```
STUDENT > # Rev 5 - Add Xm  7/29/10
STUDENT > #  in the subsequent rev, will change X to w*L
STUDENT >
STUDENT > restart; Digits:=4;
```

$$Digits := 4$$

```
STUDENT > # Construct Thevinin equivalent of R1, L1, Xm feeding
          output at mag branch
STUDENT > # Vth is with output open circuited... could also move X1
          together with X2... not done here
STUDENT > #    can move X1 and X2 to either side of Xm as convenient
STUDENT > # try the way that makes the Thev ckt simpler:
STUDENT > Vth:=Vs*I*XM/(I*XM+R1);
```

$$Vth := \frac{I\,Vs\,XM}{I\,XM + R1}$$

```
STUDENT > # Zth is Vth/Isc
STUDENT > Isc:=Vs/R1;
```

$$Isc := \frac{Vs}{R1}$$

```
STUDENT > Zth:=Vth/Isc;
```

$$Zth := \frac{I\,XM\,R1}{I\,XM + R1}$$

```
STUDENT >
STUDENT >
STUDENT > Z:=Zth+2*I*X2+R2/s;
```

$$Z := \frac{I\,XM\,R1}{I\,XM + R1} + 2\,I\,X2 + \frac{R2}{s}$$

```
STUDENT >
STUDENT > # find current I2 by ohms law
STUDENT > I2:=Vth/Z;
```

$$I2 := \frac{I\,Vs\,XM}{(I\,XM + R1)\left(\frac{I\,XM\,R1}{I\,XM + R1} + 2\,I\,X2 + \frac{R2}{s}\right)}$$

```
STUDENT > I2:=simplify(I2);
```

$$I2 := \frac{I\,s\,XM\,Vs}{I\,XM\,R1\,s - 2\,X2\,s\,XM + 2\,I\,X2\,s\,R1 + I\,R2\,XM + R2\,R1}$$

```
STUDENT > Iabs:=evalc(abs(I2));
```

$$Iabs := \left(\frac{s^2\,XM^2\,Vs^2\,(-2\,X2\,s\,XM + R2\,R1)^2}{((-2\,X2\,s\,XM + R2\,R1)^2 + (XM\,R1\,s + 2\,X2\,s\,R1 + R2\,XM)^2)^2}\right.$$

$$+\frac{s^2\, XM^2\, Vs^2\, (XM\, R1\, s+2\, X2\, s\, R1+R2\, XM)^2}{((-2\, X2\, s\, XM+R2\, R1)^2+(XM\, R1\, s+2\, X2\, s\, R1+R2\, XM)^2)^2}\Bigg)^{1/2}$$

```
STUDENT > # NEED TO CONVERT THIS TO A MAGNITUDE
STUDENT > # Find eq circuit total power P2 (Pairgap) leaving stator
          using total equivalent resistance R2/s. (note than angle
          of I is irrelevant in this particular calc.... can
          calculate real power and apparent power without ever
          dealing with current angle.
STUDENT > P2:=Iabs^2*R2/s;
```

$$P2 := \left(\frac{s^2\, XM^2\, Vs^2\, (-2\, X2\, s\, XM+R2\, R1)^2}{((-2\, X2\, s\, XM+R2\, R1)^2+(XM\, R1\, s+2\, X2\, s\, R1+R2\, XM)^2)^2} + \frac{s^2\, XM^2\, Vs^2\, (XM\, R1\, s+2\, X2\, s\, R1+R2\, XM)^2}{((-2\, X2\, s\, XM+R2\, R1)^2+(XM\, R1\, s+2\, X2\, s\, R1+R2\, XM)^2)^2}\right) R2\,/\,s$$

```
STUDENT > # The above quantity acts the same as the following
          quantity.
STUDENT > #  Find Shaft Horsepower (***neglecting motor mech
          losses...maybe need a better name) P_SHP using eq circuit
          total power (I^2*R2/s) minus rotor losses (I^2*R2)
          # = I^2*R2*(1/s - 1) =
          # = I^2*R2*(1/s - s/s)
          # = I^2*R2*(1-s)/s

STUDENT > P_SHP:=Iabs^2*R2*(1-s)/s;
```

