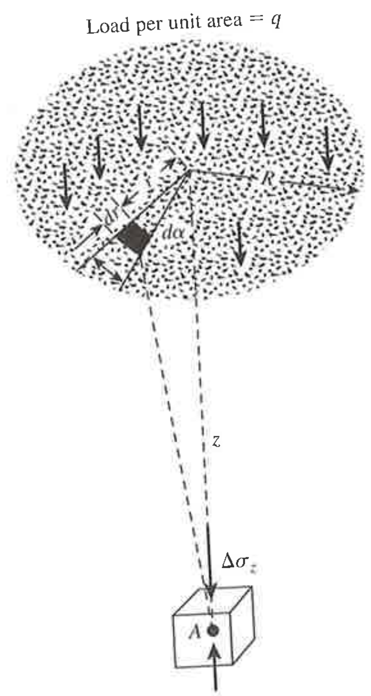

Figure 9.20 Vertical stress below the center of a uniformly loaded flexible circular area

So,

$$\Delta\sigma_z = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\} \tag{9.30}$$

The variation of $\Delta\sigma_z/q$ with z/R as obtained from Eq. (9.30) is given in Table 9.6. A plot of this also is shown in Figure 9.21. The value of $\Delta\sigma_z$ decreases rapidly with depth, and at $z = 5R$, it is about 6% of q, which is the intensity of pressure at the ground surface.

Table 9.6 Variation of $\Delta\sigma_z/q$ with z/R [Eq. (9.24)]

z/R	$\Delta\sigma_z/q$	z/R	$\Delta\sigma_z/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

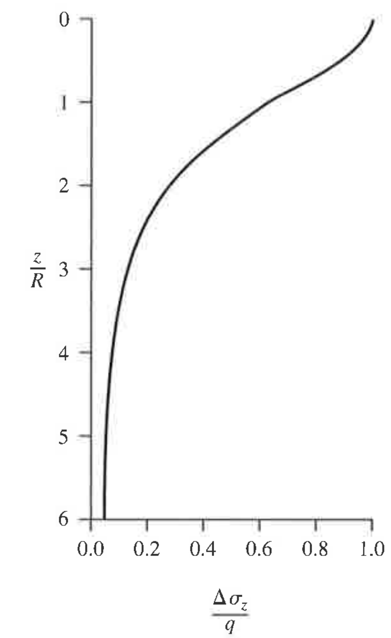

Figure 9.21 Stress under the center of a uniformly loaded flexible circular area

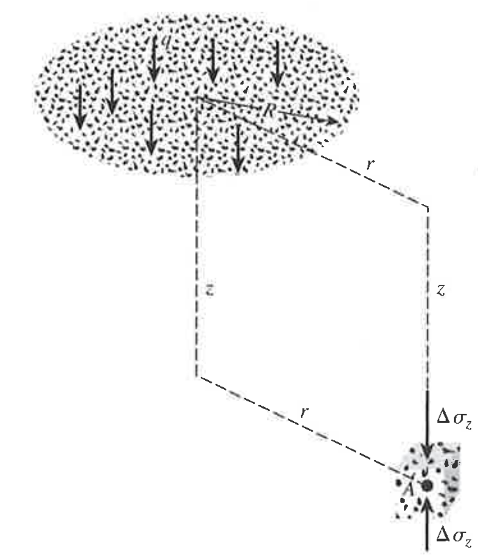

Figure 9.22 Vertical stress at any point below a uniformly loaded circular area

9.11 Vertical Stress at any Point Below a Uniformly Loaded Circular Area

A detailed tabulation for calculation of vertical stress below a uniformly loaded flexible circular area was given by Ahlvin and Ulery (1962). Referring to Figure 9.22, we find that $\Delta\sigma_z$ at any point A located at a depth z at any distance r from the center of the loaded area can be given as

$$\Delta\sigma_z = q(A' + B') \tag{9.31}$$

where A' and B' are functions of z/R and r/R. (See Tables 9.7 and 9.8.)

