CHAPTER 1

Review of Power Cable Standard Rating Methods

1.1 INTRODUCTION

Calculation of the current-carrying capability, or ampacity, of electric power cables has been extensively discussed in the literature and is the subject of several international and national standards. The main international standards are those issued by the International Electrotechnical Commission (IEC) and the Institute of Electrical and Electronic Engineers (IEEE). References at the end of this chapter list the documents either issued or sponsored by these two organizations. The calculation procedures in both standards are, in principle, the same, with the IEC method incorporating several new developments that took place after the publication of the Neher and McGrath (NM) paper (1957). Similarities in the approaches are not surprising, since during the preparation of the standard, Mr. McGrath was in touch with the Chairman of Working Group 10 of IEC Subcommittee 20A (responsible for the preparation of ampacity calculation standards). The major difference between the two approaches is the use of metric units in IEC 60287 and imperial units in NM paper (the same equations look completely different because of this). Even though the methods are similar in principle, the IEC document is more comprehensive than the NM paper. IEC 60287 not only contains all the formulas (with minor exceptions listed in Appendix F of Anders, 1997) of the NM paper, but, in several cases, it makes a distinction between different cable types and installation conditions where the NM paper does not make such a distinction. Also, the constants used in the IEC document are more up to date.

Nevertheless, the principles of heat transfer for buried cables applied in both standards are the same, and the resulting equations will be reviewed in this chapter. The ampacity calculations are usually carried out in two different ways. On the one hand, steady-state or continuous ratings are sought; and, on the other, time-dependent or transient calculations are performed. In both cases, for cables buried underground, soil dry out caused by the heat generated in the cable might be considered. The calculation methods for cables in air are slightly different in both standards and, where applicable, the differences will be brought up in this book. For cables installed in air, the presence of solar radiation and wind may have profound effects on

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the cable rating. Again, where applicable, these influences will be discussed in the subsequent chapters.

The focus of this book is on the installations that are not covered in the international standards mentioned above. However, the starting point of the analysis will be the standard methods and they will be reviewed in this chapter based on the presentation in Anders (1997).[1] This approach will make this book self-contained, thus allowing analysis of both standard and nonstandard calculation methods. The developments leading to the standard rating equations will not, however, be repeated here and the interested reader is referred to Anders (1997) for the relevant background information.

1.2 ENERGY CONSERVATION EQUATIONS

Ampacity computations of power cables require solution of the heat transfer equations, which define a functional relationship between the conductor current and the temperature within the cable and in its surroundings. In this section, we will analyze how the heat generated in the cable is dissipated to the environment. We will also show the basic heat transfer equations, and discuss how these equations are solved, thus laying the groundwork for cable rating calculations.

1.2.1 Heat Transfer Mechanism in Power Cable Systems

The two most important tasks in cable ampacity calculations are the determination of the conductor temperature for a given current loading, or conversely, determination of the tolerable load current for a given conductor temperature. In order to perform these tasks, the heat generated within the cable and the rate of its dissipation away from the conductor for a given conductor material and given load must be calculated. The ability of the surrounding medium to dissipate heat plays a very important role in these determinations, and varies widely because of several factors such as soil composition and moisture content, ambient temperature, and wind conditions. The heat is transferred through the cable and its surroundings in several ways and these are described in the following sections.

1.2.1.1 Conduction. For underground installations, the heat is transferred by conduction from the conductor and other metallic parts as well as from the insulation. It is possible to quantify heat transfer processes in terms of appropriate rate equations. These equations may be used to compute the amount of energy being transferred per unit time. For heat conduction, the rate equation is known as Fourier's law. For a wall having a temperature distribution $\theta(x)$, the rate equation is expressed as

$$q = -\frac{1}{\rho}\frac{d\theta}{dx} \tag{1.1}$$

[1]The author would like to acknowledge the permission received from the IEEE Press and the McGraw-Hill Company for extracting information from my first book for the purposes of this chapter.

The heat flux q (W/m^2) is the heat transfer rate in the x direction per unit area perpendicular to the direction of transfer, and is proportional to the temperature gradient $d\theta/dx$ in this direction. The proportionality constant ρ is a transport property known as thermal resistivity (K · m/W) and is a characteristic of the material. The minus sign is a consequence of the fact that heat is transferred in the direction of decreasing temperature.

1.2.1.2 Convection. For cables installed in air, convection and radiation are important heat transfer mechanisms from the surface of the cable to the surrounding air. Convection heat transfer may be classified according to the nature of the flow. We speak of forced convection when the flow is caused by external means, such as by wind, pump, or fan. In contrast, for free (or natural) convection, the flow is induced by buoyancy forces, which arise from density differences caused by temperature variations in the air. In order to be somewhat conservative in cable rating computations, we usually assume that only natural convection takes place at the outside surface of the cable. However, both convection modes will be considered in Chapter 6.

Regardless of the particular nature of the convection heat transfer process, the appropriate rate equation is of the form

$$q = h(\theta_s - \theta_{amb}) \tag{1.2}$$

where q, the convective heat flux (W/m^2), is proportional to the difference between the surface temperature and the ambient air temperature, θ_s and θ_{amb}, respectively. This expression is known as Newton's law of cooling, and the proportionality constant h (W/m^2 · K) is referred to as the convection heat transfer coefficient. Determination of the heat convection coefficient is perhaps the most important task in computation of ratings of cables in air. The value of this coefficient varies between 2 and 25 W/m^2 · K for free convection and between 25 and 250 W/m^2 · K for forced convection.

1.2.1.3 Radiation. Thermal radiation is energy emitted by cable or duct surface. The heat flux emitted by a cable surface is given by the Stefan–Boltzmann law:

$$q = \varepsilon\sigma_B\theta_s^{*4} \tag{1.3}$$

where θ_s^* is the absolute temperature (K) of the surface,[2] σ_B is the Stefan–Boltzmann constant ($\sigma_B = 5.67 \cdot 10^{-8}$ W/m^2 · K^4), and ε is a radiative property of the surface called the emissivity. This property, whose value is in the range $0 \leq \varepsilon \leq 1$, indicates how efficiently the surface emits compared to an ideal radiator. Conversely,

[2]Throughout this book, the temperature with an asterisk will denote absolute value in degrees Kelvin. Similarly, dimensions with an asterisk will denote the measurements in meters rather than in millimeters, as is usually the case.

if radiation is incident upon a surface, a portion will be absorbed, and the rate at which energy is absorbed per unit surface area may be evaluated from knowledge of the surface radiative property known as absorptivity, α. That is,

$$q_{abs} = \alpha q_{inc} \tag{1.4}$$

where $0 \leq \alpha \leq 1$. Equations (1.3) and (1.4) determine the rate at which radiant energy is emitted and absorbed, respectively, at a surface. Since the cable both emits and absorbs radiation, radiative heat exchange can be modeled as an interaction between two surfaces. Determination of the net rate at which radiation is exchanged between two surfaces is generally quite complicated. However, for cable rating computations, we may assume that a cable surface is small and the other surface is remote and much larger. Assuming this surface is one for which $\alpha = \varepsilon$ (a gray surface), the net rate of radiation exchange between the cable and its surroundings, expressed per unit area of the cable surface, is

$$q = \varepsilon\sigma_B(\theta_s^{*4} - \theta_{amb}^{*4}) \tag{1.5}$$

Throughout this book, we will use a notion of heat rate rather than heat flux. The heat transfer rate is obtained by multiplying heat flux by the area. Thus, the heat rate for radiative heat transfer will be given by the following equation[3]:

$$W_{rad} = \varepsilon\sigma_B A_{sr}(\theta_s^{*4} - \theta_{amb}^{*4}) \tag{1.6}$$

where A_{sr} (m^2) is the effective radiation area per meter length.

In power cables installed in air, the cable surface within the surroundings will simultaneously transfer heat by convection and radiation to the adjoining air. The total rate of heat transfer from the cable surface is the sum of the heat rates due to the two modes.[4] That is,

$$W = hA_s(\theta_s - \theta_{amb}) + \varepsilon A_{sr}\sigma_B(\theta_s^{*4} - \theta_{amb}^{*4}) \tag{1.7}$$

where A_s (m^2) is the convective area per meter length.

For some special cable installations, the ambient temperature used for heat convection can be different from the one used for heat transfer by radiation. The appropriate temperatures to be used are described in Chapter 10 of Anders (1997).

1.2.1.4 Energy Balance Equations. In the analysis of heat transfer in a cable system, the law of conservation of energy plays an important role. We will formulate this law on a rate basis; that is, at any instant, there must be a balance between all energy rates, as measured in joules per second (W). The energy conservation law can be expressed by the following equation:

[3]Throughout the book, the symbol W will be used for heat transfer rate.
[4]The heat conduction in air is often neglected in cable rating computations.

$$W_{ent} + W_{int} = W_{out} + \Delta W_{st} \tag{1.8}$$

Where W_{ent} is the rate of energy entering the cable. This energy may be generated by other cables located in the vicinity of the given cable or by solar radiation. W_{int} is the rate of heat generated internally in the cable by joule or dielectric losses and ΔW_{st} is the rate of change of energy stored within the cable. The value of W_{out} corresponds to the rate at which energy is dissipated by conduction, convection, and radiation. For underground installations, the cable system will also include the surrounding soil.

We will use the fundamental equations described in this section to develop rating equations throughout the reminder of the book.

1.2.2 Heat Transfer Equations

As we mentioned earlier, current flowing in the cable conductor generates heat, which is dissipated through the insulation, metal sheath, and cable servings into the surrounding medium. The cable ampacity depends mainly upon the efficiency of this dissipation process and the limits imposed on the insulation temperature. To understand the nature of the heat dissipation process, we need to use the relevant heat transfer equations.

1.2.2.1 Underground, Directly Buried Cables. Let us consider an underground cable located in a homogeneous soil. In such a cable, the heat is transferred by conduction through cable components and the soil. Since the length of the cable is much greater than its diameter, end effects can be disregarded and the heat transfer problem can be formulated in two dimensions only.[5]

The differential equation describing heat conduction in the soil has the following form:

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial \theta}{\partial y}\right) + W_{int} = c\frac{\partial \theta}{\partial t} \tag{1.9}$$

where

ρ = thermal resistivity, K · m/W

S = surface area perpendicular to heat flow, m^2

$\frac{\partial \theta}{\partial x}$ = temperature gradient in x direction

c = the volumetric thermal capacity of the material

For a cable buried in soil, Equation (1.9) is solved with the boundary conditions usually specified at the soil surface. These boundary conditions can be expressed in two different forms. If the temperature is known along a portion of the boundary, then

[5]Because end effects are neglected, all thermal parameters will be expressed in this book on a per-unit-length basis.

$$\theta = \theta_B(s) \tag{1.10}$$

where θ_B is the boundary temperature that may be a function of the surface length s. If heat is gained or lost at the boundary due to convection $h(\theta - \theta_{amb})$ or a heat flux q, then

$$\frac{1}{\rho}\frac{\partial \theta}{\partial n} + q + h(\theta - \theta_{amb}) = 0 \tag{1.11}$$

where n is the direction of the normal to the boundary surface, h is a convection coefficient, and θ is an unknown boundary temperature.

In cable rating computation, the temperature of the conductor is usually given and the maximum current flowing in the conductor is sought. Thus, when the conductor heat loss is the only energy source in the cable, we have $W_{int} = I^2R$, and Equation (1.9) is used to solve for I with the specified boundary conditions.

The challenge in solving Equation (1.9) analytically stems mostly from the difficulty of computing the temperature distribution in the soil surrounding the cable. An analytical solution can be obtained when a cable is represented as a line source placed in an infinite homogenous surrounding. Since this is not a practical assumption for cable installations, another assumption is often used; namely, that the earth surface is an isotherm. In practical cases, the depth of burial of the cables is on the order of ten times their external diameter, and for the usual temperature range reached by such cables, the assumption of an isothermal earth surface is a reasonable one. In cases where this hypothesis does not hold, namely, for large cable diameters and cables located close to the earth surface, a correction to the solution equation has to be used or numerical methods applied. Both are discussed in Anders (1997).

1.2.2.2 Cables in Air. For an insulated power cable installed in air, several modes of heat transfer have to be considered. Conduction is the main heat transfer mechanism inside the cable. Suppose that the heat generated inside the cable (due to joule, ferromagnetic and dielectric losses) is W_t (W/m). Another source of heat energy can be provided by the sun if the cable surface is exposed to solar radiation. Energy outflow is caused by convection and net radiation from the cable surface. Therefore, the energy balance Equation (1.83) at the surface of the cable can be written as

$$W_t + W_{sol} - W_{conv} - W_{rad} = 0 \tag{1.12}$$

where W_{sol} is the heat gain per unit length caused by solar heating, and W_{conv} and W_{rad} are the heat losses due to convection and radiation, respectively. Substituting appropriate formulas for the heat gains and losses at the surface of the cable, the following form of the heat balance equation is obtained:

$$W_t + \sigma D_e^* H - \pi D_e^* h(\theta_e^* - \theta_{amb}^*) - \pi D_e^* \varepsilon \sigma_B(\theta_e^{*4} - \theta_{amb}^{*4}) = 0 \tag{1.13}$$

where
θ_e^* = cable surface temperature, K
σ = solar absorption coefficient
H = intensity of solar radiation, W/m^2
σ_B = Stefan–Boltzmann constant, equal to $5.67 \cdot 10^{-8}$ W/m^{2}K^4
ε = emissivity of the cable outer covering
D_e^* = cable external diameter,[6] m
θ_{amb}^* = ambient temperature, K

This equation is usually solved iteratively. In steady-state rating computations, the effect of heat gain by solar radiation and heat loss caused by convection are taken into account by suitably modifying the value of the external thermal resistance of the cable. Computation of the convection coefficient h can be quite involved. Suitable approximations are summarized in Section 1.6.6.5 and, for special installations discussed in this book, are revisited in Chapter 6.

1.2.3 Analytical Versus Numerical Methods of Solving Heat Transfer Equations

Equations (1.9) and (1.13) can be solved analytically, with some simplifying assumptions, or numerically. Analytical methods have the advantage of producing current rating equations in a closed formulation, whereas numerical methods require iterative approaches to find cable ampacity. However, numerical methods provide much greater flexibility in the analysis of complex cable systems and allow representation of more realistic boundary conditions. In practice, analytical methods have found much wider application than the numerical approaches. There are several reasons for this situation. Probably the most important one is historical: cable engineers have been using analytical solutions based on either Neher/McGrath (1957) formalism or IEC Publication 60287 (1994) for a long time. Computations for a simple cable system can often be performed using pencil and paper or with the help of a hand-held calculator. Numerical approaches, on the other hand, require extensive manipulation of large matrices and have only become popular with an advent of powerful computers. Both approaches will be used in this book; analytical methods are discussed in Chapters 2 and 3, whereas the numerical approaches are dealt with in Chapter 4.

1.3 THERMAL NETWORK ANALOGS

Analytical solutions to the heat transfer equations are available only for simple cable constructions and simple laying conditions. In solving the cable heat dissipation problem, electrical engineers use a fundamental similarity between the heat flow

[6]We recall that the dimension symbols with an asterisk refer to the length in meters and without it to the length in millimeters.

due to the temperature difference between the conductor and its surrounding medium and the flow of electrical current caused by a difference of potential. Using their familiarity with the lumped parameter method to solve differential equations representing current flow in a material subjected to potential difference, they adopt the same method to tackle the heat conduction problem. The method begins by dividing the physical object into a number of volumes, each of which is represented by a thermal resistance and a capacitance. The thermal resistance is defined as the material's ability to impede heat flow. Similarly, the thermal capacitance is defined as the material's ability to store heat. The thermal circuit is then modeled by an analogous electrical circuit in which voltages are equivalent to temperatures and currents to heat flows. If the thermal characteristics do not change with temperature, the equivalent circuit is linear and the superposition principle is applicable for solving any form of heat flow problem.

In a thermal circuit, charge corresponds to heat; thus, Ohm's law is analogous to Fourier's law. The thermal analogy uses the same formulation for thermal resistances and capacitances as in electrical networks for electrical resistances and capacitances. Note that there is no thermal analogy to inductance or in steady-state analysis; only resistance will appear in the network.

Since the lumped parameter representation of the thermal network offers a simple means for analyzing even complex cable constructions, it has been widely used in thermal analysis of cable systems. A full thermal network of a cable for transient analysis may consist of several loops. Before the advent of digital computers, the solution of the network equations was a formidable numerical task. Therefore, simplified cable representations were adopted and methods to reduce a multiloop network to a two-loop circuit were developed. A two-loop representation of a cable circuit turned out to be quite accurate for most practical applications and, consequently, was adopted in international standards. In this section, we will explain how the thermal circuit of a cable is constructed, and we will show how the required parameters are computed. We will also explain how full network equations are solved.

1.3.1 Thermal Resistance

All nonconducting materials in the cable will impede heat flow away from the cables (the thermal resistance of the metallic parts in the cable, even though not equal to zero, is so small that it is usually neglected in rating computations). Thus, we can talk about material resistance to heat flow. Of particular interest is an expression for the thermal resistance of a cylindrical layer, for example, cable insulation, with constant thermal resistivity ρ_{th}. If the internal and external radii of this layer are r_1 and r_2, respectively, then the thermal resistance for conduction of a cylindrical layer per unit length is

$$T = \frac{\rho_{th}}{2\pi} \ln \frac{r_2}{r_1} \tag{1.14}$$

For a rectangular wall, we have

$$T = \rho_{th} \frac{l}{S} \tag{1.15}$$

where
ρ_{th} = thermal resistivity of a material, K · m/W
S = cross-section area of the body, m^2
l = thickness of the body, m

In analogy to electrical and thermal networks, we also can write that

$$W = \frac{\Delta\theta}{T} \tag{1.16}$$

which is the thermal equivalent of Ohm's law.

A thermal resistance may also be associated with heat transfer by convection at a surface. From Newton's law of cooling [Equation (1.2)],

$$W = h_{conv} A_s(\theta_e - \theta_{amb}) \tag{1.17}$$

where A_s is the area of the outside surface of the cable for unit length, h_{conv} is the cable surface convection coefficient, and θ_e is the cable surface temperature.

The thermal resistance for convection is then

$$T_{conv} = \frac{\theta_e - \theta_{amb}}{W} = \frac{1}{h_{conv} A_s} \tag{1.18}$$

Yet another resistance may be pertinent for a cable installed in air. In particular, radiation exchange between the cable surface and its surroundings may be important. It follows that a thermal resistance for radiation may be defined as

$$T_{rad} = \frac{\theta_e^* - \theta_{gas}^*}{W_{rad}} = \frac{1}{h_r A_{sr}} \tag{1.19}$$

where A_{sr} is the area of the cable surface effective for heat radiation for unit length of the cable and θ_{gas}^* is the temperature of the air surrounding the cable which, when cable is installed in free air, is equal to the ambient temperature θ_{amb}^*. h_r is the radiation heat transfer coefficient obtained from Equation (1.6) for radiation heat transfer rate:

$$h_r = \varepsilon\sigma_B(\theta_e^* + \theta_{gas}^*)(\theta_e^{*2} + \theta_{gas}^{*2}) \tag{1.20}$$

The total heat transfer coefficient for a cable in air is given by

$$h_t = h_{conv} + h_r \tag{1.21}$$

1.3.2 Thermal Capacitance

Many cable rating problems are time dependent. To determine the time dependence of the temperature distribution within the cable and its surroundings, we could begin by solving the appropriate form of the heat equation, for example, Equation (1.9) In the majority of practical cases, it is very difficult to obtain analytical solutions of this equation and, where possible, a simpler approach is preferred. One such approach may be used where temperature gradients within the cable components are small. It is termed the lumped capacitance method. In order to satisfy the requirement that the temperature gradient within the body must be small, some components of the cable system, for example, the insulation and surrounding soil, must be subdivided into smaller entities. This is done using a theory developed by Van Wormer (1955), application of which is briefly reviewed in this section.

As mentioned above, an equivalent thermal network will contain only thermal resistances T and thermal capacitances Q. The thermal capacitance Q can be defined as the "ability to store the heat," and is defined by

$$Q = V \cdot c \tag{1.22}$$

where
V = volume of the body, m^3
c = volumetric specific heat of the material, $J/m^3 {}^\circ C$

As an illustration, the formula for the thermal capacitance for a coaxial configuration with internal and external diameters D_1^* and D_2^* (m), respectively, which may represent, for example, a cylindrical insulation, is given by

$$Q = \frac{\pi}{4}(D_2^{*2} - D_1^{*2})c \tag{1.23}$$

Thermal capacitances and resistances are used to construct a thermal ladder network to obtain the temperature distribution within the cable and its surroundings as a function of time. This topic is discussed in the next section.

