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FOOTBRIDGE DESIGN FOR PEDESTRIAN INDUCED VIBRATIONS

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INTRODUCTION

With innovative engineering and inspiring design, footbridges have become functional works of art. However, the use of longer and lighter spans have made footbridges more susceptible to human-induced vibrations; causing discomfort to pedestrians and compromising the utility of the structure, even though the bridge is structurally sound and safe to cross. Design codes address this dynamic problem by providing limits for natural frequency and simplistic provisions to keep the footbridge experience pleasant. For slender, lightweight bridges, such as stress ribbon or cable-stayed bridges, this dynamic problem can be onerous and require a refined analysis to demonstrate that the comfort level can be satisfied.

This paper presents a guideline to determine the dynamic bridge characteristics under pedestrian loading. In addition, factors that influence a bridge's response to vibration and possible vibration mitigation measures are discussed herein.

This paper focuses on the recommended design procedure by presenting an analytical model of a concrete footbridge subjected to a dynamic load representing the effects of a stream of pedestrians crossing the structure. In the vertical direction, the peak acceleration from the pedestrian loading is compared with published acceptance criteria. In the lateral direction, the critical number of pedestrians at which the bridge response becomes unstable is calculated.

HUMAN LOCOMOTION

When a pedestrian crosses a bridge, a dynamic force is produced which has components in the vertical, lateral and longitudinal directions. These dynamic forces are described as a function of time and space, periodically repeated with regular time intervals. The dynamic actions are the displacements, velocities, accelerations and energy produced by the vibration source, which in this case, are pedestrians crossing a footbridge.

The walking force is determined by a number of factors that include the weight of the pedestrian, the step length and the walking frequency. While several parameters may affect and modify the pedestrian loading, such as gait and ground roughness, it has been proven through experimental measurements to be periodic and characterised by a fundamental parameter: frequency. According to Murray, et al., the vertical dynamic load of a single pedestrian may be represented by a sum of three harmonic loads:

$$F = P\sum_{i=1}^{3}\alpha_i \cos(2\pi i f_{step} t - \phi_i) \tag{1}$$

where *P* is the weight of an average person, which is assumed to be 700 N, α_i is the dynamic coefficient for the *i*th harmonic, f_{step} is the step frequency of walking, and Φ_i is the phase angle for the *i*th harmonic.

The dynamic coefficients for the three harmonics recommended by Murray are 0.5, 0.2 and 0.1 respectively. The step frequency is said to vary between 1.6 to 2.4 Hz for walkers and 2.0 to 3.5 Hz for runners. The phase angles in Equation 1 were determined so as to obtain a time variation of load matching the results of experiments. The load of a single pedestrian according to Equation 1 is plotted in the graph in Figure 1. The trace labelled “Dynamic Load” is used to model the dynamic load of a single individual.

