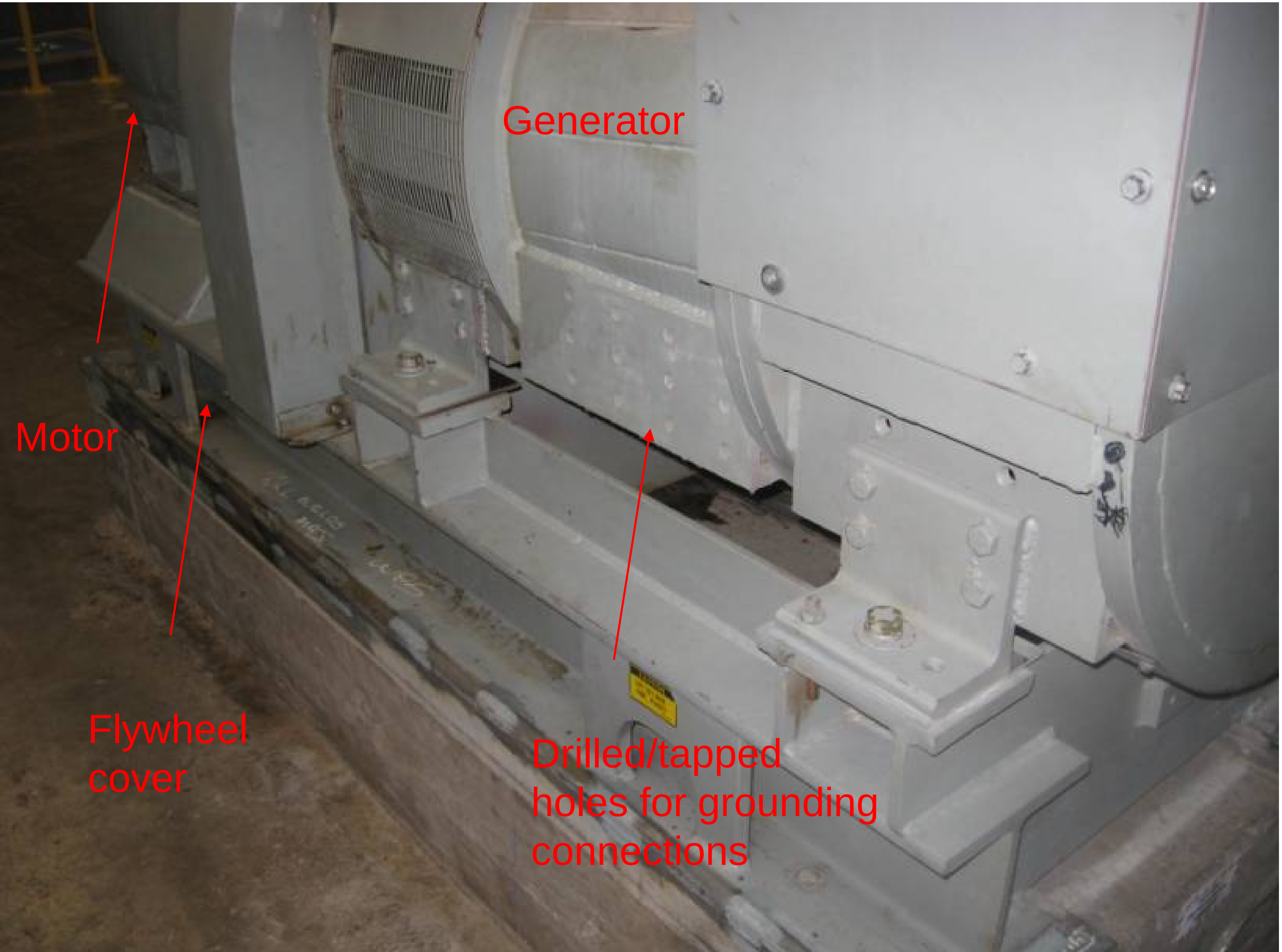
Introduction

- Dynamic Vibration Absorber
  - Case study of successful use on motor generator
  - Theory, design, application

by electricpete

- Machine Description
  - Application: Rod Drive MG Set.
  - 1800 rpm 200hp induction motor
  - Gear coupling
  - Synchronous generator
  - Flywheel overhung off inboard end of generator

Side View Of RS Generator.



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The situation

- Elevated Vibration
- Symptoms of horiz resonance moderately close to running speed”
  - horizontal vib higher than vertical by factor 2.5 - 3
  - Coastdown shows resonance ~ 1640 (running speed 1800)
  - Bump test show resonance ~ 1680
  - Temporary bracing in horizontal direction resulted in substantial vib decrease
- Cracks in foundation (not easily repairable)

Foundation Photo. Similar cracks on both sides of foundation suggest cracks all the way through (perpendicular to shaft)

Closeup of grounding pad (one on each side of generator)... pre-existing drilled tapped holes... perfect for mounting D.A.

Photo of Installed D.A.

Our MG Absorber Overview

(more complicated than needed - wanted fine tune ability)

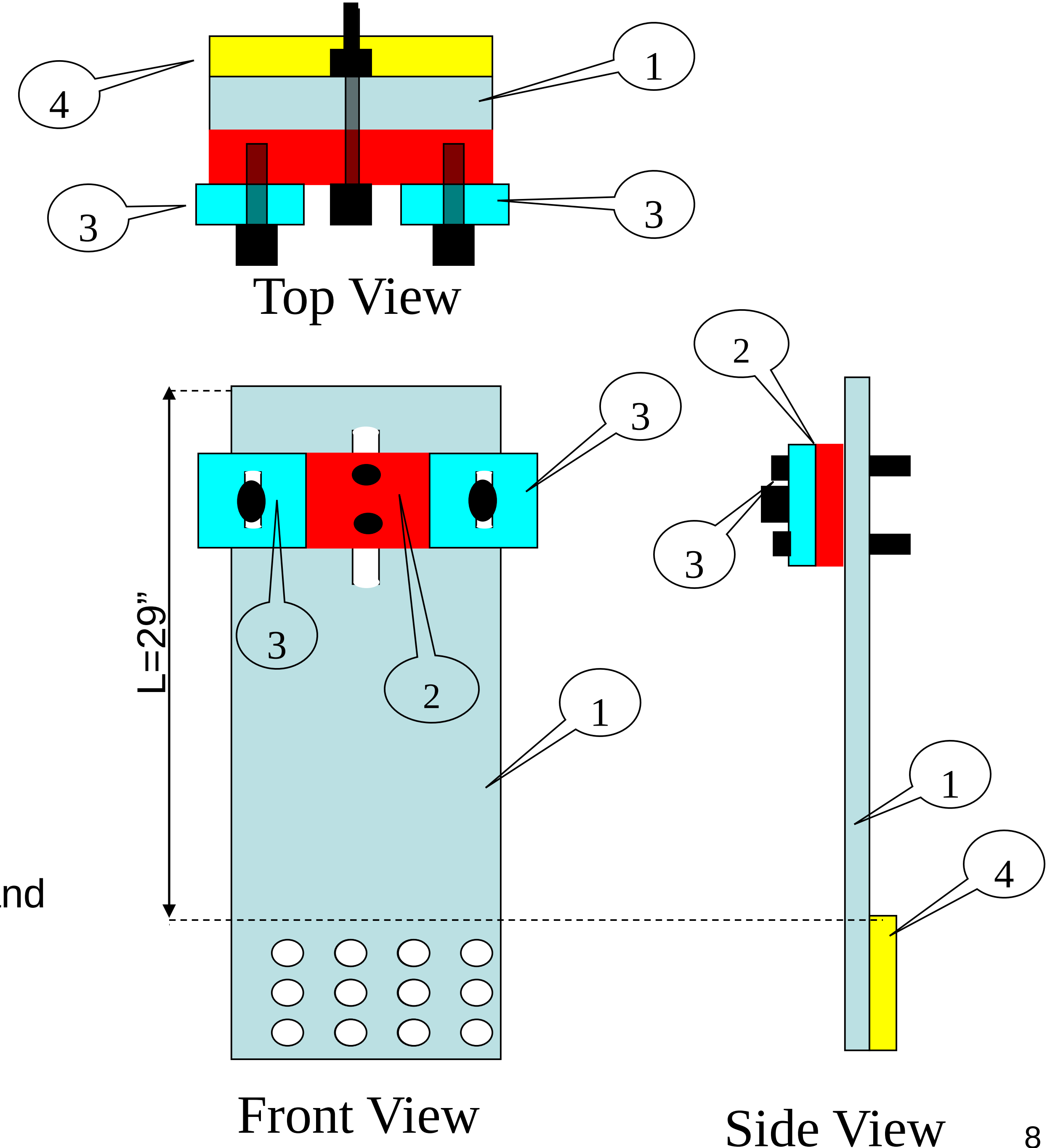
1=Main Bar
(1”x12”x..
35” total length but..
...29” effective length

2=Weight Bar (gross tuning)
(1”x12”x6”)

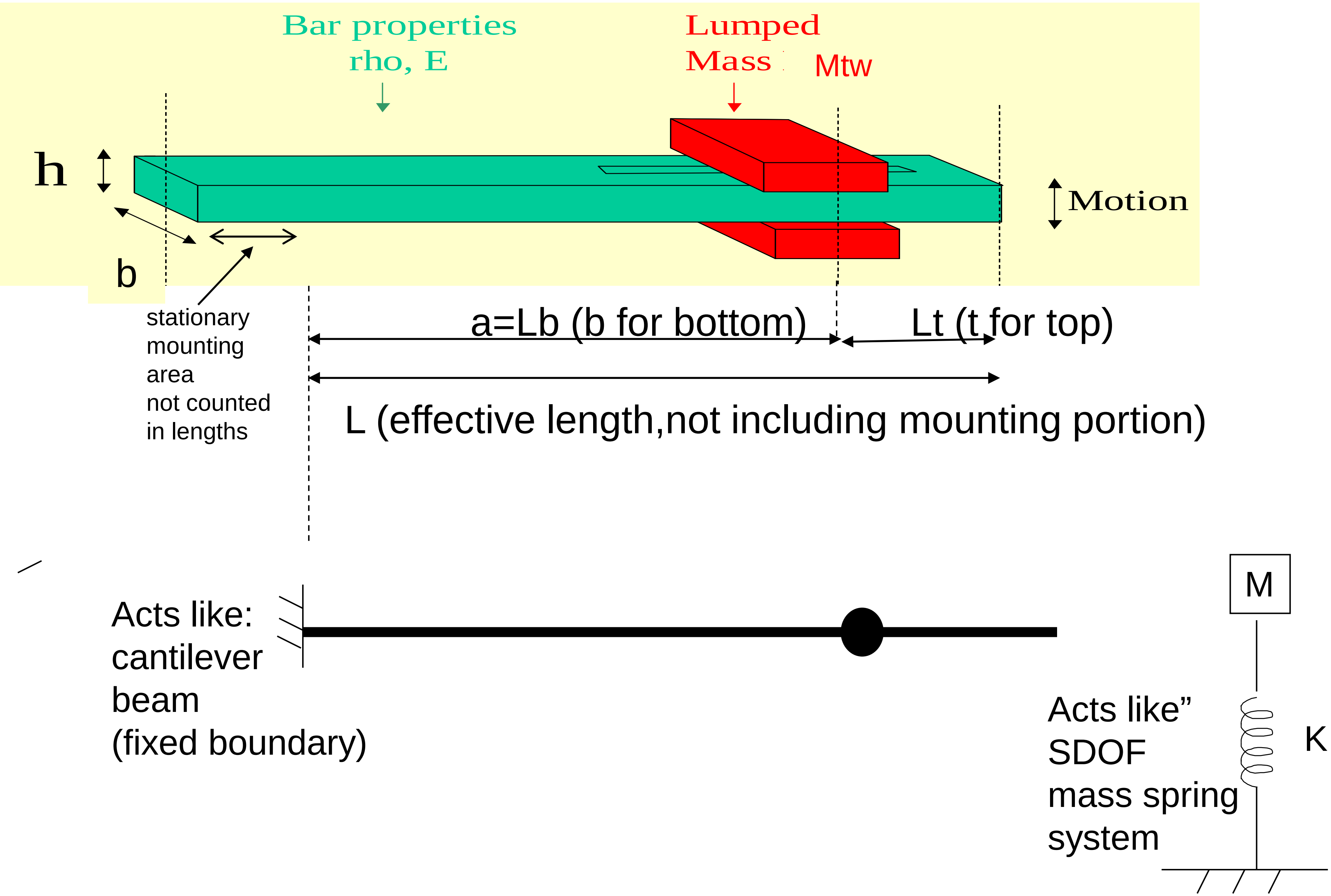
3 = Fine-Tune Blocks (2)
(1”x 6” x6”)

4 = Spacer bar - used to attain clearance between absorber and rounded contour of machine
(1”x12”x6”)

All steel: ASTM A36
All bolts: 5/8” SAE Grade 5



Simpler General Figure with dimensions symbols to be used throughout remainder of presentation



Vibration Results

	MOA	MOH	MOV	MIH	MIV	GIH	GIV	GOA	GOH	GOV	
Position:	1A	1H	1V	2H	2V	3H	3V	4A	4H	4V	Comments
Units:	ips	ips	ips	ips	ips	ips	ips	ips	ips	ips	
11/12/09	0.05	0.22	0.06	0.26	0.07	0.20	0.08	0.12	0.26	0.04	Before installing DA
11/13/09	0.07	0.16	0.07	0.15	0.07	0.07	0.07	0.08	0.1	0.04	After installing DA
CHANGE	40%	-27%	17%	-42%	0%	-65%	-13%	-33%	-62%	0%	

- Highest vibration on the generator reduced from 0.26 to 0.10
- Highest vibration on the entire machine reduced from 0.26 to 0.16
- Dramatic improvement on the highest generator positions (3H and 4H)
- Unexpected improvement in the motor positions 1H and 2H even though the absorber was installed on the generator. There must be some communication between motor and generator either through the coupling or through the base

3 Calculation Objectives / Types (deleted previous slide 11)

1. Design absorber dimensions/weights to match the resonant frequency of the absorber to the exciting frequency causing vibration.

2. Select absorber mass high enough to ensure sufficient separation of new resonant frequencies from exciting frequency

3. Analyse stress (the difficult and often-neglected calculation)

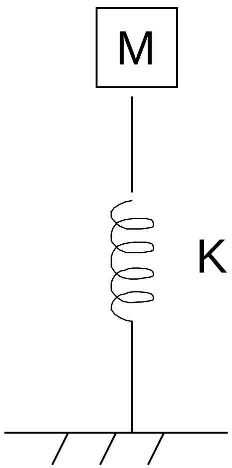
Terminology: Two different ways of expressing frequency: f and w

- f is the more familiar frequency, normally expressed in hz
- w is the radian frequency, normally expressed in radians/sec. It is less familiar in ordinary conversation, but easier to use for vib formulas. We will mostly use w, shorthand for ω (l.c.omega)
- The relationship between these two types of frequency is:

  w = 2*pi * f

Single Degree of Freedom System - Resonant Frequency

w = sqrt(k/m)



<u>SI Example:</u>

K = 10000 N/m
M = 1 KG
w = sqrt(k/m) = sqrt(10000/1) = 100 radians/sec
f = w/(2*pi) = 15.9hz

<u>Same Example In English/Imperial Units</u>

K = 57.1 lbf/inch
M = 2.2 lbm

1 by F=m*a

$$w = \sqrt{\frac{57.1 * lbf\ /\ inch}{2.2 * lbm} * \boxed{\frac{1 lbm * 32.2 * ft\ /\ \sec\text{^}2}{lbf}} * \frac{12 * inch}{ft}}$$

w = 100 radians / sec
f = w/(2*pi) = 15.9 hz

The equations in this presentation are suitable for direct computation in SI units.
That includes meters, kg, sec, and all quantities derived from those three(N, Pa, etc)
The results will be given in SI units without any unit conversions required.
These equations can also be computed in English/SI units, but you need to plug in
the units and supply appropriate conversions as you go (similar to what was done above).

