6.3.3 The deflection of vertical piles carrying lateral loads

A simple method which can be used to check that the deflections due to small lateral loads are within tolerable limits and as an approximate check on the more-rigorous methods described below, is to assume that the pile is fixed at an arbitrary depth below the ground surface and then to calculate the deflection as for a simple cantilever either free at the head, or fixed at the head but with freedom to translate.

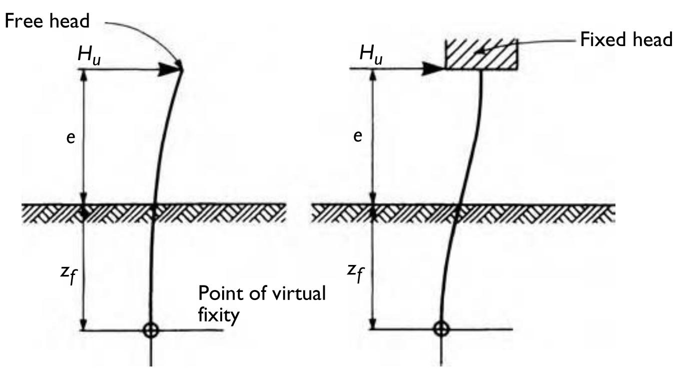

Figure 6.24 Piles under horizontal load considered as simple cantilever.

Thus from Figure 6.24

$$\text{deflection at head of free-headed pile } y = \frac{H(e + z_f)^3}{3EI} \tag{6.20}$$

and

$$\text{deflection at head of fixed-headed pile } y = \frac{H(e + z_f)^3}{12EI} \tag{6.21}$$

where E is the elastic modulus of the material forming the pile shaft, and I is the moment of inertia of the cross-section of the pile shaft. Depths which may be arbitrarily assumed for z_f are noted in Section 6.3.1.

6.3.4 Elastic analysis of laterally loaded vertical piles

The suggested procedure for using this section and Section 6.3.2 is first to calculate the ultimate load H_u for a pile of given cross-section (or to determine the required cross-sections for a given ultimate load) and then to divide H_u by an arbitrary safety factor to obtain trial working load H. The alternative procedure is to calculate the deflection y_0 at the ground surface for a range of progressively increasing loads H up to the value of H_u. The working load is then taken as the load at which y_0 is within the allowable limits. As a first approximation, H_u can be obtained by the Brinch Hansen method (Section 6.3.1) or from equations 6.18 and 6.19. A preliminary indication of the likely order of pile head deflection under this load can be obtained from equations 6.20 or 6.21 depending on the fixity conditions at the head.

It may be necessary to determine the bending moments, shearing forces, and deformed shape of a pile over its full depth at a selected working load. These can be obtained for

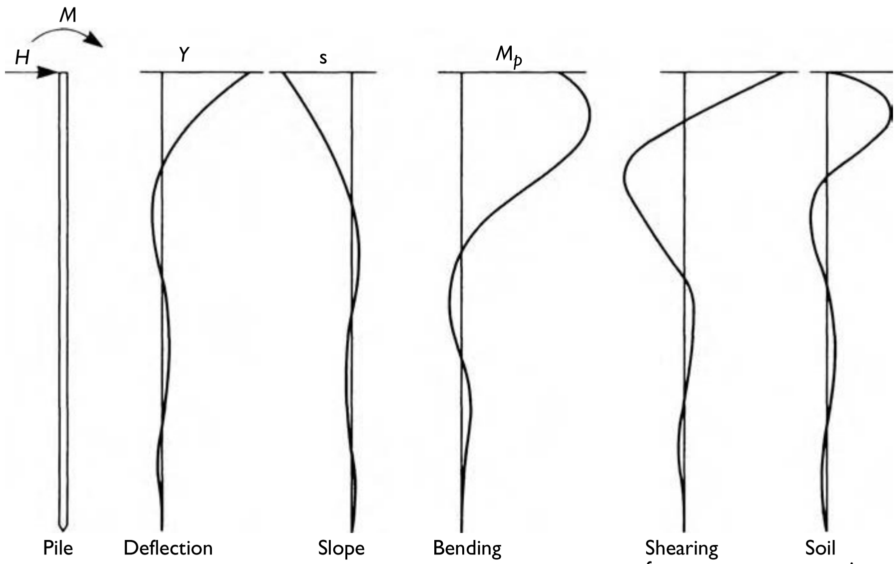

Figure 6.25 Deflections, slopes, bending moments, shearing forces, and soil reactions for elastic conditions (after Reese and Matlock[6.14]).

working-load conditions on the assumption that the pile behaves as an elastic beam on a soil behaving as a series of elastic springs. Calculations for the bending moments, shearing forces, deflections, and slopes of laterally loaded piles are necessary when considering their behaviour as energy absorbing members resisting the berthing impact of ships (see Section 8.1.1), or the wave forces in offshore platform structures (see Section 8.2).