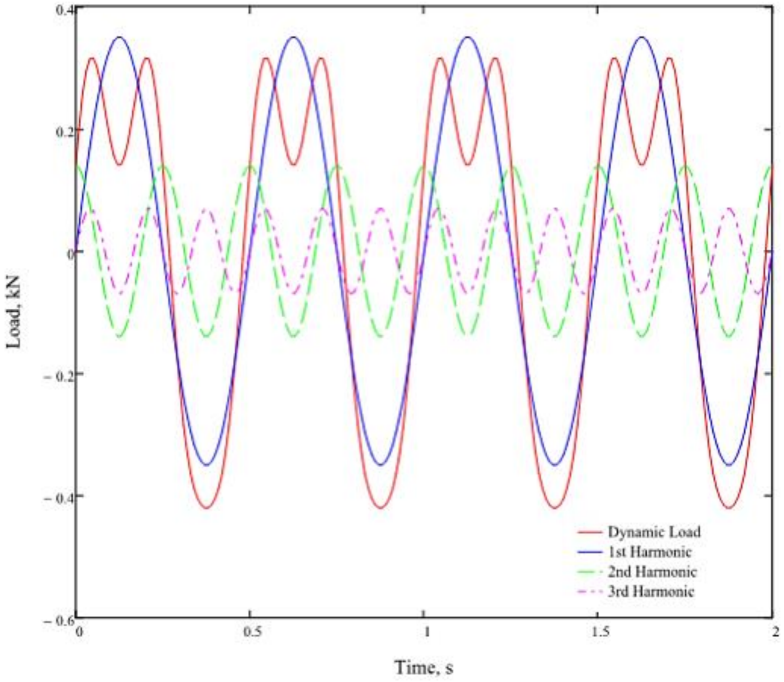


Figure 1. Dynamic load (vertical) of a single pedestrian

Compared to vertical vibration, people are known to be much more sensitive to lateral vibrations. If a person walks on a laterally vibrating bridge, he or she will try to compensate by swaying with the bridge displacement for stability. This adjustment is accompanied by an intuitive adaptation of the walking frequency and a widened gait. The swaying of the body in tune with the lateral vibration causes an increase in the lateral ground reaction forces, which introduces positive energy into the structural system of the bridge. Hence, if a footbridge vibrates slightly in the lateral direction and pedestrians synchronise their walking pattern with the bridge, then a low-damped bridge can be excited to large vibrations. This phenomenon is referred to as the “lock-in” effect. Due to the nature of walking, the lateral sway of a person is approximately half the vertical walking frequency. Thus, lateral frequencies as low as 1 Hz can be considered critical.

In a crowd, there is also a possibility that pedestrians or runners will synchronise with each other. Unfortunately, literature addressing the dynamic response of structures to crowds is not well published. However, the degree of synchronisation of pedestrians or runners with each other is limited as each pedestrian or runner can only observe and interact with the few others who are near him or her. Assuming that pedestrians or runners synchronise in pairs, rather than being independent, the response will increase by a factor of $\sqrt{2}$. Evidence in literature shows that synchronisation of pedestrians is more likely to occur in the lateral direction, and very rarely in the vertical direction.

PROPOSED DESIGN PROCEDURE

The analysis procedure recommended in this paper applies many of the recommendations defined by the French road authorities (Sétra). A study done by Zivanovic, Pavic and Ingólfsson reviewed the time-domain design procedures used for the vibration serviceability assessment of footbridges exposed to multi-person traffic as defined by Eurocode 5, ISO 10137, the UK National Annex to Eurocode 1 and Sétra. Zivanovic, et al. evaluated these different design methods for their reliability in predicting the vibration response of two as-built structures. The predicted and measured acceleration responses using the analysis procedure in Sétra showed minimal error in relation to the other published criteria in the study.

The proposed design method herein aims to demonstrate proof of comfort for vertical and lateral vibration and follows the following general procedure:

1. Define the footbridge load category and anticipated crowd density.
2. Assume a critical damping ratio for the specified bridge type.
3. Determine the natural frequency limits in the vertical and lateral direction for which the vibration serviceability limit state is satisfied.
4. For each dominate vertical and lateral mode shape, determine the bridge's natural frequency.
5. Determine if the bridge's natural frequencies imposes a risk of resonance in the vertical or lateral direction.

If the computed natural frequencies of the footbridge are outside the range of frequencies that are defined as critical, then the serviceability limit state in relation to vibration is satisfied. Otherwise, the verification procedure continues as follows:

6. Define the accepted acceleration limits that ensure pedestrian comfort.
7. Compute the dynamic response of the bridge using pedestrian loading functions.
8. Evaluate the maximum acceleration from the dynamic loading.
9. Compare the maximum acceleration with the accepted acceleration limit.

If the acceleration due to pedestrian loading is greater than the acceptable acceleration limit, then vibration-mitigation measures can be carried out to reduce the structure's vibration response.

FOOTBRIDGE LOADING CATEGORY AND CROWD DENSITY

The location and function of a pedestrian bridge influence the density of foot traffic it can expect. For this reason, footbridges are categorised by their location and expected usage, which is used to quantify maximum pedestrian crowd densities, as listed in Table 1.

