permanent uniform load superimposed on the cable. Rigorously there is only one form a cable segment can take: a catenary. For analysis it may be approximated to a parabola or a straight line.[2] The three cases are given below in detail.

2-2.1 Catenary

Consider a cable segment as shown in Fig. 2-3, hanging under its own weight, or a superimposed load q, distributed uniformly along the length of the arc. The governing equation of the arc is

$$H \frac{d^2z}{dx^2} + q \frac{dl}{dx} = 0 \tag{2-3}$$

Integrating the equation twice and substituting the boundary conditions, $z = 0$ at $x = 0$ and $z = Z$ at $x = X$, we get

$$z = \frac{H}{q}\left[\cosh \alpha - \cosh\left(\frac{2\beta x}{X} - \alpha\right)\right] \tag{2-4}$$

where $\alpha = \sinh^{-1}\left[\dfrac{\beta(Z/X)}{\sinh \beta}\right] + \beta$

$$\beta = \frac{qX}{2H}$$

From Eq. 2-4 it can be seen that the complete geometry of the cable is established if the value of H or the ordinate of any one point on the cable (for example, the cable sag at the center) is known. The

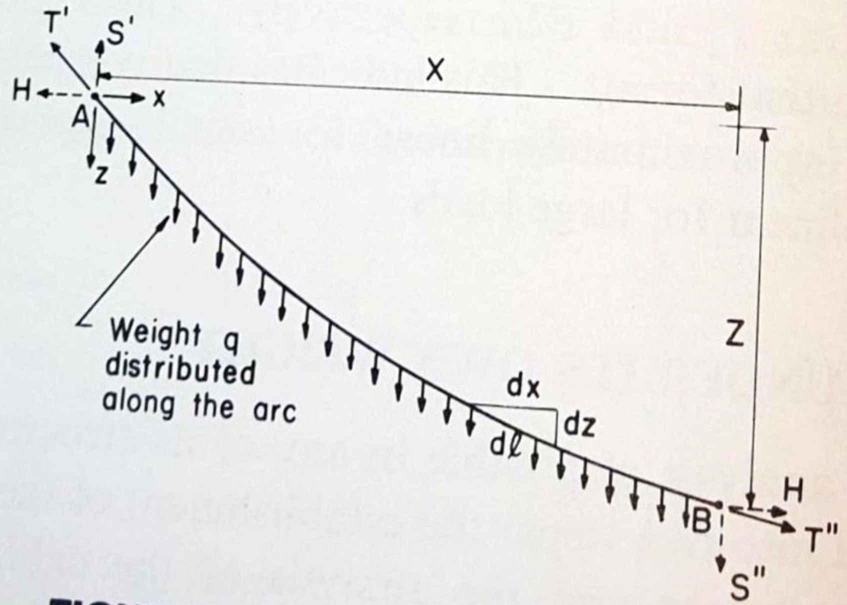

FIGURE 2-3 Segment of a catenary.

[2] Some other forms, which are perhaps only of academic interest, such as a heterogeneous cable or a catenary of uniform strength, have been treated in Ref. 80, chap. 1.

expressions for the end tensions T' and T'', reactions S' and S'', the length of the arc l, and the projections[3] X and Z in terms of these different parameters are as follows:

$$T' = H \cosh \alpha \tag{2-5}$$

$$T'' = \frac{q}{2}(l \coth \alpha - Z) \tag{2-6}$$

$$S' = H \sinh \alpha \tag{2-7}$$

$$S'' = \frac{q}{2}(Z \coth \beta - 1) \tag{2-8}$$

$$l = \frac{2H}{q} \sinh \beta \cosh(\alpha - \beta) \tag{2-9}$$

$$X = \left(\frac{H}{q}\right) \log_e\left(\frac{T' + T'' - ql}{T' + T'' + ql}\right) \tag{2-10}$$

$$Z = \left(\frac{S' + S''}{T' + T''}\right) l \tag{2-11}$$

$$(\Delta l)_e = \left(\frac{Hl}{X}\right)^2 \frac{1}{EA} \tag{2-12}$$

where $(\Delta l)_e$ = elastic stretch of the cable due to increased tension
EA = extensional rigidity of the cable

For the standard case of a cable, as shown in Fig. 2-4, with the supports A and B at the same level ($Z = 0$) and central sag f, the values of α and β reduce to

$$\alpha = \beta = \frac{qL}{2H} \tag{2-13}$$

and the expression for z (Eq. 2-4) becomes

$$z = \frac{H}{q}\left[\cosh \alpha - \cosh\left(\frac{qx}{H} - \alpha\right)\right] \tag{2-14}$$

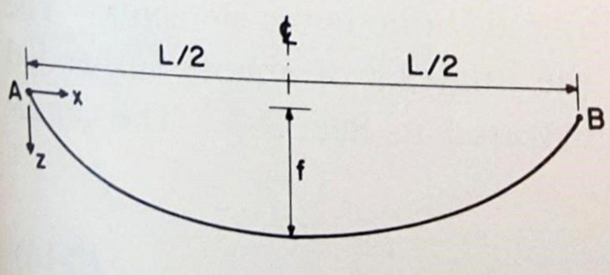

FIGURE 2-4 Simply suspended cable.

[3] The use of expressions for X and Z in terms of other properties of the cable segment as given here will be clear when studying Sec. 2-4.

Using the condition $z = f$ at $x = L/2$, Eq. 2-14 reduces to

$$f = \frac{H}{q}\,[\cosh \alpha - 1] \tag{2-15}$$

The value of H can now be computed from Eq. 2-15, and the geometry of the cable can be computed from Eq. 2-14. The parameters in Eqs. 2-5 to 2-12 can be derived for this case by substituting the values of α and β from Eq. 2-13.

2-2.2 Parabola

A cable segment whose weight q is assumed to be distributed uniformly along the horizontal length of span is shown in Fig. 2-5. The weight of a freely hanging cable of constant cross section is in fact distributed uniformly along its length, as assumed in the analysis of the catenary. However, the approximation in assuming the weight to be distributed along the cable span can be justified in the following cases:

1. When the sag/span ratio of the cable is small, the difference between the arc length and the span is very small, and the approximation makes almost negligible difference in the geometry obtained from the two analyses (see Fig. 2-7).
2. If the cable has a permanent superimposed load which is uniformly distributed along its span and is much heavier than the cable itself (as in the case of .a suspension bridge), the approximation also has justification.

It will be seen from the analysis to follow that the equations for the resulting curve, which is parabolic, are very much simpler to use than the equations for the catenary. A method for approximate analysis of pretensioned cable trusses and networks is developed in Sec. 3-5 by treating these structures as comprising parabolic cable elements. The procedure is equally applicable to the analysis of simply suspended cables under applied loading, as illustrated in Sec. 3-5. The condition of equilibrium for the curve is

$$H\frac{d^2z}{dx^2} + q = 0 \tag{2-16}$$