sufficient to develop by bond one-half the allowable stress in such bars, not less than $1 / 16$ of the clear span length, or not less than the depth of the member, whichever is greater. The tension in any bar at any section must be properly devel- oped on each side of the section by hook, lap, or embedment (see Section 906). If preferred, the bar may be bent across the .web at an angle of not less than 15 deg with the longitudinal portion of the bar and be made continuous with the re- inforcement which resists moment of opposite sign.
(a) Of the positive reinforcement in continuous beams not less than one- fourth the area shall extend along the same face of the beam intb the support a distance of 6 in.
(b) In simple beams, or at the freely supported end of continuous beams, at least one-third the required positive reinforcement shall extend along the same face of the beam into the support a distance of 6 in .

## 903-Plain bars in tension

Plain bars in tension shall terminate in standard hooks except that hooks shall not be required on the positive reinforcement at interior supports of continuous members.

## 904-Anchorage of web reinforcement

(a) The ends of bars forming simple U - or multiple stirrups shall be anchored by one of the following methods:

1. By a standard hook, considered as developing 10,000 psi, plus embedment sufficient to develop by bond the remaining stress in the bar at the unit stress specified in Table 305(a). The effective embedded length of a stirrup leg shall be taken as the distance between the middepth of the beam and the tangent of the hook.
2. Welding to longitudinal reinforcement.
3. Bending tightly around the longitudinal reinforcement through at least 180 deg .
4. Ernbedment above or below the middepth of the beam on the compression side, a distance sufficient to develop the stress to which the bar will be sub;ected at a bond stress of not to exceed $0.045 / c^{\prime}$ on plain bars nor $0.10 f c^{\prime}$ on deformed bars, but, in any case, a minimum of 24 bar diameters.
(b) Between the anchored ends, each bend in the continuous portion of a Uor multiple U-stirrup shall be made around a longitudinal bar.
(c) Hooking or bending stirrups around the longitudinal reinforcement shall be considered effective only when these bars are perpendicular to the longitudinal reinforcement
(d) Longitudinal bars bent to act as web reinforcement shall, in a region of tension, be continuous with the longitudinal reinforcement. The tensile stress in each bar shall be fully developed in both the upper and the lower half of the beam as specified in Section 904(a)l or 904 (a)4.
(a) In all cases web reinforcement shall be carried as dose to the compression surface of the beam as fireproofing regulations and the proximity of other steel will permit.

## 905-Anchorage of bars in footing slabs

(a) Plain bars in footing slabs shall be anchored by means of standard hooks. The outer faces of these hooks and the ends of deformed bars shall be not less than 3 in. nor more than 6 in. from the face of the footing.

## 906-Hooks

(a) The terms "hook" or "standard hook" as used herein shall mean either 1. A complete semicircular turn with a radius of bend on the axis of the bar of not less than three and not more than six bar diameters, plus an extension of at least four bar diameters at the free end of the bar, or
2. A 90 -deg bend having a radius of not less than four bar diameters plus an extension of 12 bar diameters, or
3. For tirrup anchorage only, a 135-deg turn witha radius on the axis of the
bar of three diameters plus an extension of at least six bar diameters at
the free end of the bar.
Hooks havinga radius of bend of more than six bar diameters shall be considered merely as extensions to the bars.
(b) No hook shall be assumed to carry a load which would producea tensile s tress ${ }^{\text {r in }}$ the bar greater than $10,000 \mathrm{psi}$.
(c) Hooks shall not be considered effective in adding to the compressive resistance (d) bars.
bay mechanical device capable of developing the strength of the bar with- out damage to the concrete may be used in lieu of a hook. Tests must be presented to show the adequacy of such devices.

## CHAPTER 10-FLAT SLABS WITH SQUARE OR RECTANGULAR PANELS

## 1000-Notation

$A=$ distance in the direction of span from center of support to the inter-. section of the centerline of the slab thickness with the extreme 45-deg diagonal line lying wholly within the concrete section of slab and column or other support, including drop panel, capital and bracket
$b=$ width of section
$c=$ effective support size [see Section 1004(c)]
$\boldsymbol{d}=$ depth from compression face of beam or slab to centroid reinforcement
$f_{c^{\prime}}=$ compressive strength of concrete at age of 28 days unless otherwise . specified
$\boldsymbol{H}=$ story height in feet of the column or support of a flat slab center to. center of slabs
$j=$ ratio of distance between centroids of compression and tension depth $d$
$L=$ span length of a flat slab panel center to center of supports
$M o=$ numerical sum of assumed positive and average negative moments' at the critical design sections of a flat slab panel [sec Section 1004(£)1]
$t=$ thickness of slab in inches at center of panel
$\mathrm{t} 1=$ thickness in inches of slabs without drop panels, or through droppanel, if any
$t_{2}=$ thickness in inches of slabs with drop panels at points beyond drop panel
${ }_{f}=$ shearing unit stress
$V=$ total shear
$\boldsymbol{w}^{\prime}=$ uniformly distributed unit dead and live load
$\boldsymbol{W}=$ total dead and live load on panel
$W_{o}=$ total dead load on panel
$W_{L}=$ total live load on panel, uniformly distributed

## 100 I-Definitions and scope

(a) Flat slab-A concrete slab reinforced in two or more directions, generallyl without beams or girders to transfer the loads to supporting members. Slabs with ${ }^{\prime \prime}$ recesses or pockets made by permanent or removable fillers between reinforcing bars may be considered flat slabs. Slabs with paneled ceilings may be considered,' as flat slabs provided the panel of reduced thickness lies entirely within the area; of intersecting middle strips, and is at least two-thirds the thickness of the re- - mainder of the slab, exclusive of the drop panel, and is not less than 4 in. thick.
(b) Column capital,:_An enlargement of the end of a column designed and $\cdot \cdot$ built to act as an integral unit with the column and flat slab. No portion of the $\backslash$ column capital shall be considered for structural purposes which lies outside of $j$ the largest righoular cone with 90-deg vertex angle that can be included with-)'
in the outlines of the column capital. Where no capital is used, the face of the column shall be considered as the edge of the capital.
(c) Drop panel-The structural portion of a flat slab which is thickened
throughout an area surrounding the column, column capital, or bracket
(d) Panel strips-A flat slab shall be considered as consisting of strips in each direction as follows:

A middle strip one-half panel in width, symmetrical about panel centerline.
A column strip consisting of the two adjacent quarter-panels either side of
the column centerline.

## 1002-Design procedures

(a) Methods of analysis-All flat slab structures shall be designed in accord- ance with a recognized elastic analysis subject to the limitations of Sections 1002 and 1003, except that the empirical method of design given in Section 1004 may be used for the design of flat slabs conforming with the limitations given therein. Flat slabs within the limitations of Section 1004, when designed by elastic analy- sis, may have resulting analytical moments reduced in such proportion that the numeric:il sum of the positive and average negative bending moments used in design procedure need not exceed $M o$ as specified under Section 1004(£).
(b) Critical sections-The slab shall be proportioned for the bending moments prevailing at every section except that the slab need. not be proportioned for a greater negative moment than that prevailing at a distance $A$ from the support centerline.

## (c) Size and tickness of slabs and drop panels

1. Subject to limitations of Section 1002(c)3, the thickness of a flat slab and the size and thickness of the drop panel, where used, shall be such that the compressive stress due to bending at any section, and the shear about the column, column capital, and drop panel shall not exceed the unit stresses allowed in concrete of the quality used. When designed under Section 1004, three-fourths of the width of the strip shall be used as the width of the section in computing compression due to bending, except that on a section through a drop panel, three-fourths of $\bullet$ the width of the drop panel shall be used. Account shall be taken of any recesses which reduce the compressive area.
2. The shearing unit stress on vertical sections which follow a periphery, $b$, at distance, $d$, beyond the edges of the column or column capital and par- allel or concentric with it, shall not exceed the following values for the con- crete when computed by the formula

$$
\begin{gathered}
V \\
--b j d
\end{gathered}
$$

a. $0.03 \mathrm{fc}^{\prime}$ but not more than 100 psi when at least 50 percht of the total
negative reinforcement required for bending in the column strip passes through the periphery.
b. $0.025 J C^{\prime}$ but not more than 85 psi when 25 percent, which is the least value permitted, of the total negative reinforcement required for bending in the column strip passes through the periphery.
c. Proportionate values of the shearing unit stress for intermediate percent--:ages of reinforcement.
3. Where drop panels are used, the shearing unit stress on vertical sections. which lie at a distance, $d$, beyond the edges of the drop panel, and parallel with them, shall not exceed $0.03 / C^{\prime}$ nor 100 psi. At least 50 percent of the total )ff: negative reinforcement required for bending in the column strip shall be ${ }^{\bullet}$;c,;if within the width of strip directly above the drop panel.
4. Slabs with drop panels whose length is at least one-third the parallel _ "1c..., span length and whose projec;tion below the slab is at least one-fourth the slab thickness shall be not less than $L / 40$ nor 4 in. in thickness.