9.12 Vertical Stress Caused by a Rectangularly Loaded Area

Boussinesq's solution also can be used to calculate the vertical stress increase below a flexible rectangular loaded area, as shown in Figure 9.23. The loaded area is located at the ground surface and has length L and width B. The uniformly distributed load per unit area is equal to q. To determine the increase in the vertical stress ($\Delta\sigma_z$) at point A, which is located at depth z below the corner of the rectangular area, we

Table 9.7 Variation of A' with z/R and r/R*

z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	1.0	1.0	1.0	1.0	1.0	0.5	0	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787	0.00851
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251	0.01680
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199	0.02440
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593	0.03118
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499	0.03701
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010	
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228	0.04558
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236	
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094	
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849	0.05185
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192	0.05260
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048	0.05116
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275	0.04496
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880	0.03787
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839	0.03150
4	0.02986	0.02976	0.02907	0.02802	0.02832	0.02749	0.02651	0.02490	0.02193
5	0.01942	0.01938				0.01835			0.01573
6	0.01361					0.01307			0.01168
7	0.01005					0.00976			0.00894
8	0.00772					0.00755			0.00703
9	0.00612					0.00600			0.00566
10								0.00477	0.00465

*After Ahlvin and Ulery (1962)

Table 9.7 *(continued)*

z/R	3	4	5	6	7	8	10	12	14
0		0	0	0	0	0	0	0	0
0.1	0.00211	0.00084	0.00042						
0.2	0.00419	0.00167	0.00083	0.00048	0.00030	0.00020			
0.3	0.00622	0.00250							
0.4									
0.5	0.01013	0.00407	0.00209	0.00118	0.00071	0.00053	0.00025	0.00014	0.00009
0.6									
0.7									
0.8									
0.9									
1	0.01742	0.00761	0.00393	0.00226	0.00143	0.00097	0.00050	0.00029	0.00018
1.2	0.01935	0.00871	0.00459	0.00269	0.00171	0.00115			
1.5	0.02142	0.01013	0.00548	0.00325	0.00210	0.00141	0.00073	0.00043	0.00027
2	0.02221	0.01160	0.00659	0.00399	0.00264	0.00180	0.00094	0.00056	0.00036
2.5	0.02143	0.01221	0.00732	0.00463	0.00308	0.00214	0.00115	0.00068	0.00043
3	0.01980	0.01220	0.00770	0.00505	0.00346	0.00242	0.00132	0.00079	0.00051
4	0.01592	0.01109	0.00768	0.00536	0.00384	0.00282	0.00160	0.00099	0.00065
5	0.01249	0.00949	0.00708	0.00527	0.00394	0.00298	0.00179	0.00113	0.00075
6	0.00983	0.00795	0.00628	0.00492	0.00384	0.00299	0.00188	0.00124	0.00084
7	0.00784	0.00661	0.00548	0.00445	0.00360	0.00291	0.00193	0.00130	0.00091
8	0.00635	0.00554	0.00472	0.00398	0.00332	0.00276	0.00189	0.00134	0.00094
9	0.00520	0.00466	0.00409	0.00353	0.00301	0.00256	0.00184	0.00133	0.00096
10	0.00438	0.00397	0.00352	0.00326	0.00273	0.00241			

Table 9.8 Variation of B' with z/R and r/R*

z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	−0.07899	−0.02672	−0.00845
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	−0.07759	−0.04448	−0.01593
0.3	0.26362	0.26787	0.28018	0.29483	0.27257	0.10757	−0.04316	−0.04999	−0.02166
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	−0.00766	−0.04535	−0.02522
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	−0.03455	−0.02651
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	−0.02101	
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	−0.00702	−0.02329
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614	
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795	
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814	−0.01005
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378	0.00023
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745	0.01385
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371	0.02836
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022	0.03429
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354	0.03511
4	0.05707	0.05666	0.05562	0.05383	0.05145	0.04773	0.04532	0.03995	0.03066
5	0.03772	0.03760				0.03384			0.02474
6	0.02666					0.02468			0.01968
7	0.01980					0.01868			0.01577
8	0.01526					0.01459			0.01279
9	0.01212					0.01170			0.01054
10								0.00924	0.00879

*Source: From "Tabulated Values for Determining the Complete Pattern of Stresses, Strains, and Deflections Beneath a Uniform Circular Load on a Homogeneous Half Space," by R. G Ahlvin and H. H. Ulery. In *Highway Research Bulletin 342,* Transportation Research Board, National Research Council, Washington, DC, 1962.

