Transactions on the Built Environment vol 6, © 1994 WIT Press, www.witpress.com, ISSN 1743-3509

# Evaluation of rail track impedance and capacitance using the electromagnetic transients program

R.J. Hill[a] & S.R. McKay[b]

[a] *School of Electonic and Electrical Engineering, University of Bath, Claverton Down, Bath, BA2 7AY, UK*

[b] *Rail Sysyems Analytics, 50 Haig Avenue, Toronto, Ontario M1N 2W1, Canada*

## Abstract

Time-domain electric circuit simulation packages designed for power systems applications may also be used for electric railway traction networks provided the track and traction line impedance and admittance are known from basic track geometry and material properties. This paper illustrates the use of the EMTP line and cable constants routines to obtain the track impedance and capacitance with special reference to running rail circuits. By comparison with practical measurements, it is shown that with care, the method is suitable for track impedance, but less precise for running rail circuit capacitance.

## 1 Introduction

Time-domain electrical circuit simulation can be a versatile tool for the design of electric railway power, traction and signalling systems, having the advantage that the programs are already validated and documented, user support is available, existing library modules may be used and circuit variables altered quickly during simulation. Simulation models of traction systems must include an accurate representation of the rail track, including parallel power cables and signalling lines, in addition to circuit models of vehicle drives and track-connected power substation and signalling equipment. An appropriate track model is that of a coupled transmission line, characterised by the line self and mutual impedances and admittances. For a complete and accurate solution, a rigorous track model addressing the frequency dependency of the line equivalent component data must be used. This paper reviews the use of the electromagnetic transients program (EMTP) to obtain the track phase impedance and capacitance by use of its line and cable constants routines.

The use of accurate track data is necessary to solve a number of problems in traction power system simulation. Examples include the effects of DC and fundamental frequency traction power transmission, low and audio frequency track signalling system design, and evaluation of electromagnetic interference between power and signalling/communications systems including the effects of conductive surfaces near the track such as iron tube tunnel walls. Previous studies of traction systems using circuit simulation have usually involved single-frequency excitation, examples being earth current studies on AC railways [1] and the prediction of higher harmonic resonant conditions in the catenary-rail network [2,3]. However, the use of frequency-dependent track data is necessary to obtain accurate simulation results of harmonic spectra and to assess effects due to impedance nonlinearities. The methodology proposed in this paper leads to the generation of frequency-dependent track impedance data, and it is shown that with care the models can be fairly accurate. However, there are shortcomings when the technique is used for modelling track admittance: the derived capacitance is frequency independent, and the method does not consider conductance, which is important in running rail circuits due to significant ohmic contact between the running rails and conducting ground.

## 2 Coupled transmission line rail track model

Rail track is essentially a coupled transmission line with parallel rails, power lines and signalling cables laid on the surface of a weakly conducting earth [4]. Its electrical equivalent circuit representation is a distributed network with frequency and current dependent self and mutual impedance and admittance. This frequency and current dependency arises from material electrical properties, environmental variations and effects such as rail grinding and dynamic track condition.

Values for the track self and mutual impedances and admittances can be obtained by experimentation, theoretical modelling or electromagnetic field modelling [5]. There is, however, considerable difficulty in establishing accurate track parameters for given conditions, and only the track differential-mode parameters may be easily verified by practical site measurements.

## 3 Track modelling with the Electromagnetic Transients Program

### 3.1 EMTP

EMTP is a circuit simulation package which was originally designed specifically for power transmission systems. It contains subroutines for modelling generators, loads and overhead lines, handling nonlinearities and dealing with non-sinusoidal signals. It works by setting up a nodal model of the electrical network under study, solving the circuit admittance matrix using the difference equation technique at each timestep to obtain time-domain dynamic operation. As the model has a fixed number of nodes, dynamic changes in the circuit topology must be made through the insertion of switches.

Within EMTP, lumped component circuits must be specified with branches comprising resistors, inductors, capacitors and current sources. The circuit node voltages are treated as state variables and the various branch currents at each node are summed. To deal with reactive components and transients, the solution process requires knowledge of the circuit state at previous timesteps.

The incorporation of transmission lines within a simulation circuit model draws on the principle of travelling waves and the finite time that waves take in passing along the line. The line acts as a delay function, with an impressed voltage at the sending end travelling at the propagation velocity until it is reflected at the far end. The transmission line equivalent circuit within the program is that of sending-end and receiving-end Norton current sources with appropriate current values to account for the signal delay along the line, and with parallel conductances relating to the line characteristic impedance and the line losses.