Table 1. Load category and anticipated crowd density

Load Category	Description	Crowd density, d (pedestrians/m^2)
I	Urban footbridge that links high pedestrian density area, is frequently used by dense crowds and subjected to very heavy traffic	1
II	Urban footbridge that links populated areas, subjected to heavy traffic and may occasionally be loaded throughout its bearing area	0.8
III	Footbridge for standard use, may occasionally be crossed by large groups of people but that will never be loaded throughout its bearing area	0.5
IV	Footbridge that is seldom used	15 persons on the whole bridge

The crowd density is assumed to be distributed over the full usable walking area of the bridge. Thus, the number of pedestrians on a loaded footbridge is:

$$N = S \cdot d \qquad (2)$$

where *S* is the usable walking area of the bridge, and *d* is the crowd density from Table 1.

CRITICAL DAMPING RATIO

Each structure possesses some capability to dissipate energy that is represented by damping. Damping is affected by many factors, which include soil stiffness, and non-structural elements fastened after construction and not modelled in the design. This makes it difficult to evaluate the damping precisely in the design phase. Critical damping coefficients typically range between 0.1% and 2.0%, and it is best not to overestimate structural damping in order to avoid under-dimensioning. Thus, small vibration critical damping should be used to assess the serviceability state response. CEB information bulletin No. 209 provides recommended minimum and average damping ratios for various deck types, listed in Table 2. Minimum values are recommended for bridges spanning up to 20 m; otherwise, average values are appropriate.

Table 2. Critical damping ratio for pedestrian bridges

Deck Type	Critical Damping Ratio (ζ)	
	Minimum	Average
Reinforced concrete	0.8%	1.3%
Prestressed concrete	0.5%	1.0%
Composite (steel and concrete)	0.3%	0.6%

MODAL PROPERTIES AND RESONANCE

For footbridges defined within the parameters of category I, II or III, it is necessary to determine the natural vibration frequency of the structure in the vertical and lateral direction. In the evaluation of the structure's natural frequency, the modal mass should include any permanent load on the bridge. If the mass of pedestrians is more than 5% of the deck modal mass, then the structure's natural frequency should also include a footbridge loaded throughout its functional area to the tune of the maximum crowd density defined in Table 1. The mean weight of a single pedestrian is assumed as 700 N, which equates to a density of 70 kg/m^2.

There are several ways to compute the natural frequency of a bridge: by use of finite element analysis or using hand formulas (closed-form solutions). A simplified method for determining the natural frequency of a single-span, homogeneous beam is represented by:

$$f_n = \frac{n^2 \pi}{2L^2}\sqrt{\frac{EI}{\rho S}} \qquad (3)$$

where f_n is the natural frequency of the n^{th} mode, *L* is the span length, *E* is the Young's Modulus of the deck material, *I* is the effective bending moment of inertia of the bridge's superstructure section (orientation to conform with bending direction), and ρS is the linear density of the deck and pedestrians in kg/m.

When human-induced frequency synchronises with one of the structural frequencies, the dynamic forces are significantly magnified and a condition known as resonance occurs. As referenced in Table 2 above, the structural damping ratio for small vibration is typically low (in the range of 0.5% to 1%). Assuming a critical damping ratio of 1% for precast concrete bridges, the amplification of the static forces can become as high as 50, as illustrated in Figure 2.

