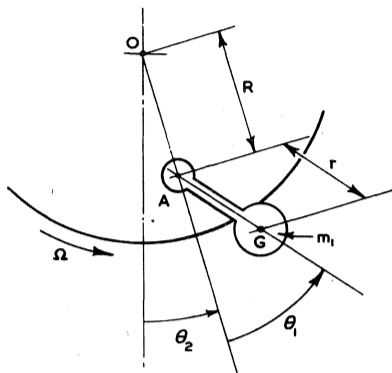# VARIOUS TYPES OF CENTRIFUGAL PENDULUM DETUNERS

Notation: Ω = phase velocity of shaft rotation, $T_{(n)}$ = excitation torque (nth order critical), $m_1 = W_1/g$ = mass of pendulum, J_G = moment of inertia of m_1 about its centre of gravity G, n = order number of critical speed [vib./rev.], θ_2 = vibration amplitude of carrier (of inertia J_2). With resonance tuning: $\theta_2 = 0$, for all pendulums considered.

	Simple ('Mathematical') pendulum	'Bifilar' pendulum	Roller-type pendulum	Ring-type pendulum	Composite-type pendulum (roller and ring)
	$\overline{OA}=R,\ \overline{AG}=r$	$\overline{OA}=R$ (where A=effective point of suspension) $\overline{AG}=r=D-d$	$\overline{OA}=R,\ r=(D-d)/2$	$\overline{OA}=R,\ r=(D-d)/2$	$\overline{OA}=R,\ r\cong D-d$

m_1 = mass of roller; D = diameter of cavity; d = diameter of roller; A = centre of cavity

m_1 = mass of ring; D = inner diameter of ring; d = pin diameter; A = centre of pin
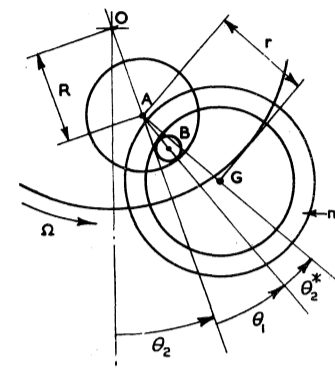
m_1 = mass of ring; D = inner diameter of ring and diameter of cavity; d = roller diameter

d = diameter of rollers; D = diameter of holes

Tuning ratio:

Simple: $n^2=\dfrac{R}{r}$

Bifilar: $n^2=\dfrac{R}{r}$

Roller: $n^2=\dfrac{R}{r}\times\dfrac{1}{\left[1+\dfrac{4J_G}{m_1 d^2}\right]}$

Ring: $n^2=\dfrac{R}{r}\times\dfrac{1}{\left[1+\dfrac{4J_G}{m_1 D^2}\right]}$

Composite: Two modes of vibration:
(I) 'Rigid-pendulum' motion (with $\overline{ABG}$ = straight line, hence $\theta_2^*=0$):
$$n_{(I)}^2=R/r$$
$$\theta_1=\frac{T_{(n)}}{m_1(R+r)\,rn^2\Omega^2}$$

(II) 'Anti-phase pendulums' motion:
$$n_{(II)}^2=\left(1+\frac{R}{r}\right)\left(\frac{m_1 D^2}{4J_G}\right)$$
$$\theta_1=-T_{(n)}\Big/\left[2J_G\frac{r}{D}n^2\Omega^2\right]$$
$$\theta_2^*=-2\theta_1$$

Note. Usually, $n_{(II)}<n_{(I)}$; in practice only $n_{(I)}$ is used because in many cases $n_{(II)}$ is unstable, with excessive pendulum amplitudes.

