Load-Carrying Capacity

Concrete Slabs-On-Grade Subject To Concentrated Loads

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The scope of this article is limited to the design of industrial concrete floor slabs on grade for concentrated loads caused by the columns of free-standing work platforms, mezzanines, or mechanical support structures. The concrete floor slab is presumed to be unreinforced. The structure supported by the floor slab is considered to be independent of the building structural system and, therefore, is outside the scope of ACI 318. Further, the spacing of the structural columns supporting these structural platforms is presumed to be sufficiently large, thereby precluding any interactive effects of neighboring columns.

For generations, structural engineers, and those who review their work, have been concerned that there was a lack of understanding of how concrete slabs-on-grade (SOG) behave, particularly under the effects of discrete concentrated loads. This has led to results that in many cases have been unrealistic and overly conservative in their application. With insufficient information, some have speculated about concrete slab-on-grade behavior, including the surmising of failures such as "punching shear," and its dire consequences in applications where none have been observed.

Earlier work done by Westergaard, and Ringo and Anderson, have long been the standard of practice. Standards by the ACI also addressed the design of slabs on grade, and Face and Al-Nasra developed a finite-element basis for the design of SOG. More recently, a definitive research paper by Shentu, Jiang, and Hsu has brought about some rigorous focus to the problem, and their analytical and confirming testing results have allowed a better understanding of the behavior and design of SOG. And even more recently, Higgins has introduced the Shentu method as an approach to a practical design method for slabs-on-grade. The significance of this work is further demonstrated by the City of Los Angeles issuing an Information Bulletin stating that this approach, among others, is an "Acceptable Design and Analysis Method for Use of Slabs-on-Grade as Foundation." The approach developed here is further cited as an acceptable design and analysis method in a recent *Guidance Document*, FEMA 460, by the Building Seismic Safety Council of the Federal Emergency Management Agency, which is focused on issues related to seismic behavior of industrial steel storage racks.

The evolution of SOG design and behavior has allowed both designers and building officials to have a better understanding of, and confidence in, the load-carrying capacities of SOG subjected to large concentrated column loads. These methods are applicable particularly to industrial warehouses and distribution centers where free-standing steel work platforms or mezzanines, typically carrying storage and equipment loads on upper levels, cause large concentrated forces to act at discrete locations on the warehouse floor on which the work platforms have been installed.

The work of Shentu and his colleagues, through a comparison of their analytical predictions and test results, has demonstrated that the load-carrying capacity, as well as settlement behavior, can be well predicted with good results. On that basis, Shentu proceeded to develop a "Simplified Analytical Method" that is the basis for this article. With this method, and the resulting equations presented in that paper, the determination of the load-carrying capacity of a SOG is found to be simple, practical, and reliable.

Extensive investigations conducted by, and on behalf of, the Storage Equipment Manufacturers Association (SMA) have, for several years, been directed to develop an acceptable and reliable design method for floor slabs on grade. This article is intended to summarize their findings. Included here is a brief description of the design parameters for subgrade properties, concrete tensile properties, and the concept of the radius of relative stiffness and its relevance to this problem. Also included is a representative design table, summarizing recommendations resulting from this work for an example six-inch slab.

Employing an Elastoplastic Model

In earlier years, the determination of the allowable concentrated load applied to an existing concrete floor slab system was solved according to the linear elastic theory of Westergaard. The elastic theory is correct as long as the load is small. However, when the ultimate load-carrying capacity is required, the elastoplastic behavior of concrete should be taken into account.

The more recent research by Shentu and Al-Nasra indicate that there is a substantial difference in the results produced by the methods presented in ACI when compared with those from an elastoplastic analysis. Floor slabs on grade can carry significant additional load after the onset of initial cracking, and it is necessary to take advantage of this additional load-carrying capacity in design procedures for engineering structures.

