was raised to 0.08. Since the 1963 Code, it has been required that bending be considered in the design of all columns, and the maximum ratio of 0.08 has been applied to both types of columns. This limit can be considered a practical maximum for bar reinforcement in terms of economy and requirements for placing.

Minimum number of bars — This section requires a minimum of six bars for circular compression members and four for rectangular compression members. For other shapes one bar should be provided at each apex or corner, and proper lateral reinforcement provided. For example, tied triangular columns should contain at least three bars.

10.9.2 — The effect of spiral reinforcement in increasing the load-carrying capacity of the concrete within the core does not come into play until the column has been subjected to a load and deformation sufficient to cause the concrete shell outside the core to spall off. The amount of spiral required by Eq. (10-3) was intended to provide additional load-carrying capacity for concentrically loaded columns equal to or slightly greater than the capacity that was lost when the shell spalled off. This principle was recommended by ACI Committee 105[10.21] and has been a part of the Code since 1963. The derivation of Eq. (10-3) is given in the report. Tests and experience show that columns containing the amount of spiral reinforcement required by this section exhibit considerable toughness and ductility.

10.10—Slenderness effects in compression members

Sections 10.10 and 10.11 dealing with slenderness provisions have been entirely rewritten, based on recommendations of ACI-ASCE Committee 441, Reinforced Concrete Columns.[10.7] This recommendation calls for the use of improved structural analysis procedures wherever possible or practical. In place of such improved analysis it provides for an approximate design method based on a moment magnifier principle and similar to the procedure used as part of the American Institute of Steel Construction specifications. After study of the normal range of variables in column design, limits of applicability were set which eliminate from consideration as slender columns a large percentage of columns in braced frames and substantial numbers of columns in unbraced frames. The accuracy of the approximate design procedure was established through a series of comparisons with analytical and test results. Over the total range of slender compression members, the proposed procedure is more rational, more accurate, and more consistent than the reduction factor method used in the 1963 Code. Because the moment magnification method calls the attention of the designer to the basic phenomenon in slender compression members and allows him to evaluate the additional moment requirements in restraining members, a superior and safer design results.

Because results of an extensive series of studies of slender compression members in frames[10.8] indicated that a somewhat modified and carefully limited reduction factor method could give reasonable accuracy in treatment of slenderness effects, such a procedure is included in this Commentary after treatment of the detailed provisions of Section 10.10 and 10.11.

10.10.1 — ACI Committee 441 endorsed the position that the slenderness effect provisions should encourage improvement in the structural analysis since the basic need for any slenderness effect provision stems from weaknesses in conventionally used methods of frame analysis. The Column Committee's studies indicated that many of the analysis shortcomings affect the short columns as much or more than slender compression members.

The following elements are regarded as minimum requirements for an adequate rational frame analysis for design of compression members under Section 10.10.1:

(a) The structure may be idealized as a plane frame of linear elements. In structures containing structural walls, a better estimate of moments and deflections will be obtained if the stiffness of the wall is considered in the analysis.

(b) Realistic moment-curvature relationships must be used to provide accurate values of deflections and secondary moments. A linear approximation of the moment-curvature relationship defined by Eq. (10-7) will be acceptable, although use of a more accurate relationship is encouraged. The effect of duration of loads on deformations must be considered.

(c) The analysis must consider the influence of the axial load on the rotational stiffness of the member.

(d) The maximum moments in the compression member must be determined considering the effects of member and frame deflections and rotations. The possibility of having a maximum moment occur at sections other than the ends of the member must be considered.

(e) Because of the complexity of the problem, any proposed analysis used under the provisions of Section 10.10.1 should be checked against the limited test results available and should show accuracy at least comparable with the more approximate provisions of Section 10.11.