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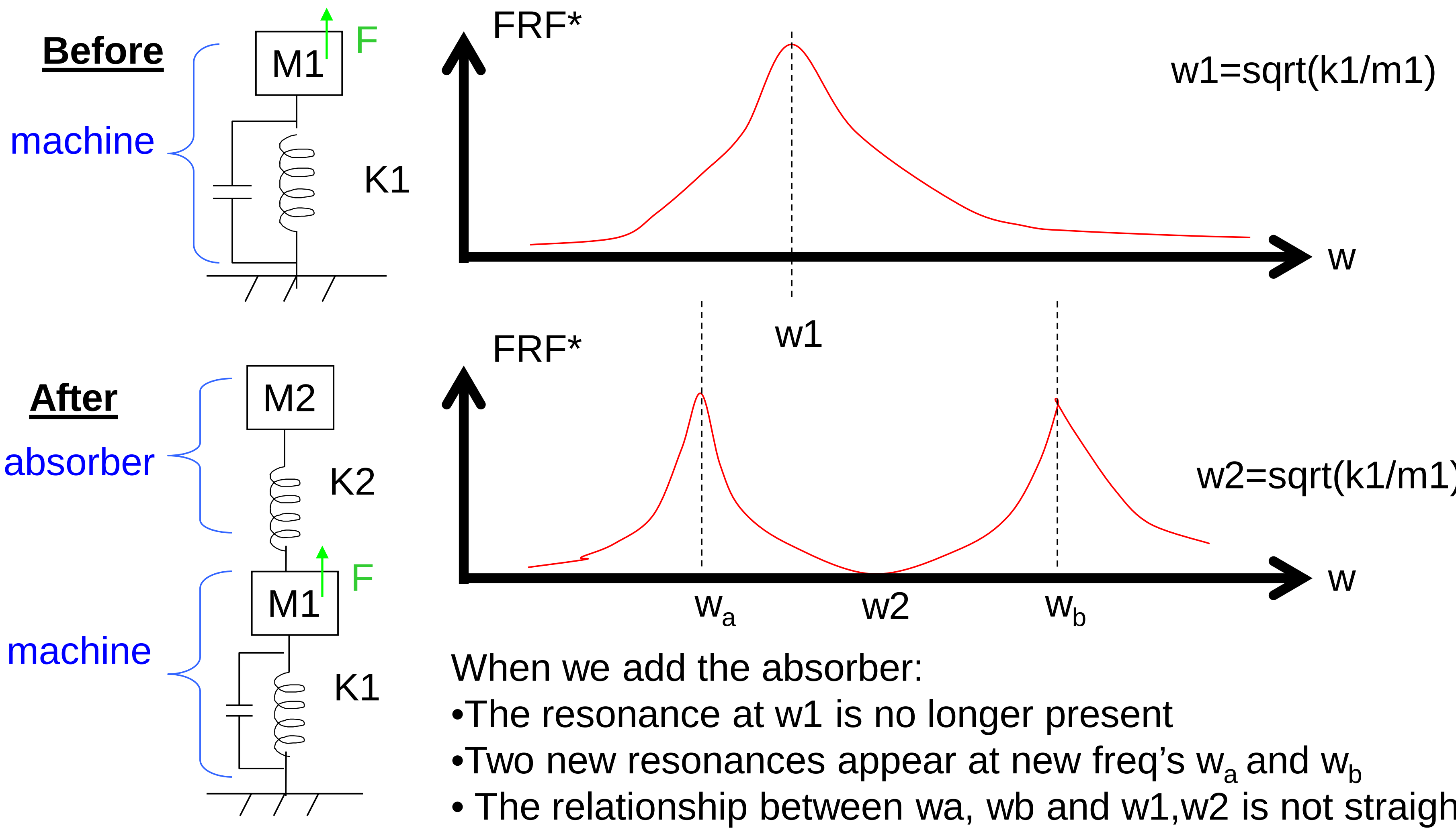
Effect of Adding Dynamic Absorber onto a SDOF system:



When we add the absorber:

- The resonance at w1 is no longer present
- Two new resonances appear at new freq's w_a and w_b
- The relationship between wa, wb and w1,w2 is not straight forward, but we know wa < w2 < wb   (more later)
- The vib response goes to zero (near zero IRL) at w = w2
- We will typically design our absorber so w2 matches exciting freq.

*Freq Resp Function
for displacement of m1
divided by force @ M1

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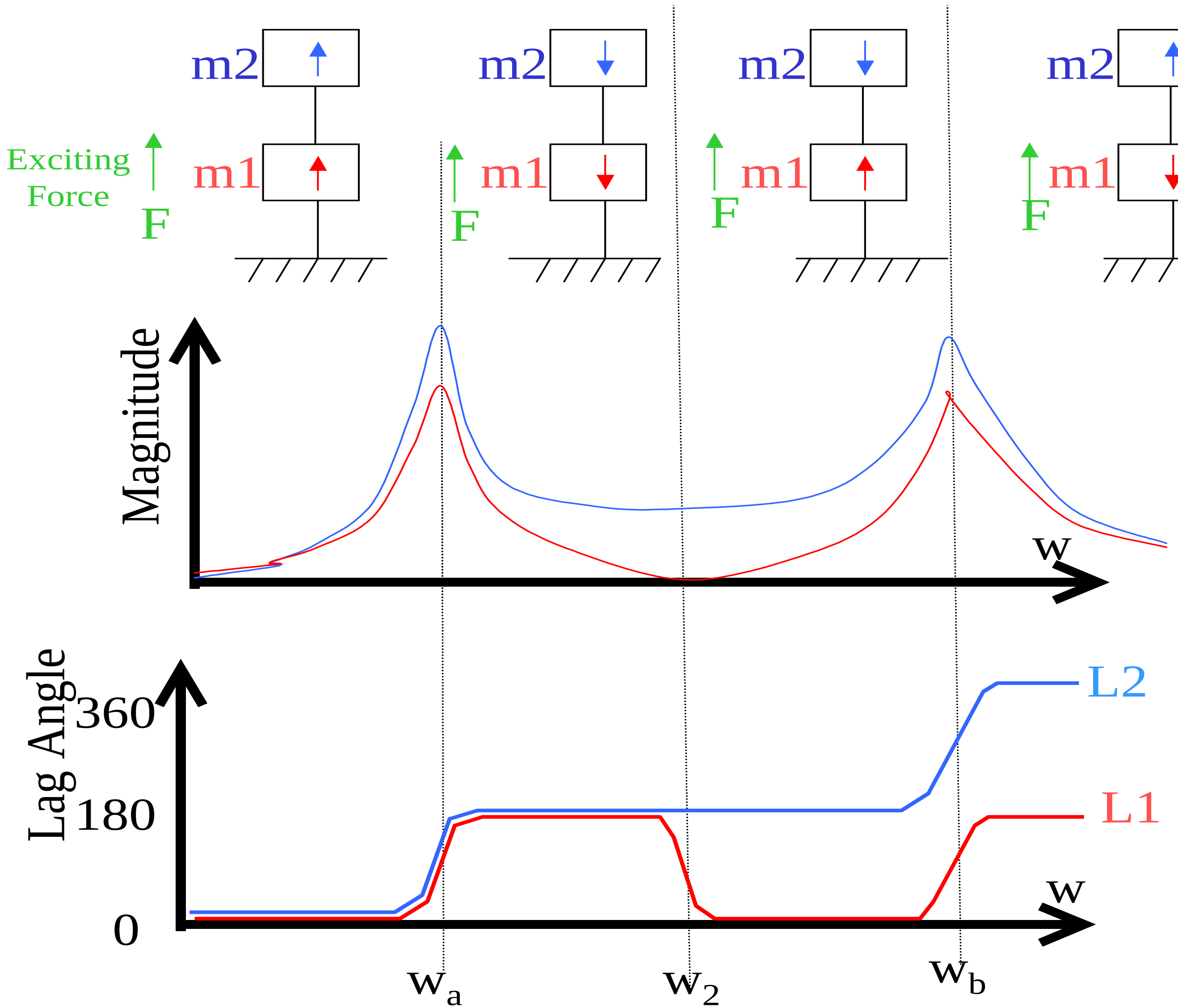
Displacements of 2DOF System with force applied at m1.



spring symbols omitted.

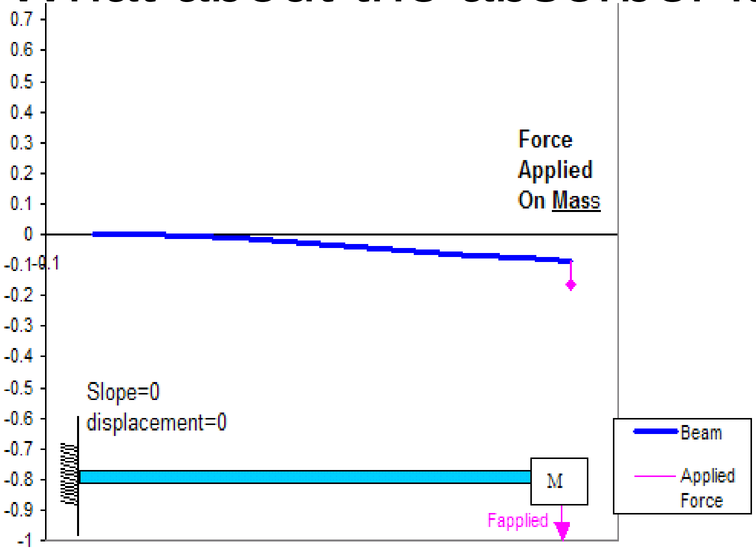
Also assumes damping present in parallel with k1 not k2.

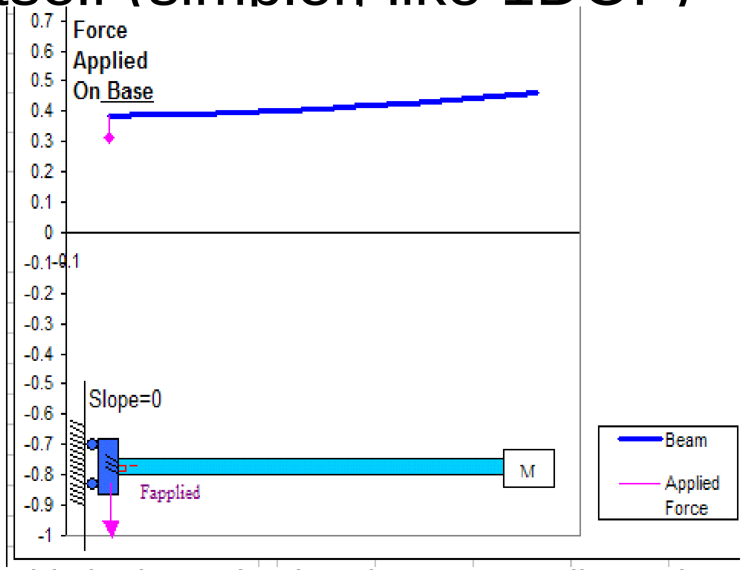
<u>Frequency Labels</u>:

- wa=lower resonant freq of composite system
- wb=upper resonant freq of composite system
- w2=sqrt(k2/m2) = tuned freq of absorber.... the frequency where m1 movement goes to 0.

Note as freq passes from below to above w2, it experiences a phase change - this corresponds to sign change as displacement passes thru 0

Previous slide focused on 2DOF (complicated)
What about the absorber itself (simpler, like 1DOF)





Double Click Above For Simulation

This is typical beam.
* Force applied at right end.
* Fixed BC at left end (base)
  (slope is 0 and disp is 0)

This is dynamic absorber. Same dimensions/weights, but..
* Force applied at left end (base)
* Unusual BC at left end (base).
  (Slope is 0, but displacement is not restrained, as if the blue wheels are magnetically attracted to base...can roll but cannot leave base)

At beam resonant frequency w2:
Displacement / Force is <u>max</u> *

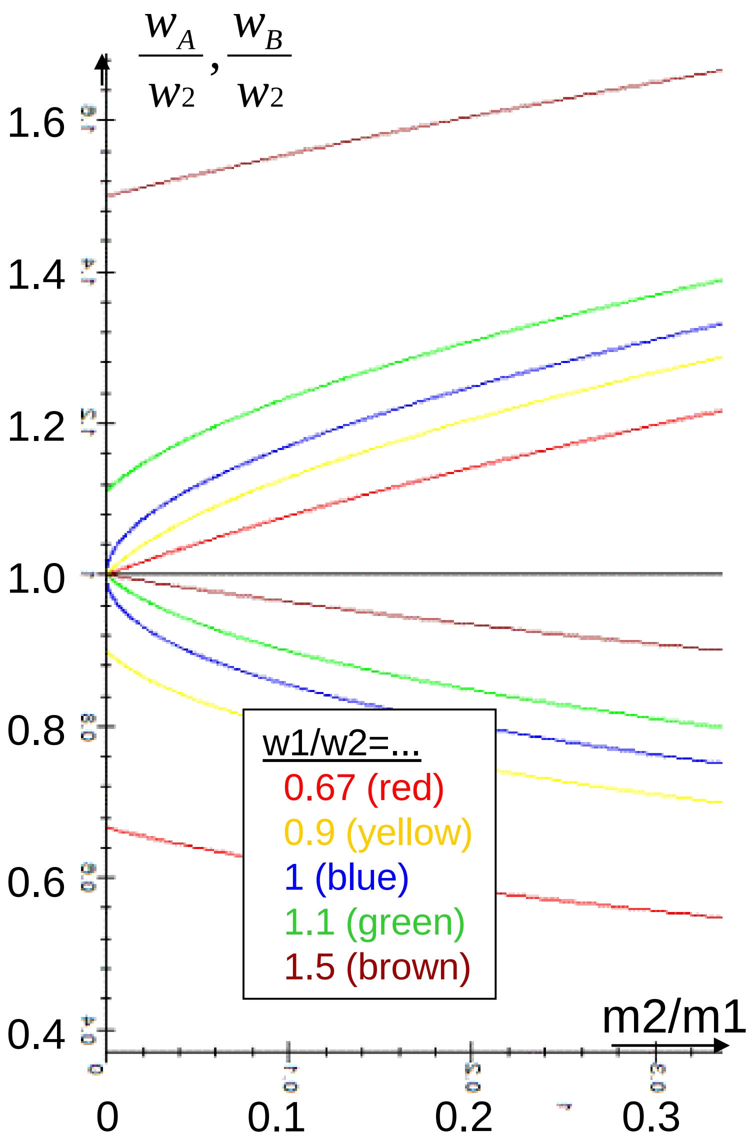
At the same frequency (w2)
Displacment / Force is <u>minimum</u> **

* this statement applies at the right end of the beam (free end)

** this statement applies at the left end of absorber (at the base)

That means for any applied force magnitude, the least movement of the base occurs when w=w2. At this frequency the absorber has a very high dynamic stiffness and can provides a large reaction force with very little movement.

Separation of resonant frequencies from w2=tuned freq
We said wa<w2<wb... but how far apart are they?



k1, m1, w1 represent machine alone.
k2, m2, w2 represent absorber (alone/rigidly mounted)
wa, wb are 2 resonant freq's of composite system

We should always tune w2 to exciting freq in order to get min response ("zero") at exciting freq
Typically the exciting freq is running speed (not necessarily w1)

We want separation between wa, wb and w2 in order to have separation between exciting frequency and composite system res. freqs ...i.e. we want w_res/w2 far from 1.
We can see this is accomplished by increasing m2/m1

For the typical case with w1~w2 (correcting a resonance), we see from blue curve:
- m2/m1=0.5% => 3% frequency separation
- m2/m1=1% => 5% frequency separation
- m2/m1 = 3% => 9% frequency separation

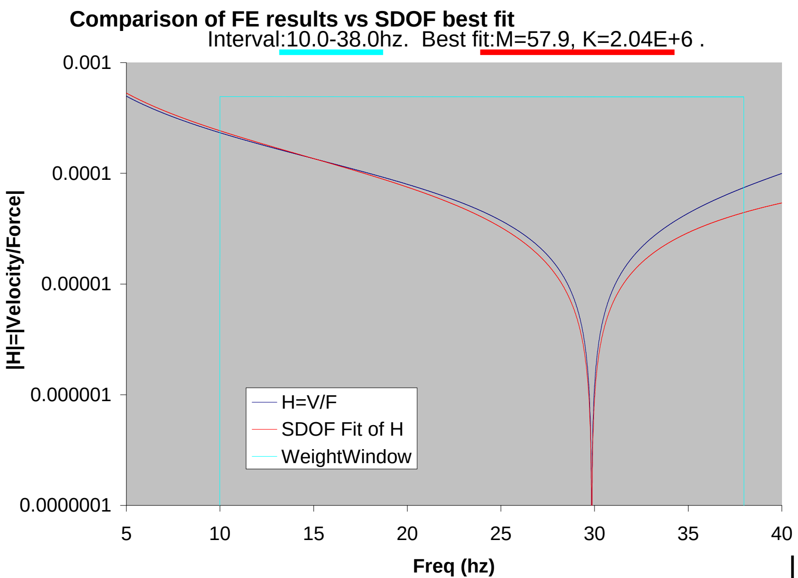
Lesson: Larger M2 makes tuning easier. 2% is a good target, but not always practical.

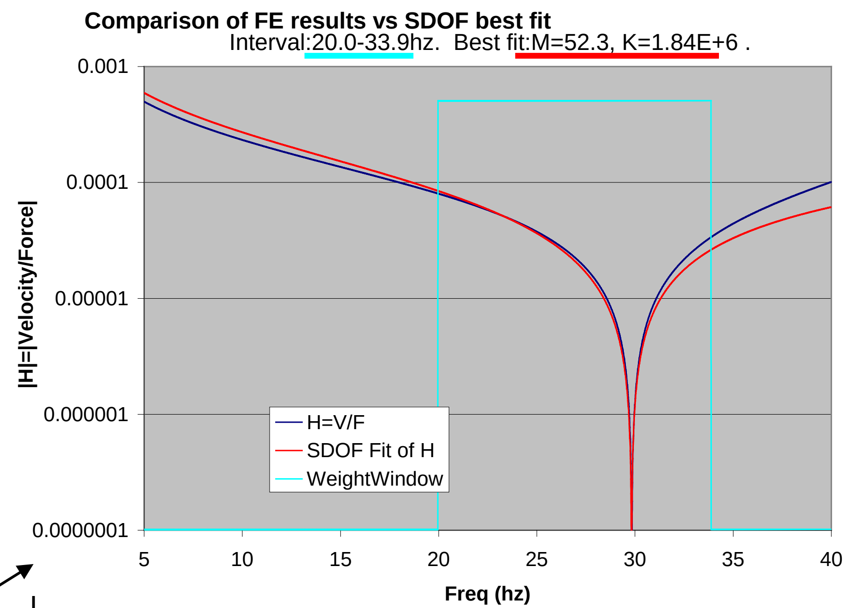
Picking high M2 satisfies the 2nd calc objective.

