Two-phase flow discharge in nozzles and pipes – a unified approach

Joseph C. Leung
Fauske & Associates, Inc., 16W070 West 83rd Street, Burr Ridge, Illinois 60521, USA

This paper presents a unified treatment on homogeneous two-phase discharge through short nozzles and long pipes. The resulting generalized solutions, cast in terms of a limited number of dimensionless variables, can adequately account for fluid properties, inlet conditions, effects due to pipe friction, gravitational change, inlet subcooling and presence of non-condensable gases. The results are also presented in the form of design charts, for practising engineers.

(Keywords: two-phase flow; flashing flow; design principles)

Releases of process material during loss-of-containment events have different characteristics depending on the prevailing (thermodynamic) conditions upstream of the rupture. A wide spectrum of flow regimes ranging from all-flashing to totally non-flashing discharges may be encountered. This paper presents a unified treatment on compressible flow discharge through nozzles and pipes. Based on the homogeneous two-phase flow model, generalized solutions were obtained for both flashing and non-flashing mixtures in these flow passages[1-4]. Although the homogeneous flow assumption may be an oversimplification, it has been found to be adequate in most engineering design. For flashing flow systems, this flow regime, together with the assumption of thermodynamic equilibrium (between the two phases), has provided the best prediction of low-quality choked flow data of various fluids[5-7]. From the viewpoint of emergency relief system designs, this flow model would yield conservative relief areas for the given vent rate requirements.

The assumptions employed in the unified treatment are: homogeneous two-phase flow; thermodynamic equilibrium in flashing mixture; thermal equilibrium in non-flashing mixture; isenthalpic expansion process for flashing mixture; isothermal expansion process for non-flashing mixture; ideal gas behaviour; and constant friction factor in pipe. Based on these simplifying assumptions, compressible flow through nozzles and pipes can be characterized by a limited number of dimensionless physical parameters.

A correlating parameter for compressible flow

The "compressible flow" parameter ω can be derived from an assumed equation of state[1]. It is given in terms of the following physical properties

$$\omega = \frac{x_o v_{vo}}{v_o} + \frac{C_p T_o P_o}{v_o}\left(\frac{v_{vlo}}{h_{vlo}}\right)^2 = \alpha_o + \rho_o C_p T_o P_o\left(\frac{v_{vlo}}{h_{vlo}}\right)^2 \quad (1)$$

This parameter, based strictly on stagnation properties, is made up of two entirely separable terms: the first reflects the compressibility of the mixture due to the existing vapour volume fraction (α_o, also known as void fraction); and the second reflects the compressibility due to flashing or phase change upon depressurization[8].

For flashing flow systems, the second term in Equation (1) is the dominating term until α_o approaches unity (all-gas inlet). Flashing choked flows of widely different fluids have been successfully correlated, based on this parameter [1]. For non-flashing flow systems, the second term vanishes (no phase change) and ω reduces simply to α_o. The fact that inlet void fraction, α_o, is the key parameter characterizing a non-flashing flow system, was confirmed by an independent theoretical study[3]. This result allows solutions developed for flashing flow to be extended to non-flashing flow[8]. The following generalized solutions are given in terms of ω, and are valid for both flashing and non-flashing systems.

Nozzle discharge

The generalized solution for flow through a nozzle is[1,2]

$$G^* = \frac{\{-2[\omega \ln \eta + (\omega - 1)(1 - \eta)]\}^{1/2}}{\omega\left(\frac{1}{\eta} - 1\right) + 1} \quad (2)$$

where $G^* = G/(P_o/v_o)^{1/2} = g/(P_o\rho_o)^{1/2}$, a dimensionless mass velocity, and $\eta = P/P_o$, the ratio of nozzle exit pressure P to upstream pressure P_o.

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Choking or critical flow condition is found by seeking a maximum in G (or G^*) as P (or η) is decreased. This condition yields the following transcendental equation for $\eta = \eta_c$, the so-called critical pressure ratio

$$\eta_c^2 + (\omega^2 - 2\omega)(1 - \eta_c)^2 + 2\omega^2 \ln \eta_c + 2\omega^2(1 - \eta_c) = 0 \quad (3)$$

After η_c is found, its value can be substituted into Equation (2) to obtain G_c^*. Alternatively, the local choking criterion as given by

$$G_c^* = \frac{\eta_c}{\sqrt{\omega}} \quad (4)$$

can be used to yield an identical result.

