

# THE DISTRIBUTION OF LOAD AND STRESS IN THE THREADS OF FASTENERS - A REVIEW

B. KENNY and E. A. PATTERSON

UNIVERSITY OF SHEFFIELD, MECHANICAL ENGINEERING DEPARTMENT,  
MAPPIN STREET, SHEFFIELD S1 3JD, U.K.

## 1. INTRODUCTION

The nut and bolt connection first appeared in the 15th century although the threaded fastener came into existence sometime earlier in the form of wood screws. Following the advent of the Industrial Revolution, nuts and bolts become commonplace. The invention of the lathe in 1800 by Henry Maudslay enabled threads of any pitch and diameter to be made with a greater degree of precision and reproducibility. The introduction of standardisation by Whitworth (1) and by Sellars led to the interchangeability of components and further enhanced their usefulness. The incompatibility of the Whitworth and Sellars system was one of the factors which resulted in the establishment of the International Organisation for Standardisation (ISO) system which is slowly being adopted worldwide.

It is important that bolts do not fail in service since this could be both dangerous and expensive. This is avoided in the majority of applications by using standard components which are larger than initial calculations indicate is necessary, since this is cheaper than designing and manufacturing special connections. However there are applications where either for reasons of space or economy of weight it is desirable to use the smallest nut and bolt that will safely perform the required purpose.

In a conventional nut-bolt connection the bolt is mainly subjected to tension, whereas the nut is subjected to compression. This produces a difference in thread pitch between the loaded bolt and the nut and since the loaded face of the nut is restrained in the axial direction, more load is taken by the threads at the loaded face of the nut. The shape of the thread load distribution is generally accepted to be parabolic (Figure 1), although direct experimental verification of this is difficult due to the problem of measuring the thread load without affecting the phenomenon being studied.

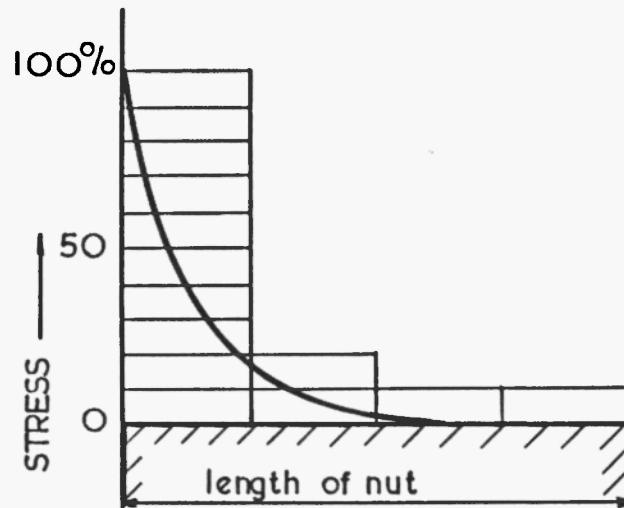


FIG.1 STRESS DISTRIBUTION ALONG BOLT DUE TO DEN HARTOG

In most nut-bolt connections, failure occurs in the bolt at the root of the first loaded thread since in cyclic loading conditions, which occur in the majority of bolted components, fatigue failures usually initiate at the position of maximum stress concentration. As explained above, the thread load tends to increase towards the loaded face of the nut, and this produces a maximum bending stress in the roots of the thread at this point. The notch effect due to the thread form, results in a stress concentration in the roots of the threads. The net result is a peak stress at the root of the first loaded thread of the bolt from the load bearing face of the nut and makes the bolt susceptible to fatigue failure at this point. Since the endurance limit for alternating tension is lower than that for alternating compression, the bolt fails before the nut. If the thread load could be distributed more uniformly along the thread helix, the maximum stress concentration in the threads would be reduced giving the connection a longer fatigue life.

Several methods have been utilised in an endeavour to evenly distribute the load along a bolt. These have included the use of bolts with differential or non-uniform thread pitch distributions compared with their mating nuts, or the use of nuts with non-uniform standard external geometry. Attempts to commercialise the use of bolts having non-uniform thread pitch distributions have not been successful, whereas the use of nuts with modified external geometry, as typified by the "tension nut", has found some favour for specialised applications.

## 2. LITERATURE REVIEW

Stromeyer (2) was one of the first to consider the stresses in threaded connections. His paper suggested in general terms, ideas about the effect of differential pitch and the bending of threads that were confirmed by later investigators. In 1929 Den Hartog (3) pointed out that when a nut and a bolt of equal pitch are loaded such that the bolt is in tension and the nut is in compression, the bolt will elongate and the nut will shorten. This results in the

mating pitches no longer being the same and produces a non-uniform distribution of load along the thread helix. He derived an expression for this distribution by equating the sum of the axial deformation of the nut and bolt to the deformation of the thread, which was treated as a cantilever subjected to bending and shear action. This resulted in a theoretical load distribution of parabolic form with the first two threads taking 45% of the load. This derivation became the basis for the more comprehensive theory published in 1948 by Sopwith (4).

Den Hartog (3) obtained experimental confirmation of his analysis using two-dimensional photoelastic models. He found by using a celluloid sheet model of four threads with a square thread form, that the first two threads took nearly the entire load, whilst the other threads were barely stressed at all. Solakian (5) used a slightly more sophisticated loading rig, in which the two-dimensional model was held in place by adjustable bearing plates so that the loading conditions bore a closer approximation to the three-dimensional situation. Although this was an improvement over the cruder clamping conditions used by Den Hartog, the way in which the model sheet was held in place could still have influenced the results. Solakian tested three different thread forms, the Sellers square thread, the American Standard and the Whitworth thread. When only one thread was in contact in each case, he found that the stress concentration in the root of this thread was 7.6, 3.8, and 2.9 respectively for the tested thread forms. As Solakian remarked, in the case of the Sellers square thread, this was considerably higher than the factor of safety that was normally assumed when designing with these threads.