Vibration amplitude of pendulum (at resonance tuning):

Simple: $\theta_1=\dfrac{T_{(n)}}{m_1(R+r)\,rn^2\Omega^2}$

Bifilar: $\theta_1=\dfrac{T_{(n)}}{m_1(R+r)\,rn^2\Omega^2}$

Roller: $\theta_1=\dfrac{T_{(n)}}{\left[m_1(R+r)-\dfrac{2J_G}{d}\right]rn^2\Omega^2}$

Ring: $\theta_1=\dfrac{T_{(n)}}{\left[m_1(R+r)+\dfrac{2J_G}{D}\right]rn^2\Omega^2}$

Response to any other nth order excitation torque $T_{(n)}$: Vibration amplitude of pendulum m_1:

Simple: $\theta_1=\dfrac{(R/r)+1}{1-\dfrac{R}{rn^2}}\times\theta_2$

Bifilar: $\theta_1=\dfrac{(R/r)+1}{1-\dfrac{R}{rn^2}}\times\theta_2$

Roller: $\theta_1=\dfrac{1+(R/r)-(2J_G/m_1 dr)}{1+\dfrac{4J_G}{m_1 d^2}-\dfrac{R}{rn^2}}\times\theta_2$

Ring: $\theta_1=\dfrac{1+(R/r)+(2J_G/m_1 Dr)}{1+\dfrac{4J_G}{m_1 D^2}-\dfrac{R}{rn^2}}\,\theta_2$

Composite: $\theta_1=-\dfrac{(R/r)+1}{1-\dfrac{R}{n^2 r}}\times\theta_2-\tfrac12\theta_2^*$

$\theta_2^*=\dfrac{-D/r}{1-\dfrac{(R/r)+1}{n^2}\times\dfrac{m_1 D^2}{4J_G}}\times\theta_2$

Vibration amplitude of carrier J_2:

Simple: $\theta_2=T_{(n)}/[K_2-\omega^2 J_2-\omega^2 J_{P\,\text{eff.}}]$, where
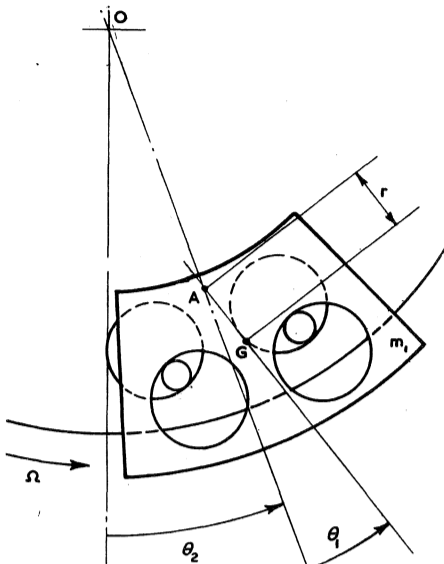$$J_{P\,\text{eff.}}=m_1(R+r)^2\times\left[1+\frac{1}{\dfrac{R}{rn^2}-1}\right]$$

Bifilar: $\theta_2=T_{(n)}/[K_2-\omega^2 J_2-\omega^2 J_{P\,\text{eff.}}]$, where
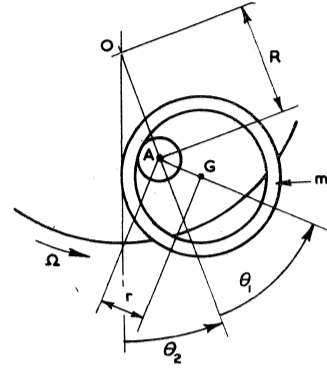$$J_{P\,\text{eff.}}=J_G+m_1\frac{(R+r)^2}{1-\dfrac{r}{R}n^2}$$

Roller: $\theta_2=T_{(n)}/[K_2-\omega^2 J_2-\omega^2 J_{P\,\text{eff.}}]$, where
$$J_{P\,\text{eff.}}=J_G+m_1(R+r)^2-\frac{m_1\left(R+r-\dfrac{2J_G}{m_1 d}\right)^2}{1+\dfrac{4J_G}{m_1 d^2}-\dfrac{R}{n^2 r}}$$