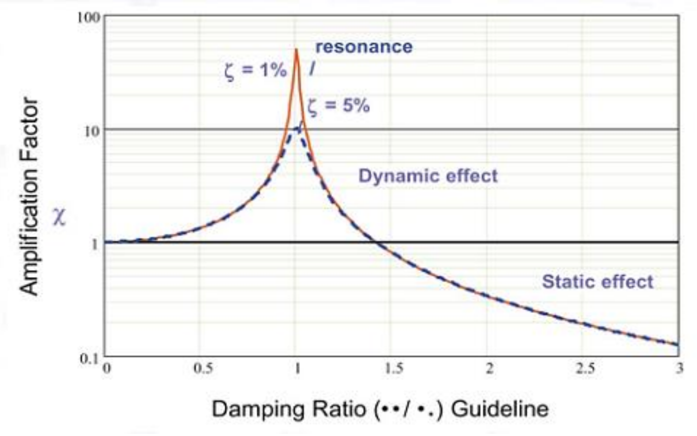


Figure 2. Dynamic amplification function [8]

The ranges in which these frequencies are arranged make it possible to assess the risk of resonance entailed by pedestrian traffic and, as a function of this, the dynamic load cases to study in order to verify the comfort criteria. In both the vertical and horizontal directions, there are four resonance risk levels as defined by Sétra, as presented in Table 3 and Table 4 below.

Table 3. Vertical vibration frequency ranges (Hz) for resonance risk

Frequency	0	1	1.7	2.1	2.6	5
1 Maximum risk			🟥			
2 Medium risk		🟧		🟧		
3 Low risk					🟨	
4 Negligible risk	🟩					🟩

Table 4. Lateral vibration frequency ranges (Hz) for resonance risk

Frequency	0	0.3	0.5	1.1	1.3	2.5
1 Maximum risk			🟥			
2 Medium risk		🟧		🟧		
3 Low risk					🟨	
4 Negligible risk	🟩					🟩

If a structure's vertical or lateral frequency is at risk of resonance, then dynamic calculations are required to check the comfort level criteria.

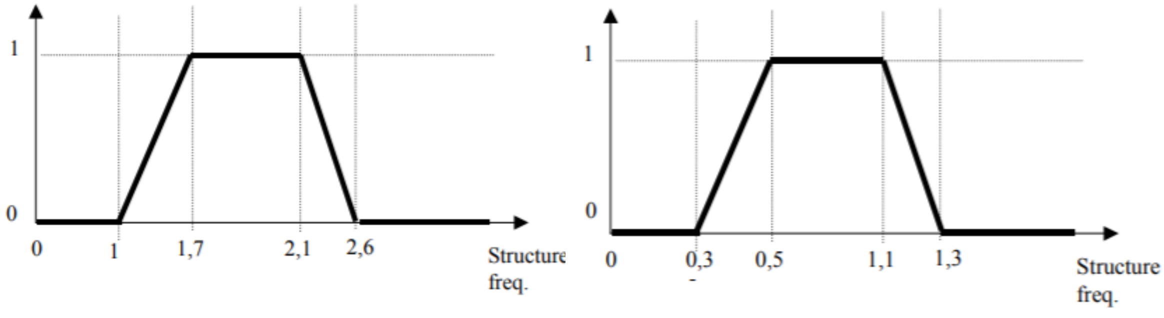
DYNAMIC LOAD OF PEDESTRIANS

In order to predict the behaviour of a bridge under human loading, a force model must be developed. The dynamic load per unit area due to vertical and lateral response is defined in the equations in Table 5 for any random crowd of pedestrians walking. The crossing of runners is relatively short and does not leave much time for a resonance phenomenon to settle; therefore, the time frame in which runners may cause discomfort to other pedestrians on the bridge is very minimum and is not considered herein. However, in the event of a foot race (i.e. marathon), the effects of running should be studied.

Table 5. Pedestrian-induced dynamic load functions

Load Category	Load Direction	Dynamic load, w (kN/m^2)	
I	Vertical	$d \times 280N \times \cos(2\pi f_n t) \times 1.85 \times (1/N)^{1/2} \times \psi$	(4)
	Lateral	$d \times 35N \times \cos(2\pi f_n t) \times 1.85 \times (1/N)^{1/2} \times \psi$	(5)
II, III	Vertical	$d \times 280N \times \cos(2\pi f_n t) \times 10.8 \times (\xi/N)^{1/2} \times \psi$	(6)
	Lateral	$d \times 35N \times \cos(2\pi f_n t) \times 10.8 \times (\xi/N)^{1/2} \times \psi$	(7)

d is the crowd density from Table 1, *N* is the number of people on the footbridge, f_n is the natural frequency, ζ is the damping ratio, and ψ is a factor that makes allowance for the risk of resonance based on the footbridge frequency as shown in Figure 3 (a) and (b) for vertical and lateral vibrations, respectively.



