ditions if the feed is subcooled to any extent (see Section 3-2-3-3 for sub-cooled liquid models).

Direct methods of solution for no-slip (HEM) cases are given in the following section. For the general case with a slip option, the complexities of phase equilibrium and slip relationships rule out an analytical integration or good short-cut methods of solution. The general approach requires numerical integration of Equation (II-15) at constant entropy and at phase equilibrium, with or without slip. Equation (II-18) is an alternative formulation for the slip case.

If the pressure drop is high enough to attain maximum flow, the conditions at the nozzle throat will satisfy the criterion of Equation (II-11). This criterion is expressed by Equation (II-12) for the slip case. For these equilibrium models, the maximum flow condition exhibited by the integrated Bernoulli equation (or the equivalent momentum form) is also the solution of Equation (II-12). Slip models for nozzles are not pursued further in this document.

3-2-2-2. Homogeneous-Equilibrium (HEM) Model. An analytical pseudo-one-component solution for the no-slip homogeneous-equilibrium (HEM) model is as follows (Section 2.2.1 of Reference [7]):

$$(G\bar{v})^2/2 = v_f(P_0 - P) + (XH_{fg}/T_0 + C_f)(T_0 - T) - C_fT \ln(T_0/T) \quad \text{(II-34)}$$

where $\bar{v} = Xv_g + (1 - X)v_f$ from Equation (II-1)

C_f = average liquid C_P over the range T to T_0
X, v_f, v_g, and H_{fg} are evaluated at T_0 and P_0.

Equation (II-34) corresponds to the line labeled "integrated at equilibrium-flash quality" on Figure II-1 (Section 3-2).

The critical flow condition is reached if the throat conditions satisfy the following no-slip formulation of Equation (II-12)

$$G_c^{-2} = Xv_g/(kP) + (v_{fg}/H_{fg})^2(C_fT - XH_{fg}) \quad \text{(II-35)}$$

where the parameters are evaluated at T and P.

Equation (II-35) corresponds to the dashed line labeled "equilibrium-flash (HEM) model" on Figure II-1 (Section 3-2). Equations (II-34) and (II-35) can be solved simultaneously for the maximum flow and the corresponding pressure at the throat (labeled "HEM model flow" on Figure II-1). Alternatively, the throat pressure in Equation (II-34) can be varied to find the maximum flow condition. This maximum flow condition satisfies Equation (II-35).

The above pseudo-one-component approach may not yield satisfactory results if mixture composition and mean properties vary significantly over

the pressure range of interest. A better approach in such cases is to fit the results of selected flash computations to some simple expression and then use the resulting expression in the flow equations. The method is described below.

The adiabatic flashing of the fluid as the pressure falls along the flow path is computed by conventional flash distillation methods (see Appendices II-A and II-B for a description of the required thermophysical property base and computational routines). In principle, the flash path should be isentropic for flow in frictionless devices (nozzles, short pipes, etc.). In practice, little error is introduced by carrying out the flash computations at constant enthalpy. The flash results can be expressed in terms of the homogeneous two-phase specific volume and a pressure drop parameter as follows [7]:

$$\bar{v}/\bar{v}_0 = a(1 - \eta)/\eta + b[(1 - \eta)/\eta]^2 + 1 \qquad \text{(II-36)}$$

where $\eta = P/P_0$

$$\bar{v} = Xv_g + (1 - X)v_f \qquad \text{(II-1)}$$

The coefficients a and b are evaluated from the results of two flash calculations from the upstream pressure. One flash is to the pressure downstream of the duct and the other is to the average of the upstream and downstream pressures. Once evaluated, Equation (II-1) provides the requisite density versus pressure results for homogeneous-equilibrium two-phase flow models. Actually, only the first term on the right of Equation (II-36) is needed for an adequate fit over the limited pressure range of nozzle flow. A flash routine is used to determine the specific volume value at 70% of the initial pressure. This value is then used to evaluate the a coefficient in Equation (II-36). The HEM flow versus pressure ratio is obtained from the following integral form of the Bernoulli equation [7,11]

$$G = \{[-2P_0/\bar{v}_0][a(\ln \eta) + (a - 1)(1 - \eta)]\}^{1/2}/\{[a(1 - \eta)/\eta] + 1\} \qquad \text{(II-37)}$$

The critical flow condition of Equation (II-12) can be represented using only the first-order term of Equation (II-36)

$$G_{c,\text{ homogeneous}} = \eta_c[P_0/(a\bar{v}_0)]^{1/2} \qquad \text{(II-38)}$$

Note that Equation (II-38) can be used to obtain a value of the critical pressure ratio if the flow rate is known. The dashed line labeled "equilibrium-flash (HEM) model" on Figure II-1 corresponds to Equation (II-38). The intersection with the Equation (II-37) line gives the maximum flow rate (labeled "HEM model flow"). The value of η obtained from the abscissa at the intersection is the critical pressure ratio. If the backpressure on a nozzle is higher than the value obtained from this critical ratio, then the flow is

subsonic. In this case, the flow is obtained from Equation (II-37) using the backpressure value for the throat pressure.