Such generalized solutions for choked flow through a nozzle can be represented graphically as in *Figure 1*, where both flashing and non-flashing flow regimes are clearly displayed. This result reduces to the isothermal gas flow solution (or $k = 1.0$) at $\omega = 1.0$, which also forms a smooth transition between these two regimes.

Horizontal pipe discharge

The generalized solution for two-phase flow discharging from a constant diameter horizontal duct is [2]

$$4f\frac{L}{D} = \frac{2}{G^{*2}}\left[\frac{\eta_1 - \eta_2}{1-\omega} + \frac{\omega}{(1-\omega)^2}\ln\frac{(1-\omega)\eta_2 + \omega}{(1-\omega)\eta_1 + \omega}\right] - 2\ln\left[\frac{(1-\omega)\eta_2 + \omega}{(1-\omega)\eta_1 + \omega}\left(\frac{\eta_1}{\eta_2}\right)\right] \quad (5)$$

where $4f\ L/D$ is the total equivalent pipe resistance, $\eta_1 = P_1/P_o$ and $\eta_2 = P_2/P_o$, the exit presure ratio. To evaluate flow discharges from a large reservoir, both the inlet and exit conditions are needed. For the inlet, G^* and η_1 are related via the ideal nozzle relationship given in Equation (2), with $\eta = \eta_1$. Non-ideal entrance effects can be incorporated into the $4f\ L/D$ term. For subsonic or unchoked exit conditions, $P_2 = P_a$, where P_a is the ambient back pressure. For exit choking conditions, Equation (4) with $\eta_c = \eta_{2c}$ provides the relation between G_c^* and the critical exit pressure ratio, η_{2c}. Thus, for a given $4f\ L/D$, we have the three equations necessary to solve the three unknowns, G^*, η_1 and η_2 (or G, P_1 and P_2).

For critical flow discharge from a horizontal pipe, *Figure 2* provides a quick but accurate solution for the exiting mass velocity G_c. Here, the ratio of G_c for pipe flow to G_{oc} corresponding to a perfect nozzle is plotted versus $4f\ L/D$ at various ω parameters, covering both flashing and non-flashing flow conditions. This ratio, G_c/G_{oc}, can be regarded as an equivalent flow discharge coefficient, C_D. The curve for $\omega = 0$ (or $\alpha_o = 0$) is in excellent agreement with the classical incompressible flow solution, which is $C_D = 1/(1 + 4f\ L/D)^{1/2}$, and the curve for $\omega = 1$ (or $\alpha_o = 1$) yields the same result as the isothermal (or $k = 1$) pipe flow solution for gas[8].

A linear relation exists between the exit (choked) pressure ratio (P_{2c}/P_o), and G_c/G_{oc}, as shown in *Figure 3*. In equation form, the exit pressure ratio is simply

$$\eta_{2c} = \eta_c\left(\frac{G_c}{G_{oc}}\right) \text{ or } P_{2c} = P_c\left(\frac{G_c}{G_{oc}}\right) \quad (6)$$

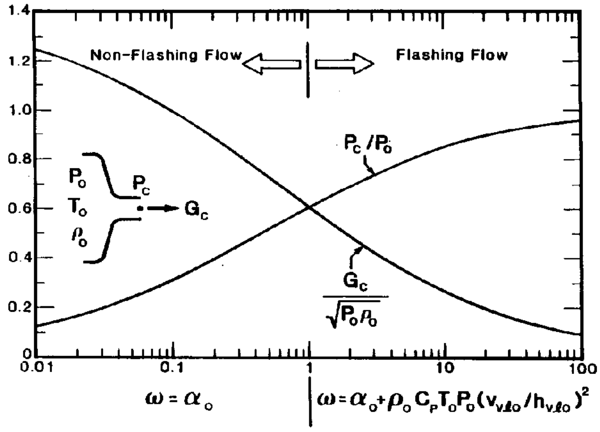

Figure 1 Flashing and non-flashing choked flow through nozzles[8]

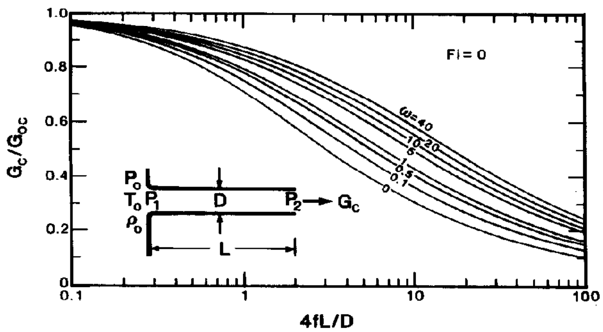

Figure 2 Choked flow discharge from horizontal pipes[8]