1.3.3 Construction of a Ladder Network of a Cable

The electrical and thermal analogy discussed in Section 1.3.1 allows the solution of many thermal problems by applying mathematical tools well known to electrical engineers. An ability to construct a ladder network is particularly useful in transient computations. To build a ladder network, the cable is considered to extend as far as the inner surface of the soil for buried cables, and to free air for cables in air.

In constructing ladder networks, dielectric losses require special attention. Although the dielectric losses are distributed throughout the insulation, it may be shown that for a single-conductor cable and also for multicore, shielded cables with round conductors, the correct temperature rise is obtained by considering for transients and steady-state that all of the dielectric loss occurs at the middle of the thermal resistance between the conductor and the sheath. For multicore belted cables, dielectric losses can generally be neglected, but if they are represented, the conductors are taken as the source of dielectric loss (Neher and McGrath, 1957).

Thermal capacitances of the metallic parts are placed as lumped quantities corresponding to their physical position in the cable. The thermal capacitances of materials with high thermal resistivity and possibly large temperature gradients across them (e.g., insulation and coverings) are allocated by the technique described below.

1.3.3.1 Representation of Capacitances of the Dielectric. To improve the accuracy of the approximate solution using lumped constants, Van Wormer (1955) proposed a simple method for allocating the thermal capacity of the insulation between the conductor and the sheath so that the total heat stored in the insulation is represented. An assumption made in the derivation is that the temperature distribution in the insulation follows a steady-state logarithmic distribution for the period of the transient. The ladder networks for short and long duration transients are somewhat different and are discussed below.

Whether the transient is long or short depends on the cable construction. For the purpose of transient rating computations, long-duration transients are those lasting longer than $\frac{1}{3}\Sigma T \cdot \Sigma Q$, where ΣT and ΣQ are the internal cable thermal resistance and capacitance, respectively. The methods for computing the values of T and Q are summarized in Section 1.6 in this chapter.

Ladder Network for Long-Duration Transients. The dielectric is represented by lumped thermal constants. The total thermal capacity of the dielectric (Q_i) is divided between the conductor and the sheath, as shown in Figure 1-1.

When screening layers are present, metallic tapes are considered to be part of the conductor or sheath, whereas semiconducting layers (including metallized carbon paper tapes) are considered part of the insulation in thermal calculations.

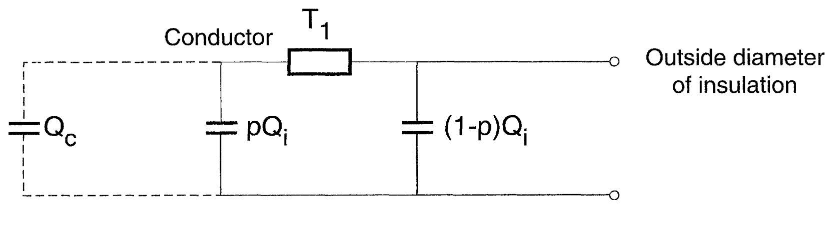


Figure 1-1 Representation of the dielectric for times greater that $\frac{1}{3}\Sigma T \cdot \Sigma Q$. T_1 = total thermal resistance of dielectric per conductor (or equivalent single-core conductor of a three-core cable; see below). Q_i = total thermal capacitance of dielectric per conductor (or equivalent single-core conductor of a three-core cable). Q_c = thermal capacitance of conductor (or equivalent single-core conductor of a three-core cable).

The Van Wormer coefficient p is given by

$$p = \frac{1}{2\ln\left(\frac{D_i}{d_c}\right)} - \frac{1}{\left(\frac{D_i}{d_c}\right)^2 - 1} \tag{1.24}$$

Equation (1.24) is also used to allocate the thermal capacitance of the outer covering in a similar manner to that used for the dielectric. In this case, the Van Wormer factor is given by

$$p' = \frac{1}{2\ln\left(\frac{D_e}{D_s}\right)} - \frac{1}{\left(\frac{D_e}{D_s}\right)^2 - 1} \tag{1.25}$$

where D_e and D_s are the outer and inner diameters of the covering.

For long-duration transients and cyclic factor computations, the three-core cable is replaced by an equivalent single-core construction dissipating the same total conductor losses (Wollaston, 1949). The diameter d_c^* of the equivalent single-core conductor is obtained on the assumption that new cable will have the same thermal resistance of the insulation as the thermal resistance of a single core of the three-core cable; that is,

$$\frac{T_1}{3} = \frac{\rho_i}{2\pi} \ln \frac{D_i^*}{d_c^*} \tag{1.26}$$

where D_i^* is the same value of diameter over dielectric (under the sheath) as for the three-core cable, and T_1 is the thermal resistance of the three-conductor cable as given in Section 1.6.6.1; ρ_i is the thermal resistivity of the dielectric.

Hence, we have

$$d_c^* = D_i^* e^{-2\pi T_1/3\rho_i} \tag{1.27}$$

Thermal capacitances are calculated on the following assumptions:

1. The actual conductors are considered to be completely inside the diameter of the equivalent single conductor, the remainder of the equivalent conductor being occupied by insulation.
2. The space between the equivalent conductor and the sheath is considered to be completely occupied by insulation (for fluid-filled cable, this space is filled partly by the total volume of oil in the ducts and the remainder is oil-impregnated paper).

Factor p is then calculated using the dimensions of the equivalent single-core cable, and is applied to the thermal capacitance of the insulation based on assumption (2) above.

Ladder Network for Short-Duration Transients. Short-duration transients last usually between 10 min and about 1 h. In general, for a given cable construction, the formula for the Van Wormer coefficient shown in this section applies when the duration of the transient is not greater than $\frac{1}{3}\Sigma T \cdot \Sigma Q$. The heating process for short-duration transients can be assumed to be the same as if the insulation were thick.

The method is the same as for long-duration transients except that the cable insulation is divided at diameter $d_x = \sqrt{D_i \cdot d_c}$, giving two portions having equal thermal resistances, as shown in Figure 1-2. The thermal capacitances Q_{i1} and Q_{i2} are defined in Section 1.6.7.

The Van Wormer coefficient is given by

$$p^* = \frac{1}{\ln\left(\frac{D_i}{d_c}\right)} - \frac{1}{\left(\frac{D_i}{d_c}\right) - 1} \tag{1.28}$$

Example 1.1

Construct a ladder network for model cable No. 4 in Appendix A for a short-duration transient.

This network is shown in Figure 1-3, where it is shown that the insulation thermal resistance is divided into two equal parts, the insulation capacitance into four parts, and the capacitance of the cable serving into two parts. ■

A three-core cable is represented as an equivalent single-core cable as described above for durations of about $\frac{1}{2}\Sigma T \cdot \Sigma Q$ or longer (the quantities ΣT and ΣQ refer to the whole cable). However, for very short transients (i.e., for durations up to the value of the product $\Sigma T \cdot \Sigma Q$, where ΣT and ΣQ now refer to the single core), the mutual heating of the cores is neglected, and a three-core cable is treated as a single-core cable with the dimensions corresponding to the one core. For durations between these two limits, $\Sigma T \cdot \Sigma Q$ for one core and $\frac{1}{2}\Sigma T \cdot \Sigma Q$ for the whole cable, the transient is assumed to be given by a straight-line interpolation in a diagram with axes of linear temperature rise and logarithmic times.

Van Wormer Coefficient for Transients Due to Dielectric Loss. In the preceding sections, it has been assumed that the temperature rise of the conductor due to

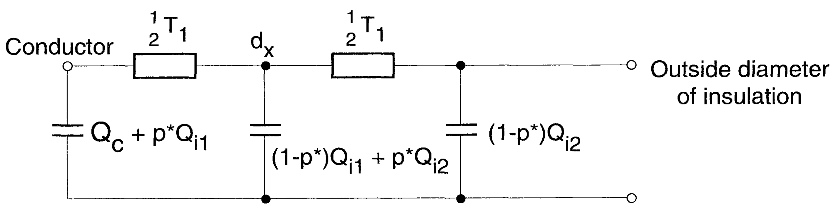


Figure 1-2 Representation of the dielectric for times less than or equal to $\frac{1}{3}\Sigma T \cdot \Sigma Q$.

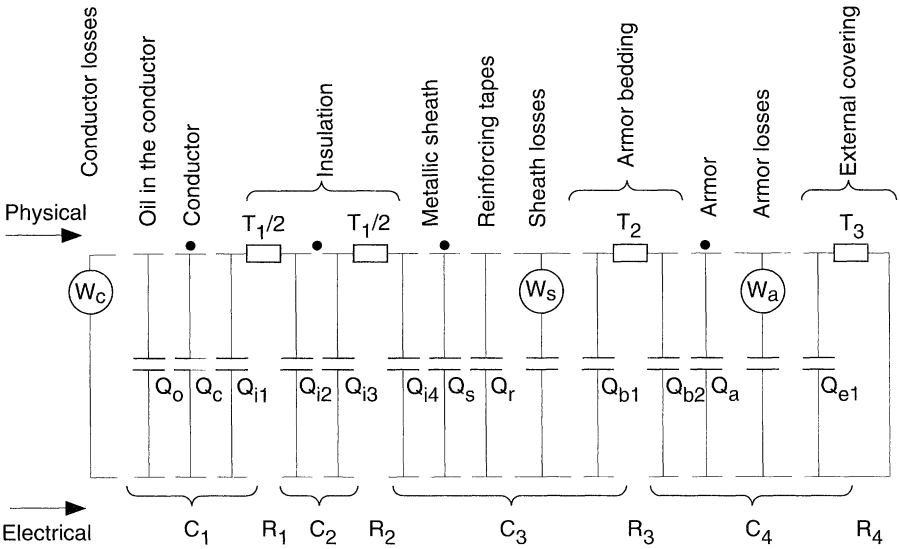


Figure 1-3 Thermal network for model cable No. 4 with electrical analogy.

dielectric loss has reached its steady state, and that the total temperature at any time during the transient can be obtained simply by adding the constant temperature value due to the dielectric loss to the transient value caused by the load current.

If changes in load current and system voltage occur at the same time, then an additional transient temperature rise due to the dielectric loss has to be calculated (Morello, 1958). For cables at voltages up to and including 275 kV, it is sufficient to assume that half of the dielectric loss is produced at the conductor and the other half at the insulation screen or sheath. The cable thermal circuit is derived by the method given above with the Van Wormer coefficient computed from equations and for long- and short-duration transients, respectively.

For paper-insulated cables operating at voltages higher than 275 kV, the dielectric loss is an important fraction of the total loss and the Van Wormer coefficient is calculated by (IEC 853-2, 1989)

$$p_d = \frac{\left[\left(\frac{D_i}{d_c}\right)^2 \ln\left(\frac{D_i}{d_c}\right)\right] - \left[\ln\left(\frac{D_i}{d_c}\right)\right]^2 - \frac{1}{2}\left[\left(\frac{D_i}{d_c}\right)^2 - 1\right]}{\left[\left(\frac{D_i^2}{d_c}\right) - 1\right]\left[\ln\left(\frac{D_i}{d_c}\right)\right]^2} \tag{1.29}$$

In practical calculations for all voltage levels for which dielectric losses are important, half of the dielectric loss is added to the conductor loss and half to the sheath loss; therefore, the loss coefficients $(1 + \lambda_1)$ and $(1 + \lambda_1 + \lambda_2)$ used to evaluate thermal resistances and capacitances are set equal to 2.

Example 1.2
We will compute the Van Wormer coefficient for dielectric losses for cable No. 3 described in Appendix A. From Table A1, we have $D_j = 67.26$ mm and $d_c = 41.45$ mm. Hence,

$$p_d = \frac{p_d = \left[\left(\frac{67.26}{41.45}\right)^2 \ln\left(\frac{67.26}{41.45}\right)\right] - \left[\ln\left(\frac{67.26}{41.45}\right)\right]^2 - \frac{1}{2}\left[\left(\frac{67.26}{41.45}\right)^2 - 1\right]}{\left[\left(\frac{67.26}{41.45}\right)^2 - 1\right]\left[\ln\left(\frac{67.26}{41.45}\right)\right]^2} = 0.585$$

An example of the transient analysis with the voltage applied simultaneously with the current is given in Example 5.4 in (Anders, 1997) ■

1.3.3.2 Reduction of a Ladder Network to a Two-Loop Circuit. CIGRE (1972, 1976) and later IEC (1985, 1989) introduced computational procedures for transient rating calculations employing a two-loop network with the intention of simplifying calculations and with the objective of standardizing the procedure for basic cable types. Even though with the advent and wide availability of fast desktop computers the advantage of simple computations is no longer so pronounced, there is some merit in performing some computations by hand, if only for the purpose of checking sophisticated computer programs. To perform hand computations for the transient response of a cable to a variable load, the cable ladder network has to be reduced to two sections. The procedure to perform this reduction is described below.

Consider a ladder network composed of v resistances and $(v + 1)$ capacitances, as shown in Figure 1-4. If the last component of the network is a capacitance, the last capacitance Q_{v+1} is short-circuited. An equivalent network, which represents the cable with sufficient accuracy, is derived with two sections T_AQ_A and $T_\beta Q_B$, as shown in Figure 1-5.

The first section of the derived network is made up of $T_A = T_\alpha$ and $Q_A = Q_\alpha$ without modification, in order to maintain the correct response for relatively short durations.

The second section $T_\beta Q_B$ of the derived network is made up from the remaining sections of the original circuit by equating the thermal impedance of the second de-

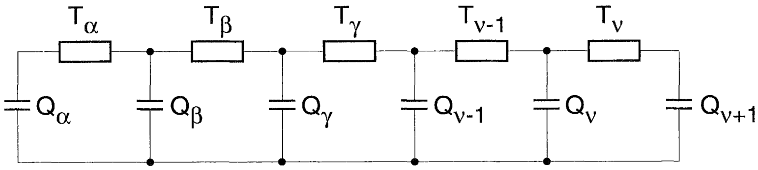


Figure 1-4 General ladder network representing a cable.

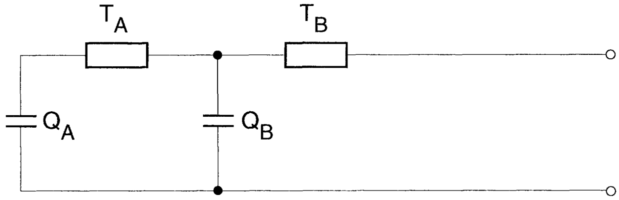


Figure 1-5 Two-loop equivalent network.

rived section to the total impedance of the multiple sections. The resulting expressions are then equal to (Anders, 1997)

$$T_B = T_\beta + T_\gamma + \ldots + T_\nu \tag{1.30}$$

$$\begin{aligned} Q_B = Q_\beta &+ \left(\frac{T_\gamma + T_\delta + \ldots + T_\nu}{T_\beta + T_\gamma + \ldots + T_\nu}\right)^2 Q_\gamma + \left(\frac{T_\delta + T_\varepsilon + \ldots + T_\nu}{T_\beta + T_\gamma + \ldots + T_\nu}\right)^2 Q_\delta + \ldots \\ &+ \left(\frac{T_\nu}{T_\beta + T_\gamma + \ldots + T_\nu}\right)^2 Q_\nu \end{aligned} \tag{1.31}$$

Even though formulas and are straightforward, a great deal of care is required when the equivalent thermal resistances and capacitances are computed in the case when sheath, armor, and pipe losses are present (IEC, 1985). This is because the location of these losses inside the original network has to be carefully taken into account. The following example illustrates this point.

Example 1.3

We will construct a two-loop equivalent network for model cable No. 1 assuming (1) a short-duration transient and (2) a long-duration transient.

1. *Short-Duration Transient*

From Table A1 we observe that for this cable, short-duration transients are those lasting half an hour or less. The diagram of the full network for a short-duration transient is shown in Figure 1-6.

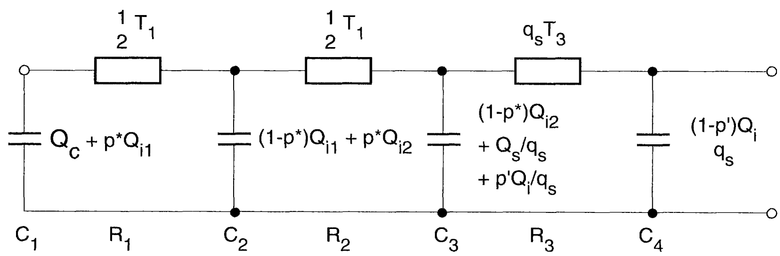


Figure 1-6 Network diagram for cable No. 1 for short-duration transients.

The method of dividing insulation and jacket capacitances into parts is discussed in Section 1.6.7. Before we apply the reduction procedure, we combine parallel capacitances into four equivalent capacitances. In the equivalent network, only conductor losses are represented. Therefore, to account for the presence of sheath losses, the thermal resistances beyond the sheath must be multiplied, and the thermal capacitances divided by the ratio of the losses in the conductor and the sheath to the conductor losses.[7] By performing these multiplications and divisions, the time constants of the thermal circuits involved are not changed. Thus,

$$Q_1 = Q_c + p^*Q_{i1} \qquad Q_2 = (1 - p^*)Q_{i1} + p^*Q_{i2}$$
$$Q_3 = (1 - p^*)Q_{i2} \qquad Q_4 = \frac{Q_s + p'Q_{j1}}{1 + \lambda_1} \qquad Q_5 = \frac{(1 - p')Q_{j2}}{1 + \lambda_1} \tag{1.32}$$

To compute numerical values, we will require expressions for Q_{i1} and Q_{i2}. These expressions are given in Section 1.6.7. The numerical values are as follows: Q_{i1} = 763 J/K · m, Q_{i2} = 453.9 J/K · m, and Q_j = 394.8 J/K · m. With these values and with the additional numerical values in Table A1, D_i = 30.1 mm, d_c = 20.5 mm, D_e = 35.8 mm, and D_s = 31.4 mm, we have

$$p^* = \frac{1}{\ln\left(\frac{D_i}{d_c}\right)} - \frac{1}{\left(\frac{D_i}{d_c}\right) - 1} = \frac{1}{\ln\frac{30.1}{20.5}} - \frac{1}{\frac{30.1}{20.5} - 1} = 0.468$$

$$p' = \frac{1}{2\ln\left(\frac{D_e}{D_s}\right)} - \frac{1}{\left(\frac{D_e}{D_s}\right)^2 - 1} = \frac{1}{2\ln\left(\frac{35.8}{31.4}\right)} - \frac{1}{\left(\frac{35.8}{31.4}\right)^2 - 1} = 0.478$$

$$Q_1 = 1035 + 0.468 \cdot 763 = 1392.1 \text{ J/K} \cdot \text{m}$$

$$Q_2 = (1 - 0.468)763 + 0.468 \cdot 453.9 = 618.3 \text{ J/K} \cdot \text{m}$$

$$Q_3 = (1 - 0.468)453.9 = 241.5 \text{ J/K} \cdot \text{m} \qquad Q_4 = \frac{4 + 0.478 \cdot 394.8}{1.09} = 176.8 \text{ J/K} \cdot \text{m}$$

$$Q_5 = \frac{(1 - 0.478)394.8}{1.09} = 189.1 \text{ J/K} \cdot \text{m}$$

The final capacitance Q_5 is omitted in further analysis because the transient for the cable response is calculated on the assumption that the output terminals on the right-hand side are short-circuited.

