EXAMPLE PROBLEMS

PROBLEM 1.1

A spacecraft's engine ejects mass at a rate of 30 kg/s with an exhaust velocity of 3,100 m/s. The pressure at the nozzle exit is 5 kPa and the exit area is 0.7 m^2. What is the thrust of the engine in a vacuum?

SOLUTION,

Given: $q = 30$ kg/s
$Ve = 3{,}100$ m/s
$Ae = 0.7\ m^2$
$Pe = 5$ kPa $= 5{,}000\ N/m^2$
$Pa = 0$

Equation (1.6),

$$F = q \times Ve + (Pe - Pa) \times Ae$$
$$F = 30 \times 3{,}100 + (5{,}000 - 0) \times 0.7$$
$$F = 96{,}500\ N$$

PROBLEM 1.2

The spacecraft in problem 1.1 has an initial mass of 30,000 kg. What is the change in velocity if the spacecraft burns its engine for one minute?

SOLUTION,

Given: $M = 30{,}000$ kg
$q = 30$ kg/s
$Ve = 3{,}100$ m/s
$t = 60$ s

Equation (1.16),

$$\Delta V = Ve \times LN[\ M / (M - qt)\ ]$$
$$\Delta V = 3{,}100 \times LN[\ 30{,}000 / (30{,}000 - (30 \times 60))\ ]$$
$$\Delta V = 192\ m/s$$

PROBLEM 1.3

A spacecraft's dry mass is 75,000 kg and the effective exhaust gas velocity of its main engine is 3,100 m/s. How much propellant must be carried if the propulsion system is to produce a total Δv of 700 m/s?

SOLUTION,

Given: $Mf = 75{,}000$ kg
$C = 3{,}100$ m/s
$\Delta V = 700$ m/s

Equation (1.20),

$$Mo = Mf \times e^{(Dv / C)}$$

$$Mo = 75{,}000 \times e^{(700 / 3{,}100)}$$
$$Mo = 94{,}000 \text{ kg}$$

Propellant mass,

$$Mp = Mo - Mf$$
$$Mp = 94{,}000 - 75{,}000$$
$$Mp = 19{,}000 \text{ kg}$$

PROBLEM 1.4

A 5,000 kg spacecraft is in Earth orbit traveling at a velocity of 7,790 m/s. Its engine is burned to accelerate it to a velocity of 12,000 m/s placing it on an escape trajectory. The engine expels mass at a rate of 10 kg/s and an effective velocity of 3,000 m/s. Calculate the duration of the burn.

SOLUTION,

Given: $M = 5{,}000$ kg
$q = 10$ kg/s
$C = 3{,}000$ m/s
$\Delta V = 12{,}000 - 7{,}790 = 4{,}210$ m/s

Equation (1.21),

$$t = M / q \times [\ 1 - 1 / e^{(Dv / C)}\ ]$$
$$t = 5{,}000 / 10 \times [\ 1 - 1 / e^{(4{,}210 / 3{,}000)}\ ]$$
$$t = 377 \text{ s}$$

PROBLEM 1.5

A rocket engine burning liquid oxygen and kerosene operates at a mixture ratio of 2.26 and a combustion chamber pressure of 50 atmospheres. If the nozzle is expanded to operate at sea level, calculate the exhaust gas velocity relative to the rocket.

SOLUTION,

Given: $O/F = 2.26$
$Pc = 50$ atm
$Pe = Pa = 1$ atm

From LOX/Kerosene Charts we estimate,

$Tc = 3{,}470$ K
$M = 21.40$
$k = 1.221$

Equation (1.22),

$$Ve = SQRT[\ (2 \times k / (k - 1)) \times (R^* \times Tc / M) \times (1 - (Pe / Pc)^{(k-1)/k})\ ]$$
$$Ve = SQRT[\ (2 \times 1.221 / (1.221 - 1)) \times (8{,}314.46 \times 3{,}470 / 21.40) \times (1 - (1 / 50)^{(1.221-1)/1.221})\ ]$$
$$Ve = 2{,}749 \text{ m/s}$$

PROBLEM 1.6

A rocket engine produces a thrust of 1,000 kN at sea level with a propellant flow rate of 400 kg/s. Calculate the specific impulse.

SOLUTION,

Given: $F = 1{,}000{,}000$ N

q = 400 kg/s

Equation (1.23),

Isp = F / (q × g)
Isp = 1,000,000 / (400 × 9.80665)
Isp = 255 s (sea level)

PROBLEM 1.7

A rocket engine uses the same propellant, mixture ratio, and combustion chamber pressure as that in problem 1.5. If the propellant flow rate is 500 kg/s, calculate the area of the exhaust nozzle throat.

SOLUTION,

Given: Pc = 50 × 0.101325 = 5.066 MPa
Tc = 3,470 K
M = 21.40
k = 1.221
q = 500 kg/s

Equation (1.27),

$Pt = Pc \times [1 + (k - 1) / 2]^{-k/(k-1)}$
$Pt = 5.066 \times [1 + (1.221 - 1) / 2]^{-1.221/(1.221-1)}$
$Pt = 2.839\ MPa = 2.839\times10^{6}\ N/m^{2}$

Equation (1.28),

Tt = Tc / (1 + (k - 1) / 2)
Tt = 3,470 / (1 + (1.221 - 1) / 2)
Tt = 3,125 K

Equation (1.26),

At = (q / Pt) × SQRT[(R* × Tt) / (M × k)]
$At = (500 / 2.839\times10^{6}) \times SQRT[(8,314.46 \times 3,125) / (21.40 \times 1.221)]$
$At = 0.1756\ m^{2}$

PROBLEM 1.8

The rocket engine in problem 1.7 is optimized to operate at an elevation of 2000 meters. Calculate the area of the nozzle exit and the section ratio.

SOLUTION,

Given: Pc = 5.066 MPa
$At = 0.1756\ m^{2}$
k = 1.221

From Atmosphere Properties,

Pa = 0.0795 MPa

Equation (1.29),

$Nm^{2} = (2 / (k - 1)) \times [(Pc / Pa)^{(k-1)/k} - 1]$
$Nm^{2} = (2 / (1.221 - 1)) \times [(5.066 / 0.0795)^{(1.221-1)/1.221} - 1]$
$Nm^{2} = 10.15$
$Nm = (10.15)^{1/2} = 3.185$

Equation (1.30),

$Ae = (At / Nm) \times [(1 + (k - 1) / 2 \times Nm^{2})/((k + 1) / 2)]^{(k+1)/(2(k-1))}$
$Ae = (0.1756 / 3.185) \times [(1 + (1.221 - 1) / 2 \times 10.15)/((1.221 + 1) / 2)]^{(1.221+1)/(2(1.221-1))}$
$Ae = 1.426\ m^{2}$

Section Ratio,

$Ae / At = 1.426 / 0.1756 = 8.12$

PROBLEM 1.9

For the rocket engine in problem 1.7, calculate the volume and dimensions of a possible combustion chamber. The convergent cone half-angle is 20 degrees.

SOLUTION,

Given: $At = 0.1756\ m^2 = 1{,}756\ cm^2$
$Dt = 2 \times (1{,}756/\pi)^{1/2} = 47.3\ cm$
$\theta = 20^o$

From Table 1,

L* = 102-127 cm for LOX/RP-1, let's use 110 cm

Equation (1.33),

$Vc = At \times L^*$
$Vc = 1{,}756 \times 110 = 193{,}160\ cm^3$

From Figure 1.7,

Lc = 66 cm (second-order approximation)

Equation (1.35),

$Dc = SQRT[(Dt^3 + 24/\pi \times \tan\theta \times Vc) / (Dc + 6 \times \tan\theta \times Lc)]$
$Dc = SQRT[(47.3^3 + 24/\pi \times \tan(20) \times 193{,}160) / (Dc + 6 \times \tan(20) \times 66)]$
Dc = 56.6 cm (four interations)

PROBLEM 1.10

A solid rocket motor burns along the face of a central cylindrical channel 10 meters long and 1 meter in diameter. The propellant has a burn rate coefficient of 5.5, a pressure exponent of 0.4, and a density of 1.70 g/ml. Calculate the burn rate and the product generation rate when the chamber pressure is 5.0 MPa.

SOLUTION,

Given: $a = 5.5$
$n = 0.4$
$Pc = 5.0$ MPa
$\rho p = 1.70$ g/ml
$Ab = \pi \times 1 \times 10 = 31.416\ m^2$

Equation (1.36),

$r = a \times Pc^n$
$r = 5.5 \times 5.0^{0.4} = 10.47$ mm/s

Equation (1.37),

$q = \rho p \times Ab \times r$
$q = 1.70 \times 31.416 \times 10.47 = 559$ kg/s

PROBLEM 1.11

Calculate the ideal density of a solid rocket propellant consisting of 68%

ammonium perchlorate, 18% aluminum, and 14% HTPB by mass.

SOLUTION,

Given: $w_{AP} = 0.68$
$w_{Al} = 0.18$
$w_{HTPB} = 0.14$

From Properties of Rocket Propellants we have,

$\rho_{AP} = 1.95$ g/ml
$\rho_{Al} = 2.70$ g/ml
$\rho_{HTPB} = \approx 0.93$ g/ml

Equation (1.38),

$\rho_p = 1 / \Sigma_i (w / \rho)_i$
$\rho_p = 1 / [(0.68 / 1.95) + (0.18 / 2.70) + (0.14 / 0.93)]$
$\rho_p = 1.767$

PROBLEM 1.12

A two-stage rocket has the following masses: 1st-stage propellant mass 120,000 kg, 1st-stage dry mass 9,000 kg, 2nd-stage propellant mass 30,000 kg, 2nd-stage dry mass 3,000 kg, and payload mass 3,000 kg. The specific impulses of the 1st and 2nd stages are 260 s and 320 s respectively. Calculate the rocket's total ΔV.