Slabs wi"thout drop panels as described above shall be not less than $L / 36$ nor 5 in. in thickness.
5. For determining reinforcement, the thickness of the drop panel below the slab shall not be assumed to be more than one-fourth of the distance from the edge of the drop panel to the edge of the column capital.
(d) Arrangement of slab reinforcement
I. The spacing of the bars at critical sections shall not exceed two times ., the slab thickness, except for those portions of the slab area which may be of ; cellular or ribbed construction. In the slab over the cellular spaces, reinforce--: ment shall be provided as required by Section 707.
2. In exterior panels, except for bottom bars adequately anchored in the ' drop panel, all positive reinforcement perpendicular to the discontinuous edge shall extend to the edge of the slab and have embedment, straight or ' hooked, of at least 6 in . in spandrel beams, walls, or columns where provided. All negative reinforcement perpendicular to $\cdot$ the discontinuous edge shall be, bent, hooked, or otherwise anchored in spandrel beams, walls, or columns. f
3. The area of reinforcement shall be determined from the bending m ments at the critical sections but shall not be less than $0.0025 b d$ at any section._
4. Required splices in bars may be made wherever convenient, but preferably away from points of maximum stress. The length of any such splice shall be at least 36 bar diameters.
(e) Openings in flat slalis-Openings of any size may be provided in flat slabs if provision is made for the total positive and negative moments and for shear: without exceeding the allowable stresses except that when design is based onSection 1004, the limitations given therein shall not be exceeded.

[^0]
## 1003-Design by elastic analysis

(a) Assumptions-In design by elastic analysis the following assumptions may be used and all sections shall be proportioned for the moments and shears thus obtained.
I. The structure may be considered divided into a number of bents, each consisting of a row of columns or supports and strips of supported slabs, each strip bounded laterally by the centerline of the panel on either side of the centerline of columns or supports. The bents shall be taken longitudinally and transversely of the building.
2. Each such bent may be analyzed in its entirety; or each. floor thereof and the roof-may- be analyzed separately with its adjacent columns as they occur above and below, the columns being assumed fixed at their remote ends. Where slabs are thus analyzed separately, it may be assumed in determining the bending at a given support that the slab is fixed at any support two panels distant therefrom beyond which the slab continues.
3. ':fhe joints between columns and slabs may be considered rigid, and this rigidity (infinite moment of inertia) may be assumed to extend in the slabs from the center of the column to the edge of the capital, and in the column from the top of slab to the bottom of the capital. The change in length of columns and slabs due to direct stress, and deflections due to shear, may be neglected.
4. Where metal column capitals are used, account may be taken of their contributions to stiffness and resistance to bending and shear.
5. The moment of inertia of the slab or column at any cross section may be assumed to be that of the cross section of the concrete. Variation in the moments of inertia of the slabs and columns along their axes shall be taken into account.
6. Where the load to be supported is definitely known, the structure shall be analyzed for that load. Where the live load is variable but-does not exceed three-quarters of the dead load, or the nature of the live load is such that all panels will be loaded simultaneously, the maximum bending may be assumed to occur at all sections under full live load. For other conditions, maximum positive bending near midspan of a panel may be assumed to occur under full live load in the panel and in alternate panels; and maximum negative bend- ing in the slab at a support may be assumed to occur under full live load in the adjacent panels only•.
(b) Critical sections-The critical section for negative bending, in both the column strip and middle strip, may be assumed as not more than the distance $\boldsymbol{A}$ from the center of the column or support and the critical negative moment shall be considered as extending over this distance.
 of each bent may be apportioned between the column strip and middle strip, as

TABLE 1003(c)-DISTRIBUTION BETWEEN COLUMN STRIPS AND MIDDLE STRIPS IN PERCENT OF TOTAL MOMENTS AT CRITICAL SECTIONS OF A PANEL

-Interpolate for intermediate ratios of beam depth to slab thickness.
Note: The total dead and Jive load reaction of a panel adjacent to a marginal beam or wall may be divided between the beam or wall and the parallel half column strip In proportion to their stiffnesses, but the moment provided in the slab shall not be less than given in Table 1003(c).
varied by not more than 10 percent of its value, but their sum for the full pand width shall not be reduced.

## 1004-Design by empirical method

(a) General limitations-Flat slab construction may be designed by the em-• pirical previsions of this section when. $\mathbf{t}$ ey conform to all of the limitations on continuity and dimensions given herein.

1. The construction shall consist of at least three continuous panels in each direction.
2. The ratio of length to width of panels shall not exceed 133 .
3. The grid pattern shall consist of approximately rectangular panels. The $\therefore$. successive span lengths in each direction shall differ by not more than 20 per- 。 cent of the longer span. Within these limitations, columns may be offset a maximum of 10 percent of the span, in direction of the offset, from either axis
4. The calculated lateral force moments from wind or earthquake may be combined with the critical moments as determined by the empirical method, and the lateral force moments shall be distributed between the column and middle strips in the same proportions as specified for the negative momentsin the strips for structures not exceeding 125 ft high with maximum story height not exceeding 12 ft 6 in.

## (b) Columns

1. The minimum dimension of any column shall be 10 in . For columns or
other supports of a flat slab, the required minimum average moment of inertia, $l e$, of the gross concrete section of the columns above and below the slab shall be determined from the following formula, and shall be not less than $1000 \mathrm{in} .{ }^{4}$ If there is rio column above the slab, the $l e$ of the column below shall be twice that given by the formula with a minimum of $1000 \mathrm{in} .^{4}$
$\qquad$
where $t$ eed not be taken greater than $t 1$ or $t z$ as determined in Section 1004 (d), $H$ is the average story height of the columns above and below the between c-erlines of successive columns. u
slab, and $W_{L}$ is the greater value of any two adjacent spans under consider-

## ation.

2. Columns supporting flat slabs designed by the empirical method shall be proportioned for the bending moments developed by unequally loaded panels, or uneven spacing of columns. Such bending moment shall be the maximum value derived from
(WL1 - WDLz) - ${ }^{1}$

L 1 and Lz being lengths of the adjacent spans ( $\mathrm{L} 2=0$ when considering an exterior column) and / is 30 for exterior and 40 for interior columns.

This moment shall be divided between the columns immediately above and below the floor or roof line under consideration in direct proportion to their stiffness and shall be applied without further reduction to the critical sections of the columns.
(c) Determination of " $c$ " (effective support size)

1. Where column capitals are used, the value of,$c$ shall be taken as the diameter of the cone described in Section 1001(6) measured at the bottom $0_{0} £$

## the slab or drop panel.

2. Where a column is without a concrete capital, the dimension $c$ shall be-;" taken as that of the column in the direction considered.
3. Brackets capable of transmitting the negative bending and the shear in : "t: the column strips to the columns without excessive unit stress may be sub- 51.$\}$ j stituted for column capitals at exterior columns. The value of _c for the span,- , \}' where a bracket is used shall be taken as twice the distance from the center $1\{$ of the column to a point where th bracket is $1 \frac{1}{2}$ in. thick, but not more than $\cdot, ; \mid: .$. the thickness of the column plus twice the depth of the bracket.
4. Where a reinforced concrete beam frames into a column without cap- ${ }^{\circ} \mathrm{j}\left\{{ }^{\prime}\right.$ ital or bracket Oil the same side with the beam, for computing bending for:;' strips parallel to the beam, the value of $c$ for the span considered may be $\boldsymbol{J} 1, ;$ taken as the width of the column plus twice the projection of the beam above 1 I.! or below the slab or drop panel.
5. The average of the values of $c$ at the two supports at the ends ofa column strip shall be used to evaluate the slab thickness $t i$ or $t 2$ as prescribed in Section 1004(d).
(d) Slab thickness
6. The slab thickness, span $L$ being the longest side of the panel, shall be at least:
L/36 for slab without drop panels conforming with Section 1004(e), or where a drop panel is omitted at any corner of the panel, but not less than 5 in . nor $t 1$ as given below.
L/40 for slabs with drop panels conforming to Section 1004(e) at all sui' ports, but not less than 4 in: nor t2 as given below.
7. The -total thickness, t 1 , in inches, of slabs without drop panels, through the drop panel if any, shall be at least

$$
\mathrm{ti}=0.028 \mathrm{~L}\left(1-\frac{2 c}{3 L}\right) \quad / \frac{w^{\prime}}{\mathrm{fc}^{\prime} / 2000}
$$

3. The total thickness, $t 2$, in inches, of slabs with drop panels, at points beyond the drop panel shall be at least

$$
\text { 2c) J } \underline{w}^{\prime}
$$

$$
{ }_{\mathrm{t} 2}=0.024 \mathrm{~L}\left(1-\quad{ }_{3} \underline{L} \quad \mathrm{~V}_{/ \mathrm{c}^{\prime} / 2000+1^{*}}\right.
$$

4. Where the exterior supports provide only negligible restraint to the slab, the values of $t i$ and $t 2$ for the exterior panel shall be increased by at least 15 . percent

