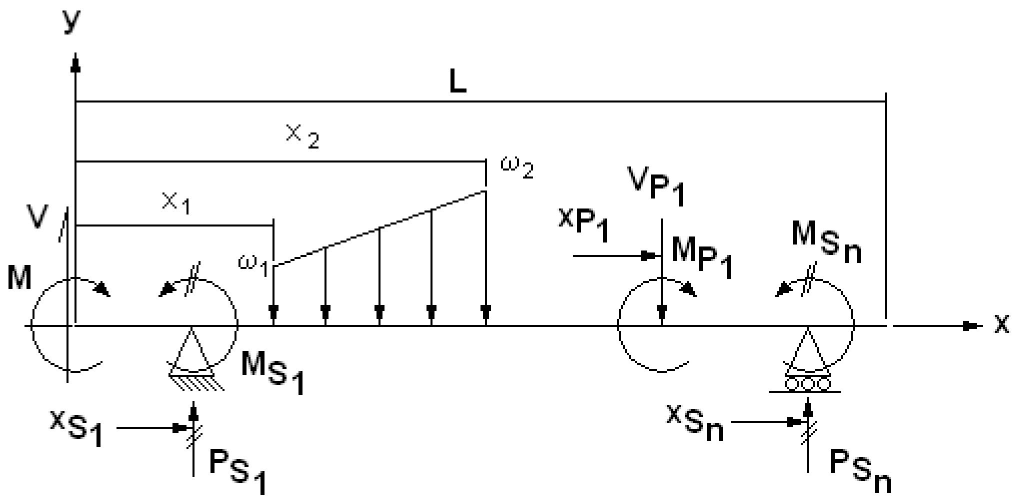
Beam Analysis



$L := 25 \cdot m$ ... beam length

$I := 659997685 \cdot mm^4$ ... beam inertia

$E := 200 \cdot GPa$ ... material modulus

Distributed Loading

$Data_{DL} :=$

"Distribution"	"Start Point"	"Load at Start"	"End Point"	"Load at End"
"No"	"(m)"	"(kN/m)"	"(m)"	"(kN/m)"
1	10	2	20	2
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0

Point Loading

$Data_{PL} :=$

"Load"	"Location"	"Point Load"	"Step Moment"
"No"	"(m)"	"(kN)"	"(kN.m)"
1	5	10	0
2	25	10	0

Supports

$$Data_S := \begin{pmatrix} \text{"Support"} & \text{"Location"} & \text{"Support"} \\ \text{"No"} & \text{"(m)"} & \text{"Condition"} \\ 1 & 0 & \text{"Fixed"} \\ 2 & 10 & \text{"Pinned"} \\ 3 & 20 & \text{"Pinned"} \end{pmatrix}$$

Condition	**Translation**	**Rotation**
Fixed	Fixed	Fixed
Pinned	Fixed	Free
Guided	Free	Fixed
Free	Free	Free

Distributed Loading

Distributed load function ...