Table 9.8 *(continued)*

z/R	3	4	5	6	7	8	10	12	14
0		0	0	0	0	0	0	0	0
0.1	−0.00210	−0.00084	−0.00042						
0.2	−0.00412	−0.00166	−0.00083	−0.00024	−0.00015	−0.00010			
0.3	−0.00599	−0.00245							
0.4									
0.5	−0.00991	−0.00388	−0.00199	−0.00116	−0.00073	−0.00049	−0.00025	−0.00014	−0.00009
0.6									
0.7									
0.8									
0.9									
1	−0.01115	−0.00608	−0.00344	−0.00210	−0.00135	−0.00092	−0.00048	−0.00028	−0.00018
1.2	−0.00995	−0.00632	−0.00378	−0.00236	−0.00156	−0.00107			
1.5	−0.00669	−0.00600	−0.00401	−0.00265	−0.00181	−0.00126	−0.00068	−0.00040	−0.00026
2	−0.00028	−0.00410	−0.00371	−0.00278	−0.00202	−0.00148	−0.00084	−0.00050	−0.00033
2.5	0.00661	−0.00130	−0.00271	−0.00250	−0.00201	−0.00156	−0.00094	−0.00059	−0.00039
3	0.01112	0.00157	−0.00134	−0.00192	−0.00179	−0.00151	−0.00099	−0.00065	−0.00046
4	0.01515	0.00595	0.00155	−0.00029	−0.00094	−0.00109	−0.00094	−0.00068	−0.00050
5	0.01522	0.00810	0.00371	0.00132	0.00013	−0.00043	−0.00070	−0.00061	−0.00049
6	0.01380	0.00867	0.00496	0.00254	0.00110	0.00028	−0.00037	−0.00047	−0.00045
7	0.01204	0.00842	0.00547	0.00332	0.00185	0.00093	−0.00002	−0.00029	−0.00037
8	0.01034	0.00779	0.00554	0.00372	0.00236	0.00141	0.00035	−0.00008	−0.00025
9	0.00888	0.00705	0.00533	0.00386	0.00265	0.00178	0.00066	0.00012	−0.00012
10	0.00764	0.00631	0.00501	0.00382	0.00281	0.00199			

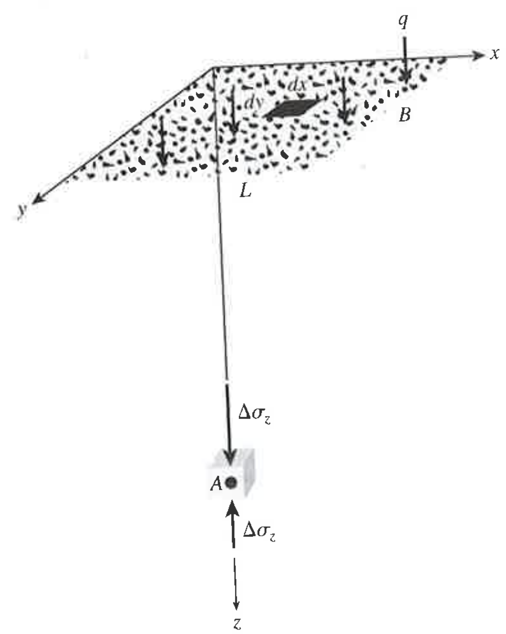

Figure 9.23 Vertical stress below the corner of a uniformly loaded flexible rectangular area

need to consider a small elemental area $dx\,dy$ of the rectangle. (This is shown in Figure 9.23.) The load on this elemental area can be given by

$$dq = q\,dx\,dy \tag{9.32}$$

The increase in the stress ($d\sigma_z$) at point A caused by the load dq can be determined by using Eq. (9.12). However, we need to replace P with $dq = q\,dx\,dy$ and r^2 with $x^2 + y^2$. Thus,

$$d\sigma_z = \frac{3q\,dx\,dy\,z^3}{2\pi(x^2 + y^2 + z^2)^{5/2}} \tag{9.33}$$

The increase in the stress, at point A caused by the entire loaded area can now be determined by integrating the preceding equation. We obtain

$$\Delta\sigma_z = \int d\sigma_z = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dx\,dy)}{2\pi(x^2 + y^2 + z^2)^{5/2}} = qI_4 \tag{9.34}$$

where

$$I_4 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1}\left(\frac{m^2 + n^2 + 2}{m^2 + n^2 + 1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2n^2 + 1}\right)\right] \tag{9.35}$$

$$m = \frac{B}{z} \tag{9.36}$$

$$n = \frac{L}{z} \tag{9.37}$$

The variation of I_4 with m and n is shown in Table 9.9 and Figure 9.24.

