# A CABLE CONFIGURATION TECHNIQUE FOR THE BALANCE OF CURRENT DISTRIBUTION IN PARALLEL CABLES 

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Key words: power system, power cable installation, parallel cables, current distribution.


#### Abstract

A cable configuration technique for the balance of current distribution in parallel-connected single-core cables is proposed. Based on the mutual inductance theorem, a closed form solution for the calculation of current distributions in parallel-connected cables is first derived. To compare the performances of different cable configurations, two indices are introduced, one for the power losses of all cables and the other for the largest cable current. A novel combinationgenerating method for the parallel cables of three phase systems is proposed to create the cable configuration patterns. The index values of the created cable configuration patterns are calculated and sorted to obtain the top configurations which have good current distribution performance. According to the sorting results and magnetic coupling theorem, some recommendations for the configuration of parallel cables are presented, which aim to achieve equal current distribution.


## I. INTRODUCTION

In industrial and commercial power distribution systems, single-core power cables are often connected in parallel to meet the high ampacity requirement of low voltage main feeders. However, parallel-connected cables have unequal current sharing between the cables of the same phase; some of the cables may be heavy loaded, while some are in light loading condition even though all of them belong to the same phase. This phenomenon of unequal current distribution may cause excessive temperature increase in the overloaded cables. It is well known that the increase of cable temperature can reduce the life expectancy of cable insulation. In this regard, many engineers and researchers in the area of power distribution

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have encountered and studied the aforementioned problem [2-8]. Figure 1 shows the phenomenon of unequal current distribution in a practical parallel-cable circuit. This phenomenon clearly indicates that an improper cable configuration would cause a highly unbalanced current distribution among parallel cables.

Some electrical codes like the National Electrical Code (USA) and Canadian Electrical Code have the requirement to specify installation requirements in order to prevent a highly unbalanced current distribution in parallel cables [1, 9]. However, these cable installation requirements and recommended configurations are not designed to obtain the most balanced current distribution, and may not meet the requirements of practical power distribution systems in which the structure of cable configuration is constrained by the available space and cable tray type. Although some important topics such as impedance models and current-distribution solution techniques for some particular cable configurations have been studied [2-8, 10, 11], there is no method yet to configure parallel cables in order to achieve current sharing balance. In an attempt to provide a useful tool to electrical engineers, a cable configuration technique for the balance of current distribution in parallel cables is proposed in this paper.

Published studies have shown that mutual inductance plays an important role in determining current distribution, and it is a function of distance between cables. Furthermore, the most effective and economical method to balance current distribution is a properly designed cable configuration when the parallel conductors are narrowly spaced [10, 11]. Hence, the influence mechanism of mutual inductance on current sharing between the cables of the same phase or different phases is analyzed first. A closed form solution is then used to solve the current distribution of parallel cables. To compare the performances of different cable configurations, two indices are introduced, one for the power losses of all cables and the other for the largest cable current. Owing to the total number of combinations for all possible cable configurations being very large, an additional constraint, the division into several groups of the power distribution cables, with each subgroup consisting of only one cable per phase, is introduced to reduce the number of combinations. According to the derived combinations, a computer program is developed to calculate and sort


Fig. 1. The unequal current distribution phenomenon of a practical feeder with 7 parallel cables per phase.
the index values of different cable configurations. Three common installation structures of parallel cables are studied to check the ability of the proposed technique. According to the calculation results and basic magnetic coupling theorem, the criteria of cable configuration for balanced current distribution are presented.

## II. MECHANISMS ASSOCIATED WITH CURRENT DISTRIBUTION

It is well known that both inter-cable distances, as that shown in Fig. 2, and the current phase angles in nearby cables have effects on the current distribution in parallel-connected cables. When the positions of the cables are specified, the distance between any two cables can be obtained and then their mutual impedance could be calculated [4]. For a group of parallel connected cables which does not have a ground return path, the self impedance of cable $j$ and its mutual impedance with cable $k$ can be given as follows:

$$
\begin{gather*}
\bar{Z}_{i j}=\left(R+j \omega\left(L+\frac{\mu_{0}}{2 \pi}\left(\ln \frac{2 s}{r}-1\right)\right)\right)  \tag{1}\\
\bar{Z}_{j k}=j \omega\left(\frac{\mu_{0}}{2 \pi}\left(\ln \frac{2 s}{D_{j k}}-1\right)\right) \tag{2}
\end{gather*}
$$