Reese and Matlock(6.14) have established a series of curves for normally consolidated and cohesion-less soils for which the elastic modulus of the soil E_s is assumed to increase from zero at the ground surface in direct proportion to the depth. The deformed shape of the pile and the corresponding bending moments, shearing forces, and soil reactions are shown in Figure 6.25.

Coefficients for obtaining these values are shown for a lateral load H on a free pile head in Figure 6.26a to e, and for a moment applied to a pile head in Figure 6.27a to e. The coefficients for a fixed pile head are shown in Figure 6.28a to c. For combined lateral loads and applied moments the basic equations for use in conjunction with Figures 6.26 and 6.27 are as follows:

$$\text{Deflection } y = y_A + y_B = \frac{A_y HT^3}{EI} + \frac{B_y M_t T^2}{EI} \quad (6.22)$$

$$\text{Slope} = s_A + s_B = \frac{A_s HT^2}{EI} + \frac{B_s M_t T}{EI} \quad (6.23)$$

$$\text{Bending moment} = M_A + M_B = A_m HT + B_m M_t \quad (6.24)$$

$$\text{Shearing force} = V_A + V_B = A_v H + \frac{B_v M_t}{T} \quad (6.25)$$

$$\text{Soil reaction} = P_A + P_B = \frac{A_p H}{T} + \frac{B_p M_t}{T^2} \quad (6.26)$$

For a fixed pile head the basic equations are as follows:

$$\text{Deflection} = y_F = \frac{F_y HT^3}{EI} \quad (6.27)$$

$$\text{Bending moment} = M_F = F_m HT \quad (6.28)$$

$$\text{Soil reaction} = P_F = F_p \frac{H}{T} \quad (6.29)$$

In equations 6.22 to 6.29, H is the horizontal load applied to the ground surface, T (a stiffness factor) $= \sqrt[5]{EI/n_h}$ (as equation 6.12), M_t is the moment applied to the head of the pile, A_y and B_y are deflection coefficients (Figures 6.26a and 6.27a), A_s and B_s are slope coefficients (Figures 6.26b and 6.27b), A_m and B_m are bending-moment coefficients (Figures 6.26c and 6.27c), A_v and B_v are shearing-force coefficients (Figures 6.26d and 6.27d), A_p and B_p are soil resistance coefficients (Figures 6.26e and 6.27e), F_y is the deflection coefficient for a fixed pile head (Figure 6.28a), F_m is the moment coefficient for a fixed pile head (Figure 6.28b), and F_p is the soil resistance coefficient for a fixed pile head (Figure 6.28c).

In Figures 6.26 to 6.28 the above coefficients are related to a depth coefficient Z for various values of Z_{max}, where Z is equal to the depth x at any point divided by T (i.e. $Z = x/T$) and Z_{max} is equal to L/T. The use of curves in Figure 6.28 is illustrated in Example 6.6.

The case of a load H applied at a distance e above the ground surface can be simulated by assuming this to produce a bending moment M_t equal to $H \times e$, this value of M_t being used