TABLE 1004(f)-MOMENTS IN FLAT SLAB PANELS IN PERCENTAGES OF Mo

| Strip | Column head | Side <br> support type | End support type | Exterior panel |  |  | Interior panel |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Exterior negative moment | Positive moment | Interior negative moment | Positive moment | Negative moment |
| $\underset{\text { strip }}{\text { Column }}$ | With drop |  | A | 44 |  |  | 20 | 50 |
|  |  |  | B | 36 | 24 | 56 |  |  |
|  |  |  | C | 6 | 36 | 72 |  |  |
|  | Without drop |  | A | 40 |  |  | 22 | 46 |
|  |  |  | B | 32 | 28 | 50 |  |  |
|  |  |  | C | 6 | 40 | 66 |  |  |
| Middle strip | With drop |  | A | 10 | 20 | $17 \cdot$ | 15 | 15• |
|  |  |  | B | 20 |  |  |  |  |
|  |  |  | C | 6 | 26 | $22 \cdot$ |  |  |
|  | Without drop |  | A | 10 | 20 | $18 \cdot$ | 16 | 16• |
|  |  |  | B | 20 |  |  |  |  |
|  |  |  | C | 6 | 28 | $24 \cdot$ |  |  |
| Halfcolumn strip adjacent to marginal beam or wall | With drop | 1 | A | 22 |  |  | 10 | 25 |
|  |  |  | B | 18 | 12 | 28 |  |  |
|  |  |  | C | 3 | 18 | 36 |  |  |
|  |  | 2 | A | 17 |  |  | 8 | 19 |
|  |  |  | B | 14 | 9 | 21 |  |  |
|  |  |  | C | 3 | 14 | 27 |  |  |
|  |  | 3 | A | 11 |  |  | 5 | 13 |
|  |  |  | B | 9 | 6 | 14 |  |  |
|  |  |  | C | 3 | 9 | 18 |  |  |
|  | $\begin{aligned} & \text { With- } \\ & \text { out } \\ & \text { drop } \end{aligned}$ | 1 | A | 20 |  |  | 11 | 23 |
|  |  |  | B | 16 | 14 | 25 |  |  |
|  |  |  | C | 3 | 20 | 33 |  |  |
|  |  | 2 | A | 15 |  |  | 9 | 18 |
|  |  |  | B | 12 | 11 | 19 |  |  |
|  |  |  | C | 3 | 15 | 25 |  |  |
|  |  | 3 | A | 10 |  |  | 6 | 12 |
|  |  |  | B | 8 | 7 | 13 |  |  |
|  |  |  | C | 3 | 10 | 17 |  |  |


-tiicrease negative moments 30 percent ot: tabulated values when middle strip
across support of type B or C. No other values need be Increased. -
Note: For intermediate proportions of total beam depth to slnb thickness. values for loads and moments may be obtained by lnterpolaUon. See also Fig. 1004 (f)a
$\cdot$ In the abovQulas, $t$, and $t$. areininches and $L$ and $c$ are in feeL.


Fig. 1004(f)a-Moments In flat slab panels in percentages of $\mathrm{M}_{0}$ - Wltho ut drops
See Table $1004(f)$ for notes and classification of conditions of end supports and side supports
*Increase negative moments, 30 percent. when middle strip Is continuous across a support of type B or C. No other values need be Increased.


Fig, $\mathbf{1 0 0 4}(\mathbf{f}) \mathrm{b}-$ Moments in flat slab panels in percentage of $\mathbf{M}_{0}$ - With drops
See Table $1004(f)$ for notes and classification of conditions of end supports and side supports

- Increase negative moments 30 percent when middle strip Is continuous across a support of type B or C. No other values need be increased,

TABLE 1004 (g)I-MINIMUM LENGTH OF NEGATIVE REINFORCEMENT

| Strip | Percentage of required reinforcing steel area to be extended at least as indicated | Minimum distance beyond centerline of support to end of straight bar or to $b$ nd point of bent bar• |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Flat slabs without drop panels |  | Flat slabs with drop panels |  |
|  |  | Straight | Bend point where bars bend down and continue as positive reinforcement | Straight | Bend point where bars bend down and continue as positive reinforce- |
| $\begin{aligned} & \text { Column } \\ & \text { strip } \\ & \text { reinforcement } \end{aligned}$ | Not less than 33 percent | 0.30Lt |  |  | ment |
|  | Not less than an additional 34 percent | 0.21 Lt |  | 0.30Lf |  |
|  | Remainder§ | 0.25 L | o.20L | 0.25 L | \| To edge of or drop but a |
| reinfortement | than 50 less percent | 0.2SL |  | $0.2 S L$ |  |
|  | Remainder§ | 0.25 L | $\underline{0.15 \mathrm{~L}}$ | 0.25 L | or $\quad \underline{0.15 \mathrm{~L}}$ |

- At eKterior supports where masonry walls or other construction provide only negligible restraint to the slab, the negative reinforcement need not be carried further than 0.20 L beyond the centerline of such support
tWhere no bent bars are used, the 0.27 L bars may be omitted, provided the 0.30 L bars are at least 50 percent of total required. 0.3 L bars may be omitted provided the 0.33 L bars pro-
vide at least 50 percent of the total required.
$\cdot$ : $\$ :
§Bara may be straight. bent, or any combination of straight and bent bars. All bars are t-o ;<,., be considered straight bars for the end under consideration unless bent at that end and con-: tinued as positive reinforcement

Note: See also Fig. 1004(g).

## Drop panels

1. The maximum total thickness at the drop panel used in computing th negative steel area for the column strip shall be 1.5 t 2 .
2. The side or diameter of the drop panel shall be at least 033 times the span in the parallel direction.
3. The minimum thickness of slabs where drop panels at wall columns arc omitted shall equal $(\mathrm{t} 1+\mathrm{t} 2) / 2$ provided the value of $c$ used in the com: putations complies with Section 1004(c).
(f) Bending moment coefficients
4. The numerical sum of the positive and negative bending moments in_th direction of either side of a rectangular panel shall be assumed as not less, than

TABLE 1004(g)2-MINIMUM LENGTH OF POSITIVE REINFORCEMENT

| Strip | Percentage of required reinforcing steel area to be extmded at least as indicated | Maximum distance from centerline of support to end of straight bar or bend point of bent bar |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $F l a \begin{gathered}\text { slabs without } \\ \text { drop panels }\end{gathered}$ |  | Flat slabs with drop panels |  |
|  |  | Straight | Bend point where bars bend up and continue as negative reinforcement | Straight | Bend point where bars bend up and continue as negative reinforcement |
| Column strip reinforcement | Not less than 33 percent | $0.125 L$ |  | Minimum embedment in drop panel of 16 bar diameters but at least 10 in . |  |
|  | Not less than 50 percent• | 3 in . | 0.25 L |  |  |
|  | Remainder• | 0.125L | $0.25 L$ | Minimum embedment in drop panel of 16 bar diameters but at least 10 in . | or $0.25 L$ |
| Middle | 50 percent | 0.15L |  | 0.15 L |  |
| :elnf rcement 7 | 50 percent• | 3 in . | 0.25L | 3 in . | or 0.25L |

Bars may be straight, bent, or any combination- of straight andbentbars.Afī bars are to be considered straight bars for the end under censideration unless bent at that end and con-
tinued as negative reinforcement.
Note: See also Fig. 1004(g).
3. The average of the values of $c$ at the two supports at the ends of a column strip shall be used to evaluate $M o$ in determining bending in the strip. The average of the values of $M o$, as determined for the two parallel half
in which $F=1.15-c / L$ but not less than 1 .
2. Unless otherwise provided, the bending moments at the critical sections ot the $\mathrm{col}=/ \mathrm{nd}$ middle strips shall be at least those given in Table 1004(f);,:
$M O=0.09$ WLF $\left(1-\frac{2 C}{3 L}\right)^{2}$.
column strips in a panel, shall be used in determining bending in the middle strip.
4. Bending in the middle strips parallel to a discontinuous edge shall he assumed the same as in an interior panel.
5. For design purposes, any of the moments determined from Table 1004(f) may be varied by not more than 10 percent, but the numerical sum of the positive and negative moments in a panel shall be not less than the amount specified.
(g) Length of reinforcement-In addition to the requirements of Section 1002(d), reinforcement shall have the minimum lengths given in Tables 1004(g)1
and $1004(\mathrm{~g}) 2$. Where adjacent spans are unequal, the extension of negative reinforcement on each side of the column centerline as prescribed in Table 1004(g)1 shall be based on the requirements of the longer span.
(h) Openings in flat slabs

1. Openings of any size may be provided in a flat slab in the area common to two intersecting middle strips provided the total positive and negative steel areas required in Section 1004(f) are maintained.

t Exterior support
t Interior support
Exterior support $\phi$.



| MINIMUM LENGTH OF BAR FROM 4, SUPPORT |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MARK | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{c}$ | $\mathbf{d}$ | $\mathbf{e}$ | f |
| 1 ENGTH | Q33L | 0.30 L | 0.27 L | 0.25 L | 0.20 L | 015 L |

Al interior supports, $L$ la longer of adjacent spans.

Fig. 1004(9)-MInl um length of flat slab reinforcement
At exterior supports, where masonry walls or other construction provide only negligible restraint to the slob, the negative reinforcement need not be carried further than 0.20 L beyond tbe centerline of such support. Any combination of straight ond bent bars may be used provided minimum requirements are met
*For bars not terminating In drop ponel use lengths shown for panels without drops.
2. In the area common to two column strips, not more than one-eighth of the width of strip in any span shall be interrupted by openings. The equiv- alent of all bars interrupted shall be provided by extra steel on all sides of the
openings. The shearing unit stresses given in Section $1002(c) 2$ shall not be exceeded.
3. In any area common to one column strip and one middle strip, openings may interrupt one-quarter of the bars in either strip. The equivalent of the bars so interrupted shall be provided by extra steel on all sides of the opening.
4. Any opening larger than described above shall be analyzed by accepted
engineering principles and shall be completely framed as required to carry the loads to the columns.