SOLUTION,

Given: $Mo_1 = 120,000 + 9,000 + 30,000 + 3,000 + 3,000 = 165,000$ kg
$Mf_1 = 9,000 + 30,000 + 3,000 + 3,000 = 45,000$ kg
$Isp_1 = 260$ s
$Mo_2 = 30,000 + 3,000 + 3,000 = 36,000$ kg
$Mf_2 = 3,000 + 3,000 = 6,000$ kg
$Isp_2 = 320$ s

Equation (1.24),

$C_1 = Isp_1 g$
$C_1 = 260 \times 9.80665 = 2,550$ m/s

$C_2 = Isp_2 g$
$C_2 = 320 \times 9.80665 = 3,138$ m/s

Equation (1.39),

$\Delta V_1 = C_1 \times LN[Mo_1 / Mf_1]$
$\Delta V_1 = 2,550 \times LN[165,000 / 45,000]$
$\Delta V_1 = 3,313$ m/s

$\Delta V_2 = C_2 \times LN[Mo_2 / Mf_2]$
$\Delta V_2 = 3,138 \times LN[36,000 / 6,000]$
$\Delta V_2 = 5,623$ m/s

Equation (1.40),

$\Delta V_{Total} = \Delta V_1 + \Delta V_2$
$\Delta V_{Total} = 3,313 + 5,623$
$\Delta V_{Total} = 8,936$ m/s

PROBLEM 3.1

Using the Barrowman method, calculate the location of the center of pressure from the leading edge of a rocket having the dimensions given below. The nose is ogive shaped.

SOLUTION,

Given: L_N = 400 mm
d = 200 mm
d_F = 200 mm
d_R = 160 mm
L_T = 120 mm
X_P = 900 mm
C_R = 240 mm
C_T = 120 mm
S = 240 mm
L_F = 247 mm
R = 80 mm
X_R = 120 mm
X_B = 1,760 mm
N = 3 each

Equations (3.1) through (3.6),

$(C_N)_N = 2$

$X_N = 0.466 \times L_N$
$X_N = 0.466 \times 400 = 186$ mm

$(C_N)_T = 2 \times [(d_R / d)^2 - (d_F / d)^2]$
$(C_N)_T = 2 \times [(160 / 200)^2 - (200 / 200)^2]$
$(C_N)_T = -0.72$

$X_T = X_P + L_T / 3 \times [1 + (1 - d_F / d_R) / (1 - (d_F / d_R)^2)]$
$X_T = 900 + 120 / 3 \times [1 + (1 - 200 / 160) / (1 - (200 / 160)^2)]$
$X_T = 958$ mm

$(C_N)_F = (1 + R / (S + R)) \times [4 \times N \times (S / d)^2 / (1 + SQRT[1 + (2 \times L_F / (C_R + C_T))^2])]$
$(C_N)_F = (1 + 80 / (240 + 80)) \times [4 \times 3 \times (240 / 200)^2 / (1 + SQRT[1 + (2 \times 247 / (240 + 120))^2])]$
$(C_N)_F = 8.01$

$X_F = X_B + X_R / 3 \times (C_R + 2 \times C_T) / (C_R + C_T) + 1/6 \times [C_R + C_T - C_R \times C_T / (C_R + C_T)]$
$X_F = 1760 + 120 / 3 \times (240 + 2 \times 120) / (240 + 120) + 1/6 \times [240 + 120 - 240 \times 120 / (240 + 120)]$
$X_F = 1,860$ mm

Equations (3.7) and (3.8),

$(C_N)_R = (C_N)_N + (C_N)_T + (C_N)_F$
$(C_N)_R = 2 - 0.72 + 8.01 = 9.29$

$X = [(C_N)_N \times X_N + (C_N)_T \times X_T + (C_N)_F \times X_F] / (C_N)_R$
$X = [2 \times 186 - 0.72 \times 958 + 8.01 \times 1860] / 9.29$
$X = 1,570$ mm

PROBLEM 4.1

Calculate the velocity of an artificial satellite orbiting the Earth in a circular orbit at an altitude of 200 km above the Earth's surface.

SOLUTION,

From Basics Constants,

Radius of Earth = 6,378.14 km
GM of Earth = 3.986005×10^{14} m^3/s^2

Given: $r = (6,378.14 + 200) \times 1,000 = 6,578,140$ m

Equation (4.6),

$v = SQRT[GM / r]$
$v = SQRT[3.986005 \times 10^{14} / 6,578,140]$
$v = 7,784$ m/s

PROBLEM 4.2

Calculate the period of revolution for the satellite in problem 4.1.

SOLUTION,

Given: $r = 6{,}578{,}140$ m

Equation (4.9),

$$P^2 = 4 \times \pi^2 \times r^3 / GM$$

$$P = \mathrm{SQRT}[\ 4 \times \pi^2 \times r^3 / GM\ ]$$
$$P = \mathrm{SQRT}[\ 4 \times \pi^2 \times 6{,}578{,}140^3 / 3.986005\times10^{14}\ ]$$
$$P = 5{,}310 \text{ s}$$

PROBLEM 4.3

Calculate the radius of orbit for a Earth satellite in a geosynchronous orbit, where the Earth's rotational period is 86,164.1 seconds.

SOLUTION,

Given: $P = 86{,}164.1$ s

Equation (4.9),

$$P^2 = 4 \times \pi^2 \times r^3 / GM$$

$$r = [\ P^2 \times GM / (4 \times \pi^2)\ ]^{1/3}$$
$$r = [\ 86{,}164.1^2 \times 3.986005\times10^{14} / (4 \times \pi^2)\ ]^{1/3}$$
$$r = 42{,}164{,}170 \text{ m}$$

PROBLEM 4.4

An artificial Earth satellite is in an elliptical orbit which brings it to an altitude of 250 km at perigee and out to an altitude of 500 km at apogee. Calculate the velocity of the satellite at both perigee and apogee.

SOLUTION,

Given: $Rp = (6{,}378.14 + 250) \times 1{,}000 = 6{,}628{,}140$ m
$Ra = (6{,}378.14 + 500) \times 1{,}000 = 6{,}878{,}140$ m

Equations (4.16) and (4.17),

$$Vp = \mathrm{SQRT}[\ 2 \times GM \times Ra / (Rp \times (Ra + Rp))\ ]$$
$$Vp = \mathrm{SQRT}[\ 2 \times 3.986005\times10^{14} \times 6{,}878{,}140 / (6{,}628{,}140 \times (6{,}878{,}140 + 6{,}628{,}140))\ ]$$
$$Vp = 7{,}826 \text{ m/s}$$

$$Va = \mathrm{SQRT}[\ 2 \times GM \times Rp / (Ra \times (Ra + Rp))\ ]$$
$$Va = \mathrm{SQRT}[\ 2 \times 3.986005\times10^{14} \times 6{,}628{,}140 / (6{,}878{,}140 \times (6{,}878{,}140 + 6{,}628{,}140))\ ]$$
$$Va = 7{,}542 \text{ m/s}$$

PROBLEM 4.5

A satellite in Earth orbit passes through its perigee point at an altitude of 200 km above the Earth's surface and at a velocity of 7,850 m/s. Calculate the apogee altitude of the satellite.

SOLUTION,

Given: Rp = (6,378.14 + 200) × 1,000 = 6,578,140 m
Vp = 7,850 m/s

Equation (4.18),

$$Ra = Rp / [2 \times GM / (Rp \times Vp^2) - 1]$$
$$Ra = 6{,}578{,}140 / [2 \times 3.986005\times10^{14} / (6{,}578{,}140 \times 7{,}850^2) - 1]$$
$$Ra = 6{,}805{,}140 \text{ m}$$

Altitude @ apogee = 6,805,140 / 1,000 - 6,378.14 = 427.0 km

PROBLEM 4.6

Calculate the eccentricity of the orbit for the satellite in problem 4.5.

SOLUTION,

Given: Rp = 6,578,140 m
Vp = 7,850 m/s

Equation (4.20),

$$e = Rp \times Vp^2 / GM - 1$$
$$e = 6{,}578{,}140 \times 7{,}850^2 / 3.986005\times10^{14} - 1$$
$$e = 0.01696$$

PROBLEM 4.7

A satellite in Earth orbit has a semi-major axis of 6,700 km and an eccentricity of 0.01. Calculate the satellite's altitude at both perigee and apogee.

SOLUTION,

Given: a = 6,700 km
e = 0.01

Equation (4.21) and (4.22),

$$Rp = a \times (1 - e)$$
$$Rp = 6{,}700 \times (1 - .01)$$
$$Rp = 6{,}633 \text{ km}$$

Altitude @ perigee = 6,633 - 6,378.14 = 254.9 km

$$Ra = a \times (1 + e)$$
$$Ra = 6{,}700 \times (1 + .01)$$
$$Ra = 6{,}767 \text{ km}$$

Altitude @ apogee = 6,767 - 6,378.14 = 388.9 km

PROBLEM 4.8

A satellite is launched into Earth orbit where its launch vehicle burns out at an altitude of 250 km. At burnout the satellite's velocity is 7,900 m/s with the zenith angle equal to 89 degrees. Calculate the satellite's altitude at perigee and apogee.