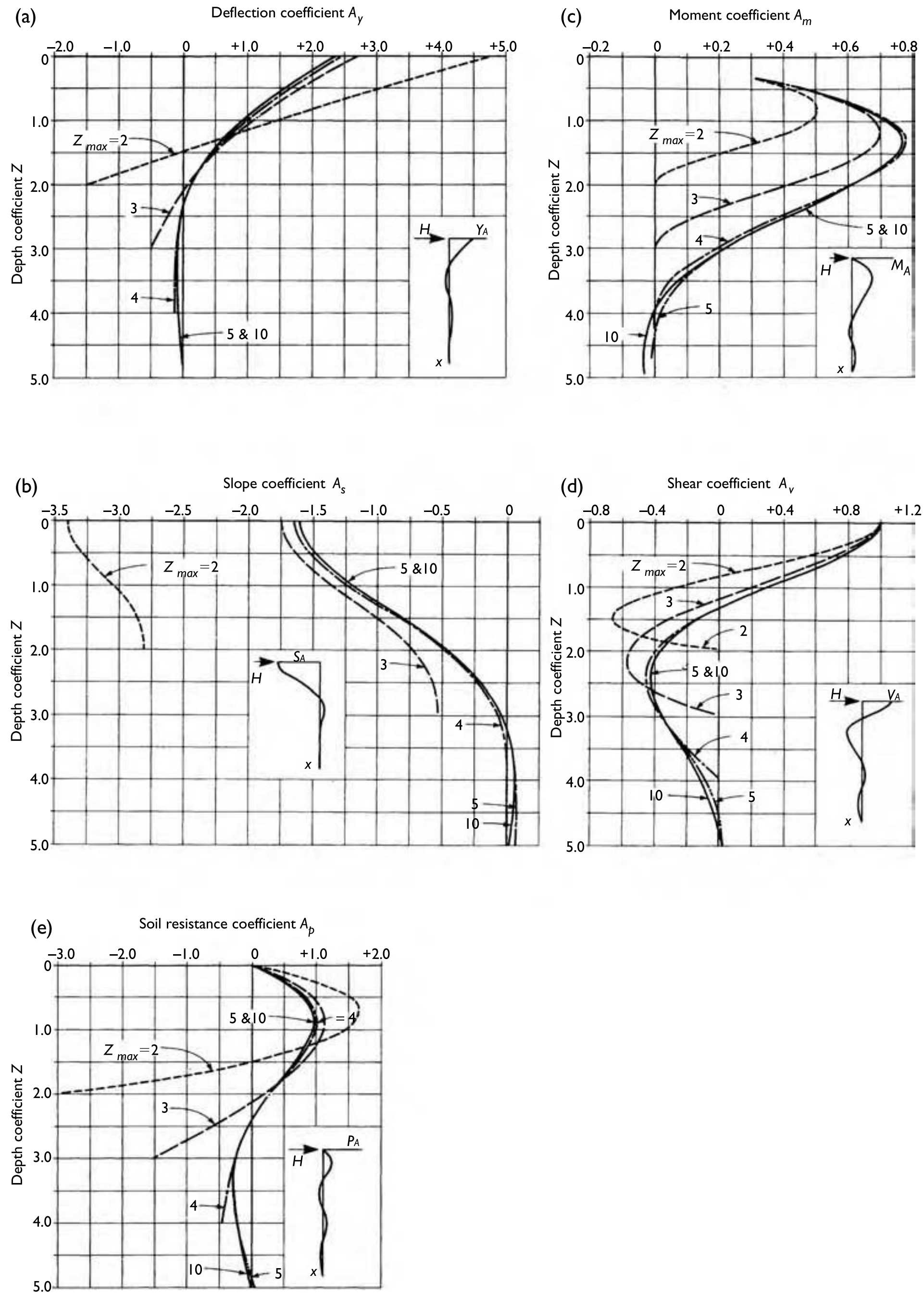

Figure 6.26 Coeffcients for laterally loaded free-headed piles in soil with linearly increasing modulus (after Reese and Matlock[(6.14)]) (a) Coefficients for deflection (b) Coefficients for slope (c) Coefficients for bending moment (d) Coefficients for shearing force (e) Coefficients for soil resistance.