10.11—Approximate evaluation of slenderness effects

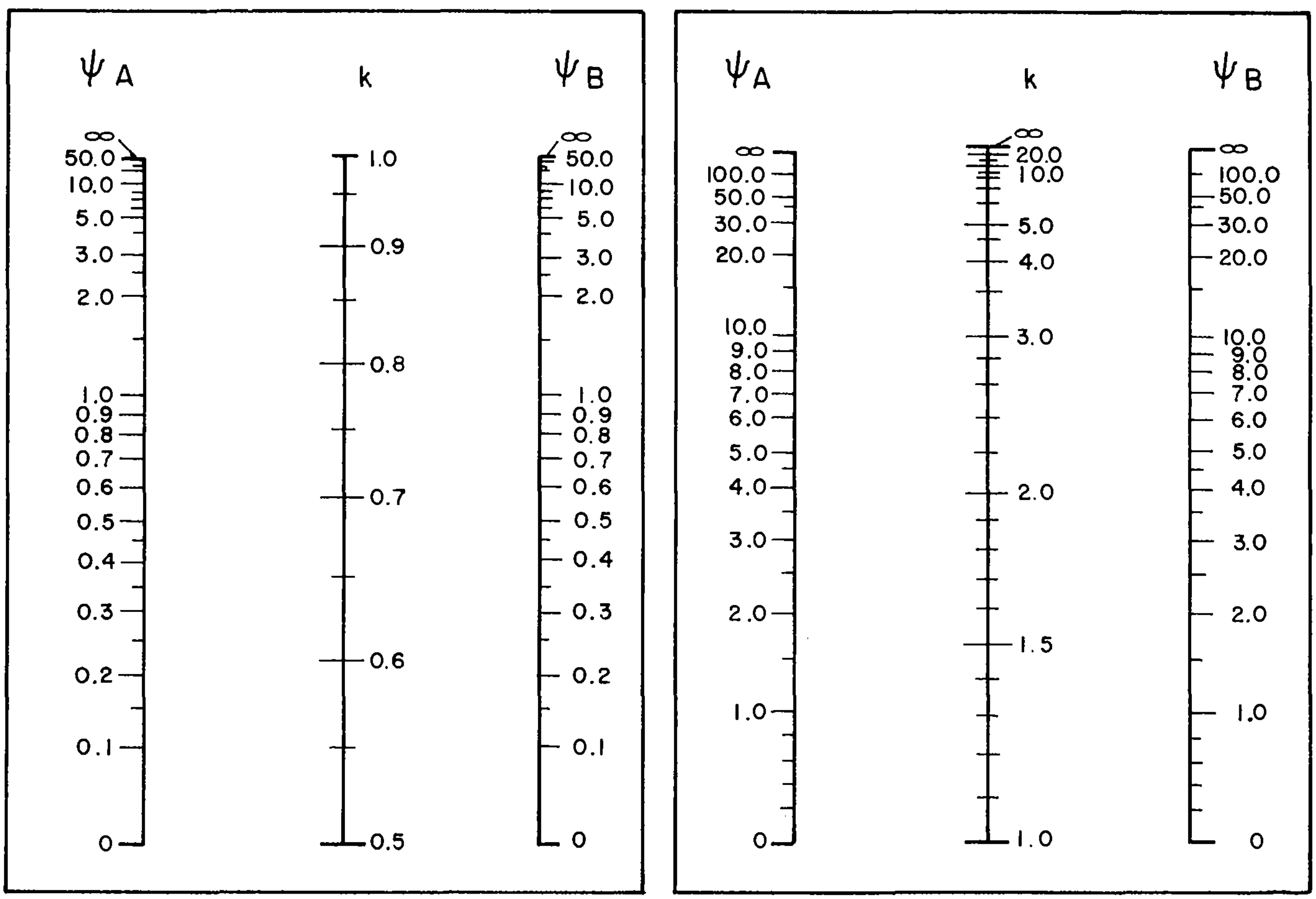
This section describes an approximate slenderness-effect design procedure based on the moment magnifier concept. The moments computed in an ordinary frame analysis are multiplied by a "moment magnifier" which is a function of the axial load P_u and the critical buckling load for the column P_c. The procedure embodies some of the main elements of the working stress design procedure for steel beam columns as included in the AISC specifications for structural steel for buildings.[10.9]

10.11.1 and 10.11.2 — These provisions are essentially unchanged from the 1963 Code, although simplified and condensed.