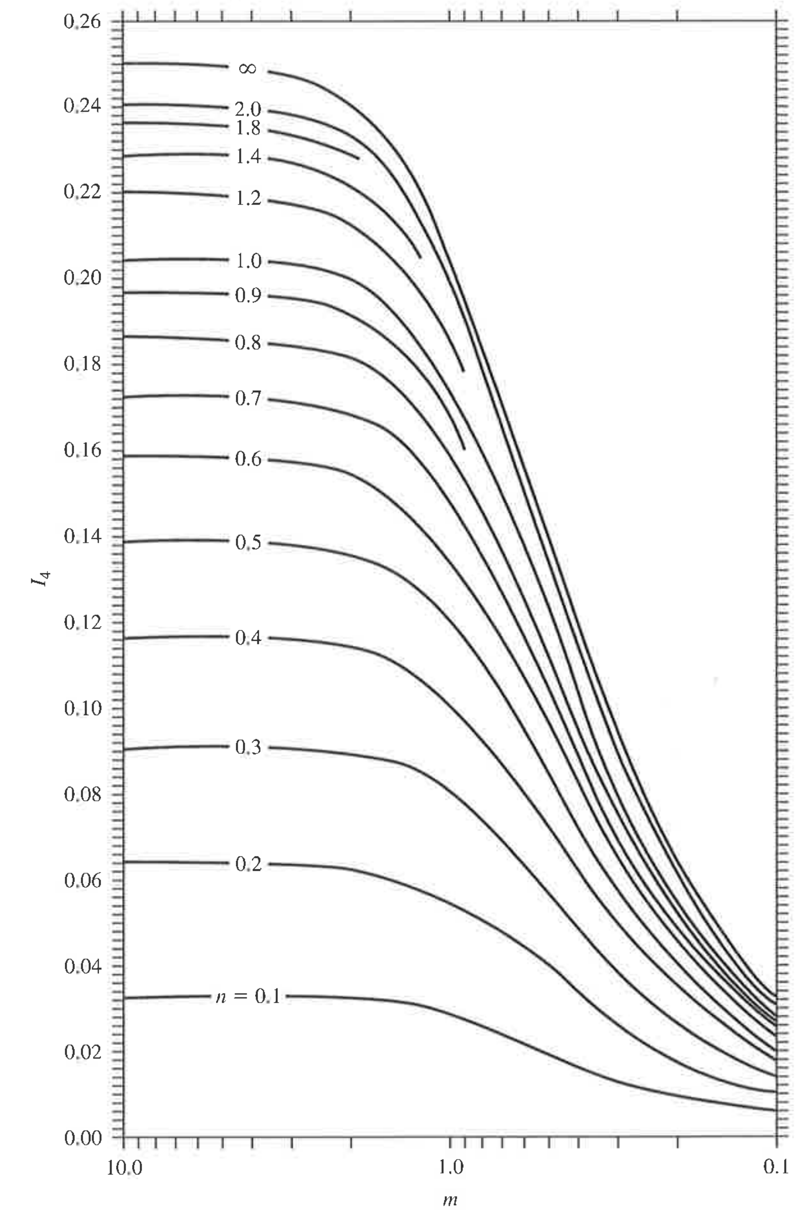

Figure 9.24 Variation of I_4 with m and n

The increase in the stress at any point below a rectangularly loaded area can be found by using Eq. (9.34). This can be explained by reference to Figure 9.25. Let us determine the stress at a point below point A' at depth z. The loaded area can be divided into four rectangles as shown. The point A' is the corner common to all four rectangles. The increase in the stress at depth z below point A' due to each rectan-