The experiment was repeated for the American Standard thread form with several threads in contact and under these conditions the maximum stress concentration was 3.95. The difference in stress concentration factors for multiple and single thread contact was less than 5% and was therefore probably not significant. However it could have been due to the combination of the effects of a loaded thread above and below the root giving a higher concentration factor than when only one thread was loaded.

Heywood (6) investigated the tensile fillet stress in loaded projections in order to optimise the thread form. He modified Lewis's (7) formula for the fillet stress by including a bending moment term. The new formula predicted stresses which were within 10% of the two-dimensional photoelastic results he obtained. Heywood concluded using the new formula, that the Whitworth threadform was remarkably close to the optimum shape. This was in agreement with Solakian's results which showed the Whitworth thread form to be the best of the three forms he tested, and it was confirmed by fatigue tests performed by Moore and Heywood (8) and which were reported by Thurston (9).

In 1940, Goodier (10) obtained the load distribution in the threads of a bolt by measuring the external deformations of the nut. He used a bolt with all but one turn of the thread removed, this single thread was positioned at a series of locations in the nut to assess the proportion of the load taken by the thread at the same location in a fully formed bolt. Goodier found that only when the

single thread was close to the loaded face of the nut, did the deformation of the nut resemble the deformation produced when a fully formed bolt thread was used. This indicated that the threads nearest the loaded face of the nut experienced the majority of the load, which could have been predicted using Den Hartog's analysis.

There is a serious inadequacy in both Goodier's experiments and the earlier photoelastic work. In order to obtain their results the investigators had to work with experimental models that did not represent the three dimensional geometry of a nut and bolt. However Hetenyi (11) (12) found the stress concentrations in a series of nuts using three dimensional photoelasticity. He used four three-dimensional double-ended bolts machined from thick plates of Bakelite, with six different designs of nuts which were, a conventional nut; a nut with an outer support; a nut with a spherical washer; a nut with a tapered thread (Figure 2a); and a nut with a tapered lip (Figure 2b). The two remaining nuts were conventional nuts to provide a check on the technique. The loaded nuts were put through a stress freezing cycle and then sliced so that the fringe patterns could be examined in the normal fashion. Hetenyi found that the maximum stress concentration in the thread roots of the bolts with the nut with the tapered lip and the nut with the tapered thread were 3.00 and 3.10 respectively compared to 3.85 in the bolts with the conventional nuts. The other designs of nut gave no significant change in the stress concentration compared to the conventional nuts.

Although Hetenyi used the Whitworth thread in his tests he predicted that for any thread form the magnitudes of the improvements achieved would be the same proportion of the maximum concentration in the conventional connection, but that for the nut with the tapered thread, the improvement was dependant on the load applied to the bolt. The explanation for this is that the tapered thread works by creating a continuously varying difference in pitch between the nut and the bolt which is greatest at the loaded face of

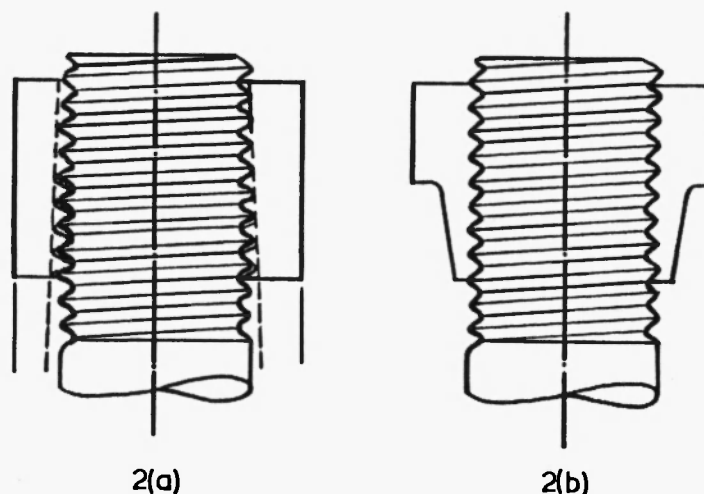


FIG. 2 (a) NUT WITH TAPERED THREAD  
(b) NUT WITH TAPERED LIP

the nut. At low bolt loads, there is then little load taken by the threads at this end, as they will only make contact as the bolt load is increased. This results in a more uniform load distribution along the bolt until normal contact is achieved all along the thread helix. Beyond the bolt load at which this occurs, the load will be distributed in the conventional manner, i.e. according to the parabolic load distribution derived by Den Hartog and inferred by the stress concentrations obtained by Hetenyi. The maximum stress concentrations found by Hetenyi in the roots of bolts fitted with conventional nuts were 25% greater than the corresponding value obtained by Solakian using a two-dimensional model. This demonstrates the inappropriateness of using the two-dimensional technique for this problem.

Hetenyi's work was the first systematic attempt to find by experiment the optimum design of nut to give the smallest maximum stress concentration and consequently the best performance in dynamic loading, and static loading at elevated temperatures. The nut with the tapered lip is now widely used and is often referred to as a tension nut. It has received much attention from investigators, while the nut with the tapered thread has received less.

The theory developed by Sopwith (4) to describe the load distribution in a bolt fitted with a conventional nut was based on Den Hartog's (3) earlier work but expanded it considerably. The theory, which has gained general acceptance, follows a theory of elasticity approach for the deformation of the nut, the bolt, and the thread, which were each considered separately and then using compatibility of displacements, were related to give a second order differential equation in which the variables were the load per unit length of thread and the distance from the loaded face of the nut.