SOLUTION,

Given: $r1 = (6,378.14 + 250) \times 1,000 = 6,628,140$ m
$v1 = 7,900$ m/s
$\gamma_1 = 89^o$

Equation (4.26),

$$(Rp / r1)_{1,2} = (-C \pm SQRT[C^2 - 4 \times (1 - C) \times -\sin^2 \gamma_1]) / (2 \times (1 - C))$$

where $C = 2 \times GM / (r1 \times v1^2)$
$C = 2 \times 3.986005\times10^{14} / (6,628,140 \times 7,900^2)$
$C = 1.927179$

$$(Rp / r1)_{1,2} = (-1.927179 \pm SQRT[1.927179^2 - 4 \times -0.927179 \times -\sin^2(89)]) / (2 \times -0.927179)$$
$$(Rp / r1)_{1,2} = 0.996019 \text{ and } 1.082521$$

Perigee Radius, $Rp = Rp_1 = r1 \times (Rp / r1)_1$

$Rp = 6,628,140 \times 0.996019$
$Rp = 6,601,750$ m

Altitude @ perigee = 6,601,750 / 1,000 - 6,378.14 = 223.6 km

Apogee Radius, $Ra = Rp_2 = r1 \times (Rp / r1)_2$

$Ra = 6,628,140 \times 1.082521$
$Ra = 7,175,100$ m

Altitude @ agogee = 7,175,100 / 1,000 - 6,378.14 = 797.0 km

PROBLEM 4.9

Calculate the eccentricity of the orbit for the satellite in problem 4.8.

SOLUTION,

Given: $r1 = 6,628,140$ m
$v1 = 7,900$ m/s
$\gamma_1 = 89^o$

Equation (4.27),

$$e = SQRT[(r1 \times v1^2 / GM - 1)^2 \times \sin^2 \gamma_1 + \cos^2 \gamma_1]$$
$$e = SQRT[(6,628,140 \times 7,900^2 / 3.986005\times10^{14} - 1)^2 \times \sin^2(89) + \cos^2(89)]$$
$$e = 0.0416170$$

PROBLEM 4.10

Calculate the angle θ from perigee point to launch point for the satellite in problem 4.8.

SOLUTION,

Given: $r1 = 6,628,140$ m
$v1 = 7,900$ m/s
$\gamma_1 = 89^o$

Equation (4.28),

$$\tan \nu = (r1 \times v1^2 / GM) \times \sin \gamma_1 \times \cos \gamma_1 / [(r1 \times v1^2 / GM) \times \sin^2 \gamma_1 - 1]$$
$$\tan \nu = (6,628,140 \times 7,900^2 / 3.986005\times10^{14}) \times \sin(89) \times \cos(89)$$
$$/ [(6,628,140 \times 7,900^2 / 3.986005\times10^{14}) \times \sin^2(89) - 1]$$
$$\tan \nu = 0.48329$$

ν = arctan(0.48329)
ν = 25.794°

PROBLEM 4.11

Calculate the semi-major axis of the orbit for the satellite in problem 4.8.

SOLUTION,

Given: r1 = 6,628,140 m
v1 = 7,900 m/s

Equation (4.32),

$a = 1 / (2 / r1 - v1^2 / GM)$
$a = 1 / (2 / 6,628,140 - 7,900^2 / 3.986005\times10^{14}))$
a = 6,888,430 m

PROBLEM 4.12

For the satellite in problem 4.8, burnout occurs 2000-10-20, 15:00 UT. The geocentric coordinates at burnout are 32° N latitude, 60° W longitude, and the azimuth heading is 86°. Calculate the orbit's inclination, argument of perigee, and longitude of ascending node.

SOLUTION,

Given: β = 86°
δ = 32°
λ_2 = -60°

From problem 4.10,

ν = 25.794°

Equation (4.33),

$\cos(i) = \cos(\delta) \times \sin(\beta)$
$\cos(i) = \cos(32) \times \sin(86)$
i = 32.223°

Equations (4.34) and (4.36),

$\tan(\ell) = \tan(\delta) / \cos(\beta)$
$\tan(\ell) = \tan(32) / \cos(86)$
ℓ = 83.630°

$\omega = \ell - \nu$
ω = 83.630 - 25.794
ω = 57.836°

Equations (4.35) and (4.37),

$\tan(\Delta\lambda) = \sin(\delta) \times \tan(\beta)$
$\tan(\Delta\lambda) = \sin(32) \times \tan(86)$
$\Delta\lambda$ = 82.483°

$\lambda_1 = \lambda_2 - \Delta\lambda$
λ_1 = -60 - 82.483
λ_1 = -142.483°

Ω = Sidereal time at -142.483 longitude, 2000-10-20, 15:00 UT
Ω = 7^h 27' 34" = 111.892°

PROBLEM 4.13

A satellite is in an orbit with a semi-major axis of 7,500 km and an eccentricity of 0.1. Calculate the time it takes to move from a position 30 degrees past perigee to 90 degrees past perigee.

SOLUTION,

Given: $a = 7{,}500 \times 1{,}000 = 7{,}500{,}000$ m
$e = 0.1$
$t_0 = 0$
$\nu_0 = 30$ deg $\times \pi/180 = 0.52360$ radians
$\nu = 90$ deg $\times \pi/180 = 1.57080$ radians

Equation (4.40),

$\cos E = (e + \cos \nu) / (1 + e \cos \nu)$

$Eo = \arccos[(0.1 + \cos(0.52360)) / (1 + 0.1 \times \cos(0.52360))]$
$Eo = 0.47557$ radians

$E = \arccos[(0.1 + \cos(1.57080)) / (1 + 0.1 \times \cos(1.57080))]$
$E = 1.47063$ radians

Equation (4.41),

$M = E - e \times \sin E$

$Mo = 0.47557 - 0.1 \times \sin(0.47557)$
$Mo = 0.42978$ radians

$M = 1.47063 - 0.1 \times \sin(1.47063)$
$M = 1.37113$ radians

Equation (4.39),

$n = \text{SQRT}[\ GM / a^3\ ]$
$n = \text{SQRT}[\ 3.986005\times10^{14} / 7{,}500{,}000^3\ ]$
$n = 0.00097202$ rad/s

Equation (4.38),

$M - Mo = n \times (t - t_0)$

$t = t_0 + (M - Mo) / n$
$t = 0 + (1.37113 - 0.42978) / 0.00097202$
$t = 968.4$ s

PROBLEM 4.14

The satellite in problem 4.13 has a true anomaly of 90 degrees. What will be the satellite's position, i.e. it's true anomaly, 20 minutes later?

SOLUTION,

Given: $a = 7{,}500{,}000$ m
$e = 0.1$
$t_0 = 0$
$t = 20 \times 60 = 1{,}200$ s
$\nu_0 = 90 \times \pi/180 = 1.57080$ rad

From problem 4.13,

$Mo = 1.37113$ rad
$n = 0.00097202$ rad/s

Equation (4.38),

$M - Mo = n \times (t - t_0)$

$M = Mo + n \times (t - t_0)$
$M = 1.37113 + 0.00097202 \times (1{,}200 - 0)$
$M = 2.53755$

METHOD #1, Low Accuracy:

Equation (4.42),

$\nu \sim M + 2 \times e \times \sin M + 1.25 \times e^2 \times \sin 2M$
$\nu \sim 2.53755 + 2 \times 0.1 \times \sin(2.53755) + 1.25 \times 0.1^2 \times \sin(2 \times 2.53755)$
$\nu \sim 2.63946 = 151.2$ degrees

METHOD #2, High Accuracy:

Equation (4.41),

$M = E - e \times \sin E$
$2.53755 = E - 0.1 \times \sin E$

By iteration, E = 2.58996 radians

Equation (4.40),

$\cos E = (e + \cos \nu) / (1 + e \cos \nu)$

Rearranging variables gives,

$\cos \nu = (\cos E - e) / (1 - e \cos E)$

$\nu = \arccos[(\cos(2.58996) - 0.1) / (1 - 0.1 \times \cos(2.58996)]$
$\nu = 2.64034 = 151.3$ degrees

PROBLEM 4.15

For the satellite in problems 4.13 and 4.14, calculate the length of its position vector, its flight-path angle, and its velocity when the satellite's true anomaly is 225 degrees.

SOLUTION,

Given: a = 7,500,000 m
e = 0.1
ν = 225 degrees

Equations (4.43) and (4.44),

$r = a \times (1 - e^2) / (1 + e \times \cos \nu)$
$r = 7{,}500{,}000 \times (1 - 0.1^2) / (1 + 0.1 \times \cos(225))$
$r = 7{,}989{,}977$ m

$\phi = \arctan[\, e \times \sin \nu / (1 + e \times \cos \nu)]$
$\phi = \arctan[\, 0.1 \times \sin(225) / (1 + 0.1 \times \cos(225))]$
$\phi = -4.351$ degrees

Equation (4.45),

$v = \mathrm{SQRT}[\, GM \times (2 / r - 1 / a)]$
$v = \mathrm{SQRT}[\, 3.986005\times10^{14} \times (2 / 7{,}989{,}977 - 1 / 7{,}500{,}000)]$
$v = 6{,}828$ m/s

PROBLEM 4.16

Calculate the perturbations in longitude of the ascending node and argument of perigee caused by the Moon and Sun for the International Space Station orbiting at an altitude of 400 km, an inclination of 51.6 degrees, and with an orbital period of 92.6 minutes.