(a) Vertical vibrations (b) Lateral vibrations

Figure 3. Factor ψ for walking (first harmonic) [11]

The dynamic loads are applied to the whole functional area of the footbridge in the direction, at any point, that produces the maximum loading effect. Therefore, the direction of the load application must be the same as the direction of the mode shape and be inverted each time the mode shape changes direction, like when passing through a node. See Figure 4 for an example.

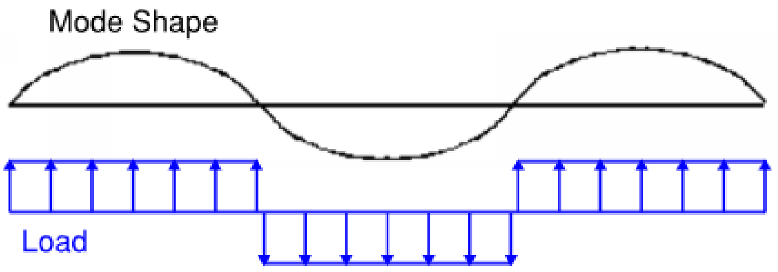


Figure 4. Sign of the amplitude of the load

ACCELERATION RESPONSE

The level of comfort depends on each individual experiencing the bridge vibrations. Some people are very sensitive to vibrations, while others are not. For designers, a common method of quantifying comfort is by means of the deck acceleration. Acceleration can be obtained by applying the pedestrian load model in Table 5 to the analysis model. Alternatively, the maximum acceleration due to vertical dynamic loading for a beam can be calculated as:

$$A_{max} = \frac{4F}{2\xi_n \pi \rho S} \tag{8}$$

where *F* is the linear load of pedestrians, which is the dynamic load from Table 5 multiplied by the usable width of the footbridge. Evaluation of acceleration takes into account the damping of the footbridge (ζ) or logarithmic decrement, which is equal to $2\pi\zeta$.

ACCEPTANCE CRITERIA FOR VERTICAL VIBRATION

Murray defines acceleration limits for vibrations due to human activities that are based on ISO 2631-2 (1989), and adjusted for structure type and occupancy [10]. Referring to Figure 5, an acceptable response for an outdoor pedestrian bridge should be less than the acceleration limit for "rhythmic activities, outdoor footbridges," which for most frequency ranges is approximately 10% g.

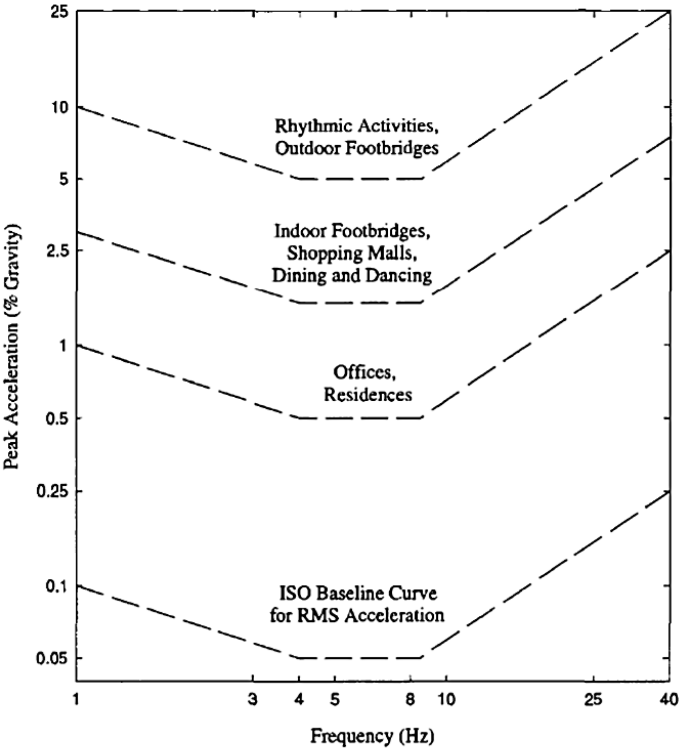


Figure 5. Acceptance comfort criteria for acceleration [10]