Sopwith compared his theoretical load distribution with the experimental results of both Goodier and Hetenyi and found reasonable agreement in both cases (Figure 3). In the case of Hetenyi's

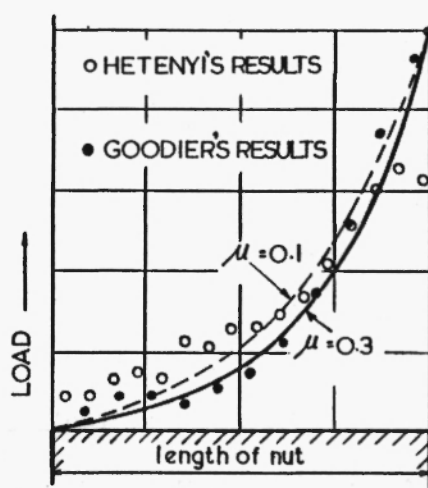


FIG. 3 COMPARISON OF SOPWITH'S THEORETICAL LOAD DISTRIBUTION WITH THE EXPERIMENTAL RESULTS OF HETENYI AND GOODIER

results, which were root stresses rather than thread loads, Sopwith assumed that the Bakelite models were stressed elastically, so that the stress concentration factor at the root of the threads would be constant and the root stresses would be proportional to the load on the thread. At the loaded face of the nut, according to Sopwith, the root stress decreases due to the proximity of the more lightly stressed full section of the shank. In fact, in this region, a marked disagreement between Sopwith's theory which gave a parabolically increasing stress up to the loaded face of the nut, and Hetenyi's results which exhibited a reduction in stress. The experimental load distribution obtained by Goodier did not show a reduction in thread load towards the loaded face of the nut, thus supporting Sopwith's hypothesis. However Goodier's loading conditions were somewhat unrealistic as only one turn of the bolt thread was modelled.

As previously stated Sopwith's theory was similar to that used by Den Hartog but in addition to considering the axial extension of the bolt, the compression of the nut, and the bending of the threads, it also considered the radial compression of the thread and the radial displacement of the nut and bolt due to the Poisson's ratio effect. Both authors agreed that a more uniform load distribution could be theoretically achieved using a tapered thread, and Den Hartog suggested that a parabolic taper would produce the best results. Sopwith also suggested that using a nut with a lower modulus of elasticity than that of the bolt, or using a tension nut would also be theoretically beneficial, as this reduced the sum of the axial strains in the nut and bolt. The superiority of aluminium alloy and magnesium alloy nuts over steel nuts has been shown in fatigue tests by Weigand (13), and Kaufmann and Janiche (14).

Following on these suggestions, Stoeckley and Macke (15) investigated both by experiment and by extending Sopwith's theory, the effect on load distribution of tapering the thread. They incorporated this in the theory by assuming that the taper caused an initial axial recession or difference in pitch and they included this effect in the differential equation formulated by Sopwith. In order to verify their theoretical analysis they determined the load distribution in a series of bolts fitted with nuts having tapered threads, by measuring the axial displacements of the bases of the threads. These measurements were obtained from dial gauges located at the base of the thread via holes drilled axially into the nut. Whilst there was some agreement between their theoretical and experimental results for small angles of taper, at large angles and or high bolt stresses, there was little correlation. However their results did support the hypotheses of Den Hartog and Sopwith, that a tapered thread improved the thread load distribution when the bolt load was equal to, or less than a critical value. Above this value the additional load is distributed in the conventional parabolic fashion thus reducing the advantage of the taper, the value of this critical load being dependant on the geometry of the taper.

Apparently independently of the theoretical work by Den Hartog, Sopwith, and Stoeckly and Macke, Zhukovskii (16) and Kolenchuk (17) also derived a parabolic curve to describe the load distribution in threads. Kolenchuk did this by dividing the nut and bolt cores into

cylindrical elements and then considering it as a planar problem. This work was originally published in Russian and was reported in English by Yakushev (18) in 1964.

Nine years after Hetenyi published his results Brown and Hickson (19) repeated his experiments for both the conventional nut and the tension nut. They used a new photoelastic material, Fosterite which they claimed was less susceptible to edge stresses than the Bakelite used by Hetenyi. They employed slices from the frozen stress models which were thinner relative to the bolt diameter, and claimed that this led to a more accurate determination of the stress concentrations in the roots of the threads. In terms of the nominal bolt diameter  $D$ , they used slices between  $0.02 D$  and  $0.03 D$  thickness compared to those used by Hetenyi which were  $0.16 D$  thick, although in fact Brown and Hickson's slices were 0.8 inches thicker. Their results are in qualitative agreement with Hetenyi's, but their absolute values of stress concentration were higher. For the Whitworth thread form, with the thread tips truncated by an amount equal to 0.4 of the normal thread depth, their second experiment gave a maximum stress concentration of approximately 9.0 in the roots of the bolt using a conventional nut, as opposed to Hetenyi's equivalent value of 3.85.

The high maximum stress concentration obtained by Brown and Hickson was probably contributed to by their excessive truncation of the threads, which would increase the bending stress in the threads, by their relatively short length of engagement which Sopwith (4) showed increased the load concentration, and by their use of a relatively small external diameter for the nut.