SOLUTION,

Given: i = 51.6 degrees
n = 1436 / 92.6 = 15.5 revolutions/day

Equations (4.46) through (4.49),

$$\Omega_{Moon} = -0.00338 \times \cos(i) / n$$
$$\Omega_{Moon} = -0.00338 \times \cos(51.6) / 15.5$$
$$\Omega_{Moon} = -0.000135 \text{ deg/day}$$

$$\Omega_{Sun} = -0.00154 \times \cos(i) / n$$
$$\Omega_{Sun} = -0.00154 \times \cos(51.6) / 15.5$$
$$\Omega_{Sun} = -0.0000617 \text{ deg/day}$$

$$\omega_{Moon} = 0.00169 \times (4 - 5 \times \sin^2 i) / n$$
$$\omega_{Moon} = 0.00169 \times (4 - 5 \times \sin^2 51.6) / 15.5$$
$$\omega_{Moon} = 0.000101 \text{ deg/day}$$

$$\omega_{Sun} = 0.00077 \times (4 - 5 \times \sin^2 i) / n$$
$$\omega_{Sun} = 0.00077 \times (4 - 5 \times \sin^2 51.6) / 15.5$$
$$\omega_{Sun} = 0.000046 \text{ deg/day}$$

PROBLEM 4.17

A satellite is in an orbit with a semi-major axis of 7,500 km, an inclination of 28.5 degrees, and an eccentricity of 0.1. Calculate the J_2 perturbations in longitude of the ascending node and argument of perigee.

SOLUTION,

Given: a = 7,500 km
i = 28.5 degrees
e = 0.1

Equations (4.50) and (4.51),

$$\Omega_{J2} = -2.06474\times10^{14} \times a^{-7/2} \times (\cos i) \times (1 - e^2)^{-2}$$
$$\Omega_{J2} = -2.06474\times10^{14} \times (7{,}500)^{-7/2} \times (\cos 28.5) \times (1 - (0.1)^2)^{-2}$$
$$\Omega_{J2} = -5.067 \text{ deg/day}$$

$$\omega_{J2} = 1.03237\times10^{14} \times a^{-7/2} \times (4 - 5 \times \sin^2 i) \times (1 - e^2)^{-2}$$
$$\omega_{J2} = 1.03237\times10^{14} \times (7{,}500)^{-7/2} \times (4 - 5 \times \sin^2 28.5) \times (1 - (0.1)^2)^{-2}$$
$$\omega_{J2} = 8.250 \text{ deg/day}$$

PROBLEM 4.18

A satellite is in a circular Earth orbit at an altitude of 400 km. The satellite has a cylindrical shape 2 m in diameter by 4 m long and has a mass of 1,000 kg. The satellite is traveling with its long axis perpendicular to the velocity vector and it's drag coefficient is 2.67. Calculate the perturbations due to atmospheric drag and estimate the satellite's lifetime.

SOLUTION,

Given: a = (6,378.14 + 400) × 1,000 = 6,778,140 m
A = 2 × 4 = 8 m^2
m = 1,000 kg
C_D = 2.67

From Atmosphere Properties,

$$\rho = 2.62\times10^{-12} \text{ kg/m}^3$$
$$H = 58.2 \text{ km}$$

Equation (4.6),

$$V = SQRT[GM / a]$$
$$V = SQRT[3.986005\times10^{14} / 6,778,140]$$
$$V = 7,669 \text{ m/s}$$

Equations (4.53) through (4.55),

$$\Delta a_{rev} = (-2 \times \pi \times C_D \times A \times \rho \times a^2) / m$$
$$\Delta a_{rev} = (-2 \times \pi \times 2.67 \times 8 \times 2.62\times10^{-12} \times 6,778,140^2) / 1,000$$
$$\Delta a_{rev} = -16.2 \text{ m}$$

$$\Delta P_{rev} = (-6 \times \pi^2 \times C_D \times A \times \rho \times a^2) / (m \times V)$$
$$\Delta P_{rev} = (-6 \times \pi^2 \times 2.67 \times 8 \times 2.62\times10^{-12} \times 6,778,140^2) / (1,000 \times 7,669)$$
$$\Delta P_{rev} = -0.0199 \text{ s}$$

$$\Delta V_{rev} = (\pi \times C_D \times A \times \rho \times a \times V) / m$$
$$\Delta V_{rev} = (\pi \times 2.67 \times 8 \times 2.62\times10^{-12} \times 6,778,140 \times 7,669) / 1,000$$
$$\Delta V_{rev} = 0.00914 \text{ m/s}$$

Equation (4.56),

$$L \sim -H / \Delta a_{rev}$$
$$L \sim -(58.2 \times 1,000) / -16.2$$
$$L \sim 3,600 \text{ revolutions}$$

PROBLEM 4.19

A spacecraft is in a circular parking orbit with an altitude of 200 km. Calculate the velocity change required to perform a Hohmann transfer to a circular orbit at geosynchronous altitude.

SOLUTION,

Given: $r_A = (6,378.14 + 200) \times 1,000 = 6,578,140$ m

From problem 4.3,

$$r_B = 42,164,170 \text{ m}$$

Equations (4.58) through (4.65),

$$a_{tx} = (r_A + r_B) / 2$$
$$a_{tx} = (6,578,140 + 42,164,170) / 2$$
$$a_{tx} = 24,371,155 \text{ m}$$

$$Vi_A = SQRT[GM / r_A]$$
$$Vi_A = SQRT[3.986005\times10^{14} / 6,578,140]$$
$$Vi_A = 7,784 \text{ m/s}$$

$$Vf_B = SQRT[GM / r_B]$$
$$Vf_B = SQRT[3.986005\times10^{14} / 42,164,170]$$
$$Vf_B = 3,075 \text{ m/s}$$

$$Vtx_A = SQRT[GM \times (2 / r_A - 1 / a_{tx})]$$
$$Vtx_A = SQRT[3.986005\times10^{14} \times (2 / 6,578,140 - 1 / 24,371,155)]$$
$$Vtx_A = 10,239 \text{ m/s}$$

$$Vtx_B = SQRT[GM \times (2 / r_B - 1 / a_{tx})]$$
$$Vtx_B = SQRT[3.986005\times10^{14} \times (2 / 42,164,170 - 1 / 24,371,155)]$$
$$Vtx_B = 1,597 \text{ m/s}$$

$$\Delta V_A = Vtx_A - Vi_A$$
$$\Delta V_A = 10,239 - 7,784$$
$$\Delta V_A = 2,455 \text{ m/s}$$

$$\Delta V_B = Vf_B - Vtx_B$$
$$\Delta V_B = 3,075 - 1,597$$
$$\Delta V_B = 1,478 \text{ m/s}$$

$\Delta V_T = \Delta V_A + \Delta V_B$
$\Delta V_T = 2{,}455 + 1{,}478$
$\Delta V_T = 3{,}933$ m/s

PROBLEM 4.20

A satellite is in a circular parking orbit with an altitude of 200 km. Using a one-tangent burn, it is to be transferred to geosynchronous altitude using a transfer ellipse with a semi-major axis of 30,000 km. Calculate the total required velocity change and the time required to complete the transfer.

SOLUTION,

Given: $r_A = (6{,}378.14 + 200) \times 1{,}000 = 6{,}578{,}140$ m
$r_B = 42{,}164{,}170$ m
$a_{tx} = 30{,}000 \times 1{,}000 = 30{,}000{,}000$ m

Equations (4.66) through (4.68),

$e = 1 - r_A / a_{tx}$
$e = 1 - 6{,}578{,}140 / 30{,}000{,}000$
$e = 0.780729$

$\nu = \arccos[(a_{tx} \times (1 - e^2) / r_B - 1) / e\ ]$
$\nu = \arccos[(30{,}000{,}000 \times (1 - 0.780729^2) / 42{,}164{,}170 - 1) / 0.780729\ ]$
$\nu = 157.670$ degrees

$\phi = \arctan[\ e \times \sin \nu / (1 + e \times \cos \nu)]$
$\phi = \arctan[\ 0.780729 \times \sin(157.670) / (1 + 0.780729 \times \cos(157.670))]$
$\phi = 46.876$ degrees

Equations (4.59) through (4.63),

$Vi_A = \mathrm{SQRT}[\ GM / r_A\ ]$
$Vi_A = \mathrm{SQRT}[\ 3.986005\times10^{14} / 6{,}578{,}140\ ]$
$Vi_A = 7{,}784$ m/s

$Vf_B = \mathrm{SQRT}[\ GM / r_B\ ]$
$Vf_B = \mathrm{SQRT}[\ 3.986005\times10^{14} / 42{,}164{,}170\ ]$
$Vf_B = 3{,}075$ m/s

$Vtx_A = \mathrm{SQRT}[\ GM \times (2 / r_A - 1 / a_{tx})]$
$Vtx_A = \mathrm{SQRT}[\ 3.986005\times10^{14} \times (2 / 6{,}578{,}140 - 1 / 30{,}000{,}000)]$
$Vtx_A = 10{,}388$ m/s

$Vtx_B = \mathrm{SQRT}[\ GM \times (2 / r_B - 1 / a_{tx})]$
$Vtx_B = \mathrm{SQRT}[\ 3.986005\times10^{14} \times (2 / 42{,}164{,}170 - 1 / 30{,}000{,}000)]$
$Vtx_B = 2{,}371$ m/s

$\Delta V_A = Vtx_A - Vi_A$
$\Delta V_A = 10{,}388 - 7{,}784$
$\Delta V_A = 2{,}604$ m/s

Equation (4.69),

$\Delta V_B = \mathrm{SQRT}[\ Vtx_B^2 + Vf_B^2 - 2 \times Vtx_B \times Vf_B \times \cos \phi\ ]$
$\Delta V_B = \mathrm{SQRT}[\ 2{,}371^2 + 3{,}075^2 - 2 \times 2{,}371 \times 3{,}075 \times \cos(46.876)]$
$\Delta V_B = 2{,}260$ m/s

Equation (4.65),

$\Delta V_T = \Delta V_A + \Delta V_B$
$\Delta V_T = 2{,}604 + 2{,}260$
$\Delta V_T = 4{,}864$ m/s

Equations (4.70) and (4.71),

$E = \arccos[(e + \cos \nu) / (1 + e \cos \nu)]$
$E = \arccos[(0.780729 + \cos(157.670)) / (1 + 0.780729 \times \cos(157.670))]$
$E = 2.11688$ radians

$TOF = (E - e \times \sin E) \times SQRT[a_{tx}^3 / GM]$
$TOF = (2.11688 - 0.780729 \times \sin(2.11688)) \times SQRT[30,000,000^3 / 3.986005 \times 10^{14}]$
$TOF = 11,931$ s = 3.314 hours

PROBLEM 4.21

Calculate the velocity change required to transfer a satellite from a circular 600 km orbit with an inclination of 28 degrees to an orbit of equal size with an inclination of 20 degrees.