where
$R=$ AC resistance of cable conductor $(\Omega)$
$L=$ internal inductance $=\mu_{0} / 8 \pi(\mathrm{H} / \mathrm{m})$
$\mu_{0}=4 \pi \times 10^{-7}(\mathrm{H} / \mathrm{m})$
$r=$ conductor radius (m)
$s=$ cable length (m)
$D_{j k}=$ distance between cable $j$ and $k(\mathrm{~m})$
To simplify the analysis, Eqs. (1) and (2) do not include skin and proximity effects. Reference [11] shows that a smaller cable radius has a lesser skin and proximity effects on cable resistance than a larger one. The sum of all cable currents is zero, so $\sum \bar{I}_{j}=0$, and the " $s$ " component and " -1 " component in (1) and (2) can be eliminated when these two equations are used to calculate the cable voltage drop [10]. Hence, the impedance equations can be simplified as follows:

$$
\begin{equation*}
\bar{Z}_{j j}=\left(R+j \omega\left(L+\frac{\mu_{0}}{2 \pi}\left(\ln \frac{1}{r}\right)\right)\right) \tag{3}
\end{equation*}
$$



Fig. 2. Distance between cables.

$$
\begin{equation*}
\bar{Z}_{j k}=j \omega\left(\frac{\mu_{0}}{2 \pi}\left(\ln \frac{1}{D_{j k}}\right)\right) \tag{4}
\end{equation*}
$$

The voltage drop of every cable then can be obtained easily through the summation of voltage due to self-impedance and the voltages induced by other cables; that is,

$$
\begin{equation*}
\bar{E}_{j}=\sum_{k=1}^{m} \bar{Z}_{j k} \times \bar{I}_{k} \tag{5}
\end{equation*}
$$

where $m$ is the number of cables in the studied system. In (5), when the angle of $\bar{I}_{k}$ is the same as that of $\bar{I}_{j}$, the addition of $\bar{Z}_{j k} \times \bar{I}_{k}$ will increase the value of voltage drop of cable $j$. However, because all of the voltage drops of the cables in the same phase are identical, the increase in mutual coupling induced voltage will decrease the self impedance voltage drop and hence decrease the amplitude of $\bar{I}_{j}$. On the contrary, when the angle of $\bar{I}_{k}$ is opposite to $\bar{I}_{j}$, and the addition of $\bar{Z}_{j k} \times \bar{I}_{k}$ will decrease the value of voltage drop of cable $j$. The amplitude of $\bar{I}_{j}$ will be raised to keep all of the cable voltage drops in the same phase identical. It is clearly shown in (4) that the mutual inductance amplitude $Z_{j k}$ has an inverse relationship with cable distance $D_{k j}$. Hence, the current in a nearby cable has a larger effect on voltage drop than a far one. The above mechanism can explain the current distribution shown in Fig. 1, where the current amplitudes in the cables located at the border between two phases are larger than other cables. This is because the phase angle difference of the cable currents belonging to two different phases are larger than that belonging to the same phase.

## III. CALCULATION OF CURRENT DISTRIBUTION

Normally, the impedance of a feeder cable is much less than the load impedance. Therefore, it is reasonable that the cable impedance can be neglected in the calculation of the three phase currents; these three phase currents are independent of cable configurations. Hence, for a three-phase system, the power source can be modeled as three current sources in the calculation of cable currents, as shown in Fig. 3. With (3) and (4), the impedance matrix of parallel connected cables can be established. Then the matrix equation for Fig. 3 can be written as


Fig. 3. Equivalent circuit for parallel cables.

$$
\begin{equation*}
[\bar{E}]=[\bar{Z}][\overline{\boldsymbol{I}}] \tag{6}
\end{equation*}
$$

where both the dimensions of $[\overline{\boldsymbol{E}}]$ and $[\overline{\boldsymbol{I}}]$ are $m \times 1$. Since the cables in the same phase are connected in parallel, their voltage drops are identical;

$$
\begin{align*}
& \bar{E}_{A 1}=\cdots=\bar{E}_{A n}=\bar{E}_{A} \\
& \bar{E}_{B 1}=\cdots=\bar{E}_{B n}=\bar{E}_{B}  \tag{7}\\
& \bar{E}_{C 1}=\cdots=\bar{E}_{C n}=\bar{E}_{C}
\end{align*}
$$