Ring: $\theta_2=T_{(n)}/[K_2-\omega^2 J_2-\omega^2 J_{P\,\text{eff.}}]$, where
$$J_{P\,\text{eff.}}=J_G+m_1(R+r)^2-\frac{m_1\left(R+r+\dfrac{2J_G}{m_1 D}\right)^2}{1+\dfrac{4J_G}{m_1 D^2}-\dfrac{R}{n^2 r}}$$

Composite: $\theta_2=T_{(n)}/[K_2-\omega^2 J_2-\omega^2 J_{P\,\text{eff.}}]$, where
$$J_{P\,\text{eff.}}=\frac{m_1(R+r)^2}{1-\dfrac{n^2 r}{R}}+\frac{J_G}{1-\dfrac{r}{R+r}\times\dfrac{4J_G}{n^2 m_1 D^2}}$$

Alternative designations:

Bifilar: Sarazin pendulum / Chilton pendulum / Taylor pendulum

Roller: Salomon pendulum

Ring: Salomon pendulum

Composite: 'Duplex' pendulum

Other remarks:

Bifilar: All holes of equal radius

Composite: Above formulae are strictly applicable when roller inertia $J_{\text{roll.}}\cong\tfrac14 m_{\text{roll.}}\times d^2$, where $m_{\text{roll.}}$ = mass of roller
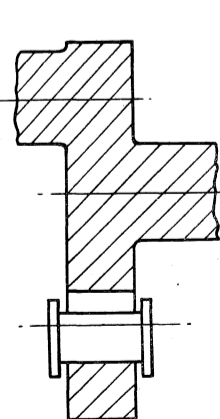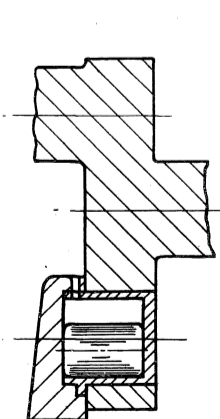
Examples of detuner arrangements:
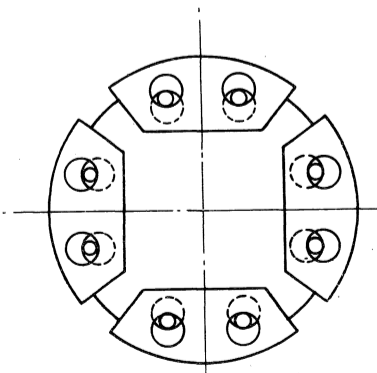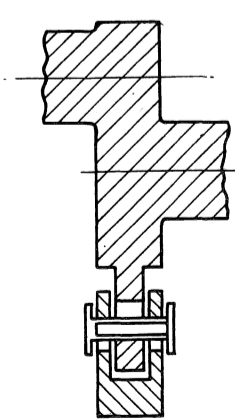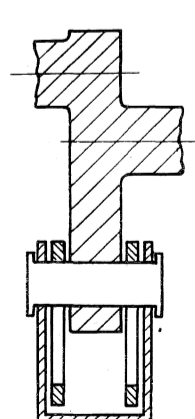

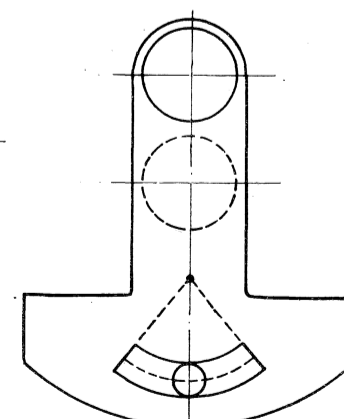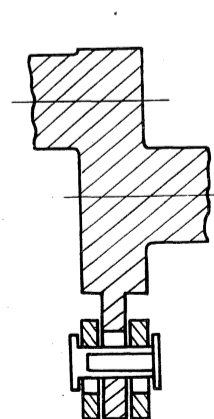Bifilar type   Bifilar type   Roller type   Roller type   Ring type   Roller type (or ball type)   Composite (roller and rings) type