# How to use the performance curves to evaluate behavior of centrifugal compressors 


#### Abstract

Pressure, temperature, compressibility, molecular weight and specific-heat ratio of the gas or gas mixture at the compressor inlet, and the machine's rotational speed affect the operating performance of single-stage compressors. Here is how to calculate the effects of changes in these factors.


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$\square$ Fluctuations of inlet conditions for a gas affect the performance of centrifugal compressors. For example, a compressor receiving inlet air at atmospheric conditions will develop higher discharge pressure on cold days than on hot days at a given rotational speed and inletvolume flow. The power requirement also will be higher.

Changes in atmospheric conditions such as relative humidity and barometric pressure will affect performance, although these factors are usually less significant than inlet temperature.

We can account for these changes and others encountered during operation by modifying the performance curve of the compressor. Manufacturers of centrifugal compressors often supply curves that define the machines' aerodynamic performance. These curves take many forms, some of which are:

- Polytropic or adiabatic head and horsepower vs. inlet volume flow.
- Discharge pressure (psia) and horsepower vs. inlet volume flow.
- Discharge pressure (in. gage, water column) and horsepower vs. inlet volume flow.

The data for the performance curve are the nameplate rating conditions, i.e., inlet pressure, inlet temperature, molecular weight, ratio of specific heats and inlet compressibility. The manufacturer will not normally supply performance curves for other than rated inlet conditions unless specifically requested.

We will develop application procedures for modifications to the performance curve for a single-stage centrifugal air compressor. However, these procedures are valid for any gases, and for multistage centrifugal compressors, with somewhat reduced accuracy.

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## Compressor-stage characteristics

First, let us consider the following equations to illustrate the procedure accounting for variations in inlet conditions:

$$
\begin{equation*}
\left(H_{a d}\right)_{s}=\mu u^{2} / g \tag{1}
\end{equation*}
$$

where: $u=N \pi d / 720$

$$
\begin{align*}
Q & =W v=W\left(Z_{1} R T_{1} / 144 P_{1}\right)  \tag{2}\\
(S H P) & =\left[\left(H_{a d}\right)_{s} W / 33,000 \eta_{a d}\right]+L_{m} \tag{3}
\end{align*}
$$

Eq. (1) shows that the head produced by an impeller is a function only of its mechanical tip speed, $u$, and head coefficient, $\mu$; that, in turn, is a function of the inlet volume flow. Therefore, the head produced by an impeller at a fixed speed and inlet volume is a constant. ${ }^{\dagger}$ This statement forms the basis upon which we can derive the procedures accounting for change in inlet conditions.

If we compare Eq. (2) and (3) at a fixed flow for the inlet volume, we find that variations in inlet conditions will affect the power requirements. Here, an increase in inlet temperature will decrease the power requirements, and an increase in inlet pressure will increase the power requirement. These power effects arise from changes in the inlet density and, hence, in the weight flow.

## Performance curves

Fig. 1 is a typical performance curve for a single-stage centrifugal compressor at rated inlet conditions, and discharge expressed as adiabatic head. Fig. 2 is a similar curve with the compressor's discharge expressed as pressure, psia.

Compressor manufacturers provide such curves to

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define the flange-to-flange performance of their machines. External hardware such as inlet and discharge piping, inlet filters, and inlet and discharge valves are not normally considered in establishing the performance curve. Hence, we must account for pressure drop due to external hardware when using the performance curve.

The term "inlet volume flow" will be used extensively in the developments that follow. Inlet volume flow is the volume flow that exists at the compressor's inlet flange.

We will develop our techniques by using the adia-batic-head curve because adiabatic head lends itself more readily to our developments. The final equations, however, are applicable to the performance curves expressed as adiabatic head or discharge pressure.

## Inlet pressures

Let us begin by considering the effects of a variation in inlet pressure. For our discussion, consider that the compressor draws atmospheric air through an inlet filter, as shown in Fig. 3. The rated compressor inlet pressure is 14.5 psia. When the filter becomes dirty, the inlet pressure at the compressor flange drops to 14.2 psia. What is the effect on discharge pressure and shaft horsepower at the rated inlet volume flow?