where $n$ is the number of parallel cables per phase. For a system with only one three-phase circuit, the relationship between $m$ and $n$ is $m=3 \times n$. In Fig. 3, cable voltage drops, $\bar{E}_{A}, \bar{E}_{B}$, and $\bar{E}_{C}$, are unknown. However, three phase currents, $\bar{I}_{A}, \bar{I}_{B}$, and $\bar{I}_{C}$ are known, and are the sum of the unknown parallel-cable currents, that is

$$
\begin{align*}
& \bar{I}_{A}=\bar{I}_{A 1}+\cdots+\bar{I}_{A n} \\
& \bar{I}_{B}=\bar{I}_{B 1}+\cdots+\bar{I}_{B n}  \tag{8}\\
& \bar{I}_{C}=\bar{I}_{C 1}+\cdots+\bar{I}_{C n}
\end{align*}
$$

Equations (6), (7), and (8) can be solved directly or using iteration techniques. However, only the direct method is introduced in this paper. For the purpose of equation derivation, expressing (6) and (7) in matrix form as

$$
\begin{equation*}
[\overline{\boldsymbol{E}}]=[\boldsymbol{K}]\left[\overline{\boldsymbol{E}}_{A B C}\right] \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\left[\overline{\boldsymbol{I}}_{A B C}\right]=[\boldsymbol{K}]^{T}[\overline{\boldsymbol{I}}] \tag{10}
\end{equation*}
$$

where

$$
[\boldsymbol{K}]_{m \times 3}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
\vdots & \vdots & \vdots \\
1 & 0 & 0 \\
\hline 0 & 1 & 0 \\
\vdots & \vdots & \vdots \\
0 & 1 & 0 \\
\hline 0 & 0 & 1 \\
\vdots & \vdots & \vdots \\
0 & 0 & 1
\end{array}\right]
$$

and

$$
\left[\overline{\boldsymbol{E}}_{A B C}\right]_{3 \times 1}=\left[\begin{array}{c}
\bar{E}_{A} \\
\bar{E}_{B} \\
\bar{E}_{C}
\end{array}\right] \quad\left[\overline{\boldsymbol{I}}_{A B C}\right]_{3 \times 1}=\left[\begin{array}{c}
\bar{I}_{A} \\
\bar{I}_{B} \\
\bar{I}_{C}
\end{array}\right]
$$

The matrix [K] is the incidence matrix which relates the cables to phases. Substituting (9) into (6), and then into (10) yields

$$
\begin{equation*}
\left[\overline{\boldsymbol{E}}_{A B C}\right]=\left([\boldsymbol{K}]^{T}[\overline{\boldsymbol{Z}}]^{-1}[\boldsymbol{K}]\right)^{-1}\left[\overline{\boldsymbol{I}}_{A B C}\right] \tag{11}
\end{equation*}
$$

The impedance item at the right side of (11) is actually the three-phase impedance matrix of the parallel-cable system with a dimension of $3 \times 3$. Finally, by applying (11) into (9), and then into (6), the closed form solution for cable currents can be obtained as

$$
\begin{equation*}
[\overline{\boldsymbol{I}}]=[\overline{\boldsymbol{Z}}]^{-1}[\boldsymbol{K}]\left([\boldsymbol{K}]^{T}[\overline{\boldsymbol{Z}}]^{-1}[\boldsymbol{K}]\right)^{-1}\left[\overline{\boldsymbol{I}}_{\boldsymbol{A B C}}\right] \tag{12}
\end{equation*}
$$

Equation (12) can be used to solve the current distribution problem for a system with multiple distribution circuits when the three-phase load currents and cable positions are provided.

## IV. PROCESSING TECHNIQUES

## 1. Pattern Generation for Cable Configurations

For three-phase three-wire systems, the number of possible configurations of $m$ cables for any given installation structure is the solution of the possible ways of assigning $m-1$ cable positions to three subsets, phase $A, B$, and, $C$, which have $m / 3-1, m / 3$, and $m / 3$ cables respectively; that is the combination function,

$$
\begin{equation*}
C\left(m-1 ; \frac{m}{3}, \frac{m}{3}, \frac{m}{3}-1\right)=\frac{(m-1)!}{\left(\frac{m}{3}\right)!\left(\frac{m}{3}\right)!\left(\frac{m}{3}-1\right)!} \tag{13}
\end{equation*}
$$