Since the first section of the network in Figure 1-6 represents the conductor, and in rating computations the conductor temperature is of interest, the equivalent

[7]These ratios are called sheath and armor loss factors and are defined in Sections 1.6.4 and 1.6.5, respectively.

network will have the first section equal to the first section of the full network; that is,

$$T_A = \tfrac{1}{2}T_1 \qquad \text{and} \qquad Q_A = Q_1 \tag{1.33}$$

From Equation (1.30), we have

$$T_B = \tfrac{1}{2}T_1 + (1 + \lambda_1)T_3 \tag{1.34}$$

Thermal capacitance of the second part is obtained by applying Equation (1.31):

$$Q_B = Q_2 + \left[\frac{(1 + \lambda_1)T_3}{\frac{1}{2}T_1 + (1 + \lambda_1)T_3}\right]^2 (Q_3 + Q_4) \tag{1.35}$$

The sheath loss factor and thermal resistances for this cable are given in Table A1 as $\lambda_1 = 0.09$, $T_1 = 0.214$ K · m/W, and $T_3 = 0.104$ K · m/W. Substituting numerical values in Equations (1.33) to (1.35), we obtain

$$T_A = 0.107 \text{ K} \cdot \text{m/W} \qquad Q_A = 1392.1 \text{ J/K} \cdot \text{m}$$

$$T_B = 0.107 + 1.09 \cdot 0.104 = 0.220 \text{ K} \cdot \text{m/W}$$

$$Q_B = 618.3 + \left(\frac{1.09 \cdot 0.104}{0.107 + 1.09 \cdot 0.104}\right)^2 (241.5 + 176.8) = 729.4 \text{ J/K} \cdot \text{m}$$

2. *Long-Duration Transients*

Long-duration transient for this cable are those lasting longer than 0.5 h. The appropriate diagram is shown in Figure 1-7. In this case, we have

$$T_A = T_1 \qquad T_B = (1 + \lambda_1)T_3 \tag{1.36}$$

The insulation and jacket are split into two parts with the Van Wormer coefficients given by Equations (1.24) and (1.25), respectively. Since the last part of the jacket capacitance is short-circuited, Q_A and Q_b are simply obtained as the sums of relevant capacitances:

$$Q_A = Q_c + pQ_i \qquad Q_B = (1 - p)Q_i + \frac{Q_s + p'Q_j}{1 + \lambda_1} \tag{1.37}$$

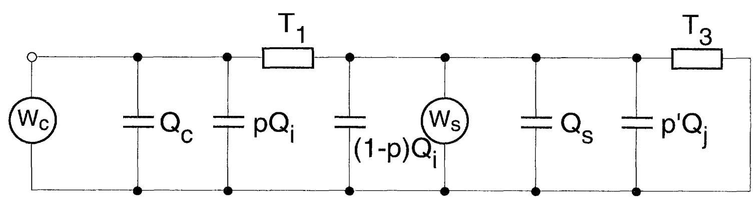


Figure 1-7 Network diagram for a long-duration transient for model cable No. 1.

Substituting numerical values, we obtain

$$p = \frac{1}{2\ln\left(\frac{D_i}{d_c}\right)} - \frac{1}{\left(\frac{D_i}{d_c}\right)^2 - 1} = \frac{1}{2\ln\left(\frac{30.1}{20.5}\right)} - \frac{1}{\left(\frac{30.1}{20.5}\right)^2 - 1} = 0.437$$

$$T_A = 0.214\ \text{K}\cdot\text{m/W} \qquad Q_A = 1035 + 0.437 \cdot 915.6 = 1435.1\ \text{J/K}\cdot\text{m}$$

$$T_B = 1.09 \cdot 0.104 = 0.113\ \text{K}\cdot\text{m/W}$$

$$Q_B = (1 - 0.437)915.6 + \frac{4 + 0.478 \cdot 394.8}{1.09} = 692.3\ \text{J/K}\cdot\text{m}$$ ■

1.4 RATING EQUATIONS—STEADY-STATE CONDITIONS

The current-carrying capability of a cable system will depend on several parameters. The most important of these are:

1. The number of cables and the different cable types in the installation under study
2. The cable construction and materials used for the different cable types
3. The medium in which the cables are installed
4. Cable locations with respect to each other and with respect to the earth surface
5. The cable bonding arrangement

For some cable constructions, the operating voltage may also be of significant importance. All of the above issues are taken into account, some of them explicitly, the others implicitly, in the rating equations summarized in this chapter. The lumped parameter network representation of the cable system is used for the development of steady-state and transient rating equations. These equations are developed for a single cable, either with one core or with multiple cores. However, they can be applied to multicable installations, for both equally and unequally loaded cables, by suitably selecting the value of the external thermal resistance, as discussed in Section 1.6.6.5.

The development of cable rating equations is quite different for steady-state and transient conditions. We will start with analysis of the steady-state conditions, which could be a result of either constant or cyclic loading. We will present only the very fundamental equations that will form the basis of the developments presented in the subsequent chapters. The parameters appearing in these equations can occasionally involve very complex calculations. We will review these calculations when a need arises before introducing modifications that are the subject of this book.

1.4.1 Buried Cables

1.4.1.1 Steady-State Rating Equation without Moisture Migration.

Steady-state rating computations involve solving the equation for the ladder network shown in Figure 1-8. With reference to Figure 1-8, W_c, W_d, W_s, and W_a (W/m) represent conductor, dielectric, sheath, and armor losses, respectively, and n denotes the number of conductors in the cable. T_1, T_2, T_3, and T_4 (K · m/W) are the thermal resistances, where T_1 is the thermal resistance per unit length between one conductor and the sheath, T_2 is the thermal resistance per unit length of the bedding between sheath and armor, T_3 is the thermal resistance per unit length of the external serving of the cable, and T_4 is the thermal resistance per unit length between the cable surface and the surrounding medium.

Since losses occur at several positions in the cable system (for this lumped parameter network), the heat flow in the thermal circuit shown in Figure 1-8 will increase in steps. Thus, the total joule loss W_I in a cable can be expressed as

$$W_I = W_c + W_s + W_a = W_c(1 + \lambda_1 + \lambda_2) \tag{1.38}$$

The quantity λ_1 is called the sheath loss factor and is equal to the ratio of the total losses in the metallic sheath to the total conductor losses. Similarly, λ_2 is called the armor loss factor and is equal to the ratio of the total losses in the metallic armor to the total conductor losses. Incidentally, it is convenient to express all heat flows caused by the joule losses in the cable in terms of the loss per meter of the conductor.

Referring now to the diagram in Figure 1-8, and remembering the analogy between the electrical and thermal circuits, we can write the following expression for $\Delta\theta$, the conductor temperature rise above the ambient temperature:

$$\Delta\theta = (W_c + \tfrac{1}{2}W_d)T_1 + [W_c(1 + \lambda_1) + W_d]nT_2 + [W_c(1 + \lambda_1 + \lambda_2) + W_d]n(T_3 + T_4) \tag{1.39}$$

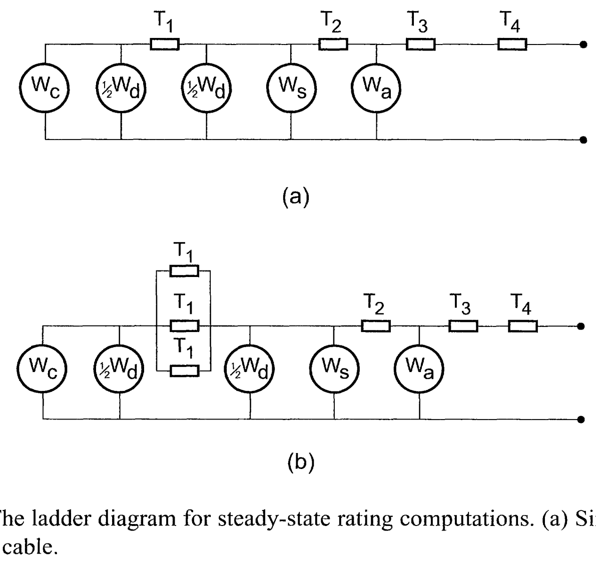


Figure 1-8 The ladder diagram for steady-state rating computations. (a) Single-core cable. (b) Three-core cable.

where W_d, W_c, λ_1, and λ_2 are defined above, and n is the number of load-carrying conductors in the cable (conductors of equal size and carrying the same load). The ambient temperature is the temperature of the surrounding medium under normal conditions at the location where the cables are installed, or are to be installed, including any local sources of heat, but not the increase of temperature in the immediate neighborhood of the cable due to the heat arising therefrom.

The unknown quantity is either the conductor current I or its operating temperature θ_c (°C). In the first case, the maximum operating conductor temperature is given, and in the second case, the conductor current is specified. The permissible current rating is obtained from Equation (1.39). Remembering that $W_c = I^2R$, we have

$$I = \left[\frac{\Delta\theta - W_d[0.5T_1 + n(T_2 + T_3 + T_4)]}{RT_1 + nR(1 + \lambda_1)T_2 + nR(1 + \lambda_1 + \lambda_2)(T_3 + T_4)} \right]^{0.5} \tag{1.40}$$

where R is the ac resistance per unit length of the conductor at maximum operating temperature.

Equation (1.40) is often written in a simpler form that clearly distinguishes between internal and external heat transfers in the cable. Denoting

$$\begin{aligned} T &= \frac{T_1}{n} + (1 + \lambda_1)T_2 + (1 + \lambda_1 + \lambda_2)T_3 \\ T_d &= \frac{T_1}{2n} + T_2 + T_3 \end{aligned} \tag{1.41}$$

Equation (1.40) becomes

$$\Delta\theta = n(W_cT + W_tT_4 + W_dT_d) \tag{1.42}$$

where W_t are the total losses generated in the cable defined by:

$$W_t = W_I + W_d = W_c(1 + \lambda_1 + \lambda_2) + W_d \tag{1.43}$$

and T computed from Equation (1.41) is an equivalent cable thermal resistance. This is an internal thermal resistance of the cable, which depends only on the cable construction. The external thermal resistance, on the other hand, will depend on the properties of the surrounding medium as well as on the overall cable diameter, as shown below.

The last term in Equation (1.42) is the temperature rise caused by dielectric losses. Denoting it by $\Delta\theta_d$,

$$\Delta\theta_d = nW_dT_d \tag{1.44}$$

1.4.1.2 Steady-State Rating Equation with Moisture Migration. The laying conditions examined in this book are particularly conducive to the formation of a dry zone around the cable. Under unfavorable conditions, the heat flux from the cable entering the soil may cause significant migration of moisture away from the cable. A dried-out zone may develop around the cable, in which the thermal conductivity can be reduced by a factor of three or more over the conductivity of the bulk. The drying-out conditions may occur in both regions of the route, but particularly in the region of high thermal resistivity.

The modeling of the dry zone around the cable is discussed in Anders (1997). For completeness, we will start by recalling the basic developments presented there. This will be followed by the modification of the expression for the conductor temperature, taking into account the drying-out conditions in the unfavorable region in Chapter 2 and in cable crossings in Chapter 3.

The current-carrying capacity of buried power cables depends to a large extent on the thermal conductivity of the surrounding medium. Soil thermal conductivity is not a constant, but is highly dependent on its moisture content. Under unfavorable conditions, the heat flux from the cable entering the soil may cause significant migration of moisture away from the cable. A dried-out zone may develop around the cable, in which the thermal conductivity is reduced by a factor of three or more over the conductivity of the bulk. This, in turn, may cause an abrupt rise in temperature of the cable sheath, which may lead to damage to the cable insulation. The likelihood of soil drying out is even greater when the route of the rated cable is crossed by another heat source.

In order to give some guidance on the effect of moisture migration on cable ratings, CIGRE (1986) has proposed a simple two-zone model for the soil surrounding loaded power cables, resulting in a minor modification of the steady-state rating equation (Anders, 1997). Subsequently, this model has been adopted by the IEC as an international standard (IEC, 1994). We will further extend this model to account for heat sources crossing the rated cable.

The concept on which the method proposed by CIGRE relies can be summarized as follows. Moist soil is assumed to have a uniform thermal resistivity, but if the heat dissipated from a cable and its surface temperature are raised above certain critical limits, the soil will dry out, resulting in a zone that is assumed to have a uniform thermal resistivity higher than the original one. The critical conditions, that is, the conditions for the onset of drying, are dependent on the type of soil, its original moisture content, and temperature.

Given the appropriate conditions, it is assumed that when the surface of a cable exceeds the critical temperature rise above ambient, a dry zone will form around it. The outer boundary of the zone is on the isotherm related to that particular temperature rise. An additional assumption states that the development of such a dry zone does not change the shape of the isothermal pattern from what it was when all the soil was moist, only that the numerical values of some isotherms change. Within the dry zone, the soil has a uniformly high value of thermal resistivity, corresponding to its value when the soil is "oven dried" at not more than 105°C. Outside the dry zone, the soil has uniform thermal resistivity corresponding to the site moisture content. The essential advantages of these assumptions are that the resistivity is uniform over each zone, and that the values are both convenient and sufficiently accurate for practical purposes.

The method presented below assumes that the entire region surrounding a cable or cables has uniform thermal characteristics prior to drying out; the only nonuniformity being that caused by drying. As a consequence, the method should not be applied without further consideration to installations where special backfills with properties different from the site soil are used.

Let θ_e be the cable surface temperature corresponding to the moist soil thermal resistivity ρ_1. Within the area between the cable surface (assumed to be isothermal) and the critical isotherm, the heat transfer equation remains the same, the only change from the uniform soil condition being the thermal resistivity of the dry zone.

Without moisture migration, we obtain the following relations, remembering that the soil thermal resistance is directly proportional to the value of resistivity:

$$nW_t = \frac{\theta_e - \theta_{amb}}{T_4} = \frac{(\theta_e - \theta_x) + (\theta_x - \theta_{amb})}{T_4} \tag{1.45}$$

and

$$nW_t = \frac{\theta_e - \theta_x}{C\rho_1} \tag{1.46}$$

where C is a constant, n is the number of cores in the cable, T_4 is the cable external thermal resistance when the soil is moist, and W_t is the total losses in a single core. θ_{amb} and θ_x are ambient temperature and the temperature of an isotherm at distance x, respectively.

If we now assume that the region between the cable and the θ_x isotherm dries out so that its resistivity becomes ρ_2, and that the power losses W_t remain unchanged, we have

$$nW_t = \frac{\theta_e' - \theta_x}{C\rho_2} \tag{1.47}$$

where θ_e' is the cable surface temperature after moisture migration has taken place.

Combining Equations (1.46) and (1.47), we obtain the following form of Equation (1.45):

$$\theta_e' - \theta_x = \frac{\rho_2}{\rho_1}(\theta_e - \theta_x) = \frac{\rho_2}{\rho_1}[(\theta_e - \theta_{amb}) - (\theta_x - \theta_{amb})] \tag{1.48}$$

After rearranging the last equation, we obtain

$$\theta_e' - \theta_{amb} = v(W_t \cdot T_4) - (v - 1)\,\Delta\theta_x \tag{1.49}$$

where

$$v = \frac{\rho_2}{\rho_1} \quad \text{and} \quad \Delta\theta_x = \theta_x - \theta_{amb} \tag{1.50}$$

The rating Equation (1.40) takes the form

$$I = \left[\frac{\theta_c - \theta_{amb} - W_d[0.5T_1 + n(T_2 + T_3 + vT_4)] + (v - 1)\Delta\theta_x}{RT_1 + nR(1 + \lambda_1)T_2 + nR(1 + \lambda_1 + \lambda_2)(T_3 + vT_4)} \right]^{0.5} \tag{1.51}$$

We can observe that Equation (1.40) has been modified by the addition of the term $(v - 1)\Delta\theta_x$ in the numerator, and the substitution of vT_4 for T_4 in both the numerator and the denominator.

1.4.2 Cables in Air

When cables are installed in free air, the external thermal resistance now accounts for the radiative and convective heat loss. For cables exposed to solar radiation, there is an additional temperature rise caused by the heat absorbed by the external covering of the cable. The heat gain by solar absorption is equal to $\sigma D_e H$, with the meaning of the variables defined below. In this case, the external thermal resistance is different than for shaded cables in air, and the current rating is computed from the following modification of Equation (1.40) (IEC 60287, 1994).

$$I = \left[\frac{\Delta\theta - W_d[0.5T_1 + n(T_2 + T_3 + T_4^*)] + \sigma D_e^* H T_4^*}{RT_1 + nR(1 + \lambda_1)T_2 + nR(1 + \lambda_1 + \lambda_2)(T_3 + T_4^*)} \right]^{0.5} \tag{1.52}$$

where

D_e^* = external diameter of the cable, m

σ = absorption coefficient of solar radiation for the cable surface

H = intensity of solar radiation, W/m^2

T_4^* = external thermal resistance of the cable in free air, adjusted to take account of solar radiation, K · m/W

1.5 RATING EQUATIONS—TRANSIENT CONDITIONS

The procedure to evaluate temperatures is the main computational block in transient rating calculations. This block requires a fairly complex programming procedure to take into account self- and mutual heating, and to make suitable adjustments in the loss calculations to reflect changes in the conductor resistance with temperature.

Transient rating of power cables requires the solution of the equations for the network in Figure 1-6. The unknown quantity in this case is the variation of the conductor temperature rise with time,[8] $\theta(t)$. Unlike in the steady-state analysis, this temperature is not a simple function of the conductor current $I(t)$. Therefore, the process for determining the maximum value of $I(t)$ so that the maximum operating conductor temperature is not exceeded requires an iterative procedure. An excep-

[8]Unless otherwise stated, in this section we will follow the notation in IEC (1989) and we will use the symbol θ to denote temperature rise, and not $\Delta\theta$ as in other parts of the book and in IEC (1994).

tion is the simple case of identical cables carrying equal current located in a uniform medium. Approximations have been proposed for this case, and explicit rating equations developed. We will discuss this case first and then we will extend the discussion to multiple cable types.

1.5.1 Response to a Step Function

1.5.1.1 Preliminaries. Whether we consider the simple cable systems mentioned above, or a more general case of several cable circuits in a backfill or duct bank, the starting point of the analysis is the solution of the equations for the network in Figure 1-6. Our aim is to develop a procedure to evaluate temperature changes with time for the various cable components. As observed by Neher (1964), the transient temperature rise under variable loading may be obtained by dividing the loading curve at the conductor into a sufficient number of time intervals, during any one of which the loading may be assumed to be constant. Therefore, the response of a cable to a step change in current loading will be considered first.

This response depends on the combination of thermal capacitances and resistances formed by the constituent parts of the cable itself and its surroundings. The relative importance of the various parts depends on the duration of the transient being considered. For example, for a cable laid directly in the ground, the thermal capacitances of the cable, and the way in which they are taken into account, are important for short-duration transients, but can be neglected when the response for long times is required. The contribution of the surrounding soil is, on the other hand, negligible for short times, but has to be taken into account for long transients. This follows from the fact that the time constant of the cable itself is much shorter than the time constant of the surrounding soil.

The thermal network considered in this work is a derivation of the lumped parameter ladder network introduced early in the history of transient rating computations (Buller, 1951; Van Wormer, 1955; Neher, 1964; CIGRE, 1972; IEC 1985, 1989). For computational purposes, Baudoux et al. (1962) and then Neher (1964) proposed to represent a cable in just two loops. Baudoux et al. provided procedures for combining several loops to obtain a two-section network, which was latter adopted by CIGRE WG 02 and published in *Electra* (CIGRE, 1972). However, transformation of a multiloop network into a two-loop equivalent not only requires substantial manual work before the actual transient computations can be performed, but also inhibits the computation of temperatures at parts of the cable other than the conductor. A procedure is given below for analytical solution of the entire network. Generally, the network will be somewhat different for short- and long-duration transients, and, usually, the limiting duration to distinguish these two cases can be taken to be 1 h. Short transients are assumed to last at least 10 min. A more detailed time division between short and long transients can be found in Table 1-1 presented later in this Chapter.

The temperature rise of a cable component (e.g., conductor, sheath, jacket, etc.) can be represented by the sum of two components: the temperature rise inside and outside the cable. The method of combining these two components, introduced by

Morello (Morello, 1958; CIGRE, 1972; IEC, 1985, 1989), makes allowance for the heat that accumulates in the first part of the thermal circuit and which results in a corresponding reduction in the heat entering the second part during the transient. The reduction factor, known as the attainment factor, $\alpha(t)$, of the first part of the thermal circuit is computed as a ratio of the temperature rise across the first part at time t during the transient to the temperature rise across the same part in the steady state. Then, the temperature transient of the second part of the thermal circuit is composed of its response to a step function of heat input multiplied by a reduction coefficient (variable in time) equal to the attainment factor of the first part. Evaluation of these temperatures is discussed below.

1.5.1.2 Temperature Rise in the Internal Parts of the Cable. The internal parts of the cable encompass the complete cable including its outermost serving or anticorrosion protection. If the cable is located in a duct or pipe, the duct and pipe (including pipe protective covering) are also included. For cables in air, the cable extends as far as the free air.