In a similar manner to the way Sopwith used Hetenyi's results to validate his theory, Brown and Hickson assumed that the root stresses were a function of the loads applied to the threads. They normalised their experimental root stresses with the mean value and replotted the results to provide a comparison with Sopwith's theory. As with Hetenyi's results, there was reasonable agreement except at the loaded face of the nut where there was a reduction in the experimental stress results which was not reproduced by the theory. Until the 1980's, no further attempts were made to validate Sopwith's theory, the main difficulty being a lack of an experimental technique for directly obtaining the load distribution in the threads.

During the late 1950's and the 1960's interest in the loads and stresses in screw threads appeared to wane. Bluhm and Flanagan (20) used an electrical analogue together with two dimensional photoelasticity, to analyse the load and stress distributions in nuts and bolts. Their results were not significantly different from those of earlier investigators. In 1968, Chalupnik (21) looked at the stresses in oversized bolts, and bolts fitted with nuts with external modifications (Figure 4). He found that the oversized bolt shanks produced a reduction in fatigue life, but that the fatigue life could also be improved by altering the external profile of the nut to form a locking section towards the free face so that at the loaded end there were a number of unloaded threads.

The advent of powerful computer techniques such as finite element analysis and the development of new experimental techniques

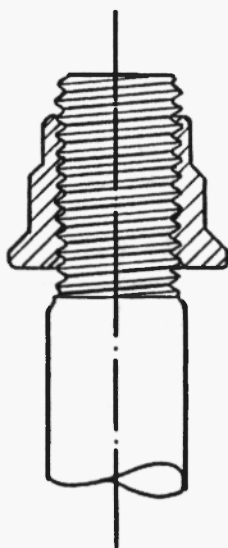


FIG.4 SPECIAL NUT TESTED BY  
CHALUPNIK

temporarily drew attention away from photoelasticity as a means of analysing the stresses in nuts and bolts.

Maruyama (22), (23) investigated the stresses in nut-bolt joints using both finite element analysis and a copper-electroplating method. For the finite element analysis it was assumed that the threads could be modelled by annular grooves, thus approximating the problem to be axially symmetric. The friction at the flank of the thread was neglected and the load per annular thread was calculated. These values were used to compute the stress concentrations in the thread roots. The computed results showed that the stress concentration in the root tends to decrease and to move to the loaded flank of the thread as the root radius is increased. This is probably because the influence of bending stresses in the thread becomes larger than that of the tensile stress at the notch of the root. Yakushev (18), Birger (24), and Walker and Finkelston (25) found by testing nuts and bolts that an increase of the thread root radius increased the fatigue life, but also slightly reduced the static strength of the connection.

The finite element results of Maruyama (22) (23) showed fairly good agreement with the experimental results from the copper-electroplating technique but differed by up to 24 percent for small root radii. Maruyama concluded that the disagreements between the stress concentration results were caused by the screw thread having too many stress concentrations to permit a reliable analysis using the finite element technique.

The copper-electroplating technique which was also used by Seika et al (26) to optimise the design of tension nuts, involved subjecting a copper-electroplated specimen to cyclic strain which caused the growth of micrograins in the copper. The strain in the roots of a test specimen were found by comparing the degree of 'flecking', or growth of the micrograins, with that of a calibrated specimen. To determine the stress, Maruyama used a bolt having the unthreaded portion of the shank tapered to form a calibration



section. When flecking first appeared in the root of interest the test was halted. The stress in the root was then assumed to be the same as that in the cross-section of the tapered shank at the plane which the flecking had reached.

Seika et al found that increasing the depth of the lip of a tension nut reduced the maximum bolt stress but also lowered the strength of the nut. The optimum depth of the lip was found to be slightly more than half the height of the nut. They also found that reducing the nut height increased the maximum bolt stress.

Motosh (27) developed a theory for the load distribution in tension nuts using a strength of materials approach. He divided the nut and bolt into a series of cylindrical elements and for each element equated the relative change in pitch between nut and bolt due to axial strains, with the effects due to radial deformation of the nut and bolt, the bending deformation of the threads and the deformation due to the bending of the nut body. The solution required an iterative procedure to determine the load distribution. The style of approach to the problem was different to that used by Sopwith, but the deformations considered were the same, with the addition of an extra term to allow for the bending of the nut. Motosh did not provide any experimental evidence to support his theory. The detailed design of tension nuts has been further considered by Doniselli and Mondina (28)

In 1977, Otaki (29) concentrated his attention on the stress distribution at the first loaded thread in a bolt by utilising an analytical approach employing complex stress functions. His method was to consider the diametral plane of the bolt to consist of a semi-infinite plate having a series of notches corresponding to the screw thread profile along its edge. The stresses in this plane were then obtained by superimposing the stresses due to the individual thread loads and those due to the notch effect of the thread profiles. It was demonstrated that the results of this theory agreed with previously published photoelastic data. The theoretical stresses were then employed to predict the fatigue behaviour of axially loaded bolts and good comparison with the reported experimental results of Yakushev (18) were obtained. General conclusions made were that, the larger the thread nominal diameter, the lower the fatigue limit, that there is an optimum thread pitch to minimise the fatigue limit, that the greater the height of the nut the higher the fatigue limit and that an optimum thread root radius exists which maximises the fatigue limit.

