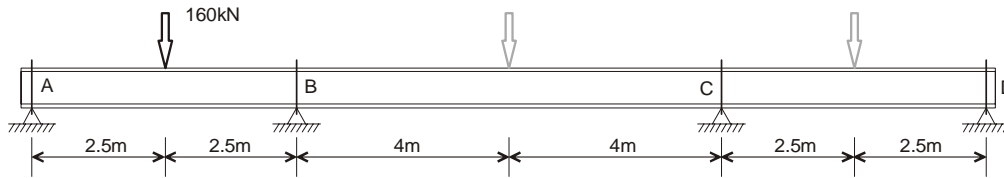


### EXAMPLE 3



The continuous beam ABCD is to be designed to carry load-factored midspan live loads of  $1.5Q=160\text{kN}$  that may act on any or all spans. The task is to select a suitable grade 300 UB section and suggest appropriate lateral restraint locations.

#### Restraints:

All supports are assumed to provide full torsional restraint. Loads provide no restraint and are applied to the top flange.

#### Analysis:

We will ignore self-weight and hope to show that its effect is negligible. This leaves 5 load cases to consider, as shown.

The results could be obtained from just two analyses (by moment distribution, for example):

- (a) Load on AB
- (b) Load on BC

All other cases can be obtained as combinations of (a) and (b). For example case (c) is simply (a)+(b), and case (d) is (a)+(b)+mirror image of (a).

Maximum design moment (all cases) = 207.1kNm.

#### First approximation:

Assume a compact section, and take  $Z_e = Z_p$ .

For  $M^* < \phi Z_e \sigma_y$

$$\text{Require } Z_p > \frac{207.1}{0.9 \times 300,000} = 767 \times 10^{-6} \text{ m}^3 = \underline{767 \times 10^3 \text{ mm}^3}$$

Try 360UB44.7,  $Z_p=777 \times 10^3 \text{ mm}^3$  (isn't quite compact, but has only slightly smaller  $Z_e=770 \times 10^3 \text{ mm}^3$ , more than compensated for by  $\sigma_y=320\text{MPa}$ , so that  $\phi Z_e \sigma_y = 222 \text{ kNm}$ ,  $> 207.1$ )

#### Check end spans AB, CD

##### Top flange

Maximum  $M^*(+ve)=178\text{kNm}$  - load case (e) - top flange critical (compr'n).

'FF' segment, 5m long.  $k_i=1$ ,  $k_L=1.4$ ,  $k_r=1$ ,  $L_e = 1.4 \times 5 = \underline{7.0\text{m}}$

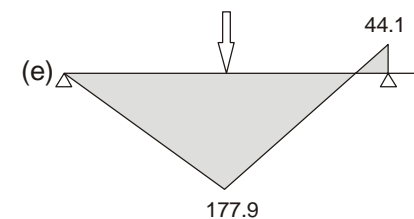
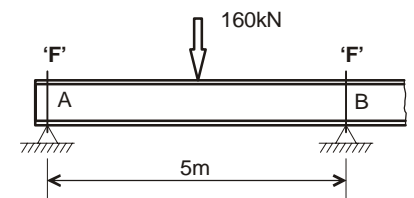
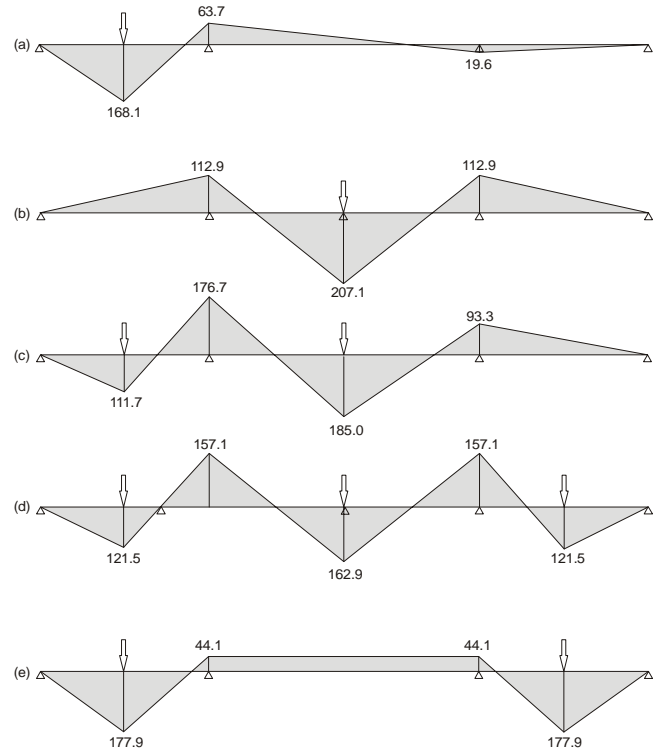
$$\alpha_s = \underline{0.293} \text{ (Table A1)}$$