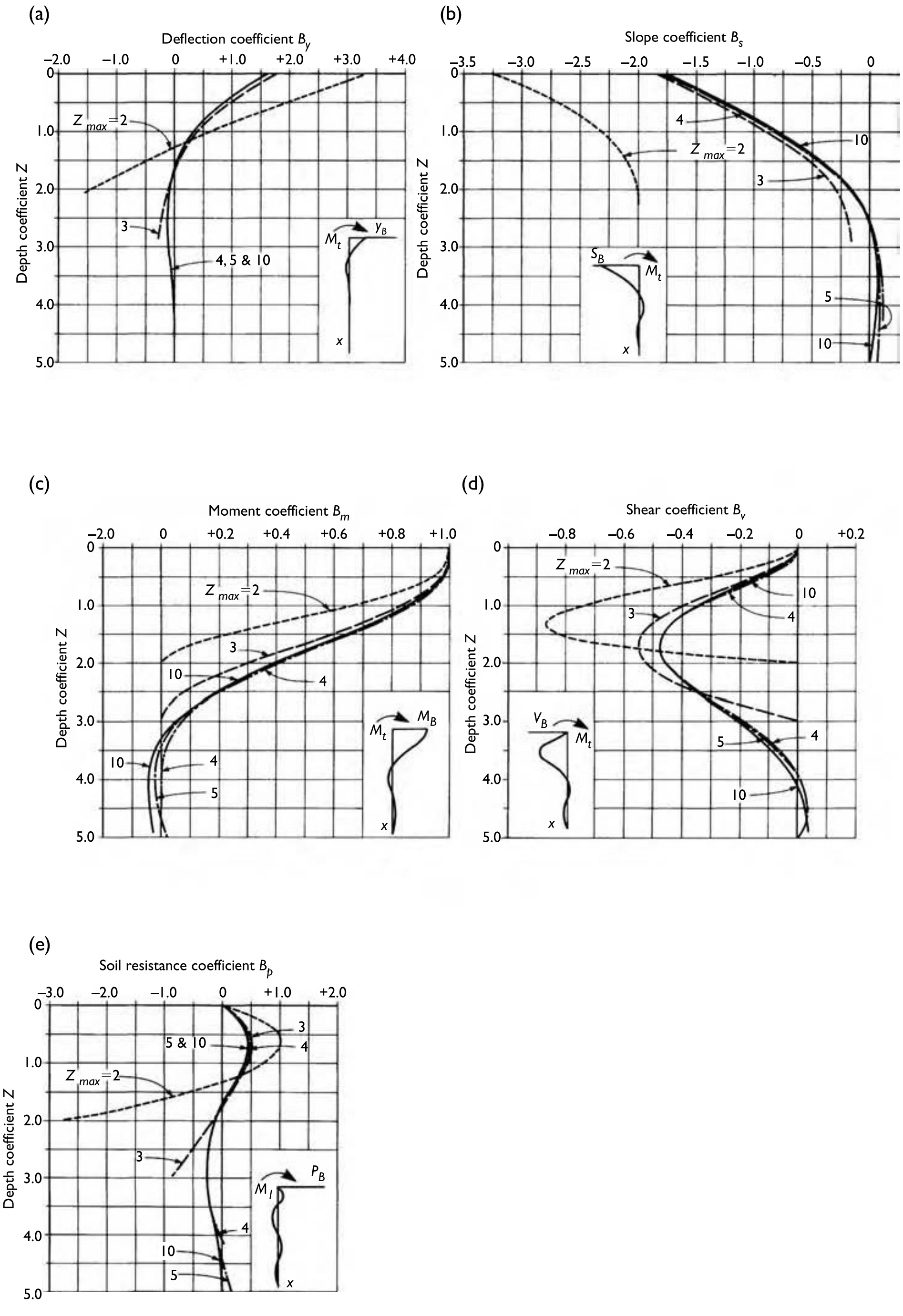

Figure 6.27 Coefficients for piles with moment at free head in soil with linearly increasing modulus (after Reese and Matlock[6.14]) (a) Coefficients for deflection (b) Coefficients for slope (c) Coefficients for bending moment (d) Coefficients for shearing force (e) Coefficients for soil resistance.