ACCEPTANCE CRITERIA FOR LATERAL VIBRATIONS

Based on investigations of the Millennium Bridge, Dallard, et. al. derived an estimate for the critical number of pedestrians above which excessive lateral vibrations of a bridge may occur. This critical number of pedestrians can be computed for a given mode as:

$$N_{cr} = \frac{4\pi\xi f_n M}{c_p \frac{1}{L}\int_0^L (|\phi(x)|^2 dx} \qquad (9)$$

where ξ is the damping ratio, f_n is the frequency of vibration of the mode considered, *M* is the modal mass, c_p is a "lateral force coefficient", *L* is the length of the bridge, and $\varphi(x)$ is the mode shape.

This formula is based on observations of the Millennium Bridge and other pedestrian bridges, which have shown that lateral vibrations are often modest at low pedestrian densities but grow rapidly in magnitude beyond a critical density of pedestrians [8]. This behaviour suggests that pedestrians synchronise their walking with the lateral motions of a bridge, thus applying lateral forces to the bridge in phase with its motion. Dallard suggests modelling this "feedback" mechanism as negative damping of the structure, with each pedestrian modelled with a lateral force coefficient c_p equal to 300 N•s/m. This coefficient corresponds to a "dashpot" where force is proportional to velocity.

VIBRATION MITIGATION MEASURES

When a bridge's acceleration does not meet the accepted comfort criteria, a number of modifications can be made to modify the structure's damping or natural frequency.

During the design of a new structure, the most economical means of modifying a structure's natural frequency is by increasing the structure's stiffness. Stiffening is best suitable for bridges that show potential pedestrian vibrations that border between acceptable and unacceptable since the overall effect of stiffening the bridge's superstructure section has a modest influence to the natural frequency. The structure should be stiffened in the appropriate direction. In addition to structural stiffening, reducing the effective span length or increasing the weight of a footbridge will also reduce the influence of human-induced vibration. However, a proportional increase in stiffness is required to maintain the natural frequency.

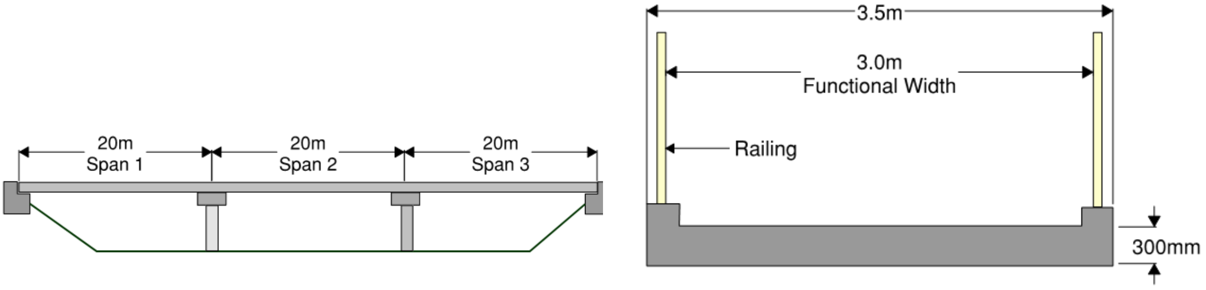
A direct method to increase a structure's damping can be achieved by use of a damping system; such as a tuned mass damper or viscous damper. Assuming the structure, new or existing, can accommodate a damping system, it may be used to increase the amount of energy that is dissipated by the structure by tuning it to a specific frequency.