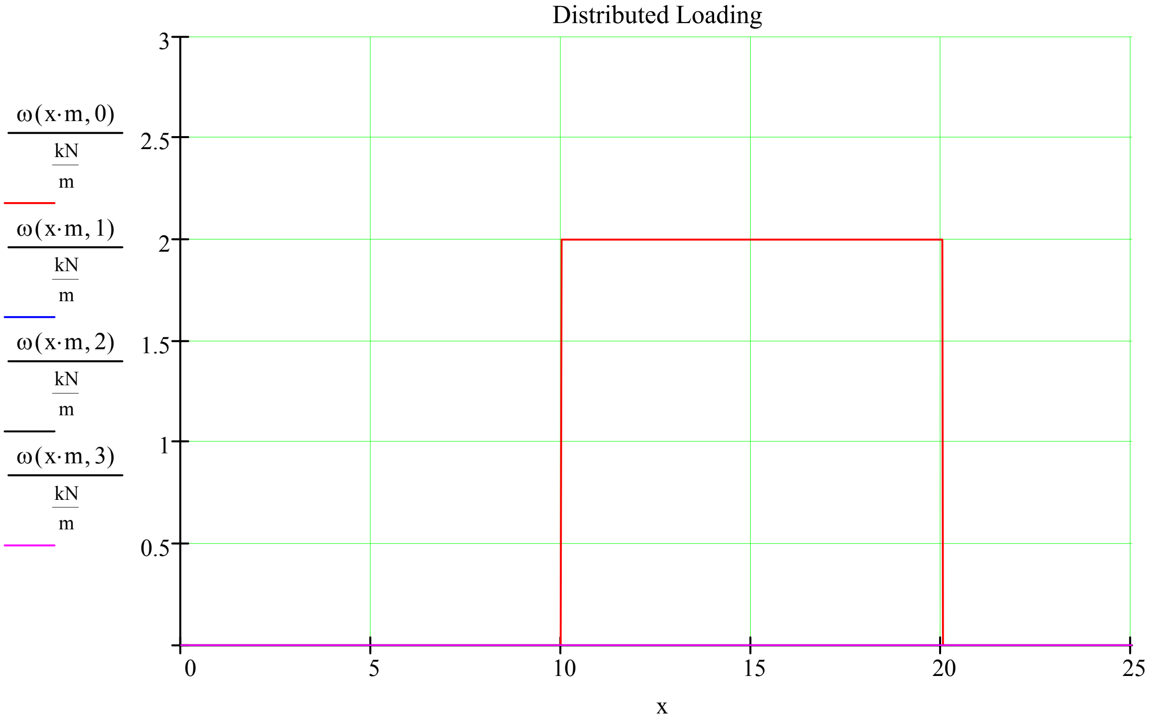
$$\omega(x,i) = \left[\omega_{1_i} + \left(\omega_{2_i} - \omega_{1_i}\right)\cdot\frac{\left(x - x_{1_i}\right)}{\left(x_{2_i} - x_{1_i}\right)}\right]\cdot\left(x_{1_i} \le x \le x_{2_i}\right)$$

Change in distribution ...

$$\Delta\omega_i = \frac{\left(\omega_{2_i} - \omega_{1_i}\right)}{\left(x_{2_i} - x_{1_i}\right)}$$

Giving ...

$$\omega(x,i) := \left[\omega_{1_i} + \Delta\omega_i\cdot\left(x - x_{1_i}\right)\right]\cdot\left(x_{1_i} \le x \le x_{2_i}\right)$$



Dist. loading force ... $\Sigma P_{\omega_i} := \int_0^L \omega(x,i)\,dx$ $\Sigma P_\omega = \begin{pmatrix} 20 \\ 0 \\ 0 \\ 0 \end{pmatrix} kN$ ... dist'n No.1 ... distrib'n No.2 etc

Dist. loading centroid ... $x_{P\omega_i} := \dfrac{\left(\Sigma P_{\omega_i} > 0\cdot N\right)}{\Sigma P_{\omega_i}} \cdot \int_0^L \omega(x,i)\cdot x\,dx$ $x_{P\omega} = \begin{pmatrix} 15 \\ 0 \\ 0 \\ 0 \end{pmatrix} m$ ... dist'n No.1 ... distrib'n No.2 etc

Shear and Moment for Distributions

Shear functions ...

$$V_\omega(x',i) = \int_{x_{1_i}}^{x'} \left[\omega_{1_i} + \Delta\omega_i\cdot\left(x - x_{1_i}\right)\right] dx = \omega_{1_i}\cdot\left(x' - x_{1_i}\right) + \frac{1}{2}\cdot\Delta\omega_i\cdot\left(x' - x_{1_i}\right)^2 \text{ ... when } \quad x_{1_i} \le x' < x_{2_i}$$

$$V_\omega(i) = \Sigma P_{\omega_i}\cdot\left(x \ge x_{2_i}\right) \quad \text{ ... when } \quad x' \ge x_{2_i}$$

Combining ... $V_\omega(x,i) := \left[\omega_{1_i}\cdot\left(x - x_{1_i}\right) + \frac{1}{2}\cdot\Delta\omega_i\cdot\left(x - x_{1_i}\right)^2\right]\cdot\left(x_{1_i} \le x < x_{2_i}\right) + \Sigma P_{\omega_i}\cdot\left(x \ge x_{2_i}\right)$

Bending moment functions ...