Equation (13) is derived based on the assumption that the system has a balanced three-phase load. With that assumption, the labels $A, B, C$ can be assigned to the subsets after the first assignment of the cable position to a subset and will not affect the calculated results of current distribution. In this paper, the subset has the first assignment is labeled as phase $A$. So, in (13), only $m-1$ cables are needed to be considered for a system with $m$ cables. It can be easily calculated from (13) that for a three-phase system with 15 cables, the number of all possible configurations is 252252 . It is very inefficient to calculate the current distributions of all possible configurations in order to determine the optimal configuration. In reality, the cable configuration is in a form of symmetrically arranged subgroups. A cable subgroup includes three or four cables for three-wire and four-wire systems, respectively. Every cable in a cable subgroup belongs to different phases. In this paper, the characteristic of grouping cables into subgroups is utilized to reduce the number of combinations of cable configurations.

To generate the combinations of cable configurations, two kinds of index bits for the control of cable arrangement are defined: the first one is named "mirror bit," while the second one is named "exchange bit." One example is illustrated in Fig. 4 for a system with four parallel cables per phase ( $m=4 \times 3=$ 12). A value of " 1 " in the mirror bit indicates that the succeeding subgroup is generated from a mirrored duplication of the preceding subgroup, while a value of " 0 " means a direct duplication of the preceding subgroup. A value of " 1 " in the exchange bit indicates the exchange of the second and third cables in the subgroup. A value of " 0 " in the exchange bit means no manipulation is done. To obtain the combinations of the subgrouped cable configurations, the binary sequences which consist of the mirror bits and exchange bits are created in sequence, for example, from 0000000 to 1111111 for a system with 12 cables as that shown in Fig. 5. The required length of the binary sequence for cable arrangement is ( $2 \mathrm{~m} / 3-1$ ) for a system with $m$ cables. With the application of the subgrouping method for cable configurations, the number of combinations for a system with $m$ cables is reduced to

$$
\begin{equation*}
2^{\left(\frac{1}{3} m-1\right)} \tag{14}
\end{equation*}
$$

Now, there are only 512 cable configurations for $m=15$. However, the information of the cable configuration provided by the binary sequence is very obscure. Hence, a matrix of position numbers with its elements corresponding to the position number of where the cable is located is introduced. The purpose of this is to record clearer information about the cable configurations. An example of mapping a binary sequence to a matrix of position numbers is shown in Fig. 6.

The system shown in Fig. 6 is a three-wire system. For a three-phase four-wire system, a neutral cable must be included in the cable subgroup. Figure 7 shows all four possible configurations for a neutral cable in a subgroup. Hence, for a three-phase four-wire system, the number of combinations is 4


Fig. 4. An example for the control of cable arrangement according to the mirror bits and exchange bits with the 3-cable subgrouping method.


Fig. 5. Binary sequences for cable arrangement.

Binary sequence for cable arrangement


Fig. 6. An example of mapping a binary sequence to a matrix of position numbers.


Fig. 7. Four possible configurations for a neutral cable in a cable subgroup.
times the number obtained from (14). In the application, one of the configurations in Fig. 7 is chosen as the first subgroup, and then the succeeding subgroups are arranged according to the method described Figs. 4 and 6 which were


Fig. 8. The calculation procedure for the cable configuration technique.
originally graphed for three-phase three-wire systems. The only difference is that a sequence change action for exchange bit $=$ " 1 " exchanges number 2 and number 3 cables shown in Fig. 7, while the position of the ' N ' cable is not shifted. Every configuration in Fig. 7 is processed in the same manner as described above.

## 2. Performance Indices and Calculation Procedure

With the purpose of searching for the top configurations which have balanced current sharing among parallel cables, two indices are defined in this paper to evaluate the performance of cable configurations. Both indices are expressed in terms of cable currents as

$$
\begin{equation*}
\delta_{T}=\frac{\sum_{p}\left(\sum_{h=1}^{n} I_{p h}^{2}\right)}{\sum_{p} n\left(\frac{I_{p}}{n}\right)^{2}}, \delta_{M}=\frac{\operatorname{Max}\left(I_{p h}\right)}{\operatorname{Max}\left(\frac{I_{p}}{n}\right)} \tag{15}
\end{equation*}
$$

where $p=\mathrm{A}, \mathrm{B}, \mathrm{C}$. In (15), index $\delta_{T}$ represents the power losses of all cables, and $\delta_{M}$ is the largest cable current. The numerators of $\delta_{T}$ and $\delta_{M}$ are the total cable losses and the largest cable current, respectively, which are calculated based on the current distribution solved by (12), while the denominators are based on the ideal uniform current distribution. For example, a value of 1.1 for $\delta_{T}$ indicates that the total cable losses of the selected cable configuration are $10 \%$ higher than when the cable current distribution is ideally uniform.

