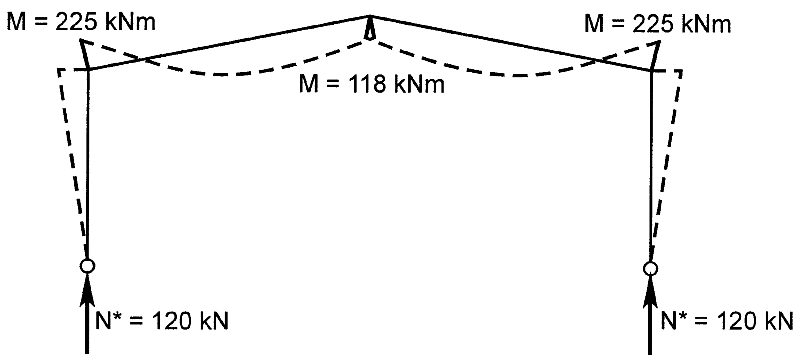
E12. Portal Frame Design Example

Although portal frames are common their structural design is relatively complex. This design example illustrates the application of the General Design Method for a typical portal frame design.

Design Parameters :

Span :	24m
Frame Spacing :	8.5m
Section Size:	410UB54 BHP Grade 300PLUS
Column:	6.0 m (baseplate to underside of rafter), pinned base
Loading:	6.38 kN/m UDL plus vertical load at eaves of 43.3 kN
Base Fixity:	Pinned

Bending moments from a simple (first-order) elastic analysis for ultimate limit state load combination 1.2G & 1.6Q are shown in the following diagram.



Axial compression forces: N^*_{column} = 120 kN; N^*_{rafter} = 35 kN min (apex)
= 55 kN max (knee)

Rafter design shear force: V^* = 63 kN

Purlins, girts spaced at approximately 1600 mm centres along rafter and column. A fly brace is provided to the rafter bottom flange at the knee joint. Others may be provided as the design example is completed.

Determine Second-Order Effects (see section 7.5.2)

Calculate elastic buckling load factor, λ_c

$$\lambda_c = \frac{3EI_{pr}}{s_r\left[N^*_{col}h_e + 0.3N^*_{rafter}s_r\right]} = \frac{3 \times 205\,\text{GPa} \times 188 \times 10^6\,\text{mm}^4}{12.4\left[120\,\text{kN} \times 6.2\,\text{m} + 0.3 \times 55\,\text{kN} \times 12.4\right]} = 9.8$$

Calculate moment amplification factor, δ_s (NZS3404, Clause 4.4.3.3.2(b))

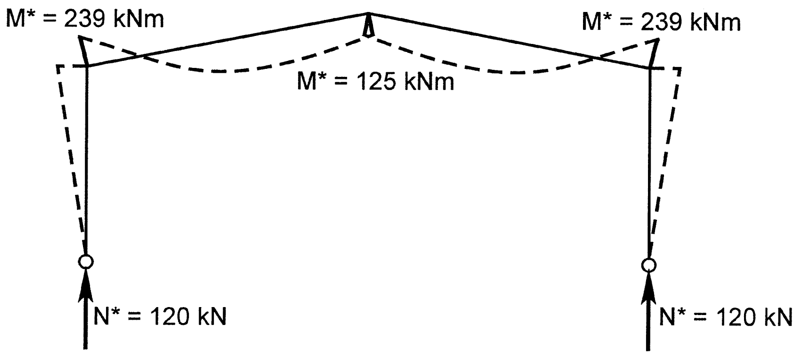
$$\delta_s = \frac{0.95}{1 - \left[\dfrac{1}{\lambda_c}\right]} \geq 1.0 \qquad = \frac{0.95}{1 - \left[\dfrac{1}{9.8}\right]} = 1.06$$

i.e. bending moments obtained from first-order analysis should be magnified by 1.06 (6 % increase) to allow for second-order effects.

Note: There is no amplification of axial forces or shear forces.

If $\lambda_c \geq 10$ then second-order effects may be ignored and no moment amplification is required. $\lambda_c = 9.8$ is just outside the exclusion limit, hence moment amplification is required. (In practice, however, such a small increase in design moments would often be ignored).

Design bending moments incorporating second-order effects are as given below. Axial forces and shear forces are unchanged.



Design of Portal Frame Rafter

The first step is to consider how the rafter should be subdivided into segments for structural design (see section 2.4.4 herein). This depends on the position and type of cross-section restraints for bending (see section 2.4.3), and identifying which flange is the critical flange. This, in turn, depends on the shape of the bending moment diagram.

Start at the end of the rafter adjacent to the knee joint because this is the position of maximum bending moment.

(In the following, all references are to section 2.3.1 in this document unless noted otherwise).

Critical Flange

For this load combination the critical flange is the bottom flange for the portion of the rafter from the eaves out to the point of contraflexure. Beyond the point of contraflexure (nearer the apex) the critical flange becomes the top flange.

Rafter Segmentation

At the first purlin connection past the point of contraflexure the rafter cross-section is provided with full twist restraint (Classification F, see HERA Report R4-92 [21], connection detail 19).

At the knee joint, the welded beam to column connection with a fly brace to the bottom flange provides full twist restraint (F) when the bottom flange is the critical flange (see [21], connection detail 23).

Therefore the rafter can be subdivided into 2 segments:

1. segment between knee joint and the first purlin cleat past the point of contraflexure for this load combination; segment length L = 4.9m (given)
2. segment between this first purlin cleat past the point of contraflexure and the apex; segment length L = 7.5 m (given)

***Note**: Because the rafter is provided with full lateral restraint at each purlin connection where the top flange is the critical flange, the 'second' portion of the rafter could be subdivided into a greater number of short segments rather than designing for the segment length of 7.5 metres. Choosing to design for the segment length of 7.5 m is quicker (to design) but ignores the beneficial effect of full lateral restraint provided by the purlins where the top flange is the critical flange.*

Check rafter section moment and shear capacity

Section moment capacity from Appendix A, Table A1
for 410UB54 Grade 300PLUS (2.3.1: Step (i))

ϕM_{sx} = 304 kNm

Check member moment capacity for segment one

Design bending moment M* = 239 kNm

Segment length L = 4900 mm

Moment modification factor α_m = 1.75 (2.3.1: Step (ii))

(lower bound estimate based on case 9, Table 5.6.1 for moment distribution on this segment).

Effective length calculation:

Cross-section twist restraints at each end of segment are F, F (from above). (2.3.1: Step (iii))

k_t = 1.0 for FF (Appendix D, Table D1)

k_l = 1.0 (See Note to Table D2 in Appendix D)

k_r = 1.0 (Appendix D, Table D3)

$\Rightarrow$ Segment effective length $L_e = k_t\, k_l\, k_r\, L$ = 4900 mm

Slenderness reduction factor (Appendix A, Table A1, 410UB54) (2.3.1: Step (iv))

L_e = 4.9 m $\Rightarrow \alpha_s$ = 0.457

Member moment capacity $\phi M_{bx} = \alpha_m\, \alpha_s\, \phi M_{sx}$ (2.3.1: Step (v))

= 1.75 x 0.457 x 304 kNm (but $\leq \phi M_{sx}$ = 304 kNm)

= 243 kNm

Capacity check $M^*_x \leq \phi M_{bx}$ (2.3.1: Step (vi))

$239 \leq 243 \Rightarrow$ OK

Rafter segment one, Flexure OK

Check shear capacity for segment one

Design shear force V* = 63 kN

ϕV_v = 529 kN (410 UB 54, Appendix A, Table A1) (3.2.1: Step (i))

Check $V^* \leq \phi V_v$ (3.2.1: Step (ii))

$63 \leq 529 \Rightarrow$ OK

(3.2.1: Step (iii))

By inspection, the design shear force is considerably less than 0.60 ϕV_v, $\Rightarrow$ no need to check moment-shear interaction.