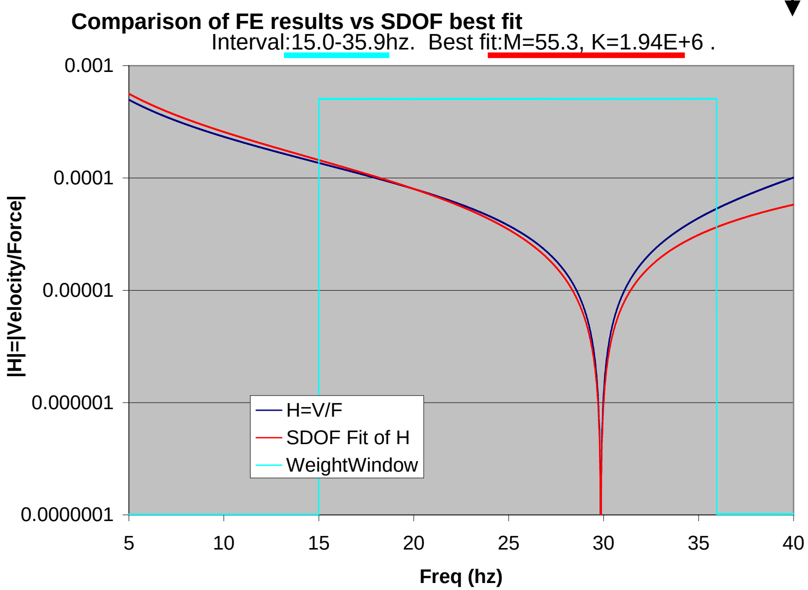
Q. What to use for M2 (when calculating M2/M1) given that my absorber includes distributed mass of the bar?

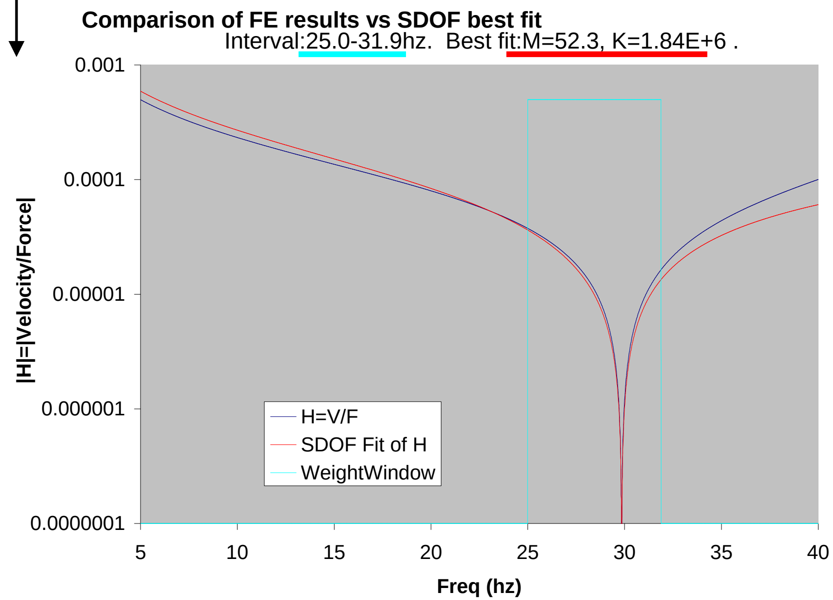
- We need to determine effective mass and stiffness of absorber that “acts” like lumped SDOF m2, k2 - not at all an easy task.
- For the MG absorber, F.E. analysis and curve fit (next page) showed effective m2=52.3kg, while actual total mass was 62.5kg
- MG absorber has relatively small tuning weight mass and large bar mass
- An absorber with relatively higher tuning weight mass will have higher ratio of effective to total mass (because if bar mass is negligible, the system is lumped and the tuning weight mass = total mass = effective mass)
- Based on the above, we conclude that total absorber weight is a reasonable estimate of effective M2 for most purposes.

MG Absorber best-fit Meffective converges to 52.3kg as zoom-in closer around 30hz. Actual total mass is 62.5kg









For a simple lumped system m2,k2, we can find w2 = sqrt(k2/m2) But our absorber is not that simple...we have distributed mass of beam which cannot be neglected. To find w2 in this situation , we need Dunkerley's approximation.

- Dunkerley's approximation to resonant frequency:
- Given: We have a spring network with masses attached: m1, m2, m3… mn
- Let w1 = resonant frequency of the spring network with only mass 1 attached
  - w2 = resonant frequency of the spring network with only mass 2 attached
  - wn = resonant frequency of the spring network with only mass n attached
- Then the resonant frequency of the entire system (all masses attached) can be estimated as $w_{dunkerly}$ which satisfies

$$\frac{1}{\left(w_{Dunkerley}\right)^2} = \frac{1}{\left(w_1\right)^2} + \frac{1}{\left(w_2\right)^2} + \dots \frac{1}{\left(w_n\right)^2}$$

If we have only 2 masses (n=2) and solve for $w_{dunkerly}$, we have:

$$w_{Dunkerley} = \sqrt{\frac{1}{\frac{1}{\left(w_1\right)^2} + \frac{1}{\left(w_2\right)^2}}}$$

The last formula is the one we will use.

Note - we are now working on calculation type 1 = sizing the D.A. to give the correct resonant frequency

Derivation of my formula for tuning weight based on Dunkerley

> # ==== PART 1 - SYMBOLS ======

E = Young's Modulus
I = area moment of inertia
mu = distributed mass per length of the bar
Mtw = mass of lumped tuning wieght attached distance "a" from fixed end
a = distance mentioned above
L = length of bar
w1 = radian nat freq of system with mass m1 only (no M2)
w2 = radian nat freq of system with mass M2 only (no m1)
wn = calculated radian natural frequency
(radian natural frequency w = 2*pi*f)

===== PART 2 - CALCULATIONS ====

1/wn^=1/w1^2+1/w2^2	# Dunkerley's approximation
w1^2=12.4*EI/(mu*L^4)	Formula for cantilever beam with uniformly distributed mass per Den Hartog App 5
w2^2=3*EI/(Mtw*a^3);	Formula for masses cantilever beam with mass on end Den Hartog App 1, eqn 2

Solve for Mtw:

Mtw = 3*EI/(a^3*wn^2) - 0.241935*mu*L^4/a^3

Use SI units, or else insert appropriate unit conversions to attain consistent units

Example Calculation of tuning weight mass for MG case

Convert everything to SI. Solve in SI. Convert result to English/Imperial

- Target frequency = 29.98hz
- Target w = 2*pi*f = 2*pi*29.98hz=188.4 rad/sec
- L=29” = 0.7366m
- a=21”=0.5334m
- b=12”=0.3048m
- E=2.9E7psi=2E11 N/m^2
- rho=0.282lbm/in^3=7805.7kg/m^3
- mu = rho*b*h=60.43kg/m
- I=b*h^3/12 = 4.162E-7 m^3
- Mtw = 3*EI/(a^3*wn^2) - 0.241935*mu*L^4/a^3
- In SI Units...
- Mtw:=3*2E11*4.162E-7/(0.5334^3*188.4^2)-0.241935*60.43*0.7366^4/0.5334^3
- Mtw = 18kg = 38.67 lbm

Comparison to other authors' frequency approaches

- For lumped-mass systems, the Dunkerley approximation can also be expressed in terms of static deflections (since static deflection has a unique relationship to resonant frequency for lumped systems).
- Eisenmann and Fox use the static deflection form of Dunkeley for this calculation. It is a misapplication imo because the static deflection does not represent the position of the continuous mass. The authors arbitrarily chose a location on the beam at which to calculate static deflection.

Example Comparison of my Dunkerley approach vs Fox

Problem Definition:

Target Frequency: Fnat = 1800 cpm (30HZ)

Rectangular Bar: b=2“ x h=1“ x L=24"

a = varies between 16 to 24" as per table...

Steel (rho=0.282 lbm/inch^3, E=2.9e7 PSI)

Comparison of Results

	Computed Mtw (lbm)		FE calculation of Fnat (HZ)	
	Method: Fox	Method: Dunkerly	Using Fox Mtw	Dunkerly Mtw
a (inch)				
24	6.314	8.123	32.80	30.08
22	7.214	10.546	34.11	30.02
20	8.393	14.036	35.51	30.00
18	9.978	19.254	37.02	30.00
16	12.177	27.415	38.67	30.05
	* - Calculated using transfer matrix program - available			

Conclusion: My Dunkerley approach creates the target frequency (30hz) much better than Fox' equation. Of course, accuracy of this calculation is not critical if sufficient room for adjustment is provided.

Also note there are errors/approximations common to both approaches:

- Euler/Bernoulli beam model - neglects shear deformation and rotary inertia.
- Treat Mtw as lumped mass - neglect its rotary inertia.
- Assume absorber is perfectly rigidly mounted to machine.
- Assume mass lost by slotting is equal to mass added by hardware.
- Assume motion of the machine is purely transverse (neglect slope from rocking)

Mounting Possibilities

- Bolt into pre-existing holes (like MG)
- Drill/tap holes
- Weld
- Remove accessories like eyebolts to create holes for allthread (see next page for photo/discussion)
- Build a clamp (photo next page) avoids having to drill/weld on the machine itself

Example Mouting Photos

Clamp – eliminated the need to
Drill/weld on the machine itself

All Thread in eyebolt hole very convenient
Cautions for this approach:
1 - All-thread has built in stress concentrations in the threads
2 - Fox and ___ caution against dynamic absorbers that have the same stiffness in both perpendicular directions (I don’t know why).

Tuning

- Leave enough room for adjustment
- Relatively small tuning weight gives a “finer” adjustment ability which increases probability of successful tuning, especially when m2 is small relative to m1 and side peaks are close in.
- Move weight inwards to raise freq, out to lower
- Can tune while running. Although this may result in initially higher vib when start and before tune
- Can pre-tune while shutdown.
  - Bump and monitor at the base of the absorber (not the top)
  - Need to tune so that the minimum response falls on the excitation frequency (NOT so that the peak falls on resonant frequency... we made this mistake!).
    - Key tricky point: Even though w2 is frequency of resonant peak for ideal absorber by itself on rigid mount, it will be a frequency of the minimum of the composite system. We want to use that minimum response to our benefit.

Can dynamic absorbers reduce vibration when the machine itself is not at resonance?

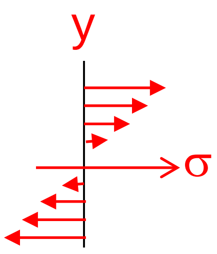
- Absolutely. D.A. should reduce vibration at tuned frequency at the mounting surface of almost any system, regardless of system resonant frequency.
- Some references imply that dynamic absorber will only reduce vibration when at resonance. (Fox, Scheffer). They are wrong.
- HOWEVER, it usually doesn’t make much sense to apply absorber unless we’re at resonance. At resonance, altering the system dynamics through D.A. stiffening etc will likely reduce stresses on machine components such as bearings. If not at resonance, we may just be fixing a symptom – hiding the vibration without reducing any forces.
- Another reason to install an absorber (besides addressing resonance) would be If it is desired to reduce vibration transmitted to foundation... either due to cracking foundation or to minimize vibration transmitted outside the machine.

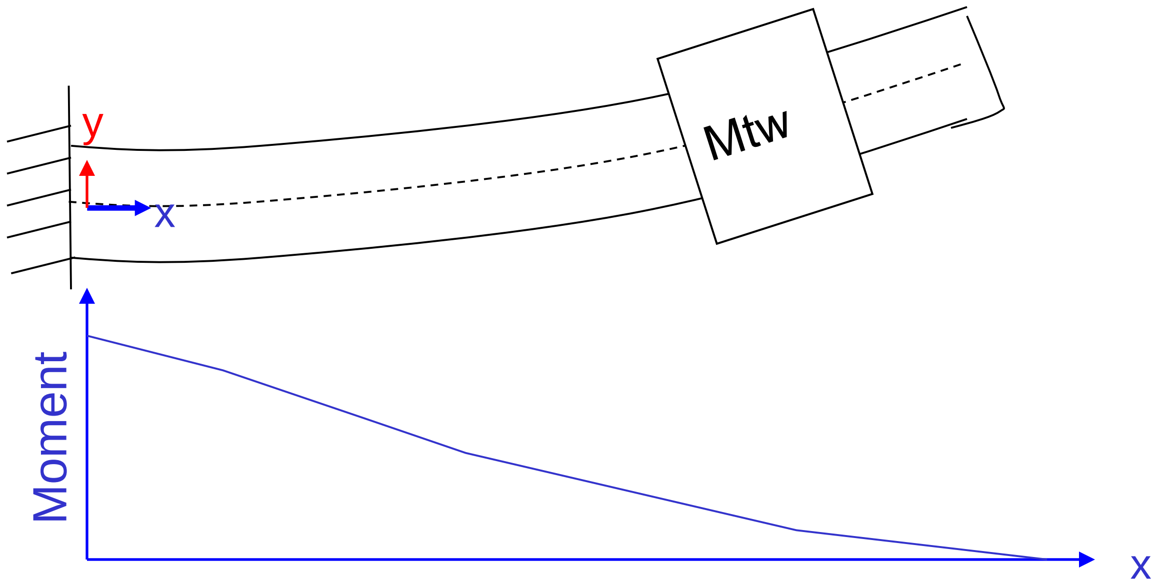
Stress and fatigue

- Dynamic absorbers have a “reputation’ for being temporary fix and reputation that absorber itself is likely to fail. I believe that is because most absorbers are designed by vibration analysts and no stress is done – for example Fox and Eisenman provide formulas for selecting Mtw to create correct tuning, but do not provide any formulas for estimating stress . I have not seen this aspect quantitatively discussed in any references – we will do it here.
- If unsure, err on the side of caution - consider attaching chain to catch absorber in case it breaks from fatigue
- Max stress generally occurs at the base of the absorber (bending stress). Calculating this stress requires knowledge of mode shape - not trivial excercize.
- Stress Analysis can be done. Two different times
  - During design stage: assume a force applied to the machine and calculate stress as a basis for relative comparison
  - After Installation: Measure absorber displacement and convert to stress.

The remainder of this presentation focuses on estimating stress (calc type 3)

Maximum bending stress at base of absorber





σ=y*Moment / I

Where σ = bending stress

I = area MOI = b*h^3 / 12

y= distance from neutral axis

ymax = h/2

σMax=(h/2)*Moment / I

Highest moment occurs at base (left end) of the beam.

Bending stress at a given cross section (x) depends on distance from centerline (y) and is max at the outer surfaces

Max bending stress occurs at the outer surfaces on the left side of the beam.

How to find moment? We need to know the mode shape

- Once we know the mode shape y(x), we can find moment as $M(x) = E*I * d^2y/dx^2$
- We will find mode shape from Euler Bernoulli model
- We could alternatively "assume" an approximate mode shape such as static deflection shape. Static deflection shape yields an estimate of stress which is about 60% higher than Euler Bernoulli for the MG case - we are not willing to accept this error

Mode shape solution for Euler Bernoulli beam

General solution form for Euler Bernoulli (ref Harris)

- $y(x) = Ca*\sin(\beta x)+Cb*\sinh(\beta x)+Cc*\cos(\beta x)+Cd*\cosh(\beta x)$

  where the constants are to be solved from the boundary conditions.

  and β will satisfy $\beta = \sqrt{\omega} * \left(\frac{\rho * A}{E * I}\right)^{1/4}$

Our solution form will be a little more complicated because:

- We need separate solutions for the top and bottom of the beam.
- Grouping the constants into sum and differences (as shown next page) will make it easier to solve the constants
- Using a coordinate L-x instead of x for the top portion of the beam will also make it easier to solve constants.