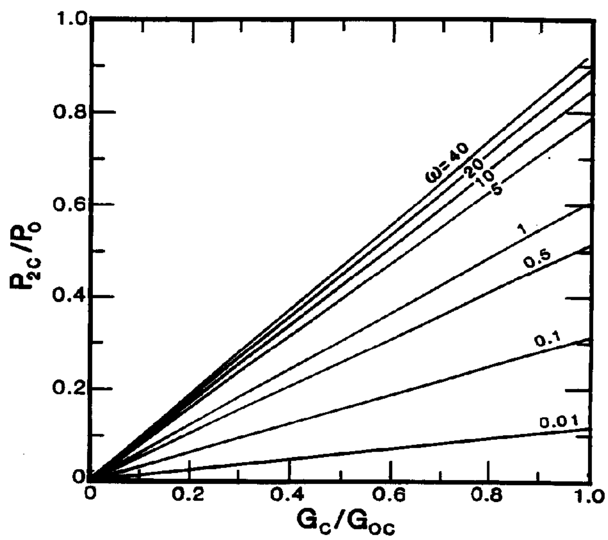

Figure 3 Relation for critical pressure ratios

where η_c is the critical pressure ratio for a choked nozzle as given by Equation (3) or *Figure 1*. This relation is valid for all pipe orientations.

Inclined pipe discharge

Generalized solutions for two-phase flow discharging from an inclined pipe have been obtained by using a dimensionless 'flow inclination' number[4]

$$\mathrm{Fi} = \frac{gD\cos\theta}{4fP_o v_o} = \frac{\rho_o g L\cos\theta}{(4f\,L/D)P_o} \tag{7}$$

where θ is the angle of inclination to the vertical. For a given pipe resistance $(4f\,L/D)$, the Fi number represents the ratio of the potential energy to the flow energy, and is a measure of the departure from the horizontal case (Fi = 0). The non-dimensionalized momentum equation to be solved is

$$4f\frac{L}{D} = -\int_{\eta_1}^{\eta_2} \frac{[(1-\omega)\eta^2 + \omega\eta]\left[1 - G^{*2}\dfrac{\omega}{\eta^2}\right]}{\dfrac{G^{*2}}{2}[(1-\omega)\eta + \omega]^2 + \eta^2(\mathrm{Fi})}\,\mathrm{d}\eta \tag{8}$$

which has a closed-form solution[4] that reduces exactly to Equation (5) for Fi = 0.

Figures 4 and *5* illustrate the effect of this Fi number (Fi = 0.1 and 0.2 respectively in upflow situations) on the exit choking mass flux. Comparison with *Figure 2* (for Fi = 0) reveals that the resulting flow rates decrease as the Fi number increases. For downflow (negative Fi number), a minimum flow behaviour

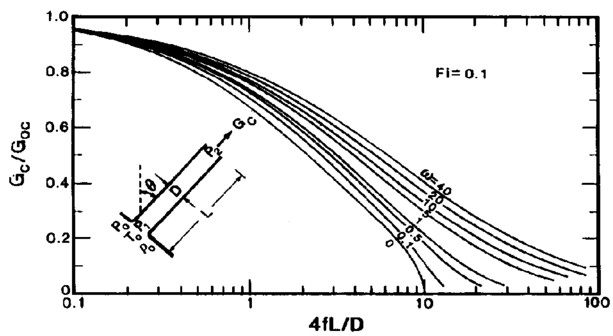

Figure 4 Choked flow discharge from inclined pipe, with Fi = 0.1

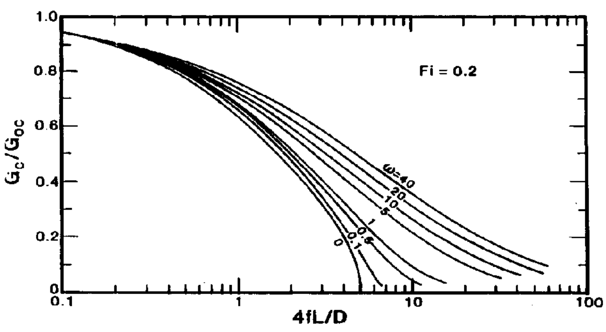

Figure 5 Choked flow discharge from inclined pipe, with Fi = 0.2

has been found, quite similar to the terminal velocity for free falling objects[4]. The flow rates in downflow situations are higher than in the horizontal flow case.