Discharge-pressure performance curve for single-stage centrifugal compressor

Fig. 2

Discharge pressure is related to the adiabatic head by:

$$
\begin{equation*}
H_{a d}=Z_{1} R T_{1} \frac{k}{k-1}\left[r_{p}^{(k-1) / k}-1\right] \tag{4}
\end{equation*}
$$

For a given inlet volume flow at a given rotational speed, the head output is constant, and because there is no change in the other inlet conditions, the pressure ratio does not vary. Therefore:

$$
\begin{align*}
& r_{p}=\left(r_{p}\right)_{r}=20.6 / 14.5=1.42 \\
& P_{2}=P_{1}\left(r_{p}\right)_{r}=14.2(1.42)=20.2 \text { psia } \tag{5}
\end{align*}
$$

Rearranging Eq. (2) yields:

$$
\begin{equation*}
W=144 Q P_{1} / Z_{1} R T_{1} \tag{6}
\end{equation*}
$$

On comparing Eq. (3) and (6), we find that shaft horsepower is proportional to weight flow or inlet pressure, or:

$$
\begin{equation*}
(S H P)=\left[\frac{P_{1}}{\left(P_{\mathbf{1}}\right)_{\tau}}\right](S H P)_{\tau} \tag{7}
\end{equation*}
$$

Eq. (7) is not strictly valid, because shaft horsepower is composed of gas horsepower and mechanical losses. The mechanical losses are approximately constant for a given speed but are generally a small part of the total

| Nomenclature |  |
| :---: | :---: |
| $C_{1}$ | Constant |
| $C_{2}$ | Constant |
| d | Impeller tip diameter, in. |
| $g$ | Gravitational constant, $32.2 \mathrm{ft}-\mathrm{lb}_{\mathrm{f}} /\left(\mathrm{lb}_{\mathrm{m}}\right)\left(\mathrm{s}^{2}\right)$ |
| $H_{a d}$ | Adiabatic head, $\mathrm{ft}-\mathrm{lb} / \mathrm{l}_{\mathrm{f}} / \mathrm{lb}_{\mathrm{m}}$ |
| $\left(H_{a d}\right)_{s}$ | Adiabatic head developed by a centrifugal compressor stage, $\mathrm{ft}^{-1 \mathrm{~b}_{\mathrm{t}}} / \mathrm{lb}_{\mathrm{m}}$ |
| $k$ | Ratio of specific heats, $c_{p} / c_{v}$ |
| $L_{m}$ | Mechanical losses |
| MW | Molecular weight |
| $N$ | Rotational speed, rpm |
| $P$ | Pressure, psia |
| $Q$ | Volume flow, icfm (inlet $\mathrm{ft}^{3} / \mathrm{min}$ ) |
| $R$ | Gas constant, $\mathrm{ft}^{-1 \mathrm{lb}_{\mathrm{f}} /\left(\mathrm{lb}_{\mathrm{m}}\right)\left({ }^{\circ} \mathrm{R}\right)}$ |
| $r_{p}$ | Pressure ratio, $P_{2} / P_{1}$ |
| (SHP) | Shaft horsepower, hp |
| $T$ | Temperature, ${ }^{\circ} \mathrm{R}$ |
| $u$ | Mechanical tip speed, ft/s |
| $v$ | Specific volume, $\mathrm{ft}^{3} / \mathrm{lb}_{\mathrm{m}}$ |
| W | Weight flow, $\mathrm{lb}_{\mathrm{m}} / \mathrm{min}$ |
| $Z$ | Compressibility |
| $\eta_{a d}$ | Adiabatic efficiency |
| $\mu$ | Head coefficient |
| Subscripts |  |
| $f$ | Fan law |
| i.c. | Inlet conditions |
| $\tau$ | Rated |
| req | Required |
| s | Stage |
| 1 | Inlet |
| 2 | Discharge |

horsepower. Therefore, ignoring mechanical losses will usually yield a useful approximation in our procedures.

The performance curve (such as Fig. 1) indicates that the power requirement for the rated conditions is 1,315 hp. Substituting in Eq. (7), we find:

$$
(S H P)=(14.2 / 14.5)(1,315)=1,290 \mathrm{hp}
$$

In this example, we have ignored the effects of system resistance downstream of the compressor discharge flange. For many applications, system resistance is small when compared to the total pressure requirements of the compressor and will therefore have minimal effect on the analysis.

In some applications, however, system resistance effects are large and these will actually define the compressor operation.

We can view system resistance as the sum of piping and system losses and utility pressure drops. These should not be thought of as "losses." As the volume flow through the system increases, frictional losses increase and require a higher pressure at the compressor's discharge flange to be overcome.