CHAPTER 11-REINFORCED CONCRETE COLUMNS AND WALLS

## 1100-Notation

$A c=$ area of core of a spirally reinforced column measured to the outside diameter of the spiral; net area of concrete section of a composite column
$A,=$ over-all or gross area of spirally reinforced or tied columns; the total area of the concrete encasement of combination columns
$A r=$ area of the steel or cast-iron core of a composite column; the area of the steel core in a combination column
A. = effective cross-sectional area of reinforcement in compression m columns
$B=$ trial factor (see Section 1109(c) and footnote thereto)
$e=$ eccentricity of the resultant load on a column, measured from the gravity axis
$F a=$ nominal allowable axial unit stress $\left(0225 / \mathrm{c}^{\prime}+f, p_{l l}\right.$ for spiral columns and 0.8 of this value for tied columns
$\mathrm{A}=$ allowable bending unit stress that would be permitted if bending stress only existed
$/ a=$ nominal axial unit stress $=$ axial load divided by area of member, All
$/ b=$ bending unit stress (actual) $=$ bending moment divided by section modulus of member
$f_{c}=$ computed concrete fiber stress in an eccentrically loaded column where the ratio of $e / t$ is greater than $2 / 3$
// = compressive strength of
$/ \mathrm{r}-=$ allowable uni core of the metal ore of a composite column
$f r^{\prime}=$ allowable unit stress on unencased steel columns and pipe columns $f_{\bullet}=$ nominal allowable stress in vertical column reinforcement
$f^{\prime}=$ useful limit
stress of spiral reinforcement
$h=$ unsupported
length of column
$K c=$ radius of
gyration of concrete in
pipe columns
K. = radius of
gyration of a metal
pipe section (in pipe
columns)
$N=$ axial load
applied to reinforced
concrete column
$p^{\prime}=$ ratio of volume
of spiral
reinforcement to the
volume of the
concrete
core (out to out
of spirals) of a
spirally
concrete column
$p_{11}=$ ratio of the
effective cross-
sectional area of
vertical reinforcement
to
the gross area
$P=$ total allowable
axial load on a
column whose length
does not exceed
ten times its
leastional
dimension
$P^{\prime}=$ total allowable
axial load on a long
column
$t=$ over-all depth of rectangular column section, or the diameter of a round column

## 1101-Limiting dimensions

(a) The following sections on reinforced concrete and composite columns, except Section 1107(a), apply to a short column for which the unsupported length
is not greater than ten times the least dimension. When the unsupported length exceeds this value, the design shall be modified as shown in Section 1107(a). Principal columns in buildings shall have a minimum diameter of 12 in ., or in the case of rectangular columns, a minimum thickness of 8 in ., and a minimum gross area of 120 sq in . Posts that are not continuous from story to story shall have a minimum diameter or thickness of 6 in.

## 1102-Unsupported length of columns

(a) For purposes of determining the limiting dimensions of columns, the unsupported length of reinforced concrete columns shall be taken as the clear distance between floor slabs, except that

1. In flat slab construction, it shall be the clear distance between the floor and the lower extremity of the capital, the drop panel or the slab, whichever is least.
2. In beam and slab construction, it shall be the clear distance between the floor and the under side of the deeper beam framing into the column in each direction at the next higher floor level.
3. In columns restrained laterally by struts, it shall be the clear distance between consecutive struts in each vertical plane; provided that to be an adequate support, two such struts shall meet the column at approximately the same level, and the angle between vertical planes through the struts shall not vary more than 15 degrees from a right angle. Such struts shall be of adequate dimensions and anchorage to restrain the column against lateral deflection.
4. In columns restrained laterally by struts or beams, with brackets used at the junction, it shall be the clear distance between the floor and the lower edge of the bracket, provided that the bracket width equals that of the beam or strut and is at least half that of the column.
(b) For rectangular columns, that length shall be considered which produces the greatest ratio of length to depth of section.

## 1103-Spirally reinforced columns

(a) Allowable load-The maximum allowable axial load, $P$, on columns withclosely spaced spirals enclosing a circular concrete core reinforced with vertical bars shall be that given by formula (11).
$P=$ All (0225/c' $+/, p l 1)$ $\qquad$
$\qquad$
Wherein $/,=$ nominal allowable stress in vertical column reinforcement, to betaken at 40 percent of the minimum specification value of the yield point; viz., 16,000 psi for intermediate grade steel and 20,000 psi for rail or hard grade steel. $\bullet$ Jil!f?
(b) Vertical reinforcement-The ratio pll shall not be less than 0.01 nor more -icllt than 0.08 . The minimum number of bars shall be six, and the minimum bar size

[^1]shall be \#5. The center to center spacing of bars within the periphery of the col- umn core shall not be less than $21 / 2$ times the diameter for round bars or three times the side dimension for square bars. The clear spacing between individual bars or between pairs of bars at lapped splices shall not be less than $1 \frac{1}{2}$ in. or $11 / 2$ times the maximum size of the coarse aggregate used. These spacing rules also apply to adjacent pairs of bars at a lapped splice; each pair of lapped bars forming a splice may be in contact, but the minimum clear spacing between one splice and the adjacent splice should be that specified for adjacent single bars.
(c) Splices in vertical reinforcement-Where lapped splices in the column verticals are used, the minimum amount of lap shall be as follows:

1. For deformed bars with concrete having a strength of 3000 psi or more, 20 diameters of bar of intermediate or hard grade steel. For bars of higher yield point, the amount of lap shall be increased one diameter for each 1000 psi by which the allowable stress exceeds $20,000 \mathrm{psi}$. When the concrete strengthsare less than 3000 psi, the amount of lap shall be one-third greater than the values given above.
2. For plain bars, the minimum amount of lap shall be twice that speci- fied for deformed bars.
3. -welded splices or other positive connections may be used instead of lapped splices. Welded splices shall preferably be used in cases where the bar size exceeds \#11. An approved welded splice shall be defined as one in which the bars are butted and welded and that will develop in tension at least the yield point stress of the reinforcing steel used.
4. Where longitudinal bars are offset at a splice, the slope of the inclined portion of the bar with the axis of the column shall not exceed 1 in 6 , and the portions of the bar above and below the offset shall be parallel to the axis of the column. Adequate horizontal support at the offset bends shall be treated as a matter of design, and may be provided by metal ties, spirals or parts of the floor construction. Metal ties or spirals so designed shall be placed near (never more than eight bar diameters from) the point of bend. The hori- zontal thrust to be resisted may be assumed as $11 / 2$ times the horizontal component of the nominal stress in the inclined portion of the bar.

Offset bars shall be bent before they are placed in the forms. No field bending of bars partially embedded in concrete shall be permitted.
(d) Spiral reinforcement-The ratio of spiral reinforcement, $p^{\prime}$, shall not be less than the value given by formula (12).

$$
\begin{equation*}
p^{\prime}=0.45 \quad(\quad-1) \ldots \tag{12}
\end{equation*}
$$

Wherein $f^{\prime}{ }^{\prime}=$ useful limit stress of spiral reinforcement, to be taken as $40,000 \mathrm{psi}$ for hot rolled rods of intermediate grade, $50,000 \mathrm{psi}$ for rods of hard grade, and 60,000 psi for cold drawn wire.
The spiral reinforcement shall consist of evenly spaced continuous spirals held firmly in place and true to line by vertical spacers, using at least two for spirals

20 in . or less in diameter, three for spirals 20 to 30 nn . in diameter, and four for spirals more than 30 in . in diameter or composed of spiral rods $1 / 4 \mathrm{in}$. or larger in
size. The spirals shall be of such size and so assembled as to permit handling and placing without being distorted from the designed dimensions. The material used in spirals shall have a minimum diameter of $1 / 4 \mathrm{in}$. for rolled bars or No. 4 AS\&W gage for drawn wire. Anchorage of spiral reinforcement shall be provided by $\mathrm{I}^{1 / 2}$ extra turns-of spiral rod or wire at each end of the spiral unit. Splices when neces- sary shall be made in spiral rod or wire by welding oi by a lap of $\mathrm{I}^{1 / 2}$ turns. The center to center spacing of the spirals shall not exceed one-sixth of the core diameter. The clear spacing between spirals shall not exceed 3 in . nor be less than $l 3 / 4 \mathrm{in}$. or $I^{1 / 2}$ times the maximum size of coarse aggregate used. The reinforcing spiral shall extend from the floor level in any story or from the top of the footing in the basement, to the level of the lowest horizontal reinforcement in the slab, drop panel or beam above. In a column with a capital, it shall extend to a plane at which the diameter or width of the capital is twice that of the column.
(e) Protection of reinforcement-The column spiral reinforcement shall be protected everywhere by a covering of concrete cast monolithically with the core, for which the thickness shall not be less than $\mathrm{I}^{1} / 2 \mathrm{in}$. nor less than $\quad \mathrm{I} 1 / 2$ times the maximum size of the coarse aggregate, nor shall it be less than required by the fire protection and weathering provisions of Section 507.
(!) Isolated column with multiple spirals-In case two or more interlocking spirals are used in a column, the outer boundary of the column shall be taken as a rectangle the sides of which are outside the extreme limits of the spiral afa distance equal to the requirements of Section 1103(e).
(g) . Limits of section of column built monolithically with wall-For a spiral column built monolithically with a concrete wall or pier, the outer boundaryof the column section shall be taken either as a circle at least $\mathrm{I}^{1} / 2 \mathrm{in}$. outside the column spiral or as a square or rectangle of which the sides are at least $\mathrm{I}^{1} / 2$ in. outside the spiral or spirals.
(h) Equivalent circular columns-As an exception to the general procedure of utilizing the full gross area of the column section, it shall be permissibleto design a circular column and to build it with a square, octagonal, or other shaped section of the same least lateral dimension. In such case, the allowable load, the gross area considered, and the required percentages of reinforcement shall be taken as those of the circular column.