Table 9.9 Variation of I_4 with m and n [Eq. (9.35)]

n \ m	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1361
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1169	0.1277	0.1365	0.1436	0.1491
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1684	0.1752
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034
4.0	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2044
6.0	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045

gular area can now be calculated by using Eq. (9.34). The total stress increase caused by the entire loaded area can be given by

$$\Delta\sigma_z = q[I_{4(1)} + I_{4(2)} + I_{4(3)} + I_{4(4)}] \tag{9.38}$$

where $I_{4(1)}, I_{4(2)}, I_{4(3)}$, and $I_{4(4)}$ = values of I_4 for rectangles 1, 2, 3, and 4, respectively.

In most cases the vertical stress increase below the center of a rectangular area (Figure 9.26) is important. This stress increase can be given by the relationship

$$\Delta\sigma_z = qI_5 \tag{9.39}$$

where

$$I_5 = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1 + m_1^2 + n_1^2}}\frac{1 + m_1^2 + 2n_1^2}{(1 + n_1^2)(m_1^2 + n_1^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2 + n_1^2}\sqrt{1 + n_1^2}}\right] \tag{9.40}$$

$$m_1 = \frac{L}{B} \tag{9.41}$$

$$n_1 = \frac{z}{b} \tag{9.42}$$

$$b = \frac{B}{2} \tag{9.43}$$

The variation of I_5 with m_1 and n_1 is given in Table 9.10.

Table 9.9 (*continued*)

n \ m	1.2	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0
0.1	0.0293	0.0301	0.0306	0.0309	0.0311	0.0314	0.0315	0.0316	0.0316	0.0316
0.2	0.0573	0.0589	0.0599	0.0606	0.0610	0.0616	0.0618	0.0619	0.0620	0.0620
0.3	0.0832	0.0856	0.0871	0.0880	0.0887	0.0895	0.0898	0.0901	0.0901	0.0902
0.4	0.1063	0.1094	0.1114	0.1126	0.1134	0.1145	0.1150	0.1153	0.1154	0.1154
0.5	0.1263	0.1300	0.1324	0.1340	0.1350	0.1363	0.1368	0.1372	0.1374	0.1374
0.6	0.1431	0.1475	0.1503	0.1521	0.1533	0.1548	0.1555	0.1560	0.1561	0.1562
0.7	0.1570	0.1620	0.1652	0.1672	0.1686	0.1704	0.1711	0.1717	0.1719	0.1719
0.8	0.1684	0.1739	0.1774	0.1797	0.1812	0.1832	0.1841	0.1847	0.1849	0.1850
0.9	0.1777	0.1836	0.1874	0.1899	0.1915	0.1938	0.1947	0.1954	0.1956	0.1957
1.0	0.1851	0.1914	0.1955	0.1981	0.1999	0.2024	0.2034	0.2042	0.2044	0.2045
1.2	0.1958	0.2028	0.2073	0.2103	0.2124	0.2151	0.2163	0.2172	0.2175	0.2176
1.4	0.2028	0.2102	0.2151	0.2184	0.2206	0.2236	0.2250	0.2260	0.2263	0.2264
1.6	0.2073	0.2151	0.2203	0.2237	0.2261	0.2294	0.2309	0.2320	0.2323	0.2325
1.8	0.2103	0.2183	0.2237	0.2274	0.2299	0.2333	0.2350	0.2362	0.2366	0.2367
2.0	0.2124	0.2206	0.2261	0.2299	0.2325	0.2361	0.2378	0.2391	0.2395	0.2397
2.5	0.2151	0.2236	0.2294	0.2333	0.2361	0.2401	0.2420	0.2434	0.2439	0.2441
3.0	0.2163	0.2250	0.2309	0.2350	0.2378	0.2420	0.2439	0.2455	0.2461	0.2463
4.0	0.2172	0.2260	0.2320	0.2362	0.2391	0.2434	0.2455	0.2472	0.2479	0.2481
5.0	0.2175	0.2263	0.2324	0.2366	0.2395	0.2439	0.2460	0.2479	0.2486	0.2489
6.0	0.2176	0.2264	0.2325	0.2367	0.2397	0.2441	0.2463	0.2482	0.2489	0.2492