$$M_{\omega 1}(x',i) = \int_{x_{1_i}}^{x'} \left[\omega_{1_i} + \Delta\omega_i\cdot\left(x - x_{1_i}\right)\right]\cdot(x' - x)\,dx = \frac{1}{6}\cdot\left(x' - x_{1_i}\right)^2\cdot\left[3\cdot\omega_{1_i} + \Delta\omega_i\cdot\left(x' - x_{1_i}\right)\right] \text{ ... when } \quad x' \ge x_{1_i}$$

$$M_{\omega 2}(x',i) = \int_{x_{2_i}}^{x'} \left[\omega_{2_i} + \Delta\omega_i\cdot\left(x - x_{2_i}\right)\right]\cdot(x' - x)\,dx = \frac{1}{6}\cdot\left(x' - x_{2_i}\right)^2\cdot\left[3\cdot\omega_{2_i} + \Delta\omega_i\cdot\left(x' - x_{2_i}\right)\right] \text{ ... when } \quad x' \ge x_{2_i}$$

Combining ... $M_{\omega}(x,i) := \frac{1}{6}\cdot\left(x - x_{1_i}\right)^2\cdot\left[3\cdot\omega_{1_i} + \Delta\omega_i\cdot\left(x - x_{1_i}\right)\right]\cdot\left(x \ge x_{1_i}\right) \ldots$

$$+ -\frac{1}{6}\cdot\left(x - x_{2_i}\right)^2\cdot\left[3\cdot\omega_{2_i} + \Delta\omega_i\cdot\left(x - x_{2_i}\right)\right]\cdot\left(x \ge x_{2_i}\right)$$

Summing shear ... $V\left(x,P_S\right) := \sum_{j=0}^{n_S} P_{S_j}\cdot\left(x \ge x_{S_j}\right) - \sum_{n=0}^{n_P} V_{P_n}\cdot\left(x \ge x_{P_n}\right) - \sum_{i=0}^{n_{dl}} V_{\omega}(x,i)$

Summing moments ... $M\left(x,P_S,M_S\right) := \sum_{j=0}^{n_S}\left[\begin{array}{l} M_{S_j}\cdot\left[\begin{array}{l}-(j>0)\ \ldots \\ +(j = 0)\end{array}\right] \ldots \\ + P_{S_j}\cdot\left(x - x_{S_j}\right)\end{array}\right]\cdot\left(x \ge x_{S_j}\right) + \sum_{n=0}^{n_P} M_{P_n}\cdot\left(x \ge x_{P_n}\right) \ldots$

$$+ -\sum_{n=0}^{n_P} V_{P_n}\cdot\left(x - x_{P_n}\right)\cdot\left(x \ge x_{P_n}\right) - \sum_{i=0}^{n_{dl}} M_{\omega}(x,i)$$

Beam Deflection

$y''_b\left(x,P_S,M_S\right) = \frac{M\left(x,P_S,M_S\right)}{E\cdot I}$ ... 2nd derivative

$y'_b\left(x',P_S,M_S,\theta_1\right) = \theta_1 + \int_0^{x'} y''_b\left(x,P_S,M_S\right)\,dx$ ... 1st derivative (gradient)

Let ... $y'_{\omega}(x') = \int_0^{x'} M_{\omega}(x,i)\,dx$ ... integral of moment due to distributed loading

$\frac{1}{6}\cdot\int_{x_{1_i}}^{x'} \left(x - x_{1_i}\right)^2\cdot\left[3\cdot\omega_{1_i} + \Delta\omega_i\cdot\left(x - x_{1_i}\right)\right]dx = \frac{1}{24}\cdot\left(x' - x_{1_i}\right)^3\cdot\left[\Delta\omega_i\cdot\left(x' - x_{1_i}\right) + 4\cdot\omega_{1_i}\right]$ ... when $x' \ge x_{1_i}$

$\frac{1}{6}\cdot\int_{x_{2_i}}^{x'} \left(x - x_{2_i}\right)^2\cdot\left[3\cdot\omega_{2_i} + \Delta\omega_i\cdot\left(x - x_{2_i}\right)\right]dx = \frac{1}{24}\cdot\left(x' - x_{2_i}\right)^3\cdot\left[\Delta\omega_i\cdot\left(x' - x_{2_i}\right) + 4\cdot\omega_{2_i}\right]$ ... when $x' \ge x_{2_i}$

Combining ... $y'_{\omega}(x,i) := \frac{1}{24}\cdot\left(x - x_{1_i}\right)^3\cdot\left[\Delta\omega_i\cdot\left(x - x_{1_i}\right) + 4\cdot\omega_{1_i}\right]\cdot\left(x \ge x_{1_i}\right) \ldots$

$+ -\frac{1}{24}\cdot\left(x - x_{2_i}\right)^3\cdot\left[\Delta\omega_i\cdot\left(x - x_{2_i}\right) + 4\cdot\omega_{2_i}\right]\cdot\left(x \ge x_{2_i}\right)$

$$y'_b\left(x, P_S, M_S, \theta_1\right) := \theta_1 + \frac{1}{E\cdot I}\cdot\left[\begin{array}{l}\sum_{j=0}^{n_S}\left[\begin{array}{l}M_{S_j}\cdot\left[\begin{array}{l}-(j>0)\ \ldots\\ +(j=0)\end{array}\right]\cdot\left(x - x_{S_j}\right)\ \ldots\\ +\frac{1}{2}\cdot P_{S_j}\cdot\left(x - x_{S_j}\right)^2\end{array}\right]\cdot\left(x \ge x_{S_j}\right) - \sum_{i=0}^{n_{dl}} y'_{\omega}(x,i)\ \ldots\\ + \sum_{n=0}^{n_P} M_{P_n}\cdot\left(x - x_{P_n}\right)\cdot\left(x \ge x_{P_n}\right) - \frac{1}{2}\cdot\sum_{n=0}^{n_P} V_{P_n}\cdot\left(x - x_{P_n}\right)^2\cdot\left(x \ge x_{P_n}\right)\end{array}\right]$$

$y_b\left(x', P_S, M_S, \theta_1, y_1\right) = y_1 + \int_0^{x'} y'_b\left(x, P_S, M_S, \theta_1\right)\,dx$ ... displacement

Let ... $y_{\omega}(x') = \int_0^{x'} y'_{\omega}(x,i)\,dx$ ... integral of gradient due to distributed loading

$\frac{1}{24}\cdot\int_{x_{1_i}}^{x'}\left(x - x_{1_i}\right)^3\cdot\left[\Delta\omega_i\cdot\left(x - x_{1_i}\right) + 4\cdot\omega_{1_i}\right]dx = \frac{1}{120}\cdot\left(x' - x_{1_i}\right)^4\cdot\left[\Delta\omega_i\cdot\left(x' - x_{1_i}\right) + 5\cdot\omega_{1_i}\right]$ ... when $x' \ge x_{1_i}$

$\frac{1}{24}\cdot\int_{x_{2_i}}^{x'}\left(x - x_{2_i}\right)^3\cdot\left[\Delta\omega_i\cdot\left(x - x_{2_i}\right) + 4\cdot\omega_{2_i}\right]dx = \frac{1}{120}\cdot\left(x' - x_{2_i}\right)^4\cdot\left[\Delta\omega_i\cdot\left(x' - x_{2_i}\right) + 5\cdot\omega_{2_i}\right]$ ... when $x' \ge x_{2_i}$

Combining ... $y_{\omega}(x,i) := \frac{1}{120}\cdot\left(x - x_{1_i}\right)^4\cdot\left[\Delta\omega_i\cdot\left(x - x_{1_i}\right) + 5\cdot\omega_{1_i}\right]\cdot\left(x \ge x_{1_i}\right) \ldots$

$+ -\frac{1}{120}\cdot\left(x - x_{2_i}\right)^4\cdot\left[\Delta\omega_i\cdot\left(x - x_{2_i}\right) + 5\cdot\omega_{2_i}\right]\cdot\left(x \ge x_{2_i}\right)$

$$y_b\left(x,P_S,M_S,\theta_1,y_1\right) := y_1 + \theta_1\cdot x + \frac{1}{E\cdot I}\cdot\left[\sum_{j=0}^{n_S}\left[\frac{1}{2}\cdot M_{S_j}\cdot\begin{bmatrix}-(j>0)\ \ldots\\+(j=0)\end{bmatrix}\cdot\left(x-x_{S_j}\right)^2\ \ldots\\ +\frac{1}{6}\cdot P_{S_j}\cdot\left(x-x_{S_j}\right)^3\right]\cdot\left(x\ge x_{S_j}\right)\ \ldots\\ +\frac{1}{2}\cdot\sum_{n=0}^{n_P} M_{P_n}\cdot\left(x-x_{P_n}\right)^2\cdot\left(x\ge x_{P_n}\right) - \sum_{i=0}^{n_{dl}} y_\omega(x,i)\ \ldots\\ +-\frac{1}{6}\cdot\sum_{n=0}^{n_P} V_{P_n}\cdot\left(x-x_{P_n}\right)^3\cdot\left(x\ge x_{P_n}\right)\right]$$

Defining Support Conditions

Defining beam deflection and / or loading to be set to zero based on support condition ...