10.11.3 — This section requires the use of effective length factors in computing slenderness effects. The fundamental equations for the design of slender compression members were derived for hinged ends and must be modified to account for the effect of end restraints. This is done by using an "effective length," kl_u, in the computation of slenderness effects, as has been used for beam-column design in the AISC specifications[10.6] since 1963. Comparisons with more precise computer solutions indicate this procedure is especially accurate in the unbraced frame.

Committee 441 proposed that the effective length be computed in a more or less standard way by use of the Jackson and Moreland Alignment Charts (Fig. 10-3), which allow graphical determination of k for a column of constant cross section in a multibay frame.[10.10,10.13] The effective length is a function of the relative stiffness at each end of the compression member and studies have indicated that the effects of widely varying beam and column reinforcement percentages and of beam cracking should be considered in determining these relative stiffnesses.

Because the behavior of braced and unbraced frames is so different, it is necessary to have one



(a)

Braced Frames

(b)

Unbraced Frames

ψ = Ratio of $\sum(EI/\ell_c)$ of compression members to $\sum(EI/\ell)$ of flexural members in a plane at one end of a compression member

k = Effective length factor

Fig. 10-3—Effective length factors

set of effective length factors for completely braced frames and another set for completely unbraced frames. In actual fact, there is rarely such a thing as a completely braced or a completely unbraced frame. For the purposes of applying Section 10.11.3, a compression member braced against sidesway is a member in a story in which the bracing elements such as shearwalls, shear trusses, or other types of lateral bracing, have a total stiffness, resisting lateral movement of a story, at least six times the sum of the stiffnesses of all the columns resisting lateral movement in the story under consideration, so that lateral deflections of the story are not large enough to significantly affect the column strength. What constitutes adequate bracing in a given case must be left to the judgment of the engineer, depending on the arrangement of the structure in question. A value of k less than 1.2 for columns not braced against sidesway, normally would not be realistic.

10.11.4 — This section provides lower and upper slenderness ratio limits for use with the moment magnification method. The lower limits indicate that many stocky and sufficiently restrained compression members can essentially develop the full cross-sectional strength. The lower limits were determined from a study of a wide range of columns and correspond to lengths for which a slender member strength of at least 95 percent of the cross-sectional strength can be developed.

While elimination of slenderness considerations for these members may result in strength inaccuracies of up to 5 percent, the designer's job is considerably simplified, since studies[10.7] of a series of actual structures indicate that slenderness effects could be neglected for about 90 percent of the columns in the braced frames and 40 percent of the columns in the sway frames studied. An upper limit is imposed on the slenderness ratio of columns designed by the moment magnification method of Section 10.11. No similar limit is imposed if design is carried out according to Section 10.10.1. The limit of $kl_u/r = 100$ represents the upper range of actual tests of slender compression members in frames.

10.11.5—This section presents the slender column approximate design equations. These equations are based on the concept of a moment magnifier δ which amplifies the column moments to account for the effect of axial loads on these moments. The column cross section is then designed for the axial load and the amplified moment. In application, δ is a function of the ratio of the axial load in the column to the assumed critical load of the column, the ratio of column end moments, and the deflected shape of the columns.

In computing δ, the factor C_m is an equivalent moment correction factor. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum applied moment occurs at one end of the column, design must be based on an "equivalent uniform moment," C_mM_2, which would lead to the same maximum moment when magnified.[10.7]

In defining the critical load, the main problem is the choice of a stiffness parameter EI, which reasonably approximates the stiffness variations due to cracking, creep, and the nonlinearity of the concrete stress-strain curve. The Design Subcommittee of Committee 441[10.7] recommended that where more precise values are not available, EI be defined by Eq. (10-7) and (10-8). These expressions approximate the lower limits of EI for practical cross sections and hence are conservative for secondary moment calculations. They were derived for small e/h values and high P_u/P_o values, where the effect of axial load is most pronounced. P_o is the theoretical axial load capacity of a short compression member. Since experimental work involved the theoretical capacity, P_u/ϕ is used in Eq. (10-5).