Fig. 4 shows a typical system resistance line superimposed on two compressor performance curves. The solid curve represents rated inlet conditions. The dashed curve shows the effects of a reduction in inlett pressure

only. Point A on the solid curve is the rated operating point.

In this example, we have assumed constant inlet volume flow and, therefore, calculated the discharge pressure at Point C. The power requirement calculated as $1,290 \mathrm{hp}$ was for operation at Point C.

If the compressor in this example operated with system resistance, it would search its new performance curve (the dashed line in Fig. 4) until it intersected system requirements. The result would be operation at Point B. Therefore, the inlet volume flow would be somewhat less than rated and the discharge pressure would be slightly higher than calculated. Referring to the horsepower curve of Fig. 1 or 2 would show that the power requirement would be less than calculated.

## Inlet temperatures

Let us assume that the inlet temperature drops to $40^{\circ} \mathrm{F}$ while other inlet conditions remain at rated values. What is the effect on discharge pressure and shaft horsepower for the compressor defined by Fig. 1 at the rated inlet volume flow?

Rearranging Eq. (4) yields:

$$
\begin{equation*}
r_{p}^{(k-1) / k}-1=\frac{H_{a d}(k-1)}{Z_{1} R T_{1} k} \tag{8}
\end{equation*}
$$

Eq. (8) indicates that a change in inlet temperature, $T_{1}$, inversely affects the pressure ratio. For a change in inlet temperature only, we can derive:

$$
\left(r_{p}\right)_{\tau}^{(k-1) / k}-1=C_{1} /\left(T_{1}\right)_{\tau}
$$

where: $C_{1}=H_{a d}(k-1) / Z_{1} R k$.

$$
\begin{aligned}
r_{p}^{(k-1) / k}-1 & =C_{1} / T_{1} \\
{\left[\frac{r_{p}^{(k-1) / k}-1}{\left(r_{p}\right)_{T}^{(k-1) / k}-1}\right] } & =\frac{\left(T_{1}\right)_{r}}{T_{1}}
\end{aligned}
$$

Solving this equation for $r_{p}$, we get:

$$
\begin{align*}
& r_{p}=\left\{\frac{\left(T_{1}\right)_{r}}{T_{1}}\left[\left(r_{p}\right)_{r}^{(k-1) / k}-1\right]+1\right\}^{k /(k-1)} \\
& P_{2}=P_{1}\left\{\frac{\left(T_{1}\right)_{r}}{T_{1}}\left[\left(r_{p}\right)_{r}^{(k-1) / k}-1\right]+1\right\}^{k /(k-1)} \tag{9}
\end{align*}
$$



Substituting the new inlet temperature of $40^{\circ} \mathrm{F}$ and the rated inlet temperature of $90^{\circ} \mathrm{F}$ into Eq. (9) yields the discharge pressure:

$$
P_{2}=14.5\left\{\frac{550}{500}\left[(1.42)^{0.286}-1\right]+1\right\}^{3.5}=21.3 \text { psia }
$$

By comparing Eq. (3) and (6), we find that shaft horsepower is inversely proportional to inlet temperature, or:

$$
\begin{align*}
& S H P=\left[\left(T_{1}\right)_{r} / T_{1}\right](S H P)_{r}  \tag{10}\\
& S H P=[550 / 500](1,315)=1,450 \mathrm{hp}
\end{align*}
$$

Again, we have neglected the effects of system resistance. Where system resistance prevails, we may refer to Fig. 5. The solid curve represents the compressor performance for the rated inlet conditions. The dashed curve represents a drop in inlet temperature with all other inlet conditions remaining at the rated values. Point A is the compressor rated point.
In our last example, we considered a constant inlet volume flow. Therefore, we calculated the performance at Point C . The compressor will search its new performance curve until it reaches Point B at its intersection with the system resistance line. As shown in Fig. 5, inlet flow will be somewhat higher than rated, and discharge pressure slightly lower than calculated. Reference to Fig. 1 or 2 will show that the power requirement will be higher than calculated.

If we compare Fig. 4 and 5, we see that a drop in inlet pressure has the net result of lowering the discharge pressure curve; while a drop in inlet temperature results in raising the curve. From this, we can deduce that we can obtain rated performance on cold days by suction

throttling of the inlet pressure. (It is also possible to obtain the same result by lowering the speed on varia-ble-speed drives. More on this later.)