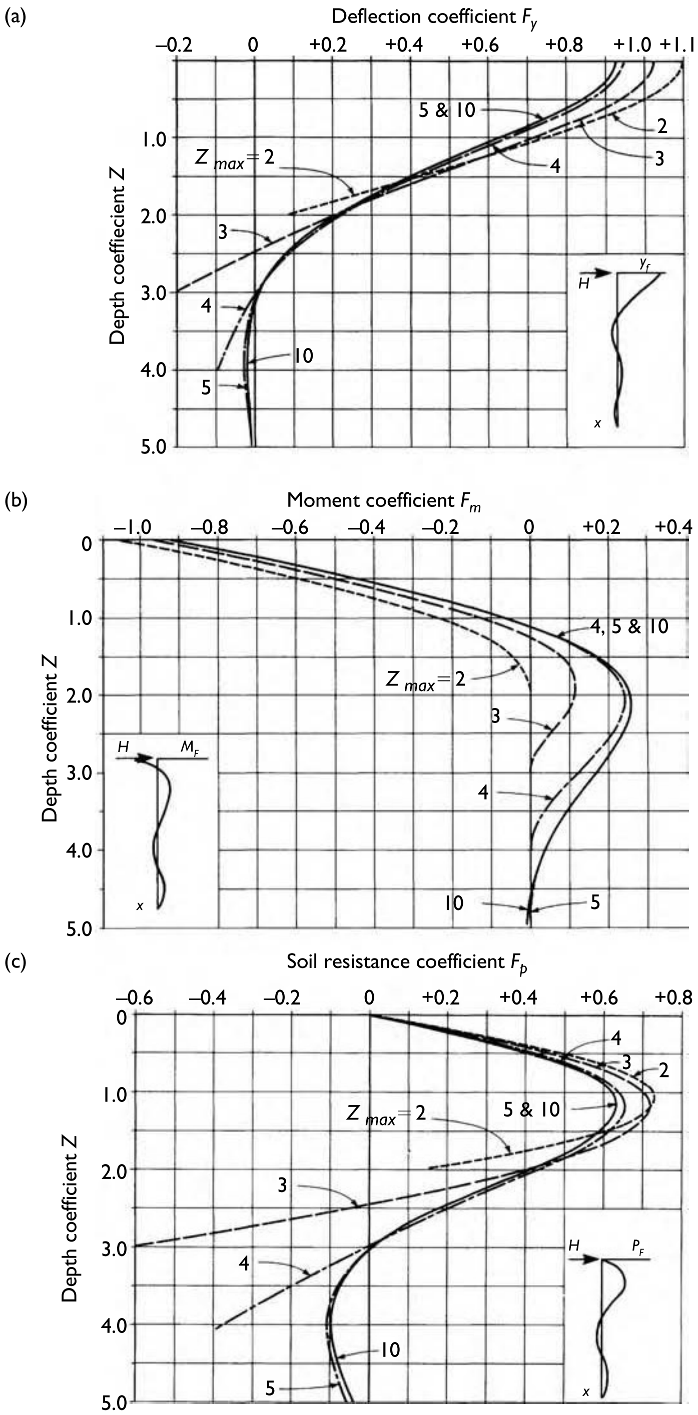

Figure 6.28 Coefficients for fixed headed piles with lateral load in soil with linearly increasing modulus (after Reese and Matlock[6.14]) (a) Coefficients for deflection (b) Coefficients for bending moment (c) Coefficients for soil resistance.

in equations 6.22 to 6.29. The moments M_a produced by load H applied at the soil surface are added arithmetically to the moments M_b produced by moment M_t applied to the pile at the ground surface. This yields the relationship between the total moment and the depth below the soil surface over the embedded length of the pile. The deflection of a pile due to a lateral load H at some distance above the soil surface is calculated in the same manner. The deflections of the pile and the corresponding slopes due to the load H at the soil surface are calculated and added to the values calculated for moment M_t applied to the pile at the surface. To obtain the deflection at the head of the pile, the deflection as for a free-standing cantilever fixed at the soil surface is calculated and added to the deflection produced at the soil surface by load H and moment M_t, together with the deflection corresponding to the calculated slope of the pile at the soil surface. This procedure is illustrated in Example 8.2.

Davisson and Gill[(6.15)] have analysed the case of elastic piles in an elastic soil of constant modulus. The bending moments and deflections are related to the stiffness coefficient R (equation 6.11) but in this case the value of K is taken as Terzaghi's subgrade modulus k_1, using the values shown in Table 6.5. The dimensionless depth coefficient Z in Figure 6.29 is equal to x/R. From these curves, deflection and bending moment coefficients are obtained for free-headed piles carrying a moment at the pile head and zero lateral load (Figure 6.29a) and for free-headed piles with zero moment at the pile head and carrying a horizontal load (Figure 6.29b). These curves are valid for piles having an embedded length L greater than $2R$ and different moment and deflection curves are shown for values of $Z_{max} = L/R$ of 2, 3, 4, and 5. Piles longer than $5R$ should be analysed for $Z_{max} = 5$. The equations to be used in conjunction with the curves in Figure 6.29 are as follows:

Load on pile head *For free-headed pile:*

Moment M; Bending moment $= MM_m$ (6.30)

Moment M; Deflection $= My_m\frac{R^2}{EI}$ (6.31)