Analyses of linear networks, such as the one in Figure 1-6, involve the determination of the expression for the response function caused by the application of a forcing function. In our case, the forcing function is the conductor heat loss and the response sought is the temperature rise above the cable surface at node i. This is accomplished by utilizing a mathematical quantity called the transfer function of the network. It turns out that this transfer function is the Fourier transform of the unit-impulse response of the network. The Laplace transform of the network's transfer function is given by a ratio

$$H(s) = \frac{P(s)}{Q(s)} \tag{1.53}$$

$P(s)$ and $Q(s)$ are polynomials, their forms depending on the number of loops in the network. Node i can be the conductor or any other layer of the cable. In terms of time, the response of this network is expressed as (Van Valkenburg, 1964)[9]

$$\theta_i(t) = W_c \sum_{j=1}^{n} T_{ij}(1 - e^{P_j t}) \tag{1.54}$$

where:
$\theta_i(t)$ = temperature rise at node i at time t, °C
W_c = conductor losses including skin and proximity effects, W/m
T_{ij} = coefficient, °Cm/W
P_j = time constant, s^{-1}
t = time from the beginning of the step, s
n = number of loops in the network

[9]Unless otherwise stated, in the remainder of this section all the temperature rises are caused by the joule losses in the cable.

i = node index
j = index from 1 to n.

The coefficients T_{ij} and the time constants P_j are obtained from the poles and zeros of the equivalent network transfer function given by Equation (1.53). Poles and zeros of the function $H(s)$ are obtained by solving equations $Q(s) = 0$ and $P(s) = 0$, respectively. From the circuit theory, the coefficients T_{ij} are given by

$$T_{ij} = -\frac{a_{(n-i)i}}{b_n} \frac{\prod_{k=1}^{n-i}(Z_{ki} - P_j)}{P_j \prod_{\substack{k=1 \\ k \neq j}}^{n}(P_k - P_j)} \tag{1.55}$$

where
$a_{(n-i)i}$ = coefficient of the numerator equation of the transfer function
b_n = first coefficient of the denominator equation of the transfer function
Z_{ki} = zeros of the transfer function
P_j = poles of the transfer function.

An algorithm for the computation of the coefficients of the transfer function equation is given in Appendix B of Anders (1997).

Example 1.4
A simple expression of Equation (1.54) is obtained for the case of $n = 2$. Construction of such a network is discussed in Section 1.3.3.2 and the network is shown in Figure 1-5.

In this simple case, the time-dependent solution for the conductor temperature can easily be obtained directly. However, to illustrate the procedure outlined above, we will compute this temperature from Equations (1.53)–(1.55).

The transfer function for this network is given by

$$H(s) = \frac{(T_A + T_B)sT_A T_B Q_B}{1 + s(T_A Q_A + T_B Q_B + T_B Q_A) + s^2 T_A Q_A T_B Q_B} \tag{1.56}$$

Since we are interested in obtaining conductor temperature, $i = 1$ and $j = 1, 2$. To simplify the notation, we will use the following substitutions:

$$T_a = T_{11} \qquad T_b = T_{12} \qquad M_0 = 0.5(T_A Q_A + T_B Q_B + T_B Q_A) \qquad N_0 = T_A Q_A T_B Q_B \tag{1.57}$$

The zeros and poles of the transfer function are easily obtained as follows:

$$Z_{11} = \frac{T_A + T_B}{T_A T_B Q_B} \qquad P_1 = -a \qquad P_2 = -b$$

where

$$a = \frac{M_0 + \sqrt{M_0^2 - N_0}}{N_0} \qquad b = \frac{M_0 - \sqrt{M_0^2 - N_0}}{N_0} \tag{1.58}$$

From Equation (1.56),

$$a_{(2-1)1} = T_A T_B Q_B \qquad b_2 = T_A T_B Q_A Q_B$$

Thus,

$$\frac{a_{11}}{b_2} = \frac{1}{Q_A}$$

From Equation (1.55), we have

$$T_a = -\frac{1}{Q_A} \frac{\dfrac{T_A + T_B}{T_A T_B Q_B + a}}{-a(-b + a)} = \frac{1}{a-b}\left(\frac{1}{Q_A} - \frac{T_A + T_B}{a T_A T_B Q_A Q_B}\right)$$

but

$$ab = \frac{1}{T_A T_B Q_A Q_B}$$

Hence,

$$T_a = \frac{1}{a-b}\left[\frac{1}{Q_A} - b(T_A + T_B)\right] \quad \text{and} \quad T_b = T_A + T_B - T_a \tag{1.59}$$

Finally, the conductor temperature as a function of time is obtained from Equation (1.54):

$$\theta_c(t) = W_c[T_a(1 - e^{-at}) + T_b(1 - e^{-bt})] \tag{1.60}$$

where W_c is the power loss per unit length in a conductor based on the maximum conductor temperature attained. The power loss is assumed to be constant during the step of the transient. Further,

$$\alpha(t) = \frac{\theta_c(t)}{W_c(T_A + T_B)} \tag{1.61}$$

■

Because the solution of network equations for a two-loop network is quite simple, IEC publications 853-1 (1985) and 853-2 (1989) recommend that this form be used in transient analysis. The two-loop computational procedure was published at a time when access to fast computers was very limited (CIGRE, 1972). Today, this limitation is no longer a problem and a full network representation is recommended in transient analysis computations. This recommendation is particularly applicable in the case when temperatures of cable components, other than the conductor, are of interest.

1.5.1.3 Second Part of the Thermal Circuit—Influence of the Soil. The transient temperature rise $\theta_e(t)$ of the outer surface of the cable can be evaluated exactly in the case when the cable is represented by a line source located in a homogeneous, infinite medium with uniform initial temperature. However, for practical applications, we have to use another hypothesis, namely, the hypothesis of Kennelly, which assumes that the earth surface must be an isotherm. Under this hypothesis, the temperature rise at any point M in the soil is, at any time, the sum of the temperature rises caused by the heat source W_t and by its fictitious image placed symmetrically with the earth surface as the axis of symmetry and emitting heat $-W_t$ (see Figure 1-9).

The temperature rise at the cable outside surface is then given by

$$\theta_e(t) = W_t \frac{\rho_s}{4\pi}\left[-Ei\left(-\frac{D_e^{*2}}{16\delta t}\right) + Ei\left(-\frac{L^{*2}}{\delta t}\right)\right] \tag{1.62}$$

where:
D_e^* = external surface diameter of cable, m
L^* = axial depth of burial of the cable, m
δ = soil diffusivity, m^2/s

The expression

$$-Ei(-x) = \int_x^{\infty} \frac{e^{-v}}{v} dv$$

is called the exponential integral. The value of the exponential integral can be developed in the series

$$-Ei(-x) = -0.577 - \ln x + x - \frac{x^2}{2 \cdot 2!} + \frac{x^3}{3 \cdot 3!} \cdots$$

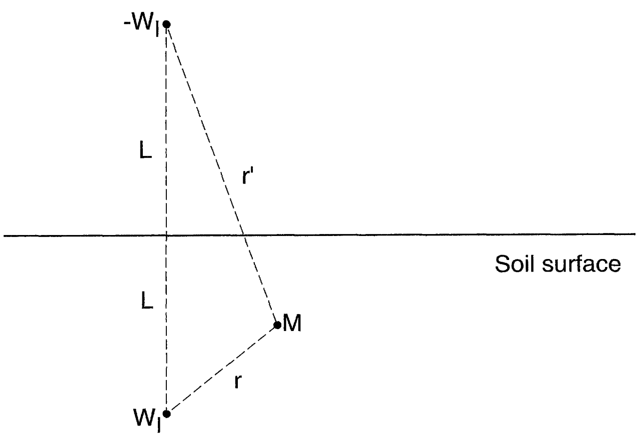


Figure 1-9 Illustration of Kennelly's hypothesis.

When $x < 0.1$,

$$-Ei(-x) = -0.577 - \ln x + x$$

to within 1% accuracy. For large x,

$$-Ei(-x) = -\frac{e^{-x}}{x}\left(1 - \frac{1}{x} + \frac{2!}{x^2} - \frac{3!}{x^3} + \cdots\right)$$

The National Bureau of Standards published in 1940 *Tables of Exponential Integrals,* Vol. 1, in which values of $-Ei(-x)$ can be found. IEC has also published nomograms from which $-Ei(-x)$ can be obtained (IEC 853-2, 1989).

Under steady-state conditions, $t \rightarrow \infty$ and x approaches zero. In this case, Equation (1.62) becomes

$$\theta_e(\infty) = W_t \frac{\rho_s}{2\pi} \ln \frac{4L^*}{D_e^*} \tag{1.63}$$

From this equation, we can define the external thermal resistance in the steady-state calculations as

$$T_4 = \frac{\rho_s}{2\pi} \ln \frac{4L^*}{D_e^*} \tag{1.64}$$

For cables in air it, is unnecessary to calculate a separate response for the cable environment. The complete transient $\theta(t)$ is obtained from Equation (1.62) but the external thermal resistance T_4, computed as described in Section 1.6.6.5, is included in the cable network.

1.5.1.4 Groups of Equally or Unequally Loaded Cables. In a typical installation, several power cables are laid in a trench. The mutual heating effect reduces the current-carrying capacity of the cables, and this effect must be taken into account in rating computations. For groups of cables, the temperature for each cable is obtained at each point in time by adding to its own temperature the temperature rise caused by other nearby cables. To achieve better accuracy in calculations with multiple time steps, the effect of other cables should be added at each time step so that their effect can be included with that caused by the temperature rise of the cable itself. Thus, the temperature rise in the cable of interest "p" due to one other adjacent cable "k" can be computed from:

$$\theta_{pk}(t) = W_{Ik} \frac{\rho_s}{4\pi}\left[-Ei\left(-\frac{d_{pk}^{*2}}{4\delta t}\right) + Ei\left(-\frac{d_{pk}^{*\prime 2}}{4\delta t}\right)\right] \tag{1.65}$$

in which W_{Ik} is the total joule losses in cable k, and d_{pk}^* and $d_{pk}^{*\prime}$ (m) denote the distance from the center of cable p to the center of cable k and its image, respectively, as shown in Figure 1-10.

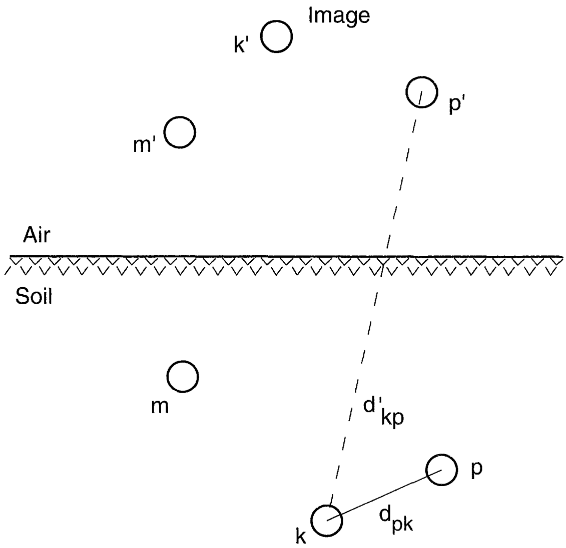


Figure 1-10 Example of cable configuration and image cables.

1.5.1.5 Total Temperature Rise. The total transient temperature rise of a cable at any time is the sum of the rise due to its own losses, given by its own network, and the rises due to mutual heating given by the networks of other cables and image sources, as appropriate. Thus, the final temperature rise at any layer of the cable of interest (that is, at any node of the equivalent network) at time t after the beginning of the load step is obtained from

$$\theta_{ptot}(t) = \theta_i(t) + \alpha(t)\theta_e(t) + \theta_d(t) + \alpha(t)\sum_{k=1}^{N-1}[\theta_{pk}(t) + \theta_{pdk}(t)] \tag{1.66}$$

where $\alpha(t)$ is the attainment factor for the transient temperature rise between the conductor and outside surface of the cable and N is the number of cables. $\theta_{pdk}(t)$ is the temperature rise in cable p caused by the dielectric losses in cable k and it is multiplied by $\alpha(t)$ only if cable k is energized at time $t = 0$. $\theta_d(t)$ is the internal temperature rise caused by the dielectric losses in cable p. In Equation (1.66), θ_{ptot} is defined for any layer of the cable, and in the above formulation, only $\theta_i(t)$ is different for each layer.

The attainment factor varies in time, and a reasonable approach for obtaining $\alpha(t)$ is to use (Morello, 1958)

$$\alpha(t) = \frac{\text{Temperature across cable at time } t}{\text{Steady-state temperature rise across the cable}} \tag{1.67}$$

The conductor attainment factor is computed from this definition using Equations (1.54) and (1.42):

$$\alpha(t) = \frac{\theta_c(t)}{\theta_c(\infty)} = \frac{W_c \sum_{j=1}^{n} T_{cj}(1 - e^{P_j t})}{W_c T} = \frac{\sum_{j=1}^{n} T_{cj}(1 - e^{P_j t})}{T} \tag{1.68}$$

The attainment factor associated with dielectric losses is obtained from a similar equation with the network parameters reflecting the presence of dielectric loss only.

1.5.2 Transient Temperature Rise under Variable Loading

In order to perform computations for variable loading, a daily load curve is divided into a series of steps of constant magnitude. For different successive steps, the computations are done repeatedly, and the final result is obtained using the principle of superposition. The temperature rise above ambient at time τ can be represented as

$$\theta(\tau) - \theta(\tau - 1) \tag{1.69}$$

1.5.3 Conductor Resistance Variations during Transients

Since the conductor electrical resistance, as well as the resistance of other metallic parts of the cable, changes with temperature, the effect of these changes should be taken into account when computing conductor and sheath losses. Goldenberg (1967, 1971) developed a technique for obtaining arbitrarily close upper and lower bounds for the temperature rise of the conductor, taking into account the changes of the resistance of metallic parts with temperature. His upper-bound formula has been adopted by CIGRE (1972, 1976) and IEC (1989) and is given by the following equation:

$$\theta_a(t) = \frac{\theta(t)}{1 + a[\theta(\infty) - \theta(t)]} \tag{1.70}$$

where:

$\theta(t)$ = conductor transient temperature rise above ambient without correction for variation in conductor loss, and is based on the conductor resistance at the end of the transient

$\theta(\infty)$ = conductor steady-state temperature rise above ambient

a = temperature coefficient of electrical resistivity of the conductor material at the start of the transient. $\alpha = 1/[\beta + \theta(0)]$, with β being a reciprocal of temperature coefficient at 0°C and $\theta(0)$ is the conductor temperature at the start of the transient

1.5.4 Cyclic Rating Factor

The complexity of cyclic rating computations varies depending on the shape of the load curve and the amount of detail known for the load cycle. If only the load-loss factor or a daily load factor is known, a method proposed by Neher and McGrath (1957) can be used. This method involves modification of the cable external thermal resistance as discussed in Anders (1997) and in Section 1.6.6.5. This modified value is then used in Equation (1.51). Neher and McGrath's approach continues to be the basis for the majority of cyclic loading computations performed in North America.

If a more detailed analysis is required, the algorithm introduced by Goldenberg (1957, 1958) and later adopted by the IEC (1985, 1989) can be used. This approach, applicable to a single cable or a cable system composed of identical, equally loaded cables located in a uniform medium, requires computation of a cyclic rating factor M by which the permissible steady-state rated current (100% load factor) may be multiplied to obtain the permissible peak value of current during a daily (24 h) cycle such that the conductor temperature attains, but does not exceed, the standard permissible maximum temperature during the cycle. A factor derived in this way uses the steady-state temperature, which is usually the permitted maximum temperature, as its reference. The cyclic rating factor depends only on the shape of the daily cycle and is independent of the actual magnitudes of the current.

Several sections in this book address the issue of cyclic rating computations under various circumstances. Therefore, the procedure for the calculation of this factor is reviewed below and Chapter 6 presents a complete discussion of the relevance of the load-loss factor used in time-dependent rating calculations.

First, we will consider a single cable and then extend the analysis to a group of identical cables. The details of the development of the equations presented here can be found in Anders (1997). We will ignore the variation of the conductor resistance with temperature. This is consistent with the IEC approach for cyclic loading calculations. Following the IEC standard approach, only the six hourly steps before the time when the temperature reaches its highest value are involved; the remaining hourly steps being represented by a representative load, as illustrated in Figure 3-13.

The cyclic rating factor for uniform soil conditions is defined as (Anders, 1997; IEC, 1989)

$$M = \frac{1}{\sqrt{\mu\left[1 - \frac{\theta_R(6)}{\theta_R(\infty)}\right] + \sum_{i=0}^{5} Y_i\left[\frac{\theta_R(i+1)}{\theta_R(\infty)} - \frac{\theta_R(i)}{\theta_R(\infty)}\right]}} \tag{1.71}$$

with the following notation, where the subscript R corresponds to the steady state rated current:

$$\frac{\theta_R(t)}{\theta_R(\infty)} = [1 - k + k\beta(t)]\alpha(t) \qquad \text{for } i \geq 1$$
$$\theta_R(0) = 0 \tag{1.72}$$

$$k = \frac{\theta_e(\infty)}{\theta_R(\infty)} = \frac{W_I T_4}{W_c T + W_I T_4} \tag{1.73}$$

$$\alpha(t) = \frac{\theta_c(t)}{\theta_c(\infty)} \tag{1.74}$$

$$\beta(t) = \frac{-Ei\left(-\frac{D_e^{*2}}{16t\delta}\right) + Ei\left(-\frac{L^{*2}}{t \cdot \delta}\right)}{2 \ln\left(\frac{4L^*}{D_e^*}\right)} \tag{1.75}$$

where

D_e^* = external diameter of the cable or duct, m

L^* = depth of laying, m

W_I = the total joule loss in the cable, W/m

δ = soil thermal diffusivity, m^2/s

T = internal thermal resistance of the cable K · m/W

μ = load loss factor

If the hourly load values are denoted by I_i, $i = 1, \ldots, 24$, then the load-loss factor is defined by:

$$\mu = \frac{1}{24} \frac{\sum_{i=0}^{23} I_i^2}{I_{\max}^2} = \frac{1}{24} \sum_{i=0}^{23} Y_i \tag{1.76}$$

(See more discussion on the definition of the load-loss factor in Chapter 6.)

Calculations are simplified considerably when the conductor attainment factor can be assumed to be equal to one. In this case, Equation (1.71) takes the form

$$M = \frac{1}{\sqrt{(1-k)Y_0 + k\{B + \mu[1 - \beta(6)]\}}} \tag{1.77}$$

where:

$$B = \sum_{i=0}^{5} Y_i \Phi_i; \qquad \Phi_m = \beta(m+1) - \beta(m) \tag{1.78}$$

IEC (1989) identifies the following cases when the internal cable capacitances can be neglected. If the period from the initiation of the thermal transient is longer than

1. 12 h for all cables,
2. The product $\Sigma T \cdot \Sigma Q$; when dealing with fluid-pressure, pipe-type cables and all types of self-contained cables where the product $\Sigma T \cdot \Sigma Q \leq 2$ h
3. The product $2\Sigma T \cdot \Sigma Q$; when dealing with gas-pressure, pipe-type cables and all types of self-contained cables where the product $\Sigma T \cdot \Sigma Q > 2$ h

where ΣT and ΣQ are the total internal thermal resistance (simple sum of all resistances) and capacitance (simple sum of all capacitances), respectively, of the cable.

Table 1-1, based on design values commonly used at present for the determination of cable dimensions, shows when cases 2 and 3 apply (IEC, 1989).

We will now consider groups of N cables with equal losses; the cables or ducts do not touch. In this case, $\theta_R(i)$ is the conductor temperature rise of the hottest cable

Table 1-1 Cases when the attainment factor can be assumed to be equal to 1

Type of cable	Case b)	Case c)
Fluid-filled cables	1) All voltages < 220 kV 2) 220 kV, sections ≤ 150 mm^2	1) 220 kV, sections > 150 mm^2 2) All voltages > 220 kV
Pipe-type, fluid-pressure cables	1) All voltages < 220 kV 2) 220 kV, sections ≤ 800 mm^2	1) 220 kV, sections > 800 mm^2 2) All voltages > 220 kV
Pipe-type, gas-pressure cables		1) ≤ 220 kV 2) Sections ≤ 1000 mm^2
Cables with extruded insulation	1) All voltages < 60 kV 2) 60 kV, sections ≤ 150 mm^2	1) 60 kV, sections > 150 mm^2 2) All voltages > 60 kV

in the group. The external thermal resistance of the hottest cable in Equations (1.73) and (1.75) will now include the effect of the other $(N-1)$ cables and will be denoted by $T_4 + \Delta T_4$. We now obtain the following new form of Equation (1.75):

$$\beta_1(t) = \frac{\rho_s}{4\pi} \frac{-Ei\left(-\frac{D_e^{*2}}{16t\delta}\right) + Ei\left(-\frac{L^{*2}}{t\delta}\right) + \sum_{\substack{k=1 \\ k\neq p}}^{N}\left[-Ei\left(-\frac{d_{pk}^2}{4t\delta}\right) + Ei\left(-\frac{d_{pk}'^2}{4t\delta}\right)\right]}{(T_4 + \Delta T_4)} \tag{1.79}$$

The value of ΔT_4 is equal to (Anders, 1997)

$$\Delta T_4 = \frac{\rho_s \ln F}{2\pi} \tag{1.80}$$

where

$$F = \frac{d_{p1}' \cdot d_{p2}' \cdot \ldots \cdot d_{pk}' \cdot \ldots \cdot d_{pN}'}{d_{p1} \cdot d_{p2} \cdot \ldots \cdot d_{pk} \cdot d_{pN}} \tag{1.81}$$

with factor d_{pp}'/d_{pp} excluded, leaving $(N-1)$ factors in Equation (1.81). The distances d_{pv} and d_{pv}' represent the distance between cables p and v and between the cable v and the image of cable p, respectively.