Suction throttling also lowers the power requirement, because horsepower is directly proportional to inlet pressure.

In this discussion, we are trying to analyze each variable independently. Hence, we have not taken into account the change in water-vapor content of the air due to the change in inlet temperature. For the rated conditions, the molecular weight of the air is 28.7. When the inlet temperature changes to $40^{\circ} \mathrm{F}$, the molecular weight becomes 28.9 (assuming relative humidity remains at $50 \%$ ). We will consider the effects of changes in water content in the next example.

## Molecular weights

The molecular weight of an air/water-vapor mixture varies with composition. The effect of this on air compressors is usually small, and can generally be ignored. However, gas compressors can operate over a wide range of molecular weights, making this variable significant. As an example of how to account for variation in molecular weight, we will consider a change in relative humidity at rated inlet temperature.

On a given day, the atmospheric temperature is $90^{\circ} \mathrm{F}$ and the relative humidity is $100 \%$. At a barometric pressure of 14.7 psia, the molecular weight is approximately 28.4 ; while at the rated conditions it is 28.7.

Let us investigate how the molecular weight enters the head equation. The term $R$ in Eq. (4) is given by:

$$
\begin{equation*}
R=1,545 / M W \tag{11}
\end{equation*}
$$



Therefore, we can rewrite Eq. (4) to include the $M W$ :

$$
H_{a d}=Z_{1}\left(\frac{1,545}{M W}\right) T_{1}\left(\frac{k}{k-1}\right)\left(r_{p}^{(k-1) / k}-1\right)
$$

Rearranging to solve for the term containing the pressure ratio yields:

$$
r_{p}^{(k-1) / 1}-1=\frac{H_{a d} M W(k-1)}{1,545 Z_{1} T_{1} k}
$$

This relationship implies that a change in molecular weight affects the pressure ratio. Therefore, for a change in molecular weight only, we can derive:

$$
\left(r_{p}\right)_{\tau}^{(k-1) / k}-1=C_{2} M W_{\tau}
$$

where: $C_{2}=H_{a \dot{d}}(k-1) / 1,545 Z_{1} T_{1} k$

$$
\begin{gather*}
r_{p}^{(k-1) / k}-1=C_{2} M W \\
r_{p}=\left\{\frac{M W}{M W}\left[\left(r_{p}\right)_{r}^{(k-1) / k}-1\right]+1\right\}^{k /(k-1)} \\
P_{2}=P_{1}\left\{\frac{M W}{M W_{r}}\left[\left(r_{p}\right)_{r}^{(k-1) / k}-1\right]+1\right\}^{k /(k-1)} \tag{12}
\end{gather*}
$$

Solving for the condition of $100 \%$ relative humidity by substituting into Eq. (12), we get:

$$
P_{2}=14.5\left\{\frac{28.4}{28.7}\left[1.42^{0.286}-1\right]+1\right\}^{3.5}=20.5 \mathrm{psia}
$$

On comparing Eq. (3), (6) and (11), we see that shaft horsepower is directly proportional to molecular weight, $M W$, or:

$$
\begin{align*}
& S H P=\left(M W / M W_{\tau}\right)\left(S H P_{r}\right)  \tag{13}\\
& S H P=(28.4 / 28.7)(1,315)=1,300 \mathrm{hp}
\end{align*}
$$



How a change in compressibility or specific-heat ratio affects performance Fig. 7

Fig. 6 illustrates the effects of system resistance on this analysis. Here, a reduction in molecular weight lowers the compressor performance curve (dashed line). Thus, the result of a change in the molecular weight is directionally the same as that of a variation in inlet pressure.

## Compressibility and specific heat

For an air compressor, the variations in inlet compressibility and specific-heat ratio are so slight that we can almost always ignore them when analyzing performance. For gases other than air, however, the changes in these parameters may be significant.

Fig. 7 shows the directional effect of a change in compressibility, $Z$, only; and in specific-heat ratio, $k$, only. The dashed curve indicates that a decrease in the inlet compressibility results in raising the compressor performance curve (when inlet volume flow is plotted against discharge pressure). The dotted curve indicates that a decrease in the ratio of specific heats also results in raising the performance curve.

Let us once again realize that our analysis is based on a given rotational speed. Hence, each inlet volume flow is associated with one and only one head value. The head curve does not change in any of the examples previously considered.