In 1979 Bretl and Cook (30) used a novel finite element technique to model the load transfer between a nut and a bolt. Rather than modelling individual threads as Maruyama (22, 23) had done (Figure 5), and thus requiring a fine mesh of elements to model the thread precisely, Bretl and Cook replaced the thread zone with a layer of elements having orthotropic properties. The principal directions of orthotropy being dependant on the thread geometry and the direction of application of the load. This permitted them to take into account the radial expansion of the nut and the radial contraction of the bolt. The bending of the threads was handled by decreasing the shear stiffness of the thread material. To avoid the support elements at the loaded face of the nut having an influence

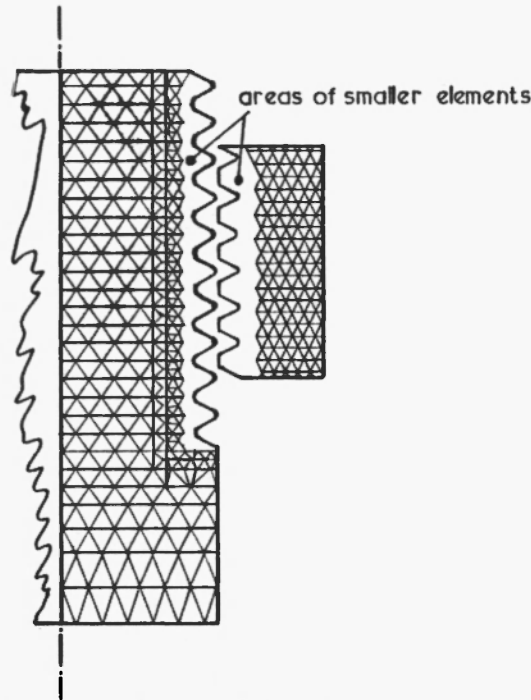


FIG. 5 SCHEMATIC ARRANGEMENT OF FINITE ELEMENT MODELLING USED BY MARUYAMA

on the radial expansion or contraction of the nut, no circumferential stress was permitted in these elements. The results of their analysis of the load distribution in a bolt fitted with a conventional nut related reasonably well with the experimental results of Stoeckly and Macke (15) and were an improvement on the finite element results of Maruyama. However the correlation decreased when tapered threads were considered. In this situation Bretl and Cook used an iterative procedure to handle the non-linearities created by the tapered threads. This was necessary, because for the tapered threads, the stiffness of the elements in the thread zone had to be changed from a known value to an assumed one to allow for the initial lack of contact between the threads.

Tanaka et al (31) made a number of changes to the method used by Maruyama but the results of their finite element analyses for conventional nut-bolt connection did not produce results which were significantly closer to available experimental results. They concluded that for the case of a nut and bolt clamping an elastic plate, the load distribution along the bolt threads was altered by varying the size of the bolt-hole, the load distribution becoming more uniform as the bolt size increased.

Subsequently (32) a finite element study of an externally loaded bolted flange coupling was reported. The effect of a transverse load on the bolt was accomplished by a two dimensional modelling of the threaded connection. To consider the phenomenon of self loosening, the threaded connection was considered as a three dimensional body which enabled a qualitative explanation to be

proposed. Finally a flange clamped by four bolts was analysed when subjected to axial loading and the deformation of the components together with the equivalent stress distribution were presented.

In 1982, Sparling (33) renewed interest in the use of tapered threads by suggesting that the heavily loaded nut threads should be formed on a conical taper, the tapered threads having the full thread form (Figure 6). The idea of this modification was that relief of the load on the threads, due to an initial difference in pitch between nut and bolt threads should be provided where it was most needed. The threads at the free end of the nut would then take a greater proportion of the bolt load than would be the case for either a conventional nut or for one with a taper for the whole of its length. Using standard fatigue testing techniques Sparling found that such a modification to the nut produced a 10% increase in the static load for failure, and a 70% increase in fatigue life for very high cyclic loading. He cut the tapered threads in a standard nut, using a tap with six threads formed on an internal conical frustum, and found that to produce the optimum improvement in fatigue life, the tap had to be turned through between 2.25 and 3.5 complete revolutions.

Miller et al (34) produced a series of finite difference equations to describe the load distribution in compression and tension nuts, their theory being based on a series of springs simulating the body of the connection, with the threads being simulated by rings linking the axial springs. Their results correlated well with Sopwith's (4) theoretical analysis, Hetenyi's (11, 12) experimental root stresses and Stoeckly and Macke's (15) experimental thread loads. However, all of these results have inherent inadequacies which have been discussed. Miller et al stated that their theory could not be extended to model connections with tapered threads.

Recently Fessler and his co-workers, Jobson (35) and Wang Jiong-Hua (36) performed some detailed photoelastic analyses of the

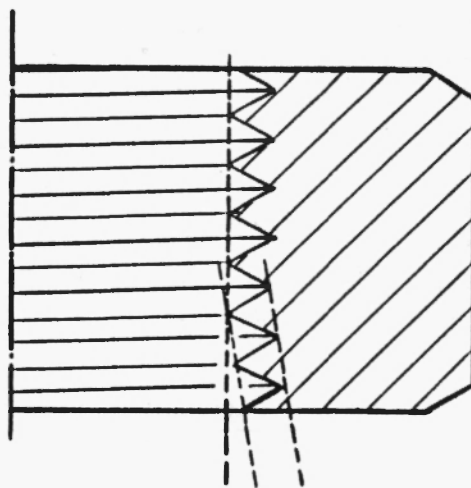


FIG. 6 HALF SECTION OF A SPARLING DESIGN OF NUT

stresses in threaded studs and blocks, loaded in both tension and combined tension and torsion. Using the three-dimensional frozen stress technique they investigated both the effect of unsymmetric blocks and of chamfering either the stud or the block at the thread runout. Fessler and Jobson (35) found that chamfering the first two threads of the block eliminated the stress concentrations at the front of the block but did not significantly reduce the stresses in the stud. The maximum stresses in the block and stud were dependent on the configuration of the thread at the start of the engagement and was independent of the block cross-sectional shape (36).