Note once again that the above equations are developed for ideal frictionless nozzles. See Section 3-2-4 for use of a discharge coefficient for real geometries.

3-2-3. Auxiliary Models

3-2-3-1. Approximate HNE and ERM Models.

The ERM model of Section 3-2-1-2 provides the basis for a rather simple method for obtaining approximate flashing flow rates at low quality levels. As can be seen from Figure II-1 (Section 3-2), the ERM critical flow is attained with little pressure drop. Thus, the use of upstream conditions in Equation (II-25) (with $N = 1$ for the ERM model) will give a value of flow, which is close to the value at the critical nozzle condition. This result is labeled "approximate ERM flow" on Figure II-1. For the zero-quality (saturated liquid) condition, this ERM approximation becomes

$$G_c^{-2} = (v_g - v_f)(C_P)_f T v_{fg}/H_{fg}^2 \qquad (II-39)$$

The specific volume change on vaporization, v_{fg}, is evaluated using the vapor composition for the condensed phase. For one component systems, the phase specific volume difference factor is the same as the density change of vaporization. For multicomponent systems, this factor is the specific volume difference of the vapor and liquid phases at their respective compositions at the maximum flow (critical) condition.

Equation (II-39) is good enough for approximating the flow for low quality one-component systems and for mixtures with a narrow boiling range. The method is not expected to be as good for mixtures with a wide boiling range, since the one-component form of the Clausius–Clapeyron equation is used in the derivation. Also, the effect of changing composition during flashing is not considered. Equation (II-23) is the rigorous form, where properties are evaluated at the equilibrium conditions attained upon flashing from upstream to throat pressures.

Equation (II-39) is used as the relief flow capacity in some DIERS simplified venting models [15]. Due to the approximate nature of this model, no distinction is made between frictional and ideal nozzle flow. However, a discharge coefficient as defined in Section 3-2-4 should be applied for consistency with other models.

A simplified form which retains the nonequilibrium factor N has been published subsequent to the DIERS contract work [16]. Strictly speaking, this form should be used only for saturated liquid flow. Application to low-quality flow conditions will yield flows on the high side. For example, the result can be high by as much as a factor of two for steam-water flow at 5%

fluids was explored in the DIERS program. Solutions of polybutadiene in hexane were used in these studies. These results show time-dependent behavior [39,40,41]. Also, large and apparently anomalous effects of pipe inlet configuration are observed. For example, placing a flow restriction at the inlet to a piping run can result in increased flow. The restriction changes the controlling behavior from that of high-viscosity pipe flow to that of a frictionless nozzle. The pipe flow regime is presumed to change from liquid-continuous to vapor-continuous. Further work was done with the ethylbenzene–polystyrene system to gain a better understanding of this effect [42]. See Chapter IV in this book for details of this work and subsequent studies sponsored by the DIERS Users Group. In any event, the results obtained using the low-shear apparent viscosity in the homogeneous-equilibrium flow model are always conservative; grossly so for the inlet-constriction case. A model for treating annular flow for power-law fluids is presented as an example approach to the problem (see Appendix A-6 of Reference [43]). See Chapter IV of this manual for other models.

The styrene–ethylbenzene–polystyrene system was used extensively for the development and verification of the DIERS system models [44]. The polymer concentration and/or molecular weight generally was low enough so that the mixture flow is described quite well by the Newtonian models at the temperature level of the tests. Further details are presented in Chapter III of this book.