EXAMPLE VIBRATION ANALYSIS OF A CONCRETE FOOTBRIDGE

Verification of the serviceability limit state related to vibration due to pedestrians following the guidelines presented herein is conducted on a model of a prestressed concrete pedestrian bridge.

The footbridge is a three-span, continuous prestressed concrete girder, with each span measuring 20m in length, as shown in Figure 6. The superstructure is supported on bearings at the abutments. The structural characteristics are defined as follows:

Bridge length	$L = 60$ m	Concrete compressive strength	$f'_c = 50$ MPa
Cross-sectional area	$S = 1.10$ m^2	Concrete modulus of elasticity	$E_c = 33234$ MPa
Moment of inertia	$Iy = 1.20$ m^4	Deck density	$\rho = 24$ kN/m^3
	$Iz = 0.01$ m^4	Railing load (each side)	$R = 1$ kN/m



(a) Elevation

(b) Cross-Section

Figure 6. Elevation and cross-section of prestressed concrete footbridge

The footbridge connects a housing community to a primary school district. The bridge can anticipate large foot traffic from students and parents, and is therefore designated as a load category II footbridge with an anticipated crowd density of 0.8 persons per square meter. The functional width of the footbridge is 3.0 m, which makes its usable area, S, 180 m^2 and maximum crowd size, N, 144 pedestrians.

$$N = 60 \text{ m} \times 3.0 \text{ m} \times 0.8 \text{ pedestrians/m}^2 = 144 \text{ pedestrians}$$

The linear density of 144 pedestrians equates to a linear mass, ρ, of 168 kg/m, which is over 5% of the mass of the deck. Therefore, the mass of the pedestrians is included in the bridge's participating modal mass for the particular mode being analysed.

$$\rho = 144 \times 70 \text{ kg}/60 \text{ m} = 168 \text{ kg/m}$$

Murray and Sétra recommend a damping ratio of 0.5% for indoor and outdoor footbridges that have spans 20m or less, which is appropriate for uncracked structures subjected to low intensity, non-seismic loading. A modal analysis of the bridge was run in finite element modelling software SAP2000, which takes into account the flexibility of the substructure. The first three frequencies obtained from this analysis are summarised in Table 6 below.

Table 6. Natural frequency of first 3 modes

Mode	Frequency (Hz)	Mode Shape
1†	1.75	Vertical
2	1.93	Vertical
3	1.98	Lateral

†80% of the mass is participating in mode 1

The first mode (illustrated in Figure 7) and second mode have a frequency of 1.75 Hz and 1.93 Hz, respectively, and may be considered potentially unstable since their frequencies have a high risk of resonance, as indicated in Table 3. Therefore, it is recommended to carry out dynamic structure calculations for the vertical response of the first harmonic, which in this case is the first mode.

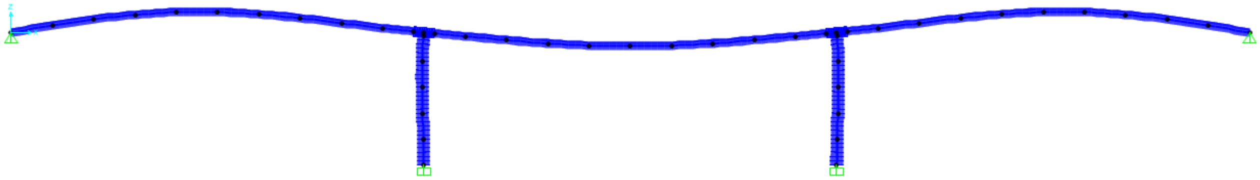


Figure 7. Vertical mode shape (mode 1) from SAP2000 modal analysis

Applying Equation (6), the surface load is equal to:

$$w = \frac{0.8}{m^2} \, 280N \, cos(2\pi \cdot 1.75Hz \cdot t) \, 10.8 \sqrt{\frac{0.005}{144}} \cdot 1 = 14.25 \, cos(11\,t) \, N/m^2$$