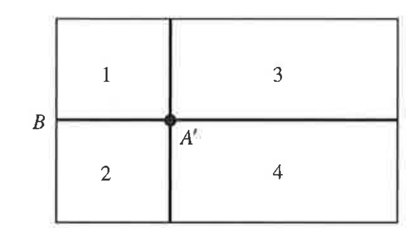

Figure 9.25 Increase of stress at any point below a rectangularly loaded flexible area

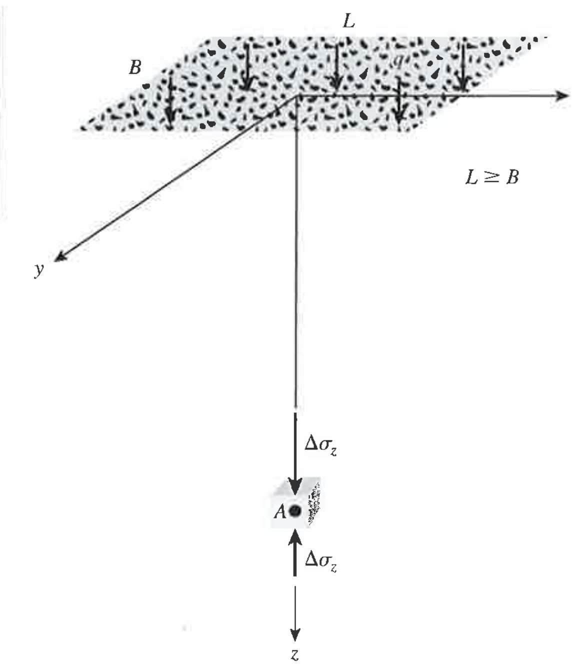

Figure 9.26 Vertical stress below the center of a uniformly loaded flexible rectangular area

Table 9.10 Variation of I_5 with m_1 and n_1 [Eq. (9.40)]

n_1	m_1 = 1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

Example 9.9

The flexible area shown in Figure 9.27a is uniformly loaded. Given that q = 150 kN/m², determine the vertical stress increase at point A.

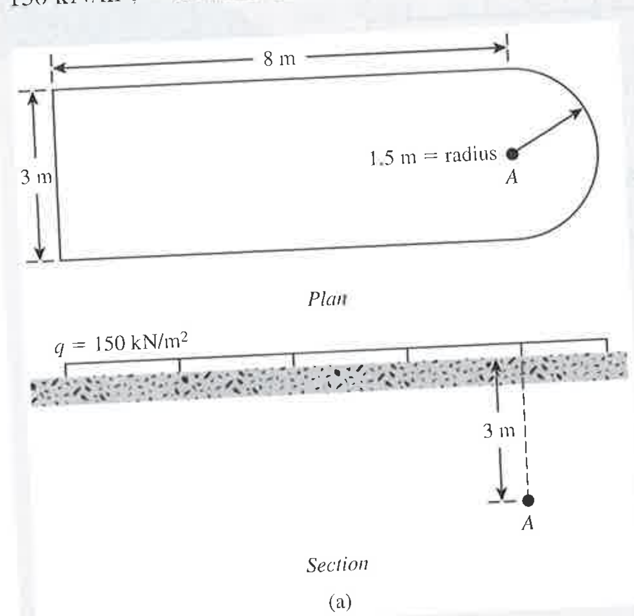

Figure 9.27 (a)

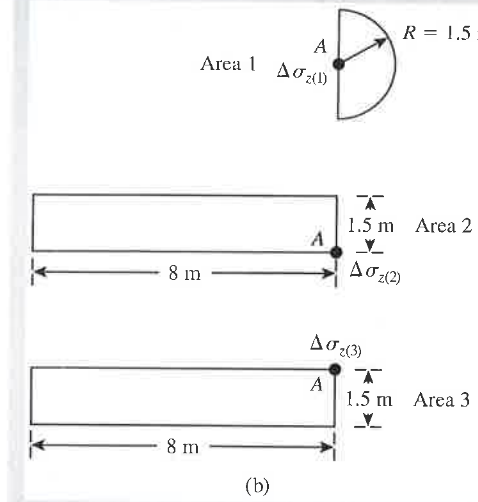

Figure 9.27 (b)