$$\text{Support}\left(P_S,M_S,\theta_1\right) := \left|\begin{array}{l}\text{for } j\in 0..n_S\\ \quad\left|\begin{array}{l}Sup^{\langle j\rangle} \leftarrow \begin{pmatrix}y'_b\left(x_{S_j},P_S,M_S,\theta_1\right)\\ y_b\left(x_{S_j},P_S,M_S,\theta_1,y_1\right)\cdot mm^{-1}\end{pmatrix} \text{ if } Supt_j = \text{"Fixed"}\\ Sup^{\langle j\rangle} \leftarrow \begin{bmatrix}M_{S_j}\cdot(N\cdot mm)^{-1}\\ y_b\left(x_{S_j},P_S,M_S,\theta_1,y_1\right)\cdot mm^{-1}\end{bmatrix} \text{ if } Supt_j = \text{"Pinned"}\\ Sup^{\langle j\rangle} \leftarrow \begin{pmatrix}P_{S_j}\cdot N^{-1}\\ y'_b\left(x_{S_j},P_S,M_S,\theta_1\right)\end{pmatrix} \text{ if } Supt_j = \text{"Guided"}\\ Sup^{\langle j\rangle} \leftarrow \begin{bmatrix}M_{S_j}\cdot(N\cdot mm)^{-1}\\ P_{S_j}\cdot N^{-1}\end{bmatrix} \text{ otherwise}\end{array}\right.\\ Sup\end{array}\right.$$

Setting support conditions to zero ...

$$\text{Conditions} := \left|\begin{array}{l}\text{for } j\in 0..n_S\\ \quad Con^{\langle j\rangle} \leftarrow \begin{pmatrix}0\\0\end{pmatrix}\\ Con\end{array}\right.$$

Setting Equations and Solving

Given $V(L, P_S) = 0 \cdot N$ ... shear at end of beam

$M(L, P_S, M_S) = 0 \cdot N \cdot mm$ ...moment at end of beam

$\text{Support}(P_S, M_S, \theta_1) = \text{Conditions}$ ... beam support conditions

Solving the above equations permit the unknowns to be derived.