Multiplying the surface load by the usable width yields a linear load of:

$$F = 42.77cos(11\,t) \, N/m$$

The maximum acceleration from the vertical loading function is equal to:

$$A_{max} = \frac{4F}{2\xi_n \pi \rho S} = \frac{4 \cdot 42.77 \, N/m}{2 \cdot 0.5\% \cdot \pi \cdot 2186 \, kg/m} = 2.5 \, m/s^2$$

The maximum vertical acceleration response is 25.4% g, which exceeds the acceptable acceleration of 8.75% g for a footbridge with a frequency of 1.75 Hz, per Murray. Had a critical damping ratio of 1% been utilised for the structure, the maximum acceleration would be 1.76 m/s^2, or 18% g, which still exceeds the allowable comfort criteria for vertical vibrations. Mitigation measures to increase the structure's natural frequency could involve increasing the structure's stiffness or decreasing the span lengths.

Mode 3, plotted in Figure 8, is the first mode with significant participation in the transverse direction, with a frequency of 1.98 Hz. A lateral frequency of 1.98 Hz has a low risk of resonance per Table 4. Still, equation (9) can be applied to estimate the critical number of pedestrians that would cause lateral instability, based on an assumed damping ratio of 0.5%.

$$N_{cr} = \frac{4\pi\xi f_o M}{c_p \frac{1}{L}\int_0^L (|\phi(x)|^2 dx} = 8867 \text{ pedestrians}$$

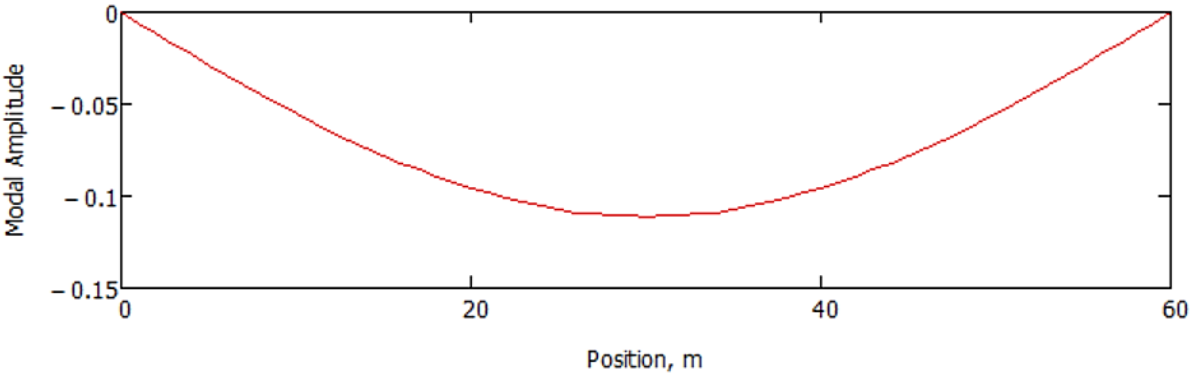


Figure 8. Lateral mode shape (mode 3)

The critical number of pedestrians that would cause instability is approximately 8867, which yields a density of approximately 49 persons per square meter. A value of 0.8 persons per square meter is assumed as a design goal for dense crowds for this particular structure. Based on an evaluation of the pedestrian bridge connectivity and usage, it is unlikely the pedestrian density of 8867 people on the bridge will be exceeded at one time.

CONCLUSION

This paper provides a guideline for analysing a concrete footbridge for pedestrian-induced vibrations, comfort acceptance criteria and possible vibration mitigation measures. A numerical example of a prestressed concrete footbridge is presented, which applies the recommended guidelines. The vertical response of the footbridge is computed by numerical simulation of its reaction to a deck loaded with the maximum number of anticipated pedestrians. In addition, the number of pedestrians that would cause a lateral “lock-in” effect is estimated.

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