Solution

The flexible area shown in Figure 9.27a is divided into three parts in Figure 9.27b. At A,

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} + \Delta\sigma_{z(3)}$$

From Eq. (9.30),

$$\Delta\sigma_{z(1)} = \left(\frac{1}{2}\right) q \left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\}$$

We know that $R = 1.5$ m, $z = 3$ m, and $q = 150$ kN/m², so

$$\Delta\sigma_{z(1)} = \frac{150}{2}\left\{1 - \frac{1}{[(1.5/3)^2 + 1]^{3/2}}\right\} = 21.3 \text{ kN/m}^2$$

We can see that $\Delta\sigma_{z(2)} = \Delta\sigma_{z(3)}$. From Eqs. (9.36) and (9.37),

$$m = \frac{1.5}{3} = 0.5$$

$$n = \frac{8}{3} = 2.67$$

From Table 9.9, for $m = 0.5$ and $n = 2.67$, the magnitude of $I_4 = 0.1365$. Thus, from Eq. (9.34),

$$\Delta\sigma_{z(2)} = \Delta\sigma_{z(3)} = qI_4 = (150)(0.1365) = 20.48 \text{ kN/m}^2$$

so

$$\Delta\sigma_z = 21.3 + 20.48 + 20.48 = \mathbf{62.26\ kN/m^2}$$

■

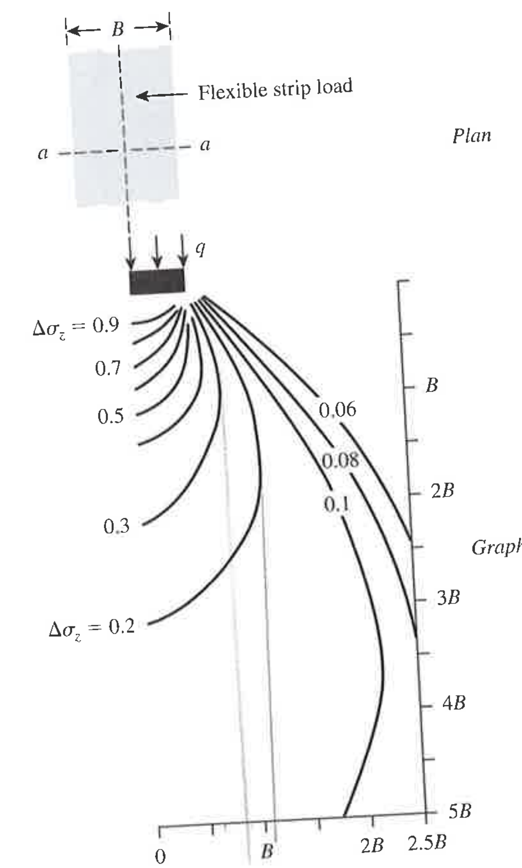

Figure 9.28 Vertical pressure isobars under a flexible strip load (*Note:* Isobars are for line *a–a* as shown on the plan.)

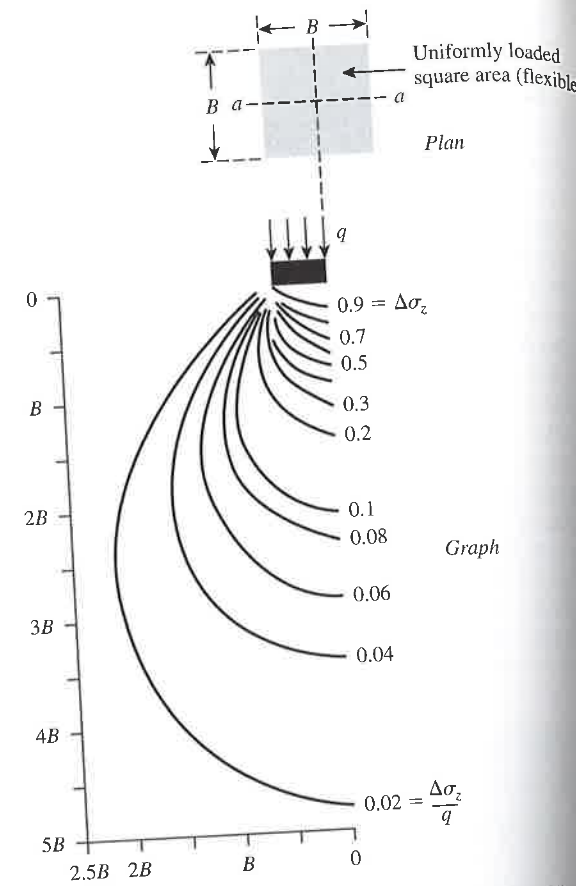

Figure 9.29 Vertical pressure isobars under a uniformly loaded square area (*Note:* Isobars are for line *a–a* as shown on the plan.)