## Constant rotational speed

We can now formulate a general equation to determine the discharge pressure that results from a variation in any or all of the inlet conditions at rated inlet volume flow for constant rotational speed.

Referring to Eq. (9), we find that a similar approach would yield a general equation for changes in inlet conditions. If we assume that the ratio of specific heats and inlet volume flow are constant, the discharge-pressure equation becomes:

$$
\begin{align*}
P_{2}= & P_{1}\left\{\left[\frac{\left(T_{1}\right)_{T}}{T_{1}}\right]\left[\frac{\left(Z_{1}\right)_{r}}{Z_{1}}\right]\left[\frac{M W}{M W_{r}}\right] \times\right. \\
& {\left.\left[\left(r_{p}\right)_{r}^{k(k-1) / k}-1\right]+1\right\}^{k /(k-1)} } \tag{14}
\end{align*}
$$



Rotational speed of the compressor affects its performance Fig. 8

If the specific heats vary, the discharge pressure is:

$$
\begin{gather*}
P_{2}=P_{1}\left\{\left[\frac{\left(T_{1}\right)_{r}}{T_{1}}\right]\left[\frac{\left(Z_{1}\right)_{r}}{Z_{1}}\right]\left[\frac{M W}{M W_{r}}\right]\left[\left(\frac{k}{k-1}\right)_{r}\right]\left[\frac{k-1}{k}\right] \times\right. \\
\left.\left[\left(r_{p}\right)_{r}^{(k-1) / k}-1\right]+1\right\}^{k /(k-1)} \tag{15}
\end{gather*}
$$

In either case, the equation for shaft horsepower is:

$$
\begin{equation*}
S H P=\left[\frac{P_{\mathrm{I}}}{\left(P_{1}\right)_{r}}\right]\left[\frac{\left(T_{1}\right)_{r}}{T_{1}}\right]\left[\frac{\left(Z_{1}\right)_{r}}{Z_{1}}\right]\left[\frac{M W}{M W_{r}}\right] S H P_{r} \tag{16}
\end{equation*}
$$

Since $k$ does not vary appreciably for different air/water-vapor mixtures, we can consider it constant, with little or no error introduced. The change in $k$ can be significant for gas compressors. Thus, we will have to consider the effects of changing values of $k$.

Let us consider the single-stage compressor as defined in Fig. 1. Our problem is to determine the discharge pressure and shaft horsepower at the rated inlet volume flow for a change in inlet conditions to:

$$
\begin{aligned}
P_{1} & =14.2 \mathrm{psia} \\
T_{1} & =40^{\circ} \mathrm{F}=500^{\circ} \mathrm{R} \\
M W & =28.4
\end{aligned}
$$

Since the ratio of specific heats is constant, we use Eq. (14) to solve for the discharge pressure, and Eq. (16) for shaft horsepower.

$$
\begin{aligned}
P_{2}^{\prime} & =14.2\left[\left(\frac{550}{500}\right)\left(\frac{1.0}{1.0}\right)\left(\frac{28.4}{28.7}\right)\left(1.42^{0.286}-1\right)+1\right]^{3.5} \\
& =20.8 \mathrm{psia}
\end{aligned}
$$

$S H P=\left(\frac{14.2}{14.5}\right)\left(\frac{550}{500}\right)\left(\frac{1.0}{1.0}\right)\left(\frac{28.4}{28.7}\right)(1,315)=1,400 \mathrm{hp}$
When necessary, we will use the system-resistance curve to make the proper adjustments.

## Constant-weight flow

Sometimes, it is necessary to consider constant-weight flow. We can use the performance curve to predict discharge pressure and shaft horsepower for this case.

Changing the inlet conditions will affect the inlet volume [see Eq. (2)]. Because we are dealing with variable inlet volume flow, the head produced by the impeller will also vary.

We will use the compressor defined by Fig. 1, and predict the discharge pressure for a change in both the inlet temperature and inlet pressure. This procedure will also be applicable to the conditions shown in Fig. 2.

Let us consider a change in the inlet temperature to $100^{\circ} \mathrm{F}$ and inlet pressure to 14.0 psia. Molecular weight, ratio of specific heats and inlet compressibility will remain at rated values, as given in Fig. 1.