Introducing the notation

$$d_f = \frac{4L^*}{F^{1/(N-1)}} \tag{1.82}$$

Equation (1.79) can be approximated by

$$\beta_1(t) = \frac{-Ei\left(-\frac{D_e^{*2}}{16t\delta}\right) + Ei\left(-\frac{L^{*2}}{t\delta}\right) + (N-1)\left[-Ei\left(-\frac{d_f^2}{16t\delta}\right) + Ei\left(-\frac{L^{*2}}{t\delta}\right)\right]}{2\ln\frac{4L^*F}{D_e^*}} \tag{1.83}$$

Also, Equation (1.73) becomes

$$k_1 = \frac{W_I(T_4 + \Delta T_4)}{W_c T + W_I(T_4 + \Delta T_4)} \tag{1.84}$$

The cyclic rating factor is given by Equation (1.71) with

$$\frac{\theta_R(t)}{\theta_R(\infty)} = [1 - k_1 + k_1 \beta_1(t)]\alpha(t) \tag{1.85}$$

1.6 EVALUATION OF PARAMETERS

Rating equations discussed in the previous sections contain many different parameters whose values need to be estimated first before ampacity calculations can proceed. For some of the parameters, analytical expressions can be derived; for others, empirical equations or curves have been proposed. In this section, we will summarize the calculation of the important parameters. For a detailed discussion on their derivation, the reader is referred again to Anders (1997).

We will start with a list of the symbols used in various expressions presented below. To facilitate the presentation, we will divide all the symbols into logical groups related to cable construction and laying conditions. This will be followed by a series of equations, tables, and charts that are needed for ampacity calculations. The order of the presentation follows the usual steps in cable rating calculations.

1.6.1 List of Symbols

1.6.1.1 General Data

f (Hz) = system frequency
U (V) = cable operating voltage (phase-to-phase)
LF = daily load factor
θ = conductor temperature[10]
θ_{amb} = ambient temperature

1.6.1.2 Cable Parameters

Conductor
S (mm²) = cross-sectional area of conductor
D_e (mm) = external diameter of cable, or equivalent diameter of a group of cores in pipe-type cable
D_e^* (m) = external diameter of cable, or equivalent diameter of cable (cables in air)
d_c (mm) = external diameter of conductor

[10]subscripts t, s, and a will be used to denote tape, sheath, and armor, respectively.

d_c' (mm) = conductor diameter of equivalent solid conductor having the same central oil duct
d_i (mm) = conductor inside diameter
n = number of conductors in a cable

Three-conductor cables
d_x (mm) = Diameter of an equivalent circular conductor having the same cross-sectional area and degree of compactness as the shaped one
c (mm) = Distance between the axes of conductors and the axis of the cable for three-core cables (= $0.55\, r_1 + 0.29\, t$ for sector-shaped conductors)
r_1 (mm) = Circumscribing radius of three-sector shaped conductors in three-conductor cable

Insulation
D_i (mm) = diameter over insulation
t_1 (mm) = insulation thickness between conductors and sheath
ρ (K · m/W) = thermal resistivity of the material[11]

Three-conductor cables
t (mm) = insulation thickness between conductors
t_i (mm) = thickness of core insulation, including screening tapes plus half the thickness of any nonmetallic tapes over the laid-up cores

Sheath
D_s (mm) = sheath diameter
d (mm) = sheath mean diameter
t_s (mm) = sheath thickness
$\left.\begin{matrix} p_2 \\ q_2 \end{matrix}\right\}$ = ratios of minor section lengths, where minor section lengths are: a, p_2a, and q_2a, and a is the shortest section
ς(Ωm) = electrical resistivity of sheath material at operating temperature

Armor or reinforcement
A (mm^2) = cross-sectional area of the armor
D_a (mm) = external diameter of armor
d_a (mm) = mean diameter of armor
d_f (mm) = diameter of armor wires
d_2 (mm) = mean diameter of reinforcement
n_a = number of armor wires
n_t = number of tapes
ℓ_a (mm) = length of lay of a steel wire along a cable
ℓ_T (mm) = length of lay of a tape
t_t (mm) = thickness of tape
w_t (mm) = width of tape

[11]The same symbol is used for thermal resistivity of various materials. The appropriate numerical value will correspond to the material considered.

Jacket/Serving
t_j (mm) = thickness of the jacket
t_3 (mm) = thickness of the serving

Pipe-type cables
D_d (mm) = external diameter of the pipe
D_o (mm) = external diameter of pipe coating
D_{sm} (mm) = moisture barrier mean diameter
D_{sw} (mm) = diameter of skid wire
D_t (mm) = diameter of moisture barrier assembly
d_d (mm) = internal diameter of pipe
n_{sw} = number of skid wires
n_t = number of moisture barrier metallic tapes
ℓ_{sw} (mm) = lay of length of skid wires
ℓ_T (mm) = lay of moisture barrier metallic tapes
t_3 (mm) = thickness of pipe coating
t_t = thickness of moisture barrier metallic tape in pipe-type cable
w_t (mm) = width of moisture barrier metallic tape

1.6.1.3 Installation Conditions

Cables in Air
H (W/m^2) = solar radiation

Buried Cables
L (mm) = depth of burial of cables
D_x (mm) = fictitious diameter at which effect of loss factor commences
ρ_s (K · m/W) = thermal resistivity of soil
s (mm) = spacing between conductors of the same circuit
s_2 (mm) = axial separation of cables; for cables in flat formation, s_2 is the geometric mean of the three spacings

Duct Bank/Thermal Backfill
L_G (mm) = distance from the soil surface to the center of a duct bank
x, y (mm) = sides of duct bank/backfill ($y > x$)
N = number of loaded cables in a duct bank/backfill
ρ_c (K · m/W) = thermal resistivity of concrete used for a duct bank or backfill
ρ_e (K · m/W) = thermal resistivity of earth surrounding a duct bank/backfill

Cables in Ducts
D_d (mm) = internal diameter of the duct
D_o (mm) = external diameter of the duct
θ_m (°C) = mean temperature of duct filling medium

1.6.2 Conductor ac Resistance

Conductor resistance is calculated in two stages. First, the dc value R' (ohm/m) is obtained from the following expression:

$$R' = \frac{1.02 \cdot 10^6 \rho_{20}}{S}[1 + \alpha_{20}(\theta - 20)]$$

In the second stage, the dc value is modified to take into account the skin and proximity effects. The resistance of a conductor when carrying an alternating current is higher than that of the conductor when carrying a direct current. The principal reasons for the increase are: skin effect, proximity effect, hysteresis and eddy current losses in nearby ferromagnetic materials, and induced losses in short-circuited nonferromagnetic materials nearby. The degree of complexity of the calculations that can economically be justified varies considerably. Except in very high voltage cables consisting of large segmental conductors, it is common to consider only skin effect, proximity effect, and in some cases, an approximation of the effect of metallic sheath and/or conduit. The relevant expressions are:

$$R = R'(1 + y_s + y_p)$$

For cables in magnetic pipes and conduits:

$$R = R'[1 + 1.5(y_s + y_p)]$$

Material properties and the expressions for the skin and proximity factors are:

Material	Resistivity $(\rho_{20}) \cdot 10^{-8}$ $\Omega \cdot$ m at 20°C	Temperature coefficient $(\alpha_{20}) \cdot 10^{-3}$ per K at 20°C
Copper	1.7241	3.93
Aluminum	2.8264	4.03

Skin and proximity factors are computed from the following expressions:

$$y_s = \frac{x_s^4}{192 + 0.8x_s^4}$$

where

$$x_s^2 = F_k \cdot k_s \qquad F_k = \frac{8\omega f \cdot 10^{-7}}{R'}$$

The proximity factor is obtained from

$$y_p = F_p\left(\frac{d_c}{s}\right)^2\left[0.312\left(\frac{d_c}{s}\right)^2 + \frac{1.18}{F_p + 0.27}\right] \qquad x_p^2 = F_k \cdot k_p \qquad F_p = \frac{x_p^4}{192 + 0.8x_p^4}$$

For sector-shaped conductors:

$$s = d_x + t$$

$$d_c = d_x$$

$$y_p = 2y_p/3$$

For oval conductors:

$$d_c = \sqrt{d_{c\text{minor}} \cdot d_{c\text{major}}}$$

The above expressions apply when $x_p \leq 2.8$ (a majority of the cases). Otherwise, Equations (7.26) or (7.27) in Anders (1997) should be used.

Constants k_s and k_p are given in Table 1-2 (IEC 60287, 1994).

1.6.3 Dielectric Losses

When paper and solid dielectric insulations are subjected to alternating voltage, they act as large capacitors and charging currents flow in them. The work required to effect the realignment of electrons each time the voltage direction changes (i.e., 50 or 60 times a second) produces heat and results in a loss of real power that is called dielectric loss, which should be distinguished from reactive loss. For a unit length of a cable, the magnitude of the required charging current is a function of the dielectric constant of the insulation, the dimensions of the cable, and the operating voltage. For some cable constructions, notably for high-voltage, paper-insulated cables, this loss can have a significant effect on the cable rating. The dielectric losses are computed from the following expression:

Table 1-2 Values of skin and proximity factors

Type of conductor	Whether dried and impregnated or not	k_s	k_p
Copper			
Round, stranded	Yes	1	0.8
Round, stranded	No	1	1
Round, compact	Yes	1	0.8
Round, compact	No	1	1
Round, segmental		0.435	0.37
Hollow, helical stranded	Yes	Eq. (7.12) in Anders (1997)	0.8
Sector-shaped	Yes	1	0.8
Sector-shaped	No	1	1
Aluminum			*
Round, stranded	either	1	
Round, 4 segment	Either	0.28	
Round, 5 segment	Either	0.19	
Round, 6 segment	Either	0.12	
Segmental with peripheral strands	Either	Eq. (7.17) in Anders (1997)	

*Since there are no accepted experimental results dealing specifically with aluminum stranded conductors, IEC 60287 recommends that the values of k_p given in this table for copper conductors also be applied to aluminum stranded conductor of similar design as the copper ones.

$$W_d = 2\pi f \cdot C \cdot U_o^2 \cdot \tan\delta$$

where the electrical capacitance and the phase-to-to-ground voltage are obtained from

$$C = \frac{\varepsilon}{18 \ln\left(\frac{D_i}{d_c}\right)} \cdot 10^{-9} \qquad U_o = \frac{U}{\sqrt{3}}$$

The dielectric constant ε and the loss factor $\tan\delta$ are taken from Table 1-3.

1.6.4 Sheath Loss Factor

Sheath losses are current dependent, and can be divided into two categories according to the type of bonding. These are losses due to circulating currents that flow in the sheaths of single-core cables if the sheaths are bonded together at two points, and losses due to eddy currents, which circulate radially (skin effect) and azimuthally (proximity effect). Eddy current losses occur in both three-core and single-core cables, irrespective of the method of bonding. Eddy current losses in the sheaths of single-core cables, which are solidly bonded are considerably smaller

Table 1-3 Values of the dielectric constant and the loss factor

Type of cable	ε	$\tan\delta$
Cables insulated with impregnated paper		
Solid type, fully impregnated, preimpregnated, or mass-impregnated nondraining	4	0.01
Oil-filled, low-pressure		
up to $U_o = 36$ kV	3.6	0.0035
up to $U_o = 87$ kV	3.6	0.0033
up to $U_o = 160$ kV	3.5	0.0030
up to $U_o = 220$ kV	3.5	0.0028
Oil-pressure, pipe-type	3.7	0.0045
Internal gas-pressure	3.4	0.0045
External gas-pressure	3.6	0.0040
Cables with other kinds of insulation		
Butyl rubber	4	0.050
EPR, up to 18/30 kV	3	0.020
EPR, above 18/30 kV	3	0.005
PVC	8	0.1
PE (HD and LD)	2.3	0.001
XLPE up to and including 18/30 (36) kV, unfilled	2.5	0.004
XLPE above 18/30 (36) kV, unfilled	2.5	0.001
XLPE above 18/30 (36) kV, filled	3	0.005
Paper–polypropylene–paper (PPL)	2.8	0.001

than circulating current losses, and are ignored except for cables with large segmental conductors.

Losses in protective armoring also fall into several categories depending on the cable type, the material of the armor, and installation methods. Armored single-core cables without a metallic sheath generally have a nonmagnetic armor because the losses in steel-wire or tape armor would be unacceptably high. For cables with nonmagnetic armor, the armor loss is calculated as if it were a cable sheath, and the calculation method depends on whether the armor is single-point bonded or solidly bonded. For cables having a metallic sheath and nonmagnetic armor, the losses are calculated as for sheath losses, but using the combined resistance of the sheath and armor in parallel and a mean diameter equal to the rms value or the armor and sheath diameters. The same procedure applies to two- and three-core cables having a metallic sheath and nonmagnetic armor. For two- and three-core cables having metallic sheath and magnetic wire armor, eddy current losses in the armor must be considered. For two- and three-core cables having steel-tape armor, both eddy current losses and hysteresis losses in the tape must be considered together with the effect of armor on sheath losses.

Submarine cables require special consideration. Single-core ac cables for submarine power connections differ in many respects from underground cables, buried directly or in ducts. In fact, submarine cables are generally armored, can be manufactured in very long lengths, and are laid with a very large distance between them. For these reasons, calculation methods described in the IEC 60287 (1994) must be supplemented and modified in some points.

1.6.4.1 Sheath Bonding Arrangements. Sheath losses in single-core cables depend on a number of factors, one of which is the sheath bonding arrangement. In fact, the bonding arrangement is the second most important parameter in cable ampacity computations after the external thermal resistance of the cable. For safety reasons, cable sheaths must be earthed, and hence bonded, at least at one point in a run. There are three basic options for bonding sheaths of single-core cables. These are: single-point bonding, solid bonding, and cross bonding (ANSI/IEEE, 1988).

In a single-point-bonded system, the considerable heating effect of circulating currents is avoided, but voltages will be induced along the length of the cable. These voltages are proportional to the conductor current and length of run, and increase as the cable spacing increases. Particular care must be taken to insulate and provide surge protection at the free end of the sheath to avoid danger from the induced voltages.

One way of eliminating the induced voltages is to bond the sheath at both ends of the run (solid bonding). The disadvantage of this is that the circulating currents that then flow in the sheaths reduce the current-carrying capacity of the cable.

Cross bonding of single-core cable sheaths is a method of avoiding circulating currents and excessive sheath voltages while permitting increased cable spacing and long run lengths. The increase in cable spacing increases the thermal independence of each cable and, hence, increases its current-carrying capacity. The cross bonding divides the cable run into three sections, and cross connects the sheaths in such a manner that the induced voltages cancel. One disadvantage of this system is that it is very expensive and, therefore, is applied mostly in high-voltage installations. Figure 1-11 gives a diagrammatic representation of the cross connections.

The cable route is divided into three equal lengths, and the sheath continuity is broken at each joint. The induced sheath voltages in each section of each phase are equal in magnitude and 120° out of phase. When the sheaths are cross connected, as shown in Figure 1-11, each sheath circuit contains one section from each phase such that the total voltage in each sheath circuit sums to zero. If the sheaths are then bonded and earthed at the end of the run, the net voltage in the loop and the circulating currents will be zero and the only sheath losses will be those caused by eddy currents.

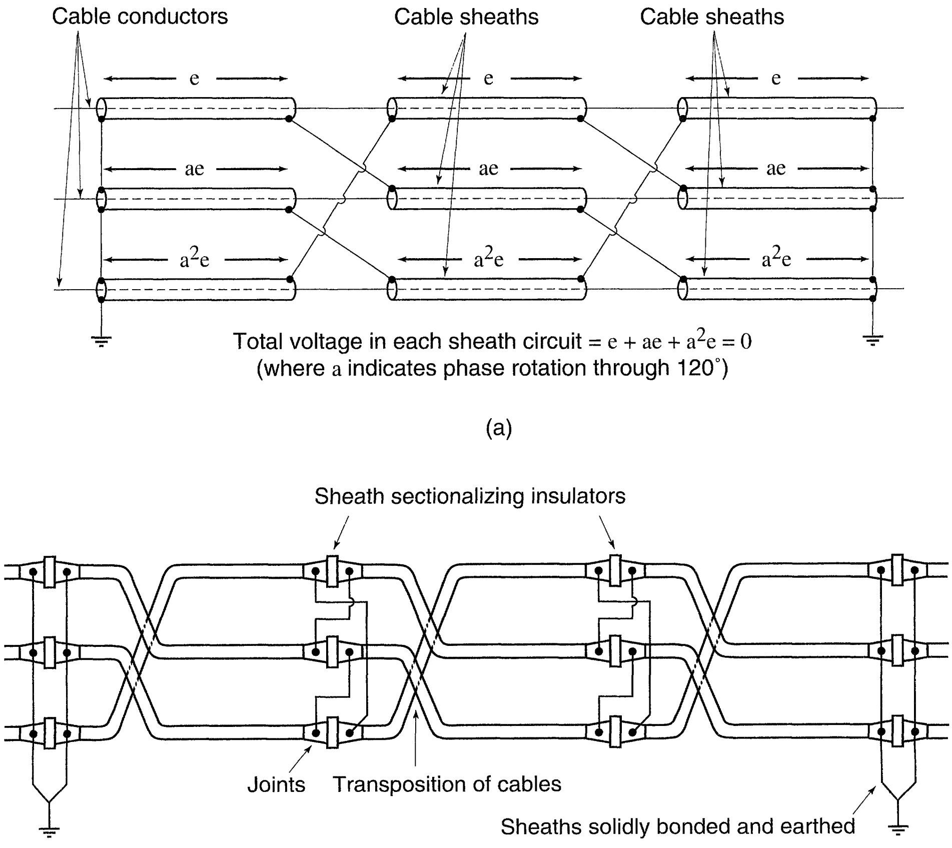


Figure 1-11 Diagrammatic representation of a cross bonded cable system. (a) Cables are not transposed. (b) Cables are transposed.

This method of bonding allows the cables to be spaced to take advantage of improved heat dissipation without incurring the penalty of increased circulating current losses. In practice, the lengths and cable spacings in each section may not be identical, and, therefore, some circulating currents will be present. The length of each section and cable spacings are limited by the voltages that exist between the sheaths and between the sheaths and earth at each cross-bonding position. For long runs, the route is divided into a number of lengths, each of which is divided into three sections. Cross bonding as described above can be applied to each length independently.

The cross-bonding scheme described above assumes that the cables are arranged symmetrically; that is, in a trefoil pattern. It is usual that single-core cables are laid in a flat configuration. In this case, it is a common practice in long-cable circuits or heavily loaded cable lines to transpose the cables as shown in Figure 1-11b so that each cable occupies each position for a third of the run.

All of the equations for sheath loss factors given in this section assume that the phase currents are balanced. The equations also require knowledge of the temperature of the sheath, which cannot be calculated until the cable rating is known, and, therefore, an iterative process is required. For the first calculation, the sheath temperature must be estimated; this estimate can be checked later after the current rating has been calculated. If necessary, the sheath losses, and, hence, the current rating, must be recalculated with the revised sheath temperature.

As discussed above, the power loss in the sheath or screen (λ_1) consists of losses caused by circulating currents (λ_1') and eddy currents (λ_1''). Thus,

$$\lambda_1 = \lambda_1' + \lambda_1'' \tag{1.86}$$

The loss factor in armor is also composed of two components: that due to circulating currents (λ_2') and, for magnetic armor, that caused by hysteresis (λ_2''). Thus,

$$\lambda_2 = \lambda_2' + \lambda_2'' \tag{1.87}$$

As mentioned above, for single-core cables with sheaths bonded at both ends of an electrical section, only losses caused by circulating currents are considered. An electrical section is defined as a portion of the route between points at which the sheaths or screens of all cables are solidly bonded. Circulating current losses are much greater than eddy current losses, and they completely dominate the calculations. Of course, there are no circulating currents when the sheaths are isolated or bonded at one point only.