The inconsistency in designing exclusively in the elastic range is apparent. Most engineering publications acknowledge the existence of shrinkage cracks in concrete floors. To use a design procedure, based on the analytical model of a floor presumed to be uncracked, for a floor slab known to be cracked, is inappropriate. The long-standing use of design methods that presume a crack-free slab, while simplifying the analytical model, has encouraged the use of methods that produce results that may not be applicable for the design of a realistic floor slab-on-grade. The design table presented later in this article is based on research results employing an elastoplastic model of concrete structural behavior.

Analysis

The design of a floor slab-on-grade involves the interaction of a concrete slab and a soil support system. The concrete is a material considered to be heterogeneous and statistically isotropic, becoming orthotropic with the development of micro-cracks. Concrete strength in compression is significantly greater than its strength in tension. Micro-cracks may form in the concrete even before loading. The soil system, in general, is also heterogeneous; its characteristics and mechanical properties may vary within a wide range.

In addition to the concrete slab thickness, the following two properties are critical to the design of a floor slab-on-grade: *subgrade strength* and concrete *tensile strength*.

Table 1: 6-inch slab.

Allowable load in Kips						
Soil	1.5 Rad Rel	Concrete PSI	10-inch baseplate	12-inch baseplate	14-inch baseplate	16-inch baseplate
50 pci	52 inch	3000	36	37	38	39
		4000	41	43	44	45
100 pci	44 inch	3000	41	44	46	48
		4000	48	50	52	55
200 pci	37 inch	3000	52	56	60	65
		4000	60	65	70	75

Example

Consider a 6-inch thick unreinforced slab made of 4000 psi concrete; sitting on a soil whose modulus of subgrade reaction, k_s, is 100 pci; and with column loads being applied through 14-inch base plates. Using *Table 1*, the allowable concentrated load, P_a, is determined to be 52 kips; and the columns should be no closer than 2x44 inches, or 88 inches, or 7.33 feet.

Soil-bearing capacity, soil compressibility, and modulus of subgrade reaction are properties of the soil system that require understanding. *Soil-bearing capacity* is a measure of soil shear failure. This value is determined by using various standardized soil tests. *Soil compressibility* is a measure of long-term settlement in a soil under load. This value is normally determined using soil consolidation tests.

Modulus of subgrade reaction is the proportionality constant, k_s, in a Winkler subgrade. Its value depends upon the kind of soil, the degree of compaction, and the moisture content. The modulus of subgrade reaction has units of *pci*; it is the pressure in *psi* per inch of soil deformation. The procedure for determining k_s is outlined in ASTM D 1196.

For the general relationship between the soil classifications and the modulus of subgrade reaction, see their depiction in Figure 3.3.5 of ACI R-92. Essentially, soils that have high compressibility and low subgrade strength will have a design k_s value of about 50 pci. Natural soils of higher subgrade strength have a design k_s value of about 200 pci.

The tensile strength of concrete is usually determined by using the split cylinder test in accordance with ASTM C 496. The tensile strength is a more variable property than the compressive strength; it is about 10 to 15 percent of the compressive strength. The tensile strength is between $6(f_c')^{0.5}$ and $7(f_c')^{0.5}$ for normal stone concrete.

The tensile strength in flexure is the modulus of rupture (ASTM C 78). The modulus of rupture is generally accepted as $7.5(f_c')^{0.5}$ for normal concrete. The magnitude of the compressive strength for concrete is generally available for use by the design engineer.

The values presented in *Table 1* are for unreinforced concrete slabs of six-inch thickness. The tabulated values represent the results for the determination of the allowable load-carrying capacities, P_a, for various concrete slabs on grade for a variety of parametric values, based on the application of the following relationship developed by Shentu:

$$P_n = 1.72\ [(k_s R_1/E_c)10^4 + 3.60]\ f_t' d^2$$

(Equation 1)

and

$$P_a = P_n / FS$$

(Equation 1a)

or, alternatively, solving for the thickness, d, and introducing a Factor of Safety (FS) yields:

$$d = [(FSxP_a)/(1.72\ [(k_s R_1/E_c)10^4 + 3.60]\ f_t'\beta)]^{0.5}$$

(Equation 2)

where

- P_n = nominal load-carrying capacity of the slab on grade, in pounds.
- P_a = allowable load-carrying capacity of the slab on grade, in pounds.
- k_s = modulus of subgrade reaction, in pci.
- R_1 = one-half the width or diameter of the column base plate, in inches.
- E_c = modulus of elasticity of concrete, in psi.
- f_t' = tensile strength in flexure of concrete, in psi.
- d = slab thickness, in inches.
- FS = factor of safety, here taken as 3.0.
- β = load reduction factor, 1.0 for $d < 7.0$ inches; 0.85 for $d \geq 7.0$ inches.