The flowchart of the calculation procedure for the proposed cable configuration technique is shown in Fig. 8. The parameters of the cable installation structure are the coordinates of the cable positions, and are used to calculated the distances between cables. Aside from the parameter input, all other steps in the procedure can be processed automatically using a software.


Fig. 9. Three common installation structures for power cables.

Table 1. Results for a $3 \phi 3 W$ 9-cable system with structure IS_0.

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCCBAABC | 1.004 | $\underline{\text { ABCCBAABC }}$ | 1.106 |
| 2 | ACBBCAACB | 1.004 | ACBBCAACB | 1.106 |
| 3 | ABCABCABC | 1.03 | ACBACBABC | 1.137 |
| 4 | ACBACBACB | 1.03 | ACBACBACB | 1.148 |
| - | AAABBBCCC | 1.446 | AAABBBCCC | 1.83 |

## V. APPLICATION EXAMPLES

To demonstrate the effectiveness of the proposed configuration technique, three common cable installation structures shown in Fig. 9, referred to as IS_0, IS_1, and IS_2 respectively, are studied and analyzed. The parameters of a $250 \mathrm{~mm}^{2}$ PVC cable and the three phase currents listed below are used in the calculation:

Cable length: 100 m
Inner diameter: 19 mm
Overall diameter: 23.8 mm
$50^{\circ} \mathrm{C}$ resistance: $0.0826 \Omega / \mathrm{km}$

$$
\bar{I}_{A}, \bar{I}_{B}, \bar{I}_{C}: 1 \angle 0^{\circ}, 1 \angle-120^{\circ}, 1 \angle 120^{\circ} \text { p.u. }
$$

In the procedure shown in Fig. 8, every created cable configuration has two calculated performance indices, $\delta_{T}$ and $\delta_{M}$. After an ascending sorting process is applied to both indices, the top four configurations for each index are listed in the result tables. For comparison purpose, an additional configuration in which all of the cables belonging to the same phase are grouped together, for example $\mathrm{AAA} / \mathrm{BBB} / \mathrm{CCC}$, is also listed in the last row of the result tables.

## 1. Three-phase Three-wire Systems

Tables 1~3 display the calculation results for the three common installation structures shown in Fig. 9 where there are 9 cables in the studied systems. Tables $4 \sim 6$ show the results of the 12 cable cases. For installation structures IS_0 and IS_1, the top 4 configurations solved based on the combinations created with the step of cable subgrouping in the calculation procedure are identical with those calculated without the step

Table 2. Results for a 3 $\mathbf{~} \mathbf{3} \mathbf{W}$ 9-cable system with structure IS_1.

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCCBAABC | 1.008 | ABCCBAABC | 1.074 |
| 2 | ACBBCAACB | 1.008 | ACBBCAACB | 1.074 |
| 3 | ABCCABBAC | 1.025 | ABCCBACAB | 1.118 |
| 4 | ABCCBAACB | 1.025 | ABCABCABC | 1.137 |
| - | AAABBBCCC | 1.288 | AAABBBCCC | 1.762 |

Table 3. Results for a $\mathbf{3} \phi \mathbf{3 W}$ 9-cable system with structure IS_2.

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCCBAABC | 1.012 | ABCCBAABC | 1.067 |
| 2 | ACBBCAACB | 1.012 | ACBBCAACB | 1.094 |
| 3 | ACBBCABCA | 1.031 | ACBACBABC | 1.171 |
| 4 | ABCCBACBA | 1.031 | ACBACBACB | 1.174 |
| - | AAABBBCCC | 1.251 | AAABBBCCC | 1.763 |

Table 4. Results for a $\mathbf{3} \phi \mathbf{3 W} \mathbf{1 2}$-cable system with structure IS_0.

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCCBAABCCBA | 1.002 | ACBBCAACBBCA | 1.06 |
| 2 | ACBBCAACBBCA | 1.002 | ABCCBAABCCBA | 1.073 |
| 3 | ABCABCABCABC | 1.024 | ACBACBACBACB | 1.109 |
| 4 | ACBACBACBACB | 1.024 | ABCABCABCABC | 1.109 |
| - | AAAABBBBCCCC | 1.687 | AAAABBBBCCCC | 2.191 |

Table 5. Results for a $3 \phi \mathbf{3 W} 12$-cable system with structure IS_1.