SOLUTION,

Given: $r = (6,378.14 + 600) \times 1,000 = 6,978,140$ m
$\theta = 28 - 20 = 8$ degrees

Equation (4.6),

$Vi = SQRT[GM / r]$
$Vi = SQRT[3.986005 \times 10^{14} / 6,978,140]$
$Vi = 7,558$ m/s

Equation (4.73),

$\Delta V = 2 \times Vi \times \sin(\theta/2)$
$\Delta V = 2 \times 7,558 \times \sin(8/2)$
$\Delta V = 1,054$ m/s

PROBLEM 4.22

A satellite is in a parking orbit with an altitude of 200 km and an inclination of 28 degrees. Calculate the total velocity change required to transfer the satellite to a zero-inclination geosynchronous orbit using a Hohmann transfer with a combined plane change at apogee.

Given: $r_A = (6,378.14 + 200) \times 1,000 = 6,578,140$ m
$r_B = 42,164,170$ m
$\theta = 28$ degrees

From problem 4.19,

$Vf_B = 3,075$ m/s
$Vtx_B = 1,597$ m/s
$\Delta V_A = 2,455$ m/s

Equation (4.74),

$\Delta V_B = SQRT[Vtx_B^2 + Vf_B^2 - 2 \times Vtx_B \times Vf_B \times \cos \theta]$
$\Delta V_B = SQRT[1,597^2 + 3,075^2 - 2 \times 1,597 \times 3,075 \times \cos(28)]$
$\Delta V_B = 1,826$ m/s

Equation (4.65),

$\Delta V_T = \Delta V_A + \Delta V_B$
$\Delta V_T = 2,455 + 1,826$
$\Delta V_T = 4,281$ m/s

PROBLEM 4.23

A spacecraft is in an orbit with an inclination of 30 degrees and the longitude of the ascending node is 75 degrees. Calculate the angle change required to change the inclination to 32 degrees and the longitude of the ascending node to 80 degrees.

SOLUTION,

Given: i_i = 30 degrees
Ωi = 75 degrees
i_f = 32 degrees
Ωf = 80 degrees

Equation (4.75),

$a1 = \sin(i_i)\cos(\Omega i) = \sin(30)\cos(75) = 0.129410$

$a2 = \sin(i_i)\sin(\Omega i) = \sin(30)\sin(75) = 0.482963$

$a3 = \cos(i_i) = \cos(30) = 0.866025$

$b1 = \sin(i_f)\cos(\Omega f) = \sin(32)\cos(80) = 0.0920195$

$b2 = \sin(i_f)\sin(\Omega f) = \sin(32)\sin(80) = 0.521869$

$b3 = \cos(i_f) = \cos(32) = 0.848048$

$\theta = \arccos(a1 \times b1 + a2 \times b2 + a3 \times b3)$
$\theta = \arccos(0.129410 \times 0.0920195 + 0.482963 \times 0.521869 + 0.866025 \times 0.848048)$
θ = 3.259 degrees

PROBLEM 4.24

Calculate the latitude and longitude of the intersection nodes between the initial and final orbits for the spacecraft in problem 4.23.

SOLUTION,

From problem 4.21,

a1 = 0.129410
a2 = 0.482963
a3 = 0.866025
b1 = 0.0920195
b2 = 0.521869
b3 = 0.848048

Equations (4.76) and (4.77),

$c1 = a2 \times b3 - a3 \times b2 = 0.482963 \times 0.848048 - 0.866025 \times 0.521869 = -0.0423757$

$c2 = a3 \times b1 - a1 \times b3 = 0.866025 \times 0.0920195 - 0.129410 \times 0.848048 = -0.0300543$

$c3 = a1 \times b2 - a2 \times b1 = 0.129410 \times 0.521869 - 0.482963 \times 0.0920195 = 0.0230928$

$lat_1 = \arctan(c3 / (c1^2 + c2^2)^{1/2})$
$lat_1 = \arctan(0.0230928 / (-0.0423757^2 + -0.0300543^2)^{1/2})$
lat_1 = 23.965 degrees

$long_1 = \arctan(c2 / c1) + 90$
$long_1 = \arctan(-0.0300543 / -0.0423757) + 90$
$long_1$ = 125.346 degrees

lat_2 = -23.965 degrees
$long_2$ = 125.346 + 180 = 305.346 degrees

PROBLEM 4.25

Calculate the escape velocity of a spacecraft launched from an Earth orbit with an altitude of 200 km.

SOLUTION,

Given: $r = (6,378.14 + 200) \times 1,000 = 6,578,140$ m

Equation (4.78),

$$V_{esc} = \text{SQRT}[\ 2 \times GM / r\ ]$$
$$V_{esc} = \text{SQRT}[\ 2 \times 3.986005\times10^{14} / 6,578,140\ ]$$
$$V_{esc} = 11,009 \text{ m/s}$$

PROBLEM 4.26

A space probe is approaching Mars on a hyperbolic flyby trajectory. When at a distance of 100,000 km, its velocity relative to Mars is 5,140.0 m/s and its flight path angle is -85.300 degrees. Calculate the probe's eccentricity, semi-major axis, turning angle, angle η, true anomaly, impact parameter, periapsis radius, and parameter p.

SOLUTION,

From Basics Constants,

$$GM \text{ of Mars} = 4.282831\times10^{13} \text{ m}^3/\text{s}^2$$

Given: $r = 100,000 \times 1,000 = 100,000,000$ m
$v = 5,140.0$ m/s
$\phi = -85.300^o$

Equations (4.30) and (4.32),

$$e = \text{SQRT}[\ (r \times v^2 / GM - 1)^2 \times \cos^2 \phi + \sin^2 \phi\ ]$$
$$e = \text{SQRT}[\ (100,000,000 \times 5,140^2 / 4.282831\times10^{13} - 1)^2 \times \cos^2(-85.3) + \sin^2(-85.3)\ ]$$
$$e = 5.0715$$

$$a = 1 / (\ 2 / r - v^2 / GM\ )$$
$$a = 1 / (\ 2 / 100,000,000 - 5,140^2 / 4.282831\times10^{13}\ )$$
$$a = -1,675,400 \text{ m}$$

Equations (4.80) through (4.85),

$$\sin(\delta/2) = 1 / e$$
$$\delta = 2 \times \arcsin(\ 1 / 5.0715\ )$$
$$\delta = 22.744^o$$

$$\cos \eta = -1 / e$$
$$\eta = \arccos(\ -1 / 5.0715\ )$$
$$\eta = 101.37^o$$

$$\nu = \arccos[\ (a \times (1 - e^2) - r) / (e \times r)\ ]$$
$$\nu = \arccos[\ (-1,675,400 \times (1 - 5.0715^2) - 100,000,000) / (5.0715 \times 100,000,000)\ ]$$
$$\nu = -96.633^o$$

$$b = -a / \tan(\delta/2)$$
$$b = 1,675.4 / \tan(22.744/2)$$
$$b = 8,330.0 \text{ km}$$

$$r_o = a \times (1 - e)$$
$$r_o = -1,675.4 \times (1 - 5.0715)$$
$$r_o = 6,821.4 \text{ km}$$

$$p = a \times (1 - e^2)$$
$$p = -1,675.4 \times (1 - 5.0715^2)$$
$$p = 41,416 \text{ km}$$

PROBLEM 4.27

The space probe in problem 4.26 has moved to a true anomaly of 75 degrees.

Calculate the radius vector, flight path angle, and velocity.

SOLUTION,

Given: $a = -1{,}675{,}400$ m
$e = 5.0715$
$\nu = 75^o$

Equations (4.43) through (4.45),

$r = a \times (1 - e^2) / (1 + e \times \cos \nu)$
$r = -1{,}675{,}400 \times (1 - 5.0715^2) / (1 + 5.0715 \times \cos(75))$
$r = 17{,}909{,}000$ m

$\phi = \arctan[e \times \sin \nu / (1 + e \times \cos \nu)]$
$\phi = \arctan[5.0715 \times \sin(75) / (1 + 5.0715 \times \cos(75))]$
$\phi = 64.729^o$

$v = \mathrm{SQRT}[GM \times (2 / r - 1 / a)]$
$v = \mathrm{SQRT}[4.282831\times10^{13} \times (2 / 17{,}909{,}000 - 1 / -1{,}675{,}400)]$
$v = 5{,}508.7$ m/s

PROBLEM 4.28

A spacecraft is launched from Earth on a hyperbolic trajectory with a semi-major axis of -36,000 km and an eccentricity of 1.1823. How long does it take to move from a true anomaly of 15 degrees to a true anomaly of 120 degrees?