In the analysis on which this article is based, tables were developed for slab thicknesses from four to eight inches; however, the loads for the seven-inch and eight-inch thick floor slabs were reduced by fifteen percent, using a load-reduction factor, β, to compensate for apparent deviation of the results of *Equation 1* from the finite-element results presented in the Shentu paper.

The earlier work of Packard (12), Pickett and Ray (13) and, more recently, by Spears and Panarese (14), and further detailed in ACI 360R-92 (4), treated the area of influence of a single concentrated load. The slab analyzed has a radius of three times the radius of relative stiffness. The radius of relative stiffness, b, is expressed as the fourth root of the result found by dividing the concrete plate stiffness by the modulus of subgrade reaction as follows:

$$b = [E_c d^3/(12\ (1 - \mu^2)\ k_s)]^{0.25}$$

(Equation 3)

where

- b = radius of relative stiffness, in inches.
- E_c = modulus of elasticity of concrete, taken here as 4,000,000 psi.
- d = slab thickness, in inches.
- μ = Poisson's ratio, taken here as 0.15.
- k_s = modulus of subgrade reaction, in pci.

Additionally, the table shows a value, in inches, which represents a distance of 1.5 times the *radius of relative curvature* for the slab/soil system. From a practical point of view, the radius of relative stiffness is used to determine the distance from the point of an applied concentrated load to a point where the load has virtually no effect on the slab stress. A load that is within a distance of 1.5 times the radius of relative stiffness from another load may have an influence on the slab stresses. Essentially, the loads shown in *Table 1* assume that the load under consideration is the only load within the distance shown on that Table.

Factor of Safety

The primary focus of this article is the analysis and design of concrete floor slabs-on-grade, in warehouses or industrial-type buildings, on which free-standing work platforms or mezzanine structures are built. These structures are normally designed for heavy storage floor or deck loads of 125 psf or more. Further, these free-standing structures are independent of the building structure and, therefore, the floor slabs are outside the scope of ACI 318.

Table 1: 4-inch Slab

Allowable load in Kips

Soil	1.5 Rad Rel	Concrete PSI	10" baseplate	12" Baseplate	14" Baseplate	16" Baseplate
50 pci	39 inch	3000	16	17	17	17
		4000	19	19	20	20
100 pci	33 inch	3000	19	19	20	21
		4000	21	22	23	24
200 pci	27 inch	3000	23	25	27	29
		4000	27	29	31	33

Table 2: 5-inch Slab

Allowable load in Kips

Soil	1.5 Rad Rel	Concrete PSI	10" baseplate	12" Baseplate	14" Baseplate	16" Baseplate
50 pci	46 inch	3000	25	26	26	27
		4000	29	30	31	31
100 pci	38 inch	3000	29	30	32	33
		4000	33	35	36	38
200 pci	32 inch	3000	36	39	42	45
		4000	41	45	48	52

Table 3: 6-inch Slab

Allowable load in Kips

Soil	1.5 Rad Rel	Concrete PSI	10" baseplate	12" Baseplate	14" Baseplate	16" Baseplate
50 pci	52 inch	3000	36	37	38	39
		4000	41	43	44	45
100 pci	44 inch	3000	41	44	46	48
		4000	48	50	52	55
200 pci	37 inch	3000	52	56	60	65
		4000	60	65	70	75