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCCBAABCCBA | 1.004 | ACBBCAACBBCA | 1.05 |
| 2 | ACBBCAACBBCA | 1.004 | ABCCBAABCCBA | 1.084 |
| 3 | ABCCBAACBBCA | 1.017 | ABCABCABCABC | 1.119 |
| 4 | ACBBCAABCCBA | 1.017 | ACBACBACBACB | 1.119 |
| - | AAAABBBBCCCC | 1.55 | AAAABBBBCCCC | 2.03 |

Table 6. Results for a 3 $\mathbf{3} \mathbf{3 W}$ 12-cable system with structure IS_2.

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCCBAABCCBA | 1.006 | ABCCBAABCCBA | 1.08 |
| 2 | ACBBCAACBBCA | 1.006 | ACBBCAACBBCA | 1.08 |
| 3 | ACBBCABACCAB | 1.019 | ABCCBACABCAB | 1.123 |
| 4 | ABCCBACABBAC | 1.019 | ABCACBBACCAB | 1.152 |
| - | AAAABBBBCCCC | 1.446 | AAAABBBBCCCC | 1.741 |

of cable subgrouping (not shown here). However, they are not identical for installation structure IS_2. Table 7 shows the top 5 configurations for structure IS_2, which are calculated based on the combinations created without cable subgrouping. The

Table 7. Results for a $3 \phi \mathbf{3 W}$ 12-cable system with structure IS_2. (with 6-cable subgrouping).

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :--- | :---: | :---: |
| 1 | $\underline{\text { AABBCCCCBBAA }}$ | 1.0 | $\underline{\text { AABBCCCCBBAA }}$ | 1.0 |
| 2 | AACCBBBBCCAA | 1.0 | AACCBBBBCCAA | 1.0 |
| 3 | ABCCBABACCAB | 1.004 | ABCCBABACCAB | 1.074 |
| 4 | ACBBCACABBAC | 1.004 | ACBBCACABBAC | 1.074 |
| 5 | $\underline{\text { ABCCBAABCCBA }}$ | 1.006 | $\underline{\text { ABCCBAABCCBA }}$ | 1.08 |



Fig. 10. The 6-cable subgrouping method for installation structure IS_2 with an even number of cables.
(A) B C (C) B (A) C


Fig. 11. Configuration ABC/CBA/ABC for structures IS_0, IS_1, IS_2.
comparison of Tables 6 and 7 shows that the top 1 configuration in Table 6 is in the $5^{\text {th }}$ place in Table 7. The reason of the difference between Table 6 and 7 is due to the vertical stacking relationship between the upper layer and the lower layer in structure IS_2. Owing to the vertical relationship, the upper and lower cables of the same column can be combined and treated as a single unit when the number of cables is even as that shown in Fig. 10; every subgroup consists of 3 units with 2 cables per unit. With the 6 -cable subgrouping method shown in Fig. 10, the calculated top 1 and 2 configurations are identical with the top 1 and 2 configurations shown in Table 7. In Tables 1~7, the configurations highlighted in boldface and underlined are the recommended cable configurations for the installation structures shown in Fig. 9. One of the recommended configurations, $\mathrm{ABC} / \mathrm{CBA} / \mathrm{ABC}$, is shown in Fig. 11.

## 2. Three-phase Four-wire Systems

For a three-phase four-wire system, the amplitude and phase of the neutral current depend on the unbalanced condition of the three-phase currents. It is not practical to evaluate all of the possible unbalanced conditions to obtain an optimal cable configuration. Hence, only two load scenarios with opposite

Table 8. Results for a $3 \phi 4 \mathrm{~W}$ 12-cable system with structure IS_0. (load scenario \#1)

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCNNCBAABCN | 1.001 | ABCNACBNACBN | 1.013 |
| 2 | ACBNNBCAACBN | 1.001 | ABCNABCNACBN | 1.013 |
| 3 | ABNCCNBAABNC | 1.005 | ACBNNBCAACBN | 1.014 |
| 4 | ACNBBNCAACNB | 1.005 | $\underline{\text { ABCNNCBAABCN }}$ | 1.014 |
| - | AAABBBCCCNNN | 1.409 | AAABBBCCCNNN | 1.651 |