Rafter segment one, Shear OK

Check member moment capacity for segment two

Design bending moment M* = 125 kNm

Segment length L = 7500 mm

Cross-section twist restraints at each end of segment (purlin as above, and apex) are FF
so $k_t = k_l = k_r = 1.0$ as for segment one

Segment effective length $L_e = k_t\, k_l\, k_r\, L_e = 7500$ mm

Moment modification factor $\alpha_m = 1.10$ (conservative estimate based on Table C1, Appendix C)

From α_s tables in Appendix A, Table A1

for $L_e = 7500$ mm $\Rightarrow \alpha_s = 0.276$

Member moment capacity $\phi M_{bx} = \alpha_m\, \alpha_s\, \phi M_{sx}$

$= 1.10 \times 0.276 \times 304$ kNm (but $\leq \phi M_{sx} = 304$ kNm)

$= 92$ kNm $<$ M* $= 125$ kNm $\Rightarrow$ no good

Re-consider the segmentation of this portion of rafter:

Because the rafter is provided with full lateral restraint at each purlin connection where the top flange is the critical flange, the rafter may be subdivided into segments of length equal to the purlin spacing. Consider that segment in this second portion of rafter which is subjected to maximum moment (M* = 125 kNm).

Segment length = 1600mm (maximum purlin spacing)

Cross-section twist restraints provided by the purlin connection at each end of segment are FF
so $k_t = k_l = k_r = 1.0$ as for segment one

segment effective length $L_e = k_t\, k_l\, k_r\, L_e = 1600$ mm

moment modification factor $\alpha_m = 1.0$ (conservative assumption)

From α_s tables in Appendix A1 for $L_e = 1600$mm $\Rightarrow \alpha_s = 0.922$

Member moment capacity $\phi M_{bx} = \alpha_m\, \alpha_s\, \phi M_{sx}$

$= 1.0 \times 0.922 \times 304$ kNm (but $\leq \phi M_{sx} = 304$ kNm)

$= 280$ kNm $>$ M* $= 125$ kNm $\Rightarrow$ OK

Rafter segment two, Flexure OK

Check combined actions for both segments

Start with segment one (highest axial force and bending moment)

Confirm whether or not axial force is 'significant' (see section 6.2 herein). Segment one was designed for bending moment on the basis of not having full lateral restraint (i.e. the product $\alpha_m\, \alpha_s < 1.0$) so need to determine ϕN_{cy}.

Assume a fly brace is provided to the rafter at the purlin cleat positioned 3.3 m from knee. Assume adequate lateral restraint against y-axis buckling in compression is provided at this fly brace (this assumption needs to be verified).

Effective length in compression $L_{ey} = 3300$ mm ($k_e = 1.0$ here for y-axis buckling).

Slenderness reduction factor $\alpha_{cy} = 0.593$ for $L_{ey} = 3300$ mm from table in Appendix B for 410UB54

ϕN_{cy} = 1075 kN and 0.05 ϕN_{cy} = 54 kN $\approx$ N* = 55 kN, so combined actions need not be checked.

***Note**: if full lateral restraint is provided to a segment and the check for combined actions involves determining ϕN_{cx}, then the segment length for compression buckling about the x-axis is that between the knee and the rafter apex, regardless of the restraints against y-axis buckling (e.g. as may be provided by fly braces).*

Check segment two

For segment length = 7500 mm, α_{cy} = 0.163, ϕN_{cy} = 295 kN

0.05 ϕN_{cy} = 15 kN < N* $\approx$ 45 kN (average), so combined actions must be checked.