Table 4: 7-inch Slab

Allowable load in Kips

Soil	1.5 Rad Rel	Concrete PSI	10" baseplate	12" Baseplate	14" Baseplate	16" Baseplate
50 pci	59 inch	3000	41	43	44	45
		4000	48	49	51	52
100 pci	49 inch	3000	48	50	53	55
		4000	55	58	61	63
200 pci	42 inch	3000	60	65	70	75
		4000	69	75	80	86

When selecting a factor of safety (FS), the following factors should be considered:

a) Will the design load be applied to the entire deck surface simultaneously? The likelihood of the design load being applied over the entire deck surface may be unlikely.
b) Will any slab failure lead to a catastrophic result?
c) Will excessive settlement under load cause problems of function or inconvenience, e.g., will windows break, will doors stick, will stored goods become unstable or dislodged, and will roofs leak due to the floor-slab settlement?
d) Will slab failure lead to costly repair?

Good engineering judgment should be exercised in the selection of any factor of safety. The tables developed in this study, such as the example presented here, in general use a factor of safety of three versus the predicted nominal load, P_n, of Equation (1). While this is considered to be very conservative, a factor of three was chosen, pending any further research results on the effects of control joints and the effects of other possibly-neighboring loads on the overall behavior and load-carrying capacity of the floor-slab system. Further, as stated earlier, the loads for seven-inch and eight-inch thick floor slabs have been reduced by approximately fifteen percent to compensate for the apparent deviation of the results of *Equation 1* from the finite-element results presented in the Shentu paper.■

Table 5: 8-inch Slab

Allowable load in Kips

Soil	1.5 Rad Rel	Concrete PSI	10" baseplate	12" Baseplate	14" Baseplate	16" Baseplate
50 pci	65 inch	3000	55	56	57	59
		4000	62	64	66	68
100 pci	55 inch	3000	62	65	69	72
		4000	72	75	79	83
200 pci	46 inch	3000	78	85	91	97
		4000	90	98	105	112

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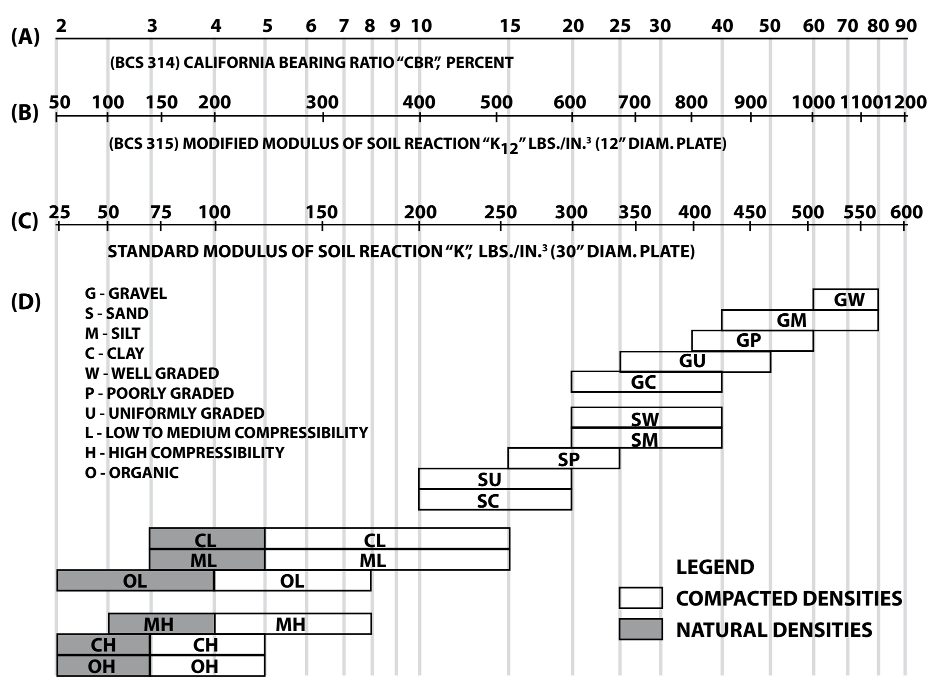

Figure 1