SOLUTION,

Given: $a = -36{,}000 \times 1{,}000 = -36{,}000{,}000$ m
$e = 1.1823$
$\nu_0 = 15^o$
$\nu = 120^o$

Equation (4.87),

$\cosh F = (e + \cos \nu) / (1 + e \cos \nu)$

$Fo = \mathrm{arccosh}[(1.1823 + \cos(15)) / (1 + 1.1823 \times \cos(15))]$
$Fo = 0.07614$

$F = \mathrm{arccosh}[(1.1823 + \cos(120)) / (1 + 1.1823 \times \cos(120))]$
$F = 1.10023$

Equation (4.86),

$t - t_0 = \mathrm{SQRT}[(-a)^3 / GM] \times [(e \times \sinh F - F) - (e \times \sinh Fo - Fo)]$
$t - t_0 = \mathrm{SQRT}[(36{,}000{,}000)^3 / 3.986005\times10^{14}] \times [(1.1823 \times \sinh(1.10023) - 1.10023)$
$- (1.1823 \times \sinh(0.07614) - 0.07614)]$
$t - t_0 = 5{,}035$ s $= 1.399$ hours

PROBLEM 4.29

A spacecraft launched from Earth has a burnout velocity of 11,500 m/s at an altitude of 200 km. What is the hyperbolic excess velocity?

SOLUTION,

Given: $V_{bo} = 11{,}500$ m/s

From problem 4.25,

$V_{esc} = 11{,}009$ m/s

Equation (4.88),

$V_\infty^2 = V_{bo}^2 - V_{esc}^2$

V_∞ = SQRT[$11{,}500^2$ - $11{,}009^2$]
V_∞ = 3,325 m/s

PROBLEM 4.30

Calculate the radius of Earth's sphere of influence.

SOLUTION,

From Basics Constants,

D_{sp} = 149,597,870 km
M_P = 5.9737×10^{24} kg
M_S = 1.9891×10^{30} kg

Equation (4.89),

$R_{Earth} = D_{sp} \times (M_P / M_S)^{0.4}$
$R_{Earth} = 149{,}597{,}870 \times (5.9737\times10^{24} / 1.9891\times10^{30})^{0.4}$
R_{Earth} = 925,000 km

PROBLEM 5.1

Using a one-tangent burn, calculate the change in true anomaly and the time-of-flight for a transfer from Earth to Mars. The radius vector of Earth at departure is 1.000 AU and that of Mars at arrival is 1.524 AU. The semi-major axis of the transfer orbit is 1.300 AU.

SOLUTION,

Given: r_A = 1.000 AU
r_B = 1.524 AU
a_{tx} = 1.300 AU × 149.597870×10^9 m/AU = 194.48×10^9 m

From Basics Constants,

GM of Sun = 1.327124×10^{20} m^3/s^2

Equations (4.66) and (4.67),

$e = 1 - r_A / a_{tx}$
e = 1 - 1.0 / 1.3
e = 0.230769

$\nu = \arccos[(a_{tx} \times (1 - e^2) / r_B - 1) / e]$
$\nu = \arccos[(1.3 \times (1 - 0.230769^2) / 1.524 - 1) / 0.230769]$
ν = 146.488 degrees

Equations (4.70) and (4.71),

$E = \arccos[(e + \cos \nu) / (1 + e \cos \nu)]$
E = arccos[(0.230769 + cos(146.488)) / (1 + 0.230769 × cos(146.488))]
E = 2.41383 radians

$TOF = (E - e \times \sin E) \times SQRT[a_{tx}^3 / GM]$
$TOF = (2.41383 - 0.230769 \times \sin(2.41383)) \times SQRT[(194.48\times10^9)^3 / 1.327124\times10^{20}]$
TOF = 16,827,800 s = 194.77 days

PROBLEM 5.2

For the transfer orbit in problem 5.1, calculate the departure phase angle, given that the angular velocity of Mars is 0.5240 degrees/day.

SOLUTION,

Given: $\nu_2 - \nu_1 = 146.488^o$
$t_2 - t_1 = 194.77$ days
$\omega_t = 0.5240^o$/day

Equation (5.1),

$\gamma_1 = (\nu_2 - \nu_1) - \omega_t \times (t_2 - t_1)$
$\gamma_1 = 146.488 - 0.5240 \times 194.77$
$\gamma_1 = 44.43^o$

PROBLEM 5.3

A flight to Mars is launched on 2020-7-20, 0:00 UT. The planned time of flight is 207 days. Earth's postion vector at departure is 0.473265**X** - 0.899215**Y** AU. Mars' postion vector at intercept is 0.066842**X** + 1.561256**Y** + 0.030948**Z** AU. Calculate the parameter and semi-major axis of the transfer orbit.

SOLUTION,

Given: t = 207 days
$\mathbf{r_1} = 0.473265\mathbf{X} - 0.899215\mathbf{Y}$ AU
$\mathbf{r_2} = 0.066842\mathbf{X} + 1.561256\mathbf{Y} + 0.030948\mathbf{Z}$ AU
$GM = 1.327124\times10^{20}\ m^3/s^2$
$= 1.327124\times10^{20} / (149.597870\times10^9)^3 = 3.964016\times10^{-14}\ AU^3/s^2$

From vector magnitude,

r_1 = SQRT[$0.473265^2 + (-0.899215)^2$]
$r_1 = 1.016153$ AU

r_2 = SQRT[$0.066842^2 + 1.561256^2 + 0.030948^2$]
$r_2 = 1.562993$ AU

From vector dot product,

$\Delta\nu$ = arccos[(0.473265 × 0.066842 - 0.899215 × 1.561256) / (1.016153 × 1.562993)]
$\Delta\nu = 149.770967^o$

Equations (5.9), (5.10) and (5.11),

$k = r_1 \times r_2 \times (1 - \cos \Delta\nu)$
k = 1.016153 × 1.562993 × (1 - cos(149.770967))
k = 2.960511 AU

$\ell = r_1 + r_2$
ℓ = 1.016153 + 1.562993
ℓ = 2.579146 AU

$m = r_1 \times r_2 \times (1 + \cos \Delta\nu)$
m = 1.016153 × 1.562993 × (1 + cos(149.770967))
m = 0.215969 AU

Equations (5.18) and (5.19),

p_i = k / (ℓ + SQRT(2 × m))
p_i = 2.960511 / (2.579146 + SQRT(2 × 0.215969))
p_i = 0.914764 AU

p_{ii} = k / (ℓ - SQRT(2 × m))
p_{ii} = 2.960511 / (2.579146 - SQRT(2 × 0.215969))
p_{ii} = 1.540388 AU

Since $\Delta\nu < \pi$, $0.914764 < p < \infty$

Equation (5.12),

Select trial value, p = 1.2 AU

$a = m \times k \times p / [(2 \times m - \ell^2) \times p^2 + 2 \times k \times \ell \times p - k^2]$
$a = 0.215969 \times 2.960511 \times 1.2$
$/ [(2 \times 0.215969 - 2.579146^2) \times 1.2^2 + 2 \times 2.960511 \times 2.579146 \times 1.2 - 2.960511^2]$
$a = 1.270478$ AU

Equations (5.5), (5.6) and (5.7),

$f = 1 - r_2 / p \times (1 - \cos \Delta\nu)$
$f = 1 - 1.562993 / 1.2 \times (1 - \cos(149.770967))$
$f = -1.427875$

$g = r_1 \times r_2 \times \sin \Delta\nu / \mathrm{SQRT}[\ GM \times p\ ]$
$g = 1.016153 \times 1.562993 \times \sin(149.770967) / \mathrm{SQRT}[\ 3.964016\times10^{-14} \times 1.2\ ]$
$g = 3{,}666{,}240$

$\dot{f} = \mathrm{SQRT}[\ GM / p\ ] \times \tan(\Delta\nu/2) \times [(1 - \cos \Delta\nu) / p - 1/r_1 - 1/r_2\ ]$
$\dot{f} = \mathrm{SQRT}[\ 3.964016\times10^{-14} / 1.2\ ] \times \tan(149.770967/2)$
$\times [(1 - \cos(149.770967)) / 1.2 - 1/1.016153 - 1/1.562993\ ]$
$\dot{f} = -4.747601\times10^{-8}$

Equation (5.13),

$\Delta E = \arccos[\ 1 - r_1 / a \times (1 - f)\ ]$
$\Delta E = \arccos[\ 1 - 1.016153 / 1.270478 \times (1 + 1.427875)\ ]$
$\Delta E = 2.798925$ radians

Equation (5.16),

$t = g + \mathrm{SQRT}[\ a^3 / GM\ ] \times (\Delta E - \sin \Delta E)$
$t = 3{,}666{,}240 + \mathrm{SQRT}[\ 1.270478^3 / 3.964016\times10^{-14}\ ] \times (2.798925 - \sin(2.798925))$
$t = 21{,}380{,}951$ s = 247.4647 days

Select new trial value of p and repeat above steps,

p = 1.300000 AU, a = 1.443005 AU,   t = 178.9588 days

Equation (5.20),

$p_{n+1} = p_n + (t - t_n) \times (p_n - p_{n-1}) / (t_n - t_{n-1})$
$p_{n+1} = 1.3 + (207 - 178.9588) \times (1.3 - 1.2) / (178.9588 - 247.4647)$
$p_{n+1} = 1.259067$ AU

Recalculate using new value of p,

p = 1.259067 AU, a = 1.336197 AU,   t = 201.5624 days

Perform additional iterations,

p = 1.249221 AU, a = 1.318624 AU,   t = 207.9408 days
p = 1.250673 AU, a = 1.321039 AU,   t = 206.9733 days
p = 1.250633 AU, a = 1.320971 AU,   t = 206.9999 days <-- close enough

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PROBLEM 5.4

For the Mars transfer orbit in Problem 5.3, calculate the departure and intecept velocity vectors.