Effect of subcooling

The effect of subcooling on flashing flow discharge of an initially subcooled liquid (as a result of imposed pad pressure or hydrostatic head) can be assessed by first defining[9] a suitable correlating parameter, ω_s

$$\omega_s = \rho_{lo} C_p T_o P_s \left(\frac{v_{vlo}}{h_{vlo}}\right)^2 \tag{9}$$

where P_s is the saturation pressure corresponding to the inlet stagnation temperature, T_o. The generalized solutions are divided into low and high subcooling regions, delineated by the following inequality

$$\eta_s \geqslant 1 - \frac{1}{2\omega_s} \tag{10}$$

where $\eta_s = P_s/P_o$. In the low subcooling region, where this inequality is satisfied, the fluid attains flashing (two-phase) flow before reaching the exit location. The normalized mass velocity in a nozzle flow situation is[9]

$$G^* = \frac{G}{\sqrt{P_o\rho_{lo}}} = \frac{\left\{2(1-\eta_s) + 2\left[\omega_s\eta_s \ln\left(\dfrac{\eta_s}{\eta}\right) - (\omega_s - 1)(\eta_s - \eta)\right]\right\}^{1/2}}{\omega_s\left(\dfrac{\eta_s}{\eta} - 1\right) + 1} \tag{11}$$

and for choking conditions to occur, the following transcendental equation[9], yields the critical pressure ratio, η_c

$$\frac{\left(\omega_s + \dfrac{1}{\omega_s} - 2\right)}{2\eta_s}\eta_c^2 - 2(\omega_s - 1)\eta_c + \omega_s\eta_s \ln\left(\frac{\eta_c}{\eta_s}\right) + \frac{3}{2}\omega_s\eta_s - 1 = 0 \tag{12}$$

Equations (11) and (12) reduce exactly to Equations (2) and (3), respectively, for the saturated inlet case when $\eta_s = 1$ (no subcooling). In the high subcooling region, no vapour is formed until the exit location is reached. In this case, the solution reduces to the familiar Bernoulli-type equation

$$G_c^* = [2(1-\eta_s)]^{0.5} \quad \text{or} \quad G_c = [2\rho_{lo}(P_o - P_s)]^{0.5} \tag{13}$$

and the critical pressure ratio is simply

$$\eta_c = P_s/P_o \tag{14}$$

Figures 6 and *7* illustrate these results for both G_c^* and η_c respectively[9].

Effect of noncondensables

The presence of noncondensable gas in a flashing flow mixture can be considered as a hybrid system. The