The approximate nature of these expressions is shown in Fig. 10-4, where they are compared with values derived from load-moment-curvature diagrams for the case of no sustained load ($\delta_d = 0$). Eq. (10-7) represents the lower limit of the practical range of stiffness values. This is especially true for the heavily reinforced columns. Eq. (10-8) is

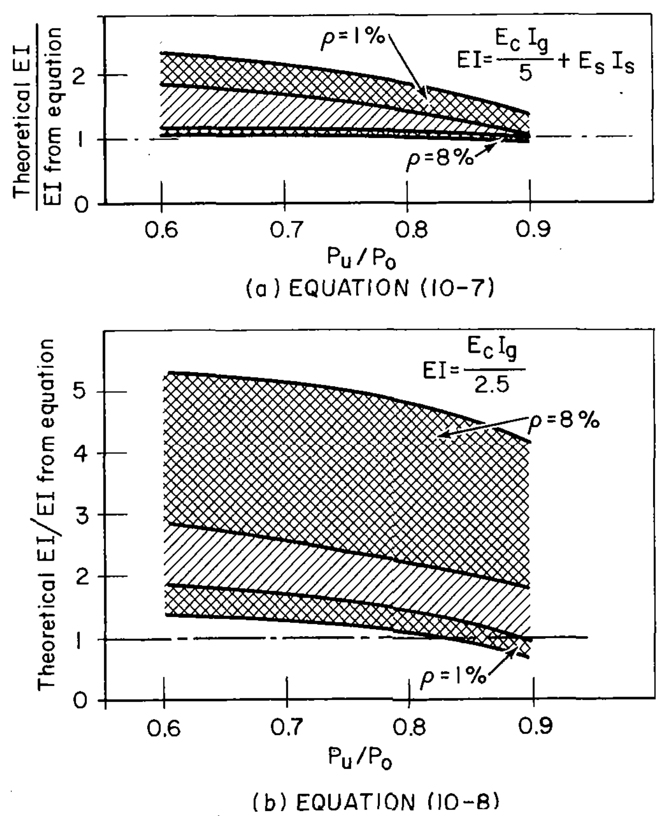


Fig. 10-4—Comparison of equations for EI with EI values from moment-curvature diagrams

simpler to use but greatly underestimates the effect of reinforcement in heavily reinforced columns. However, in many cases, when reinforcement percentages are low, or slenderness effects not very substantial, its relative simplicity may be desirable.

Creep due to sustained loads tends to reduce the effective value of EI. This is taken into account by dividing the EI term by $(1 + \beta_d)$ where β_d is the ratio of dead load moment to total load moment. This factor gives a correct trend when compared to both analyses and tests of columns under sustained loads.

Note that the Code states that EI in Eq. (10-6) may be taken as either value obtained from Eq. (10-7) or (10-8) in lieu of a more precise calculation. In this respect, the Code refers to a more accurate value of EI as obtained from moment-curvature relationships, based on the integration of acceptable nonlinear stress-strain diagrams for concrete in flexure. Any stress-strain function which provides agreement with test data may be used (see Code Section 10.2.6). The more accurate values of EI may be used for designing columns or walls under the provisions stated in Chapter 10.

When the alternate design method of Section 8.10 is used, P_u/ϕ in Eq. (10-5) is taken as $2.5P$ when gravity loads govern and as $1.875P$ when lateral loads with gravity loads govern the design, where P is the unfactored design load in the compression member.

10.11.5.1 When a story of a structure fails in a lateral instability mode, one floor translates relative to another as a unit. Thus, the deflections and hence the amount of moment magnification must be related for all compression members in the story. This section provides a procedure for computing an effective moment magnifier for the entire story. However, since any individual compression member in the story could also be overloaded while being braced against lateral instability by the other members, it is also necessary to check individual heavily loaded members using the effective length factors for braced frames.

10.11.5.2 When biaxial bending occurs in a compression member, the component moments about each of the principal axes must be magnified. The magnification factors (δ) are computed considering the buckling load P_c about each axis separately, based on the appropriate effective lengths (kl_u) and the related stiffness (EI). The clear column height may differ in each direction, and the stiffness ratios $\Sigma_{cols}/\Sigma_{beams}$ may also differ. Thus, the different buckling capacities about the two axes are reflected in different magnification factors.