9.13 Stress Isobars

In Section 9.7, we developed the relationship to estimate $\Delta\sigma_z$ at any point due to a vertical strip loading. Also, Section 9.12 provides the relationships to calculate $\Delta\sigma_z$ at any point due to a vertically and uniformly loaded rectangular area. These relationships for $\Delta\sigma_z$ can be used to calculate the stress increase at various grid points below the loaded area. Based on those calculated stress increases, stress isobars can be plotted. Figures 9.28 and 9.29 show such stress isobars under uniformly loaded (vertically) strip and square areas.

9.14 Influence Chart for Vertical Pressure

Equation (9.30) can be rearranged and written in the form

$$\frac{R}{z} = \sqrt{\left(1 - \frac{\Delta\sigma_z}{q}\right)^{-2/3} - 1} \tag{9.44}$$

Table 9.11 Values of R/z for Various Pressure Ratios [Eq. (9.44)]

$\Delta\sigma_z/q$	R/z	$\Delta\sigma_z/q$	R/z
0	0	0.55	0.8384
0.05	0.1865	0.60	0.9176
0.10	0.2698	0.65	1.0067
0.15	0.3383	0.70	1.1097
0.20	0.4005	0.75	1.2328
0.25	0.4598	0.80	1.3871
0.30	0.5181	0.85	1.5943
0.35	0.5768	0.90	1.9084
0.40	0.6370	0.95	2.5232
0.45	0.6997	1.00	∞
0.50	0.7664		

Note that R/z and $\Delta\sigma_z/q$ in this equation are nondimensional quantities. The values of R/z that correspond to various pressure ratios are given in Table 9.11.

Using the values of R/z obtained from Eq. (9.44) for various pressure ratios, Newmark (1942) presented an influence chart that can be used to determine the vertical pressure at any point below a uniformly loaded flexible area of any shape.

Figure 9.30 shows an influence chart that has been constructed by drawing concentric circles. The radii of the circles are equal to the R/z values corresponding to $\Delta\sigma_z/q = 0, 0.1, 0.2, \ldots, 1$. (*Note:* For $\Delta\sigma_z/q = 0$, $R/z = 0$, and for $\Delta\sigma_z/q = 1$, $R/z = \infty$, so nine circles are shown.) The unit length for plotting the circles is $\overline{AB}$. The circles

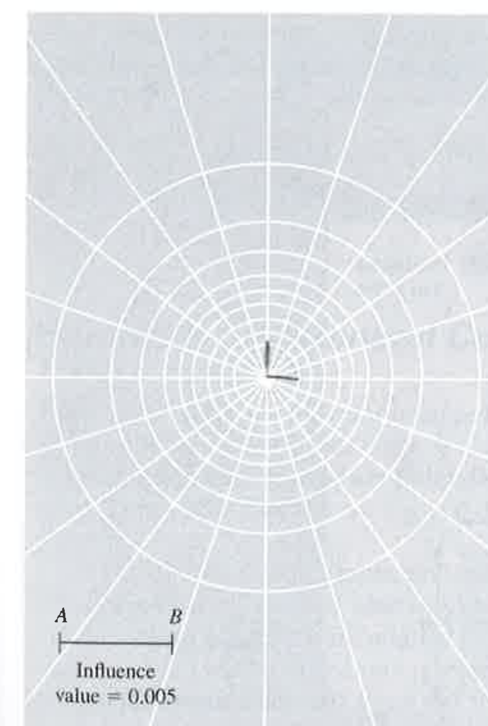

Figure 9.30 Influence chart for vertical pressure based on Boussinesq's theory (After Newmark, 1942)