## 1104-Tied columns

(4) Allowable load-The maximum allowable axial load on columns rein-'? 't: forced. with longitudinal bars and separate lateral tie.s shall e 8 ? percent of
that given by formula (11). The ratio, $P u$, to be consld re m tl d columns : shall not be less than 0.01 nor more than 0.04 . The long1tud10al remforcement shall consist of at least four bars, of minimum bar size of .J.t.:.. Splices. in rein-
forcing bars shall be made as described in Section 1103(c). The spacing require- ments for vertical reinforcement in Section 1103(6) shall also apply for all tied columns.
0. Combined axial and bending load-Foe tied columns which are designed to withstand combined axial and bending stresses, the limiting steel ratio of. 0.04 may be increased to 0.08 . The amount of steel spliced by lapping shall not exceed a steel ratio of 0.04 in any $3-\mathrm{ft}$ length of column. The size of the column designed under this provision shall in no case be less than that required to withstand the axial load alone with a steel ratio of 0.04 .
(1) Lateral ties-Lateral ties shall be at least $1 / 4 \mathrm{in}$. in diameter and shall be spaced apart not over 16 bar diameters, 48 tie diameters, or the least dimension of the column. When there are more than four vertical bars, additional ties shall be provided so that every longitudinal bar is held firmly in its designed position and has lateral support equivalent to that provided by a 90-deg corner of a tie.
(4) Limits of column section-In a tied column which for architectural reasons has a larger cross section than required by considerations of loading, a reduced effective area, $A u$, not less than one-half of the total area may be used in applying the provisions of Section 1104(a).

## 11OS-Composite columns

(a) Allowable load-The allowable load on a composite column, consistingof a structural steel or cast iron column thoroughly encased in concrete rein- forced with both longitudinal and spiral reinforcement, shall not exceed that gi en by formula (13).

$$
\begin{equation*}
P=0.225 A d e^{\prime}+f . A .+f, A \tag{13}
\end{equation*}
$$

Wherein $/$, $=$ allowable unit stress in metal core, not to exceed $16,000 \mathrm{psi}$ for a steel core; or $10,000 \mathrm{psi}$ for a cast-iron core.
(b) Details of metal core and rein/ orcement-The cross-sectional area of the metal core shall not exceed 20 percent of the gross area of the column. If a hollow metal core is used it shall be filled with concrete. The amounts of longi- tudinal and spiral reinforcement and the requirements as to spacing of bars, details of splices and thickness of protective shell outside the spiral shall con- form to the limiting values specified in Section 1103(6), (c), (d), and (e). A clearance of at least 3 in. shall be maintained between the spiral and the metal core at all points except that when the core consists of a structural steel H-column, the minimum clearance may be reduced to 2 in .
(c) Splices and connections of metal cores-Metal cores in composite columns shall be accurately milled at splices and positive provision shall be made for alignment of one core above another. At the column base, provision shall be made to transfer the load to the footing at safe unit stresses in accordance with Section 305(a). The base of the metal section shall be designed to transfer the load from the entire composite column to the footing, or it may be designed to transfer the load from the metal section only, provided it is so placed in
the pier or pedestal as to leave ample section of conc ete above the base for the
transfer of load from the reinforced concrete section of the column by means of bond on the vertical reinforcement and by direct compression on the concrete. Transfer of loads to the metal core shall be provided for by the use of bearing members such as billets, brackets or other positive connections; these shall be provided at the top of the metal core and at intermediate floor levels where required. The column as a whole shall satisfy the requirements of formula
(13) at any point; in addition to this, the reinforced concrete portion shall be designed to carry, in accordance with formula (11), all floor loads brought onto the column at levels between the metal brackets or connections. In applying formula (11), the value of $A u$ shall be interpreted as the area of the concrete section outside the metal core, and the allowable load on the reinforced concrete section shall be further limited to $035 \mathrm{fc} \mathrm{c}^{\prime \prime} \mathrm{A}_{9}$. Ample section of concrete and continuity of reinforcement shall be provided at the junction with beams or girders.
(d) Allowable load on metal core only-The metal cores of composite columns shall be designed to carry safely any construction or other loads to be placed upon them prior to their encasement in concrete.

## 1106-Combination columns

(a) Steel columns encased in concrete-The all wable load on a structural steel column which is encased in concrete at least $2 \frac{1}{2}$ in. thick over all metal (except rivet heads) reinforced as hereinafter specified, shall be computed by formula (14).

$$
\begin{equation*}
\mathrm{P} \quad A^{\prime} ; f r\left[1+\underline{10} \hat{o}^{A} \underline{A} \underline{A_{-}}\right] \tag{14}
\end{equation*}
$$

The concrete used shall develop a compressive strength, $f C^{\prime}$, of at least 2000 psi at 28 days. The concrete shall be reinforced by the equivalent of welded wire mesh having wires of No. 10 AS\&W gage, the wires encircling the column being spaced not more than 4 in . apart and those parallel to the column axis not more than 8 in. apart. This mesh shall extend entirely around the column at a distance of 1 in . inside the outer concrete surface and shall be lapspliced at least 40 wire diameters and wired at the splice. Special bracketsshall be used to receive the entire floor load at each floor level. The steel columnshall be designed to carry safely any construction or other loads to be placedupon it prior to its encasement in concrete.
(b) .Pipe columns-The allowable load on columns consisting of steel pip.e,
filled with concrete shall be determined by formula (15).

$$
\begin{equation*}
P=025 / c^{\prime} \quad\left(\mathrm{I}-\quad 0.000025 \underline{h^{\prime}}\right) \tag{15}
\end{equation*}
$$

The value of $/, '$ shall be given by formula (16) when the pipe has a yield strength of at least $33,000 \mathrm{psi}$, and an $h / K$. ratio equal to or less than 120 .

$$
/ r^{\prime}=17,000-0.485 \underline{h^{\prime}}:
$$

## 1107-Long columns

(a) The maximum allowable load, $P^{\prime}$, on axially loaded reinforced concrete or composite columns having an unsupported length, $h$, greater than ten times the least lateral dimension, $t$, shall be given by formula (17).

$$
\begin{equation*}
P^{\prime}=P[13-0.03 h / t] \tag{17}
\end{equation*}
$$

where $P$ is the allowable axial load on a short column as given by Sections 1103, 1104 , and 1105.

The maximum allowable load, $P^{\prime}$, on eccentrically loaded columns in which $h / t$ exceeds 10 shall also be given by formula (17), in which $P$ is the allowable eccentrically applied load ori a short column as determined by the provisions of Section 1109. In long columns subjected to definite bending stresses, as deter- mined in Section 1108 , the ratio $h / t$ shall not exceed 20.

## 11OS-Bending moments in columns

(a) The bending moments in the columns of all reinforced concrete structures shall be determined on the basis of loading conditions and restraint and shall be provided for in the design. When the stiffness and strength of the columns are utilized to reduce moments in beams, girders, or slabs, as in the case of rigid frames, or in other forms of continuous construction wherein column moments are unavoidable, they shall be provided for in the design. In building frames, particular attention shall be given to the effect of unbalanced floor loads on both exterior and interior columns and of eccentric loading due to other causes. In computing moments in columns, the far ends may be con- sidered fixed. Columns shall be designed to resist the axial forces from loads on all floors, plus the maximum bending due to loads on a single adjacent span of the. floor under consideration.

Resistance to bending moments at any floor level shall be provided by distributing the moment between the columns immediately above and below the given floor in proportion to their relative stiffnesses and conditions of restraint.

## 1109-Columns subiected to axial load and bending-

(a) Members subject to an axial load and bending in one principal plane, but with the ratio of eccentricity to depth e/t no greater than $2 / 3$, shall be so proportioned that

$$
\begin{equation*}
\underline{l \mathbf{a}}+\frac{b}{F, .} \quad \mathrm{A} \text { does not exceed umty. } \tag{18}
\end{equation*}
$$

(b) When bending exists on both of the principal axes, formula (18) becomes
where $f b z$ and $/ b u$ are the bending moment components about the $x$ and $y$ principal
axes divided by the section modulus of the transformed section relative to the respective axes, provided that the ratio $e / t$ is no greater than $2 / 3$ in either direction.
(c) In designing a column subject to both axial load and bending, the pre- liminary selection of the column may be made by use of an equivalent axial load given by formula (20).

$$
\begin{equation*}
P=N \quad 1+\beta e)^{*} \tag{20}
\end{equation*}
$$

When bending exists on both of the principal axes, the quantity $B e / t$ is the numerical sum of the Be/t quantities in the two directions.
(d) For columns in which the load, $N$, has an eccentricity," $e$, greater than $2 / 3$ the column depth, $t$, the determination of the fiber stress $f_{c}$ shall be made by use of recognized theory for cracked sections, based on the assumption that the concrete does not resist tension. In such cases the modular ratio for the compressive reinforcement shall be assumed as double the value given in Section 601; however the stress in the compressive reinforcement when calculated on this basis, shall not be greater than the allowable stress in tension. The maximum com- bined compressive stress in the concrete shall not exceed $0.45 / c^{\prime}$. For such cases the tensile steel stress shall also be investigated.