Transactions on the Built Environment vol 6, © 1994 WIT Press, www.witpress.com, ISSN 1743-3509

### 3.2 Rail track modelling with EMTP

The existing EMTP transmission line model functions at constant frequency. Because the rail track line parameters vary with frequency, the main difficulty in simulating track is in obtaining a model that is sufficiently accurate over the entire frequency range of interest. Existing models of frequency dependency [6] deal with the integration of transient effects and do not address the issue of harmonic frequency response. Moreover, most EMTP applications involve overhead or ground-coupled lines whose geometry and material properties are known fairly precisely - in these overhead lines, shunt losses are negligible and series losses can be approximated accurately by the skin effect in the line conductors. The problem of accurate modelling of the line series and shunt losses is particularly important for rail track because of the ohmic contact of the running rails with the conductive ground.

The value of using a full transmission line model for the track and traction line is apparent when the physical nature of the system is considered. Even for a single track with no electrification or parallel signal cables, the concept of a two-conductor transmission line over a conductive earth surface must be used to account for both differential mode and common mode effects [7].

### 3.3 The line and cable constants routines

EMTP provides utility programs for the evaluation of transmission line phase data at each frequency of interest using analytical models such as the skin effect and Carson's equations. The cable constants routine is intended to model buried cables in the earth with or without a screening enclosure, and the line constants routine is intended to model overhead lines. Using the programs requires knowledge of geometrical and material data of the system under study and gives the line phase inductance, resistance and capacitance matrices.

## 4 Modelling rail track impedance

### 4.1 The track phase impedance matrix

The phase impedance matrix is given by

$$[\mathbf{Z}] = \begin{bmatrix} z_{11} & z_{12} & z_{13} & \cdots\cdots & z_{1n} \\ z_{21} & z_{22} & z_{23} & \cdots\cdots & z_{2n} \\ z_{31} & z_{32} & z_{33} & \cdots\cdots & z_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ z_{1n} & \cdots & \cdots & \cdots & z_{nn} \end{bmatrix} \quad (1)$$

where the diagonal elements $z_{ii}$ represent the line self-impedances and the off-diagonal elements $z_{ij}$ represent the mutual impedance per unit length of line i with respect to line j with earth return.

In rail track, the elements are difficult to measure individually. However, loop impedance measurements between any two lines can be used as a partial check of the accuracy of the impedance matrix coefficients. For example, the running rail loop impedance is

$$Z_{loop12} = 2(Z_{11} - Z_{12}) \quad (2)$$

For any set of conductors, the absolute values of $z_{ij}$ will vary according to the ground reference chosen. Remote ground is usually taken as the absolute reference potential (but is physically inaccessible for measurements).

### 4.2 Track modelling with EMTP cable constants routine

The cable constants routine is based on analysis of a circular multiconductor cable where the cores need not be symmetric. Input data required are the system geometry and material properties (resistivity, permeability and permittivity of each region) and the DC resistance of each conductor. The program calculates the skin effect radius from the inner and outer radii of the conductors.

For (n-1) cores, the routine produces [(n-1)×(n-1)] impedance and capacitance matrices if the exterior screen is earthed. Alternatively, the screen can be treated as a separate conductor with respect to remote earth, in which case the matrices are [n×n].

**Tube tunnel track** The cable constants routine was applied to two running rails with a grounded tunnel with the dimensions and material data shown in Fig. 1. The tube was of London Underground (LU) dimensions with bullhead rail with properties taken from reference [8]. The routine was run to find the running rail track impedance matrix coefficients at 4 kHz for three conditions:

Running rails only, with tunnel wall grounded:

$$[\mathbf{R}] = \begin{bmatrix} 4.56 & 0.0798 \\ 0.0798 & 4.56 \end{bmatrix} \ \Omega/\text{km} \qquad [\mathbf{L}] = \begin{bmatrix} 0.717 & 0.020 \\ 0.020 & 0.717 \end{bmatrix} \ \text{mH/km}$$

Running rails only, with tunnel wall not grounded:

$$[\mathbf{R}] = \begin{bmatrix} 9.67 & 5.19 & 5.11 \\ 5.19 & 9.67 & 5.11 \\ 5.11 & 5.11 & 5.11 \end{bmatrix} \ \Omega/\text{km} \qquad [\mathbf{L}] = \begin{bmatrix} 1.33 & 0.630 & 0.609 \\ 0.630 & 1.32 & 0.609 \\ 0.609 & 0.609 & 0.609 \end{bmatrix} \ \text{mH/km}$$

Running and power rails (4-rails in total), tunnel wall not grounded:

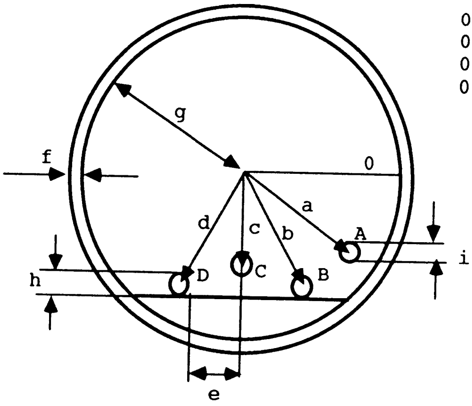
$$[\mathbf{R}] \ (\Omega/\text{km}) = \qquad\qquad [\mathbf{L}] \ (\text{mH/km}) =$$

$$\begin{bmatrix} 9.03 & 5.00 & 4.88 & 4.86 & 4.80 \\ 5.00 & 1.77 & 5.00 & 4.94 & 4.80 \\ 4.88 & 5.00 & 9.03 & 5.18 & 4.80 \\ 4.86 & 4.94 & 5.18 & 1.81 & 4.80 \\ 4.80 & 4.80 & 4.80 & 4.80 & 4.80 \end{bmatrix} \qquad \begin{bmatrix} 1.22 & 0.615 & 0.548 & 0.538 & 0.528 \\ 0.615 & 1.75 & 0.615 & 0.566 & 0.528 \\ 0.548 & 0.615 & 1.22 & 0.656 & 0.528 \\ 0.538 & 0.566 & 0.656 & 1.61 & 0.528 \\ 0.528 & 0.528 & 0.528 & 0.528 & 0.528 \end{bmatrix}$$

Although the absolute values of the matrix coefficients differ in the above cases, the loop impedance represents differences and remains the same in all cases. Thus the running rail loop resistance and inductance are approximately 8.96 Ω/km and 1.39 mH/km respectively.

The output from cable constants was compared with practical LU track measurements of loop impedance. These measurements are regarded as accurate only to about 10% due to ignoring the current dependency in rail impedance. The results, shown in Fig. 2, indicate good agreement between the model and practical experimentation.

The cable constants routine was also used to obtain the track self and mutual capacitances. With a relative permittivity of 10, the running rail self and mutual capacitances are constant with frequency at 0.208 and -0.0068 $\mu$F/km, giving an effective rail-rail capacitance value of (0.208/2 + 0.0068) = 0.11 $\mu$F/km.



0-A: 46.5°
0-B: 60°
0-C: 90°
0-D: 120°

a: 1.645 m
b: 1.528 m
c: 1.257 m
d: 1.528 m
e: 0.6925 m
f: 0.1 m
g: 1.898 m
h: 0.046 m
i: 0.03 m

Tube, running rail $\mu$: 0.5 mH/m
Tube, running rail $\mu r$: 379.9
Power rail $\mu$: 2 mH/m
Power rail $\mu r$: 1592

Steel conductivity: 4.44E+6 S/m
Steel resistivity: 0.2252 $\mu$ohm-m
Earth conductivity: 0.007-0.077 S/m
Earth resistivity: 12.987 $\mu$ohm-m

*Fig. 1: Tube tunnel track geometry and material properties*

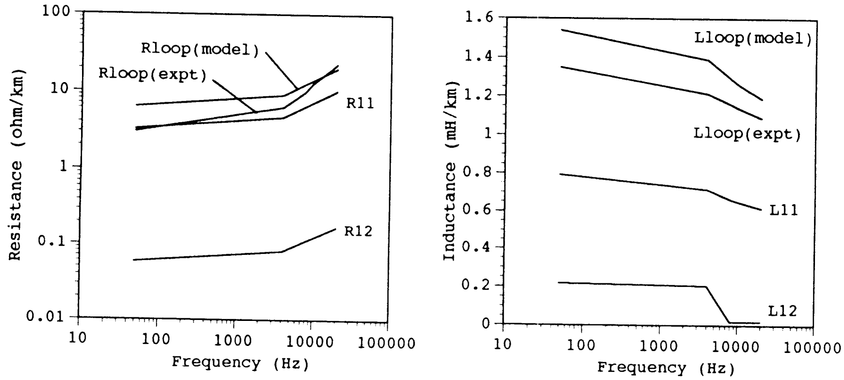


*Fig. 2: EMTP cable constants routine model: tube tunnel track self, mutual and loop impedance and comparison with experimental measurements*