In 1985, Fukuoka et al (37) published the results of a finite element analysis for the case of a plate being clamped by a nut and bolt. They considered that the problem was axisymmetric and were concerned with analysing the effect of both the axial bolt load, and the torsional loading produced by the frictional forces which were present in the screw threads and at the interface of the base of the nut and the clamped plate. They treated each of these effects separately and combined the stress distributions to obtain the resultant stresses. The axial loading case was used to provide details of the normal stresses at all the mating surfaces of the threads, and from these stresses, the distributed torque was estimated. The crucial problem of the contact interface conditions at the mating threads of the nut and bolt was treated by an iterative method to produce matching displacements at the threads. The effect on the stress concentrations, produced by varying the coefficient of friction between the mating parts was found to be an increase of stress with increase of coefficient of friction. A pseudo three-dimensional finite element solution was obtained for the axisymmetric model by modifying the boundary forces on the threads in accordance with the three-dimensional boundary conditions existing at each thread. Similarity with the axisymmetric solution was obtained. The results of the approximate analysis was shown to be in accordance with the generally accepted evidence that the first loaded thread of the bolt sustained the highest stresses.

In this same period, Kenny and Patterson published a series of photoelastic frozen stress (38, 39, 40) studies concerning the load distribution along mating nut and bolt threads together with detailed stress analysis of the roots of axially loaded bolt threads. Their use of a recording microdensitometer in conjunction with a photoelastic fringe multiplying polariscope (38), enabled the axial shear stress components along the bases of bolt threads to be integrated to give the loads carried by individual bolt threads. The successful stress analysis of any component relies on a correct specification of boundary loads and/or displacements. Sopwith's (4) theory for the thread load distribution in fasteners has gained general acceptance as being the most realistic analytical prediction of the load distribution. Kenny and Patterson (39), through frozen stress studies of full scale models of 30 mm ISO nuts and bolts, obtained good comparison with Sopwith's theory (Figure 7). They further showed that it was incorrect to assume that normalised maximum root stresses were equivalent to thread load as had been previously used (4, 19) and that the employment of nut deformations to deduce thread load distributions (10) was unsatisfactory.

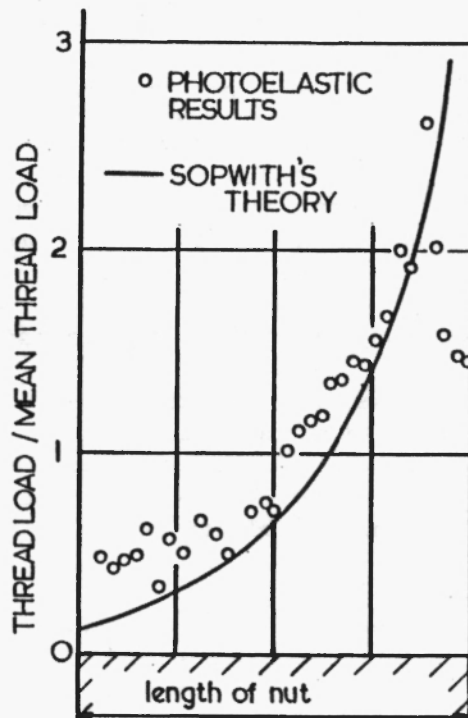


FIG. 7 COMPARISON OF SOPWITH'S THEORETICAL LOAD DISTRIBUTION WITH THE PHOTOELASTIC RESULTS OF KENNY AND PATTERSON.

The use of tapered or truncated thread forms have both been advocated in the literature as a means of improving the stress in loaded threads and some limited results using these devices were obtained by Kenny and Patterson (40). They considered four modifications to the first three threads at the load bearing faces of the nuts, these being a complete straight taper of the full thread form and three angles of truncation of the tips of the threads by a straight taper. They concluded that the maximum stress in the bolt always occurred within one pitch of the load bearing face of the nut, that truncating the threads increased the maximum bolt stresses and that tapering the thread form for the first three threads as advocated by Sparling (33) produced a more uniform load distribution in the bolt-threads.

A substantial photoelastic study was reported by Patterson and Kenny (41) which was aimed at optimising the improvement in the stress distributions reported above when using a fully tapered thread for part of a nut. The investigation concerned 30 mm ISO coarse series nuts and it was found that a two degree taper for five and a half threads from the loaded face of the nut, reduced the maximum normalised tensile stress in the bolt by 41%. One disadvantage of this technique for improving the stress distributions was found to be that if the angle of thread taper was too large, the stress concentration produced at the junction of the parallel and tapered threads could negate the beneficial effects of the taper geometry.

When comparing experimental thread load distributions with the smooth analytical result of Sopwith, it is found that a discrepancy occurs at the loaded face of the nut. This may be explained by the fact that the nut thread in the first pitch from the loaded face is not fully formed and exhibits a continuously changing cross section. Thus the first nut thread exhibits a lower stiffness and load bearing capacity than a fully formed thread. Patterson and Kenny (42) described a modification to Sopwith's analysis whereby the variable stiffness of the first loaded nut thread was obtained by means of a finite element study of various partly formed threads. This process enabled deflection factors to be obtained for a selected number of points along the non-fully formed thread. Cubic splines were then fitted to this data and the numerical representation of these splines was then used in a numerical solution of the governing equation of the problem. The results of the modified theory compared well with those from three-dimensional photoelastic analyses of the loads in the threads of two bolts fitted with conventional nuts.