Table 9. Results for a $3 \phi 4 \mathrm{~W}$ 12-cable system with structure IS_1. (load scenario \#1)

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCNNCBAABCN | 1.006 | ACNBBNCAACNB | 1.026 |
| 2 | ACBNNBCAACBN | 1.006 | ACBNABCNNCBA | 1.027 |
| 3 | ANBCCBNAANBC | 1.006 | ACBNNBCAABCN | 1.029 |
| 4 | NABCCBANNABC | 1.006 | ABCNACBNABCN | 1.03 |
| - | AAABBBCCCNNN | 1.247 | AAABBBCCCNNN | 1.525 |

Table 10. Results for a $3 \phi 4 \mathrm{~W} \mathbf{1 2}$-cable system with structure IS_2. (load scenario \#1)

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | NABCCBANNABC | 1.008 | ACBNABCNNCBA | 1.025 |
| 2 | NACBBCANNACB | 1.008 | ACNBBNACBNCA | 1.034 |
| 3 | ANBCCBNAANBC | 1.008 | ACBNABCNACBN | 1.038 |
| 4 | ANCBBCNAANCB | 1.008 | ABCNACBNABCN | 1.038 |
| - | AAABBBCCCNNN | 1.212 | AAABBBCCCNNN | 1.478 |

Table 11. Results for a $\mathbf{3} \phi \mathbf{4 W} \mathbf{1 6}$-cable system with structure IS_0. (load scenario \#1)

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ABCNNCBAABCNNCBA | 1.001 | NACBBCANNACBBCAN | 1.003 |
| 2 | ACBNNBCAACBNNBCA | 1.001 | ANCBBCNAANCBBCNA | 1.003 |
| 3 | ACNBBNCAACNBBNCA | 1.001 | ABCNACBNABCNACBN | 1.005 |
| 4 | ABNCCNBAABNCCNBA | 1.001 | ACBNNBCAACBNNBCA | 1.006 |
| - | AAAABBBBCCCCNNNN | 1.63 | AAAABBBBCCCCNNNN | 1.983 |

Table 12. Results for a $\mathbf{3} \phi \mathbf{4 W}$ 16-cable system with structure IS_1. (load scenario \#1)

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | ANCBBCNAANCBBCNA | 1.001 | ACNBBNCAACNBBNCA | 1.005 |
| 2 | ANCBBCNAANCBBCNA | 1.001 | NABCCBANNABCCBAN | 1.01 |
| 3 | ANBCCBNAANBCCBNA | 1.001 | ANBCCBNAANBCCBNA | 1.01 |
| 4 | NABCCBANNABCCBAN | 1.001 | ANCBBCNAANCBBCNA | 1.014 |
| - | AAAABBBBCCCCNNNN | 1.468 | AAAABBBBCCCCNNNN | 1.776 |

neutral-current directions are used in the calculation for illustration purpose, and they are

Table 13. Results for a $\mathbf{3} \phi \mathbf{4 W}$ 16-cable system with structure IS_2. (load scenario \#1)

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | NABCCBANNABCCBAN | 1.001 | NABCCBANNACBBCAN | 1.007 |
| 2 | ANCBBCNAANCBBCNA | 1.001 | NACBBCANNABCCBAN | 1.007 |
| 3 | NACBBCANNACBBCAN | 1.001 | ANBCCBNAANCBBCNA | 1.007 |
| 4 | ANBCCBNAANBCCBNA | 1.001 | ANCBBCNAANBCCBNA | 1.007 |
| - | AAAABBBBCCCCNNNN | 1.377 | AAAABBBBCCCCNNNN | 1.684 |

Table 14. Results for a $3 \phi 4 \mathrm{~W}$ 16-cable system with structure IS_0. (load scenario \#2)

| No. | Configurations | $\delta_{T}$ | Configurations | $\delta_{M}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | NACBBCANNACBBCAN | 1.0 | NABCCBANNABCCBAN | 1.001 |
| 2 | NABCCBANNABCCBAN | 1.0 | NACBBCANNACBBCAN | 1.003 |
| 3 | ANCBBCNAANCBBCNA | 1.004 | NABCNABCNABCNABC | 1.009 |
| 4 | ANBCCBNAANBCCBNA | 1.004 | NABCNABCCBANCBAN | 1.01 |
| - | AAAABBBBCCCCNNNN | 1.799 | AAAABBBBCCCCNNNN | 1.883 |


IS_0
IS_1

IS_2

Fig. 12. Configuration ABCN/NCBA/ABCN for structures IS_0, IS_1, IS_2.