Assumed form of solution for top and bottom of the beam

$$
\begin{aligned}
yb(x) = \;& Cb1*[\cos(\beta*x)+\cosh(\beta*x)] \\
&+Cb2*[\cos(\beta*x)-\cosh(\beta*x)] \\
&+Cb3*[\sin(\beta*x)+\sinh(\beta*x)] \\
&+Cb4*[\sin(\beta*x)-\sinh(\beta*x)]
\end{aligned}
$$

$$
\begin{aligned}
yt(x) = \;& Ct1\ [\cos(\beta*(L - x)) + \cosh(\beta*(L - x))] \\
&+ Ct2\ [\cos(\beta*(L - x)) - \cosh(\beta* (L - x))] \\
&+ Ct3\ [\sin(\beta* (L - x)) + \sinh(\beta* (L - x))] \\
&+ Ct4\ [\sin(\beta* (L - x)) - \sinh(\beta* (L - x))]
\end{aligned}
$$

where

x = longitudinal coordinate

yb, yt are modeshape of the bottom and top section

Cb_ constants apply to bottom and Ct__ apply to top

We can find slope, moment, and shear force by differentitaing displacement (and multiplying by EI between slope and moment)

Bottom section variables:

$$\mathrm{yb}(x) := Cb2\,(\cos(\beta\,x) - \cosh(\beta\,x)) + Cb4\,(\sin(\beta\,x) - \sinh(\beta\,x))$$

$$\mathrm{sb}(x) := -Cb2\,\sin(\beta\,x)\,\beta - Cb2\,\sinh(\beta\,x)\,\beta + Cb4\,\cos(\beta\,x)\,\beta - Cb4\,\cosh(\beta\,x)\,\beta$$

$$\mathrm{mb}(x) := EI\,(-Cb2\,\cos(\beta\,x)\,\beta^2 - Cb2\,\cosh(\beta\,x)\,\beta^2 - Cb4\,\sin(\beta\,x)\,\beta^2 - Cb4\,\sinh(\beta\,x)\,\beta^2)$$

$$\mathrm{vb}(x) := EI\,\beta^3\,Cb2\,\sin(\beta\,x) - EI\,\beta^3\,Cb2\,\sinh(\beta\,x) - EI\,\beta^3\,Cb4\,\cos(\beta\,x) - EI\,\beta^3\,Cb4\,\cosh(\beta\,x)$$

Top section variables:

$$\mathrm{yt}(x) := Ct1\,(\cos(\beta\,(L-x)) + \cosh(\beta\,(L-x))) + Ct3\,(\sin(\beta\,(L-x)) + \sinh(\beta\,(L-x)))$$

$$\mathrm{st}(x) := Ct1\,\sin(\beta\,(L-x))\,\beta - Ct1\,\sinh(\beta\,(L-x))\,\beta - Ct3\,\cos(\beta\,(L-x))\,\beta - Ct3\,\cosh(\beta\,(L-x))\,\beta$$

$$\mathrm{mt}(x) := EI\,(-Ct1\,\cos(\beta\,(L-x))\,\beta^2 + Ct1\,\cosh(\beta\,(L-x))\,\beta^2 - Ct3\,\sin(\beta\,(L-x))\,\beta^2 + Ct3\,\sinh(\beta\,(L-x))\,\beta^2)$$

$$\mathrm{vt}(x) := -EI\,\beta^3\,Ct1\,\sin(\beta\,(L-x)) - EI\,\beta^3\,Ct1\,\sinh(\beta\,(L-x)) + EI\,\beta^3\,Ct3\,\cos(\beta\,(L-x)) - EI\,\beta^3\,Ct3\,\cosh(\beta\,(L-x))$$

Where

y = displacement
s = slope
m = moment
v = shear

8 Boundary Conditions (BC's)

- BC1: yb(0)=0 [displacement at base is 0]
- BC2: sb(0) = 0 [slope at base is 0]
- BC3: mt(L)=0 [moment at top is 0]
- BC4: vt(L)=0 [shear at top is 0]
- BC5: yb(Lb) = yt(Lb) [continuity of disp @ location of Mtw]
- BC6: sb(Lb) = st(Lb) [continuity of slope @ location of Mtw]
- BC7: mb(Lb) = mt(Lb) [continuity of moment @ Mtw]
- BC8: vb(Lb) = vt(Lb)- w^2*Mtw *yb(Lb)
  - BC8 reflects that shear at location of the tuning weight changes by an amount needed to accelerate the tuning weight.
  - We could have used yb(Lb) or yt(Lb) in BC 8 with same results

The first 4 B.C.'s result in 4 constants going to 0 *

- BC1 $\Rightarrow$ Cb1 = 0
- BC2 $\Rightarrow$ Cb3 = 0
- BC3 $\Rightarrow$ Ct2 = 0
- BC4 $\Rightarrow$ Ct4 = 0

* This was not just luck. It is a result of the way we set up the equations as sum/difference and using L-x as argument for the functions in yt

The next 4 constants are a **<u>little</u>** bit harder to solve!

Rewrite our equations setting those first four constants to 0

Bottom section variables:

$$\mathrm{yb}(x) := Cb2\,(\cos(\beta\,x) - \cosh(\beta\,x)) + Cb4\,(\sin(\beta\,x) - \sinh(\beta\,x))$$

$$\mathrm{sb}(x) := -Cb2\,\sin(\beta\,x)\,\beta - Cb2\,\sinh(\beta\,x)\,\beta + Cb4\,\cos(\beta\,x)\,\beta - Cb4\,\cosh(\beta\,x)\,\beta$$

$$\mathrm{mb}(x) := EI\,(-Cb2\,\cos(\beta\,x)\,\beta^2 - Cb2\,\cosh(\beta\,x)\,\beta^2 - Cb4\,\sin(\beta\,x)\,\beta^2 - Cb4\,\sinh(\beta\,x)\,\beta^2)$$

$$\mathrm{vb}(x) := EI\,\beta^3\,Cb2\,\sin(\beta\,x) - EI\,\beta^3\,Cb2\,\sinh(\beta\,x) - EI\,\beta^3\,Cb4\,\cos(\beta\,x) - EI\,\beta^3\,Cb4\,\cosh(\beta\,x)$$

Top section variables:

$$\mathrm{yt}(x) := Ct1\,(\cos(\beta\,(L-x)) + \cosh(\beta\,(L-x))) + Ct3\,(\sin(\beta\,(L-x)) + \sinh(\beta\,(L-x)))$$

$$\mathrm{st}(x) := Ct1\,\sin(\beta\,(L-x))\,\beta - Ct1\,\sinh(\beta\,(L-x))\,\beta - Ct3\,\cos(\beta\,(L-x))\,\beta - Ct3\,\cosh(\beta\,(L-x))\,\beta$$

$$\mathrm{mt}(x) := EI\,(-Ct1\,\cos(\beta\,(L-x))\,\beta^2 + Ct1\,\cosh(\beta\,(L-x))\,\beta^2 - Ct3\,\sin(\beta\,(L-x))\,\beta^2 + Ct3\,\sinh(\beta\,(L-x))\,\beta^2)$$

$$\mathrm{vt}(x) := -EI\,\beta^3\,Ct1\,\sin(\beta\,(L-x)) - EI\,\beta^3\,Ct1\,\sinh(\beta\,(L-x)) + EI\,\beta^3\,Ct3\,\cos(\beta\,(L-x)) - EI\,\beta^3\,Ct3\,\cosh(\beta\,(L-x))$$

Note: substituting x=0 into mb(x) gives the moment at the base which is what we will need later on when we try to calculate that max stress at the outside of the base:

$$mb(0) = 2{*}E{*}I{*}Cb2{*}\beta^2$$

We have 4 equations left, but really how many unknowns?

- At this stage we only want a mode "shape". The ratio between constants C is all that is important....
- So we will use terminology
  - Cb2 is our magnitude normalizing coefficient. Divide the others by Cb2 and using subscript "o" to remind us o stands for "over Cb2"
  - Cb4o =Cb4/Cb2
  - Ct1o = Ct1/Cb2
  - Ct3o = Ct3/Cb2
- β and w=w2 are related by $\omega = \beta^2 \sqrt{\dfrac{E*I}{\rho*A}}$ $\beta = \sqrt{\omega} * \left(\dfrac{\rho*A}{E*I} \right)^{1/4}$
- Strictly speaking, β and w are unknown and we need the 4th equation to solve one of them. However we have chosen a value of Mtw which will give w very close to our target, so we know w well enough that we don't need to solve β and w.
- Therefore we can drop one of the last 4 equations with negligible error.

We choose to drop BC5.

We are left with three BC's or equations: 6,7, 8

BC 6:

$-Cb2 \sin(\beta\, Lb) - Cb2 \sinh(\beta\, Lb) + Cb4 \cos(\beta\, Lb) - Cb4 \cosh(\beta\, Lb)$

$= -Ct1 \sin(\beta\, (Lb - L)) + Ct1 \sinh(\beta\, (Lb - L)) - Ct3 \cos(\beta\, (Lb - L))$

$- Ct3 \cosh(\beta\, (Lb - L))$

BC 7:

$-Cb2 \cos(\beta\, Lb) - Cb2 \cosh(\beta\, Lb) - Cb4 \sin(\beta\, Lb) - Cb4 \sinh(\beta\, Lb)$

$= -Ct1 \cos(\beta\, (Lb - L)) + Ct1 \cosh(\beta\, (Lb - L)) + Ct3 \sin(\beta\, (Lb - L))$

$- Ct3 \sinh(\beta\, (Lb - L))$

BC 8:

EI*ß^3*Cb2*sin(ß*Lb) - EI*ß^3*Cb2*sinh(ß*Lb) - EI*ß^3*Cb4*cos(ß*Lb) - EI*ß^3*Cb4*cosh(ß*Lb)
= -EI*ß^3*Ct1*sin(ß*(L-Lb)) - EI*ß^3*Ct1*sinh(ß*(L-Lb)) + EI*ß^3*Ct3*cos(ß*(L-Lb))
-EI*ß^3*Ct3*cosh(ß*(L-Lb)) - w^2*Mtw*Cb2*cos(ß*Lb) + w^2*Mtw*Cb2*cosh(ß*Lb)
-w^2*Mtw*Cb4*sin(ß*Lb) + w^2*Mtw*Cb4*sinh(ß*Lb)

If we move all the terms involving Cb4, Ct1, Ct3 to the left and all the terms involving Cb2 to the right, then divide through by Cb2 and substitute our normalized constants, we can rewrite these three equations in matrix form: A * C = B where...

A=

$$\begin{bmatrix} \cos(\beta\, Lb) - \cosh(\beta\, Lb) & \sin(\beta\,(-L + Lb)) - \sinh(\beta\,(-L + Lb)) & \cos(\beta\,(-L + Lb)) + \cosh(\beta\,(-L + Lb)) \\ -\sin(\beta\, Lb) - \sinh(\beta\, Lb) & \cos(\beta\,(-L + Lb)) - \cosh(\beta\,(-L + Lb)) & -\sin(\beta\,(-L + Lb)) + \sinh(\beta\,(-L + Lb)) \\ -\dfrac{\cos(\beta\, Lb)\,\beta^3\, EI + \cosh(\beta\, Lb)\,\beta^3\, EI - wa^2\, Mtw\, \sin(\beta\, Lb) + wa^2\, Mtw\, \sinh(\beta\, Lb)}{\beta^3\, EI} & -\sin(\beta\,(-L + Lb)) - \sinh(\beta\,(-L + Lb)) & -\cos(\beta\,(-L + Lb)) + \cosh(\beta\,(-L + Lb)) \end{bmatrix}$$

C= $$\begin{bmatrix} Cb4o \\ Ct1o \\ Ct3o \end{bmatrix}$$

Note:
A is 3x3
B and C are 3x1

B= $$\begin{bmatrix} \sin(\beta\, Lb) + \sinh(\beta\, Lb) \\ \cos(\beta\, Lb) + \cosh(\beta\, Lb) \\ \dfrac{-\sin(\beta\, Lb)\,\beta^3\, EI + \sinh(\beta\, Lb)\,\beta^3\, EI - wa^2\, Mtw\, \cos(\beta\, Lb) + wa^2\, Mtw\, \cosh(\beta\, Lb)}{\beta^3\, EI} \end{bmatrix}$$

It is not practical to simplify the expression further algebraically. We must instead plug in numerical values to populate the A and B matrix, and then compute the the matrix solution C = A^{-1} * B.

Example stress calculation using numerical values of MG case

Previous Input data (same as the Mtw calculation)

- Target frequency = 29.98hz
- Target w = 2*pi*f = 2*pi*29.98hz=188.4 rad/sec
- L=29” = 0.7366m
- a=21”=0.5334m
- b=12”=0.3048m
- h=1” = 0.0254m
- E=2.9E7psi=2E11 N/m^2
- rho=0.282lbm/in^3=7805.7kg/m^3
- mu = rho*b*h=60.43kg/m
- I=b*h^3/12 = 4.162E-7 m^3
- Mtw = 18kg (previously calcualted by Dunkerley’s approach)

New Input Data

- Vibration at top of absorber was 3.7 ips pk/0

We have already everything needed to populate the A and B matrices listed on the previous slide except for beta which is calculated as follows:

$$\beta = \sqrt{\omega} * \left(\frac{\rho * A}{E * I} \right)^{1/4} = \sqrt{188.4} * \left(\frac{7805.3 * 0.3048 * 0.0254}{2E11 * 4.162E-7} \right)^{1/4} = 2.25 m^{-1}$$

Plug into previous formula's for A and C:

$$A = \begin{bmatrix} -1.453338128 & .03201066870 & 2.003664555 \\ -2.445983598 & -.2097242054 & -.03201066870 \\ -2.562818809 & .9161927901 & .2097242054 \end{bmatrix}$$

$$B = \begin{bmatrix} 2.445983598 \\ 2.174207362 \\ 1.553484229 \end{bmatrix}$$

$$C = A^{-1} B = \begin{bmatrix} -.8308709754 \\ -.7728785700 \\ .6304386060 \end{bmatrix}$$

Cb4o=Cb4/Cb2 = -0.8308709754
Ct1o = Ct1/Cb2 = -0.7728785700
Ct3o = Ct3/Cb2 = 0.6304386060

Solve for the value of Cb2 that will recreate the vibration that we measured at the top (x=L)

3.7 ips pk/0 at 1800cpm is 0.0394” pk/0 = 0.0005 m pk/0

$$yt(x) := Ct1\,(\cos(\beta\,(L-x)) + \cosh(\beta\,(L-x))) + Ct3\,(\sin(\beta\,(L-x)) + \sinh(\beta\,(L-x)))$$

For x=L, this simplifies to:

yt(L) = 2*Ct1 = 2*Ct1o * Cb2

Solve for Cb2:

Cb2 = yt(L) / [2*Ct1o] = 0.0005/[2*-0.7728785700] = 0.0003235m (drop the - sign)

Use Cb2 to find max moment.. at base.. mb(0):

mb(0) = 2*E*I*Cb2*β^2 = 2*2E11 * 4.162E-7 *0.0003235 * 2.25^2

mb(0) = 272.6 N*m (moment at the base)

Convert to max bending stress (at outside surfaces of the base)

σMax=(h/2)*Moment / I = (0.0254/2) * 272.6 / 4.162E-7 =0.8318x10^7 N/m^2

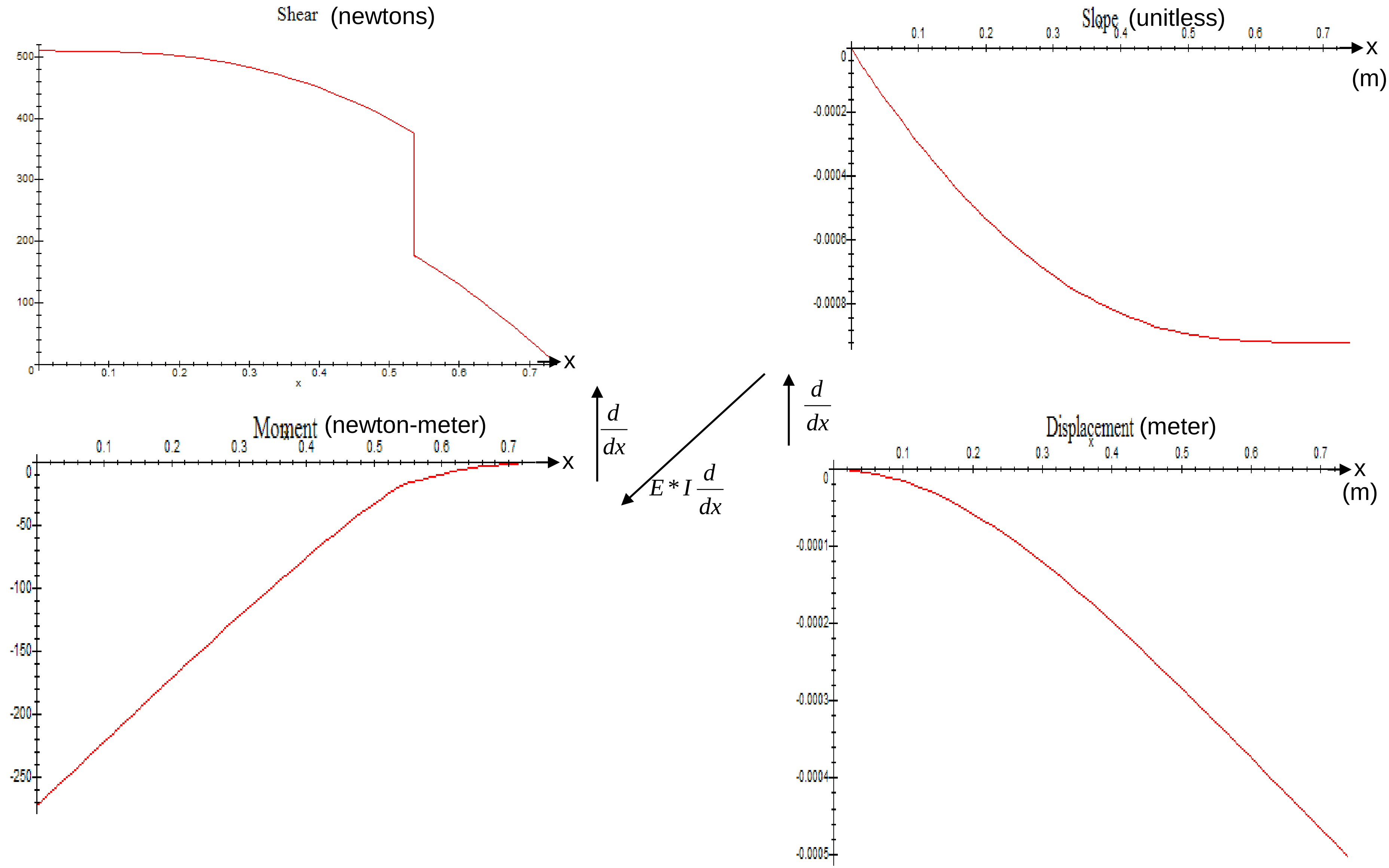
Convert to PSI:

σMax=1206 psi

Interpretation

- The computed maximum stress approx 1200 psi is a factor of ~24 below the endurance limit of A36 steel (which is 29,000 psi).
- We did not consider added contribution from shear stress at the base [vb(0)/(b*h)], but calculations show that contribution is generally small.
- We did not consider any stress concentrations, but a safety factor of 24 should be more than adequate to cover any stress concentrations. We expect the absorber will operate reliably with no concern for fatigue as long as vibration level on the tip does not change.
- Will monitor the absorber vibration periodically as part of routine vibration rounds on the machine.