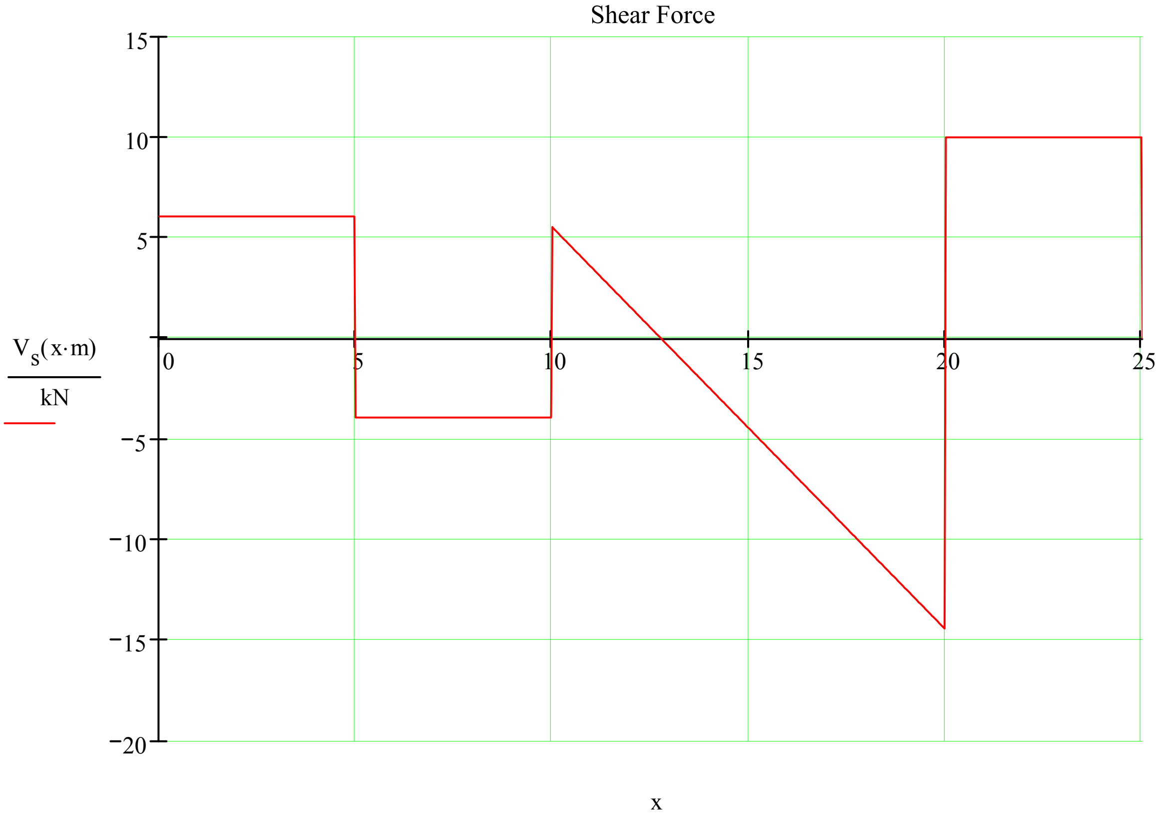
Beam Loading and Deflection Equations

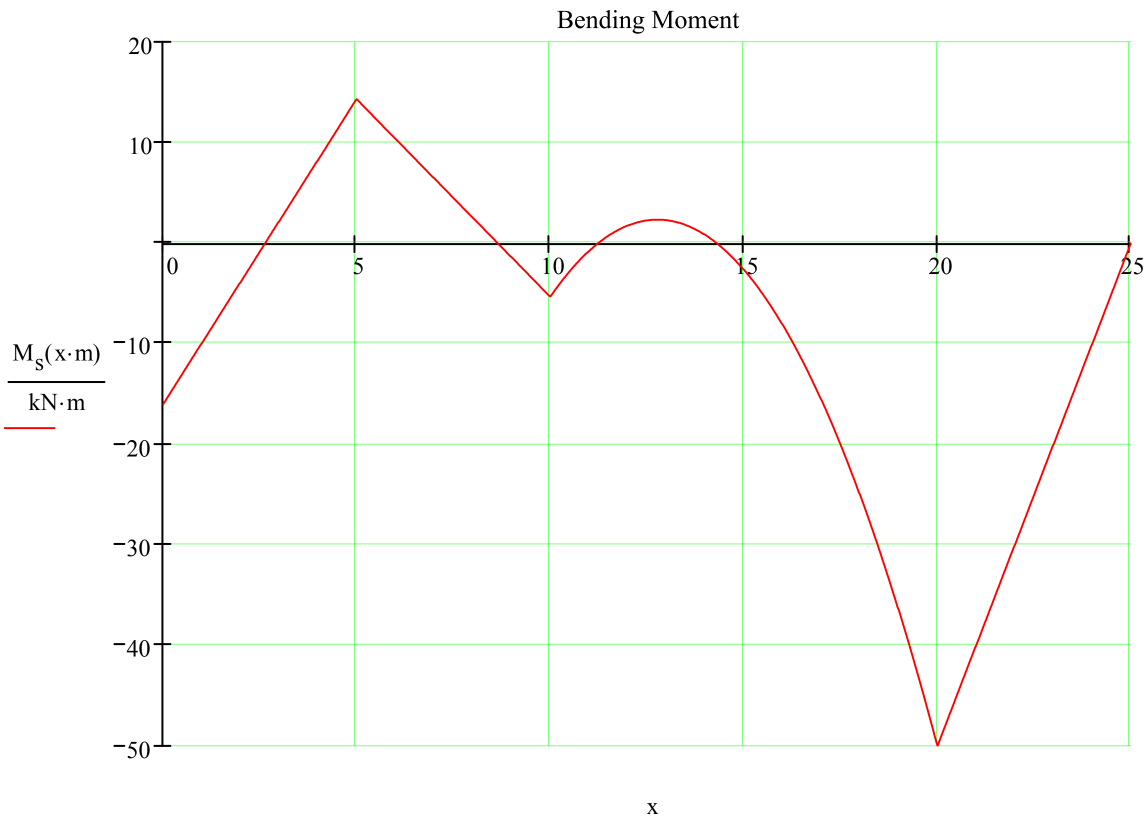