SOLUTION,

Given: $\mathbf{r_1}$ = 0.473265**X** - 0.899215**Y** AU
$\mathbf{r_2}$ = 0.066842**X** + 1.561256**Y** + 0.030948**Z** AU
r_1 = 1.016153 AU
r_2 = 1.562993 AU
p = 1.250633 AU
a = 1.320971 AU
$\Delta\nu = 149.770967^o$

Equations (5.5), (5.6) and (5.7),

$f = 1 - r_2 / p \times (1 - \cos \Delta\nu)$
$f = 1 - 1.562993 / 1.250633 \times (1 - \cos(149.770967))$
$f = -1.329580$

$g = r_1 \times r_2 \times \sin \Delta\nu$ / SQRT[GM × p]
$g = 1.016153 \times 1.562993 \times \sin(149.770967)$ / SQRT[$3.964016\times10^{-14} \times 1.250633$]
$g = 3,591,258$

$\dot{f}$ = SQRT[GM / p] × tan($\Delta\nu$/2) × [(1 - cos $\Delta\nu$) / p - 1/r_1 - 1/r_2]
$\dot{f}$ = SQRT[3.964016×10^{-14} / 1.250633] × tan(149.770967/2)
× [(1 - cos(149.770967)) / 1.250633 - 1/1.016153 - 1/1.562993]
$\dot{f} = -8.795872\times10^{-8}$

$\dot{g} = 1 - r_1 / p \times (1 - \cos \Delta\nu)$
$\dot{g} = 1 - 1.016153 / 1.250633 \times (1 - \cos(149.770967))$
$\dot{g} = -0.514536$

Equation (5.3),

$\mathbf{v_1} = (\mathbf{r_2} - f \times \mathbf{r_1}) / g$

$\mathbf{v_1}$ = [(0.066842 + 1.329580 × 0.473265) / 3,591,258] **X**
+ [(1.561256 + 1.329580 × -0.899215) / 3,591,258] **Y**
+ [(0.030948 + 1.329580 × 0) / 3,591,258] **Z**

$\mathbf{v_1}$ = 0.000000193828**X** + 0.000000101824**Y** + 0.00000000861759**Z** AU/s × 149.597870×10^{9}
$\mathbf{v_1}$ = 28996.2**X** + 15232.7**Y** + 1289.2**Z** m/s

Equation (5.4),

$\mathbf{v_2} = \dot{f} \times \mathbf{r_1} + \dot{g} \times \mathbf{v_1}$

$\mathbf{v_2}$ = [-8.795872×10^{-8} × 0.473265 - 0.514536 × 0.000000193828] **X**
+ [-8.795872×10^{-8} × -0.899215 - 0.514536 × 0.000000101824] **Y**
+ [-8.795872×10^{-8} × 0 - 0.514536 × 0.00000000861759] **Z**

$\mathbf{v_2}$ = -0.000000141359**X** + 0.0000000267017**Y** - 0.00000000443406**Z** AU/s × 149.597870×10^{9}
$\mathbf{v_2}$ = -21147.0**X** + 3994.5**Y** - 663.3**Z** m/s

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PROBLEM 5.5

For the Mars transfer orbit in Problems 5.3 and 5.4, calculate the orbital elements.

SOLUTION,

Problem can be solved using either $\mathbf{r_1}$ & $\mathbf{v_1}$ or $\mathbf{r_2}$ & $\mathbf{v_2}$ – we will use $\mathbf{r_1}$ & $\mathbf{v_1}$.

Given: $\mathbf{r_1}$ = (0.473265**X** - 0.899215**Y** AU) × 149.597870×10^{9} m/AU
= 7.079944×10^{10}**X** - 1.345206×10^{11}**Y** m
$r_1 = 1.016153 \times 149.597870\times10^{9} = 1.520144\times10^{11}$ m
GM = 1.327124×10^{20} m^3/s^2

From problem 5.4,

$\mathbf{v_1}$ = 28996.2**X** + 15232.7**Y** + 1289.2**Z** m/s

Also,

v = SQRT[$28996.2^2 + 15232.7^2 + 1289.2^2$] = 32,779.2 m/s

Equations (5.21) and (5.22),

$\mathbf{h} = (r_Y v_Z - r_Z v_Y)\mathbf{X} + (r_Z v_X - r_X v_Z)\mathbf{Y} + (r_X v_Y - r_Y v_X)\mathbf{Z}$
$\mathbf{h} = (-1.345206\times10^{11} \times 1289.2 - 0 \times 15232.7)\mathbf{X} + (0 \times 28996.2 - 7.079944\times10^{10} \times 1289.2)\mathbf{Y}$
$+ (7.079944\times10^{10} \times 15232.7 + 1.345206\times10^{11} \times 28996.2)\mathbf{Z}$
$\mathbf{h} = -1.73424\times10^{14}\mathbf{X} - 9.12746\times10^{13}\mathbf{Y} + 4.97905\times10^{15}\mathbf{Z}$

$\mathbf{n} = -h_Y\,\mathbf{X} + h_X\,\mathbf{Y}$
$\mathbf{n} = 9.12746\times10^{13}\mathbf{X} - 1.73424\times10^{14}\mathbf{Y}$

Also,

h = SQRT[$(-1.73424\times10^{14})^2 + (-9.12746\times10^{13})^2 + (4.97905\times10^{15})^2$] = 4.98291×10^{15}

n = SQRT[$(9.12746\times10^{13})^2 + (1.73424\times10^{14})^2$] = 1.95977×10^{14}

Equation (5.23),

$\mathbf{e} = [(v^2 - GM / r) \times \mathbf{r} - (\mathbf{r} \bullet \mathbf{v}) \times \mathbf{v}] / GM$

$v^2 - GM / r = 32779.2^2 - 1.327124\times10^{20} / 1.520144\times10^{11} = 2.01451\times10^8$

$\mathbf{r} \bullet \mathbf{v} = 7.079944\times10^{10} \times 28996.2 - 1.345206\times10^{11} \times 15232.7 + 0 \times 1289.2 = 3.80278\times10^{12}$

$\mathbf{e} = [2.01451\times10^8 \times (7.079944\times10^{10}\mathbf{X} - 1.345206\times10^{11}\mathbf{Y})$
$- 3.80278\times10^{12} \times (28996.2\mathbf{X} + 15232.7\mathbf{Y} + 1289.2\mathbf{Z})] / 1.327124\times10^{20}$
$\mathbf{e} = 0.106639\mathbf{X} - 0.204632\mathbf{Y} - 0.000037\mathbf{Z}$

Equations (5.24) and (5.25),

$a = 1 / (2 / r - v^2 / GM)$
$a = 1 / (2 / 1.520144\times10^{11} - 32779.2^2 / 1.327124\times10^{20})$
$a = 1.97614\times10^{11}$ m

e = SQRT[$0.106639^2 + (-0.204632)^2 + (-0.000037)^2$]
$e = 0.230751$

Equations (5.26) though (5.30),

$\cos i = h_Z / h$
$\cos i = 4.97905\times10^{15} / 4.98291\times10^{15}$
$i = 2.255^o$

$\cos \Omega = n_X / n$
$\cos \Omega = 9.12746\times10^{13} / 1.95977\times10^{14}$
$\Omega = 297.76^o$

$\cos \omega = \mathbf{n} \bullet \mathbf{e} / (n \times e)$
$\cos \omega = (9.12746\times10^{13} \times 0.106639 - 1.73424\times10^{14} \times (-0.204632) + 0 \times (-0.000037))$
$/ (1.95977\times10^{14} \times 0.230751)$
$\omega = 359.77^o$

$\cos \nu_o = \mathbf{e} \bullet \mathbf{r} / (e \times r)$
$\cos \nu_o = (0.106639 \times 7.079944\times10^{10} - 0.204632 \times (-1.345206\times10^{11}) - 0.000037 \times 0)$
$/ (0.230751 \times 1.520144\times10^{11})$
$\nu_o = 0.226^o$

$\cos u_o = \mathbf{n} \bullet \mathbf{r} / (n \times r)$
$u_o = 0$ (launch point = ascending node)

Equations (5.31) and (5.32),

$\Pi = \Omega + \omega$
$\Pi = 297.76 + 359.77$
$\Pi = 297.53^o$

$\ell_o = \Omega + \omega + \nu_o$
$\ell_o = 297.76 + 359.77 + 0.23$
$\ell_o = 297.76^o$

PROBLEM 5.6

For the spacecraft in Problems 5.3 and 5.4, calculate the hyperbolic excess velocity at departure, the injection ΔV, and the zenith angle of the departure asymptote. Injection occurs from an 200 km parking orbit. Earth's velocity vector at departure is 25876.6**X** + 13759.5**Y** m/s.