## 1110-Wind and earthquake stresses

(a) When the allowable stress in columns is modified to provide for com- bined axial load and bending, and the stress due to wind or earthquake loads is also added, the total shall still come within the allowable values specified for wind or earthquake loads in Section 603(c).

## 1111-Reinforced concrete walls

(a) The allowable stresses in reinforced concrete bearing walls with minimum reinforcement as required by Section $1111(\mathrm{~h})$, shall be $025 / c^{\prime}$ for walls having; a ratio of height to thickness of ten or less, and shall be reduced proportionally- to $0.15 / \mathrm{c}^{\prime}$ for walls having a ratio of height to thickness of 25 . When the rein-• forcement; in bearing walls is designed, placed, and anchored in position as for tied colwnns, the allowable stresses shall be on the basis of Section 1104, as for columns. In the case of concentrated loads, the length of the wall to be con- sidered as effective for each shall not exceed the center to center distance between loads, nor shall it exceed the width of the bearing plus four times the wall thickness. The ratio pll shall not exceed 0.04 .

[^2](b) Walls shall be designed for any lateral or other pressure to which they an;: subjected. Proper provision shall be made for eccentric loads and wind stresses. In such designs the allowable stresses shall be as given in Section 305(a) and 603(c).
(c) Panel and enclosure walls of reinforced concrete shall have a thick- ness of not less than 4 in . and not less than $1 / 30$ the distance between the support- ing or enclosing members.
(d) Reinforced concrete bearing walls of buildings shall be not less than 6 in . thick for the uppermost 15 ft of their height; and for each successive 25 ft downward, or fraction thereof, the minimum thickness shall be increased
1 in . Reinforced concrete bearing walls of twer-story dwellings may be 6 in . thick throughout their height.
(e) Exterior basement walls, foundation walls, fire walls, and party walls shall not be less than 8 in. thick whether reinforced or not.
(/) Reinforced concrete bearing walls shall have a thickness of at least $1 / 25$ of the unsupported height or width, whichever is the shorter.
(g) Reinforced concrete walls shall be anchored to the floors, or to the columns, pilasters, buttresses, and intersecting walls with reinforcement at least equivalent to \#3 bars 12 in . on centers, for each layer of wall reinforcement.
(h) ) The area of the horizontal reinforcement of reinforced concrete walls shall be not less than 0.0025 and that of the vertical reinforcement not less than 0.0015 times the area of the reinforced section of the wall if of bars, and not less than three-fourths as much if of welded wire fabric. The wire of the welded fabric shall be of not less than No. 10 AS\&W gage. Walls more than 10 in. thick, except for basement walls, shall have the reinforcement for each direction placed in two layers parallel with the faces of the wall. One layer consisting of not less than onehalf and not more than ewer-thirds the total required shall be placed p.ot less than 2 in. nor more than one-third the thickness of the wall from the exterior surface. The other layer, comprising the balance of the required reinforcement, shall be placed not less than $3 / 4 \mathrm{in}$. and not more than one-third the thick- ness of the wall from the interior surface. Bars, if used, shall not be less than \#3 bars, nor shall they be spaced more than 18 in. on centers. Welded wire reinforce- ment for walls shall be in flat sheet form.
(i) In addition to the minimum as prescribed in Section 1111(h) there shall be not less than two \#5 bars around all window or door openings. Such bars shall extend at least 24 in . beyond the corner of the openings.
(i) Where reinforced concrete bearing walls consist of studs or ribs tied to- gether by reinforced concrete members at each floor level, the studs may be con- sidered as columns, but the restrictions as to minimum diameter or thickness of columns shall not apply.
(k) The limits of thicknesses and quantity of reinforcement may be waived where structural analysis shows adequate strength and stability•

## CHAPTER 12-FOOTINGS

## 1201-Scope

(a) The requirements prescribed in Sections 1202 to 1209 apply only to isolated footings.

## 1202-Loads and reactions

(a) Footings shall be proportioned to sustain the applied loads and induced reactions without exceeding the allowable stresses as prescribed in Sections 305 and 306, and as further provided in Sections 1205, 1206, and 1207.
(b) In cases where the footing is concentrically loaded and the member being supported does not transmit any moment to the footing, computations for moments and shears shall be based on an upward reaction assumed to be uniformly distributed per unit area or per pile and a downward applied load assumed to be uniformly distributed over the area of the footing covered by the column, pedestal,
wall, or metallic column base.
(c) In cases where the footing is eccentrically loaded and/or the member being supported transmits a moment to the footing, proper allowance shall be made for any variation that may exist in the intensities of reaction and applied load consistent with the magnitude of the applied load and the amount of its actual or virtual eccentricity.
(d) In the case of footings on piles, computations for moments and shears may be based on the assumption that the reaction from any pile is concentrated at the center of the pile.

## 1203-Sloped or stepped footings

(a) In sloped or stepped footings, the angle of slope or depth and location of
steps shall be such that the allowable stresses are not exceeded at any section.
(b) In sloped or stepped footings, the effective cross section in compression shall be limited by the area above the neutral plane.
(c) Sloped or stepped footings shall be cast as a unit.

## 1204-Bending moment

(a) The external moment on any section shall be determined by passing through the section a vertical plane which extends completely across the footing, and computing the moment of the forces acting over the entire area of the footing on one side of said plane.
(b) The greatest bending moment to be used in the design of an isolated footing shall be the moment computed in the manner prescribed in Section 1204(a) at sections located as follows:

1. At the face of the column, pedestal or wall, for footings supporting a concrete column, pedestal or wall.

[^3]2. Halfway between the middle and the edge of the wall, for footings under masonry walls.
3. Halfway between the face of the column or pedestal and the edge of the metallic base, for footings under metallic bases.
(c) The width resisting compression at any section shall be assumed as the entire width of the top of the footing at the section under consideration.
(d) In one-way reinforced footings, the total tc;nsile reinforcement at any section shall provide a moment of resistance at least equal to the moment computed in the manner prescribed in Section 1204(a); and the reinforcement thus determined shall be distributed uniformly across the full width of the section.
(e) In two-way reinforced footings, the total tensile reinforcement at any section shall provide a moment of resistance at least equal to 85 percent of the moment computed in the manner prescribed in Section 1204(a); and thci total reinforcement thus determined shall be distributed across the corresponding resisting
section in the manner prescribed for the square footings in Section 1204(f), and for rectangular footings in Section 1204(g).
(/) In two-way square footings, the reinforcement extending in each direction shall be distributed uniformly across the full width of the footing.
(g) In two-way rectangular footings, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing. In the case of the reinforcement in the short direction, that portion determined by formula (21) shall be uniformly distributed across a band-width $(B)$ centered with respect to the centerline of the column or pedestal and having a width equal to the length of the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portions of the footing.
\[

$$
\begin{gather*}
\underline{\text { Reinforcement in band-width }(B)}-\frac{\underline{2}}{}  \tag{21}\\
\text { Total reinforcement in short,direction }
\end{gather*}
$$(\mathrm{S}+1)
\]

In formula (21), $S$ is the ratio of the long side to the short side of the footing.

## 1205-Shear and bond

(a) The critical section for shear to be used as a measure of diagonal tension shall be assumed as a vertical section obtained by passing a series of vertical planes through the footing, each of which is parallel to a corresponding face of the column, pedestal, or wall and located a distance therefrom equal to the depth $d$ for footings on soil, and one-half the depth $d$ for footings on piles.
(b) Each face of the critical section as defined in Section 1205(a) shall be considered as resisting an external shear equal to the load on an area bounded by said face of the critical section for shear, two diagonal lines drawn from the column or pedestal corners and making 45-deg angles with the principal axes of the foot- ing, and that p rtion of the correspq ding edge or edges of the footing intercepted between the two diagonals.
(c) Critical sections for bond shall be assumed at the same planes as those prescribed for bending moment in Section 1204(6); also at all other vertical planes where changes of section or of reinforcement occur.
(d) Computation for shear to be used as a measure of bond shall be based on the same section and loading as prescribed for bending moment in Section 120-f(a). .
(e) The total tensile reinforcement at any section shall provide a bond resis- tance at least equal to the bond requirement as computed from the following per- centages of the external shear at the section:

1. In one-way reinforced footings, 100 percent.
2. In two-way reinforced footings, 85 percent.
(/) In computing the external shear on any section through a footing sup . ported on piles, the entire reaction from any pile whose center is located 6 in . or more outside the section shall be assumed as producing shear on the section; the reaction from any pile whose .center is located 6 in . or more inside the section shall be assumed as producing no shear on the section. For intermediate positions of the pile center, the portion of the pile reaction to be assumed as producing shear on the section shall be based on straight-line interpolation between full value at 6 in. outside the section and zero value at 6 in . inside the section.
(g) For allowable shearing stresses, see Section 305 and 809
(/z) For allowable bond stresses, see Section 305 and 901 to 905.