$$\beta_m = \frac{44.1 \times 16}{3 \times 160 \times 5} = 0.294 \text{ (Table 5.6.1, case 4)}$$

$$\alpha_m = 1.35 + 0.15 \times 0.29 = \underline{1.39}$$

$$\phi M_b = 1.39 \times 0.293 \times 222 = \underline{90.4\text{kNm}} < M_{\text{max}}^* (178\text{kNm}) \Rightarrow \text{NG} \otimes$$

#### Try lateral (L) restraint to top flange, mid-span



This creates two segments, 2.5m long. Considering the left hand segment:

'FL' segment, 2.5m long.  $k_t=1$ ,  $k_L=1$  (load outside seg),  $k_r=1$ ,  $L_e = 1.0 \times 2.5 = 2.5\text{m}$   
 $\alpha_s = 0.778$  (Table A1).

$\alpha_m = 1.75$  (Table 5.6.1, case 1,  $\beta_m = 0$ , or case 9).

$$\phi M_b = 1.75 \times 0.778 \times 222 = 302\text{kNm},$$

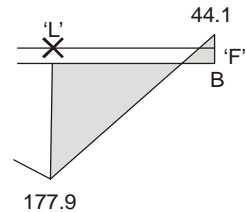
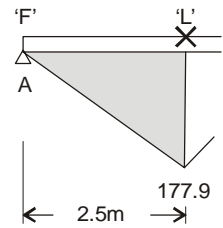
However  $\phi M_b$  must be  $\leq \phi M_s = 222\text{kNm} > M_{\max}^* (178\text{kNm}) \Rightarrow \text{OK} \text{ 😊}$

Right segment certain to be less critical as bending moment pattern will give rise to a higher  $\alpha_m$ :

$$\beta_m = \frac{44.1}{178} = 0.25 \text{ (Table 5.6.1, case 1).}$$

$$\alpha_m = 1.75 + 1.05(0.25) + 0.3(0.25)^2 = 3.78 = 2.5(\text{max}).$$

$$\alpha_m \alpha_s > 1, \text{ so } \phi M_b \text{ still} = \phi M_s = 222\text{kNm} \Rightarrow \text{OK} \text{ 😊}$$



### Bottom flange

Top flange restraint does not restrain bottom flange. For loading case (b) bottom flange is critical so spans AB and BC revert to 5m segments.

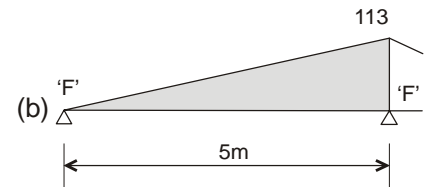
Maximum  $M^*(-ve)=113\text{kNm}$

'FF' segment, 5m long.  $k_t=1$ ,  $k_L=1$ ,  $k_r=1$ ,  $L_e = 5.0\text{m}$

$\alpha_s = 0.436$  (Table A1)

$\alpha_m = 1.75$  (Table 5.6.1, case 1,  $\beta_m = 0$ ).

$$\phi M_b = 1.75 \times 0.436 \times 222 = 169\text{kNm} > M_{\max}^* (113\text{kNm}) \Rightarrow \text{OK} \text{ 😊}$$



### Check middle span, BC

#### Top flange

Top flange is critical flange for load cases (b), (c) and (d).

Maximum  $M^*(+ve)=207\text{kNm}$  - load case (b) - top flange critical.

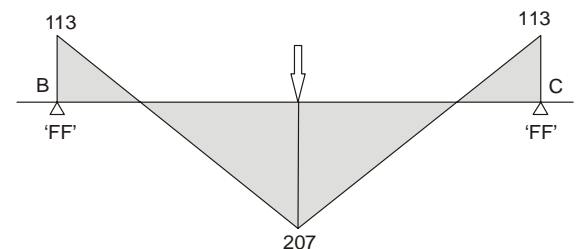
'FF' segment, 8m long.  $k_t=1$ ,  $k_L=1.4$ ,  $k_r=1$ ,  $L_e = 1.4 \times 8 = 11.2\text{m}$