The moments about each of the two axes are magnified separately, and the cross section is then proportioned. References 10.1, 10.2, and 10.17 provide guidance in this respect. Note that the design moment, $M_c = \delta M_2$, refers to the "larger end moment" with respect to bending about one axis. It will usually be necessary, therefore, to magnify the moments at both ends of a column subjected to biaxial bending, and to investigate both conditions at both ends.

In the case of compression members which are subject to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of M_2 in Eq. (10-4). In accordance with the last sentence of Section 10.11.5, C_m must be taken as 1.0 for this case.

10.11.6 — This provision (similar to one in ACI 318-63) allows computed moments to be used in determining conditions of curvature and restraint when design must be based on minimum eccentricity This eliminates what would otherwise be a discontinuity between columns with computed eccentricities less than minimum eccentricity and columns with computed eccentricities equal to or greater than minimum eccentricity.

10.11.7 — The strength of a laterally unbraced frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beam, the structure approaches a mechanism and its axial load capacity is drastically reduced. This section provides that the designer make certain that the restraining flexural members have the capacity to resist the amplified column moments. The ability of the moment magnification method to provide a good approximation of the actual magnified moments at the member ends in a sway frame is a significant improvement over the 1963 Code reduction factor method.

Modified R Method*

The 1963 Code used a column reduction factor R (Section 916) and an effective length h' for unbraced columns [Section 915(d)]. The modified R values listed below, within the limits noted, lead to an accuracy equal to that of the "moment magnifier" method of Section 10.11.5. Hence they may be used as an alternate method within the stated limits. (Note that for design both the axial load and the moment must be divided by the appropriate factor R.)

If relative lateral displacement of the ends of the member is prevented and the ends of the member are fixed or definitely restrained such that a point of contraflexure occurs between the ends, no

*This section is based on the methods of ACI 318-63, so the notation remains as used in that Code while other notation in both Code and Commentary for ACI 318-71 agrees with "Preparation of Notation for Concrete (ACI 104-71)."

correction for length need be made unless h/r exceeds 54, where h is the actual unsupported length of column and r is the radius of gyration of gross concrete area of a column. For h/r between 54 and 100, the following factor from ACI 318-63 may be used:

$$R = 1.32 - 0.006h/r \leq 1.0$$

If relative lateral displacement of the ends of the members is prevented and the member is bent in single curvature, the following factor, more liberal than ACI 318-63, may be used where the nominal eccentricity does not exceed $0.10t$ where t is the overall thickness of the column

$$R = 1.23 - 0.008h/r \leq 1.0$$

If the nominal eccentricity exceeds $0.10t$, the factor as in ACI 318-63 should be

$$R = 1.07 - 0.008h/r \leq 1.0$$

In both the above paragraphs, no increase in R is justified where tension governs the design, unless axial load is less than $0.10f_c'bt$. Then, the transition to $R = 1$ for flexure without axial load may be patterned after Section 9.2.1.2(c).

For members where (1) relative lateral displacement of the ends is not prevented, (2) with h'/r not exceeding 40, and (3) with bracing beams having a negative moment steel ratio of at least $p = 0.01$, the reduction factor, where design is governed by lateral loads of short duration, should be

$$R = 1.07 - 0.008h'/r \leq 1.0$$

For other loads of longer duration, the factor should be

$$R = 0.97 - 0.008h'/r \leq 1.0$$

These R values generally are more restrictive than those in ACI 318-63 and are primarily for use with columns restrained at each end where $h' = h(0.78 + 0.22r') \leq h$ and r' is the average of ΣK of columns to ΣK of floor members taken at the two ends of the column.

For the restraining beams in both these cases, the design should be based on taking from the column additional lateral load total moment of

$$M = \text{Col. } M_L = \text{Nom. } P_L e_N (1 - P_L/P_o) / (R - P_L/P_o)$$

where

M_L = long column end moment

P_L = long column ultimate load

e_N = nominal eccentricity, as for a short column

P_o = theoretical axial load capacity of short column

This equation is based on similar triangles from Fig. 10-5.

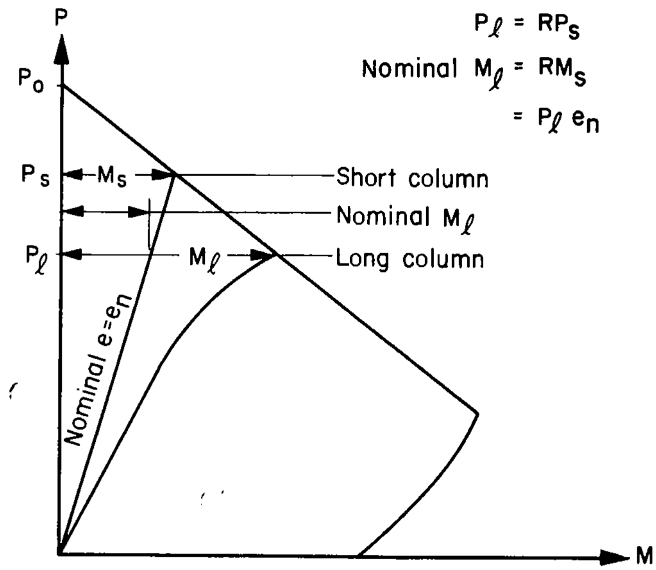


Fig. 10-5—Approximation for M_l for use in determining beam design moment

10.13—Transmission of column loads through floor system

The requirements of this section are based on a paper on the effect of floor concrete strength on column strength.[10.20] The provisions mean that where the column concrete strength does not exceed the floor concrete strength by more than 40 percent, no special precautions need be taken. For higher column concrete strengths, methods in Paragraphs (a) or (b) must be used for corner or edge columns and methods in Paragraphs (a), (b), or (c) for interior columns with adequate restraint on all four sides.

10.14—Bearing

This section deals with bearing stresses on concrete supports which are not laterally reinforced to resist splitting stresses. The provisions are similar to but more liberal than the bearing provisions of ACI 318-63. Work by Hawkins[10.12] indicates the liberalization to be justified. (See also Section 15.6.)

10.14.2 — When the supporting area is wider than the loaded area on all sides, the surrounding concrete confines the bearing area, resulting in an increase in the permissible bearing stress.

This section gives no minimum depth for the supporting pier. The supporting pier should satisfy the shear provisions of Section 11.10, which will control the minimum depth of the support.

10.14.3 — When the top of the support is sloped or stepped, advantage may still be taken of the fact that the supporting pier is larger than the loaded area, provided that the pier does not slope away at too great an angle. Fig. 10-6 illustrates the application of the frustum to find A_2. The frustum