The first step in computing the sheath loss factor is to obtain the sheath resistance and reactance. If the sheath is reinforced with a nonmagnetic tape or tapes, a parallel combination of both resistances is required. Once the sheath resistance is obtained, the sheath reactance needs to be computed. The appropriate formulae for the calculation of the sheath reactance are as follows.

For single-conductor and pipe-type cables:

$$X = 4\pi f \cdot 10^{-7} \cdot \ln \frac{2s}{d}$$

For single-conductor cables in flat formation, regularly transposed, sheaths bonded at both ends:

$$X_1 = 4\pi f \cdot 10^{-7} \cdot \ln\left[2 \cdot \sqrt[3]{2}\left(\frac{s}{d}\right)\right]$$

For single-conductor cables in flat configuration with sheaths solidly bonded at both ends, the sheath loss factor depends on the spacing. If it is not possible to maintain the same spacing in the electrical section (i.e., between points at which the sheaths of all cables are bonded), the following allowances should be made.

1). If the spacings are known, the value of X is computed from

$$X = \frac{l_a X_a + l_b X_b + \ldots l_n X_n}{l_a + l_b + \ldots l_n}$$

where

$l_a, l_b, \ldots, l_n$ are lengths with different spacing along an electrical section

$X_a, X_b, \ldots, X_n$ are the reactances per unit length of cable, given by equations for X and X_1, where appropriate values of spacing $s_a, s_b, \ldots, s_n$ are used

2. If the spacings are not known, the value of λ'_1 calculated below should be increased by 25%:

$$X_m = 8.71 \cdot 10^{-7} \cdot f$$

The following equations provide sheath loss factors for various bonding arrangements and cable configurations.

1.6.4.2 Loss Factors for Single-Conductor Cables

1. Sheath bonded both ends, triangular configuration:

$$\lambda'_1 = \frac{R_s}{R} \cdot \frac{1}{1 + \left(\frac{R_s}{X_1}\right)^2}; \qquad \lambda''_1 = 0.$$

2. Sheath bonded both ends, flat configuration, regular transposition:

$$\lambda'_1 = \frac{R_s}{R} \cdot \frac{1}{1 + \left(\frac{R_s}{X}\right)^2}; \qquad \lambda''_1 = 0$$

3. Sheath bonded both ends, flat configuration, no transposition. Center cable equidistant from other cables:

$$\lambda'_{11} = \frac{R_s}{R}\left[\frac{\frac{1}{4}Q^2}{R_s^2 + Q^2} + \frac{\frac{3}{4}P^2}{R_s^2 + P^2} - \frac{2R_sPQX_m}{\sqrt{3}(R_s^2 + Q^2)(R_s^2 + P^2)}\right] \quad \text{in the leading phase}$$

$$\lambda'_{1m} = \frac{R_s}{R}\,\frac{Q^2}{R_s^2 + Q^2} \quad \text{in the middle cable}$$

$$\lambda'_{12} = \frac{R_s}{R}\left[\frac{\frac{1}{4}Q^2}{R_s^2 + Q^2} + \frac{\frac{3}{4}P^2}{R_s^2 + P^2} + \frac{2R_sPQX_m}{\sqrt{3}(R_s^2 + Q^2)(R_s^2 + P^2)}\right] \quad \text{in the lagging phase}$$

$$\lambda''_1 = 0$$

where P and Q are defined by

$$P = X_m + X \qquad Q = X - \frac{X_m}{3}$$

Ratings for cables in air should be calculated using λ'_{11}.

Large Segmental Conductors. When the conductor proximity effect is reduced, for example, by large conductors having insulated segments, λ''_1 cannot be ignored and is calculated by multiplying the value of the eddy current sheath loss factor calculated below by F:

$$F = \frac{4M^2N^2 + (M + N)^2}{4(M^2 + 1)(N^2 + 1)}$$

where

$$M = N = \frac{R_s}{X}$$

for cables in trefoil formation, and

$$M = \frac{R_s}{X + X_m} \qquad N = \frac{R_s}{X - \frac{X_m}{3}}$$

for cables in flat formation with equidistant spacing.

Sheaths Single-Point Bonded or Cross Bonded. The sheath loss factor is obtained in this case from the following formula:

$$\lambda''_1 = \frac{R_s}{R}\left[g_s\lambda_0(1 + \Delta_1 + \Delta_2) + \frac{(\beta_1 t_s)^4}{12} \cdot 10^{-12}\right]$$

and the parameters in this expression depend on the cable arrangement, as follows. For lead-sheathed cables.

$$\beta_1 = 0 \qquad g_s = 1$$

For corrugated sheaths, the mean outside diameter should be used:

$$\beta_1 = \sqrt{\frac{4\pi\omega}{10^7 \varsigma}}$$

$$\omega = 2\pi f$$

$$g_s = 1 + \left(\frac{t_s}{D_s}\right)^{1.74} (\beta_1 D_s \times 10^{-3} - 1.6)$$

$$m = \frac{2\pi f}{R_s} \cdot 10^{-7}$$

$$\text{If } m \leq 0.1, \Delta_1 = 0, \Delta_2 = 0$$

Three Single-Conductor Cables in Triangular Configuration

$$\lambda_0 = 3\left(\frac{d}{2s}\right)^2 \frac{m^2}{1+m^2} \qquad \Delta_1 = (1.44m^{2.45} + 0.33)\left(\frac{d}{2s}\right)^{0.92m+1.66} \qquad \Delta_2 = 0$$

Three Single-Conductor Cables in Flat Configuration

1. Center cable:

$$\lambda_0 = 6\left(\frac{d}{2s}\right)^2 \frac{m^2}{1+m^2} \qquad \Delta_1 = 0.86m^{3.08}\left(\frac{d}{2s}\right)^{1.4m+0.7} \qquad \Delta_2 = 0$$

2. Outer cable leading phase:

$$\lambda_0 = 1.5\left(\frac{d}{2s}\right)^2 \frac{m^2}{1+m^2} \qquad \Delta_1 = 4.7m^{0.7}\left(\frac{d}{2s}\right)^{0.16m+2} \qquad \Delta_2 = 21m^{3.3}\left(\frac{d}{2s}\right)^{1.47m+5.06}$$

3. Outer cable lagging phase:

$$\lambda_0 = 1.5\left(\frac{d}{2s}\right)^2 \frac{m^2}{1+m^2} \qquad \Delta_1 = \frac{0.74(m+2)m^{0.5}}{2+(m-0.3)^2}\left(\frac{d}{2s}\right)^{m+1} \qquad \Delta_2 = 0.92m^{3.7}\left(\frac{d}{2s}\right)^{m+2}$$

Sheaths Cross Bonded. The ideal cross-bonded system will have equal lengths and spacing in each of the three sections. If the section lengths are different, the induced voltages will not sum to zero and circulating currents will be present. These circulating currents are taken account of by calculating the circulating current loss factor λ_1', assuming the cables were not cross bonded, and multiplying this value by a factor to take account of the length variations. This factor F_c is given by[12]

$$F_c = \frac{p_2^2 + q_2^2 + 1 - p_2 - p_2 q_2 - q_2}{(p_2 + q_2 + 1)^2}$$

where:
$p_2 a$ = length of the longest section
$q_2 a$ = length of the second longest section
a = length of the shortest section

This formula deals only with differences in the length of minor sections. Any deviations in spacing must also be taken into account.

Where lengths of the minor sections are not known, IEC 287-2-1 (1994) recommends that the value for λ_1' based on experience with carefully installed circuits be

$\lambda_1' = 0.03$ for cables laid directly in the ground
$\lambda_1' = 0.05$ for cables installed in ducts

1.6.4.3 Three-Conductor Cables. The sheath loss factor depends in this case on the value of the sheath resistance as follows.

Round or Oval Conductors in Common Sheath, No Armor

$R_s \leq 100\ \mu\Omega/\text{m}$

$$\lambda_1'' = \frac{3R_s}{R}\left[\left(\frac{2c}{d}\right)^2 \frac{1}{1 + \left(\frac{R_s 10^7}{\omega}\right)^2} + \left(\frac{2c}{d}\right)^4 \frac{1}{1 + 4\left(\frac{R_s 10^7}{\omega}\right)^2}\right]$$

$R_s > 100\ \mu\Omega/\text{m}$

$$\lambda_1'' = \frac{3.2\omega^2}{RR_s}\left(\frac{2c}{d}\right)^2 10^{-14}$$

Sector-Shaped Conductors

$$\lambda_1'' = 0.94\frac{R_s}{R}\left(\frac{2r_1 + t}{d}\right)^2 \frac{1}{1 + \left(\frac{R_s 10^7}{\omega}\right)^2}$$

Three-Conductor Cables with Steel-Tape Armor. The value for λ_1' calculated above should be multiplied by the factor F_t:

[12]This formula is somewhat different from the one appearing in the latest IEC Standard 60287 from 1994. It was developed recently in EDF and will be introduced into the standard during the next maintenance cycle.

$$F_t = \left[1 + \left(\frac{d}{d_a}\right)^2 \frac{1}{1 + \frac{d_a}{\mu\delta_0}}\right] \qquad \delta_0 = \frac{A}{\pi d_a}$$

μ is usually taken as 300.

Cables with Separate Lead Sheath (SL Type) with Armor

$$\lambda_1' = \frac{R_s}{R} \frac{1.5}{1 + \left(\frac{R_s}{X}\right)^2}$$

1.6.4.4 Pipe-Type Cables. Nonmagnetic core screens, copper or lead, of pipe-type cables will carry circulating currents induced in the same manner as in the sheaths of solidly bonded single-core cables. As with SL type cables, there is an increase in screen losses due to the presence of the steel pipe. The loss factor is given by:

$$\lambda_1' = \frac{R_s}{R} \frac{1.5}{1 + \left(\frac{R_s}{X}\right)^2}$$

If additional reinforcement is applied over the core screens, the above formula is applied to the combination of sheath and reinforcement. In this case, R_s is replaced by the resistance of the parallel combination of sheath and reinforcement, and the diameter is taken as the rms diameter d', where

$$d' = \sqrt{\frac{d^2 + d_2^2}{2}}$$

with
d = mean diameter of the screen or sheath, mm
d_2 = mean diameter of the reinforcement, mm

1.6.5 Armor Loss Factor

Armored single-core cables for general use in ac systems usually have nonmagnetic armor. This is because of the very high losses that would occur in closely spaced single-core cables with magnetic armor. On the other hand, when magnetic armor is used, losses due to eddy currents and hysteresis in the steel must be considered.

The armoring or reinforcement on two-core or three-core cables can be either magnetic or nonmagnetic. These cases are treated separately in what follows. Steel wires or tapes are generally used for magnetic armor.

When nonmagnetic armor is used, the losses are calculated as a combination of sheath and armor losses. The equations set out above for sheath losses are applied, but the resistance used is that of the parallel combination of sheath and armor, and the sheath diameter is replaced by the rms value of the mean armor and sheath diameters.

For nonmagnetic tape reinforcement where the tapes do not overlap, the resistance of the reinforcement is a function of the lay length of the tape. The advice given in IEC 60287 to deal with this is as follows.

1. If the tapes have a very long lay length, that is, are almost longitudinal tapes, the resistance taken is that of the equivalent tube, that is, a tube having the same mass per unit length and the same internal diameter as the tapes.
2. If the tapes are wound at about 54° to the axis of the cable, the resistance is taken to be twice the equivalent tube resistance.
3. If the tapes are wound with a very short lay, the resistance is assumed to be infinite; hence, the reinforcement has no effect on the losses.
4. If there are two or more layers of tape in contact with each other and having a very short lay, the resistance is taken to be twice the equivalent tube resistance. This is intended to take account of the effect of the contact resistance between the tapes.

The loss factor is then given by the following expressions.

1.6.5.1 Single-Conductor Cables

With Nonmagnetic Wire Armor

$$\lambda_1 = \frac{R_A}{R} \cdot \chi \qquad \lambda_2 = \frac{R_s}{R} \cdot \chi$$

where

$$\chi = \frac{R_s R_A}{(R_s + R_A)^2}$$

With Magnetic Wire Armor. The armor losses are lowest when the armor and sheath are bonded together at both ends of a run; thus, this condition is selected for the calculations below. The method gives a combined sheath and armor loss for cables that are very widely spaced (greater than 10 m). It has been applied for submarine cables where the cable spacing may be very wide and there is a need for the mechanical protection provided by the steel wire armor. The following method does not take into account the possible influence of the surrounding media, which may be appreciable, in particular for cables laid under water. It gives values of sheath and armor losses that are usually higher than the actual ones, so that ratings are on the safe side.

The ac resistance of the armor wires will vary between about 1.2 and 1.4 times its dc resistance, depending on the wire diameter, but this variation is not critical because the sheath resistance is generally considerably lower than that of the armor wires. For magnetic wire, the armor loss factor is obtained from

$$\lambda_1' = \lambda_2 = \frac{R_e}{R}\left[\frac{B_2^2 + B_1^2 + R_e B_2}{(R_e + B_2)^2 + B_1^2}\right]$$

where

$$B_1 = \omega(H_s + H_1 + H_3) \qquad B_2 = \omega H_2$$

and

$$H_s = 2 \times 10^{-7} \ln\left(\frac{2s_2}{d}\right) \qquad H_1 = \pi\mu_e\left(\frac{n_a d_f^2}{\ell_a d_a}\right)10^{-7} \sin\beta \cos\gamma$$

$$H_2 = \pi\mu_e\left(\frac{n_a d_f^2}{\ell_a d_a}\right)10^{-7} \sin\beta \sin\gamma \qquad H_3 = 0.4(\mu_t \cos^2\beta - 1)\left(\frac{d_f}{d_a}\right)10^{-6}$$

Average values for magnetic properties of armor wires with diameters in the range of 4–6 mm and tensile strengths on the order of 40 Mpa:

$\mu_e = 400$
$\mu_t = 10$ for armor wires in contact
$\mu_t = 1$ for armor wires that are spaced
$\gamma = \pi/4$

1.6.5.2 Three-Conductor Cables, Steel-Wire Armor. The loss factor depends on the armor construction as follows.

Round Conductor Cable

$$\lambda_2 = 1.23\frac{R_A}{R}\left(\frac{2c}{d_a}\right)^2 \frac{1}{\left(\frac{2.77\, R_A\, 10^6}{\omega}\right)^2 + 1}$$

For the SL type cables, λ_2 calculated above should be multiplied by $(1 - \lambda_1')$, where λ_1' is calculated in the section on the sheath loss factor for SL type cables.

Sector-Shaped Conductors

$$\lambda_2 = 0.358\frac{R_A}{R}\left(\frac{2r_1}{d_a}\right)^2 \frac{1}{\left(\frac{2.77\, R_A\, 10^6}{\omega}\right)^2 + 1}$$

1.6.5.3 Three-Conductor Cables, Steel-Tape Armor or Reinforcement.

The two components of the armor loss factor are computed as follows:

$$\lambda_2' = \frac{s^2k^2 10^{-7}}{Rd_a\delta_0} \qquad \lambda_2'' = \frac{2.23s^2k^2\delta_0 10^{-8}}{Rd_a}$$

with

$$\delta_0 = \frac{A}{\pi d_a} \qquad k = \frac{1}{1 + \dfrac{d_a}{\mu\delta_0}} \times \left(\frac{f}{50}\right)$$

μ is usually taken as 300.

Finally,

$$\lambda_2 = \lambda_2' + \lambda_2''$$

1.6.5.4 Pipe-Type Cables in Steel Pipe, Pipe Loss Factor.

For a pipe type cable, the loss factor in the metallic pipe depends on the cable arrangement in the pipe. The loss factor is given by

$$\lambda_2 = \left(\frac{f}{60}\right)^{1.5}\left(\frac{A \cdot s + B \cdot d_d}{R}\right)10^{-5}$$

where constants A and B are taken from Table 1-4.

1.6.6 Thermal Resistances

The internal thermal resistances and capacitances are characteristics of a given cable construction and were defined in Section 1.3. Without loss of accuracy, we will assume that these quantities are constant and independent of the component temperature. Where screening layers are present, we will also assume that for thermal calculations, metallic tapes are part of the conductor or sheath, whereas semiconducting layers (including metallized carbon-paper tapes) are part of insulation.

The units of the thermal resistance are K/W for a specified length. Since the length considered here is 1 m, the thermal resistance of a cable component is expressed in K/W per meter, which is most often written as K · m/W. This should be

Table 1-4 Constants for loss factor calculations in pipe-type cables

Configuration	A	B
Cradled	0.00438	0.00226
Triangular bottom of pipe	0.0115	–0.001485
Mean between trefoil and cradled	0.00794	0.00039
Three-core cable	0.0199	–0.001485

distinguished from the unit of thermal resistivity, which is also expressed as K · m/W. In transient computations discussed in Section 1.5, thermal capacitances associated with the same parts of the cable were identified. The unit of thermal capacitance is J/K · m and the unit of the specific heat of the material is J/K · m^3. The thermal resistivities and specific heats of materials used for insulation and for protective coverings are given in Table 1-5.

Table 1-5 Values of thermal resistivity and capacity for cable materials (IEC 60287, 1994)

Material	Thermal resistivity (ρ) (K · m/W)	Thermal capacity ($c \cdot 10^{-6}$) [J/(m^3 · K)]
Insulating materials*		
Paper insulation in solid-type cables	6.0	2.0
Paper insulation in oil-filed cables	5.0	2.0
Paper insulation in cables with external gas pressure	5.5	2.0
Paper insulation in cables with internal gas pressure		
a) preimpregnated	6.5	2.0
b) mass impregnated	6.0	2.0
PE	3.5	2.0
XLPE	3.5	2.0
Polyvinyl chloride		
up to and including 3 kV cables	5.0	1.7
greater than 3 kV cables	6.0	1.7
EPR		
up to and including 3 kV cables	3.5	2.0
greater than 3 kV cables	5.0	2.0
Butyl rubber	5.0	2.4
Rubber	5.0	2.4
Paper–polypropylene–paper (PPL)	6.5	2.0
Protective coverings		
Compounded jute and fibrous materials	6.0	2.0
Rubber sandwich protection	6.0	2.0
Polychlroprene	5.5	2.0
PVC		
up to and including 35 kV cables	5.0	1.7
greater than 35 kV cables	6.0	1.7
PVC/bitumen on corrugated aluminum sheaths	6.0	1.7
PE	3.5	2.4
Materials for duct installations		
Concrete	1.0	2.3
Fiber	4.8	2.0
Asbestos	2.0	2.0
Earthenware	1.2	1.8
PVC	6.0	1.7
PE	3.5	2.4

*For the purpose of current rating computations, the semiconducting screening materials are assumed to have the same thermal properties as the adjacent dielectric materials.

The thermal resistances of the insulation and the external environment of a cable have the greatest influence on cable rating. In fact, for the majority of buried cables, the external thermal resistance accounts for more than 70% of the temperature rise of the conductor. For cables in air, the external thermal resistance has a smaller effect on cable rating than in the case of buried cables.

The calculation of thermal resistances of the internal components of cables for single-core cables, whether based on rigorous mathematical computations or empirical investigations, is straightforward. For three-core cables the calculations are somewhat more involved. Also, the calculation of the external thermal resistance requires particular attention. The relevant formulae are given below.

1.6.6.1 Thermal Resistance of the Insulation

Single-Conductor Cables. The thermal resistance between one conductor and the sheath is computed from

$$T_1 = \frac{\rho}{2\pi} \ln\left(1 + \frac{2t_1}{d_c}\right)$$

For oval-shaped conductors, the diameter over the insulation is the geometric mean of the minor and major diameters over the insulation.

For corrugated sheaths, t_1 is based on the mean internal diameter of the sheath which is equal to $[(D_{it} + D_{oc})/2] - t_s$.

The dimensions of the cable occur in $\ln[1 + (2t_1/d_c)]$ and, therefore, this expression plays the role of a geometric factor or shape modulus, and has been termed the geometric factor.

Three-Conductor Belted Cables. The computation of the internal thermal resistance of three-core cables is more complicated than for the single-core case. Rigorous mathematical formulas cannot be determined, although mathematical expressions to fit the conditions have been derived either experimentally or numerically. The general method of computation employs geometric factor (G) in place of the logarithmic term in the equation given above; that is,

$$T_1 = \frac{\rho}{2\pi} G$$

The value of G is obtained from Figure 1-12.