SOLUTION,

Given: $r_o = (6,378.14 + 200) \times 1,000 = 6,578,140$ m
$\mathbf{r} = 0.473265\mathbf{X} - 0.899215\mathbf{Y}$ AU
$\mathbf{V_P} = 25876.6\mathbf{X} + 13759.5\mathbf{Y}$ m/s

From problem 5.4,

$\mathbf{V_S}$ = 28996.2**X** + 15232.7**Y** + 1289.2**Z** m/s

Equation (5.33),

$\mathbf{V_{S/P}}$ = $(Vs_X - Vp_X)$**X** + $(Vs_Y - Vp_Y)$**Y** + $(Vs_Z - Vp_Z)$**Z**
$\mathbf{V_{S/P}}$ = (28996.2 - 25876.6)**X** + (15232.7 - 13759.5)**Y** + (1289.2 - 0)**Z**
$\mathbf{V_{S/P}}$ = 3119.6**X** + 1473.2**Y** + 1289.2**Z** m/s

Equation (5.34),

$V_{S/P}$ = SQRT[Vs/p_X^2 + Vs/p_Y^2 + Vs/p_Z^2]
$V_{S/P}$ = SQRT[3119.6^2 + 1473.2^2 + 1289.2^2]
$V_{S/P}$ = 3,683.0 m/s

V_∞ = $V_{S/P}$ = 3,683.0 m/s

Equations (5.35) and (5.36),

V_o = SQRT[V_∞^2 + 2 × GM / r_o]
V_o = SQRT[$3,683.0^2$ + 2 × 3.986005×10^{14} / 6,578,140]
V_o = 11,608.4 m/s

ΔV = V_o - SQRT[GM / r_o]
ΔV = 11,608.4 - SQRT[3.986005×10^{14} / 6,578,140]
ΔV = 3,824.1 m/s

Equation (5.37),

r = SQRT[0.473265^2 + $(-0.899215)^2$ + 0^2]
r = 1.01615 AU

γ = arccos[(r_X × v_X + r_Y × v_Y + r_Z × v_Z) / (r × v)]
γ = arccos[0.473265 × 3119.6 - 0.899215 × 1473.2 + 0 × 1289.2) / (1.01615 × 3683.0)]
γ = 87.677°

PROBLEM 5.7

For the spacecraft in Problems 5.3 and 5.4, given a miss distance of +18,500 km at arrival, calculate the hyperbolic excess velocity, impact parameter, and semi-major axis and eccentricity of the hyperbolic approach trajectory. Mars' velocity vector at intercept is -23307.8**X** + 3112.0**Y** + 41.8**Z** m/s.

SOLUTION,

Given: d = 18,500 km / 149.597870×10^6 = 0.000123664 AU
r = 0.066842**X** + 1.561256**Y** + 0.030948**Z** AU
$\mathbf{V_P}$ = -23307.8**X** + 3112.0**Y** + 41.8**Z** m/s

From Basics Constants,

GM of Mars = 4.282831×10^{13} m^3/s^2

From problem 5.4,

$\mathbf{V_S}$ = -21147.0**X** + 3994.5**Y** - 663.3**Z** m/s

Equation (5.33),

$\mathbf{V_{S/P}}$ = $(Vs_X - Vp_X)$**X** + $(Vs_Y - Vp_Y)$**Y** + $(Vs_Z - Vp_Z)$**Z**
$\mathbf{V_{S/P}}$ = (-21147.0 + 23307.8)**X** + (3994.5 - 3112.0)**Y** + (-663.3 - 41.8)**Z**
$\mathbf{V_{S/P}}$ = 2160.8**X** + 882.5**Y** - 705.1**Z** m/s

Equation (5.34),

$V_{S/P}$ = SQRT[Vs/p_X^2 + Vs/p_Y^2 + Vs/p_Z^2]
$V_{S/P}$ = SQRT[2160.8^2 + 882.5^2 + $(-705.1)^2$]
$V_{S/P}$ = 2,438.2 m/s

V_∞ = $V_{S/P}$ = 2,438.2 m/s

Equations (5.38.A) and (5.38.B),

$d_x = -d \times r_y / \text{SQRT}[\ r_x^2 + r_y^2\ ]$
$d_x = -0.000123664 \times 1.561256 / \text{SQRT}[\ 0.066842^2 + 1.561256^2\ ]$
$d_x = -0.000123551$ AU

$d_y = d \times r_x / \text{SQRT}[\ r_x^2 + r_y^2\ ]$
$d_y = 0.000123664 \times 0.066842 / \text{SQRT}[\ 0.066842^2 + 1.561256^2\ ]$
$d_y = 0.0000052896$ AU

Equation (5.39),

$\theta = \arccos[(d_x \times v_x + d_y \times v_y) / (d \times v)]$
$\theta = \arccos[(-0.000123551 \times 2160.8 + 0.0000052896 \times 882.5) / (0.000123664 \times 2{,}438.2)]$
$\theta = 150.451^o$

Equations (5.40) through (5.42),

$b = d \times \sin \theta$
$b = 18{,}500 \times \sin(150.451)$
$b = 9{,}123.6$ km

$a = -GM / V_\infty^2$
$a = -4.282831\times10^{13} / 2{,}438.2^2$
$a = -7.2043\times10^6$ m = -7,204.3 km

$e = \text{SQRT}[\ 1 + b^2 / a^2\ ]$
$e = \text{SQRT}[\ 1 + 9{,}123.6^2 / -7{,}204.3^2\ ]$
$e = 1.6136$

PROBLEM 5.8

As a spacecraft approaches Jupiter, it has a velocity of 9,470 m/s, a flight path angle of 39.2 degrees, and a targeted miss distance of -2,500,000 km. At intercept, Jupiter's velocity is 12,740 m/s with a flight path angle of 2.40 degrees. Calculate the spacecraft's velocity and flight path angle following its swing-by of Jupiter.

Given: V_P = 12,740 m/s
$\phi_P = 2.40^o$
V_{S_i} = 9,470 m/s
$\phi_{S_i} = 39.2^o$
d = -2,500,000 km

From Basics Constants,

GM of Jupiter = 1.26686×10^{17} m^3/s^2

Equations (5.44) and (5.45),

$\mathbf{V_P} = (V_P \times \cos \phi_P)\mathbf{X} + (V_P \times \sin \phi_P)\mathbf{Y}$
$\mathbf{V_P} = (12740 \times \cos(2.40))\mathbf{X} + (12740 \times \sin(2.40))\mathbf{Y}$
$\mathbf{V_P} = 12729\mathbf{X} + 533\mathbf{Y}$ m/s

$\mathbf{V_{S_i}} = (V_{S_i} \times \cos \phi_{S_i})\mathbf{X} + (V_{S_i} \times \sin \phi_{S_i})\mathbf{Y}$
$\mathbf{V_{S_i}} = (9470 \times \cos(39.2))\mathbf{X} + (9470 \times \sin(39.2))\mathbf{Y}$
$\mathbf{V_{S_i}} = 7339\mathbf{X} + 5985\mathbf{Y}$ m/s

Equations (5.46) and (5.47),

$\mathbf{V_{S/P_i}} = ((V_{S_i})_X - V_{P_X})\mathbf{X} + ((V_{S_i})_Y - V_{P_Y})\mathbf{Y}$
$\mathbf{V_{S/P_i}} = (7339 - 12729)\mathbf{X} + (5985 - 533)\mathbf{Y}$
$\mathbf{V_{S/P_i}} = -5390\mathbf{X} + 5452\mathbf{Y}$ m/s

$V_{S/P} = \text{SQRT}[\ (V_{S/P_i})_X^2 + (V_{S/P_i})_Y^2\ ]$
$V_{S/P} = \text{SQRT}[\ (-5390)^2 + 5452^2\ ]$
$V_{S/P}$ = 7,667 m/s

$V_\infty = V_{S/P}$ = 7,667 m/s

Equation (5.48),

$\theta_i = \arctan[\ (V_{S/P_i})_Y\ /\ (V_{S/P_i})_X\ ]$
$\theta_i = \arctan[\ 5452\ /\ -5390\ ]$
$\theta_i = 134.67^o$

Equations (5.40) through (5.42),

$b = d \times \sin\theta$
$b = -2{,}500{,}000 \times \sin(134.67)$
$b = -1{,}777{,}900$ km

$a = -GM\ /\ V_\infty^{\ 2}$
$a = -1.26686{\times}10^{17}\ /\ 7667^2$
$a = -2.1552{\times}10^9$ m = -2,155,200 km

$e = \mathrm{SQRT}[\ 1 + b^2\ /\ a^2\ ]$
$e = \mathrm{SQRT}[\ 1 + (-1{,}777{,}900)^2\ /\ (-2{,}155{,}200)^2\ ]$
$e = 1.2963$

Equation (5.49.B),

$\delta = -2 \times \arcsin(\ 1\ /\ e\ )$
$\delta = -2 \times \arcsin(\ 1\ /\ 1.2963\ )$
$\delta = -100.96^o$

Equation (5.50),

$\theta_f = \theta_i + \delta$
$\theta_f = 134.67 + (-100.96)$
$\theta_f = 33.71^o$

Equation (5.51),

$\mathbf{V_{S/P_f}} = (V_{S/P} \times \cos\theta_f)\mathbf{X} + (V_{S/P} \times \sin\theta_f)\mathbf{Y}$
$\mathbf{V_{S/P_f}} = (7667 \times \cos(33.71))\mathbf{X} + (7667 \times \sin(33.71))\mathbf{Y}$
$\mathbf{V_{S/P_f}} = 6378\mathbf{X} + 4255\mathbf{Y}$ m/s

Equations (5.52) and (5.53),

$\mathbf{V_{S_f}} = ((V_{S/P_f})_X + V_{P_X})\mathbf{X} + ((V_{S/P_f})_Y + V_{P_Y})\mathbf{Y}$
$\mathbf{V_{S_f}} = (6378 + 12729)\mathbf{X} + (4255 + 533)\mathbf{Y}$
$\mathbf{V_{S_f}} = 19107\mathbf{X} + 4788\mathbf{Y}$ m/s

$V_{S_f} = \mathrm{SQRT}[\ (V_{S_f})_X^{\ 2} + (V_{S_f})_Y^{\ 2}\ ]$
$V_{S_f} = \mathrm{SQRT}[\ 19107^2 + 4788^2\ ]$
$V_{S_f} = 19{,}698$ m/s

Equation (5.54),

$\phi_{S_f} = \arctan[\ (V_{S_f})_Y\ /\ (V_{S_f})_X\ ]$
$\phi_{S_f} = \arctan[\ 4788\ /\ 19107\ ]$
$\phi_{S_f} = 14.07^o$