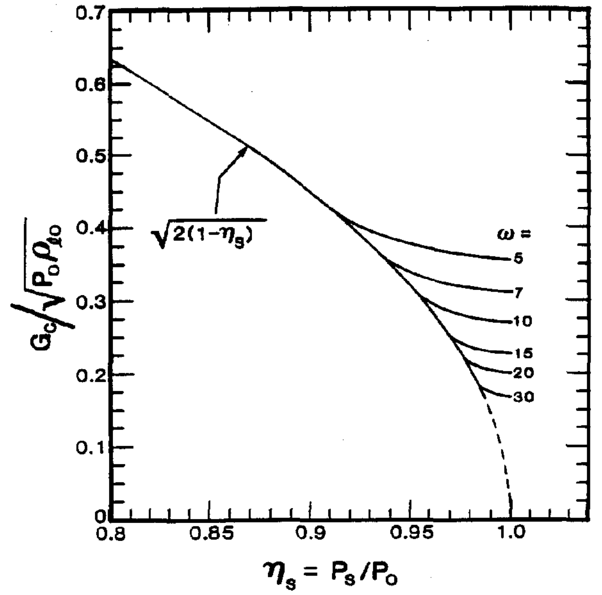

Figure 6 Subcooled inlet choked flow through nozzles - mass velocity

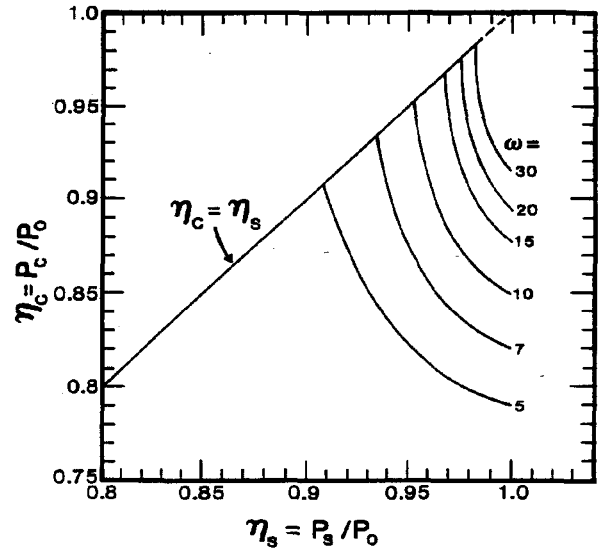

Figure 7 Subcooled inlet choked flow through nozzles - critical pressure ratio

solution for nozzle flow has been obtained[10] and can be characterized by the following dimensionless physical groups:

$$\omega = \alpha_o + (1-\alpha_o)\,\rho_{lo} C_p T_o P_{vo}\left(\frac{v_{vlo}}{h_{vlo}}\right)^2$$

$$= \alpha_o + (1-\alpha_o)\omega_s \qquad (15)$$

α_o (inlet void fraction)

$$y_{go} = P_{go}/P_o \text{ (gas mole fraction)} \qquad (16)$$

$$G^* = \frac{G}{\sqrt{P_o \rho_o}}$$

Here, ω uses P_{vo} ($= P_s$ in Equation (9)) instead of P_o as being more appropriate for the flashing component, while the gas component is characterized by α_o. The relative amount is conveyed by the gas mole fraction, y_{go}. The generalized solution for the mass velocity in a nozzle[10] bears some resemblance to Equation (2), (see Appendix).

$$G^* = \left\{2\left[-\alpha_o y_{go}\ln\frac{P_g}{P_{go}} + (1-\alpha_o)y_{go}\left(1-\frac{P_g}{P_{go}}\right) - \omega(1-y_{go})\ln\frac{P_v}{P_{vo}} + (1-\omega)(1-y_{go})\left(1-\frac{P_v}{P_{vo}}\right)\right]\right\}^{1/2} \div \left(\omega\left(\frac{P_{vo}}{P_v}-1\right)+1\right) \qquad (17)$$

This formula reduces to flashing and non-flashing flow limits at $y_{go} = 0$ and $y_{go} = 1.0$, respectively. Before Equation (17) can be solved for G^* (or G), an expression relating the two partial pressures, P_v and P_g, during the expansion process is required[10]

$$\alpha_o\left[\frac{P_{go}}{P_g} - 1\right] = \omega\left[\frac{P_{vo}}{P_v} - 1\right] \qquad (18)$$

This result states that during the acceleration process the two individual pressure ratios, P_g/P_{go} and P_v/P_{vo}, are solely governed by the term (α_o/ω), which is simply the ratio of the two key correlating parameters for non-flashing and flashing flow systems. *Figure 8* illustrates the relationship between these two pressure ratios at different values of α_o/ω. At low α_o, and therefore small α_o/ω, most of the initial pressure drop is taken up by the rapid decay of gas partial pressure.

The final expression governing exit choking conditions is[10]

$$-\alpha_o y_{go}\ln\frac{P_{gc}}{P_{go}} + (1-\alpha_o)y_{go}\left(1-\frac{P_{gc}}{P_{go}}\right) - \omega(1-y_{go})\ln\frac{P_{vc}}{P_{vo}} + (1-\omega)(1-y_{go})\left(1-\frac{P_{vc}}{P_{vo}}\right)$$

$$= \frac{1}{2}\left[\frac{y_{go}}{\alpha_o}\left(\frac{P_{gc}}{P_{go}}\right)^2 + \frac{(1-y_{go})}{\omega}\left(\frac{P_{vc}}{P_{vo}}\right)^2\right]\left[\omega\left(\frac{P_{vo}}{P_{vc}}-1\right)+1\right]^2 \qquad (19)$$