In 1986, Fukuoka et al (43) published a further application of their axisymmetric finite element analysis to study the case of hollow bolts. The thread form chosen was a coarse metric thread and the bolt was subjected to a uniform axial stress in the shank portion remote from the threads. Three sizes of bolt were studied, namely M6, M24 and M48 with inner bore diameter to nominal bolt diameter ratios, 0, 1/16, 1/3 and 1/2. They concluded that the load concentration on the first loaded thread increased as the bore of the hollow bolt increased and as the coefficient of friction on the threads increased. This effect was more marked on the smaller sizes of screw threads. However, their analysis indicated that the stress concentration at the first thread decreased as the bore of the hollow bolt increased, increased with increase of coefficient of friction and became more important as the nominal diameter of the bolt increased.

Tanaka and Yamada further developed their finite element analyses and a technique for the analysis of externally loaded threaded fasteners was published (44). Both an axisymmetric and a three-dimensional finite element model were used to obtain force ratios for a through bolt, a tap bolt, a stud and a T-type flange. It was concluded that the finite element method produced force ratios which differed from those obtained when using traditional methods. Subsequently (45) the boundary element method was applied to the case of a bolted flat plate. It was concluded that for this specific problem the method required simple input data and that a part partitioning technique also made it possible to analyse axisymmetric and three-dimensional fastener problems when subjected to actual service loading.

Recently, a further photoelastic study by Patterson & Kenny (46) was concerned with modifying the external shape of a nut such that when axially loaded, it suffered beneficial displacements of the first three loaded threads of the nut. This approach to improve the stress distributions along the threads was pursued because the manufacture of tapered threads, although feasible, was not economically attractive for the mass production industry whereas modifying

the external geometry of a standard nut would not be as costly. The modifications tested consisted of an external circumferential groove together with a conical bearing face to the nut. These modifications resulted in a reduction of 26% in the maximum bolt stresses.

### 3. DISCUSSION

It is clear that the stress analysis of threaded connections is of continuing interest. A purely analytical approach to the problem is beset by so many problems associated with complex geometry and boundary conditions at individual threads that it is doubtful that this will ever prove to be a fruitful exercise.

The use of displacement measurements taken on the exterior or interior of an actual threaded connection does not lead to reliable results for the stresses in individual threads. The most useful experimental stress analysis technique for this problem would still appear to be the frozen stress technique of photoelasticity. Although this technique relies on the production of an accurate model in an epoxy resin material, it does automatically provide the natural boundary conditions for the complicated thread mating geometry. The major drawback of this technique however is the fact that the stresses obtained from such a study are elastic stresses. In practice, most threaded connections are tightened initially to yield stress conditions, if not in the shank of the bolt then certainly in the threads of the bolt.

Of the computer based methods using numerical analysis, the finite element and boundary element method are the most versatile. In theory, the finite element method is capable of modelling a threaded connection in full three dimension form, yet few if any such analyses have been reported in the literature. Most studies tend to treat the problem in two dimensions only and manipulate boundary conditions into a pseudo three-dimensional state to simplify the analysis. It is clear from photoelastic analyses that the stresses in the roots of the threads vary cyclically around the thread helix and so the problem is not one of two dimensions. The potential advantage of the finite element method lies in its ability to model elastic-plastic behaviour. Evidence of this type of analysis in the literature is sparse. The new boundary element method would appear to be useful for studying the behaviour of clamped components but at the moment it cannot be applied to study the detailed stresses in individual threads. Perhaps the recent developments of combining the finite element method with the boundary element method will produce a combined technique capable of solving this difficult problem.

## REFERENCES

1. WHITWORTH, J., 'On a uniform system of screw threads', Inst. Civil Eng. Proc., 1: 157-160. 1841
2. STROMEYER, C. E., 'Stress distribution in bolts and nuts', Inst. Naval Arch. Trans., 60: 112-122 and plates XIII and XV. 1918
3. DEN HARTOG, J. P., 'The mechanics of plate rotors for turbo-generators', Trans. ASME, Paper no. APM 51-1. 51: 1-10. 1929
4. SOPWITH, D. G., 'The distribution of load in screw threads', Inst. Mech. Engrs., App. Mech. Proc., 159: 373-383. 1948
5. SOLAKIAN, A. G., 'Why threaded parts fail?' Product Engineering, 4(7): 261-262. 1933
6. HEYWOOD, R. B., 'Tensile fillet stresses in loaded projections', Proc. Inst. Mech. Engrs., 159: 384-391. 1948
7. LEWIS, W., Proc. Eng. Club., Philadelphia, vol. 10, p.16. 1893
8. MOORE, H. F., and HEYWOOD, P. E., 'The strength of screw threads under repeated tension', University of Illinois, Engineering Experimental Station, Bulletin no. 264. 1934
9. THURSTON, R. C. A., 'The fatigue strength of threaded connections', Trans. ASME, Paper no. 51-SA-11, 73(11): 1085-1092. 1951
10. GOODIER, J. H., 'Distribution of load on threads of screws', Trans. ASME, 62: A10-A16. 1940
11. HETENYI, M., 'The distribution of stress in threaded connections', Expt. Stress Anal., 1(1): 147-156. 1943
12. HETENYI, M., 'A photoelastic study of bolt and nut fastenings', Jnl. Appl. Trans. ASME, 65: A93-A100. 1943
13. WEIGLAND, H., 'Mutterwerkstoff und Dauerhaltbarkeit der Schraubenverbindung', Zeitschrift VDI, vol. 83, p.64. 1939
14. KAUFMANN, F., and JANICHE, W., 'Fatigue strength of steel bolts and studs fitted with steel or magnesium alloy nuts', Zeitschrift VDI, 85:504. 1941
15. STOECKLY, E. E., and MACKE, H. J., 'Effect of taper on screw thread load distribution' ASME, Trans., 74(1): 103-112. 1952
16. ZHUKOVSKII, N. E., 'Load distribution in the threads of a screw and nut', Bulletin Politekhnikheskogo obshchestva, 1. 1902