$\alpha_s < 0.190$  (Table A1;  $\alpha_s = 0.19$  is for  $L_e=10\text{m}$ )

$$\beta_m \frac{FL}{8} = 113; \quad \beta_m = \frac{113 \times 8}{160 \times 8} = 0.706 \text{ (Table 5.6.1, case 4)}$$

$$\alpha_m = 1.35 + 0.36 \times 0.706 = 1.60$$

$$\phi M_b = 1.6 \times 0.19 \times 222 = 67.5\text{kNm} < M_{\max}^* (207\text{kNm}) \Rightarrow \text{NG} \text{ ☹}$$



#### Try lateral (L) restraint to top flange, mid-span

This creates two segments, 4m long. Considering the left hand segment:

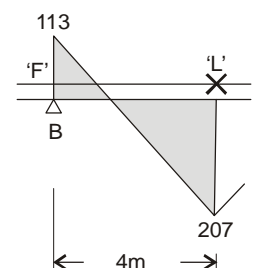
'FL' segment, 4m long.  $k_t=1$ ,  $k_L=1$  (load outside seg),  $k_r=1$ ,  $L_e = 1.0 \times 4 = 4\text{m}$

$\alpha_s = 0.55$  (Table A1).

$$\beta_m = \frac{113}{207} = 0.546 \text{ (Table 5.6.1, case 1).}$$

$$\alpha_m = 1.75 + 1.05(0.55) + 0.3(0.55)^2 = 2.42.$$

$$\alpha_m \alpha_s = 1.33, \text{ so } \phi M_b = \phi M_s = 222\text{kNm} > M_{\max}^* (207\text{kNm}) \Rightarrow \text{OK} \text{ 😊}$$



Bottom flange

Predominantly -ve bm in loading cases (e) and (a) results in critical bottom flange. Since bottom flange is unrestrained it becomes an 8m segment.

Load case (e):

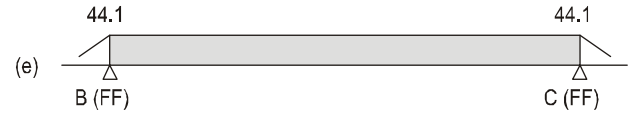
Maximum  $M^*(-ve)=44 \text{ kNm}$

'FF' segment, 8m long.  $k_t=1, k_L=1, k_r=1, L_e = 8.0\text{m}$

$\alpha_s = 0.248$  (Table A1)

$\alpha_m = 1.0$  (Table 5.6.1, case 8)

$\phi M_b = 1.0 \times 0.248 \times 222 = 55 \text{ kNm} > M_{max}^* (44 \text{ kNm}) \Rightarrow \text{OK} \text{ 😊}$



Load case (a):

Maximum  $M^*(-ve)=64 \text{ kNm}$

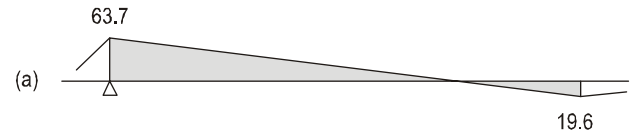
'FF' segment, 8m long.  $k_t=1, k_L=1, k_r=1, L_e = 8.0\text{m}$

$\alpha_s = 0.248$  (Table A1)

$\beta_m = \frac{19.6}{63.7} = 0.31$  (Table 5.6.1, case 1).

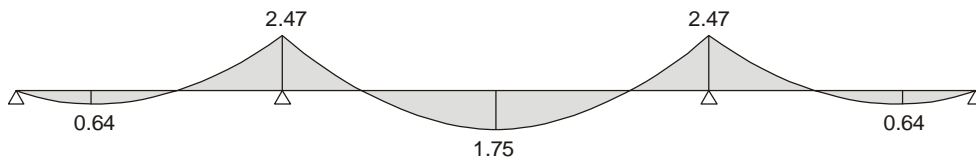
$\alpha_m = 1.75 + 1.05(0.31) + 0.3(0.31)^2 = 2.1$

$\phi M_b = 2.1 \times 0.248 \times 222 = 115 \text{ kNm} > M_{max}^* (64 \text{ kNm}) \Rightarrow \text{OK} \text{ 😊}$



Check that dead load IS negligible:

Dead load is 44.7 kg/m giving  $w_{dead} = 1.2 \times 0.45 = 0.53 \text{ kN/m}$ . Applying a uniform spread load of 0.53kN/m gives the bm's shown below.



These are insignificant compared with the 1.6Q (live load) bm's which ranged up to 207 kNm.

Final configuration:

