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An Approach to $\phi_u = 0$ Analysis for Stage Construction

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Introduction

In the two-stage construction of an embankment, if the construction is rapid in the second stage, as is often the case, a stability analysis by the total stress method (4), instead of the effective stress analysis (1,7), can be carried out for the finished height of the embankment. This can be done if the gain in the undrained strength of the subsoil due to dissipation of pore pressure under the sustained first-stage loading or preload (2) is known. A method of estimating this gain in strength is indicated in the following.

Theoretical Consideration

Let the in-situ effective stresses prior to construction, at a depth within a saturated subsoil, be $\bar{p}_0$ and $K_0\bar{p}_0$ while the pore pressure is u_n. The corresponding Mohr's circle is represented by circle 1 in Fig. 1. The first-stage fill is now placed at a height at which the subsoil, or at least a large zone within it, is almost on the verge of failure. Let the increase in total stresses under this condition be Δp_1 and Δp_3. The vertical and horizontal total stresses then become

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TABLE 1.—Pore Pressure Coefficients A_{f_1} and A_{f_2}

Stress history of samples (1)	Overconsolidation ratio. (2)	Pore Pressure Coefficient, as an Average of Samples at Three Different Depths: A_{f1} (3)	A_{f2} (4)
Normally consolidated	1.0	0.85	0.93
Over-consolidated	1.5	0.22	0.66
	2.0	0.17	0.28
	3.0	0.13	0.24

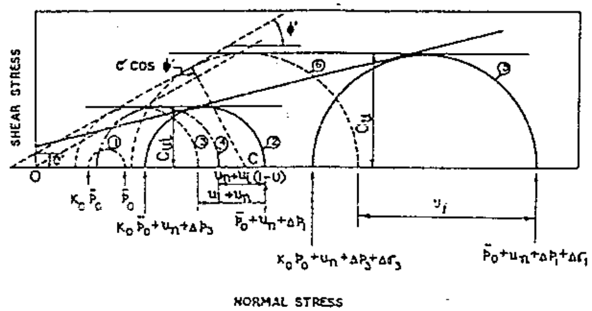

FIG. 1.—Mohr's Circles of Stresses at Different Stages of Construction

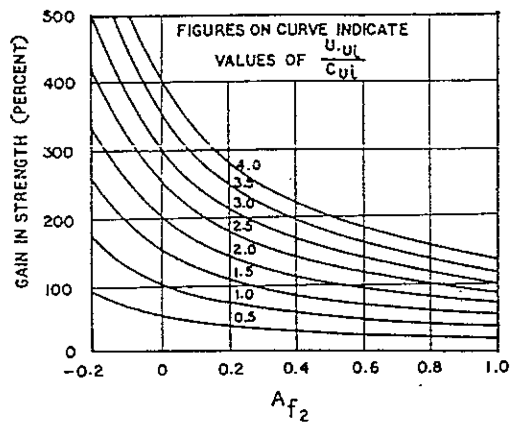

FIG. 2.—Gain in Strength, as a percentage, Versus Pore Pressure Coefficient A_{f_2} for $\phi' = 30°$

$$\left.\begin{aligned}\sigma_1 &= \bar{p}_0 + \Delta p_1 + u_n \\ \sigma_3 &= K_0\bar{p}_0 + \Delta p_3 + u_n\end{aligned}\right\} \qquad (1)$$

and the excess pore pressure

$$u_i = \Delta p_3 + A_{f_1}(\Delta p_1 - \Delta p_3) \qquad (2)$$

in which A_{f_1} = Skempton's pore pressure coefficient at failure (5). The total and effective stress circles at this stage are shown by circles 2 and 3 in Fig. 1.

After a pause in construction, the residual pore pressure becomes

$$u = u_i - U u_i + u_n \qquad (3)$$

in which U = the degree of consolidation after the pause. The effective stresses then become

$$\left.\begin{aligned}(\bar{\sigma}_1)\, t_p &= \bar{p}_0 + \Delta p_1 - u_i + U u_i \\ (\bar{\sigma}_3)\, t_p &= K_0\bar{p}_0 + \Delta p_3 - u_i + U u_i\end{aligned}\right\} \qquad (4)$$

The corresponding effective stress circle is represented by circle 4 in Fig. 1. Due to this dissipation of pore pressure, let the increase in undrained strength be such that failure would be reached under total stress increments $\Delta\sigma_1$ and $\Delta\sigma_3$. The stress condition would then be represented by circles 5 and 6 in Fig. 1 while the total stresses become

$$\left.\begin{aligned}\sigma_{1_f} &= \bar{p}_0 + \Delta p_1 + u_n + \Delta\sigma_1 \\ \sigma_{3_f} &= K_0\bar{p}_0 + \Delta p_3 + u_n + \Delta\sigma_3\end{aligned}\right\} \qquad (5)$$

and the pore pressure

$$u_f = u_i - U u_i + u_n + [\Delta\sigma_3 + A_{f2}(\Delta\sigma_1 - \Delta\sigma_3)] \qquad (6)$$

in which A_{f_2} = pore pressure coefficient at failure in the second stage of construction.