This is a transcendental equation for either P_{gc}/P_{go} or P_{vc}/P_{vo} as it is to be solved simultaneously with Equation (18) for these 'critical' pressure ratios. Once these ratios are found, the overall critical pressure ratio is simply given by

$$\frac{P_c}{P_o} = y_{go}\frac{P_{gc}}{P_{go}} + (1-y_{go})\frac{P_{vc}}{P_{vo}} \qquad (20)$$

Figure 9 illustrates how G_c^* and P_c/P_o depends on α_o at various gas mole fraction, y_{go}. At a fixed $\omega_s = 10$ (where ω_s is the value of the coefficient that multiplies $(1-\alpha_o)$ in Equation (15)), *Figure 9* displays the shape of the G_c^* versus α_o curves and the P_c/P_o versus α_o curves for the entire range of y_{go} values, i.e. from the pure flashing flow limit ($y_{go} = 0$) to the totally non-flashing two-phase flow limit ($y_{go} = 1.0$). At the limit of $\alpha_o = 0$ (absence of both vapour and gas in the inlet), the current solutions are in perfect agreement with the choked flow solution for subcooling liquid inlet conditions.

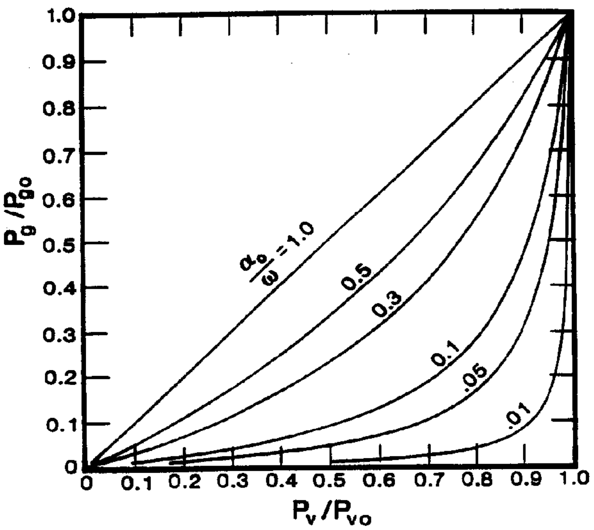

Figure 8 Partial pressure relationship during expansion in hybrid system

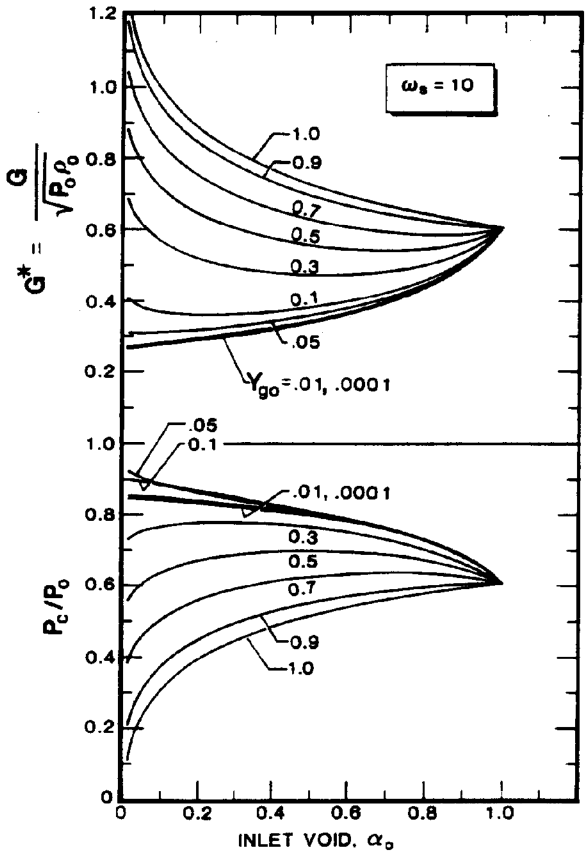

Figure 9 Choked flow discharge through nozzles for hybrid system

Effect of multicomponents

Extension of this model to multicomponent flashing mixtures has been found to yield accurate results[11]. For low-quality flow, the mixture latent heat of vaporization may be approximated by

$$h_{vl} = \Sigma\, Y_i h_{vli} \tag{21}$$

where Y_i is the vapour mass fraction of the ith component. Likewise

$$v_{vl} = \Sigma Y_i v_{vli} \tag{22}$$

These values correspond to inlet stagnation conditions (subscript o has been dropped for simplicity), which are generally known. Otherwise an adiabatic flash calculation would be required to yield the stagnation composition and mixture properties. The mixture liquid specific heat is simply the mass weighed average property, i.e.

$$C_p = \Sigma X_i C_{pi} \tag{23}$$

where X_i is the liquid mass fraction of the ith component.