Let ... $V_s(x) := V(x, P_S)$

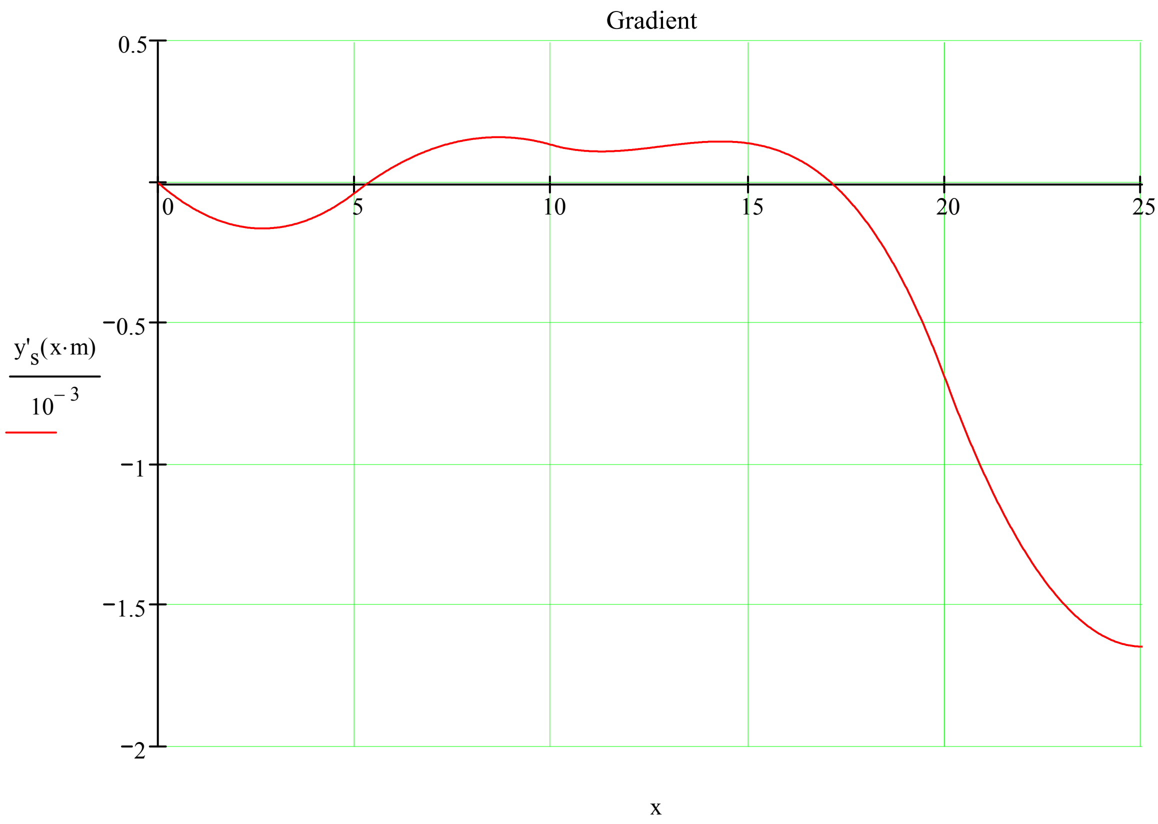
$M_s(x) := M(x, P_S, M_S)$ ... 2 equations for beam loading and ...