$$P\_SHP := \left(\frac{s^2\, XM^2\, Vs^2\, (-2\, X2\, s\, XM+R2\, R1)^2}{((-2\, X2\, s\, XM+R2\, R1)^2+(XM\, R1\, s+2\, X2\, s\, R1+R2\, XM)^2)^2} + \frac{s^2\, XM^2\, Vs^2\, (XM\, R1\, s+2\, X2\, s\, R1+R2\, XM)^2}{((-2\, X2\, s\, XM+R2\, R1)^2+(XM\, R1\, s+2\, X2\, s\, R1+R2\, XM)^2)^2}\right) R2\,(1-s)\,/\,s$$

```
STUDENT > # In the linear range of small s where R2 dominates the
          denomiator:
STUDENT > # P APPROX EQUAL V2^2*s/R2 (not just prop)
STUDENT > # Slope of P_SHP vs s is Papprox/s = V^2/R2
STUDENT >
STUDENT > # find rotational speed w
STUDENT > w:=wsync*(1-s);
```

$$w := wsync\,(1-s)$$

```
STUDENT >
STUDENT > # find Torque T
STUDENT > T:=P_SHP/w;
```

$$T := \left(\frac{s^2\, XM^2\, Vs^2\, (-2\, X2\, s\, XM + R2\, R1)^2}{\left((-2\, X2\, s\, XM + R2\, R1)^2 + (XM\, R1\, s + 2\, X2\, s\, R1 + R2\, XM)^2\right)^2} + \frac{s^2\, XM^2\, Vs^2\, (XM\, R1\, s + 2\, X2\, s\, R1 + R2\, XM)^2}{\left((-2\, X2\, s\, XM + R2\, R1)^2 + (XM\, R1\, s + 2\, X2\, s\, R1 + R2\, XM)^2\right)^2} \right) R2 \,/\, (s\, wsync)$$

```
STUDENT > #  in the range where R2/s dominates the denominator (the
          linear range of T vs s), the following hold:
STUDENT > # T linear with s
STUDENT > # T prop to V^2
STUDENT > # T prop to (1/R)
STUDENT > # T APPROX EQUAL to (V^2*s)/(R2wsync) (not just prop)
STUDENT > # T prop to V^2 ANY SPEED (not just the linear range with
          s)
STUDENT > # is there a more diret way to find T directly (without
          going throuGH p_shp)
STUDENT >
STUDENT >
STUDENT >
STUDENT > subs1:={R1=1,R2=0.15,X1=0.5,X2=0.5,Vs=1,wsync=60,XM=20};
```

$$subs1 := \{R2 = .15,\ R1 = 1,\ X1 = .5,\ XM = 20,\ wsync = 60,\ Vs = 1,\ X2 = .5\}$$

```
STUDENT > subs2:={R1=1,R2=0.7,X1=0.5,X2=0.5,Vs=1,wsync=60,XM=20};
```

$$subs2 := \{R1 = 1,\ X1 = .5,\ XM = 20,\ wsync = 60,\ Vs = 1,\ X2 = .5,\ R2 = .7\}$$

