Chapter 7

Convection: External Flow

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Introduction

In Chapter 6 we obtained a non-dimensional form for the heat transfer coefficient, applicable for problems involving the formation of a boundary layer:

$$\boxed{Nu_L = f(x^*, \mathrm{Re}_L, \mathrm{Pr})} \quad \boxed{\overline{Nu}_L = f(\mathrm{Re}_L, \mathrm{Pr})}$$

- In this chapter we will obtain convection coefficients for different flow geometries, involving external flows:
  - Flat plates
  - Spheres, cylinders, airfoils, blades
- ➢ In such flows, boundary layers develop freely
- Two approaches:
  - Experimental or empirical: Experimental heat transfer measurements are correlated in terms of dimensionless parameters
  - Theoretical approach: Solution of boundary layer equations.

Empirical correlations

- Generally

$$T_f = \frac{T_s + T_\infty}{2} \qquad (7.1)$$

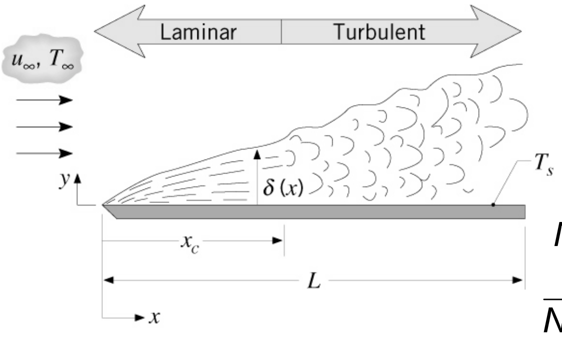
- ➢ Fluid properties are usually evaluated at the film temperature:

$$\overline{Nu}_L = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n \qquad (7.2)$$

- ➢ Sometimes all properties are evaluated at T_∞ and the RHS of eq. (7.1) is multiplied by

$$(\mathrm{Pr}_\infty/\mathrm{Pr}_s)^r \quad \text{or} \quad (\mu_\infty/\mu_s)^r$$

Flat Plate in Parallel Flow



Laminar Flow

- ➢ Blasius solution:

$$Nu_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}, \quad \mathrm{Pr} \ge 0.6 \quad (7.3)$$

$$\overline{Nu}_x = \frac{\overline{h}_x x}{k} = 0.664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}, \quad \mathrm{Pr} \ge 0.6 \quad (7.4)$$

- ➢ For Pr<0.05, Pe=Re Pr >100:

$$Nu_x = 0.565\ Pe_x^{0.5} \qquad (7.5)$$

- ➢ For all Pr numbers, Churchill and Ozoe correlation

$$Nu_x = \frac{0.3387\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \quad \mathrm{Pe}_x \ge 100$$

$$\overline{Nu}_x = 2Nu_x \qquad (7.6)$$

Flat Plate in Parallel Flow

Turbulent Flow

$$Nu_x = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}, \quad 0.6 < \mathrm{Pr} < 60 \qquad (7.7)$$

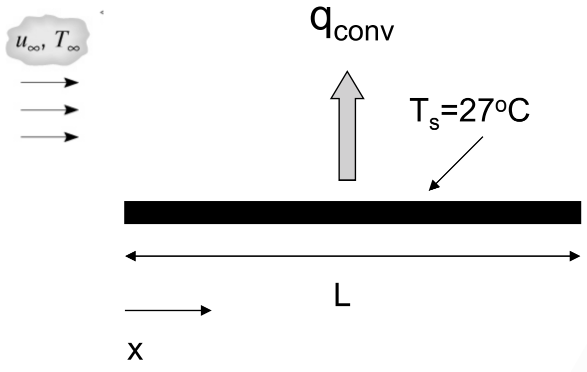
Mixed boundary layer conditions

$$\overline{Nu}_L = (0.037\,\mathrm{Re}_L^{4/5} - 871)\,\mathrm{Pr}^{1/3} \quad 0.6 < \mathrm{Pr} < 60,\ 5\times10^5 < \mathrm{Re}_L < 10^8 \qquad (7.8a)$$