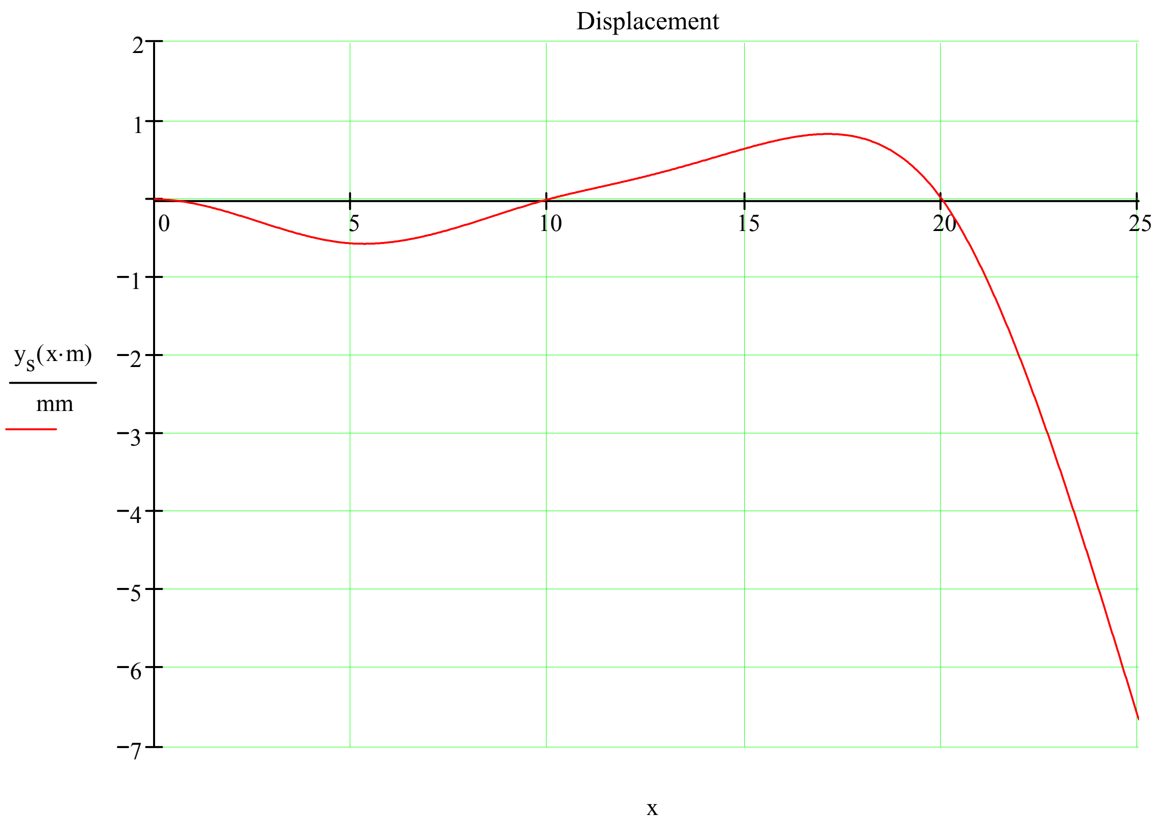
$y'_s(x) := y'_b(x, P_S, M_S, \theta_1)$

$y_s(x) := y_b(x, P_S, M_S, \theta_1, y_1)$ ... 2 equations for beam deflection


Shear Force
15
10
5
−5
−10
−15
−20
0
5
10
15
20
25
$\frac{V_s(x \cdot m)}{kN}$
x


Bending Moment
20
10
−10
−20
−30
−40
−50
0
5
10
15
20
25
$\frac{M_s(x \cdot m)}{kN \cdot m}$
x


Gradient
0.5
−0.5
−1
−1.5
−2
0
5
10
15
20
25
y'S(x·m)
10−3
x


Displacement
2
1
−1
−2
−3
−4
−5
−6
−7
0
5
10
15
20
25
yS(x·m)
mm
x

Results

Reaction forces ... $P_S = \begin{pmatrix} 6.071 \\ 9.464 \\ 24.464 \end{pmatrix} kN$ ... at ... $x_S = \begin{pmatrix} 0.000 \\ 10.000 \\ 20.000 \end{pmatrix} m$

Check ... $\sum P_S - \sum V_P - \sum \Sigma P_\omega = 0\,N$

Reaction moments ... $M_S = \begin{pmatrix} -16.071 \\ 0.000 \\ 0.000 \end{pmatrix} kN \cdot m$

Max. BM occurs at ... $x_{BM\_max} = 5.000\,m$ ... with a value of ... $M_{max} = 14.286\,kN \cdot m$

Min. BM occurs at ... $x_{BM\_min} = 20.000\,m$ ... with a value of ... $M_{min} = -49.998\,kN \cdot m$

Max. deflection occurs at ... $x_{y\_max} = 25.000\,m$ ... with a value of ... $y_{max} = -6.65136\,mm$

Deflection and gradient ... at supports and point load locations

$y_s\left(x_{S_j}\right) = \begin{pmatrix} 0.00000 \\ 0.00000 \\ -0.00000 \end{pmatrix} mm$ $\quad y'_s\left(x_{S_j}\right) = \begin{pmatrix} 0.00000 \\ 1.35282 \\ -6.98956 \end{pmatrix} 10^{-4}$

$y_s\left(x_{P_n}\right) = \begin{pmatrix} -0.56367 \\ -6.65136 \end{pmatrix} mm$ $\quad y'_s\left(x_{P_n}\right) = \begin{pmatrix} -0.33820 \\ -16.45929 \end{pmatrix} 10^{-4}$

$y_s(0 \cdot mm) = 0.00000\,mm$ $\quad y'_s(0 \cdot mm) = 0.00000$

$y_s(L) = -6.65136\,mm$ $\quad y'_s(L) = -1.64593\ 10^{-3}$