## 1206-Transfer of stress at base of column

(a) The stress in the longitudinal reinforcement of a column or pedestal shall be transferred to its supporting pedestal or footing either by extending the longi- tudinal bars into the supporting member, or by dowels.
(b) In case the transfer of stress in the reinforcement is accomplished by ex- tension of the longitudinal bars, they shall extend into the supporting member the distance required to transfer to the concrete, by allowable bond stress, their full working value.
(c) In cases where dowels are used, their total sectional area shall be not less than the sectional area of the longitudinal reinforcement in the member from which the stress is being transferred. In no case shall the number of dowels per . member be less than four and the diameter of the dowels shall not exceed the diameter of the column bars by more than $1 / 4 \mathrm{in}$.
(d) Dowels shall extend up into the column or pedestal a distance at least : ;'j\{ equal to that required for lap of longitudinal column bars (see Section 1103) and $<? f 1$ down into the supporting pedestal or footing the distance required to transfer to: the concrete, by allowable bond stress, the full working value of the dowel [see Section 906(c) ].
. (e) The compressive stress in the concrete at the base of a coll!ffin or pedestal shall be considered as being transferred by bearing to the top of the supporting pedestal or footing. The unit compressive stress on the loaded area shall not ex-
ceed. the bearing stress allowable for the quality of concrete in the supporting member as limited by the ratio of the loaded area to the supporting area.
(/) For allowable bearing stresses see Table 305(a), Section 305.
(g) In sloped r stepped footings, the supporting area for bearing may be taken as the top horizontal surface of the footing, or assumed as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base the area actually loaded, and having side slopes of one vertical to two horizontal.

## 1207-Pedestals and footings (plain concrete)

(a) The allowable compressive unit stress on the gross area of a concentrically loaded pedestal shall not exceed $0.25 / \mathrm{c}^{\prime}$. Where this stress is exceeded, rein- forcement sha_ll be provided and the member designed as a reinforced concrete column.
(b) The depth and width of a pedestal or footing of plain concrete shall be such that the tension in the concrete shall not exceed $0.03 / \mathrm{c}^{\prime}$, and the average shear- ing stress shall not exceed 0.02/c' taken on sections as prescribed in Section 1204 and 1205 for reinforced concrete footings.

## 1208-Footings supporting round columns

(a) In computing the stresses in footings which support a round or octagonal concrete column or pedestal, the "face" of the column or pedestal shall be taken as the side of a square having an area equal to the area enclosed within the perimeter of the column or pedestal.

## 1209-Minimum _ edge-thickness

(a) In reinforced concrete footings, the thickness above the reinforcement at the edge shall be not less than 6 in. for footings on soil, nor less than 12 in. for footings on piles.
(b) In plain concrete footings, the thickness at the edge shall be not less than 8 in . for footings on soil, nor less than 14 in . above the tops of the piles for foot- ings on piles.

## CHAPTER 13-PRECAST CONCRETE

## 1301-Scope

(a) All provisions of this code shall apply to precast concrete except for the specific variations given in this chapter.

## 1302-Aggregates

(a) The maximum size of aggregate shall not be larger than one-third of the least dimension of the member.

## 1303-Concrete protection for reinforcement

(a) At surfaces not exposed to weather, all reinforcement shall be protected
by concrete equal to the nominal diameter of bars but not less than $1 / 4 \mathrm{in}$.

## 1304-Details

(a) '.All details of jointing, inserts, and anchors shall be shown on the drawings.

## 1305-Curing

(a) Curing by high-pressure steam, steam vapor, or other accepted processes may be employed to accelerate the hardening of the concrete and to reduce the time of curing required by Section 405 provided that the compressive strength
of the concrete at the time of use be at least equal to the specified design strength.

## 1306-Identification and marking

(a) All precast concrete members shall be plainly marked to indicate the top of the member and its location and orientation in the structure. Identification marks shall be duplicated on the placing plans.

## 1307-Transportation, storage, and erection

(a) Units shall be so stored, transported, and placed that they will not be over-stressed or damaged.
(b) Precast concrete units shall be adequately braced and supported during. erection to insure proper alignment and safety and such bracing or support shall _ be maintained until there are adequate permanent connections.

## APPENDIX

## ABSTRACT OF REPORT OF ACI-ASCE JOINT COMMITTEE ON ULTIMATE

## STRENGTH DESIGN*

## A600-Notation

> (a) Loads and load factors
> $u \quad=$ ultimate strength capacity of section
> B $\quad=$ effect of basic load consisting of dead load plus volume change due to creep, elastic action,
> _ shrinkage, and temperature
> $L \quad=\quad$ effect of live load plus impact
> W = effect of wind load
> $E \quad=$ effect of earthquake forces
(b) Cross-sectional constants .

A, $=$ gross area of section
A. $=$ area of tensile reinforcement
$A_{;} \quad=$ area of compressive reinforcement
$A_{,_{1}}=$ steel area to develop compressive strength of cv::rhanging flange in T-section:i, defined by Eq. (A5)
$A, \quad=$ total area of longitudinal rein:(orcement
b = width of a rectangular section or over-all width of flange in Tsections
$b^{\prime} \quad=$ width of web in T-section
$D \quad=$ total diameter of circular sec tion
D, $=$ diameter of circle circumscribing the longitudinal reinforcement in circular section
$d=$ distance from extreme compres sive fiber to centroid of tensile reinforcement
$d^{\prime} \quad=$ distance from extreme compressive fiber to centroid of compressive reinforcement
$e=$ eccentricity of axial load measHeqeffromcthrefentroid of ten-

$e .^{\prime}=$ eccentricity of load $P$. meas- ured
$I .^{\prime} \quad=\quad \begin{aligned} & \text { from plastic centroid of section } \\ & \text { 28-day cyliinder } \\ & \text { strength }\end{aligned}$
f. $\quad=$ stress 'in tensile reinforcement at ultimate strength
I, = yield point of reinforcement, not to be taken greater than

60,000 psi
k. $\quad$ defined by $k . \Phi$ distance from extreme compressive fiber to neutral axis at ultimate strength
$k$, $\quad=$ ratio of average compressive stress to $0.85 f$.'
k. $\quad$ ratio of distance between ex treme compressive fiber and resultant of compressive stresse3 tQ distance between extreme .fiber and neutral axis
= f,/0.85 I. ${ }^{\prime}$
$m^{\prime}=m-1$
$p=A, / b d$
$P^{\prime}=A, ' / b d$
Pt $\quad$ = $A, l / b^{\prime} d$
= $A,, ; A$.
三 ppr; $;$ b! $\cdot d$ also total depth of rectangular
-For full report see Proceedings, ASCE, V. 81, Paper No. 809, Oct. 1955. Also •see ACI 30UIINAL, Jan. 1956, Proc. V. 52, pp. 505-524.

## A601-Definitions and scope

(a) This appendix presents recommendations for design of reinforced concrete structures by ultimate strength theories. The term "ultimate strength design" indicates a method of design based on the ultimate strength of a re- inforced concrete cross section in simple bending, combined bending and axial load on the basis of inelastic action.
(b) These recommendations are confined to design of sections. It is assumed that external moments and forces acting in a structure will be determined by the theory of elastic frames. With the specified load factors, stresses under service loads will remain within safe limits.

## A602-General requirements

(a) The American Concrete Institute "Building Code Requirements for Reinforced Concrete" shall apply to the design of members by ultimate strength ., theory except where otherwise provided in this appendix.
(b) Analysis of indeterminate structures, _such as continuous girders and ;!,' arches, shall be based on the theory of elastic frames. For buildings of usual types of construction, spans, and story heights, approximate methods such as $\cdot, f$. the use of coefficients recommended in the ACI Building Code are acceptable for determination of moments and shears.
(c) Bending moments in compression members shall be taken into account "Ji
in the calculation of their required strength.
(d) In arches the effect of shortening of the arch axis, temperature, shrink- : $\cdot \mathbf{1}$ : age, and secondary moments due to deflection -shall be considered.
(e) Attention shall be given to the deflection of members, inch1ding the , effect of creep, especially whenever the net ratio of reinforcement which is $t>^{\prime} . t^{\prime}$ defined as $\left(p-p^{\prime}\right)$ or ( $p^{\circ}-p r$ ) in any section of a flexural member exceeds 0.18/c'//11-
(/) Controlled concrete should be used and shall meet the following require- ments. The quality of concrete shall be such that not. more than one test in ten shall have an average strength less than the strength assumed in the design/ and the average of any three consecutive tests shall not be less than the as- sumed design strength. Each test shall consist of not less than three sfandard cyI'm ers.

## A603-Assumptions

Ultimat: strength esign of reinforced concrete members shall be based on the followmg assumpaons:
(a) Plane sections normal to the axis remain plane after bending.
(b) Tensil_e strength in concrete is neglec ed in sections subj ct to bend g.
(c) At ultimate strength, stresses and strams are not proportlonal. The dia- , $\mathrm{li}^{\prime} \ll / i c$ gram of compressive concrete stress distribution may be assumed a rectangle, '\%"'t : trapezoid, parabola, or : $1{ }^{\circ} \mathrm{Y}$ other shap_e which results in ultimate strength in reasonable agreement with comprehensive tests.

(d) Ma,cimum fiber stress in concrete docs not exceed $0.85 / \mathrm{c}^{\prime}$.
(e) Stress in tensile and compressive reinforcement at ultimate load shall nuc be assumed greater than the yield point or $60,000 \mathrm{psi}$, whichever is smaller.