Simplifies to: $\overline{Nu}_L = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}$ for $\mathrm{Re}_L >> 5\times10^5$ (7.8b)

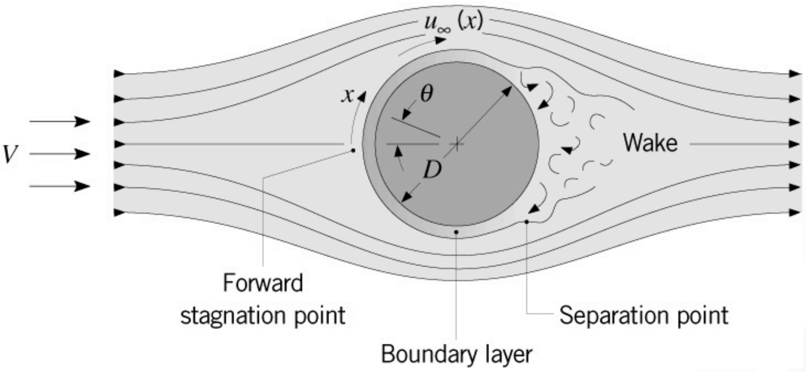
Example 7.1

Air at a pressure of 6 kN/m^2 and a temperature of 300^{o}C flows with a velocity of 10 m/s over a flat plate, 0.5 m long. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface temperature of 27^{o}C.



Flow around Cylinders and Spheres

- Flow around cylinders and spheres is characterized by boundary layer development and separation.
- Heat transfer coefficients are strongly influenced by the nature of boundary layer development at the surface.



- Laminar boundary layer for $\mathrm{Re}_D = \dfrac{\rho V D}{\mu} < 2\times10^5$

Crossflow around Cylinders

1.Zhukauskas correlation:

$$\overline{Nu_D} = \frac{\bar{h}D}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4} \qquad 0.7 < \mathrm{Pr} < 500, 1 < \mathrm{Re_D} < 10^6 \qquad (7.9)$$

where C and m are listed in Table 7.4, (n=0.37 for 10≥Pr) and (n=0.36 for 10<Pr). Properties evaluated at T_∞ , except Pr_s which is evaluated at T_s.

2. Churchill and Bernstein correlation, for all Re_D and Pr>0.2

$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad (7.10)$$

Properties evaluated at film temperature

3. Hilpert correlation, can be used for cross flow around other non-circular shapes – see Table 7.3 for values of C and m

$$\overline{Nu_D} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3} \qquad (7.11)$$

Crossflow around Spheres

- Whitaker correlation:

$$\overline{Nu_D} = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4} \qquad \begin{array}{l} 0.71 < \mathrm{Pr} < 380 \\ 3.5 < \mathrm{Re}_D < 7.6\times10^4 \end{array} \qquad (7.12)$$

where properties are evaluated at T_∞ , except μ_s which is evaluated at T_s

- Correlation by Ranz and Marshall for heat transfer from freely falling liquid drops:

$$\overline{Nu_D} = 2 + 0.6\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3} \qquad (7.13)$$

- ➢ At Re_D=0, equations (7.12) and (7.13) reduce to:

$$\overline{Nu_D} = 2$$

❖ Applicable for heat transfer to a stationary infinite medium around the surface

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Procedure for Calculations

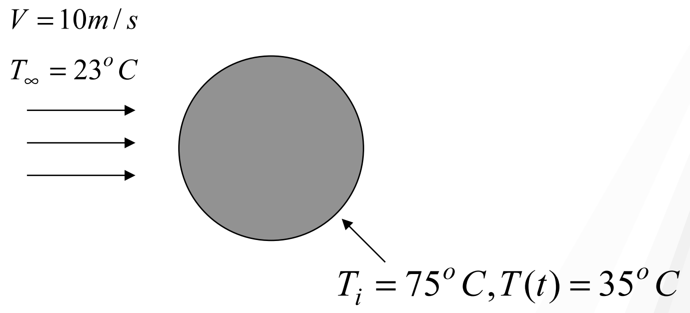
- Begin by recognizing the flow geometry (i.e. flat plate, sphere, cylinder etc.)
- Specify appropriate reference temperature for evaluation of fluid properties (usually film temperature, equation 7.2)
- Calculate Reynolds number – determine whether flow is laminar or turbulent
  - ➢ Reminder: Transition criteria:

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu} = 5\times10^5 \quad \textit{Flat plates} \qquad \mathrm{Re}_D = \frac{\rho V D}{\mu} < 2\times10^5 \quad \textit{Cylinders and spheres}$$

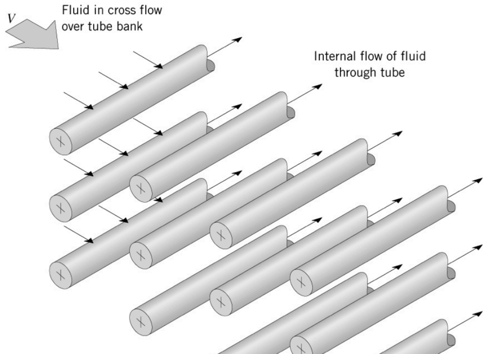
- Decide whether a local or average heat transfer coefficient is required
- Use appropriate correlation to determine heat transfer coefficient
- Proceed with other calculations, such as determination of heating or cooling rate

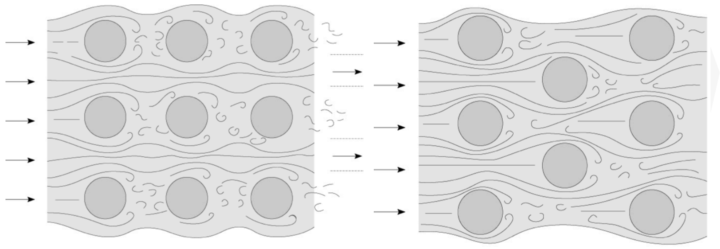
Example 7.5

The decorative plastic film on a copper sphere of 10-mm diameter is cured in an oven at 75°C. Upon removal from the oven, the sphere is subjected to an airstream at 1 atm and 23°C, having a velocity of 10 m/s. Estimate how long it will take to cool the sphere to 35°C.



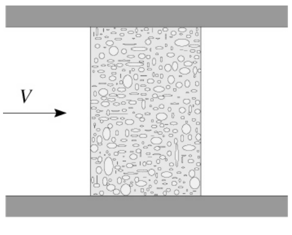
Other Applications



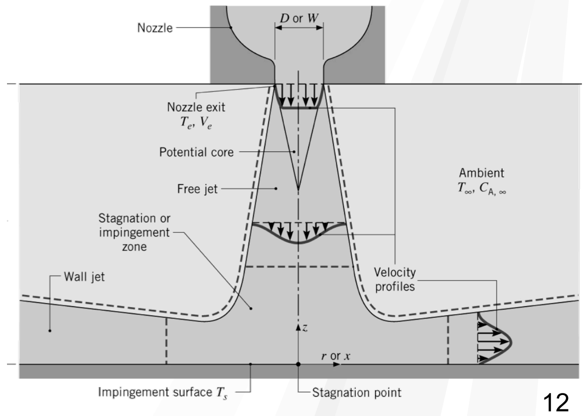


Flow around tube banks

Packed beds



Impinging jets



Flow across Banks of Tubes

- Several correlations exist (textbook section 7.6)
- Usually of the form

$$\overline{Nu_D} = C \, \mathrm{Re}_{D,\max}^{m} \, \mathrm{Pr}^{b}$$

where C and m can be found in tables (7.5-7.8)

Summary

- Convection heat transfer coefficients in external flows depend on the nature of boundary layer development.
- There are numerous correlations available for describing convection heat transfer for external flows
- Technologically important cases include flows around flat plates, cylinders, spheres, tube banks, packed beds, impinging jets etc.

➢ Comprehensive summary of correlations provided in Table 7.9, textbook