For cables with sector-shaped conductors,

$$G = F_1 \ln\left(\frac{d_a}{2r_1}\right) \qquad \text{and} \qquad F_1 = 3 + \frac{9t}{2\pi(d_x + t) - t}$$

Also, d_a = external diameter of the belt insulation, mm.

All three-core cables require fillers to fill the space between insulated cores and the belt insulation or a sheath. In the past, when impregnated paper was used to insulate the conductors, the resistivity of the filler material very closely matched that of the paper (around 6 K · m/W). Because polyethylene insulation has much lower thermal resistivity (3.5 K · m/W), the higher thermal resistivity of the filler may have a significant influence on the overall value of T_1 and, hence, on the cable rating. For these cables, the following approximating formula is applied:

$$T_1^{filler} = \frac{\rho_i}{2\pi} G + 0.031(\rho_f - \rho_i)e^{0.67t/d_c}$$

where ρ_f and ρ_i are the thermal resistivities of filler and insulation, respectively, and G is the geometric factor obtained from Figure 1-12, assuming $\rho_f = \rho_i$.

Three-Conductor Shielded Cables. Screening reduces the thermal resistance of a cable by providing additional heat paths along the screening material of high thermal

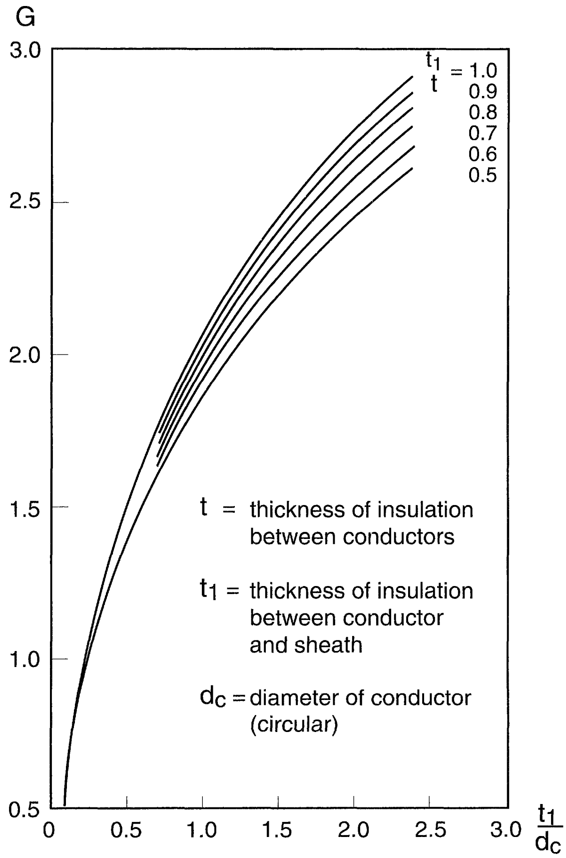


Figure 1-12 Geometric factor for three-core belted cables with circular conductors (IEC 60287, 1994).

conductivity, in parallel with the path through the dielectric. The thermal resistance of the insulation is thus obtained in two steps. First, the cables of this type are considered as belted cables for which $t_1/t = 0.5$. Then, in order to take account of the thermal conductivity of the metallic screens, the results are multiplied by a factor K, called the screening factor, values of which are obtained from Figure 1-13. Thus, we have

$$T_1 = K\frac{\rho}{2\pi}G$$

As discussed above, filler resistivity may have a significant influence on the value of T_1 for plastic-insulated cables. However, for screened cables, IEC advises that no additional modifications of the above formula be taken.

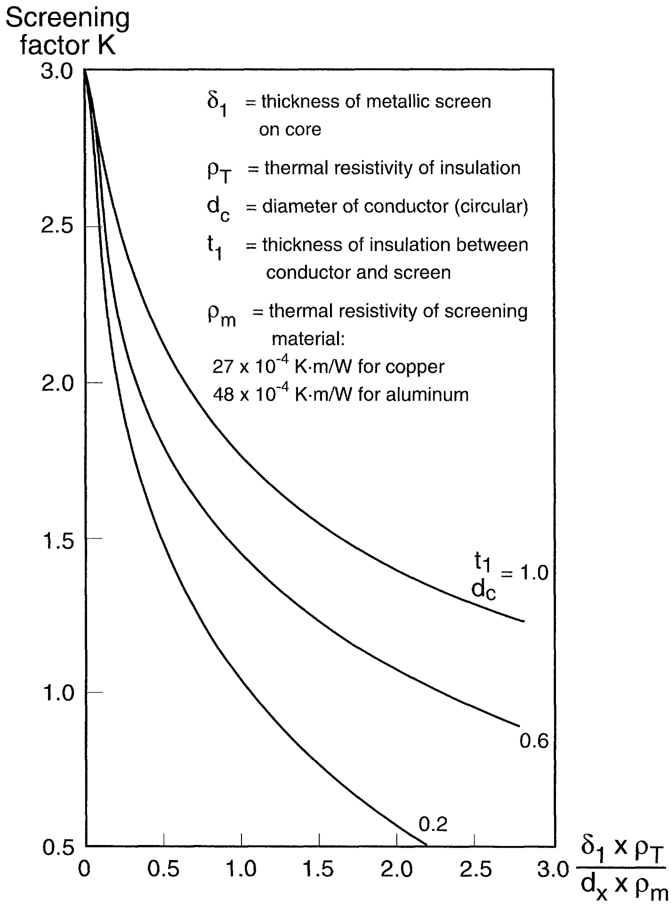


Figure 1-13 Thermal resistance of three-core screened cables with circular conductors compared to that of a corresponding unscreened cable (IEC 287, 1994).

For cables with sector-shaped conductors,

$$G = F_1 \ln\left(\frac{d_a}{2r_1}\right) \quad \text{and} \quad F_1 = 3 + \frac{9t}{2\pi(d_x + t) - t}$$

The screening factor is obtained from Figure 1-14.

Oil-Filled Cables with Round Conductors and Round Oil Ducts Between Cores. In cables of this construction, oil ducts are provided by laying up an open spiral duct of metal strip in each filler space. The expression for the thermal resistance between one core and the sheath was obtained experimentally:

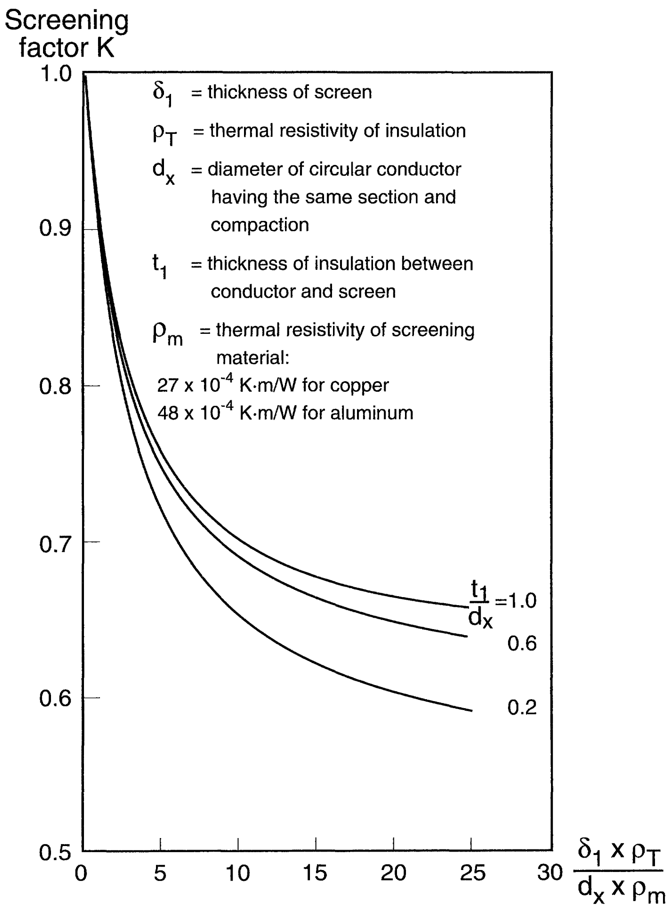


Figure 1-14 Thermal resistance of three-core screened cables with sector-shaped conductors compared to that of a corresponding unscreened cable (IEC 287-2-1, 1994).

$$T_1 = 0.358\rho\left(\frac{2t_i}{d_c + 2t_i}\right)$$

where t_i (mm) is the thickness of core insulation, including carbon black and metallized paper tapes plus half of any nonmetallic tapes over the three laid-up cores. This equation assumes that the space occupied by the metal ducts and the oil inside them has a very high thermal conductance compared with that of the insulation; the equation, therefore, applies irrespective of the metal used to form the duct or its thickness.

Three-Core Cables with Circular Conductors and Metal-Tape Core Screens and Circular Oil Ducts between the Cores. The thermal resistance between one conductor and the sheath is

$$T_1 = 0.35\rho\left(0.923 - \frac{2t_i}{d_c + 2t_i}\right)$$

where t_i (mm) is the thickness of core insulation, including the metal screening tapes plus half of any nonmetallic tapes over the three laid-up cores.

SL Type Cables. In SL type cables the lead sheath around each core may be assumed to be isothermal. The thermal resistance T_1 is calculated the same way as for single-core cables.

1.6.6.2 Thermal Resistance between Sheath and Armor, T_2

Single-Core, Two-Core, and Three-Core Cables Having a Common Metallic Sheath. The thermal resistance between sheath and armor is obtained from Equation (1.14), representing thermal resistance of any concentric layer. With the notation applicable to this part of the cable, we have

$$T_2 = \frac{\rho}{2\pi}\ln\left(1 + \frac{2t_2}{D_s}\right)$$

SL-Type Cables. In these cables, the thermal resistance of the fillers between sheaths and armoring is given by

$$T_2 = \frac{\rho}{6\pi}\overline{G}$$

where $\overline{G}$ is the geometric factor given in Figure 1-15.

1.6.6.3 Thermal Resistance of Outer Covering (Serving), T_3. The external servings are generally in the form of concentric layers, and the thermal resistance T_3 is given by

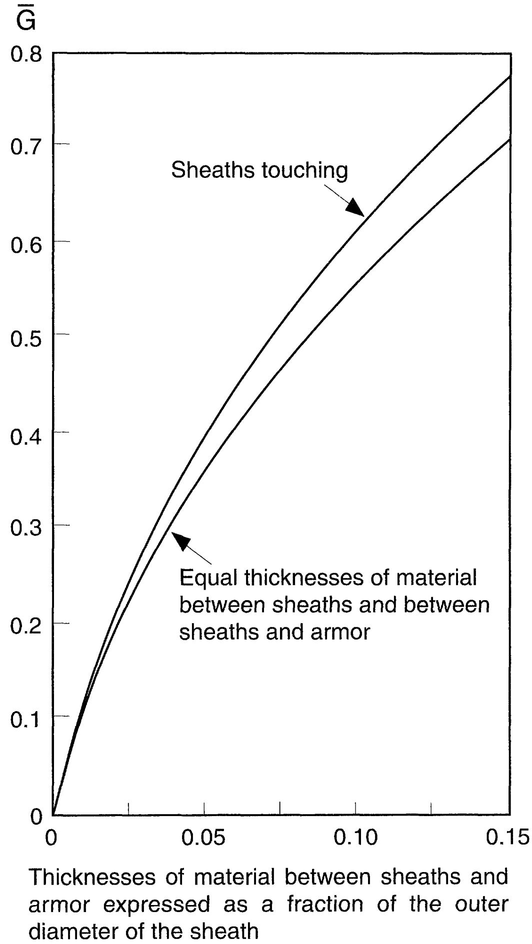


Figure 1-15 Geometric factor for obtaining the thermal resistances of the filling material between the sheaths and armor of SL-type cables (IEC 287-1-1, 1994).

$$T_3 = \frac{\rho}{2\pi} \ln\left(1 + \frac{2t_3}{D_a'}\right)$$

For corrugated sheaths,

$$T_3 = \frac{\rho}{2\pi} \ln\left[\frac{D_{oc} + 2t_3}{\left(\frac{D_{oc} + D_{it}}{2}\right) + t_s}\right]$$

1.6.6.4 Pipe-Type Cables. For these three-core cables, the following computational rules apply:

1. The thermal resistance T_1 of the insulation of each core between the conductor and the screen is calculated from the equation for single-core cables.
2. The thermal resistance T_2 is made up of two parts:
   (a) The thermal resistance of any serving over the screen or sheath of each core. The value to be substituted for part of T_2 in the rating equation is the value per cable; that is, the value for a three-core cable is one-third of the value of a single core. The value per core is calculated by the method given in Section 1.6.6.2 for the bedding of single-core cables. For oval cores, the geometric mean of the major and minor diameters $\sqrt{d_M d_m}$ is used in place of the diameter for a circular core assembly.
   (b) The thermal resistance of the gas or liquid between the surface of the cores and the pipe. This resistance is calculated in the same way as part T_4, which is between a cable and the internal surface of a duct, as given in Section 1.6.6.5. The value calculated will be per cable and should be added to the quantity calculated in (a) above, before substituting for T_2 in the rating equation .
3. The thermal resistance T_3 of any external covering on the pipe is dealt with as in Section 1.6.6.3. The thermal resistance of the metallic pipe itself is negligible.

1.6.6.5 External Thermal Resistance. The current-carrying capability of cables depends to a large extent on the thermal resistance of the medium surrounding the cable. For a cable laid underground, this resistance accounts for more than 70% of the temperature rise of the conductor. For underground installations, the external thermal resistance depends on the thermal characteristics of the soil, the diameter of the cable, the depth of laying, mode of installation (e.g., directly buried, in thermal backfill, in pipe or duct, etc.), and on the thermal field generated by neighboring cables. For cables in air, the external thermal resistance has a smaller effect on the cable rating. For aerial cables, the effect of installation conditions (e.g., indoors or outdoors, proximity of walls and other cables, etc.) is an important factor in the computation of the external thermal resistance. In the following sections, we will describe how the external thermal resistance of buried and aerial cables is computed.

External Thermal Resistance of Buried Cables—Directly Buried Cables. For buried cables, two values of the external thermal resistance are calculated: T_4, corresponding to dielectric losses (100% load factor); and $T_{4\mu}$, the thermal resistance corresponding to the joule losses, where allowance is made for the daily load factor (LF) and the corresponding loss factor μ (a more detailed discussion about the load loss factor is offered in Chapter 5):

$$\mu = 0.3 \cdot (LF) + 0.7 \cdot (LF)^2$$

The effect of the loss factor is considered to start outside a diameter D_x, defined as $D_x = 61200\sqrt{\delta(\text{length of cycle in hours})}$ where δ is soil diffusivity (m^2/h). For a daily load cycle and typical value of soil diffusivity of 0.5×10^{-6} m^2/s, D_x is equal to 211 mm (or 8.3 in). The value of D_x is valid even when the diameter of the cable or pipe is greater than D_x.

The following expressions are used to calculate the external thermal resistance of a single cable or a group of equally loaded identical cables:

$$T_2 = \frac{\rho_s}{2\pi} \ln \frac{4L \cdot F}{D_e} \qquad T_{4\mu} = \frac{\rho_s}{2\pi}\left(\ln \frac{D_x}{D_e} + \mu \cdot \ln \frac{4L \cdot F}{D_x}\right)$$

For cables in pipes, one should use D_o in place of D_e in the above formulas.

A factor F accounts for the mutual heating effect of the other cables or cable pipes in a system of equally loaded, identical cables or cable pipes. For several loaded cables placed underground, we must deal with superimposed heat fields. The principle of superposition is applicable if we assume that each cable acts as a line source and does not distort the heat field of the other cables. Therefore, in the following subsections, we will assume that the cables are spaced sufficiently apart so that this assumption is approximately valid. The axial separation of the cables should be at least two cable diameters. The case in which the superposition principle is not applicable is discussed in Section 9.6.3 of Anders (1997).

The distances needed to compute factor F are defined in Figure 1-16. These are center-to-center distances.

For cable p:

$$F = \left(\frac{d'_{p1}}{d_{p1}}\right)\left(\frac{d'_{p2}}{d_{p2}}\right)\cdots\left(\frac{d'_{pk}}{d_{pk}}\right)\cdots\left(\frac{d'_{pq}}{d_{pq}}\right)$$

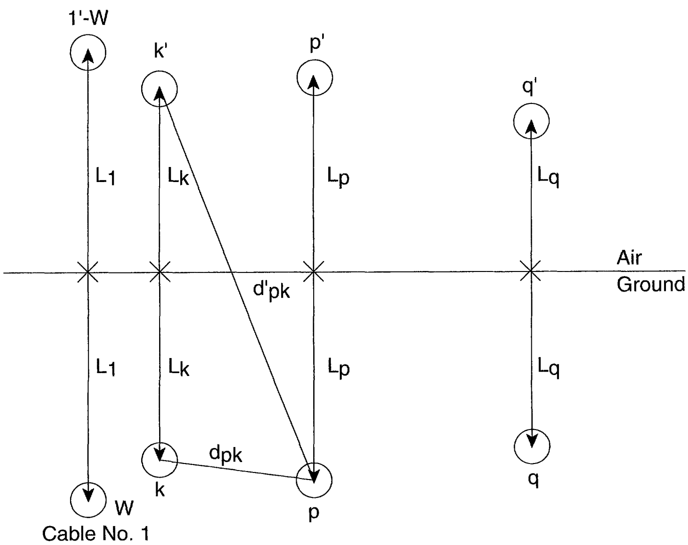


Figure 1-16 Illustration of the development of an equation for the external thermal resistance of a single cable buried under an isothermal plane.

There are $(q - 1)$ terms, with the term d'_{pp}/d_{pp} excluded. The rating of the cable system is determined by the rating of the hottest cable or cable pipe, usually the cable with the largest ratio, L/D_o. For a single isolated cable or cable pipe, $F = 1$.

When the losses in the sheaths of single-core cables laid in a horizontal plane are appreciable, and the sheaths are laid without transposition and/or are bonded at all joints, the inequality of losses affects the external thermal resistance of the cables. In such cases, the value of the F factor used to calculate T_4 and $T_{4\mu}$ is modified by first computing the sheath factor (SHF):

$$(SHF) = \frac{1 + 0.5(\lambda'_{11} + \lambda'_{12})}{1 + \lambda'_m}$$

and then calculating

$$F' = F^{(SHT)}$$

External Thermal Resistance of Buried Cables—Cables or Cable Pipes Buried in Thermal Backfill. In many North American cities, medium- and low-voltage cables are often located in duct banks in order to allow a large number of circuits to be laid in the same trench. The ducts are first installed in layers with the aid of distance pieces, and then a bedding of filler material is compacted after each layer is positioned. Concrete is the material most often used as a filler. High- and extra-high-voltage cables are, on the other hand, often placed in an envelope of well-conducting backfill to improve heat dissipation. What both methods of installation have in common is the presence of a material that has a different thermal resistivity from that of the native soil. The first attempt to model the presence of a duct bank or a backfill was presented by Neher and McGrath (1957) and later adopted in IEC Standard 60287 (1982). Later, the basic method of Neher and McGrath was extended to take into account backfills and duct banks of elongated rectangular shapes, and to remove the assumption that the external perimeter of the rectangle is isothermal.

The application of a thermal backfill is discussed in some detail in Chapter 4 of this book. The formulae presented below and used in Chapter 4 apply to the case in which the longer to shorter length ratio is smaller than 3.

When the cables are installed in a backfill or duct bank, the following modification of the expressions for the external thermal resistance are applied:

$$T_4 = \frac{\rho_c}{2\pi} \ln \frac{4L \cdot F}{D_e} + \frac{N}{2\pi}(\rho_e - \rho_c)G_b$$

$$T_{4\mu} = \frac{\rho_c}{2\pi}\left(\ln \frac{D_x}{D_e} + \mu \ln \frac{4L \cdot F}{D_x}\right) + \mu\frac{N}{2\pi}(\rho_e - \rho_c)G_b$$

where

$$G_b = \ln[u + \sqrt{(u^2 - 1)}] \qquad u = \frac{L_G}{r_b} \quad \text{and} \quad \ln r_b = \frac{1}{2}\frac{x}{y}\left(\frac{4}{\pi} - \frac{x}{y}\right)\ln\left(1 + \frac{y^2}{x^2}\right) + \ln\frac{x}{2}$$

External Thermal Resistance of Buried Cables—Cables in Ducts and Pipes. This section deals with the external thermal resistance of cables in ducts or pipes filled with air or a liquid. Cables in ducts that have been completely filled with a pumpable material having a thermal resistivity not exceeding that of the surrounding soil, either in the dry state or when sealed to preserve the moisture content of the filling material, may be treated as directly buried cables.