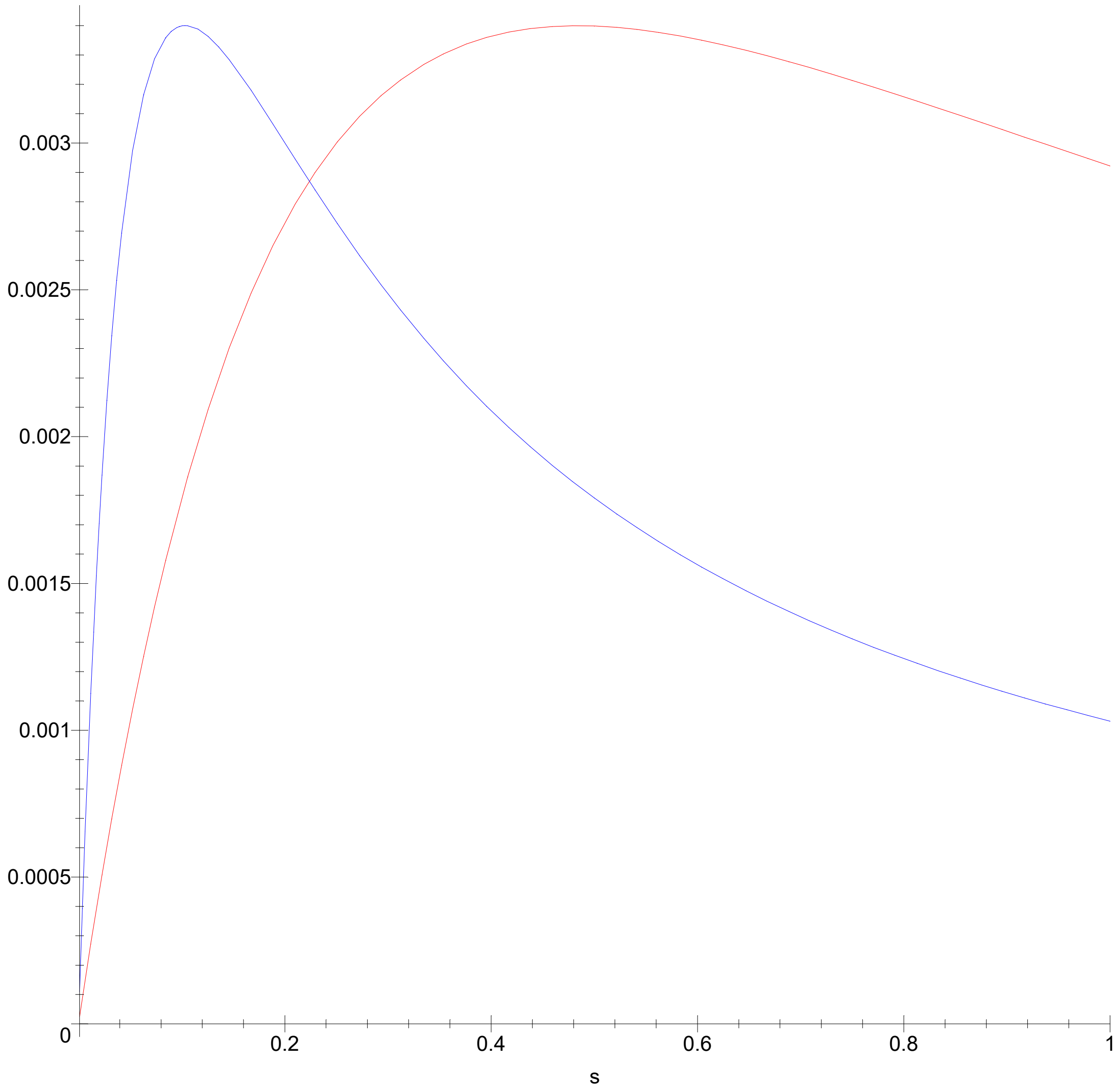
```
STUDENT > T1:=subs(subs1,T);
```

$$T1 := .002501\, \frac{400\, \dfrac{s^2\, (-20.0\, s + .15)^2}{\left((-20.0\, s + .15)^2 + (21.0\, s + 3.00)^2\right)^2} + 400\, \dfrac{s^2\, (21.0\, s + 3.00)^2}{\left((-20.0\, s + .15)^2 + (21.0\, s + 3.00)^2\right)^2}}{s}$$

```
STUDENT > T2:=subs(subs2,T);
```

$$T2 := .01167\, \frac{400\, \dfrac{s^2\, (-20.0\, s + .7)^2}{\left((-20.0\, s + .7)^2 + (21.0\, s + 14.0)^2\right)^2} + 400\, \dfrac{s^2\, (21.0\, s + 14.0)^2}{\left((-20.0\, s + .7)^2 + (21.0\, s + 14.0)^2\right)^2}}{s}$$

```
STUDENT > plot({T1,T2},s=0.001..1,color=[red,blue],thickness=[0,4]);
          #T2=2nd one has higher rotor resistance
```



```
STUDENT > # Find slip at the location of Tmax (s_Tmax)
STUDENT >
STUDENT >
STUDENT > dT_ds:=(diff(T,s));
```

$$
\begin{aligned}
dT\_ds := \Bigg(& 2\,\frac{s\,XM^2\,Vs^2\,\%2^2}{\left(\%2^2+\%1^2\right)^2} - 4\,\frac{s^2\,XM^3\,Vs^2\,\%2\,X2}{\left(\%2^2+\%1^2\right)^2} \\
& - 2\,\frac{s^2\,XM^2\,Vs^2\,\%2^2\left(-4\,\%2\,X2\,XM + 2\,\%1\,(XM\,R1 + 2\,X2\,R1)\right)}{\left(\%2^2+\%1^2\right)^3} + 2\,\frac{s\,XM^2\,Vs^2\,\%1^2}{\left(\%2^2+\%1^2\right)^2}
\end{aligned}
$$

$$+2\,\frac{s^2\,XM^2\,Vs^2\,\%1\,(XM\,R1+2\,X2\,R1)}{(\%2^2+\%1^2)^2}$$

$$\left.-2\,\frac{s^2\,XM^2\,Vs^2\,\%1^2\,(-4\,\%2\,X2\,XM+2\,\%1\,(XM\,R1+2\,X2\,R1))}{(\%2^2+\%1^2)^3}\right)R2\,/\,(s\,wsync)$$