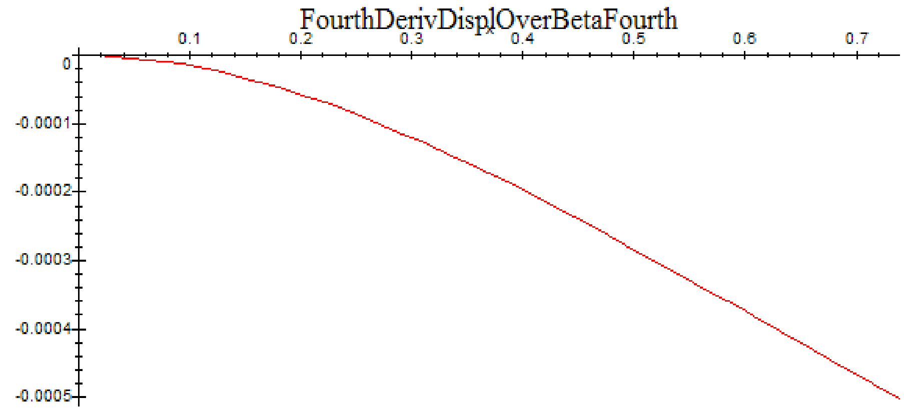
Complete Solution of Shear, Moment, Slope, Displacement of MG using previously-solved coefficients. Note displacement solution is continuous at Mtw, even though we “discarded” the boundary condition 5 requiring continuity of displacement⇒ The solution passes one “sanity check”



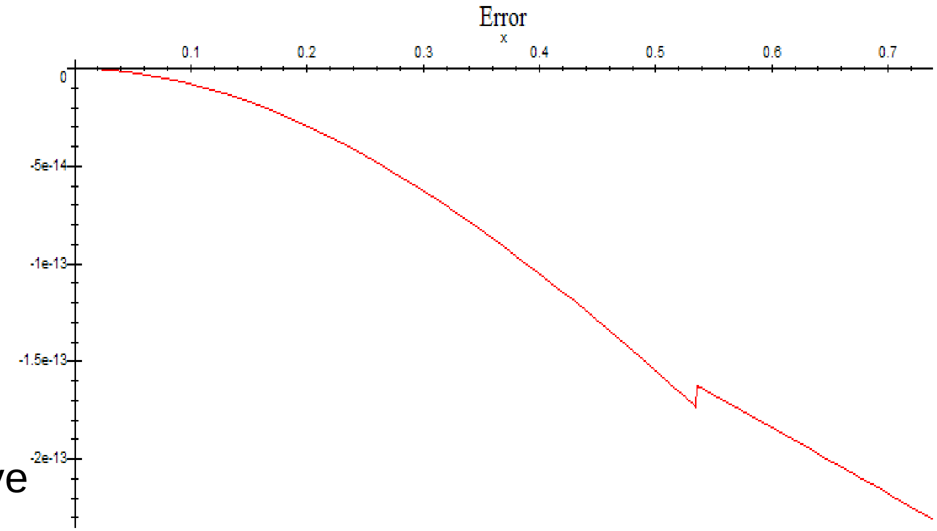
Double-check that the solution matches the differential equation.

The Euler Bernoulli Mode Shape solution requires $y(x) = \dfrac{\dfrac{d^4}{dx^4}[y(x)]}{\beta^4}$

We compute the quantity on RHS and see by eyeball we can't tell any difference from previous plotted y(x) (previous slide) ===>



Numerical comparision at right shows the max error is on the order 2e-13m which is < 1E-6 of the max displacement. Errors presumably arose out of floating point numerical errors as well as substituting our approximate value of w rather than solving the exact w using the 4th equation.



Based on this outstanding agreement, we have every reason to trust the solution (no errors were made in finding the coefficients)

Another approach to stress analysis - before the absorber is built

- The force transmitted to the absorber from the machin ewill be constant regardless of absorber design.
- Sometimes we can estimate the force. If not, guess a force, and use it to estimate stress, it gives a means for judging relative performance of various designs
- Once we have assumed a value of force, we can use it to solve Cb2 by evaluating vb(x) at x=0

$$\mathrm{vb}(x) := EI\,\beta^3\,Cb2\,\sin(\beta\,x) - EI\,\beta^3\,Cb2\,\sinh(\beta\,x) - EI\,\beta^3\,Cb4\,\cos(\beta\,x) - EI\,\beta^3\,Cb4\,\cosh(\beta\,x)$$

|vb(0)| = Force = |- 2*EI*beta^3*Cb4| = 2*EI*beta^3*Cb4o*Cb2

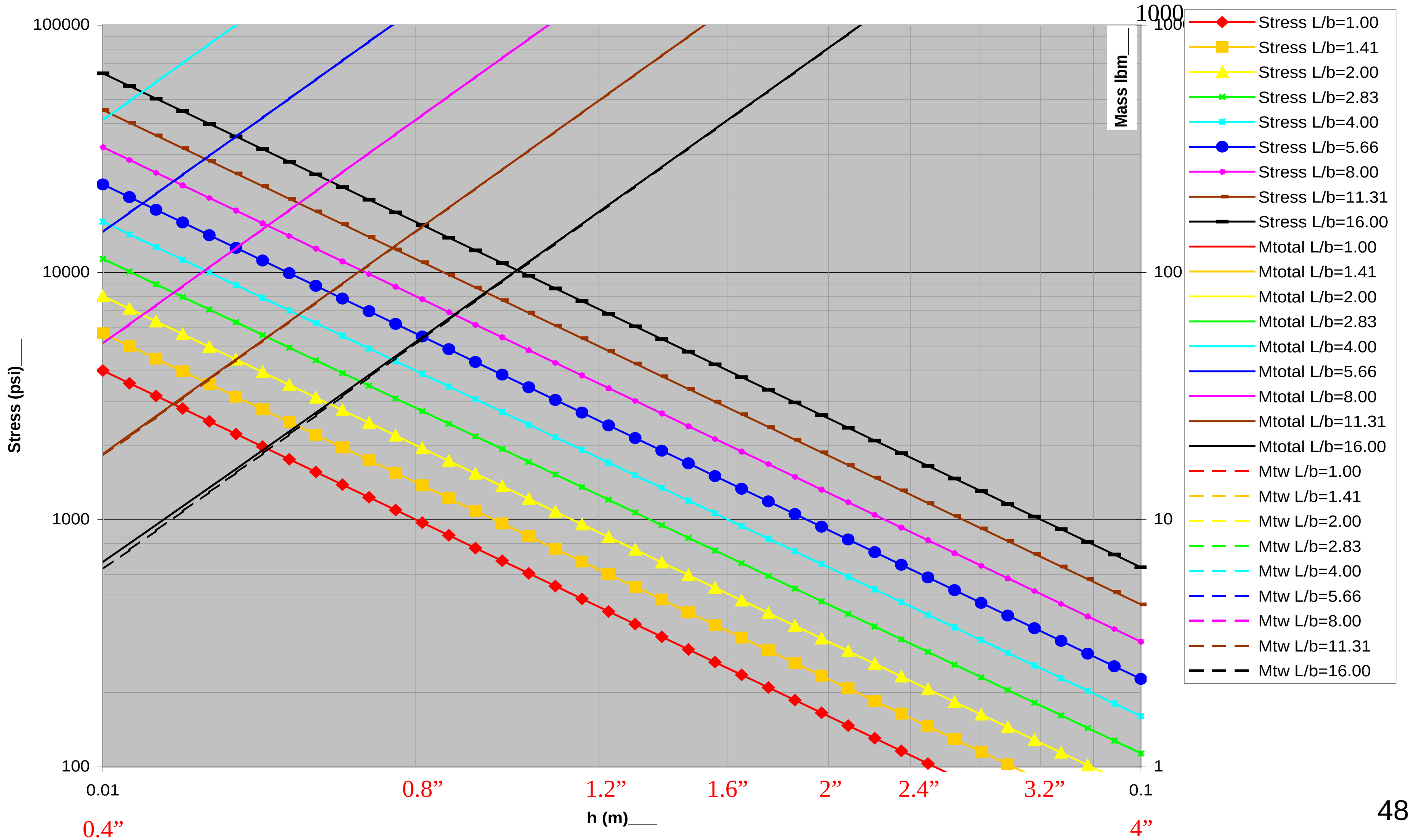
Solve Cb2
Cb2 = Force / (2*EI*beta^3*Cb4o)

Cb4o can be determined from absorber dimensions as previously. When combined with Force estimate, this allows us to estimate Cb2 and using similar calculations as before we can estimate max stress.

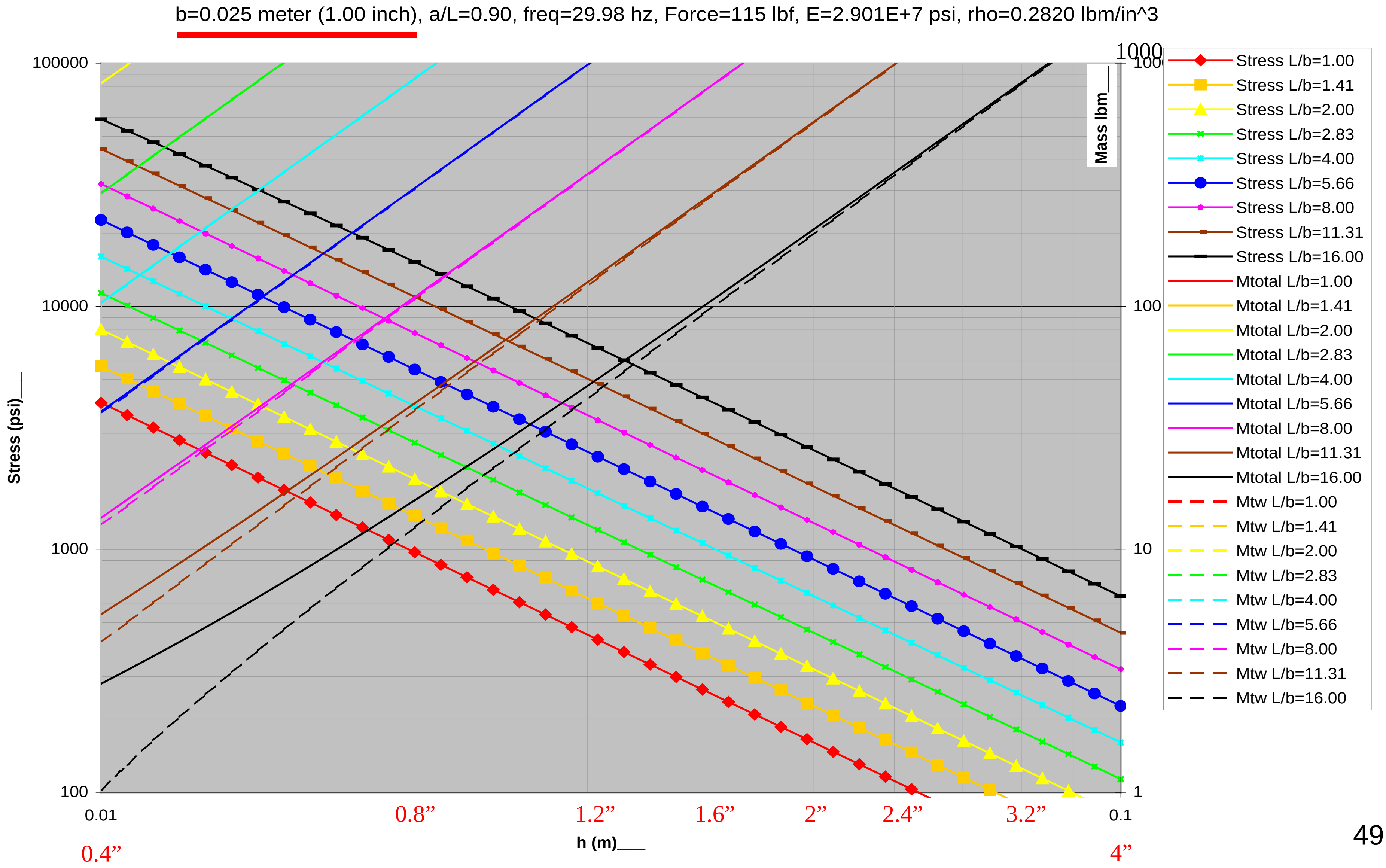
This enables us to compare relative performance of e objective is to compare various designs. Graphical tabulation assuming force = 115lbf in the following curves:

b=0.5”

b=0.013 meter (0.50 inch), a/L=0.90, freq=29.98 hz, Force=115 lbf, E=2.901E+7 psi, rho=0.2820 lbm/in^3

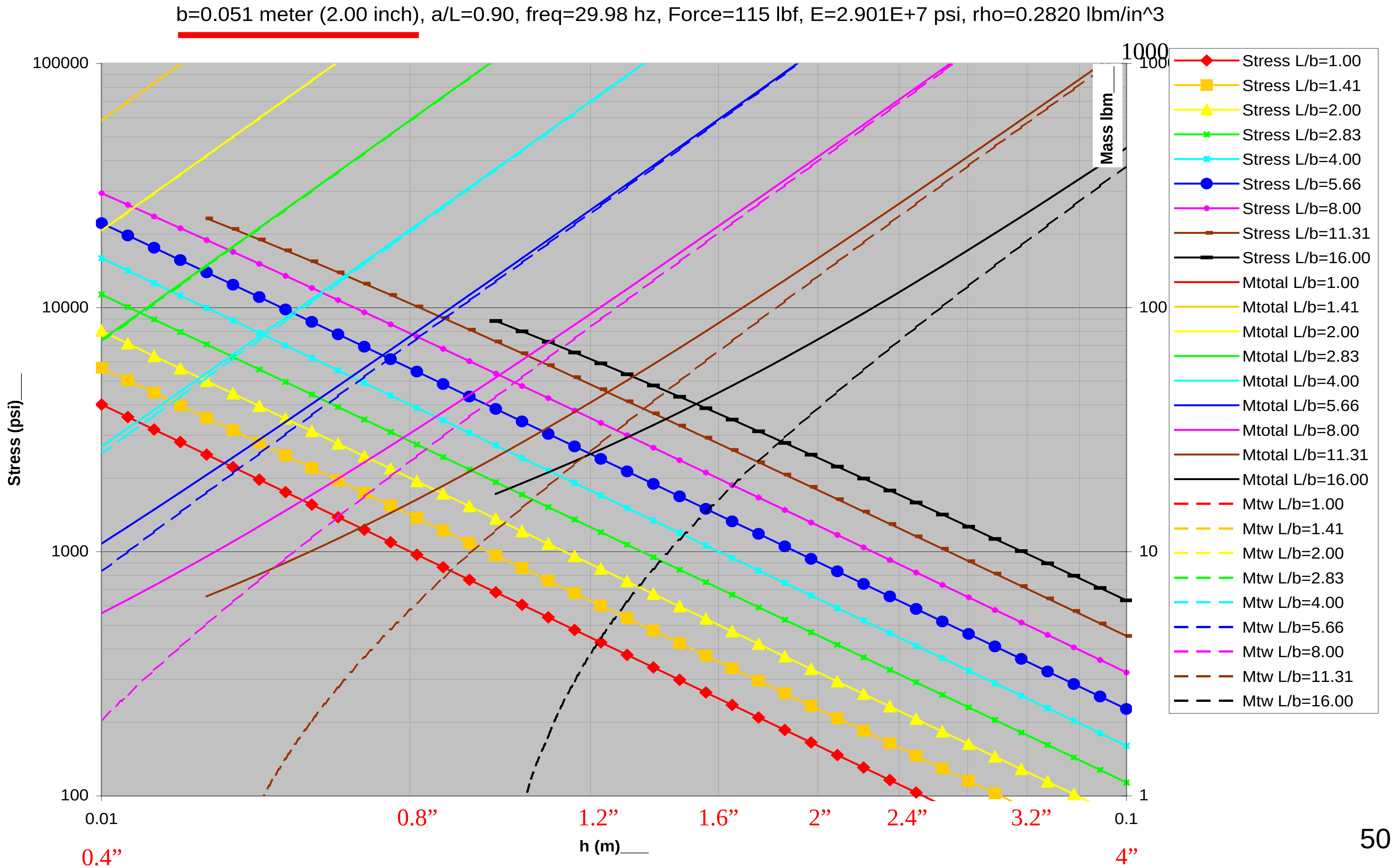


b=1.0”