From Eq. 5

$$\Delta\sigma_1 - \Delta\sigma_3 = 2\,C_u - \bar{p}_0(1 - K_0) - (\Delta p_1 - \Delta p_3) \qquad (7)$$

in which $C_u = (\sigma_{1_f} - \sigma_{3_f})/2$ = increased undrained shear strength of the subsoil due to dissipation of pore pressure during the pause in construction. Substituting Eq. 7 into Eq. 6

$$u_f = u_i - U u_i + u_n + \Delta\sigma_3 + A_{f_2}[2C_u - \bar{p}_0(1 - K_0) - (\Delta p_1 - \Delta p_3)] \qquad (8)$$

Now, from the geometry of effective stress circle 6 (Fig. 1),

$$C_u = \frac{\bar{\sigma}_{1_f} - \bar{\sigma}_{3_f}}{2} = OC \sin\phi' + c' \cos\phi' \qquad (9a)$$

$$C_u = \frac{\bar{\sigma}_{1_f} - \bar{\sigma}_{3_f}}{2} \sin\phi' + (\bar{\sigma}_{3_f} \sin\phi') + c' \cos\phi' \qquad (9b)$$

$$C_u = C_u \sin\phi' + (\sigma_{3_f} - u_f) \sin\phi' + c' \cos\phi' \qquad (9c)$$

From Eqs. 5, 8, and 9

$$C_u = C_u \sin\phi' + [K_0\bar{p}_0 + \Delta p_3 - (1 - U)u_i$$

$$- A_{f_2}\{2C_u - \bar{p}_0(1 - K_0) - (\Delta p_1 - \Delta p_3)\}] \sin\phi' + c'\cos\phi' \quad \ldots\ldots (10)$$

Since the subsoil is stressed to near failure condition in the first stage of loading, the original undrained strength is given by

$$\left.\begin{array}{l} C_{ui} = \dfrac{\sigma_1 - \sigma_3}{2} \\ C_{ui} = \dfrac{1}{2}(\bar{p}_0 + \Delta p_1 - K_0\bar{p}_0 - \Delta p_3) \\ \text{or} \quad \Delta p_1 - \Delta p_3 = 2C_{ui} - \bar{p}_0(1 - K_0) \end{array}\right\} \quad \ldots\ldots (11)$$

From Eqs. 2 and 11

$$\Delta p_3 = u_i - A_{f_1}[2C_{ui} - \bar{p}_0(1 - K_0)] \quad \ldots\ldots (12)$$

Combining Eqs. 10, 11, and 12, and simplifying

$$C_u = \frac{c'\cos\phi' + [K_0\bar{p}_0 + Uu_i - 2C_{ui}(A_{f_1} - A_{f_2}) + \bar{p}_0(1 - K_0)A_{f_1}]\sin\phi'}{1 + (2A_{f_2} - 1)\sin\phi'} \quad (13)$$

Now, the original undrained strength of the soil is given by (6)

$$C_{ui} = \frac{c'\cos\phi' + \bar{p}_0\sin\phi'[K_0 + A_{f_1}(1 - K_0)]}{1 + (2A_{f_1} - 1)\sin\phi'} \quad \ldots\ldots (14)$$

Combining Eqs. 13 and 14 and simplifying

$$C_u = C_{ui}\left[1 + \frac{Uu_i}{C_{ui}}\,\frac{\sin\phi'}{1 + (2A_{f_2} - 1)\sin\phi'}\right] \quad \ldots\ldots (15)$$

The gain in strength of the subsoil, as a percentage, can then be expressed as

$$\frac{C_u - C_{ui}}{C_{ui}}\ \% = \frac{Uu_i}{C_{ui}}\,\frac{\sin\phi'}{1 + (2A_{f_2} - 1)\sin\phi'} \times 100 \quad \ldots\ldots (16)$$

Note, from Eq. 16, that the gain in strength depends primarily on the magnitude of pore pressure that will dissipate during the pause in construction and the pore pressure coefficient, A_{f_2}. For $\phi' = 30°$, Fig. 2 shows a typical relationship between the gain in strength, as a percentage, and the pore pressure coefficient, A_{f_2}, for various values of Uu_i/C_{ui}.

Also, Table 1 shows typical values of A_{f_1} and A_{f_2}, from a preliminary investigation (3) carried out on simulated field elements of normally consolidated and overconsolidated kaolin. Note that for the normally consolidated soil, the coefficients, A_{f_1} and A_{f_2}, are nearly equal, while for the overconsolidated soil [illegible] considerably greater than A_{f_1}.

A_{f_2} is

Practical Implications and Conclusions

Use of Eq. 16 in determining the increase in undrained strength makes it possible to analyze the stability of stage constructions by $\phi_u = 0$ method. While the values of u_i and U are to be estimated or obtained through field measurements, the value of the coefficient, A_{f_2}, also needs to be ascertained. For normally consolidated clays, the value of A_{f_2} may be estimated to be somewhat higher than the value of A_{f_1} determined from Eq. 14. As indicated by the limited test data, a value of A_{f_2} in the vicinity of 1.0 would seem to be appropriate for such soils. Fig. 2 shows that the gain in strength is not very sensitive to A_{f_2} for values of the latter in the vicinity of 1.

Note, however, that the foregoing analysis is valid beneath the center line of a circular loaded area, or if the effect of intermediate principal stress is ignored, applicable to conditions beneath the center line of an embankment where the horizontal and vertical stresses are the principal stresses. Consequently, it does not consider the effect of rotation of the principal stresses at points away from the center line, where undrained strength anisotropy may assume importance.

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