Load scenario \#1:
$\bar{I}_{A}, \bar{I}_{B}, \bar{I}_{C}, \bar{I}_{N}: 0.5 \angle 0^{\circ}, 1 \angle-120^{\circ}, 1 \angle 120^{\circ}, 0.5 \angle 0^{\circ}$ p.u.
Load scenario \#2:
$\bar{I}_{A}, \bar{I}_{B}, \bar{I}_{C}, \bar{I}_{N}: 1.5 \angle 0^{\circ}, 1 \angle-120^{\circ}, 1 \angle 120^{\circ}, 0.5 \angle 180^{\circ}$ p.u.
The top 4 configurations for the 12 -cable systems, 3 cables per phase, with load scenario \#1 are shown in Tables 8~10. Tables 11~13 show the calculation results for the 16 -cable cases. For load scenario \#2, only the results of a 16 -cable system in IS_0 configuration are given in Table 14 for comparison. The comparison of Table 14 to Table 11 shows that the neutral current does not have a great effect on the selection of cable configuration for the balanced current distribution of the studied cases. Similarly, the configurations highlighted in boldface and underlined in Tables 8~14 are the recommended cable configurations, and one of them is shown in Fig. 12.

## 3. Comments for the Studied Examples

Based on the calculation results and the current distributing mechanisms for parallel cables, several comments for the configuration of parallel cables are made as follows:


Fig. 13. Recommended configurations for installation structure IS_2 with an even number of cables per phase.
A. Putting all of the cables of the same phase together will introduce a highly unbalanced current distribution. With this type of configuration, more parallel cables will cause a more unbalanced current distribution.
B. The rule of thumb for the configuration of parallel cables: include only one cable per phase in a cable subgroup and have them installed symmetrically.
C. A configuration with its adjacent subgroups arranged in mirror symmetry can obtain a very balanced current distribution. Figures 11 and 12 are the examples of the application of the configurations of $\mathrm{ABC} / \mathrm{CBA} / \mathrm{ABC}$ and ABCN/NCBA/ABCN, respectively, for structures IS _0, IS _1, and IS_2.
D. For installation structure IS_2 with an even number of cables per phase $(\geq 4)$, the two cables of the same column can be treated as a single cable, and the method described in item C and Fig. 10 can be used to arrange the cable configurations to obtain the most balanced current distribution. Figure 13 shows the configurations of $\mathrm{AABBCC} /$ CCBBAA and AABBCCNN/NNCCBBAA for three-phase three-wire systems and three-phase three-wire systems, respectively.
E. In general, the configurations with repeated cable subgroups, such as $A B C / A B C$ for three-wire systems and $\mathrm{ABCN} / \mathrm{ABCN}$ for four-wire systems, can also obtain acceptable current distributions. This kind of arrangement method is recommended for users who want a simple cable configuration.
F. For $3 \phi 4 \mathrm{~W}$ systems, the load scenarios with different neutral currents could affect the solution of cable configuration pattern. Hence, it's better that the load scenario used in the computer calculation is the actual load currents flowing through the parallel cables rather than a randomly selected one. However, in power distribution systems, the neutral current varies from time to time. The load currents in heaviest load condition are suggested to be used as the simulated load scenario.
G. Both the performance index and installation structure can influence the solution of cable configuration pattern. However, the installation structure has a greater effect on the solution than the performance index has. Hence, when the installation structure is quite different with the installation structures studied in this paper, a computer simulation based on the proposed technique may need to be performed instead of using the suggested rule of thumb.

## VI. CONCLUSION

A cable configuration technique for the balance of current distribution in parallel cables is proposed in this paper. The technique is a multi-step procedure which includes configuration pattern generation, current distribution calculation, and performance index sorting. The proposed technique can be implemented easily and processed automatically using a software, and is applicable to any cable installation structure. Owing to the use of a novel combination-generating method for cable configuration patterns in the calculation procedure, the top configurations can find three-phase systems very quickly. The application of the technique on three common installation structures for parallel cables demonstrates that the technique can be used to design a cable configuration which has superior performance in terms of current distribution balance. Based on the results from the example cases and the current distribution mechanisms for parallel cables, it is shown that dividing the cables into subgroups with only one cable per phase and arranging the adjacent subgroups in mirror symmetry can achieve a balanced current distribution.

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