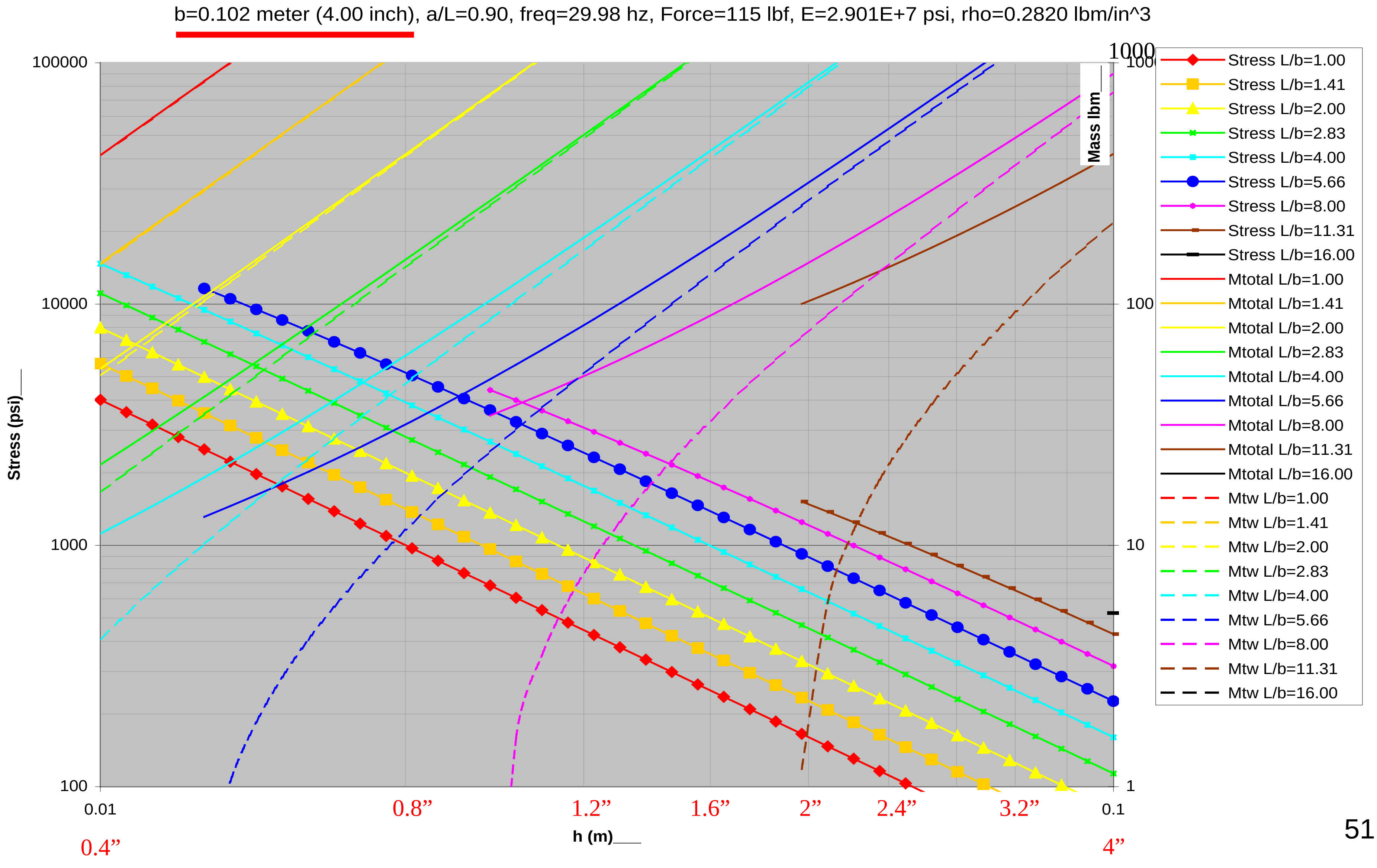
b=0.025 meter (1.00 inch), a/L=0.90, freq=29.98 hz, Force=115 lbf, E=2.901E+7 psi, rho=0.2820 lbm/in^3
Stress (psi)___
100000
10000
1000
100
Mass lbm___
1000
100
10
1
h (m)___
0.01
0.1
0.4”
0.8”
1.2”
1.6”
2”
2.4”
3.2”
4”
Stress L/b=1.00
Stress L/b=1.41
Stress L/b=2.00
Stress L/b=2.83
Stress L/b=4.00
Stress L/b=5.66
Stress L/b=8.00
Stress L/b=11.31
Stress L/b=16.00
Mtotal L/b=1.00
Mtotal L/b=1.41
Mtotal L/b=2.00
Mtotal L/b=2.83
Mtotal L/b=4.00
Mtotal L/b=5.66
Mtotal L/b=8.00
Mtotal L/b=11.31
Mtotal L/b=16.00
Mtw L/b=1.00
Mtw L/b=1.41
Mtw L/b=2.00
Mtw L/b=2.83
Mtw L/b=4.00
Mtw L/b=5.66
Mtw L/b=8.00
Mtw L/b=11.31
Mtw L/b=16.00

b=2.0”



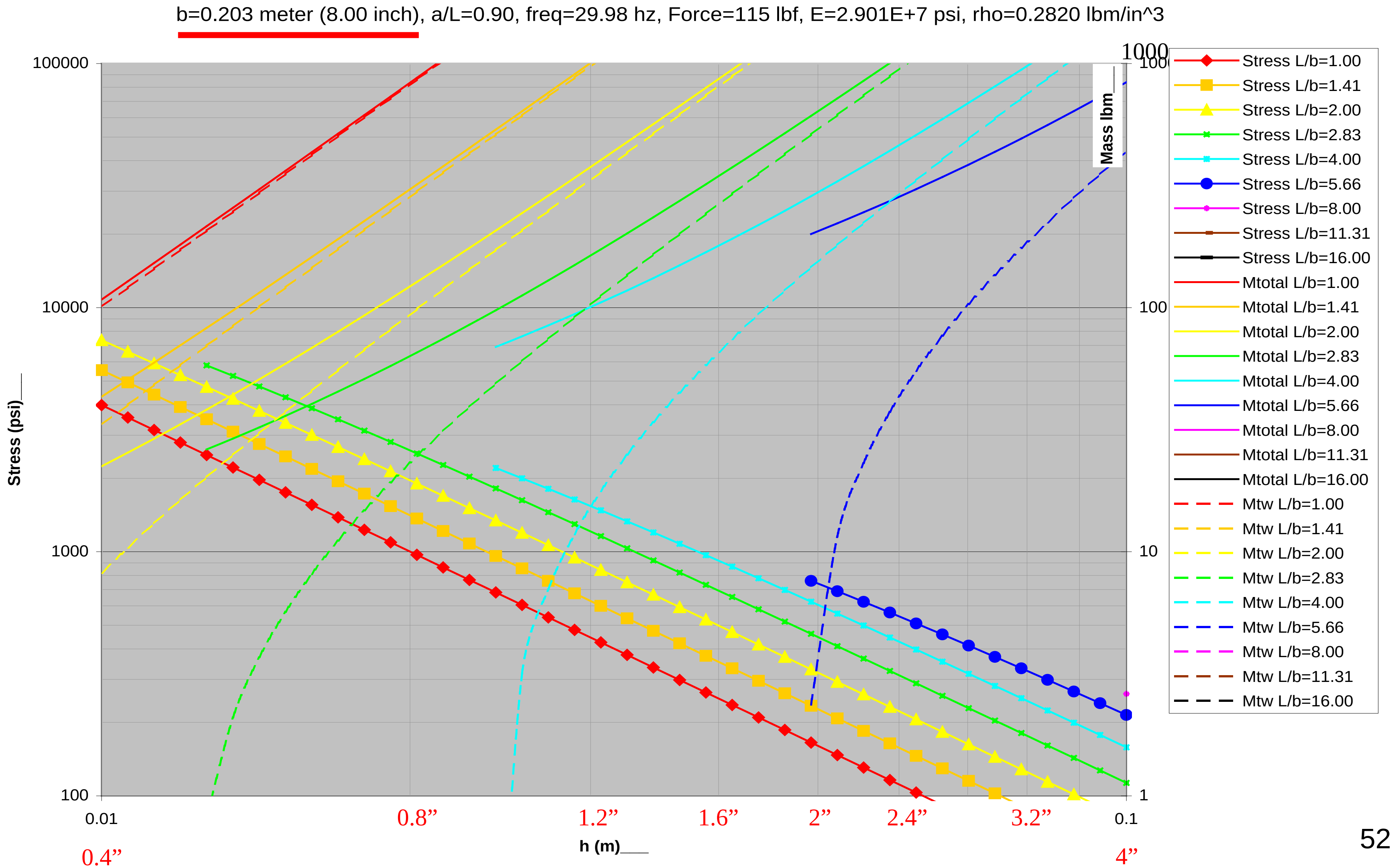
b=0.051 meter (2.00 inch), a/L=0.90, freq=29.98 hz, Force=115 lbf, E=2.901E+7 psi, rho=0.2820 lbm/in^3
Stress (psi)
100000
10000
1000
100
Mass lbm
1000
100
10
1
0.01
0.1
h (m)
0.4”
0.8”
1.2”
1.6”
2”
2.4”
3.2”
4”
Stress L/b=1.00
Stress L/b=1.41
Stress L/b=2.00
Stress L/b=2.83
Stress L/b=4.00
Stress L/b=5.66
Stress L/b=8.00
Stress L/b=11.31
Stress L/b=16.00
Mtotal L/b=1.00
Mtotal L/b=1.41
Mtotal L/b=2.00
Mtotal L/b=2.83
Mtotal L/b=4.00
Mtotal L/b=5.66
Mtotal L/b=8.00
Mtotal L/b=11.31
Mtotal L/b=16.00
Mtw L/b=1.00
Mtw L/b=1.41
Mtw L/b=2.00
Mtw L/b=2.83
Mtw L/b=4.00
Mtw L/b=5.66
Mtw L/b=8.00
Mtw L/b=11.31
Mtw L/b=16.00

b=4.0”

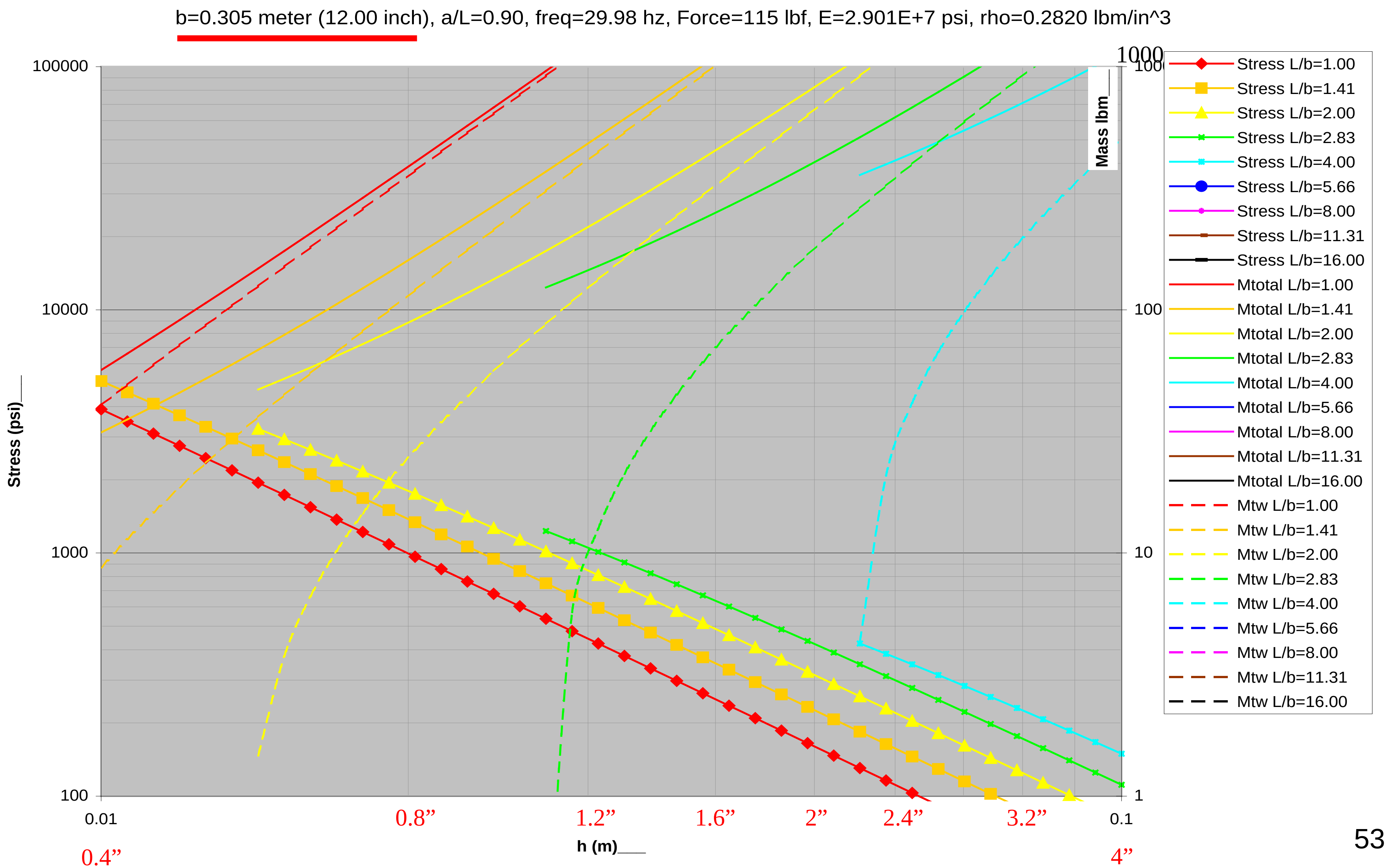


b=0.102 meter (4.00 inch), a/L=0.90, freq=29.98 hz, Force=115 lbf, E=2.901E+7 psi, rho=0.2820 lbm/in^3
Stress (psi)
100000
10000
1000
100
Mass lbm
1000
100
10
1
0.01
0.1
h (m)
0.4”
0.8”
1.2”
1.6”
2”
2.4”
3.2”
4”
Stress L/b=1.00
Stress L/b=1.41
Stress L/b=2.00
Stress L/b=2.83
Stress L/b=4.00
Stress L/b=5.66
Stress L/b=8.00
Stress L/b=11.31
Stress L/b=16.00
Mtotal L/b=1.00
Mtotal L/b=1.41
Mtotal L/b=2.00
Mtotal L/b=2.83
Mtotal L/b=4.00
Mtotal L/b=5.66
Mtotal L/b=8.00
Mtotal L/b=11.31
Mtotal L/b=16.00
Mtw L/b=1.00
Mtw L/b=1.41
Mtw L/b=2.00
Mtw L/b=2.83
Mtw L/b=4.00
Mtw L/b=5.66
Mtw L/b=8.00
Mtw L/b=11.31
Mtw L/b=16.00

b=8”