17. KOLENCHUK, K. I., 'Load distribution in the threads of bolt-nut pair', Collection of articles 'Bolted connexions', Mashgiz.  
1949
18. YAKUSHEV, A. I., Effect of manufacturing technology and basic thread parameters on the strength of threaded connections, Pergamon Press, London. 1964
19. BROWN, A. F. C., and HICKSON, V. M., 'A photoelastic study of stress in screw threads', Proc. I.Mech.Eng., IB: 605-612. 1952
20. BLUHM, J. L. and FLANAGAN, J. H., 'A procedure for the elastic stress analysis of threaded connectors including the use of an electrical analogue', Expt. Stress Anal., XV (1): 85-100. 1957
21. CHALUPNIK, J. D., 'Stress concentrations in bolt thread roots', Expt. Stress. Anal., XXV(2): 398-404. 1968
22. MARUYAMA, K., 'Stress analysis of a nut-bolt joint by the finite element method and the copper-electroplating method (1st report)', Bull. JSME. 16 (94), 671-78. 1973
23. MARUYAMA, K., 'Stress analysis of a bolt-nut joint by the finite element method and the copper-electroplating method', Bull. JSME., 17(106), 442-450. 1974
24. BIRGER, I. A., 'Extending life of threaded joints by increasing thread root radius', Russ. Eng. Jnl., XLVIII(10):50-52. 1968
25. WALKER, R. A., and FINKELSTON, R. J., 'Effect of basic thread parameters on fatigue life', SAE, Trans., vol. 79, paper no. 700351, pp. 12. 1970
26. SEIKA, M., SASAKI, S., and HOSONO, K., 'Measurement of stress concentrations in threaded connections', Bull. JSME, 17(111): 1151-1156. 1974
27. MOTOSH, N., 'Load distribution on threads of titanium tension nuts and steel bolts', J.Eng. Ind. Trans. ASME, 97(B): 162-166. 1975
28. DONISELLI, C. and MONDINA, A., 'Influence of shape of nut on stresses in threaded connections', 6th Canadian Eng. App. Mech., Vancouver, B. C., Ed. V. J. MODI. Cancam, Vancouver. 1977
29. OTAKI, H., 'Stress distribution at a bolt thread root in a bolt and nut coupling', Bull. JSME, 28. 1-29. 1977
30. BRETL, J. L., and COOK, R. D., 'Modelling the load transfer in threaded connections by the finite element method', Int. J. Numer. Methods Eng., 14(9): 1359-1377. 1979

31. TANAKA, M., MIYAZAWA, H., ASABA, E., and HONGO, E., 'Application of the finite element method to bolt-nut joints - Fundamental studies on analyses of bolt-nut joints', Bull. JSME, 24(192): 1064-1071. 1981
32. TANAKA, M., HONGO, K., ASABA, E., 'Finite element analysis of the threaded connections subjected to external loads', Bull. JSME, 25(200): 291-298. 1982
33. SPARLING, L. G. M., 1982, 'Improving the strength of screw fasteners', Chartered Mech. Eng. 29(4) 58-59. 1982
34. MILLER, D. L., MERSHEK, K. M., and NAJI, M. R., 'Determination of load distribution in a threaded connection', Mech. and Machine Theory, 18(6): 421-430. 1983
35. FESSLER, H., and JOBSON, P. K., 'Stresses in a bottoming stud assembly with chamfers at the ends of the threads', Jnl. Strain Anal., 18(1): 15-22. 1983
36. FESSLER, H., and WANG JIONG-HUA, 'Stress analysis of some unsymmetric screwed connections', Jnl. Strain Anal., 19(2): 111-119. 1984
37. FUKUOKA, T., YAMASAKI, N., KITAGAWA, H., HAMADA, M., 'A stress analysis of threaded portions in fastenings', Bull. JSME, 28(244) 2247-2253. 1985
38. KENNY, B., and PATTERSON, E. A., 'The use of a microdensitometer in photoelastic analyses', Strain JBSSM, 21(1) 13-18. 1985
39. KENNY, B., and PATTERSON, E. A., 'Load and stress distribution in screw threads', Exptl. Mech., (25) 208-213. 1985
40. KENNY, B., and PATTERSON, E. A., 'Stress analysis of some nut-bolt connections with modifications to the nut thread form', Jnl. Strain Anal. 20 (1) 35-40. 1985
41. PATTERSON, E. A., and KENNY, B., 'The optimisation of the design of nuts with partly tapered threads', Jnl. Strain Anal., 21 (2) 71-84. 1986
42. PATTERSON, E. A., and KENNY, B., 'A modification to the theory for the load distribution in conventional nuts and bolts'. Jnl. Strain Anal., 21 (1) 17-23. 1986
43. FUKUOKA, T., YAMASAKI, N., KITAGAWA, H., HAMADA, M., 'Ratio of flank loads of screw threads in a hollow bolt', Bull. JSME, 29(247), 265-272. 1986
44. TANAKA, M., and YAMADA, A., 'The behaviour of fasteners under external loading', Bull. JSME, 29(248), 617-624. 1986

45. TANAKA, M., and YAMADA, A., 'Application of the boundary element method to threaded connections', Bull. JSME, 29(251), 1628-1634. 1986
46. PATTERSON, E. A. and KENNY, B., 'Stress analysis of some nut-bolt connections with modifications to the external shape of the nut', Jnl. Strain Anal., 22(4), 187-193. 1987