Eq. (2) implies that for a constant-weight-flow process, the inlet-volume flow is directly proportional to the inlet temperature, and inversely proportional to the inlet pressure: Hence:

$$
\begin{align*}
& Q=\left[\frac{T_{1}}{\left(T_{1}\right)_{r}}\right]\left[\frac{\left(P_{1}\right)_{r}}{P_{1}}\right] Q_{r}  \tag{17}\\
& Q=\left(\frac{560}{550}\right)\left(\frac{14.5}{14.0}\right)(42,200)=44,500 \mathrm{icfm}
\end{align*}
$$

where icfm $=$ inlet $\mathrm{ft}^{3} / \mathrm{min}$.
From Fig. 1, we find that at the rated inlet volume flow of $42,200 \mathrm{icfm}$, the head produced is $11,000 \mathrm{ft}-$ $\mathrm{lb}_{\mathrm{f}} / \mathrm{lb}_{\mathrm{m}}$. The head produced at the new flow of $44,500 \mathrm{icfm}$ is $10,900 \mathrm{ft}-\mathrm{lb}_{\mathrm{f}} / \mathrm{lb}_{\mathrm{m}}$. From Eq. (4), we establish that the expression containing the rated pressure ratio is directly proportional to the adiabatic head and inversely proportional to the inlet temperature. Hence:

$$
\begin{align*}
P_{2} & =P_{1}\left\{\left[\frac{H_{a d}}{\left(H_{a d}\right)_{r}}\right]\left[\frac{\left(T_{1}\right)_{r}}{T_{1}}\right] \times\right. \\
& {\left.\left[\left(r_{p}\right)_{r}^{(k-1) / k}-1\right]+1\right\}^{k /(k-1)} }  \tag{18}\\
P_{2} & =14.0\left[\left(\frac{10,900}{11,000}\right)\left(\frac{550}{560}\right)\left(1.42^{0.286}-1\right)+1\right]^{3.5} \\
& =19.7 \text { psia }
\end{align*}
$$

Shaft horsepower for rated inlet conditions at a flow of $44,500 \mathrm{icfm}$ can be determined from the performance curve (Fig. 1) as $1,380 \mathrm{hp}$. From Eq. (16), we find:

$$
(\mathrm{SHP})=\left(\frac{14.0}{14.5}\right)\left(\frac{550}{560}\right)(1,380)=1,310 \mathrm{hp}
$$

The procedure for using Fig. 2 follows from the preceding. In this case, the discharge pressure at the new inlet volume flow for rated inlet temperature is obtained directly from the performance curve. For example, the discharge pressure at $44,500 \mathrm{icfm}$ and $90^{\circ} \mathrm{F}$ is 20.5 psia from Fig. 2. We will consider this new discharge pressure "rated" and use the previously estab-
lished relationship [Eq. (9)] to correct for the new temperature.

## Rotational speed

Centrifugal-compressor performance varies with the rotational speed. If the variation in speed is not too large-say $90 \%$ to $105 \%$ of rated speed-we can predict the compressor's performance from the fan-law relationships. These state that the adiabatic head, $H_{a d}$, varies as the square of the speed, $N^{2}$, and the inlet-volume flow, $Q$, varies directly as the speed, $N$. If we use the rated points as reference, we can write:

$$
\begin{align*}
H_{a d} & =\left(H_{a d}\right)_{r}\left(N / N_{r}\right)^{2}  \tag{19}\\
Q & =Q_{r}\left(N / N_{r}\right) \tag{20}
\end{align*}
$$

From Eq. (19) and (20), we can predict the speed required to overcome the effects of changes in inlet conditions. The procedure that will be outlined can be used for any variations in inlet conditions and for any type of performance curve.
Let us consider the earlier example involving a change in the inlet pressure only. The drop in inlet pressure caused the discharge pressure to decrease to 20.2 psia. What speed is required to raise the discharge pressure to the rated value of 20.6 psia at rated inlet volume conditions?
The pressure ratio at rated speed for the inlet conditions under consideration is determined from the procedures previously developed. For this example, it is:

$$
\left(r_{p}\right)_{i . c .}=20.2 / 14.2=1.42
$$

The required pressure ratio is:

$$
\left(r_{p}\right)_{r}=20.6 / 14.2=1.45
$$

For any given set of inlet conditions, we have:

$$
\begin{aligned}
r_{p}^{(k-1) / k}-1 & \sim H_{a d} \\
H_{a d} & \sim N^{2} \\
r_{p}^{(k-1) / k}-1 & \sim N^{2}
\end{aligned}
$$