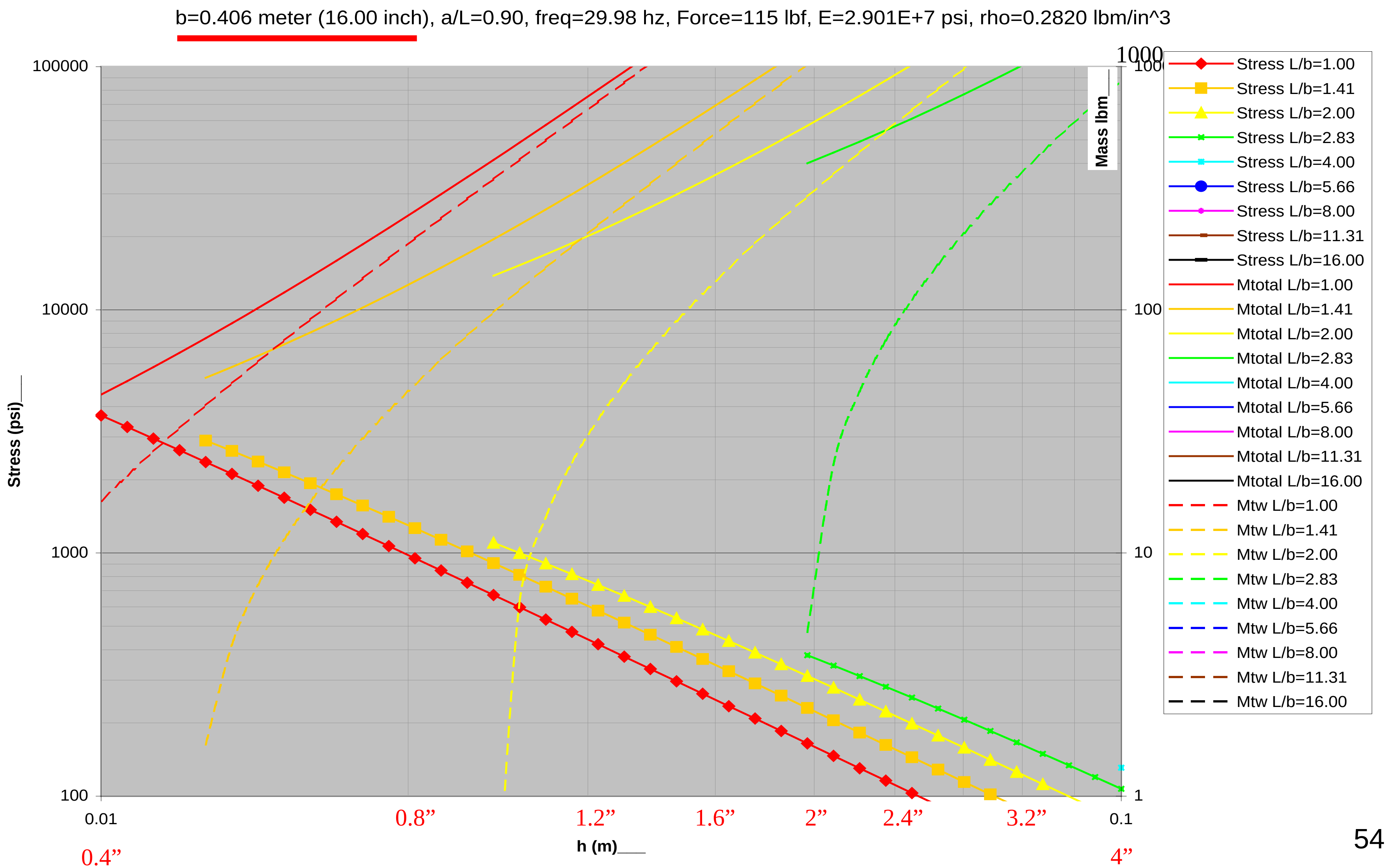

b=0.203 meter (8.00 inch), a/L=0.90, freq=29.98 hz, Force=115 lbf, E=2.901E+7 psi, rho=0.2820 lbm/in^3
Stress (psi)___
100000
10000
1000
100
Mass lbm___
1000
100
10
1
0.01
0.1
h (m)___
0.4”
0.8”
1.2”
1.6”
2”
2.4”
3.2”
4”
Stress L/b=1.00
Stress L/b=1.41
Stress L/b=2.00
Stress L/b=2.83
Stress L/b=4.00
Stress L/b=5.66
Stress L/b=8.00
Stress L/b=11.31
Stress L/b=16.00
Mtotal L/b=1.00
Mtotal L/b=1.41
Mtotal L/b=2.00
Mtotal L/b=2.83
Mtotal L/b=4.00
Mtotal L/b=5.66
Mtotal L/b=8.00
Mtotal L/b=11.31
Mtotal L/b=16.00
Mtw L/b=1.00
Mtw L/b=1.41
Mtw L/b=2.00
Mtw L/b=2.83
Mtw L/b=4.00
Mtw L/b=5.66
Mtw L/b=8.00
Mtw L/b=11.31
Mtw L/b=16.00

b=12”



b=0.305 meter (12.00 inch), a/L=0.90, freq=29.98 hz, Force=115 lbf, E=2.901E+7 psi, rho=0.2820 lbm/in^3
Stress (psi)
100000
10000
1000
100
Mass lbm
1000
100
10
1
0.01
0.1
h (m)
0.4”
0.8”
1.2”
1.6”
2”
2.4”
3.2”
4”
Stress L/b=1.00
Stress L/b=1.41
Stress L/b=2.00
Stress L/b=2.83
Stress L/b=4.00
Stress L/b=5.66
Stress L/b=8.00
Stress L/b=11.31
Stress L/b=16.00
Mtotal L/b=1.00
Mtotal L/b=1.41
Mtotal L/b=2.00
Mtotal L/b=2.83
Mtotal L/b=4.00
Mtotal L/b=5.66
Mtotal L/b=8.00
Mtotal L/b=11.31
Mtotal L/b=16.00
Mtw L/b=1.00
Mtw L/b=1.41
Mtw L/b=2.00
Mtw L/b=2.83
Mtw L/b=4.00
Mtw L/b=5.66
Mtw L/b=8.00
Mtw L/b=11.31
Mtw L/b=16.00

b=16”

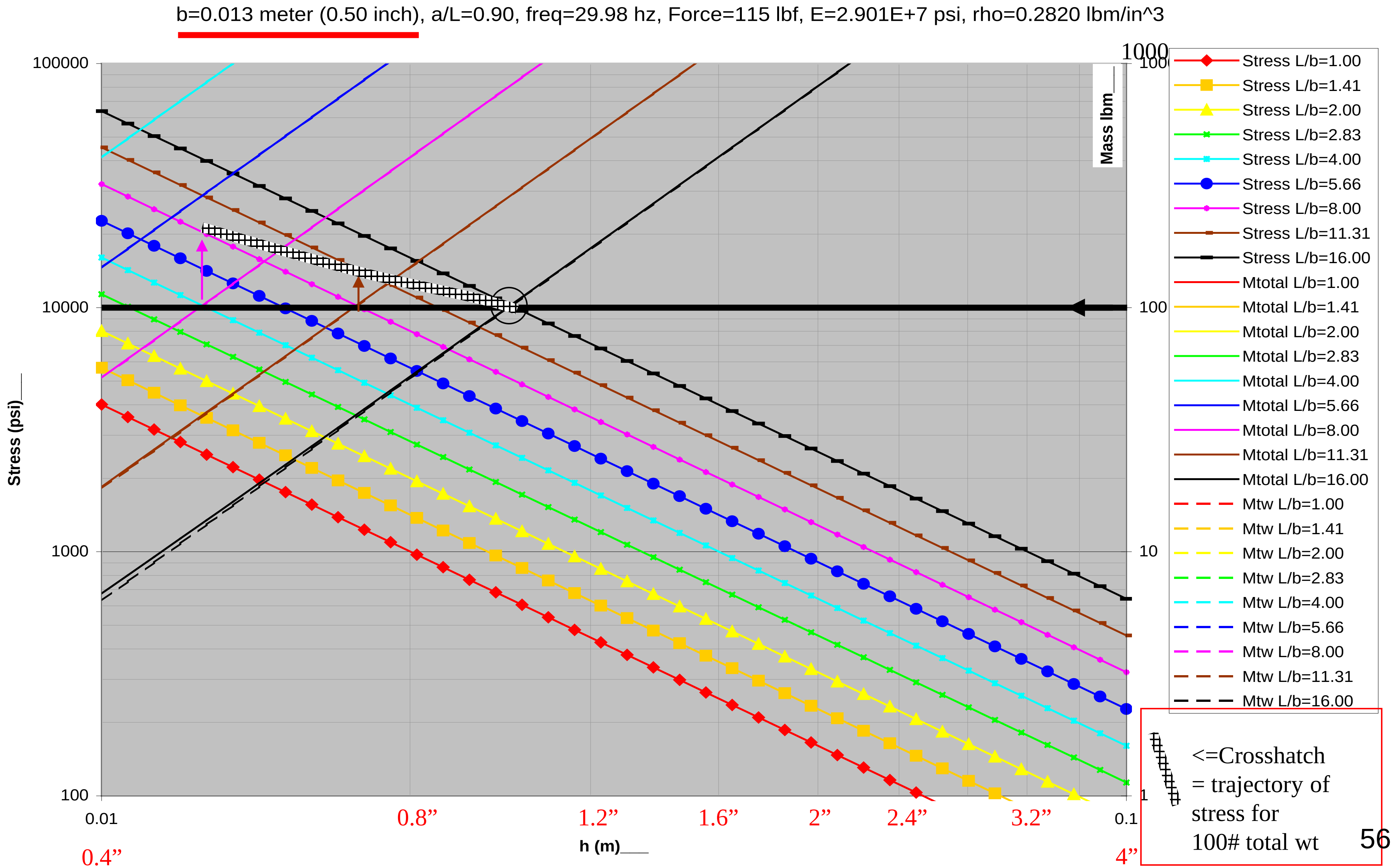


b=0.406 meter (16.00 inch), a/L=0.90, freq=29.98 hz, Force=115 lbf, E=2.901E+7 psi, rho=0.2820 lbm/in^3
Stress (psi)
100000
10000
1000
100
Mass lbm
1000
100
10
1
h (m)
0.01
0.1
0.4”
0.8”
1.2”
1.6”
2”
2.4”
3.2”
4”
Stress L/b=1.00
Stress L/b=1.41
Stress L/b=2.00
Stress L/b=2.83
Stress L/b=4.00
Stress L/b=5.66
Stress L/b=8.00
Stress L/b=11.31
Stress L/b=16.00
Mtotal L/b=1.00
Mtotal L/b=1.41
Mtotal L/b=2.00
Mtotal L/b=2.83
Mtotal L/b=4.00
Mtotal L/b=5.66
Mtotal L/b=8.00
Mtotal L/b=11.31
Mtotal L/b=16.00
Mtw L/b=1.00
Mtw L/b=1.41
Mtw L/b=2.00
Mtw L/b=2.83
Mtw L/b=4.00
Mtw L/b=5.66
Mtw L/b=8.00
Mtw L/b=11.31
Mtw L/b=16.00

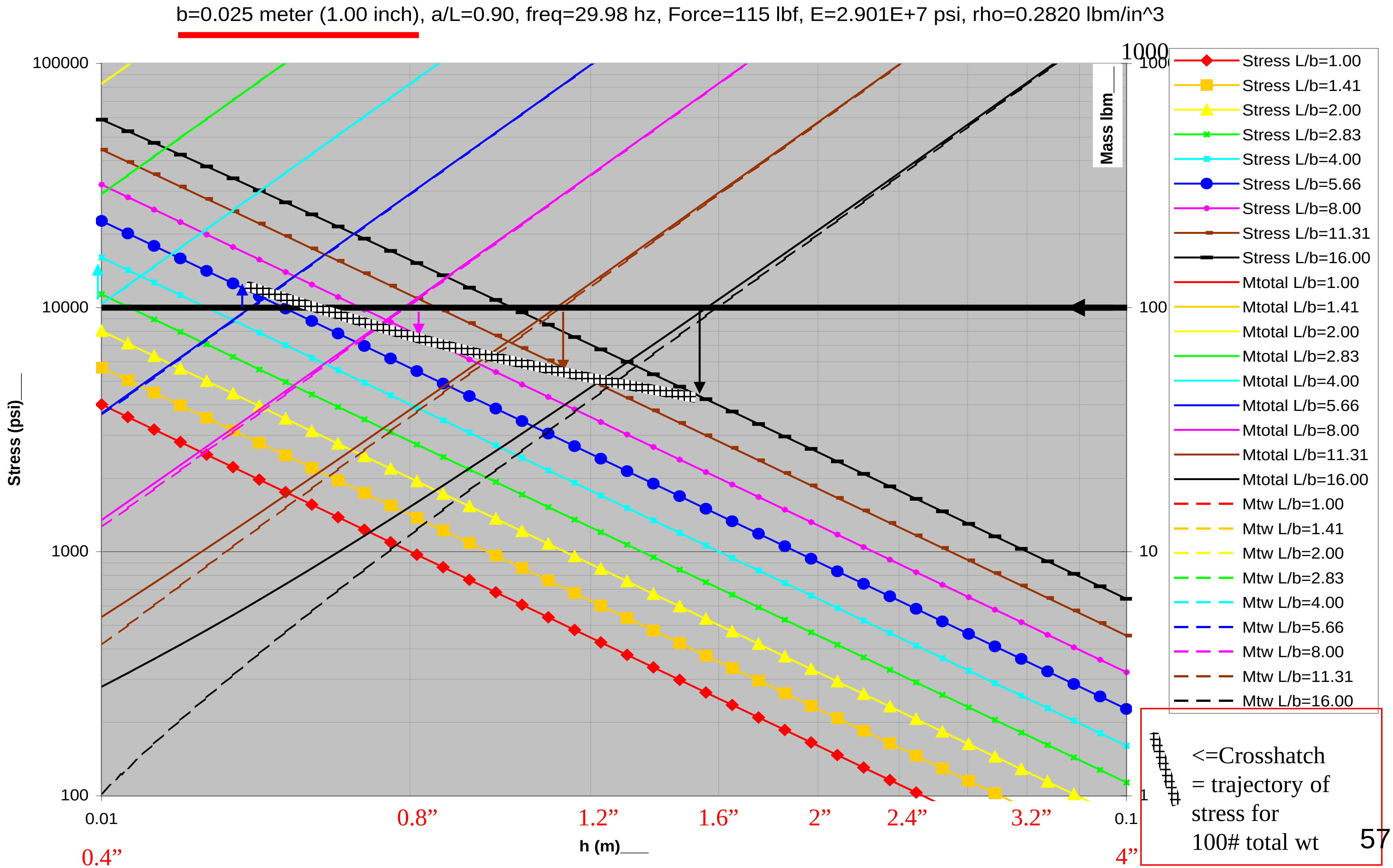
Simplifying assumption: Compare among options with the same total weight

- We want a large total weight to give high m2/m1 in order to improve separation of the resonant frequencies. (Choose for example M2=0.02*M1 if possible)
- As weight increases, cost tends to increase
- Comparing options with same weight will give comparable frequency separation and comparable cost