## A604-Load factors

(a) Members shall be so proportioned that an ample factor of safety is pro- vided against an increase in live load beyond that assumed in design; and strains under service loads should not be so large as to cause excessive cracking. These criteria are satisfied by the following formulas:

1. For structures in which, due to location or proportions, the effects of wind and earthquake loading can be properly neglected:

$U=1.2 \mathrm{~B}+2.4 L$

$U=K(B+L)$
2. For structures in which wind loading must be considered:
$U=12 B+2.4 L+0.6 W$
$U=12 \mathrm{~B}+0.6 L+2.4 W$.
$U=K(B+L+1 / 2 W)$
$U=K(B+1 / 2 L+W)$ $\qquad$ -
3. For those structures in which earthquake loading must be considered, substitute $E$ for $W$ in the preceding equations.
(b) The load factor, $K$, shall be taken equal to 2 for columns and members subjected to combined bending and axial load, and equal to 1.8 for beams and girders subjec $t$ to bending only.

## A605-Rectangular beams with tensile reinforcement only

(a) The ultimate capacity of an under-reinforced section is approached when the tensile steel begins to yield. The steel shall then be assumed to elon-gate plastically at its yield point stress, thereby reducing the concrete area in compression until crushing takes place. The ultimate strength so obtained is con- trolled by tension.
(b) The computed ultimate moment shall not exceed that given by:
$M, .=b d^{2} f c^{\prime} q(1-059 q)$.
in which $q=p / 11 / f c^{\prime}$.
. (c) In Eq. (Al), the maximum ratio of reinforcement shall be so limited that $p$ does not exceed:

$$
p=0.40 \mathrm{fc} / / 11 \ldots . .
$$

! .
. strength in excess of 5000 psi .

## A606..;..Rectangular beams with compressive reinforcement

(a) The ultimate moment shall not exceed that computed by: $M u=(A .-A) f v d.\left(1-0.59\left(p-p^{\prime}\right) f v / f c^{\prime}\right]+A .^{\prime} f v^{\prime \prime \prime}(d-a)$ $\qquad$
(b) In Eq. (A3), the maximum ratio of reinforcement shall be so limited that ( $p-p^{\prime}$ ) does not exceed the values given by Eq. (A2).

## A607-T-sections

(a) When the flange thickness equals or exceeds the depth to the neutral axis given by $k u d=130 q d$ or the depth of the equivalent stress block ( 1.18 $\_q d$ ), the section may be designed by Eq. (Al), with $q$ computed as for a rectangular beam with a width equal to the over-all flange width.
(b) When the flange thickness is less than kud or less than the depth of the equivalent stress block, the ultimate moment shall not exceed that computed by: $M u=(A s-A s t) f v d\left[\mathrm{I}-0.59(p w-P t)!11 / f c^{\prime}\right]+$ Astfy (d-0.5t) (A4)
in which $A, t$, the steel area necessary to develop the compressive strength of the overhanging portions of the flange, is:

$$
\begin{equation*}
A s t=0.85\left(\mathrm{~b}-b^{\prime}\right) t f c^{\prime} / f v \tag{A5}
\end{equation*}
$$

(c) In Eq. (A4), the maximum ratio of reinforcement shall be so limited tha ( $p u$ - $P t$ ) does not exceed the values given by Eq. (A2).

## A608-Concentrically loaded short columns

(a) All members subject to axial loads shall be designed for at least a mum eccentricity:

For spirally reinforced columns, the minimum eccentricity measured from centroidal axis of column shall be 0.05 times the depth of the column sectio
For tied columns, the minimum eccentricity shall be 0.10 times the depth. ',j
(b) The maximum load capacity for concentric loads for use in Eq. (Al is given by the formula:

$$
P a=0.85 f \mathrm{fc}^{\prime}(A / J-A s t)+\text { Ast fy }
$$

$\qquad$

## A609-Bending and axial load: Rectangular section

(a) The ultimate strength of members subject to combined bending axial load shall be computed from the equations of equilibrium, which. w
$k u$ is less than unity may be expressed as follows:

$$
P u=0.85 \mathrm{fc}^{\prime} b d k u k l+A,,^{\prime} f v^{\prime \prime \prime}-A a f \bullet
$$

(b) It shall be assumed that the maximum concrete strain is limited to 0.003 so that the section is controlled by tension when:

$$
\begin{equation*}
\underline{P u<} P b=0.85 \mathrm{kl} \quad\left(\underset{90,000+!11}{\left.\frac{90,000}{}\right) / c^{\prime} b d+A . ' f, I '-A, / l l}\right. \tag{A8}
\end{equation*}
$$

$\qquad$
$k i$ being limited as for Eq. (A7a) and (A7b). The section is controlled by compression when $P u$ exceeds A.
(c) When the section is controlled by tension, the ultimate strength shall not exceed that computed by:

$$
\begin{aligned}
P u= & \frac{0.85 f c^{\prime} b d\left\{p^{\prime} m^{\prime}-p m+\frac{(1-e / d)}{}\right.}{} \begin{aligned}
& \left.+\mathrm{y}(1-\mathrm{e} / \mathrm{d})^{2}+2\left[(e / d)\left(p m-p^{\prime} m^{\prime}\right)+p^{\prime} m^{\prime}\left(/-\quad \mathrm{d}^{\prime} / \mathrm{d}\right)\right]\right\} \ldots(\mathrm{A} 9)
\end{aligned}
\end{aligned}
$$

(d) When the section is controlled by compression, a linear relationship between axial load and moment may be assumed for: values of $P u$ between that given as A by Eq. (A8) and the concentric ultimate strength $P a$ given by Eq (A6). For this range the ultimate strength may be computed by either Eq. (AlO) or (All):

## A610-Bending and axial load: Circular sedions

(a) The ultimate strength of circular sections subject to combined bend- ing and axial load may be computed on the basis of the equations of equi- librium taking into account inelastic deformations, or by the empirical formulas Eq. (A12) and (AB):

When tension controls:

$$
\left.\left(0 ; \underline{e}^{\prime} \quad 038\right)\right] \quad \ldots \quad(\mathrm{A} 12)
$$

$$
p, .=0.85 / c^{\prime} D_{[ }^{\prime} \mathbf{V} D^{1 / D^{0.85 \mathrm{e}^{\prime}}-038^{)^{2}}+\underline{p, m D}, \underline{25 D-}}
$$

When compression controls: $A / J f c^{\prime}$

$$
\begin{align*}
& P u=\overline{3} e^{\prime} \quad+-\overline{9.6 D e^{\prime}} \quad-\quad  \tag{A13}\\
& \left.\bar{D}^{+I} \quad \overline{(0 .} \overline{8} \bar{D} \overline{+} \overline{0} . \overline{67 D} .\right)^{1} \geq 18
\end{align*}
$$

$$
\begin{align*}
& P_{l l}=\quad P a \\
& 1+((\mathrm{Po} / \mathrm{A})-\mathrm{I}] e^{\prime} / e,{ }^{\prime} \\
& \text { A. } / 11 \\
& b t f c^{\prime} \\
& P, \underset{e^{\prime} /\left(d^{\prime}-d^{\prime}\right)+\mathbf{1} / 2}{ }+{ }_{\left(3 t e^{\prime} / d^{\prime}\right)+1.18} \tag{All}
\end{align*}
$$

$\operatorname{InEq}(\mathrm{A} 7 \mathrm{a})$ and $(\mathrm{A} 7 \mathrm{~b}), k 2 / k 1$ shall not be taken as less than 0.5 , and $\mathrm{k} 1 \mathrm{~s}!\mathrm{i} \bullet$
not be taken greater than 0.85 for $f c^{\prime} \leq 5000 \mathrm{psi}$. The coefficient 0.85 is $\mathrm{tO},$, ; reduced at the rate of 0.05 per 1000 psi concrete strength in excess of 5000
$\cdot$ Correction for concrete area displaced by compressive reinforcement may be made subtracting $0.85 \mathrm{f}-$-<rom $/ \mathrm{r}$ in this term only

## A611-Long members

(a) When the unsupported length, $L$, of an axially loaded member is greater than 15 times its least lateral dimension, the maximum axial load, $P$,,', shall be determined by one of the following methods:

1. $P, .{ }^{\prime}=P a(l .6-0.04 L / t)$ $\qquad$ ':' $\qquad$
$\bullet$ correction for concrete area displaced by compressive reinforcement may $\mathbf{b}$ de bysubtracting $0.85 / \bullet^{\prime}$ from $/$, in this term only.
2. A stability determination for $P,$. ' may be made with an apparent reduced modulus of elasticity used for sustained loads, such as the method recommended in the report of ACI Committee 312, "Plain and Reinforced Concrete Arches" \{ACI journal, May 1951, Proc. V.47, p. 681).

For such discussion of this standard as may develop please see Part 2, December 1956 Journal. In Proceedings V. 52 discussion immediately follows the June 1956 Journal pages.


[^0]:    (/) Design of columns
    I. All olumns supporting flat slabs shall be designed as provided Chapte the additional requirements of this chapter.

[^1]:    *Nominal allowable stresses for reinforcement of higher yield point may be established at 40 percent of the yield point stress. but not more than $30,000 \mathrm{psi}$. when the properties of such reinforcing steels have been definitely specified by standards of ASTM designation. If this Is done, the lengthosplice required by Section 1103(c) shall be increased accordingly.

[^2]:    *For trial computations $B$ may be taken from 3 to $31 / 2$ for rectangular tied columns. the lower value being used for columns with the minimum amount of reinforcement. Similarly for circular spiral columns. the value of $B$ from 5 to 6 mav be used.

[^3]:    -The committee Is not prepared at this time to make recommendations for combined footings -those supporting more than one column or wall.