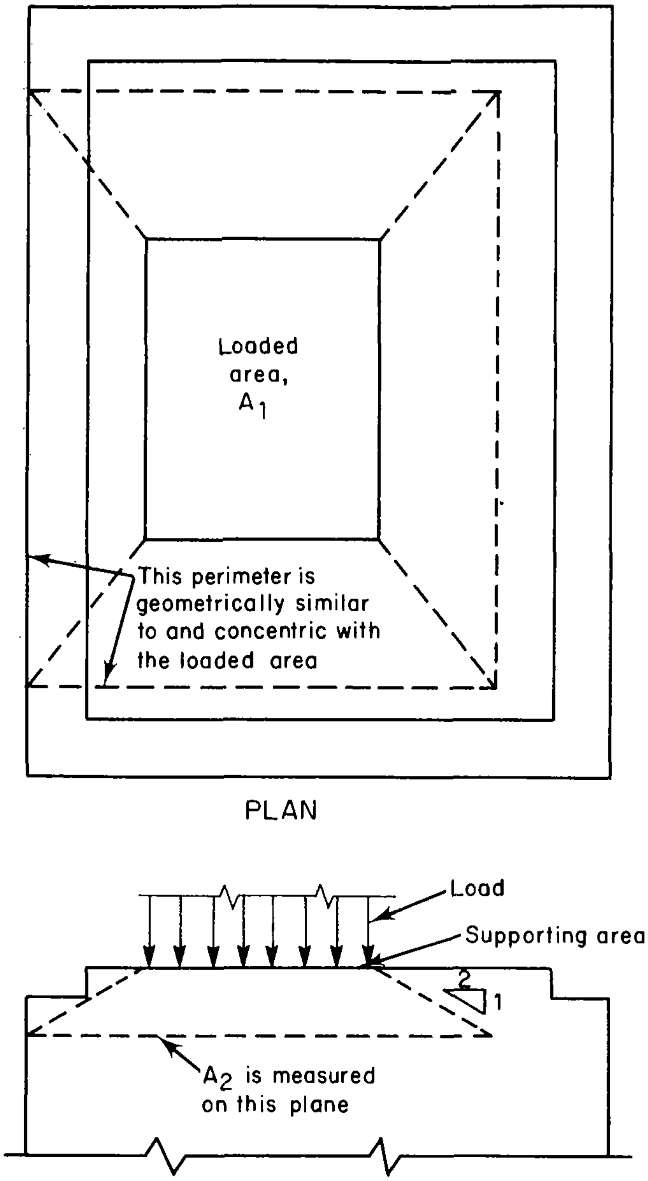


Fig. 10-6—Application of frustum to find A_2 in stepped or sloped supports

should not be confused with the path by which a load spreads out as it travels downward through the support. Such a load path would have steeper sides. However, the frustum described has somewhat flat side slopes, to insure that there is concrete immediately surrounding the zone of high stress at the bearing.

10.14.4 — Post-tensioning anchorages are normally laterally reinforced, in accordance with Section 18.11.3.

10.15—Composite compression members

10.15.1 — Composite columns are defined without reference to obsolete classifications of combination, composite, or concrete-filled pipe column. Reference to other metals used for reinforcement has been omitted because they are seldom used now with concrete in construction.

10.15.2-10.15.3 — These sections give rules for determining the strength of composite cross sections. The same rules that are used for computing ultimate load interaction functions for reinforced concrete sections can be applied to composite sections. Interaction charts for concrete-filled tubing would have a form identical to those of ACI SP-7[10.11] and the *Ultimate Strength Design Handbook, V. 2, Columns,*[10.13] but with γ (formerly g) slightly greater than 1.0.

The requirement that loads assigned to concrete must be developed by direct bearing against the concrete effectively eliminates the old combination column as a composite column under the new definition. Direct bearing can be developed through lugs, plates, or reinforcing bars welded to the structural shape or tubing before the concrete is cast. Flexural compression stress need not be considered a part of direct compression load to be developed by bearing. Simply wrapping concrete around a structural steel shape would stiffen the shape, but it would not necessarily increase its strength.

The rules of Section 10.11.2 for estimating the radius of gyration are overly conservative for concrete-filled tubing, and an alternate procedure is provided in this section. The *EI* formula suggested is consistent with Section 10.11.5, and provides a conservative estimate of the concrete stiffness. It leads to excess moment magnification and conservative estimates of strength.

10.15.4 — Steel encased, concrete sections should have a metal wall thickness large enough to maintain longitudinal yield stress before buckling outward.

10.15.5 — Concrete encasement that is laterally contained by a spiral is obviously useful for carrying load, and the size of spiral required can be regulated on the basis of the strength of the concrete *outside* the spiral by means of the same reasoning that applies for columns reinforced only with longitudinal bars. The radial pressure guaranteed by the spiral insures interaction among concrete, reinforcing bars, and a steel core such that longitudinal bars will both stiffen and strengthen the cross section.

10.15.6 — Concrete encasement that is laterally contained by tie bars is likely to be rather thin along at least one face of a steel core section, and complete interaction among the core, the concrete, and any longitudinal reinforcement should not be assumed. Concrete will probably separate from smooth faces of the steel core. To maintain the concrete encasement, it is reasonable to require more lateral tie steel than that needed for ordinary reinforced concrete columns. Due to the probable separation at high strains between the steel core and the concrete encasement, longitudinal bars will be ineffective in stiffening cross