$$-\frac{\left(\dfrac{s^2\,XM^2\,Vs^2\,\%2^2}{(\%2^2+\%1^2)^2}+\dfrac{s^2\,XM^2\,Vs^2\,\%1^2}{(\%2^2+\%1^2)^2}\right)R2}{s^2\,wsync}$$

$$\%1 := XM\,R1\,s+2\,X2\,s\,R1+R2\,XM$$

$$\%2 := -2\,X2\,s\,XM+R2\,R1$$

```
STUDENT >
STUDENT > dT_ds:=simplify(diff(T,s));
```

$$dT\_ds := -(XM^2\,R1^2\,s^2+4\,X2^2\,s^2\,XM^2-R2^2\,XM^2+4\,XM\,R1^2\,s^2\,X2+4\,X2^2\,s^2\,R1^2-R2^2\,R1^2)\,XM^2\,Vs^2\,R2\Big/\big((4\,X2^2\,s^2\,XM^2+R2^2\,R1^2+XM^2\,R1^2\,s^2+4\,XM\,R1^2\,s^2\,X2+2\,XM^2\,R1\,s\,R2+4\,X2^2\,s^2\,R1^2+R2^2\,XM^2)^2\,wsync\big)$$

```
STUDENT > # Exampine dT/ds at slip =0 (slope of torque speed near
          zero
STUDENT > simplify(subs(s=0,dT_ds));
```

$$\frac{Vs^2\,XM^2}{R2\,wsync\,(XM^2+R1^2)}$$

```
STUDENT >
STUDENT >
STUDENT >
STUDENT > s_Tmax:=solve(dT_ds=0,s); # as can see entire behavior
          with R1, R2, X.  Loc of peak goes to lower slip (higher
          speed) as R2 dec, R1 and X inc
```

$$s\_Tmax := \frac{\sqrt{\%1\,(XM^2+R1^2)}\,R2}{\%1},\,-\frac{\sqrt{\%1\,(XM^2+R1^2)}\,R2}{\%1}$$

$$\%1 := 4\,X2^2\,XM^2+XM^2\,R1^2+4\,XM\,R1^2\,X2+4\,X2^2\,R1^2$$

```
STUDENT > # Choose the positive root
STUDENT > equation1:=sTmax=s_Tmax[1];
```

$$equation1 := sTmax=\frac{\sqrt{(4\,X2^2\,XM^2+XM^2\,R1^2+4\,XM\,R1^2\,X2+4\,X2^2\,R1^2)\,(XM^2+R1^2)}\,R2}{4\,X2^2\,XM^2+XM^2\,R1^2+4\,XM\,R1^2\,X2+4\,X2^2\,R1^2}$$

```
STUDENT > R2:=solve(equation1,R2);
```

$$R2 := \frac{sTmax\,(4\,X2^2\,XM^2+XM^2\,R1^2+4\,XM\,R1^2\,X2+4\,X2^2\,R1^2)}{\sqrt{(4\,X2^2\,XM^2+XM^2\,R1^2+4\,XM\,R1^2\,X2+4\,X2^2\,R1^2)\,(XM^2+R1^2)}}$$

```
STUDENT > # Above is the value of R2 which gives
```

```
              speed corresponding to slip sTmax
STUDENT > 
STUDENT > 
STUDENT > s_Tmax1:=evalf(subs(subs1,s_Tmax[1]));
```

$$s\_Tmax1 := sTmax$$

```
STUDENT > Tmax:=subs(s=s_Tmax[1],T);
          # can see Tmax dec as R2 inc.
          # CAN ALSO SEE TMAX prop to V^2
```

$$Tmax := \left(\frac{sTmax^2\, XM^2\, Vs^2 \left(-2\, X2\, sTmax\, XM + \frac{sTmax\, \%1\, R1}{\sqrt{\%1\, (XM^2 + R1^2)}} \right)^2}{\left(\left(-2\, X2\, sTmax\, XM + \frac{sTmax\, \%1\, R1}{\sqrt{\%1\, (XM^2 + R1^2)}} \right)^2 + \%2^2 \right)^2} + \frac{sTmax^2\, XM^2\, Vs^2\, \%2^2}{\left(\left(-2\, X2\, sTmax\, XM + \frac{sTmax\, \%1\, R1}{\sqrt{\%1\, (XM^2 + R1^2)}} \right)^2 + \%2^2 \right)^2} \right) \%1 \Big/ \left(\sqrt{\%1\, (XM^2 + R1^2)}\; wsync \right)$$