b=0.5 Options for 100 pound total absorber weight

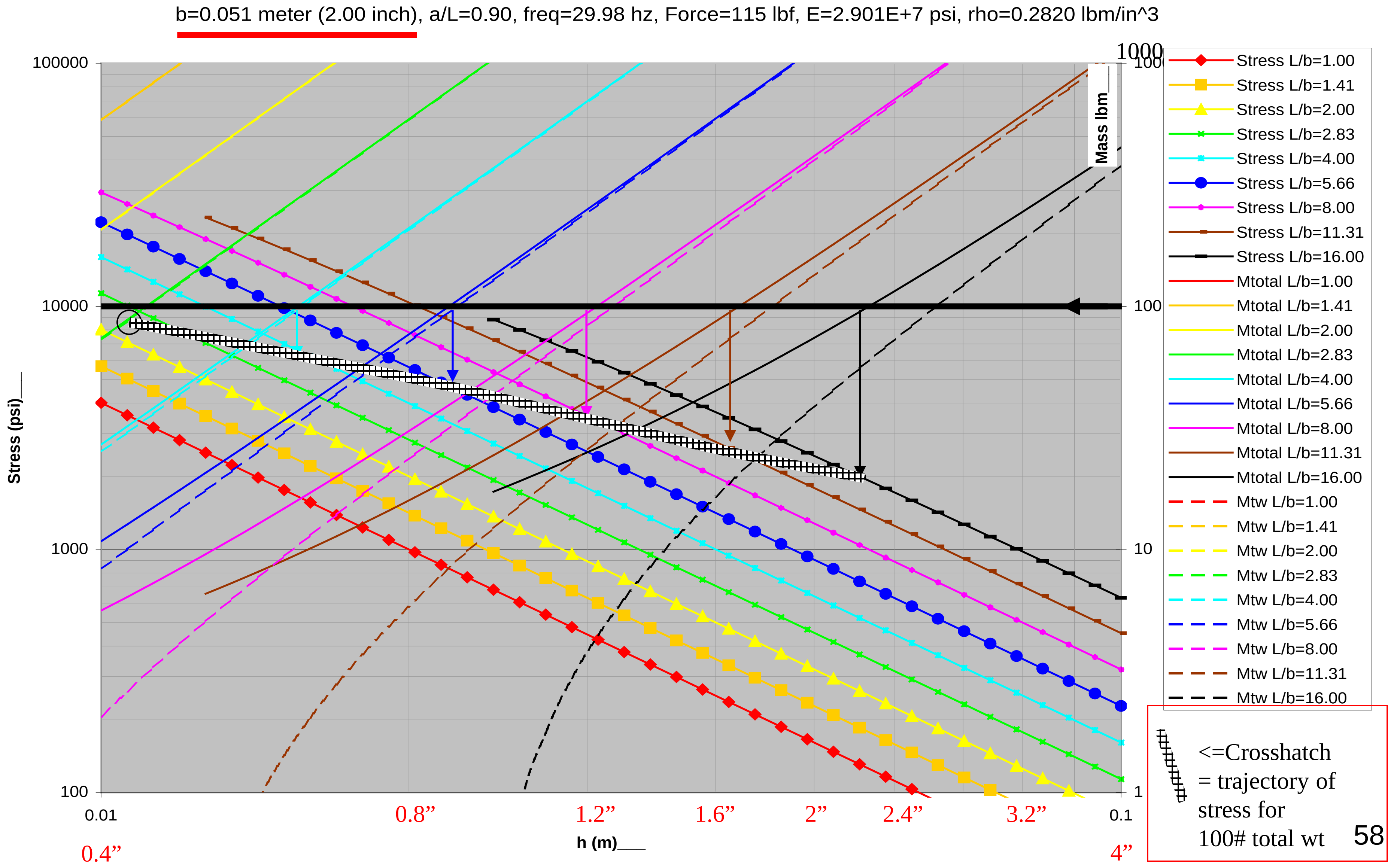


b=1.0 Options for 100 pound total absorber weight



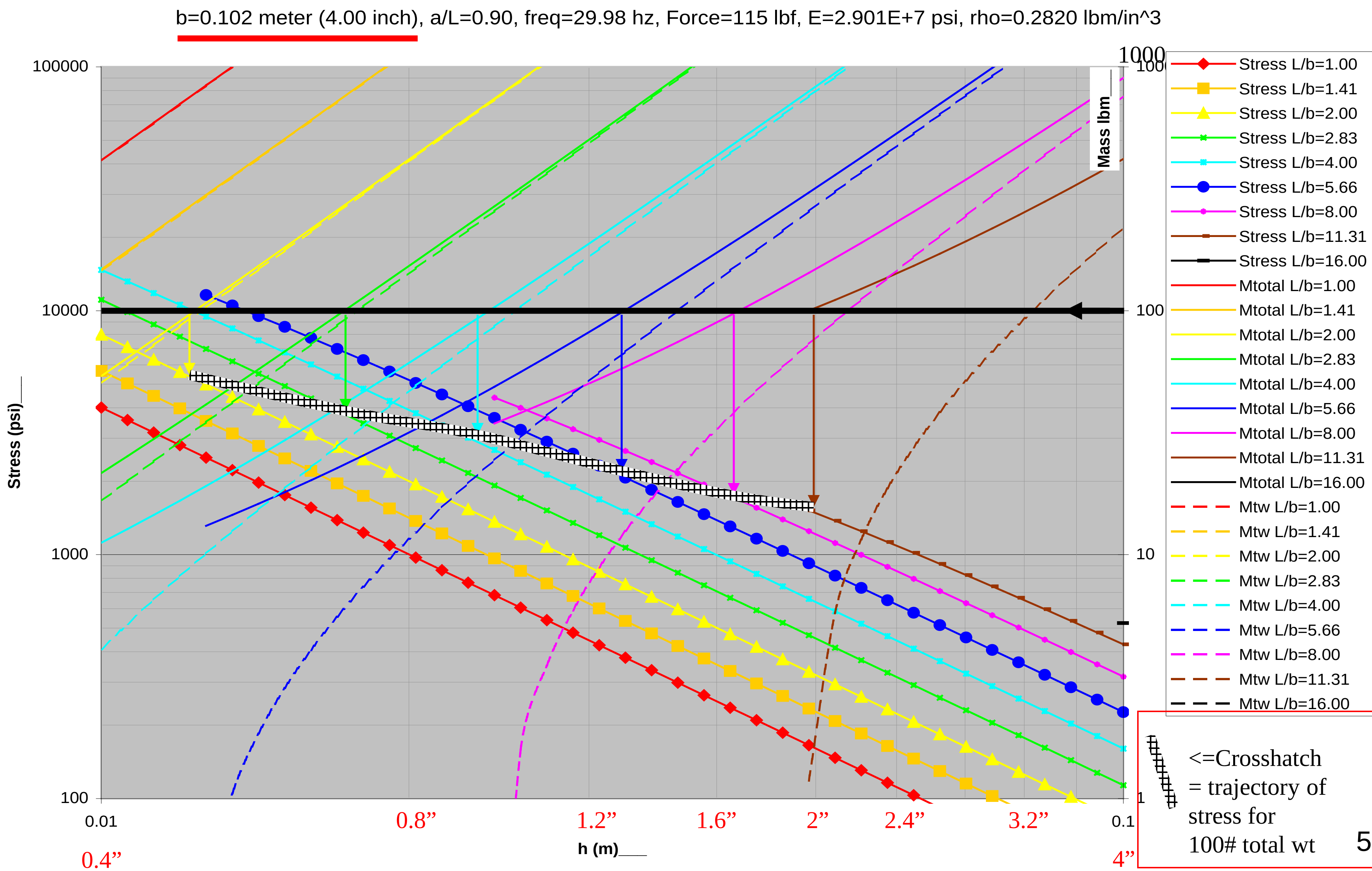
<=Crosshatch = trajectory of stress for 100# total wt

b=2.0” Options for 100 pound total absorber weight

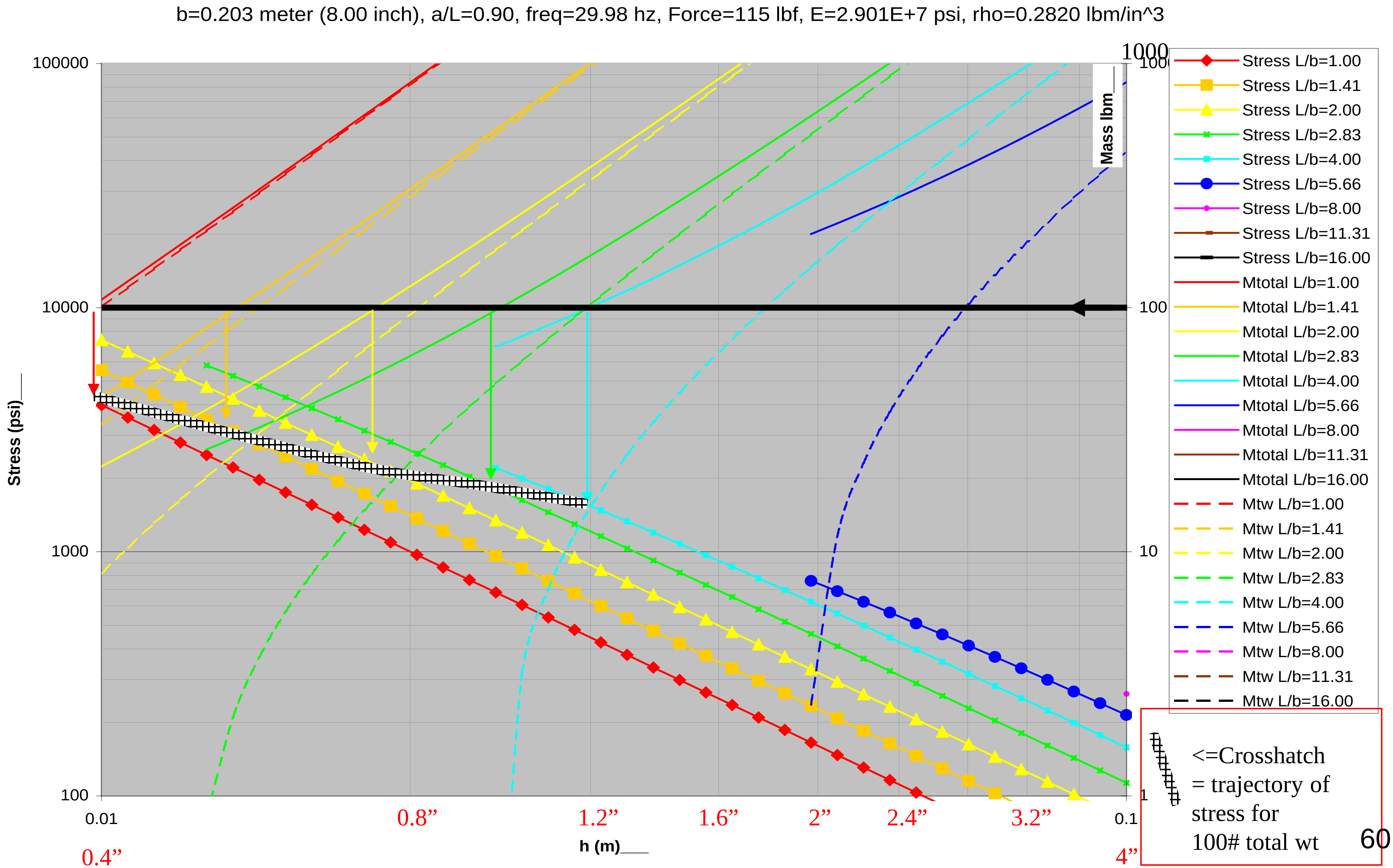


<=Crosshatch = trajectory of stress for 100# total wt

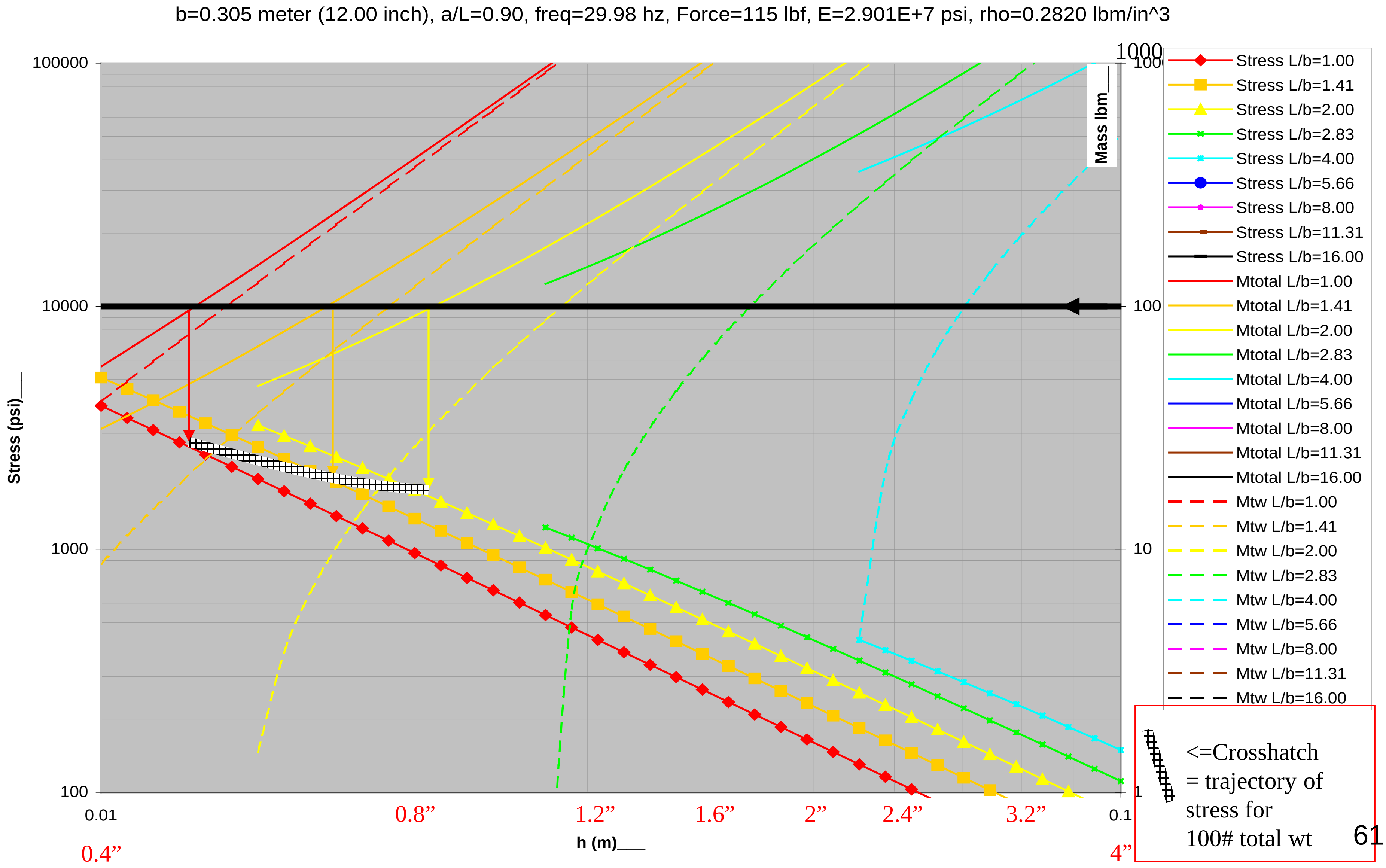
b=4.0 Options for 100 pound total absorber weight:



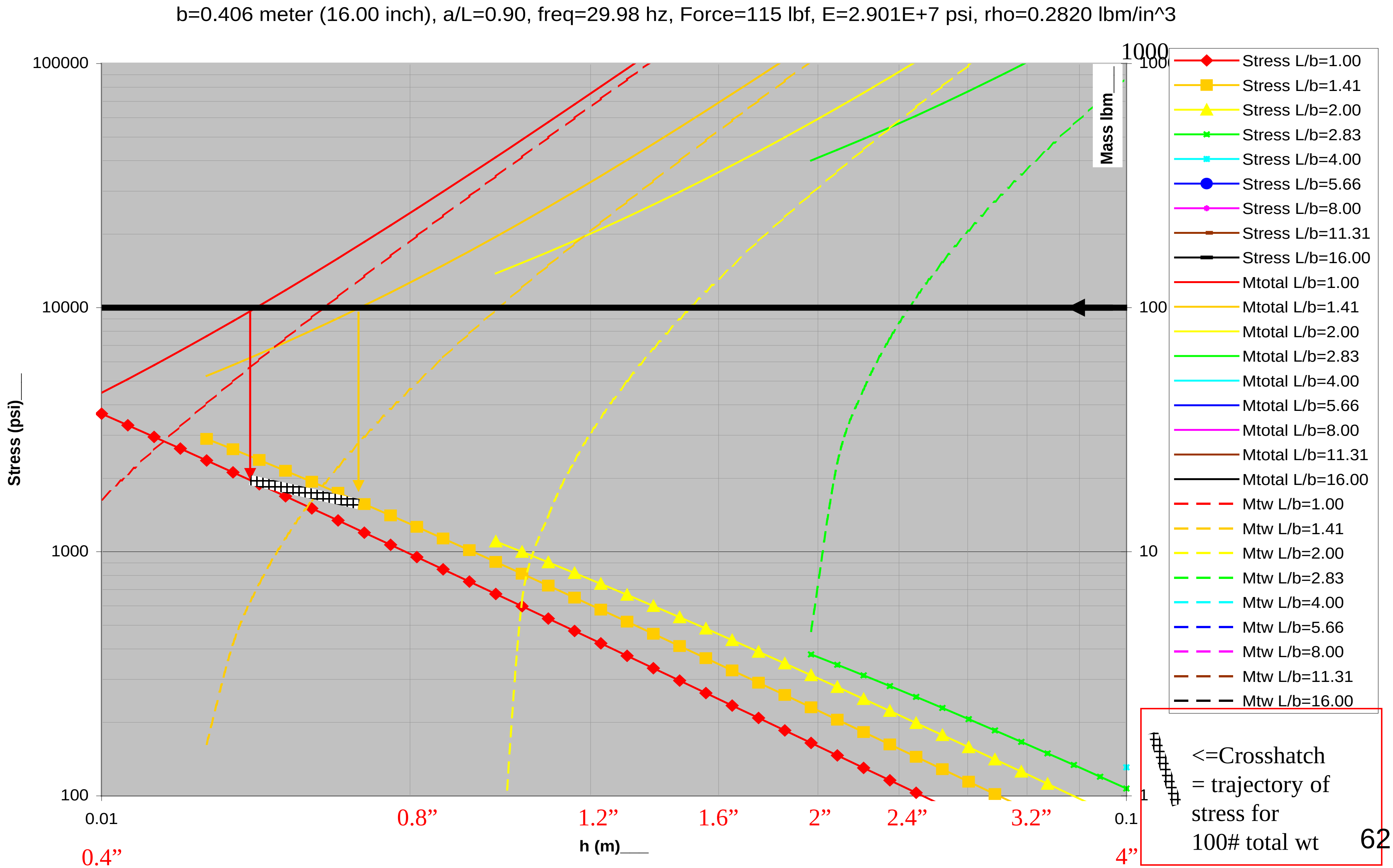
b=8” Options for 100 pound total absorber weight



b=12 Options for 100 pound total absorber weight



b=16 Options for 100 pound total absorber weight



Conclusions from constant-weight comparison

- Using this approach (choosing among designs that have the same total weight), the optimum designs tend to have most of the weight in the bar and a fairly small fraction of weight in the Mtw (similar to our MG DA design)
  - This design also tends to make tuning easier since movement of mass is more of fine-tune. However for small tuning mass we also need larger tuning slot length to account for frequency errors.

References

- Fox: “Dynamic Absorbers for Solving Resonance Problems”, Randy Fox, Entek
- Eisenmann: “Machinery Monitoring, Diagnosis and Correction” (a great book!)
- Scheffer - "Practical Machinery Vibration Analysis and Predictive Maintenance" by Girdhar C. Scheffer ISBN 0 7506 6275 1
- Harris: Shock & Vib Handbook
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