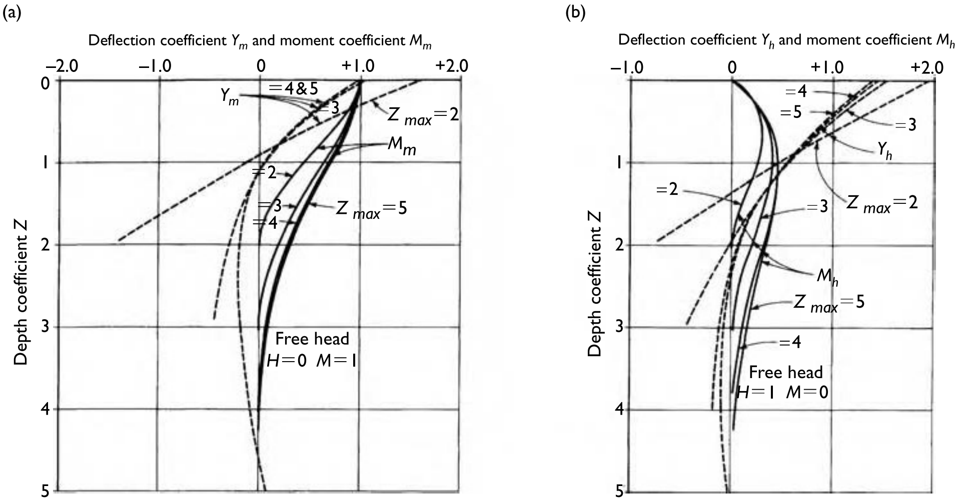

Figure 6.29 Coefficients for free headed piles carrying lateral load or moment at pile head in soil of constant modulus (after Davisson and Gill[(6.15)]) (a) Coefficients for deflection and bending moment for piles carrying moment at head and zero lateral (b) Coefficients for deflection and bending moment for piles carrying horizontal load at head and zero moment.

Horizontal load H; Bending moment $= HM_hR$ (6.32)

Horizontal load H; Deflection $= Hy_h\frac{R^3}{EI}$ (6.33)

The effect of fixity at the pile head can be allowed for by plotting the deflected shape of the pile from the algebraic sum of the deflections (equations 6.31 and 6.33) and then applying a moment to the head which results in zero slope for complete fixity, or the required angle of slope for a given degree of fixity. The deflection for this moment is then deducted from the calculated value for the free-headed pile. The use of the curves in Figure 6.29 is illustrated in Example 8.2. Conditions of partial fixity occur in jacket-type offshore platform structures where the tubular jacket member only offers partial restraint to the pile that extends through it to below sea-bed level.

Where marine structures are supported by long piles ($L \geq 4T$) , Matlock and Reese[6.16] have simplified the process of calculating deflections by re-arranging equation 6.27 to incorporate a deflection coefficient C_y. Then

$$y = C_y\frac{HT^3}{EI} \tag{6.34}$$

where

$$C_y = A_y + \frac{M_tB_y}{HT} \tag{6.35}$$

Values of C_y are plotted in terms of the dimensionless depth factor $Z(= x/T)$ for various values of M_t/HT in Figure 6.30. Included in these curves are the fixed-headed case (i.e. $M_t/HT = -0.93$) and the free-headed case (i.e. $M_t = 0$).

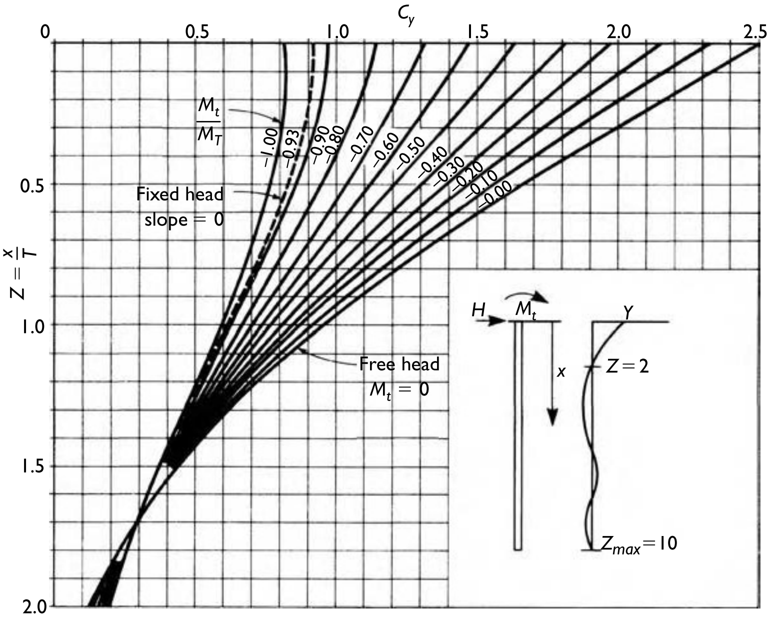

Figure 6.30 Coefficients for calculating deflection of pile carrying both moment and lateral load (after Matlock and Reese[6.16]).

The elastic deflections of piles in layered soils, each soil layer having its individual constant modulus, have been analysed by Davisson and Gill[6.15] who have produced design charts for this condition.