdot MPa$ | Clamp length | $l := 30 \cdot mm$ |
| | | Preload fraction | $f_p := 0.59$ |



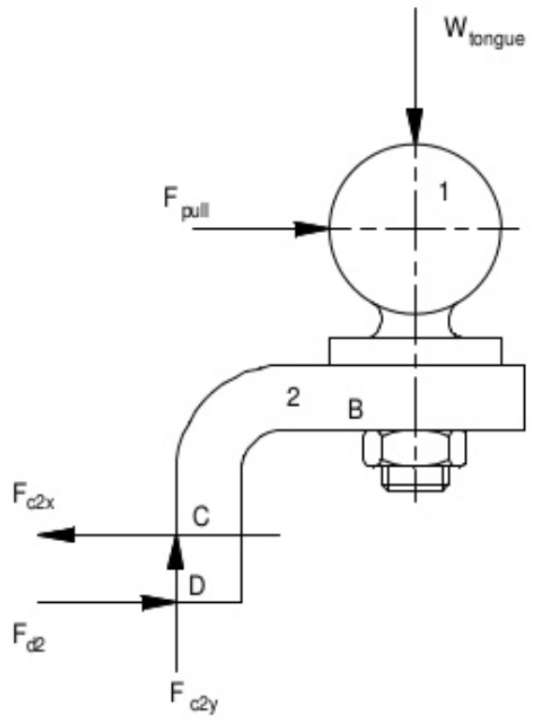



**FIGURE S15-4**
Dimensions and Free Body Diagram for Problem 15-4

**Solution:** See Figure S15-4 and Mathcad file P1504.

1. The weight on the tongue is

$$W_{tongue} := M_{tongue} \cdot g \qquad W_{tongue} = 0.981 \cdot kN$$

2. The FBD of the hitch and bracket assembly is shown in Figure 3-4. The known external forces that act on the ball are $F_{pull}$ and $W_{tongue}$. The reactions on the bracket are at points $C$ and $D$. The bolts at $C$ provide tensile ($F_{c2x}$) and shear ($F_{c2y}$) forces, and the bracket resists rotation about point $D$ where the reaction force $F_{d2}$ is applied by the channel to which the bracket is bolted.



3. Solving for the reactions by summing the horizontal and vertical forces and the moments about $D$:

$$\Sigma F_x: \quad (F_{pull} - F_{c2x}) + F_{d2} = 0 \tag{1}$$

$$\Sigma F_y: \quad F_{c2y} - W_{tongue} = 0 \tag{2}$$

$$\Sigma M_D: \quad F_{c2x} \cdot d - F_{pull} \cdot (a + t + b + d) - W_{tongue} \cdot c = 0 \tag{3}$$

4. Solving equation (3) for $F_{c2x}$

machine-design-problems

$$\Sigma F_y: \quad F_{c2y} - W_{tongue} = 0 \tag{2}$$

$$\Sigma M_D: \quad F_{c2x} \cdot d - F_{pull} \cdot (a + t + b + d) - W_{tongue} \cdot c = 0 \tag{3}$$

4. Solving equation (3) for $F_{c2x}$

$$F_{c2x} := \frac{F_{pull} \cdot (a + t + b + d) + W_{tongue} \cdot c}{d} \qquad F_{c2x} = 30.41 \cdot kN \tag{4}$$

5. Substituting into (1) and solving for $F_{d2}$

$$F_{d2} := F_{c2x} - F_{pull} \qquad F_{d2} = 25.505 \cdot kN \tag{5}$$

6. Solving (2) for $F_{c2y}$

$$F_{c2y} := W_{tongue} \qquad F_{c2y} = 0.981 \cdot kN \tag{6}$$

7. The loads applied to the two bolts that attach the bracket to the channel are:

| | | |
|---|---|---|
| Axial force on two bolts | $F_{c2x} = 30.4 \cdot kN$ | $P_{tot} := F_{c2x}$ |
| Shear force taken by friction | $F_{c2y} = 0.98 \cdot kN$ | |

8. Determine the load per bolt. $P := \frac{P_{tot}}{N_{bolts}}$ $P = 15.20 \cdot kN$

9. Get the tensile stress area from Table 15-2. $A_t := 84.27 \cdot mm^2$

$N_{bolts}$

9. Get the tensile stress area from Table 15-2. $A_t := 84.27 \cdot mm^2$

10. Calculate the preload. $F_i := f_p \cdot S_p \cdot A_t$ $F_i = 29.83 \cdot kN$

11. Determine the relevant ratios for this joint from equations 15-18*a* and *b*.

Joint aspect ratio: $j := \frac{d_b}{l}$ $j = 0.400$

Plate to bolt modulus: $r := \frac{E_{memb}}{E_{bolt}}$ $r = 1$

12. Calculate $C_r = C$ using equation 15.19 and the coefficients $p_i$ from Table 15-8 for $j = 0.4$ .

Coefficients from Table 15-8: $p_0 := 0.7351$

$p_1 := -1.2612$

$p_2 := 1.1111$

$p_3 := -0.3779$

Joint stiffness constant: $C := p_3 \cdot r^3 + p_2 \cdot r^2 + p_1 \cdot r + p_0$ $C = 0.207$





13. The portions of the applied load P felt by the bolt and the material can now be found from equations 15.13.

$P_b := C \cdot P$ $P_b = 3.15 \cdot kN$

$P_m := (1 - C) \cdot P$ $P_m = 12.1 \cdot kN$

14. Find the resulting loads in bolt and material after the load P is applied.

$F_b := F_i + P_b$ $F_b = 32.98 \cdot kN$

$F_m := F_i - P_m$ $F_m = 17.78 \cdot kN$

15. The maximum tensile stress in the bolt is

$\sigma_b := \frac{F_b}{A_t}$ $\sigma_b = 391.4 \cdot MPa$

16. This is a uniaxial stress situation, so the principal stress and von Mises stress are identical to the applied tensile stress. The safety factor against yielding for class 8.8 with $S_y = 660 \cdot MPa$ is then

$N_y := \frac{S_y}{\sigma_b}$ $N_y = 1.7$

17. The load required to separate the joint and the safety factor against joint separation are found from equations 15.14*c* and 15.14*d*.

$P_0 := \frac{F_i}{1 - C}$ $P_0 = 37.6 \cdot kN$

$$\sigma_b := \frac{P}{A_t} \qquad \sigma_b = 391.4 \cdot MPa$$

16. This is a uniaxial stress situation, so the principal stress and von Mises stress are identical to the applied tensile stress. The safety factor against yielding for class 8.8 with $S_y = 660 \cdot MPa$ is then

$$N_y := \frac{S_y}{\sigma_b} \qquad N_y = 1.7$$

17. The load required to separate the joint and the safety factor against joint separation are found from equations 15.14*c* and 15.14*d*.

$$P_0 := \frac{F_i}{1 - C} \qquad P_0 = 37.6 \cdot kN$$

$$N_{sep} := \frac{P_0}{P} \qquad N_{sep} = 2.5$$