**Open track** The cable constants routine can also be used for open track by omitting the cable sheath in the model. The program can be used to determine the effect of parametric variations, such as rail permeability, ground conductivity and geometrical dimensions. Studies were made to determine the effect of rail radius and the distance of the rails with respect to the ground surface using a two-running-rail and earth model. Fig. 3 shows the self and mutual impedances obtained as functions of frequency, and Fig. 4 compares the loop impedances with experimental measurements. The cases are with the rails buried 0.2 m beneath the ground surface (25 mm rail radius, $\rho_{ground}$ = 12.98 Ω-m, $\mu_{rail}$ = 397.9 and $\rho_{rail}$ = 225.2 $10^{-9}$ Ω-m) and the rails located at the ground surface, 150 mm over the surface and 300 mm over the surface (the latter all with 46 mm rail radius, $\rho_{(ground)}$ = 1000 Ω-m, $\mu_{(rail)}$ = 65 and $\rho_{(rail)}$ = 225.2 $10^{-9}$ Ω-m).

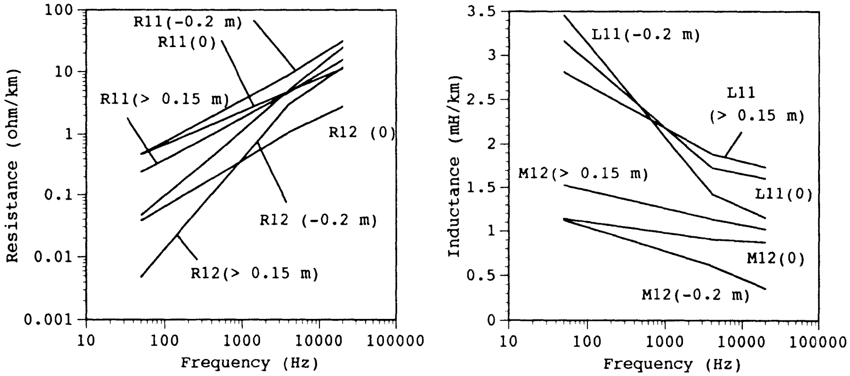


*Fig. 3: EMTP cable constants routine model: open section track self and mutual impedance matrix elements, with rails at variable height above ground*

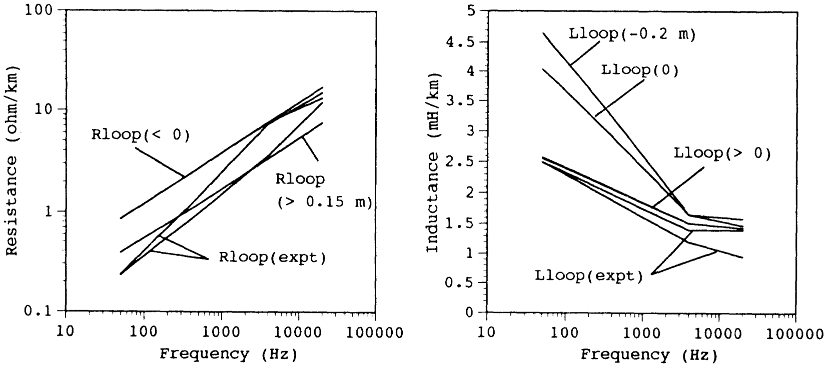


*Fig. 4: EMTP cable constants routine model: open section track loop impedance, with rails at variable height and comparison with experimental results*

The results for the capacitance matrix elements are independent of frequency and are given in Table 1. Since it is known that practical values of rail-rail capacitance range from 0.1 to 100 $\mu$F/km, the method clearly does not give reasonable results.

Transactions on the Built Environment vol 6, © 1994 WIT Press, www.witpress.com, ISSN 1743-3509

*Table 1: Track capacitance using EMTP cable constants*

| Rail-ground distance (m) | -0.20 | 0 | +0.15 | +0.30 |
|---|---|---|---|---|
| $C_{11}$ ($\mu$F/km) | 2780 | 0.0106 | 0.0114 | 0.0104 |
| $C_{12}$ ($\mu$F/km) | 0 | -0.0059 | -0.00574 | - 0.00574 |
| $C_{rail\text{-}rail}$ ($\mu$F/km) | 1390 | 0.0112 | 0.0114 | 0.0109 |

### 4.3 Track modelling with EMTP line constants routine

The line constants routine is used for modelling overhead power lines and will thus gives good results for an overhead catenary electrification system. It requires the DC resistance of each conductor as input, from which it models the frequency behaviour through the skin effect. Investigations were made to find the effect of the height above ground and DC resistance on running rail self and mutual impedances with material properties as $\rho_{(ground)} = 1000\ \Omega$-m, $\rho_{(rail)} = 0.23\ \mu\Omega$-m and $A_{(rail)} = 0.0075\ m^2$. The results are given in Table 2. Line constants does not give good results for resistance, although the inductance results are adequate.