Hence, we can derive the following rotational-speed equation:

$$
\begin{align*}
& N_{\text {req }}=N_{r}\left[\frac{\left(r_{p}^{(k-1) / k}-1\right)_{\text {req }}}{\left(r_{p}^{(k-1) / k}-1\right)_{i . c .}}\right]^{1 / 2}  \tag{21}\\
& N_{\text {req }}=4,350\left[\frac{(1.45)^{0.286}-1}{(1.42)^{0.286}-1}\right]^{1 / 2}=4,490 \mathrm{rpm}
\end{align*}
$$

At $4,490 \mathrm{rpm}$, the compressor will produce the required head. However, the increase in speed will also affect the volume flow. Here, the new flow will become:

$$
Q=42,200(4,490 / 4,350)=43,560 \mathrm{icfm}
$$

Reduction of the flow to rated conditions at the new speed of $4,490 \mathrm{rpm}$ would result in too much head.

Fig. 8 shows the effect on inlet volume flow and head for an increase in rotational speed. The solid curve represents rated conditions. Point A is the rated operation point. Also plotted in Fig. 8 is the line for the fan law. Following this line to $4,490 \mathrm{rpm}$ shows a shift in performance to Point B, which represents the desired compressor head. Reducing the inlet volume flow to rated conditions results in performance at Point C .

The volume-flow effect can best be handled by determining the percentage change in head that occurs from a change in relative loading, as follows:

$$
\begin{align*}
& \% Q=\frac{Q_{\text {req }} N_{r}}{Q_{r} N_{\text {req }}} \times 100  \tag{22}\\
& \% Q=\frac{(42,200)(4,350)}{(42,200)(4,490)} \times 100=96.9 \%
\end{align*}
$$

By referring to Fig. 1, the percentage change in head as a result of going from $100 \% Q$ to $96.9 \% Q$ may be determined (i.e., flow is 42,200 icfm at $100 \% Q_{r}$, and $40,900 \mathrm{icfm}$ at $96.9 \% Q_{r}$ ).
At $42,900 \mathrm{icfm}, H_{a d}=11,000 \quad \mathrm{ft}-\mathrm{lb}_{\mathrm{f}} / \mathrm{lb}_{\mathrm{m}}$. At $40,900 \mathrm{icfm}, H_{a d}=11,075 \mathrm{ft}-\mathrm{lb}_{\mathrm{f}} / \mathrm{lb}_{\mathrm{m}}$. Then:

$$
\% H_{a d}=\frac{11,075}{11,000} \times 100=100.7 \%
$$

We can now revise Eq. (21) to:

$$
\begin{align*}
N_{r e q} & =N_{r}\left[\frac{\left(r_{p}^{(k-1) / k}-1\right)_{\text {req }}}{\left(r_{p}^{(k-1) / k}-1\right)_{i . c}\left(\% H_{a d} / 100\right)}\right]^{1 / 2}  \tag{23}\\
N_{\text {req }} & =4,350\left[\frac{(1.45)^{0.286}-1}{\left[(1.42)^{0.286}-1\right](1.007)}\right]^{1 / 2} \\
& =4,470 \mathrm{rpm}
\end{align*}
$$

The result is operation at Point D in Fig. 8, which is the desired operating point.
The last step in our analysis of speed variation may seem a little difficult for a performance curve plotted against discharge pressure. In using a curve such as that in Fig. 2, we should remember that for any given set of inlet conditions:

$$
\% H_{a d}=\%\left(r_{p}^{(k-1) / k}-1\right)
$$

## Summary

With these procedures, we should be able to accurately predict the performance for any single-stage centrifugal compressor. Furthermore, we should be able to analyze plant-air centrifugal compressors by applying these techniques to each stage between coolers.
The use of these techniques on centrifugal units having several impellers between cooling points will yield useful qualitative results. However, accuracy of the results will deteriorate in proportion to the number of impellers between coolers and differences in the molecular weights of the gases involved.


## The author

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[^0]:    *This article was written by R. P. Lapina while an employee of Elliott, a United Technologies company, and adapted from a prize-winning article in the CAGI (Compressed Air and Gas Institute) technical article program.

[^1]:    ${ }^{\text {t}}$ This statement is not rigorously true, due to volume ratio effects. Variations in inlet conditions will affect the value of $\mu$. However such deviations are normally small, and we can safely ignore them for our purposes.