The external thermal resistance of a cable in duct or pipe consists of three parts:

1. The thermal resistance of the air or liquid between the cable surface and the duct internal surface, T'_4.
2. The thermal resistance of the duct itself, T''_4 The thermal resistance of a metal pipe is negligible.
3. The external thermal resistance of the duct, T''_4

The value of T_4 to be substituted in the current rating equation will be the sum of the individual parts; that is:

$$T_4 = T'_4 + T''_4 + T'''_4 \qquad T_{4\mu} = T'_4 + T''_4 + T'''_{4\mu}$$

The constituent thermal resistances are computed from the following formulae:

$$T'_4 = \frac{U}{1 + 0.1(V + Y\theta_m)D_e}$$

Constants U, V, and Y are read from Table 1-6.

Table 1-6 Values of contants U, V, and Y (IEC 60287, 1994)

Installation condition	U	V	Y
In metallic conduit	5.2	1.4	0.011
In fiber duct in air	5.2	0.83	0.006
In fiber duct in concrete	5.2	0.91	0.010
In asbestos cement			
duct in air	5.2	1.2	0.006
duct in concrete	5.2	1.1	0.011
Earthenware ducts	1.87	0.28	0.0036
Gas pressure cable in pipe	0.95	0.46	0.0021
Oil-pressure, pipe-type cable	0.26	0.0	0.0026

$$T_4'' = \frac{\rho}{2\pi} \ln \frac{D_o}{D_d}$$

where ρ is the thermal resistivity of duct material. For metal ducts, $T_4'' = 0$ and

$$T_4''' = \frac{\rho_c}{2\pi}\left(\ln \frac{D_x}{D_d} + \mu \ln \frac{4L \cdot F}{D_x}\right) + \mu \frac{N}{2\pi}(\rho_e - \rho_c)G_b$$

External Thermal Resistance of Buried Cables—Unequally Loaded or Dissimilar Cables. The method suggested for the calculation of ratings of a group of cables set apart is to calculate the temperature rise at the surface of the cable under consideration caused by the other cables of the group, and to subtract this rise from the value of $\Delta\theta$ used in Equation (1.40) for the rated current. An estimate of the power dissipated per unit length of each cable must be made beforehand, and this can be subsequently amended as a result of the calculation where it becomes necessary.

For cable j, the losses are

$$W_j = n[I_j^2 R_j(1 + \lambda_1 + \lambda_2)\mu_j + W_{dj}]$$

The thermal resistance between cable j and cable i, the cable being studied, is

$$T_{ij} = \frac{\rho_s}{2\pi} \ln \frac{d'_{ij}}{d_{ij}} \qquad \text{for directly buried cables}$$

$$T_{ij} = \frac{\rho_c}{2\pi} \ln \frac{d'_{ij}}{d_{ij}} + \frac{N}{2\pi}(\rho_e - \rho_c)G_b \qquad \text{for cables in backfill or duct bank}$$

The temperature rise at the surface of cable i due to the losses in cable j is given by

$$\Delta\theta_{ij} = W_j \cdot T_{ij}$$

and the temperature rise at the surface of cable i due to all other cables in the group is computed from

$$\Delta\theta_{\text{int}} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \Delta\theta_{ij}$$

External Thermal Resistance of Cables in Air—Simple Configurations. Heat transfer phenomena are more complex for cables installed in free air than for those located underground. The external thermal resistance of cables in air can be written as

$$T_4 = \frac{1}{\pi D_e^* h_t}$$

where h_t is the total heat transfer coefficient defined in Equation (1.21). This coefficient is a nonlinear function of the cable surface temperature and one approximation, proposed in IEC 60287 (1994), is

$$T_4 = \frac{1}{\pi D_e^* h(\Delta\theta_s)^{1/4}}$$

where h (W/m$^2K^{5/4}$) is the heat transfer coefficient embodying convection, radiation, conduction, and mutual heating and is given by the following analytical expression:

$$h = \frac{Z}{(D_e^*)^g} + E$$

Constants Z, E, and g are given in Table 1-7. Served cables and cables having a nonmetallic surface are considered to have a black surface. Unserved cables, either plain lead or armored, should be given a value of h equal to 88% of the value for the black surface.

The cable surface temperature rise is obtained iteratively from the following equation:

$$(\Delta\theta_s)_{n+1}^{1/4} = \left[\frac{\Delta\theta + \Delta\theta_d}{1 + K_A(\Delta\theta_s)_n^{1/4}}\right]^{1/4}$$

The iterations are started by setting the initial value of $(\Delta\theta_s)^{1/4} = 2$ and reiterating until $(\Delta\theta_s)_{n+1}^{1/4} - (\Delta\theta_s)_n^{1/4} \le 0.001$.

The variable K_A is defined as

$$K_A = \frac{\pi D_e^* h}{1 + \lambda_1 + \lambda_2}\left[\frac{T_1}{n} + (1 + \lambda_1)T_2 + (1 + \lambda_1 + \lambda_2)T_3\right]$$

$\Delta\theta_d$ is the temperature rise caused by dielectric losses and is obtained from Equations (1.41) and (1.44) as

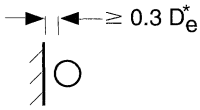
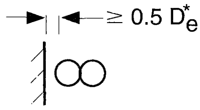
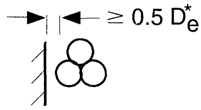
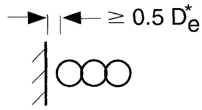
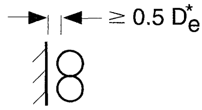
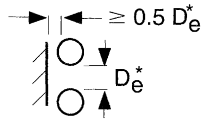
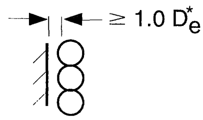
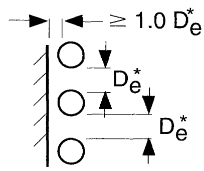
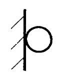
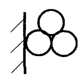
$$\Delta\theta_d = W_d\left[\left(\frac{1}{1 + \lambda_1 + \lambda_2} - \frac{1}{2}\right)T_1 - \frac{n\lambda_2 T_2}{1 + \lambda_1 + \lambda_2}\right]$$

If the dielectric losses are neglected, $\Delta\theta_d = 0$.

When the cable is exposed to solar radiation, the following expression applies:

$$(\Delta\theta_s)_{n+1}^{1/4} = \left[\frac{\Delta\theta + \Delta\theta_d + \sigma H K_A/\pi h}{1 + K_A(\Delta\theta_s)_n^{1/4}}\right]^{1/4}$$

Table 1-7 Values for constants Z, E, and g for black surfaces of cables

No.	Installation	*Z*	*E*	*G*	Mode
1	Single cable[a]	0.21	3.94	0.60	
2	Two cables touching, horizontal	0.21	2.35	0.50	
3	Three cables trefoil pattern	0.96	1.25	0.20	
4	Three cables touching, horizontal	0.62	1.95	0.25	
5	Two cables touching, vertical	1.42	0.86	0.25	
6	Two cables spaced D_e^*, vertical	0.75	2.80	0.30	
7	Three cables touching, vertical	1.61	0.43	0.20	
8	Three cables spaced D_e^*, vertical	1.31	2.00	0.20	
9	Single cable	1.69	0.63	0.25	
10	Three cables in trefoil	0.94	0.79	0.20	

[a]Values for a "single cable" also apply to each cable of a group when they are spaced horizontally with a clearance between cables of at least 0.75 times the cable overall diemeter.

The solar absorption coefficients for various covering materials are given in Table 1-8.

Plain lead or unarmored cables should be assigned a value of h equal to 80% of the value for a cable with a black surface.

Ambient temperature should be increased by $\Delta\theta_{sr}$, where

$$\Delta\theta_{sr} = \sigma D_e^* H T_4$$

External Thermal Resistance of Cables in Air—Derating Factors for Groups of Cables in IEC 601042 (1991). An approach to dealing with groups of identical cables in air, shown in Figure 1-17, is presented in IEC60287-2-2 (1995) and discussed in Anders (1997). The following limitations apply in this case:

1. A maximum of nine cables in a square formation (the last arrangement in Figure 1-17a)
2. A maximum of six circuits, each comprising three cables mounted in a trefoil pattern, with up to three circuits placed side by side or two circuits placed one above the other (the last arrangement in Figure 1-17b)
3. Cables for which dielectric losses can be neglected (usually, only lower voltage polymeric cables are installed in groups).

When cables are installed in groups as shown in Figure 1-17, the rating of the hottest cable will be lower than in the case when the same cable is installed in isolation. This reduction is caused by mutual heating. A simple method to account for this mutual heating effect is to calculate the rating of a single cable or circuit using the method described in the previous section and apply a reduction factor. This is defined as follows:

$$I_g = F_g \cdot I_1$$

where
I_g = rating of the hottest cable in the group, A
I_1 = rating of the same cable or circuit isolated, A
F_g = group reduction factor

Table 1-8 Absorption coefficient of solar radiation values for cable surfaces

Material	σ
Bitumen/jute serving	0.8
Polychloroprene	0.8
PVC	0.6
PE	0.4
Lead	0.6

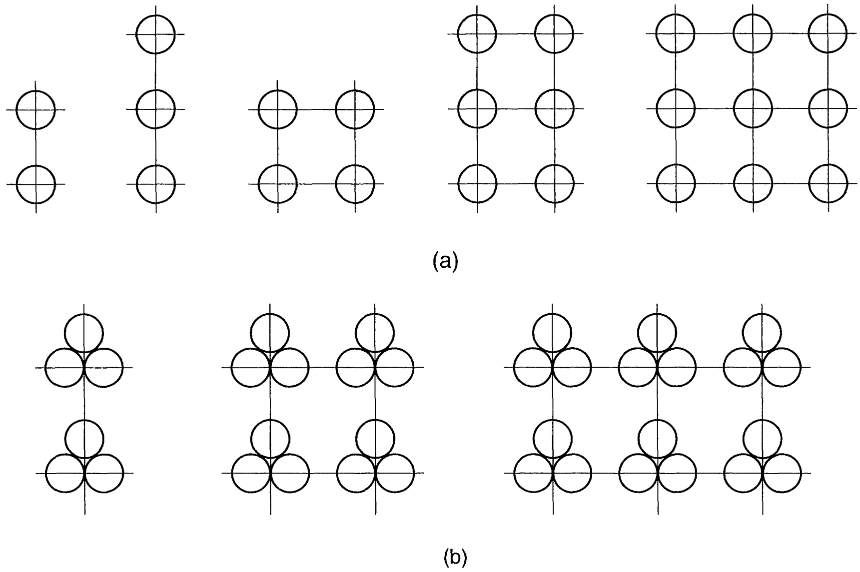


Figure 1-17 Typical groups of (a) multicore cables, and (b) trefoil circuits.

The reduction in current is a result of an increase in the external thermal resistance of a cable or circuit in a group as compared to the case in which the cable or circuit is installed in isolation. A basic criterion is that the temperature rise above ambient of the conductor be the same for the grouped and isolated cases.

The reduction factor is computed from

$$F_g = \sqrt{\frac{1}{1 - k_1 + k_1(T_{4g}/T_{4I})}}$$

where

$$k_1 = \frac{W_t \cdot T_{4I}}{\theta_c - \theta_{amb}}$$

and

T_{4g} = external thermal resistance of the hottest cable in the group, K · m/W

T_{4I} = external thermal resistance of one cable, assumed to be isolated, used to compute the rated current I, K · m/W

W_t= power loss from one multicore cable or one single-core cable mounted in a trefoil pattern, assumed to be isolated, when carrying the current I_1, W/m

The term T_{4g}/T_{4I} can be calculated from the ratio (h_I/h_g) by using the iterative relationship

$$\left(\frac{T_{4g}}{T_{4I}}\right)_{n+1} = \frac{h_I}{h_g}\left[\frac{1-k_1}{(T_{4g}/T_{4I})_n} + k_1\right]^{0.25}$$

and starting with $(T_{4g}/T_{4I})_1 = (h_I/h_g)$. The above equation converges quickly, and one iteration with $(T_{4g}/T_{4I})_1 = (h_I/h_g)$ is usually sufficient. Alternatively, when (h_I/h_g) is less than 1.4, it is sufficient to substitute (h_I/h_g) for (T_{4g}/T_{4I}) in the equation for F_g.

Values for the ratio (h_I/h_g) have been obtained empirically and are given in of Table 1-9 (IEC 1042, 1991).

If a clearance exceeding the appropriate value given in column 2 of Table 1-9 cannot be maintained with confidence throughout the length of the cable, the reduction coefficient is determined as follows.

1. For horizontal clearances, we assume that the cables are touching each other or the vertical surface. Appropriate values are given in column 4 of Table 1-9.
2. For vertical clearances, the reduction coefficient due to grouping is derived according to the value of the expected clearance:
   a) Where the clearance is less than the appropriate value given in column 2 of Table 1-9, but can be maintained at a value equal or exceeding the minimum given in column 3, the appropriate value of (h_I/h_g) is obtained from the formula in column 4.
   b) Where the clearance is less than the minimum given in column 3, we assume that the cables are touching each other. Suitable values of (h_I/h_g) are provided in column 4 of Table 1-9.

The values in column 4 are the average values for cables having diameters from 13 to 76 mm. More precise values for multicore cables may be evaluated for a specific cable diameter, both inside and outside this range, by consulting Table 1-7.

1.6.7 Thermal Capacitances

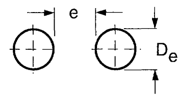
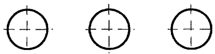
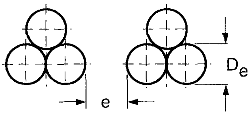
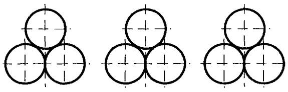
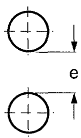
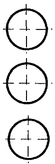
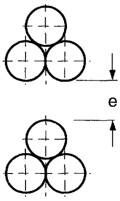
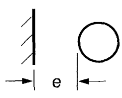
In Section 1.3.1.2, thermal capacitance was defined as the product of the volume of the material and its specific heat. In the following section, formulas for special and concentric layers are presented. In all formulas, Q is the thermal capacitance (J/K · m) and c is the specific heat of the material (J/K · m^3).

1.6.7.1 Oil in the Conductor. The thermal capacitance of oil inside the conductor is obtained from

$$Q_o = \left(\frac{\pi d_i^{*2}}{4} + S \cdot F\right)c$$

where F is a factor representing the unfilled cross section (usually 0.36).

Table 1-9 Data for calculating reduction factors for grouped cables

Arrangement of cable		Thermal proximity effect is negligible if e/D_e is greater then or equal to:	Thermal proximity effect is not negligible: If e/D_e is less then	Thermal proximity effect is not negligible: Average value of h_l/h_g[a,b]
Side by Side				
2 multicore		0.5	0.5	1.41
3 multicore		0.75	0.75	1.65
2 trefoils		1.0	1.0	1.2
3 trefoils		1.5	1.5	1.25
One above the other				
2 multicore		2	2 or 0.5	$1.085\,(e/D_e)^{-0.128}$ or 1.35
3 multicore		4	4 or 0.5	$1.19\,(e/D_e)^{-0.136}$ or 1.57
2 trefoils		4	4 or 0.5	$1.106\,(e/D_e)^{-0.078}$ or 1.39
Near to a vertical surface or to a horizontal surface below the cable		0.5	0.5	1.23

[a]The formulae for (h_l/h_g) given in column 4 of this table shall not be used for values of e/D_e less than 0.5 or greater than the appropriate values given in column 1.
[b]Average values for cables having diameter from 13 mm to 76 mm. More precise values of (h_l/h_g) for multicore cables may be evaluated for a specific cable diameter, both inside and outside this range, by consulting Table V of IEC 287.

1.6.7.2 Conductor. The thermal capacitance of the conductor is given by

$$Q_c = S \cdot c$$

1.6.7.3 Insulation. Representation of the capacitance of the insulation depends on the duration of the transient. For transients lasting longer than about 1 h, the capacitance of the insulation is divided between the conductor and the sheath positions according to a method given by Van Wormer (1955) (see Section 1.3.1.2). This assumes logarithmic temperature distribution across the dielectric during the transient, as in the steady state. The total thermal capacitance is obtained as

$$Q_i = \frac{\pi}{4}(D_i^{*2} - d_c^{*2})c$$

The portion pQ_i is placed at the conductor and the portion $(1 - p)Q_i$ at the sheath, where p is the Van Wormer coefficient, defined in Equation (1.24) as

$$p = \frac{1}{2\ln\left(\frac{D_i}{d_c}\right)} - \frac{1}{\left(\frac{D_i}{d_c}\right)^2 - 1}$$

From the thermal point of view, the thickness of the dielectric includes any nonmetallic semiconducting layer either on the conductor or on the insulation.

For shorter durations (less than 1 h), it has been found necessary to divide the insulation into two portions having equal thermal resistances. The thermal capacitance of each portion of the insulation is then assumed to be located at its boundaries, using the Van Wormer coefficient to split the capacitances as shown in Figure 1-2. The thermal capacitance of the first part of the insulation is given by

$$Q_{I1} = \frac{\pi}{4}(D_i^* \cdot d_c^* - d_c^{*2})c$$

This capacitance is split into two parts using the Van Wormer coefficient as follows:

$$Q_{i1} = p^* Q_{I1}, \qquad Q_{i2} = (1 - p^*) Q_{I1} \qquad p^* = \frac{1}{\ln\frac{D_i}{d_c}} - \frac{1}{\frac{D_i}{d_c} - 1}$$

The total thermal capacitance of the second part is given by

$$Q_{I2} = \frac{\pi}{4}(D_i^{*2} - D_i^* \cdot d_c^*)c$$

which leads to the third and fourth part of the thermal capacitance of the insulation, defined as

$$Q_{i3} = p^* Q_{I2}, \qquad Q_{i4} = (1 - p^*) Q_{I2}$$

In the case of dielectric losses, the cable thermal circuit is the same as shown in Figure 1-2, with the Van Wormer coefficient apportioning to the conductor a fraction of the dielectric thermal capacitance given by Equation (1.24). For cables at voltages higher than 275 kV, the Van Wormer coefficient is given by Equation (1.29).

1.6.7.4 *Metallic Sheath or Any Other Concentric Layer.* The thermal capacitance of all other concentric layers of cable components such as sheath, armor, jacket, armor bedding, or serving is computed by using Equation (1.23). However, one should remember that the thermal capacitances of nonmetallic layers have to be divided into two parts using the Van Wormer factor given by Equation (1.25). The appropriate dimensions for the inner and outer diameters must be used in order to attain sufficient accuracy for short-duration transients.

1.6.7.5 *Reinforcing Tapes.* The thermal capacitance of the reinforcing tapes over the sheath is

$$Q_T = n_t\left(w_t^* t_t^* \sqrt{\ell_t^{*2} + (\pi d_2^*)^2}\right)c$$

1.6.7.6 *Armor.* The thermal capacitance of the armor is obtained from

$$Q_a = n_1\left(\frac{\pi d_f^{*2}}{4} \sqrt{\ell_a d_a^*}\right)c$$

1.6.7.7 *Pipe-Type Cables.* The thermal capacitances of cable components are computed as described above, and the thermal capacitance of the oil in the pipe is obtained from

$$Q_{op} = \frac{\pi}{4}(D_d^{*2} - 3D_d^{*2})c$$

The capacitance of the oil is divided into two equal parts.

The thermal capacitance of the skid wires is generally neglected, but may be computed using the equation for the capacitance of the armor, if needed. The thermal capacitance of the metallic pipe and of the external covering are computed using Equation (1.23).

1.7 CONCLUDING REMARKS

The rating calculations described in this chapter will serve as a basis for the new developments presented in the following chapters. In many applications discussed further, the external thermal resistance of the cable will be modified. In Chapter 2, we will distinguish the thermal properties of two different regions through which the cable route may pass, whereas in Chapter 3 we will look at the mutual thermal resistance between the rated cable and an external heat source. The probabilistic nature of the external thermal resistivity of buried cables will be explored in Chapter 4. In the same chapter, we will investigate the optimal dimensions of a thermal backfill. The calculation of the load loss factor appearing in the equation for the external thermal resistance with a nonunity load factor is addressed in Chapter 5. For cables in air, discussed in Chapter 6, several alternative approaches to the computation of the convective and radiative heat transfer coefficients will be introduced.

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