$$\%1 := 4\, X2^2\, XM^2 + XM^2\, R1^2 + 4\, XM\, R1^2\, X2 + 4\, X2^2\, R1^2$$

$$\%2 := XM\, R1\, sTmax + 2\, X2\, sTmax\, R1 + \frac{sTmax\, \%1\, XM}{\sqrt{\%1\, (XM^2 + R1^2)}}$$

```
STUDENT > Tmax1:=evalf(subs(subs1,Tmax));
```

$$Tmax1 := .003402$$

```
STUDENT > # Note that Tmax is independent of R2.  Increasing R2
          always moves the peak Tmax toward lower speed but doesn't
          change the magnitude of the peak
STUDENT > # Also note that we would predict this from the change in
          slope _____
STUDENT > 
STUDENT > # find starting Torque by evaluating torque at s=1
STUDENT > T_start:=subs(s=1,T);
```

$$T\_start := \left(\frac{XM^2\, Vs^2 \left(-2\, X2\, XM + \frac{sTmax\, \%1\, R1}{\sqrt{\%1\, (XM^2 + R1^2)}} \right)^2}{\left(\left(-2\, X2\, XM + \frac{sTmax\, \%1\, R1}{\sqrt{\%1\, (XM^2 + R1^2)}} \right)^2 + \%2^2 \right)^2} + \frac{XM^2\, Vs^2\, \%2^2}{\left(\left(-2\, X2\, XM + \frac{sTmax\, \%1\, R1}{\sqrt{\%1\, (XM^2 + R1^2)}} \right)^2 + \%2^2 \right)^2} \right)$$

$$sTmax\ \%1 \Big/ \left(\sqrt{\%1\ (XM^2 + R1^2)}\ wsync\right)$$

$$\%1 := 4\ X2^2\ XM^2 + XM^2\ R1^2 + 4\ XM\ R1^2\ X2 + 4\ X2^2\ R1^2$$

$$\%2 := XM\ R1 + 2\ X2\ R1 + \frac{sTmax\ \%1\ XM}{\sqrt{\%1\ (XM^2 + R1^2)}}$$

```
STUDENT > # starting torque is heavily dependent on R2.  High R2
          gives good start torque and low start current  - both
          great for starting but lousy for run efficiency =>
          deep-bar design or variable resistance on wound-rotor
          rotor.
STUDENT > # starting torque is also propr to V^2 based on above.
STUDENT > # check that this gives the correct starting torque value
          for a specific set of circuit values.
STUDENT > subs(subs1,T_start);
```

$$.02414 \left(400 \frac{(-20.0 + 1.448\ sTmax)^2}{\left((-20.0 + 1.448\ sTmax)^2 + (21.0 + 28.96\ sTmax)^2\right)^2} + 400 \frac{(21.0 + 28.96\ sTmax)^2}{\left((-20.0 + 1.448\ sTmax)^2 + (21.0 + 28.96\ sTmax)^2\right)^2} \right) sTmax$$

```
STUDENT > # Summary picture of changing R2 (see earlier graph). As
          R2 increases, TslopevsS(~V^2/R2) decreases, peak moves to
          left (same height), starting T increases.  There is a
          crossover
STUDENT >
STUDENT >
STUDENT > # Look at slip vs power in the linear range
STUDENT > # first look at slope of T vs N in linear range near s=0
STUDENT > RunningTSlope:=subs(s=0,dT_ds);
```

$$RunningTSlope := -\frac{\left(-\frac{sTmax^2\ \%1\ XM^2}{XM^2 + R1^2} - \frac{sTmax^2\ \%1\ R1^2}{XM^2 + R1^2}\right) XM^2\ Vs^2\ sTmax\ \%1}{\sqrt{\%1\ (XM^2 + R1^2)} \left(\frac{sTmax^2\ \%1\ R1^2}{XM^2 + R1^2} + \frac{sTmax^2\ \%1\ XM^2}{XM^2 + R1^2}\right)^2 wsync}$$

$$\%1 := 4\ X2^2\ XM^2 + XM^2\ R1^2 + 4\ XM\ R1^2\ X2 + 4\ X2^2\ R1^2$$

```
STUDENT > # This is same thing we came up with previously.
STUDENT > # High R2  gives lower slope and higher slip (moves peak
          toward lower speeds)
STUDENT >
STUDENT > #Also note that V^2 gives lower slope and higher slip.
STUDENT >
STUDENT >
```

```
STUDENT > #as a close approximation - speed is constant and "slope"
          of power vs s curve is related to V^2 and (1/R2)
STUDENT > # And actual value of slip (for given horsepower) is
          directly related to the slope.
STUDENT > 
STUDENT > # Summary - ** slip prop to R2/V^2.  Note that R2 is
          F(Temperature)
STUDENT > 
STUDENT > 
STUDENT > 
```