4. NOMENCLATURE

The designated units apply for the parameters in equations. Numerical values may be given in related units (temperature in Celsius = K − 273.2, absolute pressure in bar = $N/m^2 \times 10^{-5}$, time in minutes, quality in weight percent, etc.).

a	general constant
A	cross-sectional flow area, m^2
A'	first Antoine constant of component (pressure in N/m^2)
A_{ij}	coefficient in liquid-phase activity relationship
b	general constant
B'	second Antoine constant of component (temperature in K)
c	general constant
C'	third Antoine constant of component, K
C_a	fluid jet area coefficient
C_d	fluid jet loss coefficient
C_f	liquid specific heat, $J/kg \cdot K$ (at constant pressure unless otherwise defined in text)
C_P	specific heat at constant pressure, $J/kg \cdot K$

C_D, C_{DO}	discharge coefficient for nozzle models (subscript DO denotes values for sharp-edged orifices)
D	diameter of flow path, m (*Note*: D in relative roughness ratio is in same units as roughness value)
f	Fanning friction factor
F_{jk}	weight of the jk class of components, expressed as a fraction of the total weight of the two-phase mixture
g	gravitational acceleration, m/s^2 (ft/s^2 in English units)
g_c	gravitational constant, (ft-lb mass/lb force/s^2 in English units)
G	mass flow rate, $kg/m^2 \cdot s$ (total flow if not subscripted "f" or "g")
H	specific enthalpy, J/kg
k	isentropic expansion exponent
k_j	mass-weighted-mean k for class of components ($j = I$ for class Ig, $j = V$ for class Vg)
K	loss factor (fittings, etc.)
L	duct length, m (L_{eq} is equivalent length including fittings)
n	polytropic exponent
N_{Re}	Reynolds number
p	partial pressure of component in vapor, N/m^2
P	absolute pressure N/m^2 (ΔP is pressure drop, upstream minus downstream values)
P'	pure component vapor pressure, N/m^2
q	heat added to fluid, J/kg
R_L	volume fraction liquid holdup
s	slip ratio (ratio of vapor velocity to liquid velocity)
S	specific entropy, $J/kg \cdot K$
t	temperature, Celsius
T	temperature, K
u	linear velocity, m/s
v	specific volume, m^3/kg
$\bar{v}$	weight-flow-mean two-phase specific volume, m^3/kg
$\bar{v}_{jk}$	mean specific volume of components of class jk, m^3/kg (additive volume rule)
v_e	effective kinetic energy two-phase specific volume, m^3/kg
v_h	holdup-mean two-phase specific volume, m^3/kg
v_m	effective momentum two-phase specific volume, m^3/kg
v'	specific volume, $m^3/kmol$
w	work done by fluid, J/kg
W	weight rate of flow, kg/s
x	mole fraction of a component in liquid phase
X	weight-flow fraction vapor in mixture (weight fraction vapor if no flow)
z	general exponent
Z	elevation, m
α	volume fraction vapor

β	ratio
γ	activ
Δ	finite
ϵ	pipe
η	ratio
Λ	Wilso
μ	visco
μ'	polyr
Σ	summ
ϕ	volur
	inelli
χ	Lock
ψ	gene

Subscripts

av	arith
B	Burn
c	at ch
d	point
e	at va
ex	of ex
f	liquid
	natio
fg	chang
	value
fr	frictic
ftt	turbu
g	gas (
	stagn
i, j, k, jk	indice
Ig, If	inert
0	stagn
s	subco
S	at co
sat	at sat
t	at thr
tf	two-p
tp	two-p
T	at tra
u	point
v	at ve
Vg, Vf	distri
1, 2	upstr

β	ratio of smaller to larger duct diameters
γ_i	activity coefficient of component i
Δ	finite difference
ϵ	pipe surface roughness (used as dimensionless ratio with D)
η	ratio of nozzle throat pressure to inlet pressure
Λ	Wilson constants
μ	viscosity, $N \cdot s/m^2$
μ'	polymer–solvent interaction parameter
Σ	summation
ϕ	volume fraction of components in liquid phase (or Lockhart-Martinelli two-phase multipliers if subscript includes f or g)
χ	Lockhart-Martinelli correlating parameter
ψ	general function designator (defined where used)

Subscripts

av	arithmetic average value over increment
B	Burnell model
c	at choke (critical) condition
d	point downstream of flow expansion
e	at vapor-liquid equilibrium condition
ex	of expansion
f	liquid phase; "fe" at phase equilibrium conditions, "fo" at stagnation conditions
fg	change in property on vaporization (vapor value minus liquid value)
fr	frictional
ftt	turbulent–turbulent regime, based on liquid
g	gas (vapor) phase; "ge" at phase equilibrium conditions, "go" at stagnation conditions, "gt" at nozzle throat conditions.
i, j, k, jk	indices
Ig, If	inert gas, inert liquid components
0	stagnation (negligible velocity) condition
s	subcooled liquid
S	at constant entropy
sat	at saturation
t	at throat of nozzle (or minimum flow area)
tf	two-phase friction
tp	two-phase flow
T	at transition point
u	point upstream of flow expansion
v	at vena contracta
Vg, Vf	distributed (volatile) component in vapor, liquid
1, 2	upstream and downstream stations