Using the simple combined actions check (see section 6.3 herein)

Let ϕM_{bx} = 282 kNm (for the segment with greatest flexural effective length L_e = 1600 mm)

$$\phi M_{ox} = \phi M_{bx}\left(1-\frac{N^*}{\phi N_{cy}}\right)$$

$$= 280\ \text{kNm}\left(1-\frac{45\ \text{kN}}{295\ \text{kN}}\right)$$

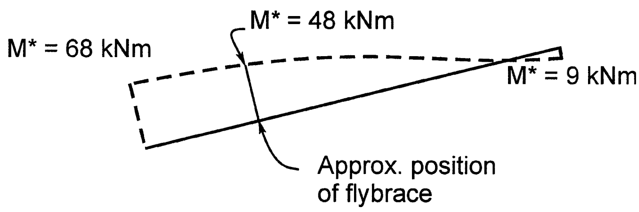
$$= 237\ \text{kNm}$$

ϕM_{ox} = 237 kNm > M* = 125 kNm $\Rightarrow$ combined actions OK

Rafter, 1.2G & 1.6Q load case OK

Design of portal frame rafter for wind uplift

To demonstrate the design of the rafter for the case where the shape of the bending moment is reversed, the load combination incorporating wind uplift is considered (0.9G & W_u):



Critical Flange

For this load combination and this section of rafter the critical flange is the bottom flange between the knee and the point of contraflexure near the apex.

Member segmentation

Some restraint is necessary to the critical flange to achieve a full or partial restraint of the cross-section. Normally this would be provided with a fly brace. The fly brace at the knee joint provides restraint to the cross-section at the end of the rafter. However, assume a fly brace is also provided to the rafter at the purlin cleat positioned 3.3 m from knee. This provides 'partial' twist restraint (P) to the rafter cross-section if the purlin is classified as 'flexible' (see HERA Report R4–92, connection detail 21 Case 2).

Full twist restraint (F) is provided at the knee. Therefore rafter can be subdivided into 2 segments:

1. Segment between knee and fly brace; segment length L $\approx$ 3.3 m
2. Segment between fly brace and the apex; segment length L $\approx$ 9.1 m

Check member moment capacity for segment one

Design bending moment M* = 68 kNm

Segment length L = 3300 mm

Moment modification factor $\alpha_m \approx 1.0$ (case 8, Table C1. for moment distribution on this segment)

Effective length calculation:

Cross-section twist restraints at each end of segment are F,P

$$k_t = 1 + \left[\frac{403}{3300} \times \left(\frac{10.9}{2 \times 7.6} \right)^3 \right] = 1.04 \quad \text{for FP} \quad (\text{Appendix D, Table D1})$$

k_l = 1.0 (load height 'below' shear centre)

k_r = 1.0 (conservative – see Appendix D, Table D3)

Segment effective length L_e = $k_t\, k_l\, k_r\, L$ = 3450 mm

Slenderness reduction factor from Appendix A, Table A1. for L_e = 3450 $\Rightarrow \alpha_s$ = 0.641

Member moment capacity $\phi\, M_{bx}$ = $\alpha_m\, \alpha_s\, \phi M_{sx}$

= 1.0 x 0.641 x 304 kNm (but $\leq \phi\, M_{sx}$ = 304 kNm)

= 195 kNm > M* = 68 kNm $\Rightarrow$ OK

Check member moment capacity for segment two

Design bending moment M* = 48 kNm at position of fly-brace (end of segment)

Segment length L = 9100 mm

Moment modification factor $\alpha_m \approx$ 1.75 (case 1, Table C1, for moment distribution on this segment)

Effective length calculation:

Cross-section twist restraints are P,F

k_t = 1.02 (Appendix D, Table D1)

k_l = k_r = 1.0 as above

Segment effective length L_e = $k_t\, k_l\, k_r\, L$ = 9250mm

Slenderness reduction factor from Appendix A, Table A1. for L_e = 9250 $\Rightarrow \alpha_s$ = 0.212

Member moment capacity $\phi\, M_{bx}$ = $\alpha_m\, \alpha_s\, \phi M_{sx}$

= 1.75 x 0.212 x 304 kNm (but $\leq \phi\, M_{sx}$ = 304 kNm)

= 113 kNm > M* = 48 kNm $\Rightarrow$ OK

Rafter check for wind uplift, OK