*Table 2: Running rail self and mutual impedances using EMTP line constants*

| Frequency Hz | $R_{11}$ Ω/km | $R_{12}$ Ω/km | $R_{loop}$ Ω/km | $L_{11}$ mH/km | $L_{12}$ mH/km | $L_{loop}$ mH/km | $R_{DC}$ mΩ/km | Height mm |
|---|---|---|---|---|---|---|---|---|
| 50 | 0.0819 | 0.0493 | 0.065 | 7.55 | 4.68 | 1.827 | 30 | 400 |
| 50 | 0.0913 | 0.0493 | 0.084 | 2.402 | 1.492 | 1.820 | 40 | 400 |
| 50 | 0.1312 | 0.0493 | 0.164 | 1.967 | 1.057 | 1.820 | 30 | 150 |
| 2000 | 2.139 | 1.97 | 0.338 | 2.033 | 0.112 | 1.821 | 40 | 400 |

The track capacitance was also investigated, Table 3 showing results which are the correct order of magnitude.

*Table 3: Running rail self and mutual capacitances using EMTP line constants*

| Height mm | $C_{11}$ $\mu$F/km | $C_{12}$ $\mu$F/km | $C_{rail\text{-}rail}$ $\mu$F/km |
|---|---|---|---|
| 150 | 21.66 | -0.1293 | 11.03 |
| 400 | 15.69 | -0.1442 | 8.21 |

## 5 Conclusions

This study has demonstrated the possibilities and limitations of using the EMTP cable and line constants routines to model the frequency-dependent values of rail track impedance and capacitance. Although the routines are not precise and have oversimplified frequency characteristics, they can give reasonable results if care is taken in defining the track material properties and geometry.

Preliminary results have shown that the cable constants routine is adequate for track impedance modelling, whereas the line constants routine shows more promise for track capacitance. Parametric studies have been reported which show that the exact position of the rails over the ground surface does not appear to be as critical for impedance as for capacitance.

The difficult physical structure of rail track means that care must be taken in constructing track models with the routines. Problems arise because the rails are ferromagnetic and are laid on the ground surface; the ground conductivity model necessary for the EMTP subroutines must be precise enough to deal with ground eddy current effects beneath the track.

Problems with modelling rail track admittance arise from the fact that the rails have appreciable self and mutual conductance since they are laid on the surface of, and in contact with, the earth. The EMTP routines do not consider conductance, which for the running rails cannot be neglected. Development of appropriate routines for track conductance is thus necessary.

## 6 References

1. Bozkaya H., Shao Z.Y., Allen J., Mellitt B. & Goodman C.J. A comparison between auto transformers and booster transformers for AC traction supply. *Proceedings of International Conference on Electric Railway Systems for a New Century*, London, 22-25 September 1987, 114-118.
2. Holtz J. & Klein H.-J. The propagation of harmonic currents generated by inverter-fed locomotives in the distributed overhead supply system. *IEEE Transactions on Power Electronics*, April 1989, **4**, (2), 168-174.
3. Coles P.C., Fracchia M., Hill R.J., Pozzobon P. & Sciutto G. Modelling and simulation of supply current interference in traction systems arising from multi-level converters in high-power locomotive drives. *Proceedings 5th European Conference on Power Electronics and Applications*, Brighton, 13-16 September 1993, **7**, 24-28.
4. Hill R.J. & Carpenter D.C. In-situ determination of rail track electrical impedance and admittance matrix elements. *IEEE Transactions on Instrumentation and Measurement*, October 1992, **41**, (5), 666-673.
5. Hill R.J. & Carpenter D.C. Rail track transmission line distributed impedance and admittance: theoretical modeling and experimental results. *IEEE Transactions on Vehicular Technology*, **42**, (2), May 1993, 225-241.
6. Marti J.R. Accurate modelling of frequency-dependent transmission lines in electromagnetic transient simulations. *IEEE Transactions on Power Apparatus and Systems*, January 1982, **PAS-101**, (1), 147-155.
7. Hill R.J., Carpenter D.C., Berova M. & Meecham D.G.J. Using the circuit simulation package EMTDC for audio frequency track circuit design. *Proceedings Institution of Railway Signal Engineers International Conference: ASPECT 91*, London, October 1991, 456-469.
8. Hill R.J. & Carpenter D.C. Determination of rail internal impedance for electric railway traction system simulation. *IEE Proceedings, Part B: Electric Power Applications*,November 1991, **138**, (6), 311-321.