A four-component flashing mixture with a wide range of boiling points was selected as an example[11]. For simplicity, ideal solutions (obeying Dalton's and Raoult's laws) were assumed throughout the calculations. The results for five mixtures with widely differing compositions and ω parameters (as calculated using the above mixture properties) are shown in *Table 1*. For this particular illustration, the stagnation temperature is fixed so that the component saturation properties are unaltered. Also shown for comparison are the results obtained from the DIERS computer code, SAFIRE[12,13], which makes use of rigorous flash calculations. The nozzle choked flow correlation, based on Equations (2) and (3), yields results that are within 2% of the detailed calculations, thus lending support to its application for multicomponent mixtures.

Conclusions

A unified approach has been summarized for evaluating the compressible flow of two-phase mixtures

Table 1 Multicomponent flashing flow prediction[a]

Conposition (weight)				P_o	ω	Calculated G (kg m^{-2} s^{-1})		G_{Correl} ·
H_2O	EG	EtOH	MeOH	(kPa)		Correl.	SAFIRE	G_{SAFIRE}
0.50	0.50	0.00	0.00	160.3	40.3	1516	1511	1.003
0.25	0.25	0.25	0.25	323.8	19.2	3050	3100	0.984
0.25	0.49	0.25	0.01	195.3	30.1	1940	1960	0.990
0.25	0.25	0.45	0.05	273.1	20.8	2690	2710	0.993
0.05	0.25	0.45	0.25	398.6	13.6	3960	4020	0.985

[a] T_o = 120 °C, ρ_o = 682 kg m^{-3}; EB = ethylene glycol

through nozzles and pipes. The resulting generalized solutions can be cast concisely in terms of key dimensionless variables relevant to these flow processes. As such, effects due to friction, gravitational change, inlet subcooling and presence of noncondensable gases can be adequately accounted for.

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Nomenclature

C_D discharge coefficient
C_p liquid specific heat at constant pressure
D pipe diameter
f Fanning friction factor
Fi flow inclination number, from Equation (7)
g gravitational acceleration
G mass velocity or flux
G_{oc} maximum (critical) mass velocity for a nozzle
G^* normalized mass velocity
h_{vl} latent heat of vaporization
k isentropic exponent or specific heat ratio
L equivalent length of pipe
M_w molecular weight
P total pressure
P_i partial pressure of phase i
T_o stagnation temperature
v specific volume
x_o inlet quality
X_i liquid phase mass fraction of ith component
Y_i vapour phase mass fraction of ith component
y_{go} gas mole fraction in vapour phase, from Equation (16)
α_o stagnation inlet void fraction
η pressure ratio
ω correlating parameter, from Equations (1) or (15)
ω_s all-liquid correlating parameter, from Equations (9) and (15)
ρ_o density at stagnation inlet
θ angle of inclination to the vertical

Subscripts

a ambient or outside
c critical or choked
g gas
l liquid
o stagnation (inlet) condition
s saturated liquid condition
vl difference between vapour and liquid properties
v vapour
1 entrance of a constant diameter pipe
2 exit end of a constant diameter pipe

Appendix

Equation (17) for hybrid flow can be rearranged to give

$$G^* = \sqrt{y_{go}G_g^{*2} + (1 - y_{go})G^{*2}_v} \tag{A-1}$$

where

$$G_g^* = \frac{\left\{-2\left[\alpha_o \ln \frac{P_g}{P_{go}} + (\alpha_o - 1)\left(1 - \frac{P_g}{P_{go}}\right)\right]\right\}^{1/2}}{\alpha_o\left(\frac{P_{go}}{P_g} - 1\right) + 1} \tag{A-2}$$

and

$$G_v^* = \frac{\left\{-2\left[\omega \ln \frac{P_v}{P_{vo}} + (\omega - 1)\left(1 - \frac{P_v}{P_{vo}}\right)\right]\right\}^{1/2}}{\omega\left(\frac{P_{vo}}{P_v} - 1\right) + 1} \tag{A-3}$$

Thus equations for both G_g^* and G_v^* are similar in form to Equation (2), with appropriate substitution of η by their respective partial pressure ratios.